# Introduction to Parton Showers

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Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

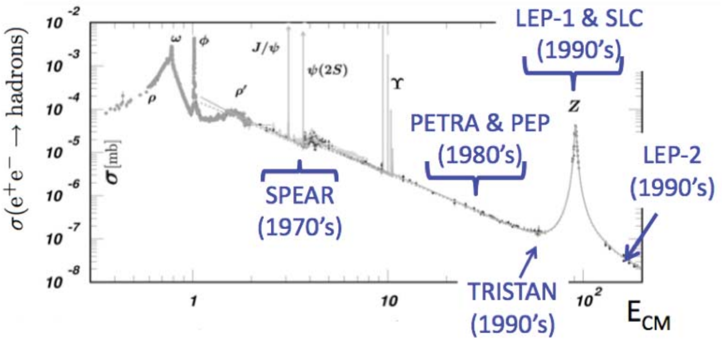
# Suggested reading

- R. K. Ellis, W. J. Stirling, B. R. Webber
  **QCD and Collider Physics**
  Cambridge University Press, 2003
- R. D. Field
  **Applications of Perturbative QCD**
  Addison-Wesley, 1995
- T. Sjöstrand, S. Mrenna, P. Z. Skands
  **PYTHIA 6.4 Physics and Manual**
  JHEP 05 (2006) 026
- L. Dixon, F. Petriello (Editors)
  **Journeys Through the Precision Frontier**
  Proceedings of TASI 2014, World Scientific, 2015

# Outline of lectures

- Introduction
  - Historical context
  - Collider observables
  - Event generators
- Parton showers
  - Leading-order formalism
  - Assessment of formal precision
  - Going beyond the leading order
- Combination with fixed-order calculations
  - Matching to NLO calculations
  - LO-Merging of multiplicities
  - Combination of matched results

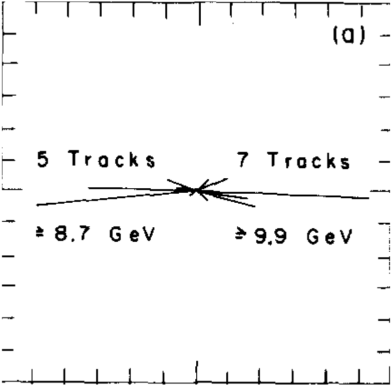
# QCD in $e^+e^-$ annihilation

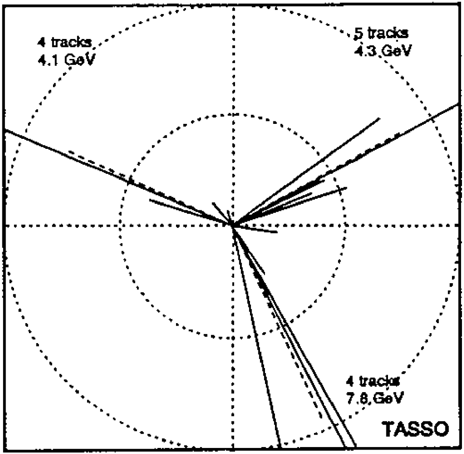



- SPEAR (SLAC): Discovery of quark jets
- PETRA (DESY) & PEP (SLAC): First high energy (>10 GeV) jets
  Discovery of gluon jets (PETRA) & pioneering QCD studies
- LEP (CERN) & SLC (SLAC): Large energies → more reliable QCD calculations, smaller hadronization uncertainties
  Large data samples → precision tests of QCD

# Discovery of the gluon

[TASSO] PLB86(1979)243 & Proc. Neutrino '79, Vol.1, p.113

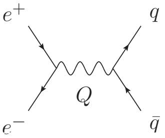


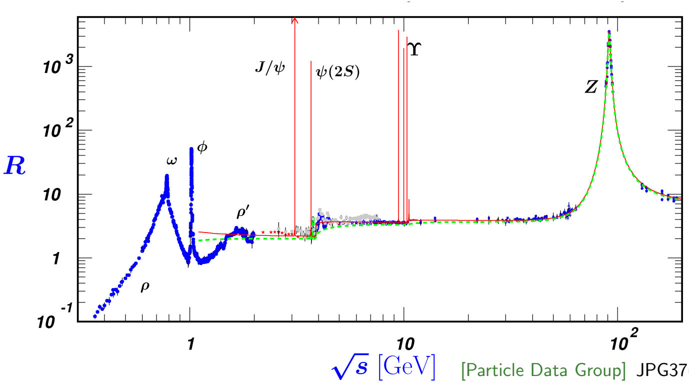




- Gluon discovery at the PETRA collider at DESY
- Typical three-jet event (right) vs. two-jet event (left)

# Basic process for $e^+e^- \to$hadrons

- Prediction for $e^+e^- \to q\bar{q}$ at leading perturbative order differs from $e^+e^- \to \mu^+\mu^-$ only by quark charges
- Define ratio $R = \dfrac{\sigma_{e^+e^-\to\text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} \xrightarrow{\text{LO}} 3\sum_i e_{q,i}^2$



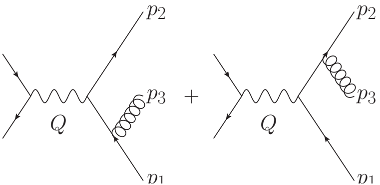




[Particle Data Group] JPG37(2010)075021

# Three-jet cross section & corrections to $e^+e^- \to$ hadrons

- Kinematic variables $x_i = \frac{2p_i \cdot Q}{Q^2}$

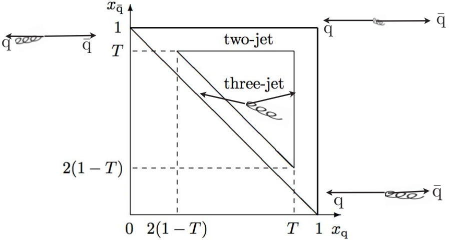
  $\to x_i < 1, \quad x_1 + x_2 + x_3 = 2$




- Differential cross section

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_0 \frac{\alpha_s}{\pi} C_F \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

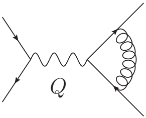
- Divergent as
  - $x_1 \to 1 \ (p_3 \parallel p_1)$
  - $x_2 \to 1 \ (p_3 \parallel p_2)$
  - $(x_1, x_2) \to (1, 1) \ (x_3 \to 0)$



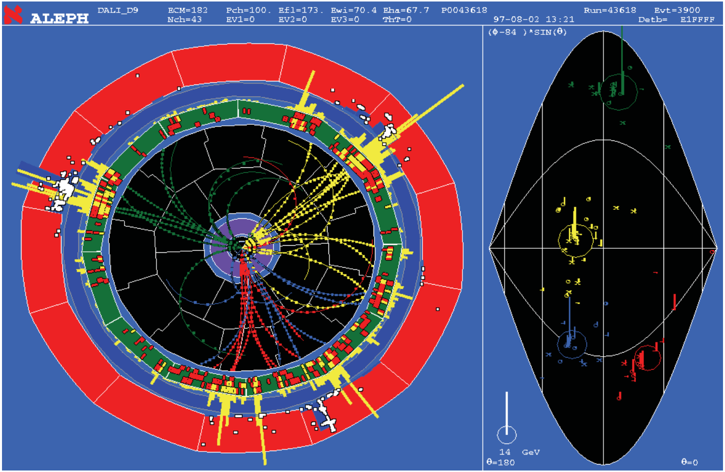

- Divergences canceled by virtual correction

  Total correction to $e^+e^- \to$ hadrons:

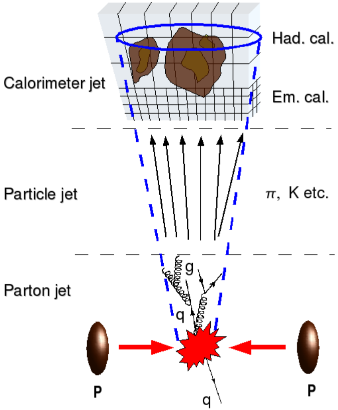
$$\sigma^{\mathrm{NLO}} = \sigma_0 \left(1 + \frac{3}{4} C_F \frac{\alpha_s}{\pi}\right)$$



[ALEPH]


ALEPH
DALI_D9
ECM=182
Nch=43
Pch=100.
EV1=0
Efl=173.
EV2=0
Ewi=70.4
EV3=0
Eha=67.7
ThT=0
P0043618
97-08-02 13:21
Run=43618
Evt=3900
Detb= E1FFFF
(ϕ-84 )*SIN(θ)
14 GeV
θ=180
θ=0

# Jet algorithms



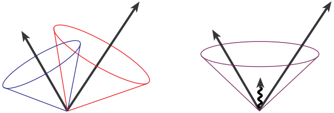

- Identify hadronic activity in experiment with partonic activity in pQCD theory

$\Rightarrow$ Requirements

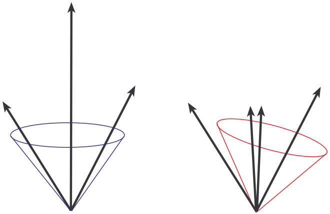
- Applicable both to data and theory
  - partons
  - stable particles
  - measured objects (calorimeter objects, tracks, etc.)
- Gives close relationship between jets constructed from any of the above
- Independent of the details of the detector, e.g. calorimeter granularity

Further requirements from QCD

- Infrared safety → no change when adding a soft particle

  Counterexample: 

- Collinear safety → no change when substituting particle with two collinear particles

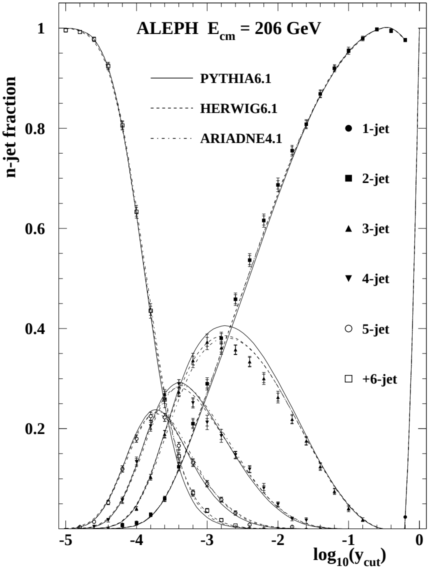
  Counterexample: 

# Jet algorithms

- Most widely used jet algorithms today of sequential recombination type
- Example: Durham algorithm
  1. Start with a list of preclusters
  2. For each pair of preclusters calculate

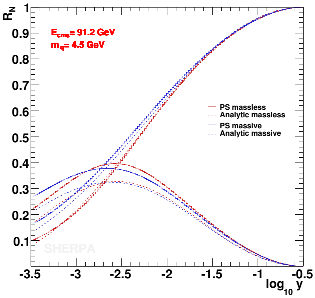
$$y_{ij} = \frac{2}{E_{cm}^2} \min\left\{E_i^2, E_j^2\right\} (1 - \cos\theta_{ij}) \approx \frac{k_T^2}{E_{cm}^2}$$

  3. Identify $y_{kl} = \min\{ y_{ij} \}$
  4. If $y_{kl} < y_{\rm cut}$, define all preclusters as jets and stop
     else merge preclusters $k$ and $l$ and continue at step 1
- Ambiguities:
  - Distance measure $y_{ij}$ (e.g. Jade algorithm $y_{ij} \to 2p_i p_j / E_{cm}^2$)
  - Recombination scheme (e.g. four-momentum addition $p_{kl} = p_k + p_l$)
  - Resolution criterion $y_{\rm cut}$
- For hadron collider algorithms, see [Salam] arXiv:0906.1833




[ALEPH] CERN-EP-2003-084

- Can compute $n$-jet rate in coherent branching formalism
  [Catani, Olsson, Turnock, Webber] PLB269(1991)432
- Alternatively simulate with MC event generators



- Shape variables characterize event as a whole
- Thrust (introduced 1978 at PETRA)

  $$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

  - $T \to 1$ – back-to-back event
  - $T \to 1/2$ – spherically symmetric event

  Vector for which maximum is obtained $\to$ thrust axis $\vec{n}_T$
- Jet broadening

  $$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}$$

  Computed for two hemispheres w.r.t. $\vec{n}_T$, then
  - $B_W = \max(B_1, B_2)$ – Wide jet broadening
  - $B_N = \min(B_1, B_2)$ – Narrow jet broadening
- C-Parameter

  Linearized momentum tensor

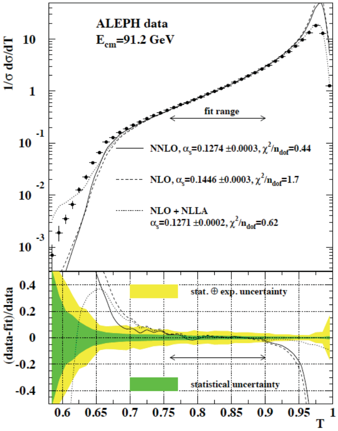
  $$\Theta^{\alpha\beta} = \frac{1}{\sum_j |\vec{p}_j|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|} ,$$

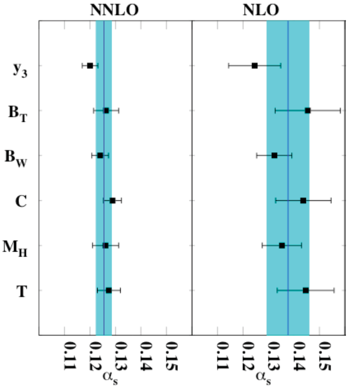
  Eigenvalues $\lambda_i$ define $C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$

# Application of event shape variables

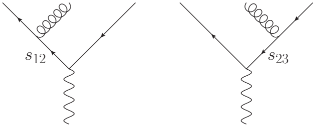
- Discovery of quark and gluon jets – Sphericity & Oblateness
- Measurement of strong coupling constant – $T$, $C$, $B$, $M_H$, jet rates

[Dissertori et al.] arXiv:0906.3436



# Parton evolution




- Consider $e^+e^- \to 3$ partons

$$\frac{1}{\sigma_{2\to 2}} \frac{\mathrm{d}\sigma_{2\to 3}}{\mathrm{d}\cos\theta \mathrm{d}z} \sim C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta} \frac{1+(1-z)^2}{z}$$

  $\theta$ - angle of gluon emission
  $z$ - fractional energy of gluon

- Divergent in
  - Collinear limit: $\theta \to 0, \pi$
  - Soft limit: $z \to 0$
- Separate into two independent jets

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

- Independent evolution with $\theta$

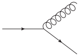
$$\mathrm{d}\sigma_3 \sim \sigma_2 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} \mathrm{d}z \, \frac{1+(1-z)^2}{z}$$

# Parton evolution

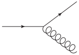
- Same equation for any variable with same limiting behavior
  - Transverse momentum $k_T^2 = z^2(1-z)^2\theta^2 E^2$
  - Virtuality $t = z(1-z)\theta^2 E^2$
- Call this the "evolution variable"

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}k_T^2}{k_T^2} = \frac{\mathrm{d}t}{t} \qquad \leftrightarrow \qquad \text{collinear divergence}$$

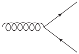
- Absorb $z$-dependence into flavor-dependent splitting kernel $P_{ab}(z)$



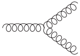
$$= C_F \frac{1+z^2}{1-z}$$



$$= C_F \frac{1+(1-z)^2}{z}$$



$$= T_R \left[ z^2 + (1-z)^2 \right]$$



$$= C_A \left[ \frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right]$$

- Branching equation emerges, but so far only pQCD, no hadrons

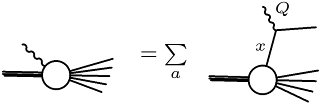
$$\mathrm{d}\sigma_{n+1} \sim \sigma_n \sum_{\text{jets}} \frac{\mathrm{d}t}{t} \mathrm{d}z \, \frac{\alpha_s}{2\pi} \, \frac{1}{2} P_{ab}(z)$$

# The DGLAP equation

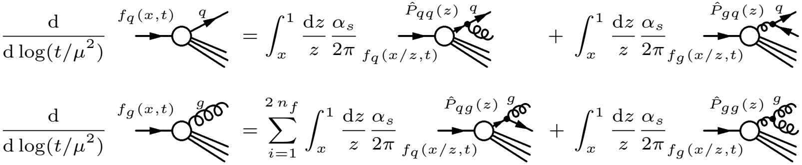
[Altarelli,Parisi] NPB126(1977)298

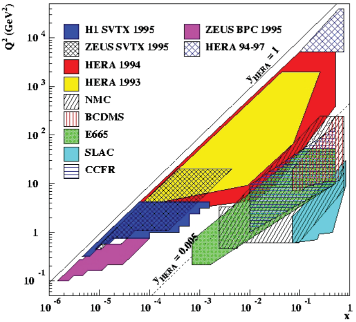
- Hadronic cross section factorizes into perturbative & non-perturbative piece

$$\sigma = \sum_{a=q,g} \int \mathrm{d}x\, f_a(x,\mu_F^2)\hat{\sigma}_a(\mu_F^2)$$

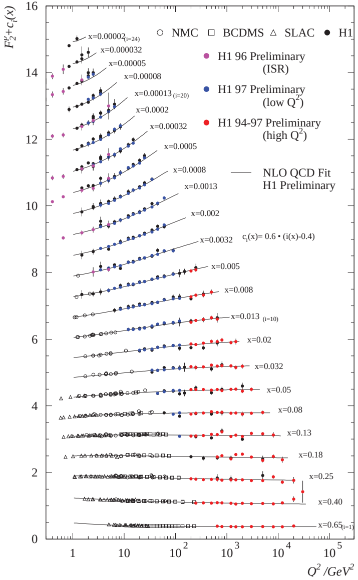



- Evolution from previous slide turns into evolution equation for $f_a(x,\mu_F^2)$
- $f_a(x,\mu_F^2)$ cannot be predicted as a function of $x$, but dependence on $\mu_F^2$ can be computed order by order in pQCD due to invariance of $\sigma$ under change of $\mu_F$
- DGLAP equation $\leftrightarrow$ renormalization group equation



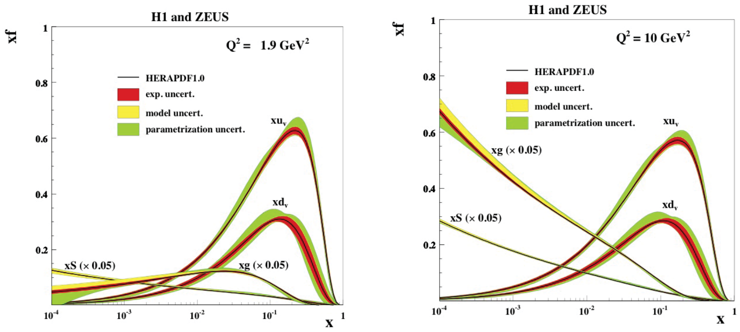


Q² (GeV²)
H1 SVTX 1995
ZEUS SVTX 1995
HERA 1994
HERA 1993
NMC
BCDMS
E665
SLAC
CCFR
ZEUS BPC 1995
HERA 94-97
y HERA = 1
y HERA = 0.005
x

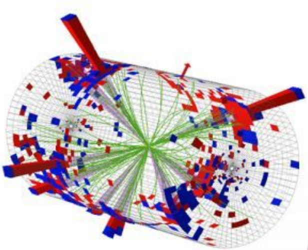



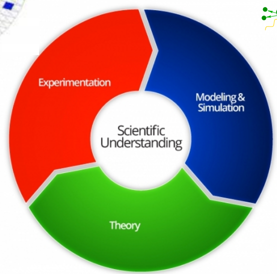
NMC
BCDMS
SLAC
H1
H1 96 Preliminary (ISR)
H1 97 Preliminary (low Q²)
H1 94-97 Preliminary (high Q²)
NLO QCD Fit H1 Preliminary
c_i(x)= 0.6 • (i(x)-0.4)
Q² /GeV²

# PDF measurements

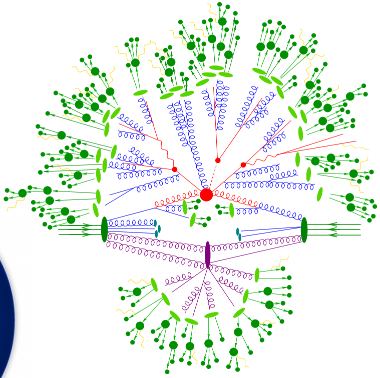


# How event generators fit in



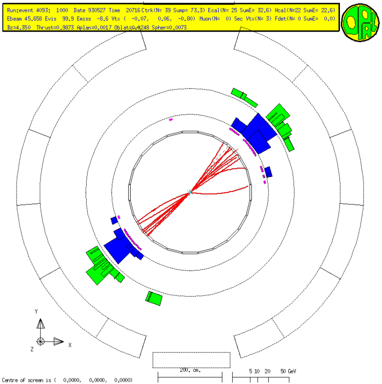


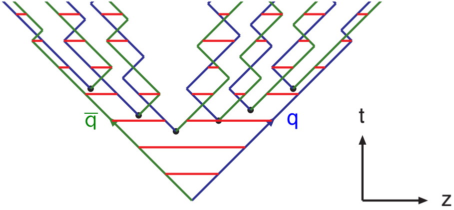

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + h.c.$$



# Event generators in 1978

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31




- Lund string model: ~ like rubber band that is pulled apart and breaks into pieces, or like a magnet broken into smaller pieces.
- Complete description of 2-jet events in $e^+e^-$ →hadrons

# Event generators in 1978

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

```
      SUBROUTINE LIST(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      WRITE(6,110)
      DO 100 I=1,N
      IF(K(I,1).GT.0) C1=CHA1(K(I,1))
      IF(K(I,1).LE.0) IC1=-K(I,1)
      C2=CHA2(K(I,2))
      C3=CHA3((47-K(I,2))/20)
      IF(K(I,1).GT.0) WRITE(6,120) I, C1, C2, C3, (P(I,J), J=1,5)
  100 IF(K(I,1).LE.0) WRITE(6,130) I, IC1, C2, C3, (P(I,J), J=1,5)
      RETURN
  110 FORMAT(////T11,'I',T17,'ORI',T24,'PART',T32,'STAB',
     &T44,'PX',T56,'PY',T68,'PZ',T80,'E',T92,'M'/)
  120 FORMAT(10X,I2,4X,A2,1X,2(4X,A4),5(4X,F8.1))
  130 FORMAT(10X,I2,4X,1X,I2,2(4X,A4),5(4X,F8.1))
      END
```

```
      SUBROUTINE DECAY(IPD,I)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      DIMENSION U(3), BE(3)
C 1 DECAY CHANNEL CHOICE, GIVES DECAY PRODUCTS
      TBR=RANF(0)
      IDC=IDCO(K(IPD,2)-7)
  100 IDC=IDC+1
      IF(TBR.GT.CBR(IDC)) GOTO 100
      ND=(59+KDP(IDC,3))/20
      DO 110 I1=I+1,I+ND
      K(I1,1)=-IPD
      K(I1,2)=KDP(IDC,I1-I)
  110 P(I1,5)=PMAS(K(I1,2))
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
      IF(ND.EQ.2) GOTO 130
      SA=(P(IPD,5)+P(I+1,5))**2
      SB=(P(IPD,5)-P(I+1,5))**2
      SC=(P(I+2,5)+P(I+3,5))**2
      SD=(P(I+2,5)-P(I+3,5))**2
      TDU=(SA-SD)*(SB-SC)/(4.*SQRT(SB*SC))
      IF(K(IPD,2).GE.11) TDU=SQRT(SB*SC)*TDU**3
  120 SX=SC+(SB-SC)*RANF(0)
      TDF=SQRT((SX-SA)*(SX-SB)*(SX-SC)*(SX-SD))/SX
      IF(K(IPD,2).GE.11) TDF=SX*TDF**3
      IF(RANF(0)*TDU.GT.TDF) GOTO 120
      P(100,5)=SQRT(SX)
C 3 TWO-PARTICLE DECAY IN CM, TWICE TO SIMULATE THREE-PARTICLE DECAY
  130 DO 160 IL=1,ND-1
      IO=(IL-1)*100-(IL-2)*IPD
      I1=I+IL
      I2=(ND-IL-1)*100-(ND-IL-2)*(I+IL+1)
      PA=SQRT((P(IO,5)**2-(P(I1,5)+P(I2,5))**2)*
     &(P(IO,5)**2-(P(I1,5)-P(I2,5))**2))/(2.*P(IO,5))
  140 U(3)=2.*RANF(0)-1.
      PHI=6.2832*RANF(0)
      U(1)=SQRT(1.-U(3)**2)*COS(PHI)
      U(2)=SQRT(1.-U(3)**2)*SIN(PHI)
      TDA=1.-(U(1)*P(IO,1)+U(2)*P(IO,2)+U(3)*P(IO,3))**2/
     &(P(IO,1)**2+P(IO,2)**2+P(IO,3)**2)
      IF(K(IPD,2).GE.11.AND.IL.EQ.2.AND.RANF(0).GT.TDA) GOTO 140
      DO 150 J=1,3
      P(I1,J)=PA*U(J)
  150 P(I2,J)=-PA*U(J)
      P(I1,4)=SQRT(PA**2+P(I1,5)**2)
  160 P(I2,4)=SQRT(PA**2+P(I2,5)**2)
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
      DO 190 IL=ND-1,1,-1
      IO=(IL-1)*100-(IL-2)*IPD
      DO 170 J=1,3
  170 BE(J)=P(IO,J)/P(IO,4)
      GA=P(IO,4)/P(IO,5)
      DO 190 I1=I+IL,I+ND
      BEP=BE(1)*P(I1,1)+BE(2)*P(I1,2)+BE(3)*P(I1,3)
      DO 180 J=1,3
  180 P(I1,J)=P(I1,J)+GA*(GA/(1.+GA)*BEP+P(I1,4))*BE(J)
  190 P(I1,4)=GA*(P(I1,4)+BEP)
      I=I+ND
      RETURN
      END
```

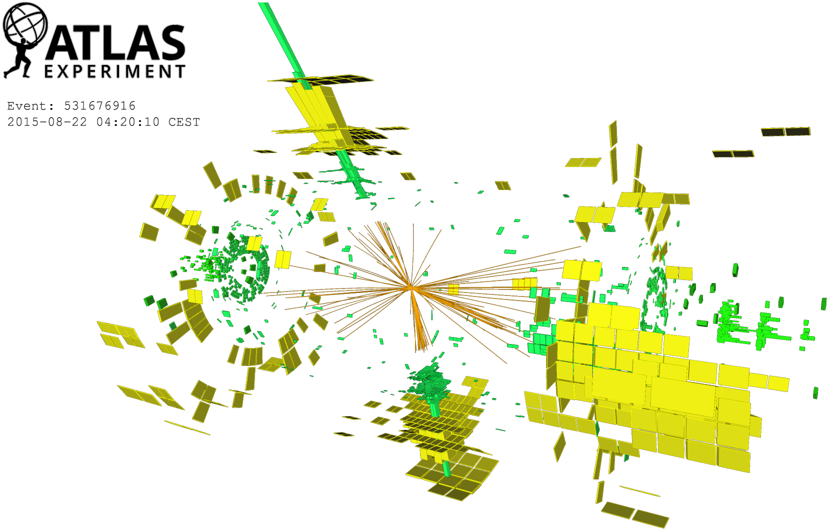
≈ 200 punched cards

Fortran code

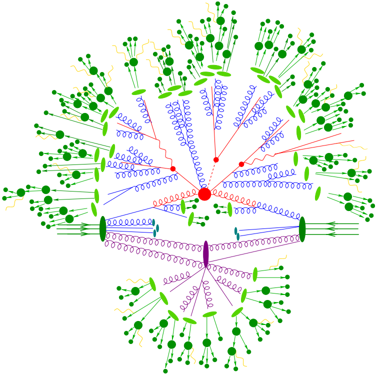
```
      SUBROUTINE EDIT(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      REAL ROT(3,3), PR(3)
C 1 THROW AWAY NEUTRALS OR UNSTABLE OR WITH TOO LOW PZ OR P
      I1=0
      DO 110 I=1,N
      IF(ITHROW.GE.1.AND.K(I,2).GE.8) GOTO 110
      IF(ITHROW.GE.2.AND.K(I,2).GE.6) GOTO 110
      IF(ITHROW.GE.3.AND.K(I,2).EQ.1) GOTO 110
      IF(P(I,3).LT.PZMIN.OR.P(I,4)**2-P(I,5)**2.LT.PMIN**2) GOTO 110
      I1=I1+1
      K(I1,1)=IDIM(K(I,1),0)
      K(I1,2)=K(I,2)
      DO 100 J=1,5
  100 P(I1,J)=P(I,J)
  110 CONTINUE
      N=I1
C 2 ROTATE TO GIVE JET PRODUCED IN DIRECTION THETA, PHI
      IF(THETA.LT.1E-4) GOTO 140
      ROT(1,1)=COS(THETA)*COS(PHI)
      ROT(1,2)=-SIN(PHI)
      ROT(1,3)=SIN(THETA)*COS(PHI)
      ROT(2,1)=COS(THETA)*SIN(PHI)
      ROT(2,2)=COS(PHI)
      ROT(2,3)=SIN(THETA)*SIN(PHI)
      ROT(3,1)=-SIN(THETA)
      ROT(3,2)=0.
      ROT(3,3)=COS(THETA)
      DO 130 I=1,N
      DO 120 J=1,3
  120 PR(J)=P(I,J)
      DO 130 J=1,3
  130 P(I,J)=ROT(J,1)*PR(1)+ROT(J,2)*PR(2)+ROT(J,3)*PR(3)
C 3 OVERALL LORENTZ BOOST GIVEN BY BETA VECTOR
  140 IF(BETA(1)**2+BETA(2)**2+BETA(3)**2.LT.1E-8) RETURN
      GA=1./SQRT(1.-BETA(1)**2-BETA(2)**2-BETA(3)**2)
      DO 160 I=1,N
      BEP=BETA(1)*P(I,1)+BETA(2)*P(I,2)+BETA(3)*P(I,3)
      DO 150 J=1,3
  150 P(I,J)=P(I,J)+GA*(GA/(1.+GA)*BEP+P(I,4))*BETA(J)
  160 P(I,4)=GA*(P(I,4)+BEP)
      RETURN
      END
```

```
      BLOCK DATA
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      DATA PUD/0.4/, PS1/0.5/, SIGMA/350./, CX2/0.77/,
     &EBEG/10000./, WFIN/100./, IFLBEG/1/
      DATA ITHROW/1/, PZMIN/0./, PMIN/0./, THETA,PHI,BETA/5*0./
      DATA MESO/7,1,3,2,8,5,4,6,9,7,2,4,1,8,6,3,5,9/
      DATA CMIX/2*0.5,1.,2*0.5,1.,2*0.25,0.5,2*0.,1./
      DATA PMAS/0.,2*139.6,2*493.7,2*497.7,135.,548.8,957.6,
     &2*765.9,2*892.2,2*896.3,770.2,782.6,1019.6/
      DATA IDCO/0,1,6,11,12,13,15,17,19,21,22,25/
      DATA CBR/1.,0.381,0.681,0.918,0.969,1.,0.426,0.662,0.959,
     &0.980,1.,1.,1.,0.667,1.,0.667,1.,0.667,1.,0.667,1.,1.,
     &0.899,0.987,1.,0.486,0.837,0.984,1./
      DATA KDP/1,1,8,2,1,1,2,8,1,1,1,2,3,6,4,7,5,4,6,5,7,2,2,
     &1,2,4,6,2,1,1,1,8,3,2,1,3,8,17,18,1,8,8,2,8,3,8,3,8,2,8,
     &3,3,8,3,5,7,3,9,0,0,8,8,3,8,9,9,14*0,8,4*0,8,0/
      DATA CHA1/'UD','DU','US','SU','DS','SD','UU','DD','SS'/
      DATA CHA2/'GAMM','PI+','PI-','K+','K-','K0','KB0','PI0','ETA',
     &'ETAP','RHO+','RHO-','K*+','K*-','K*0','KB*0','RHO0','OMEG','PHI'/
      DATA CHA3/'    ','STAB'/
      END
```

# Experimental situation in 2016



# Event generators in 2016



**Need to cover large dynamic range**

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

**Divide and Conquer**

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

# General-purpose event generators for LHC physics

SLAC

[Buckley et al.] arXiv:1101.2599

**Herwig**

- Originated in coherent shower studies → angular ordered PS
- Front-runner in development of MC@NLO and POWHEG
- Simple in-house ME generator & spin-correlated decay chains
- Original framework for cluster fragmentation

**Pythia**

- Originated in hadronization studies → Lund string
- Leading in development of multiple interaction models
- Pragmatic attitude to ME generation → external tools
- Extensive PS development and earliest ME⊕PS matching

**Sherpa**

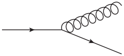
- Started with PS generator APACIC++ & ME generator AMEGIC++
- Current MPI model and hadronization pragmatic add-ons
- Leading in development of automated ME⊕PS merging
- Automated framework for NLO calculations and MC@NLO

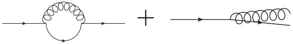
# Leading-order parton showers

# Radiative corrections as a branching process

[Marchesini,Webber] NPB238(1984)1, [Sjöstrand] PLB157(1985)321

- Make two well motivated assumptions
  - Parton branching can occur in two ways

 - observed

 - unobserved

  - Evolution conserves probability
- The consequence is Poisson statistics
  - Let the decay probability be $\lambda$
  - Assume indistinguishable particles $\rightarrow$ naive probability for $n$ emissions

$$P_{\rm naive}(n,\lambda) = \frac{\lambda^n}{n!}$$

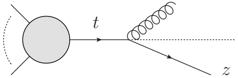
  - Probability conservation (i.e. unitarity) implies a no-emission probability

$$P(n,\lambda) = \frac{\lambda^n}{n!}\exp\{-\lambda\} \qquad \longrightarrow \qquad \sum_{n=0}^{\infty} P(n,\lambda) = 1$$

- In the context of parton showers $\Delta = \exp\{-\lambda\}$ is called Sudakov factor

- Decay probability for parton state in collinear limit

$$\lambda \to \frac{1}{\sigma_n}\int_t^{Q^2} \mathrm{d}\bar{t}\,\frac{\mathrm{d}\sigma_{n+1}}{\mathrm{d}\bar{t}} \approx \sum_{\text{jets}} \int_t^{Q^2} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int \mathrm{d}z\, \frac{\alpha_s}{2\pi} P(z)$$




Parameter $t$ identified with evolution "time"

- Splitting function $P(z)$ spin & color dependent

$$P_{qq}(z) = C_F \left[ \frac{2}{1-z} - (1+z) \right] \qquad P_{gq}(z) = T_R \left[ z^2 + (1-z)^2 \right]$$

$$P_{gg}(z) = C_A \left[ \frac{2}{1-z} - 2 + z(1-z) \right] + (z \leftrightarrow 1-z)$$

- Matching to soft limit will requires some care, because full soft emission probability present in all collinear sectors

$$\frac{1}{t}\frac{2}{1-z} \xrightarrow{z\to 1} \frac{p_i p_k}{(p_i q)(q p_k)}$$

Soft double counting problem [Marchesini,Webber] NPB310(1988)461

- Let us first see how to compute the Poissonian in practice

- Pseudo-random number generators produce uniform numbers
- The probability to draw a point in $[x, x+\Delta x]$ is $\Delta x$

  hence we can compute integrals as expectation values:
- Let the integrand be $f(x)$. Then

$$I = \int_a^b \mathrm{d}x\, f(x) = \frac{b-a}{N}\sum_{i=1}^{N} f(x_i) = [\,b-a\,]\langle f\rangle$$

The statistical uncertainty on this number is

$$\sigma_I = [\,b-a\,]\sqrt{\frac{\langle f^2\rangle - \langle f\rangle^2}{N-1}}\;, \qquad \text{where} \qquad \langle f^2\rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i)^2$$

We call this the Monte-Carlo error of the integral

# Monte-Carlo methods: Importance sampling

- So far we used uniformly distributed random numbers
- Assume we want points following the distribution $g(x)$ and that $g(x)$ has a known primitive $G(x) = \int^x \mathrm{d}x' g(x')$
- Probability of producing point in $[x, x + \mathrm{d}x]$ should be $g(x)\,\mathrm{d}x$
- This can be achieved by solving the following equation for $x$

$$\int_a^x \mathrm{d}x'\, g(x') = R \int_a^b \mathrm{d}x'\, g(x')$$

where $R$ is a uniform random number in $[0, 1]$

$$x = G^{-1}\Big[G(a) + R\left(G(b) - G(a)\right)\Big]$$

# Monte-Carlo methods: Importance sampling

SLAC

- In many cases we can approximate the unknown integral of a function $f(x)$ with some known function $g(x)$ such that primitive $G(x)$ is known
- This amounts to a variable transformation

$$I = \int_a^b \mathrm{d}x\, g(x)\, \frac{f(x)}{g(x)} = \int_{G(a)}^{G(b)} \mathrm{d}G(x)\, w(x) \quad \text{where} \quad w(x) = \frac{f(x)}{g(x)}$$

- Integral and error estimate are

$$I = \big[G(b) - G(a)\big]\langle w \rangle \qquad \sigma = \big[G(b) - G(a)\big]\sqrt{\frac{\langle w^2 \rangle - \langle w \rangle^2}{N-1}}$$

  $N$ - Number of MC events (points)
- Note: $I$ is independent of $g(x)$, but $\sigma$ is not
  $\rightarrow$ suitable choice of $g(x)$ can be used to minimize error

- Assume nuclear decay process described by $g(x)$
- Nucleus can decay only if it has not decayed already
  Must account for survival probability $\leftrightarrow$ Poisson distribution

$$\mathcal{G}(x) \;=\; g(x)\Delta(x,b) \qquad \text{where} \qquad \Delta(x,b) \;=\; \exp\left\{-\int_x^b \mathrm{d}x'\, g(x')\right\}$$

- If $G(x)$ is known, then we also know the integral of $\mathcal{G}(x)$

$$\int_x^b \mathrm{d}x'\mathcal{G}(x') \;=\; \int_x^b \mathrm{d}x'\,\frac{\mathrm{d}\Delta(x',b)}{\mathrm{d}x'} \;=\; 1-\Delta(x,b)$$

- Can generate events by requiring $1-\Delta(x,b) = 1-R$

$$x = G^{-1}\Big[G(b) + \log R\Big]$$

SLAC

- Importance sampling for Poisson distributions
  - Generate event according to $\mathcal{G}(x)$
  - Accept with $w(x) = f(x)/g(x)$
  - If rejected, continue starting from $x$
- Probability for immediate acceptance

$$\frac{f(x)}{g(x)}\, g(x) \exp\left\{-\int_x^b \mathrm{d}x'\, g(x')\right\}$$

- Probability for acceptance after one rejection

$$\frac{f(x)}{g(x)}\, g(x) \int_x^b \mathrm{d}x_1 \exp\left\{-\int_x^{x_1} \mathrm{d}x'\, g(x')\right\} \left(1 - \frac{f(x_1)}{g(x_1)}\right) g(x_1) \exp\left\{-\int_{x_1}^b \mathrm{d}x'\, g(x')\right\}$$

- For $n$ intermediate rejections we obtain $n$ nested integrals $\int_x^b \int_{x_1}^b \ldots \int_{x_{n-1}}^b$
- Disentangling yields $1/n!$ and summing over all possible rejections gives

$$f(x) \exp\left\{-\int_x^b \mathrm{d}x'\, g(x')\right\} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\int_x^b \mathrm{d}x' \left[g(x') - f(x')\right]\right]^n = f(x) \exp\left\{-\int_x^b \mathrm{d}x'\, f(x')\right\}$$

# Monte-Carlo method for parton showers

- Start with set of $n$ partons at scale $t'$, which evolve collectively
  Sudakovs factorize, schematically

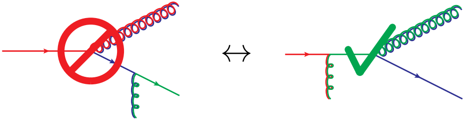
$$\Delta(t,t') = \prod_{i=1}^{n} \Delta_i(t,t')\;, \qquad\qquad \Delta_i(t,t') = \prod_{j=q,g} \Delta_{i\to j}(t,t')$$

- Find new scale $t$ where next branching occurs using veto algorithm
  - Generate $t$ using overestimate $\alpha_s^{\max} P_{ab}^{\max}(z)$
  - Determine "winner" parton $i$ and select new flavor $j$
  - Select splitting variable according to overestimate
  - Accept point with weight $\alpha_s(k_T^2) P_{ab}(z) / \alpha_s^{\max} P_{ab}^{\max}(z)$
- Construct splitting kinematics and update event record
- Continue until $t$ falls below an IR cutoff

# Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved if gluon wavelength larger than dipole size → emission off "mother"

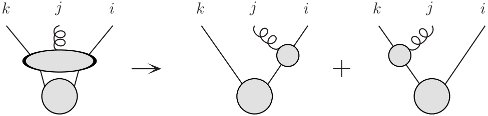


- Net effect is destructive interference outside cone with opening angle set by emitting color dipole → phase space for soft radiation halved

[Gustafsson,Pettersson] NPB306(1988)746

- Alternative description of effect in terms of dipole evolution
- Modern approach is to partial fraction soft eikonal and match to collinear sectors [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j}$$



# Color coherence and the dipole picture

SLAC

- Splitting kernels become dependent on anti-collinear direction usually defined by color spectator in large-$N_c$ limit
- Singularity confined to soft-collinear region only captures all coherence effects at leading color, NLL

$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- Complete set of leading-order splitting functions now given by

$$P_{qq}(z,\kappa^2) = C_F \left[ \frac{2\,(1-z)}{(1-z)^2+\kappa^2} - (1+z) \right]$$

$$P_{qg}(z,\kappa^2) = C_F \left[ \frac{1+(1-z)^2}{z} \right], \qquad P_{gq}(z,\kappa^2) = T_R \left[ z^2 + (1-z)^2 \right]$$

$$P_{gg}(z,\kappa^2) = 2\,C_A \left[ \frac{1-z}{(1-z)^2+\kappa^2} + \frac{1}{z} - 2 + z(1-z) \right]$$

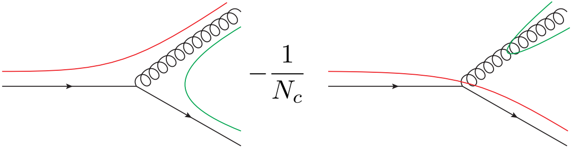
# Color flow

- Parton showers replace gluon propagators by means of the identity

$$\underbrace{\delta^{ab}}_{\text{standard}} = 2\,\mathrm{Tr}(T^a T^b) = 2\,T^a_{ij}T^b_{ji} = T^a_{ij}\;\underbrace{2\,\delta_{ik}\delta_{jl}}_{\text{parton shower}}\;T^b_{lk}$$

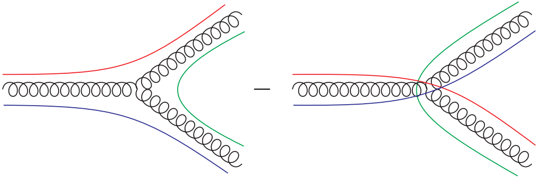
- Quark-gluon vertex

$$T^a_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$



- Gluon-gluon vertex

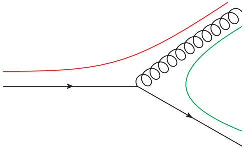
$$f^{abc}T^a_{ij}T^b_{kl}T^c_{mn} = \delta_{il}\delta_{kn}\delta_{mj} - \delta_{in}\delta_{ml}\delta_{kj}$$



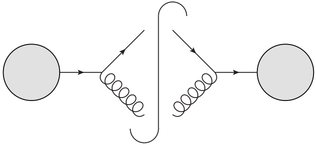
# Color flow

- Typically, parton showers also make the leading-color approximation

$$T^a_{ij} T^a_{kl} \to \frac{1}{2}\delta_{il}\delta_{jk} \qquad \leftrightarrow$$



- If used naively, this would overestimate the color charge of the quark: Consider process $q \to qg$ attached to some larger diagram



$$\propto \qquad T^a_{ij} T^a_{jk} = C_F \delta_{ik}$$

but now we have $\dfrac{1}{2}\,\delta_{il}\delta_{jm}\delta_{mj}\delta_{lk} = \dfrac{C_A}{2}\,\delta_{ik}$

- While color assignments in the parton shower are made at leading color the color charge of quarks is actually kept at $C_F$

# Color flow

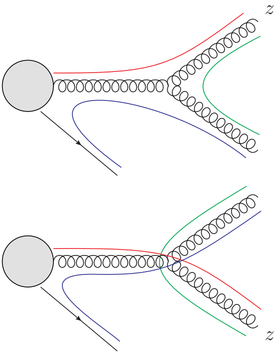
SLAC

- Having matched the eikonal to two collinear sectors implies that in $g \to gg$ splittings color and kinematics are entangled

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i + p_k) p_j} + \ldots \to \frac{1}{p_i p_j}\frac{1-z}{(1-z)^2 + \kappa^2} \ldots$$

- There is only one possible color assignment for each leading-color dipole

$$\frac{1-z}{(1-z)^2 + \kappa^2} \qquad \leftrightarrow$$



# Kinematics: Final state radiation

SLAC

- Want to construct three (massless) on-shell momenta from two, corresponding to branching process $\widetilde{ij} \to i, j$ in presence of $\tilde{k} \to k$
- Calculate $p_{ij}^2$ and $\tilde{z} = (p_i\tilde{p}_k)/(\tilde{p}_{ij}\tilde{p}_k)$ from PS variables $t$ and $z$
- First generate the propagator mass by rescaling

$$p_{ij}^\mu = \tilde{p}_{ij}^\mu + \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\,\tilde{p}_k^\mu\,, \qquad p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

- Then branch off-shell momentum into two on-shell momenta

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\,\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$

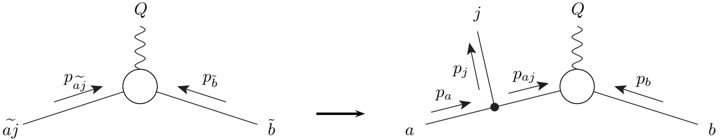
- On-shell conditions require that

$$\vec{k}_T^2 = p_{ij}^2\,\tilde{z}(1-\tilde{z}) \qquad \leftrightarrow \qquad \tilde{z}_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4\vec{k}_T^2}{p_{ij}^2}}\right)$$

$\to$ for any finite $\vec{k}_T$ we have $0 < \tilde{z} < 1$

# Kinematics: Initial-state radiation

- Initial-state kinematics slightly more involved as recoil should not be taken by opposite-side beam




- Compute new beam momentum by rescaling to new partonic cms energy

$$p_a^\mu = \frac{2\,p_a p_b}{2\,\tilde{p}_{aj}\tilde{p}_b}\,\tilde{p}_{aj}^\mu$$

- Compute final-state momentum and internal momentum as

$$p_{aj}^\mu = \tilde{z}\,p_a^\mu + \frac{p_{aj}^2}{2p_b p_a}\,p_b^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,p_a^\mu - \frac{p_{aj}^2}{2p_b p_a}\,p_b^\mu - k_\perp^\mu$$

- Recoil is taken by complete final state via Lorentz transformation

$$p_i^\mu = p_{\tilde{\imath}}^\mu - \frac{2\,p_{\tilde{\imath}}(K+\tilde{K})}{(K+\tilde{K})^2}\,(K+\tilde{K})^\mu + \frac{2\,p_{\tilde{\imath}}\tilde{K}}{\tilde{K}^2}\,K^\mu\;,$$

where $K^\mu = p_a^\mu - p_j^\mu + p_b^\mu$ and $\tilde{K}^\mu = p_{\widetilde{aj}}^\mu + p_b^\mu$

# Properties of splitting kernels

SLAC

- At leading order, splitting functions are probability densities
  They obey a special symmetry relation ($\varepsilon > 0$)

$$\sum_{b=q,g} \int_0^{1-\varepsilon} \mathrm{d}\zeta\, \zeta\, P_{qb}(\zeta) = \int_\varepsilon^{1-\varepsilon} \mathrm{d}\zeta\, P_{qq}(\zeta) + \mathcal{O}(\varepsilon)$$

$$\sum_{b=q,g} \int_0^{1-\varepsilon} \mathrm{d}\zeta\, \zeta\, P_{gb}(\zeta) = \int_\varepsilon^{1-\varepsilon} \mathrm{d}\zeta \left[ \frac{1}{2} P_{gg}(\zeta) + n_f\, P_{gq}(\zeta) \right] + \mathcal{O}(\varepsilon)$$

  Can thus replace $1/2 \to z$ in branching equations
- Physical sum rules must hold at any order

$$\int_0^1 \mathrm{d}\zeta\, \hat{P}_{qq}(\zeta) = 0 \qquad \to \qquad \text{flavor sum rule}$$

$$\sum_{c=q,g} \int_0^1 \mathrm{d}\zeta\, \zeta\, \hat{P}_{ac}(\zeta) = 0 \qquad \to \qquad \text{momentum sum rule}$$

$\to$ defines regularized DGLAP splitting functions $\hat{P}_{ab}$ as

$$\hat{P}_{ab}(z) = \lim_{\varepsilon \to 0} \left[ P_{ab}(z)\Theta(1-\varepsilon-z) - \delta_{ab} \frac{\Theta(z-1+\varepsilon)}{\varepsilon} \sum_{c=q,g} \int_0^{1-\varepsilon} d\zeta\, \zeta\, P_{ac}(\zeta) \right]$$

SLAC

- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}\,xD_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\,\frac{\alpha_s}{2\pi}\left[zP_{ab}(z)\right]_+ \tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Refine plus prescription $\left[zP_{ab}(z)\right]_+ = \lim_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-\varepsilon-z) - \delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

- Rewrite for finite $\varepsilon$

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t} = -\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta) + \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is derivative of Sudakov factor $\Delta = \exp\{-\lambda\}$

$$\Delta_a(t,Q^2) = \exp\left\{-\int_t^{Q^2}\frac{\mathrm{d}\bar{t}}{\bar{t}}\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)\right\}$$

# Relation between parton shower and DGLAP evolution

- Use generating function $\Pi_a(x,t,Q^2) = D_a(x,t)\Delta_a(t,Q^2)$ to write

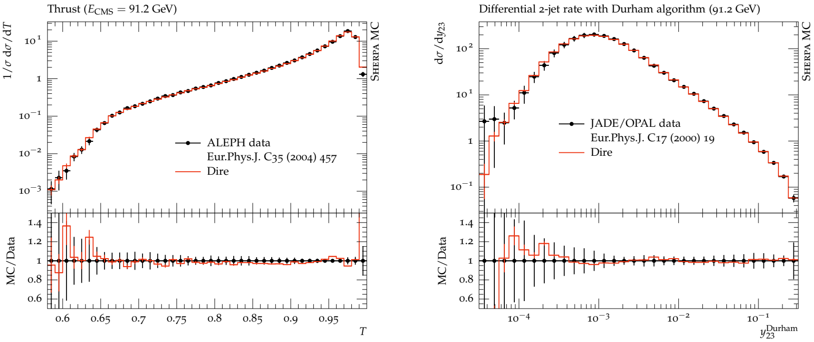
$$\frac{\mathrm{d}\ln\Pi_a(x,t,Q^2)}{\mathrm{d}\ln t/Q^2} = \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}\;.$$

- If hadron not resolved, obtain

$$\frac{\mathrm{d}}{\mathrm{d}\ln t/Q^2}\ln\left(\frac{\Pi_a(x,t,Q^2)}{D_a(x,t)}\right) = \frac{\mathrm{d}\Delta_a(t,Q^2)}{\mathrm{d}\ln t/Q^2} = \sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)$$

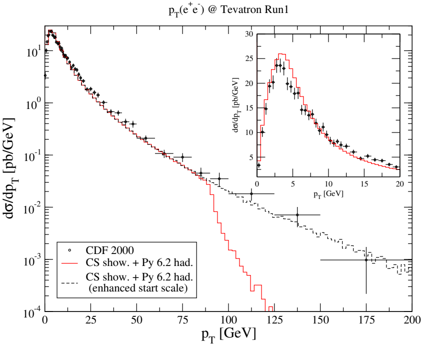
- Survival probabilities for one parton between scales $t_1$ and $t_2$:
  - $\dfrac{\Pi_a(x,t_2,Q^2)}{\Pi_a(x,t_1,Q^2)}$ Resolved hadron ↔ constrained (backward) evolution
  - $\dfrac{\Delta_a(t_2,Q^2)}{\Delta_a(t_1,Q^2)}$ No resolved hadron ↔ unconstrained (forward) evolution
- Parton-showers draw $t_2$-points starting from $t_1$ based on these probabilities

# Effects of the parton shower



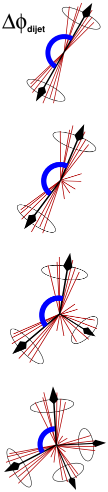

- Thrust and Durham $2 \to 3$-jet rate in $e^+e^- \to$hadrons
- Hadronization region to the right (left) in left (right) plot

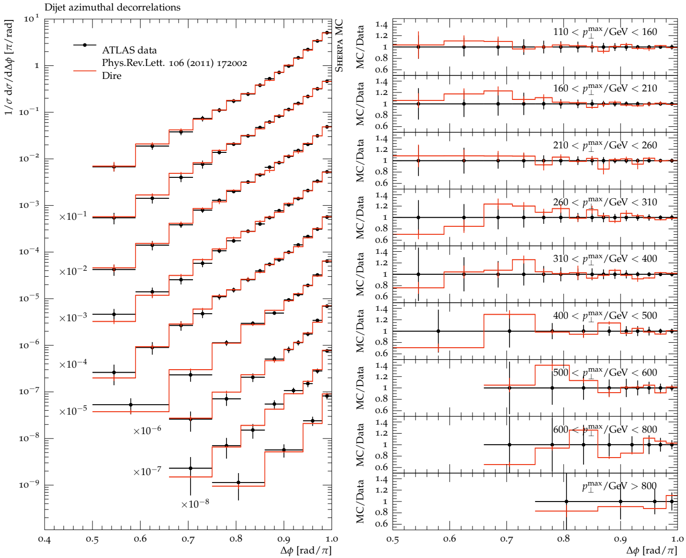
# Effects of the parton shower




- Drell-Yan lepton pair production at Tevatron
- If hard cross section computed at leading order, then parton shower is only source of transverse momentum

# Effects of the parton shower



# Hands on tutorials

- Great resource for learning parton showers:
  "Hackathons" at CTEQ/MCnet schools
  `http://www.slac.stanford.edu/~shoeche/cteq17`
  `svn co svn://svn.slac.stanford.edu/mc/ps`

## Tutorial on MC event generators

Held by the MCnet collaboration at CTEQ 2017.

### Instructions

PS coding tutorial
MC running tutorial

### TASI Lectures

arXiv:1411.4085

## Tutorial on Parton Showers and Matching

### 1 Introduction

In this tutorial we will discuss the construction of a parton shower, the implementation of on-the-fly uncertainty estimates, and of matrix-element corrections, and matching at next-to-leading order. At the end, you will be able to run your own parton shower for $e^+e^- \to$hadrons at LEP energies and compare its predictions to results from the event generator Sherpa (using a simplified setup). You will also have constructed your first MC@NLO and POWHEG generator.

### 2 Getting started

In order to run this tutorial you should install PyPy and Rivet on your PC. The following command will

# Formal precision of parton showers

# How to assess formal precision?

SLAC

- PS proven to be NLL accurate for simple observables, provided that soft double-counting removed ($\nearrow$ before) and 2-loop cusp anomalous dimension included [Catani,Marchesini,Webber] NPB349(1991)635
- Not entirely clear what this means numerically, because
  - Parton shower is momentum conserving, NLL is not
  - Parton shower is unitary, NLL approximations break this
- Differences can be quantified by
  - Designing an MC that reproduces NLL exactly
  - Removing NLL approximations one-by-one
- Employ well-established NLL result as an example
  - Observable: Thrust in $e^+e^- \to$hadrons
  - Method: Caesar [Banfi,Salam,Zanderighi] hep-ph/0407286

- This discussion will be technical, but it is needed to show that equivalence at NLL does not mean identical numerics Please bear with me and ask questions as needed to clarify!

# NLL resummation for simple additive observables

SLAC

[Banfi,Salam,Zanderighi] hep-ph/0407286

- Contribution of one emission with momentum $k$ to observable $v$

$$V(k) = \left(\frac{k_{T,l}}{Q}\right)^a e^{-b_l \eta_l} \qquad \rightarrow \qquad V(\{p\},\{k\}) = \sum_i V(k_i)$$

where $k^\mu = (1-z)p_l^\mu + \beta n^\mu + k_{T,l}^\mu$ is soft-gluon momentum

- On-shell condition $\beta = k_T^2/Q^2/(1-z) \rightarrow \eta = \log((1-z)Q/k_T)$
- Define "evolution" variable $\xi = Q^2 v^{2/(a+b)} = k_T^2 (1-z)^{-2b/(a+b)}$
- Integrated one-emission probability for $\xi > Q^2 v^{2/(a+b)}$

$$R_{\rm NLL}(v) = 2\int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz\, \frac{\alpha_s\big(\xi(1-z)^{\frac{2b}{a+b}}\big)}{2\pi} \frac{2\,C_F}{1-z}\Theta(\eta) - \frac{\alpha_s(\xi)}{\pi}\,C_F B_q\right]$$

- Cumulative cross section $\Sigma(v) = 1/\sigma \int^v d\bar{v}(d\sigma/d\bar{v})$ given by

$$\Sigma_{\rm NLL}(v) = e^{-R_{\rm NLL}(v)} \mathcal{F}(v)$$

$\mathcal{F}(v) = \lim_{\epsilon\to 0} \mathcal{F}_\epsilon(v)$ is pure NLL, accounting for multiple emissions

$$\mathcal{F}_\epsilon(v) = e^{R'_{\rm NLL}(v)\ln\epsilon} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m} R'_{\rm NLL}(v) \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i}\right) \Theta\left(1 - \sum_{j=1}^{m} \zeta_j\right)$$

# Parton shower for simple additive observables

- Integrated one-emission probability in parton shower

$$R_{\rm PS}(v) = 2\int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\rm min}}^{z_{\rm max}} dz\, \frac{\alpha_s\left(\xi(1-z)^{\frac{2b}{a+b}}\right)}{2\pi} C_F \left[\frac{2}{1-z} - (1+z)\right]\Theta(\eta)$$

$z$-limits from momentum conservation, $\Theta(\eta)$ removes soft double-counting

- $\Sigma_{\rm PS}(v)$ determined by unitarity (i.e. Poisson statistics)
- One can find a unified NLL/PS expression for $R(V)$ and $\Sigma(v)$

$$\Sigma\left(v\right) = \exp\left\{-\int_v \frac{d\xi}{\xi} R'_{>v}(\xi) - \int_{v_{\rm min}}^{v} \frac{d\xi}{\xi} R'_{<v}(\xi)\right\}$$
$$\times \sum_{m=0}^{\infty} \frac{1}{m!}\left(\prod_{i=1}^{m} \int_{v_{\rm min}} \frac{d\xi_i}{\xi_i} R'_{<v}(\xi_i)\right)\Theta\left(v - \sum_{j=1}^{m} V(\xi_i)\right)$$

where

$$R'_{\lessgtr v}(\xi) = \frac{\alpha_s^{\lessgtr v,{\rm soft}}(\mu^2_{\lessgtr})}{\pi}\int_{z_{\rm min}}^{z^{\rm max}_{\lessgtr v,{\rm soft}}} dz\, \frac{C_{\rm F}}{1-z} - \frac{\alpha_s^{\lessgtr v,{\rm coll}}(\mu^2_{\lessgtr v})}{\pi}\int_{z_{\rm min}}^{z^{\rm max}_{\lessgtr v,{\rm coll}}} dz\, C_{\rm F}\,\frac{1+z}{2}$$

# Differences between pure NLL and parton shower

[Reichelt,Siegert,SH] arXiv:1711.03497

- Isolated differences in terms of resolved/unresolved splitting probability:

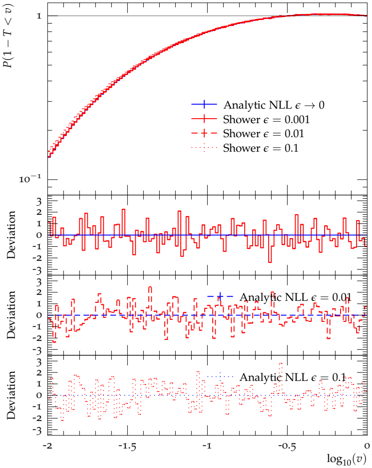
$$R'_{\lessgtr v}(\xi) = \frac{\alpha_s^{\lessgtr v,\mathrm{soft}}(\mu^2_{\lessgtr})}{\pi} \int_{z_{\min}}^{z^{\max}_{\lessgtr v,\mathrm{soft}}} dz\, \frac{C_F}{1-z} - \frac{\alpha_s^{\lessgtr v,\mathrm{coll}}(\mu^2_{\lessgtr v})}{\pi} \int_{z_{\min}}^{z^{\max}_{\lessgtr v,\mathrm{coll}}} dz\, C_F \frac{1+z}{2}$$

| | NLL | Parton Shower |
|---|---|---|
| $z^{\max}_{>v,\mathrm{soft}}$ | $1-(\xi/Q^2)^{\frac{a+b}{2a}}$ | |
| $\mu^2_{>v,\mathrm{soft}}$ | $\xi(1-z)^{\frac{2b}{a+b}}$ | |
| $\alpha_s^{>v,\mathrm{soft}}$ | 2-loop CMW | |
| $z^{\max}_{<v,\mathrm{soft}}$ | $1-v^{\frac{1}{a}}$ | $1-(\xi/Q^2)^{\frac{a+b}{2a}}$ |
| $\mu^2_{<v,\mathrm{soft}}$ | $Q^2 v^{\frac{2}{a+b}}(1-z)^{\frac{2b}{a+b}}$ | $\xi(1-z)^{\frac{2b}{a+b}}$ |
| $\alpha_s^{<v,\mathrm{soft}}$ | 1-loop | 2-loop CMW |

| | NLL | Parton Shower |
|---|---|---|
| $z^{\max}_{>v,\mathrm{coll}}$ | 1 | $1-(\xi/Q^2)^{\frac{a+b}{2a}}$ |
| $\mu^2_{>v,\mathrm{coll}}$ | $\xi$ | $\xi(1-z)^{\frac{2b}{a+b}}$ |
| $\alpha_s^{>v,\mathrm{coll}}$ | 1-loop | 2-loop CMW |
| $z^{\max}_{<v,\mathrm{coll}}$ | 0 | $1-(\xi/Q^2)^{\frac{a+b}{2a}}$ |
| $\mu^2_{<v,\mathrm{coll}}$ | n.a. | $\xi(1-z)^{\frac{2b}{a+b}}$ |
| $\alpha_s^{<v,\mathrm{coll}}$ | n.a. | 2-loop CMW |

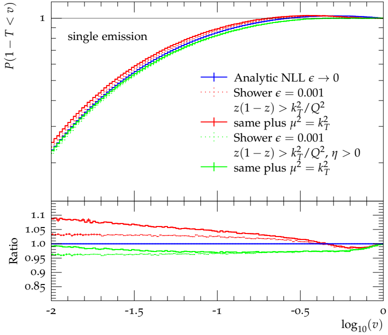
- Can cast pure NLL into PS language by using NLL expressions in PS
- Can study each effect in detail by reverting changes back to PS

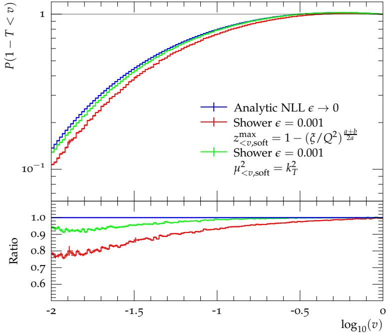
# Baseline for comparison




- Modified parton shower exactly reproduces pure NLL result
- $E_{\mathrm{cms}}$=91.2 GeV, $\alpha_s(M_Z) = 0.118$ fixed flavor $n_f = 5$

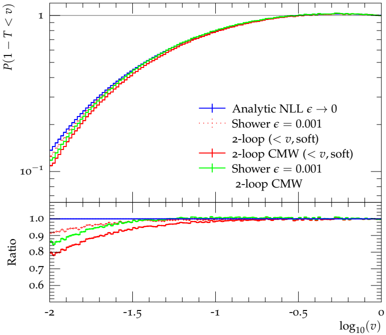
# Local momentum conservation and unitarity

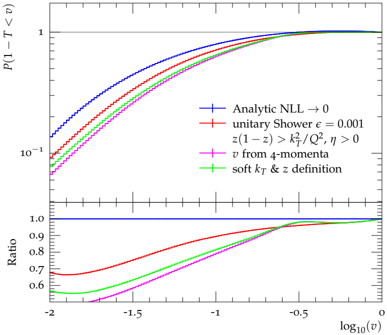



- NLL→PS in $z_{\mathrm{min/max}}$ (4-momentum conservation)
- NLL→PS in $z^{\mathrm{coll}}_{>v,\mathrm{max}}$ (phase-space sectorization)
- NLL→PS in $\mu^2_{>v,\mathrm{coll}}$ (conventional)
- NLL→PS in $z^{\mathrm{soft}}_{<v,\mathrm{max}}$ (from PS unitarity)
- NLL→PS in $\mu^2_{<v,\mathrm{soft}}$ (from PS unitarity)

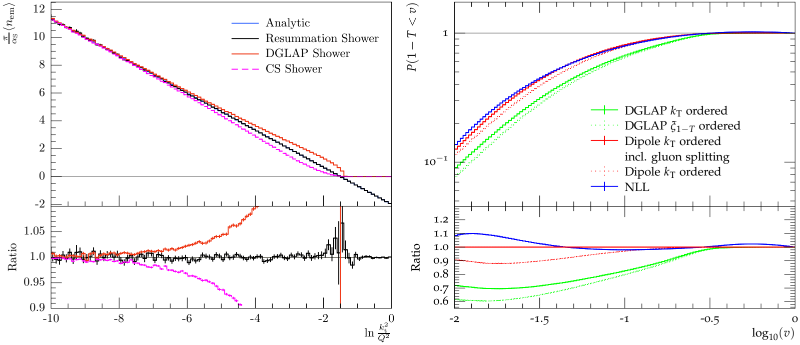
# Running coupling and global momentum conservation




- NLL→PS in 2-loop CMW $< v$, soft (from PS unitarity)
- NLL→PS in 2-loop CMW overall (conventional)
- NLL→PS in observable (use experimental definition)
- NLL→PS in evolution variable

# Overall comparison NLL / PS / Dipole Shower

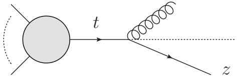



- Tuned comparison of differences between formally equivalent calculations
- Simplest process and simplest observable, but still large differences
- Origin of differences traced to treatment of kinematics & unitarity
- At NLL accuracy, none of the methods is formally better than another
  $\rightarrow$ Difference is a systematic uncertainty & needs to be kept in mind

# Parton showers at NLO

# Quick and dirty introduction to DGLAP

- Compute $e^+e^- \to q\bar{q}g$ in collinear limit




- Phase space factor for one additional parton in collinear limit, $D = 4 - 2\varepsilon$
  Note: $y = t/Q^2$, see for example [Catani,Seymour] hep-ph/9605323

$$\mathrm{d}\Phi_{+1} = \frac{Q^{2-2\varepsilon}}{16\pi^2}\,\frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)}\,\mathrm{d}y\,\mathrm{d}z\,\big[y\,z(1-z)\big]^{-\varepsilon}$$

- Factorized matrix element squared in collinear limit

$$|M_{n+1}|^2 = |M_n|^2\,\frac{2g_s^2\mu^{2\varepsilon}}{Q^2 y}\,P_{qq}(z) \xrightarrow{\overline{\mathrm{MS}}} |M_n|^2\,8\pi\alpha_s(\mu^2)\,\frac{e^{\varepsilon\gamma_E}}{(4\pi)^\varepsilon}\,\frac{\mu^{2\varepsilon}}{Q^2 y}\,P_{qq}(z)$$

- Combine into branching probability at fixed $x$, where $x < 1$

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma_3}{\mathrm{d}x} = \int_0^1 \frac{\mathrm{d}y}{y^{1+\varepsilon}}\,\frac{\alpha_s(\mu^2)}{2\pi}\,\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\left(\frac{Q^2}{\mu^2}\right)^{-\varepsilon}\big[x(1-x)\big]^{-\varepsilon}P_{qq}(x)$$

$$= -\frac{1}{\varepsilon}\,\frac{\alpha_s(\mu^2)}{2\pi}\,\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\left(\frac{Q^2}{\mu^2}\right)^{-\varepsilon}\big[x(1-x)\big]^{-\varepsilon}P_{qq}(x)$$

- Upon adding virtual corrections $\sigma_{2,V}$ we obtain

$$\int_0^1 \mathrm{d}x \, \frac{\mathrm{d}\sigma_3}{\mathrm{d}x} + \sigma_{2,V} = \sigma_2 \, \frac{\alpha_s}{\pi}$$

- Alternatively we can write

$$\int_0^1 \mathrm{d}x \, \left\{ \frac{\mathrm{d}\sigma_3}{\mathrm{d}x} + \left( \sigma_{2,V} - \sigma_2 \, \frac{\alpha_s}{\pi} \right) \delta(1-x) \right\} = \int_0^1 \mathrm{d}x \, \left[ \frac{\mathrm{d}\sigma_3}{\mathrm{d}x} \right]_+ = 0$$

- From previous slide we obtain by expanding in $\varepsilon$

$$\frac{1}{\sigma_2} \left[ \frac{\mathrm{d}\sigma_3}{\mathrm{d}x} \right]_+ = \frac{\alpha_s(\mu^2)}{2\pi} \hat{P}_{qq}(x) \log \frac{Q^2}{\mu^2} + \alpha_s f^{e^+e^-}(x)$$

- Now we compute the single-hadron inclusive cross section
  $\rightarrow$ At LO, just multiply $\sigma_2$ with bare fragmentation function $D^h_{0,q}(x)$

$$\frac{\mathrm{d}\sigma^h(x,Q^2)}{\mathrm{d}x} = \sum_{i=1}^{n_f} \sigma_{2,q_i} \left[ D^h_{0,q_i}(x) + D^h_{0,\bar{q}_i}(x) \right]$$

# Quick and dirty introduction to DGLAP

SLAC

- At NLO this becomes a convolution due to $x$-conservation

$$\frac{\mathrm{d}\sigma^h(x,Q^2)}{\mathrm{d}x} = \int_x^1 \frac{\mathrm{d}z}{z} \sum_{i=1}^{n_f} \sigma_{2,q_i} \left[ D^h_{0,q_i}(x/z) + D^h_{0,\bar{q}_i}(x/z) \right] \left[ \left(1+\frac{\alpha_s}{\pi}\right)\delta(1-z) + \frac{\alpha_s}{2\pi}\hat{P}_{qq}(z)\log\frac{Q^2}{\mu^2} + \ldots \right] + 2\sum_{i=1}^{n_f}\sigma_{2,q_i}\, D^h_{0,g}(x/z) \left[ \frac{\alpha_s}{2\pi}\hat{P}_{qg}(z)\log\frac{Q^2}{\mu^2} + \ldots \right]$$

- Observable fragmentation functions at NLO are now introduced as

$$D^h_a(x,\mu_F^2) = D^h_{0,a}(x) + \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}\sum_{b=q,g}\hat{P}_{ab}(z)\log\frac{\mu_F^2}{\mu^2}\, D^h_{0,b}(x/z)$$

- This implies that $D^h_a$ obeys a renormalization group equation

$$\frac{\mathrm{d}D^h_a(x,\mu_F^2)}{\mathrm{d}\log(\mu_F^2/Q^2)} = \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s(\mu^2)}{2\pi}\sum_{b=q,g}\hat{P}_{ab}(z)\, D^h_b(x/z,\mu_F^2)\ ,$$

- Eventually we can write the single-hadron cross section as

$$\frac{\mathrm{d}\sigma^h(x,Q^2)}{\mathrm{d}x} = \left(1+\frac{\alpha_s}{\pi}\right)\sum_{i=1}^{n_f}\sigma_{2,q_i}\left[ D^h_{q_i}(x,Q^2) + D^h_{\bar{q}_i}(x,Q^2) \right]$$

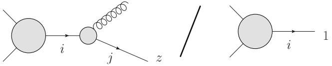
# Parton evolution at the next-to-leading order

[Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
[Floratos,Kounnas,Lacaze] NPB192(1981)417

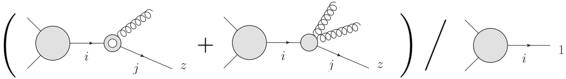
- Higher-order differential cross sections for partonic final states exhibit IR divergences after regularization and UV renormalization
- IR poles removed in matching to fragmentation functions and PDFs
- Coefficients can be computed from differential cross sections

$$D_{ji}^{(0)}(z,\mu)=\delta_{ij}\delta(1-z) \quad \leftrightarrow$$


$$D_{ji}^{(1)}(z,\mu)=-\frac{1}{\varepsilon}P_{ji}^{(0)}(z) \quad \leftrightarrow$$




$$D_{ji}^{(2)}(z,\mu)=-\frac{1}{2\varepsilon}P_{ji}^{(1)}(z)+\frac{\beta_0}{4\varepsilon^2}P_{ji}^{(0)}(z)+\frac{1}{2\varepsilon^2}\int_z^1\frac{\mathrm{d}x}{x}P_{jk}^{(0)}(x)P_{ki}^{(0)}(z/x)$$

$\leftrightarrow$ 

# Parton evolution at the next-to-leading order

- Individual splitting kernels $P_{ji}^{(n)}$ not probabilities, but sum rules hold
  In particular: Momentum sum rule identical between LO & NLO

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-\varepsilon-z) - \delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

  → PS implements renormalization group equation if Sudakov defined as

$$\Delta_a(t,Q^2) = \exp\left\{-\int_t^{Q^2}\frac{\mathrm{d}\bar{t}}{\bar{t}}\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)\right\}$$

- Negative weights accommodated by modified veto algorithm
  [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
  Standard probability for one acceptance with $n$ rejections

$$\frac{f(t)}{g(t)}\,g(t)\,\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t'}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)g(t_i)\,\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\right]$$

  Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\,g(t)\,\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t'}\mathrm{d}t_i\left(1-\frac{f(t_i)}{h(t_i)}\right)g(t_i)\,\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\right]$$

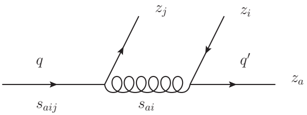
$$w(t,t_1,\ldots,t_n) = \frac{h(t)}{g(t)}\prod_{i=1}^{n}\frac{h(t_i)}{g(t_i)}\,\frac{g(t_i)-f(t_i)}{h(t_i)-f(t_i)}$$

# Parton evolution at the next-to-leading order

SLAC

[Prestel,SH] arXiv:1705.00742

- Fully exclusive simulation requires computing splitting functions on the fly using differential NLO calculation & IR renormalization
- Schematically very similar to Catani-Seymour dipole subtraction
- Simplest example: Flavor-changing configuration $q \to q'$




Tree-level expression[1] $\leftrightarrow$ real-emission correction in CS

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[ -\frac{t^2_{ai,j}}{s_{ai} s_{aij}} + \frac{4\, z_j + (z_a - z_i)^2}{z_a + z_i} + (1-2\varepsilon) \left( z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

Subtraction term $\leftrightarrow$ differential subtraction term in CS

$$\tilde{P}_{qq'} = C_F T_R \frac{s_{aij}}{s_{ai}} \left( \frac{1+\tilde{z}_j^2}{1-\tilde{z}_j} - \varepsilon(1-\tilde{z}_j) \right) \left( 1 - \frac{2}{1-\varepsilon} \frac{\tilde{z}_a \tilde{z}_i}{(\tilde{z}_a + \tilde{z}_i)^2} \right) + \ldots$$

[1] $(z_a + z_i)\, t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

# Parton evolution at the next-to-leading order

[Prestel,SH] arXiv:1705.00742

- Complete NLO result schematically given by

$$P_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1}\Big[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\Big]$$

- Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$ $\nearrow$ previous slide
  Difference finite in 4 dimensions $\to$ amenable to MC simulation
- Must add integrated subtraction and renormalization counterterms

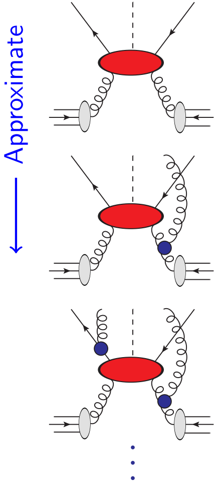
$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x}\left(P^{(0)}_{qg}(x) + \varepsilon \mathcal{J}^{(1)}_{qg}(x)\right)\frac{1}{\varepsilon}P^{(0)}_{gq}(z/x)$$

$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F\left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x\right)$$

- Analytical computation of $\mathrm{I}$ not needed, as $\mathrm{I} + \mathcal{P}/\varepsilon$ finite
  generate as endpoint at $s_{ai} = 0$, starting from integrand at $\mathcal{O}(\varepsilon)$
- All components of $P_{qq'}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

# Parton showers in a nutshell



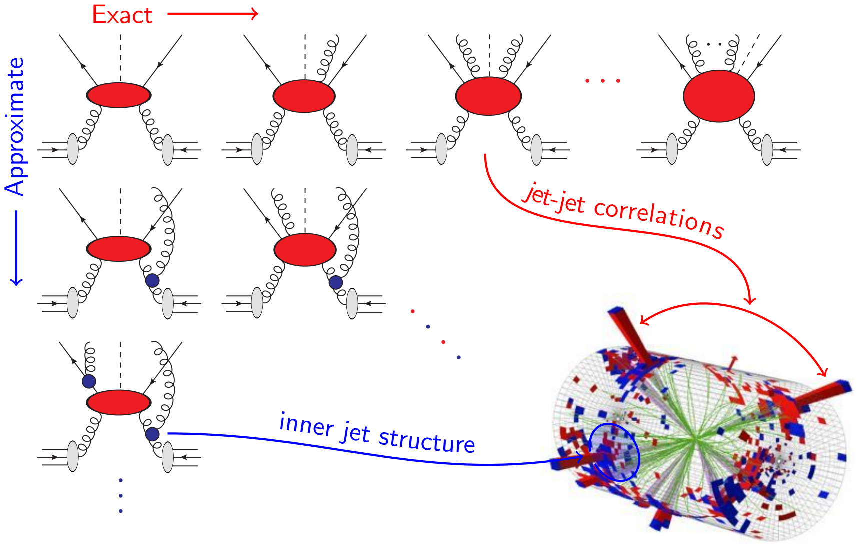

$$\sigma_{\text{incl}} \Bigg[ \Delta(t_c, Q^2)$$

$$+ \int_{t_c}^{Q^2} \frac{\mathrm{d}t}{t} \int \mathrm{d}z \frac{\alpha_s}{2\pi} P(z)\, \Delta(t, Q^2)$$

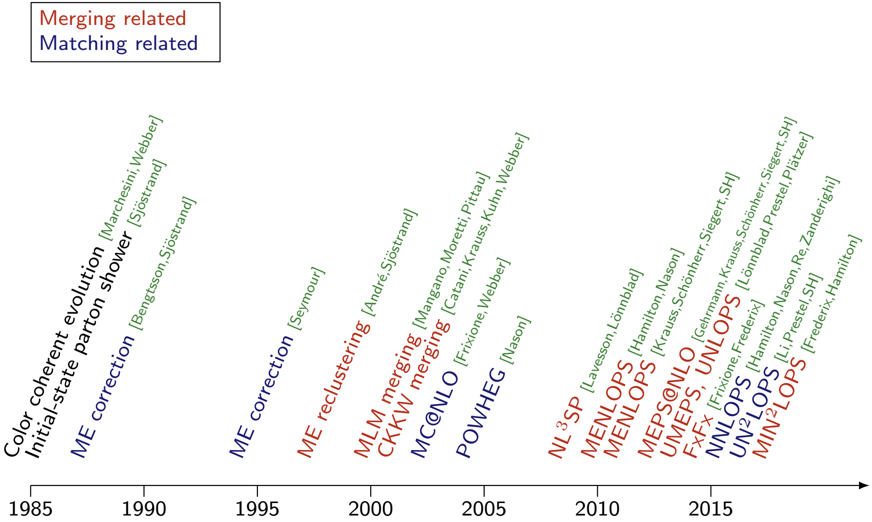
$$+ \frac{1}{2} \left( \int_{t_c}^{Q^2} \frac{\mathrm{d}t}{t} \int \mathrm{d}z \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(t, Q^2)$$

$$+ \ldots$$

# Parton-shower matching & merging



# Parton-shower matching & merging



# Fixed-Order NLO & IR Subtraction

# Toy model for IR subtraction at NLO

[Frixione,Webber] hep-ph/0204244

- Assume system of charges radiating "photons" of fractional energy $x$.
- Predicting observables at NLO amounts to computing expectation value

$$\langle O \rangle = \lim_{\varepsilon \to 0} \int_0^1 \mathrm{d}x \, x^{-2\varepsilon} \left[ \left( \frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_{\mathrm{B}} O_0 + \left( \frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_{\mathrm{V}} O_0 + \left( \frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_{\mathrm{R}} O_1(x) \right]$$

- Born, virtual and real-emission contributions given by

$$\left( \frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_{\mathrm{B,V,R}} = \quad \mathrm{B}\,\delta(x), \qquad \left( \mathrm{V}_f + \frac{\mathrm{B}\mathrm{V}_s}{2\varepsilon} \right) \delta(x), \qquad \frac{\mathrm{R}(x)}{x}$$

KLN cancellation theorem: $\lim_{x \to 0} \mathrm{R}(x) = \mathrm{B}\mathrm{V}_s$
Infrared safe observable: $\lim_{x \to 0} O_1(x) = O_0$

Virtual correction $\begin{cases} \mathrm{V}_f & \text{– finite piece} \\ \mathrm{B}\mathrm{V}_s/2\varepsilon & \text{– singular piece} \end{cases}$

Implicit: All higher-order terms proportional to coupling $\alpha$

# Toy model for IR subtraction at NLO

SLAC

- Add and subtract approximation of real correction in soft limit

$$\langle O\rangle_R = \mathrm{BV}_s\, O(0) \int_0^1 \mathrm{d}x \frac{x^{-2\varepsilon}}{x} + \int_0^1 \mathrm{d}x\, \frac{\mathrm{R}(x)\, O(x) - \mathrm{BV}_s\, O(0)}{x^{1+2\varepsilon}}$$

- Second integral non-singular $\rightarrow$ set $\varepsilon = 0$

$$\langle O\rangle_R = -\frac{\mathrm{BV}_s}{2\varepsilon}\, O(0) + \int_0^1 \mathrm{d}x\, \frac{\mathrm{R}(x)\, O(x) - \mathrm{BV}_s\, O(0)}{x}$$

- Combine everything with Born and virtual correction

$$\langle O\rangle = \Big(\mathrm{B} + \mathrm{V}_f\Big)\, O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \Big[\mathrm{R}(x)\, O(x) - \mathrm{BV}_s\, O(0)\Big]$$

  Both terms separately finite
- Rewrite for future reference

$$\langle O\rangle = \Big(\mathrm{B} + \mathrm{V} + \mathrm{I}\Big)\, O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \Big[\mathrm{R}(x)\, O(x) - \mathrm{S}\, O(0)\Big]$$

  $\mathrm{I} = -\mathrm{BV}_s/2\varepsilon \rightarrow$ Integrated subtraction term
  $\mathrm{S} = \mathrm{BV}_s \rightarrow$ Real subtraction term

# IR subtraction at NLO

SLAC

- QCD subtraction more cumbersome:
  - Soft limit color dependent [Bassetto,Ciafaloni,Marchesini] PR100(1983)201

$$|\mathcal{M}(1,\ldots,j,\ldots,n)|^2 \stackrel{j\to\text{soft}}{\longrightarrow} -\sum_{i,k\neq i}\frac{8\pi\mu^{2\varepsilon}\alpha_s}{p_ip_j}$$

$$\times\,{}_m\langle 1,\ldots,i,\ldots,k,\ldots,n|\frac{\mathbf{T}_i\mathbf{T}_k\;p_ip_k}{(p_i+p_k)p_j}|1,\ldots,i,\ldots,k,\ldots,n\rangle_m$$

    $\mathbf{T}_i$ - color insertion operator for parton $i$
    $|1,\ldots,i,\ldots,k,\ldots,n\rangle_m$ - $m$-parton Born amplitude

  - Collinear limit spin dependent [Altarelli,Parisi] NPB126(1977)298

$$|\mathcal{M}(1,\ldots,i,\ldots,j,\ldots,n)|^2 \stackrel{i,j\to\text{coll}}{\longrightarrow} \frac{8\pi\mu^{2\varepsilon}\alpha_s}{2p_ip_j}$$

$$\times\,{}_m\langle 1,\ldots,ij,\ldots,n|\hat{P}_{(ij)i}(z,k_T,\varepsilon)\,|1,\ldots,ij,\ldots,n\rangle_m$$

    $\hat{P}_{(ij)i}(z,k_T,\varepsilon)$ - Spin-dependent DGLAP kernel

- Basic features surviving from toy model are phase-space mapping and subtraction terms as products of Born times splitting operator
- Commonly used techniques: Dipole method & FKS method

  [Catani,Seymour] NPB485(1997)291, [Catani,Dittmaier,Seymour,Trocsanyi] NPB627(2002)189
  [Frixione,Kunszt,Signer] NPB467(1996)399

# Matching NLO & PS

# Matching schemes

Two major techniques to match NLO calculations and parton showers

**Additive** (MC@NLO-like)

[Frixione,Webber] hep-ph/0204244

- Use parton-shower splitting kernel as an NLO subtraction term
- Multiply LO event weight by Born-local K-factor including integrated subtraction term and virtual corrections
- Add hard remainder function consisting of subtracted real-emission correction

**Multiplicative** (POWHEG-like)

[Nason] hep-ph/0409146

- Use matrix-element corrections to replace parton-shower splitting kernel by full real-emission matrix element in first shower branching
- Multiply LO event weight by Born-local NLO K-factor (integrated over real corrections that can be mapped to Born according to PS kinematics)

# Toy model for modified subtraction

[Frixione,Webber] hep-ph/0204244

- Revisit toy model for NLO

$$\langle O\rangle = \Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big)\,O(0) + \int_0^1 \frac{\mathrm{d}x}{x}\Big[\mathrm{R}(x)\,O(x) - \mathrm{S}\,O(0)\Big]$$

- In parton showers, any number of "photons" can be emitted
- Emission probability controlled by Sudakov form factor

$$\Delta(x_1,x_2) = \exp\left\{-\int_{x_1}^{x_2}\frac{\mathrm{d}x}{x}\,\mathrm{K}(x)\right\}$$

Evolution kernel behaves as $\lim_{x\to 0}\mathrm{K}(x) = \lim_{x\to 0}\mathrm{R}(x)/\mathrm{B} = \mathrm{V}_s$

- Define generating functional

$$\mathcal{F}^{(n)}_{\mathrm{MC}}(x,O) = \Delta(x_0,x)\,O_n(x) + \int_{x_0}^{x}\frac{\mathrm{d}\bar{x}}{\bar{x}}\frac{\mathrm{d}\Delta(\bar{x},x)}{\mathrm{d}\ln\bar{x}}\,\mathcal{F}^{(n+1)}_{\mathrm{MC}}(\bar{x},O)$$

- $\mathcal{F}^{(n)}_{\mathrm{MC}}(x,O)$ now replaces observable $O$ $\to$ Naively:

$O(0)$ $\Leftrightarrow$ start MC with 0 emissions $\to$ $\mathcal{F}^{(0)}_{\mathrm{MC}}(1,O)$

$O(x)$ $\Leftrightarrow$ start MC with 1 emission $\to$ $\mathcal{F}^{(1)}_{\mathrm{MC}}(x,O)$

# Toy model for modified subtraction

- Combined generating functional would be

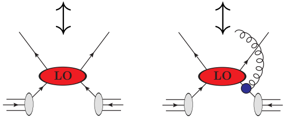
$$\left[\Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big)-\int_0^1\frac{\mathrm{d}x}{x}\mathrm{S}\right]\mathcal{F}^{(0)}_{\mathrm{MC}}(1,O)+\int_0^1\frac{\mathrm{d}x}{x}\,\mathrm{R}(x)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(x,O)$$

- This is wrong because

$$\mathcal{F}^{(0)}_{\mathrm{MC}}(O)=\Delta(x_c,1)\,O(0)+\int_{x_c}^1\frac{\mathrm{d}x}{x}\mathrm{K}(x)\Delta(x,1)\,O(x)+\ldots$$

- So $\mathrm{B}\,\mathcal{F}^{(0)}_{\mathrm{MC}}$ generates an $\mathcal{O}(\alpha)$ term that spoils NLO accuracy

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{MC}}O(x)=\mathrm{B}\left[-\frac{\mathrm{K}(x)}{x}O(0)+\frac{\mathrm{K}(x)}{x}O(x)\right]$$



# Toy MC@NLO

[Frixione,Webber] hep-ph/0204244

- The proper matching is obtained by subtracting this $\mathcal{O}(\alpha)$ contribution

$$\langle O \rangle = \Big[ \underbrace{\Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big) + \int_0^1 \frac{\mathrm{d}x}{x} \Big(\mathrm{BK}(x) - \mathrm{S}\Big)}_{\text{NLO-weighted Born cross section}} \Big] \mathcal{F}^{(0)}_{\mathrm{MC}}(1, O)$$

$$+ \int_0^1 \frac{\mathrm{d}x}{x} \underbrace{\Big[\mathrm{R}(x) - \mathrm{BK}(x)\Big]}_{\text{hard remainder}} \mathcal{F}^{(1)}_{\mathrm{MC}}(x, O)$$

- Like at fixed order, both terms are separately finite
- We call events from the first term $\mathbb{S}$-events ($\mathbb{S}$tandard) and events from the second term $\mathbb{H}$-events ($\mathbb{H}$ard)
- For further reference, define $\mathrm{D}^{(\mathrm{K})}(x) := \mathrm{BK}(x)$ as well as

$$\bar{\mathrm{B}}^{(\mathrm{K})} = \Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big) + \int_0^1 \frac{\mathrm{d}x}{x} \Big(\mathrm{D}^{(\mathrm{K})}(x) - \mathrm{S}\Big)\,, \quad \mathrm{H}^{(\mathrm{K})}(x) = \mathrm{R}(x) - \mathrm{D}^{(\mathrm{K})}(x)$$

$\to$ compact notation

$$\langle O \rangle = \bar{\mathrm{B}}^{(\mathrm{K})}\, \mathcal{F}^{(0)}_{\mathrm{MC}}(O) + \int_0^1 \frac{\mathrm{d}x}{x}\, \mathrm{H}^{(\mathrm{K})}(x)\, \mathcal{F}^{(1)}_{\mathrm{MC}}(x, O)$$

# Modified subtraction in QCD

[Frixione,Webber] hep-ph/0204244

- Leading-order calculation for observable $O$

$$\langle O \rangle = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O \rangle = \int \mathrm{d}\Phi_B \left\{ \mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B) \right\} O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\, O(\Phi_R)$$

- Parton-shower result until first emission

$$\langle O \rangle = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[ \Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t(\Phi_1))\, O(\Phi_R) \right]$$

$$\stackrel{\mathcal{O}(\alpha_s)}{\rightarrow} \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left\{ 1 - \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \right\} O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_B \mathrm{d}\Phi_1\, \mathrm{B}(\Phi_B)\, \mathrm{K}(\Phi_1)\, O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\, \mathrm{d}z\, \mathrm{d}\phi$

Splitting functions: $\mathrm{K}(t,z) \to \alpha_s/(2\pi t) \sum \mathrm{P}(z)\, \Theta(\mu_Q^2 - t)$

Sudakov factors: $\Delta^{(\mathrm{K})}(t) = \exp\left\{ -\int_t \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \right\}$

# Modified subtraction in QCD

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from NLO result

$$\int \mathrm{d}\Phi_B \left\{ \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B) \int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \right\} \ldots$$

$$+ \int \mathrm{d}\Phi_R \left\{ \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right\} \ldots$$

- In DLL approximation both terms finite $\rightarrow$
  MC events in two categories, $\mathbb{S}$tandard and $\mathbb{H}$ard

$$\mathbb{S} \;\rightarrow\; \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B) \int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)$$

$$\mathbb{H} \;\rightarrow\; \mathrm{H}^{(\mathrm{K})} = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1)$$

- Color & spin correlations $\rightarrow$ NLO subtraction needed
  $1/N_c$ corrections can be faded out in soft region by smoothing function

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \left[ \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) - \mathrm{S}(\Phi_R) \right] f(\Phi_1)$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \left[ \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right] f(\Phi_1)$$

# Dealing with color and spin

SLAC

Method 1 [Frixione,Webber] hep-ph/0204244

- $f(\Phi_1) \to 0$ in soft-gluon limit
- Full NLO in hard / collinear region
- Subleading color terms not $\phi_1$-dependent in soft domain

Method 2 [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220

- Replace $\mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1) \to \mathrm{S}(\Phi_R)$, includes color & spin correlations
- Can lead to non-probabilistic $\Delta^{(\mathrm{S})}(t)$
  $\to$ requires modification of veto algorithm

# MC@NLO

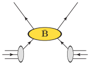
[Frixione,Webber] hep-ph/0204244

- Add parton shower, described by generating functional $\mathcal{F}_{\mathrm{MC}}$

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\, \mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_R)\, \mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

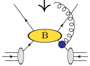
  Probability conservation: $\mathcal{F}_{\mathrm{MC}}(t,1) = 1 \rightarrow$ cross section correct at NLO
- Expansion of matched result until first emission

$$\langle O\rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) \Big[ \Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) \quad \leftrightarrow$$

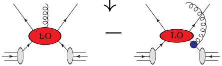



$$+ \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \Delta^{(\mathrm{K})}(t(\Phi_1))\, O(\Phi_r) \Big] + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_{n+1})\, O(\Phi_R)$$

$\updownarrow$




$\updownarrow$

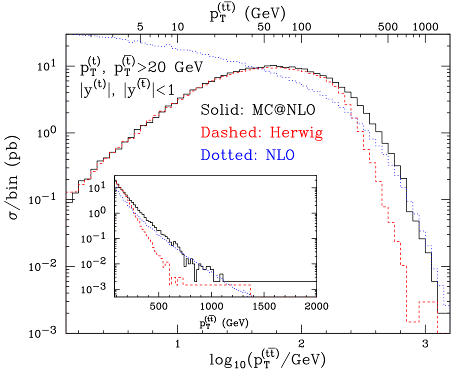



- Parametrically $\mathcal{O}(\alpha_s)$ correct
- Preserves logarithmic accuracy of PS

# MC@NLO – Features

[Nason,Webber] arXiv:1202.1251

$pp \to t\bar{t} + X$ @ 14 TeV

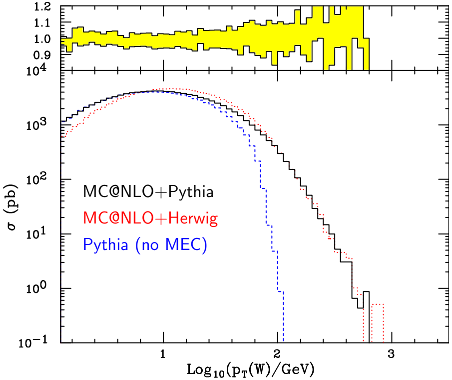



- MC@NLO interpolates smoothly between real-emission ME and PS

# MC@NLO – Features

[Torrielli,Frixione] arXiv:1002.4293

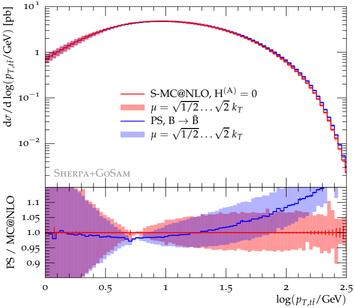
$pp \to W + X$ @ 14 TeV

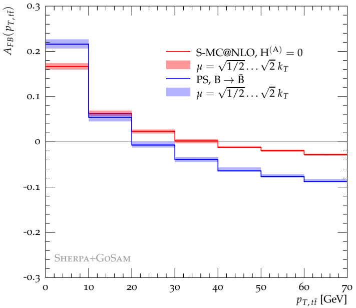



- MC@NLO with different PS agree at high $p_T$ $\leftrightarrow$ NLO
- Differences at low $p_T$ due to differences in PS

# MC@NLO – Features

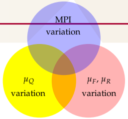
[Huang,Luisoni,Schönherr,Winter,SH] arXiv:1306.2703

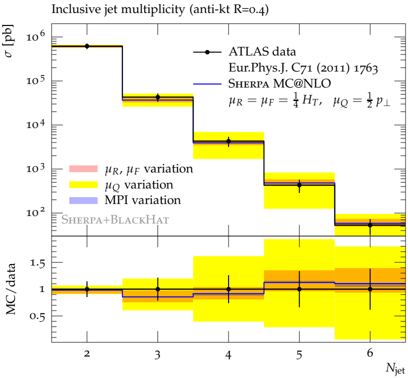


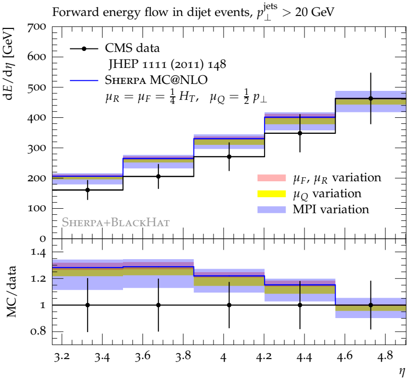




- Leading color appropriate for sufficiently inclusive observables
- Full vs leading color has larger impact on $A_{FB}$ $\rightarrow$ explained by kinematics effects using arguments of [Skands,Webber,Winter] arXiv:1205.1466

# Matching – Uncertainties

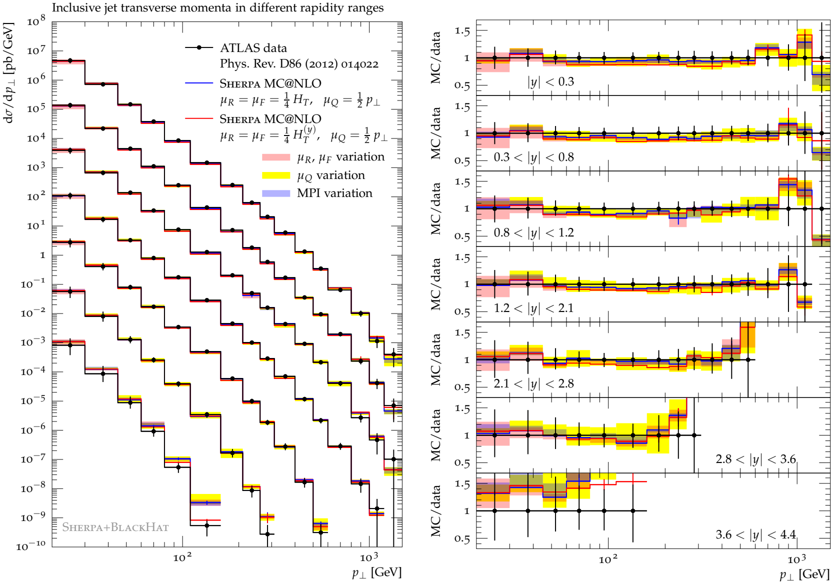



- Jet multiplicity $\rightarrow$ uncertainty due to choice of $\mu_Q^2$
- Forward energy flow $\rightarrow$ major uncertainty from underlying event

# Matching – Uncertainties



[Nason] hep-ph/0409146

- Aim of the method: Eliminate negative weights from MC@NLO
- Replace $\mathrm{BK} \to \mathrm{R} \Rightarrow$ no $\mathbb{H}$-events $\quad \Rightarrow \quad \bar{\mathrm{B}}^{(\mathrm{R})}$ positive in physical region
- Expectation value of observable is

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B) \Bigg[ \Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg]$$

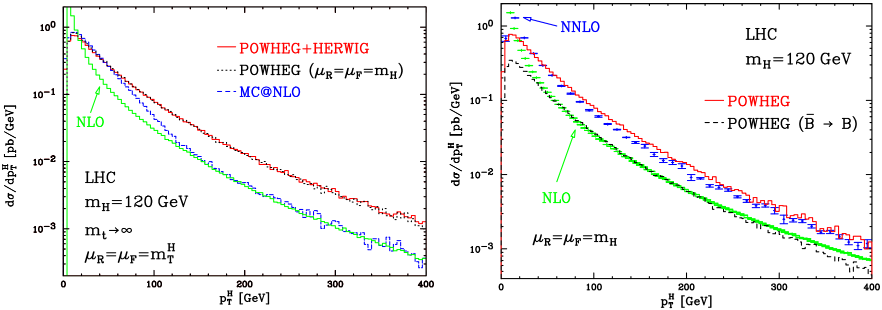
- $\mu_Q^2$ has changed to hadronic centre-of-mass energy squared, $s_{\mathrm{had}}$, as full phase space for real-emission correction, $\mathrm{R}$, must be covered
- Absence of $\mathbb{H}$-events leads to enhancement of high-$p_T$ region by

  $$K = \frac{\bar{\mathrm{B}}}{\mathrm{B}} = 1 + \mathcal{O}(\alpha_s)$$

  Formally beyond NLO, but sizeable corrections in practice

# POWHEG – Features

[Alioli,Nason,Oleari,Re] arXiv:0812.0578




- Large enhancement at high $p_{T,h}$
- Can be traced back to large NLO correction
- Fortunately, NNLO correction is also large $\to \sim$ agreement

# Improved POWHEG

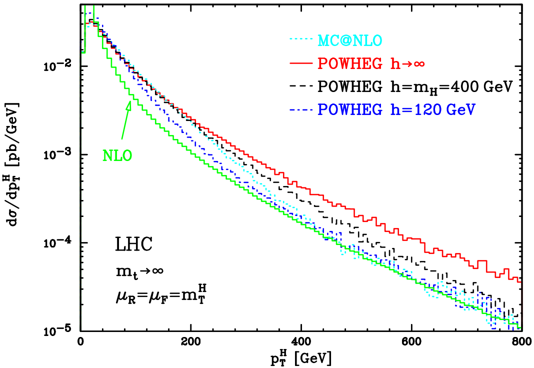
SLAC

- To avoid problems in high-$p_T$ region, split real-emission ME into singular and finite parts as $\mathrm{R} = \mathrm{R}^s + \mathrm{R}^f$
- Treat singular piece in $\mathbb{S}$-events and finite piece in $\mathbb{H}$-events
Similar to MC@NLO with redefined PS evolution kernels
- Differential event rate up to first emission

$$\langle O\rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R^s})}(\Phi_B)\bigg[\Delta^{(\mathrm{R^s})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}^s(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R^s})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R)\bigg] + \int \mathrm{d}\Phi_R\, \mathrm{R}_n^f(\Phi_R)$$

# POWHEG – Features

[Alioli,Nason,Oleari,Re] arXiv:0812.0578




- Singular real-emission part here defined as

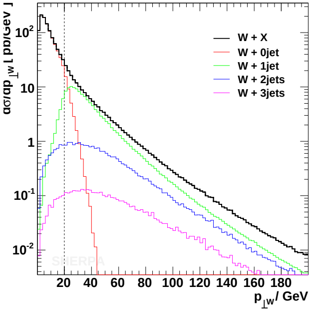
$$\mathrm{R}^s = \mathrm{R}\,\frac{h^2}{p_T^2 + h^2}$$

- Can "tune" NNLO contribution by varying free parameter $h$

# Multi-jet merging

# Basic idea of merging

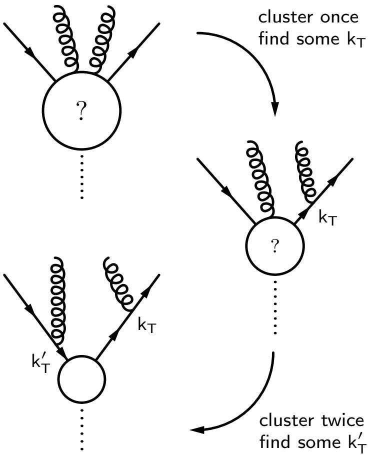
- Separate phase space into "hard" and "soft" region
- Parton shower populates soft domain
- N$^x$LO real corrections replace PS emission term in hard domain
- Need criterion to define "hard" & "soft" $\rightarrow$ jet measure $Q$ and corresponding cut, $Q_{\mathrm{cut}}$



# Parton shower histories

- Start with some "core" process for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive It sets the resummation scale $\mu_Q^2$
- Higher-multiplicity ME can be reduced to core by clustering
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until core process reached

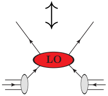
[André,Sjöstrand] hep-ph/9708390



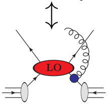
# Basic idea of merging

- MC@LO split into $Q < Q_{\rm cut}$ (PS) and $Q > Q_{\rm cut}$ (ME) region
  PS expression replaced by real-emission matrix-element in ME region

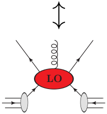
$$\langle O \rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B) \bigg[ \Delta^{(\mathrm{K})}(t_c, \mu_Q^2)\, O(\Phi_B)$$




$$+ \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t(\Phi_1), \mu_Q^2)\, \Theta(Q_{\rm cut} - Q)\, O(\Phi_R) \bigg]$$




$$+ \int \mathrm{d}\Phi_R \mathrm{R}(\Phi_R)\, \Delta^{(\mathrm{K})}(t(\Phi_R), \mu_Q^2)\, \Theta(Q - Q_{\rm cut})\, O(\Phi_R)$$




- Jet veto in PS / Jet cut on ME
- To match $\mathrm{K}(\phi_1)$, weight $\mathrm{R}(\phi_1)$ by $\alpha_s(k_T^2)/\alpha_s(\mu_R^2)$

# Truncated vetoed parton showers

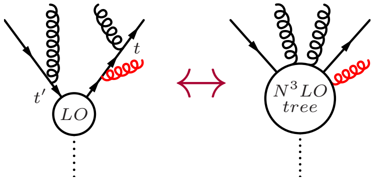
[Lönnblad] hep-ph/0112284

- In hard region $\Delta(t(\Phi_R), \mu_Q^2)$ is additional weight
- Most efficiently computed using pseudo-showers

  Recall PS no-emission probability: Constrained: $\Pi(x, t_2, \mu_Q^2)/\Pi(x, t_1, \mu_Q^2)$

  Unconstrained: $\Delta(t_2, \mu_Q^2)/\Delta(t_1, \mu_Q^2)$

- Start PS from core process
- Evolve until predefined branching
  $\leftrightarrow$ truncated parton shower
- Emissions that would produce additional hard jets lead to event rejection (veto)



# Truncated unvetoed parton showers

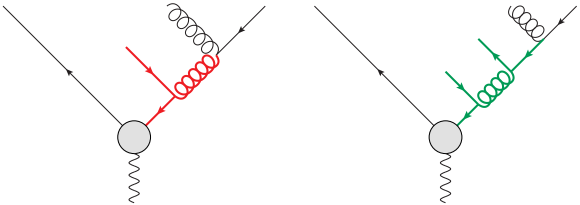
[Nason] hep-ph/0409146

- For $t \neq Q$, PS may generate emissions between $\mu_Q^2$ and $t(\Phi_R)$, as

$$\Delta(t, \mu_Q^2) = \Delta(t, \mu_Q^2; > Q_{\rm cut})\, \Delta(t, \mu_Q^2; < Q_{\rm cut})$$

$$\Delta(t, \mu_Q^2; > Q_{\rm cut}) = \exp\left\{ -\int_t^{\mu_Q^2} {\rm d}\Phi_1\, K(\Phi_1)\, \Theta(Q - Q_{cut}) \right\}$$

- Momentum and flavor conserving implementation non-trivial
  Example: Two emissions may be allowed, while one may be not



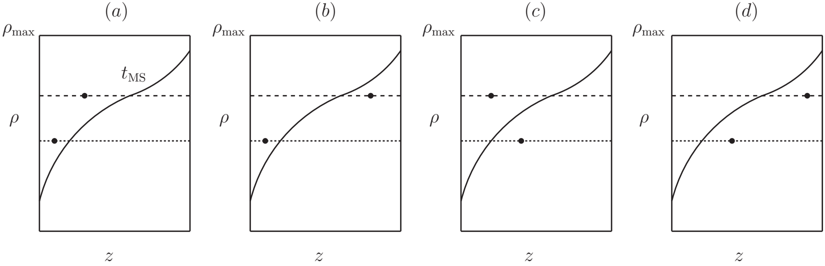
- Effects of non-trivial terms formally suppressed
  Better algorithm may be easier to implement

## Evading truncated unvetoed parton showers

SLAC

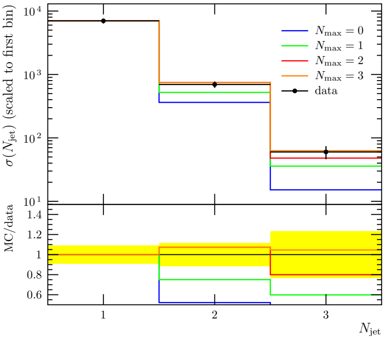
[Lönnblad] hep-ph/0112284

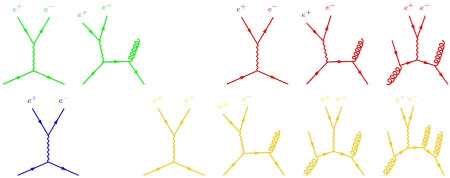
- Generate truncated unvetoed configurations with parton shower
  effective redefinition of $Q$, assuming PS ordering parameter $\sim$ "hardness"
- Schematic illustration of phase space coverage




from 2-jet ME | from 0-jet ME | from 1-jet ME | from 0-jet ME

- Straightforward implementation, no reshuffling of kinematics or flavor

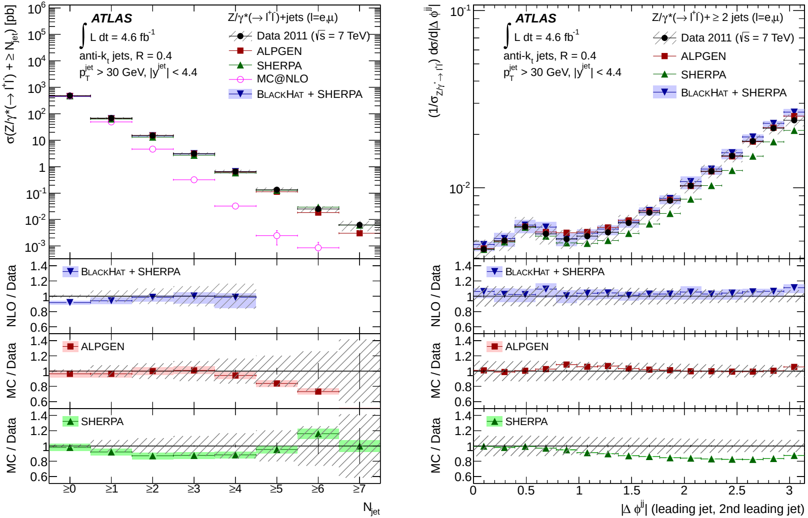


- MC predictions for exclusive $n$-jet rates match data well as long as corresponding final states are described by matrix elements

# Effects of merging - $Z$+jets at the LHC

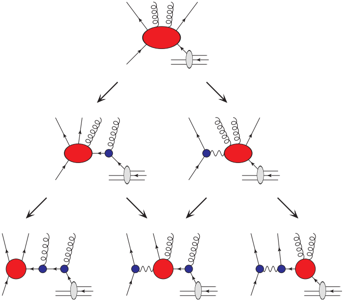
[ATLAS] arXiv:1304.7098

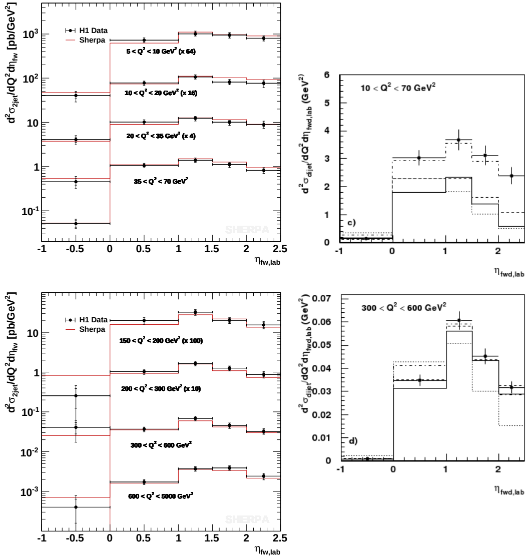


[Carli,Gehrmann,SH] arXiv:0912.3715

Simulation often too focused on resonant contributions

Need be inclusive to describe DIS, low-mass Drell-Yan or photon / diphoton production





# Unitarization

SLAC

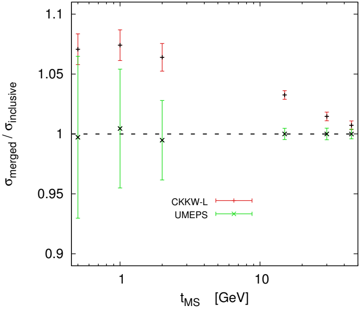
[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467
[Bellm,Gieseke,Plätzer] arXiv:1705.06700

- Unitarity condition of PS:

$$1 = \Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t)$$

- ME+PS(@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$\mathrm{K}(\Phi_1) \to \frac{\mathrm{R}(\Phi_1,\Phi_B)}{\mathrm{B}(\Phi_B)}$$



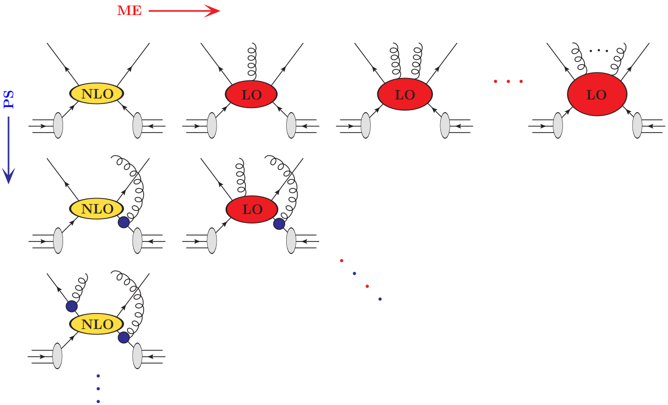

- Can be corrected by explicit subtraction

$$1 = \underbrace{\left\{\Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1) - \frac{\mathrm{R}(\Phi_1,\Phi_B)}{\mathrm{B}(\Phi_B)}\right]\Theta(Q-Q_{\mathrm{cut}})\,\Delta^{(\mathrm{K})}(t)\right\}}_{\text{unresolved emission / virtual correction}}$$

$$+ \underbrace{\int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1)\Theta(Q_{\mathrm{cut}}-Q) + \frac{\mathrm{R}(\Phi_1,\Phi_B)}{\mathrm{B}(\Phi_B)}\Theta(Q-Q_{\mathrm{cut}})\right]\Delta^{(\mathrm{K})}(t)}_{\text{resolved emission}}$$

# Combining Matching and Merging
# NLO Merging

# Combining Matching and Merging

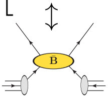


# Combined matching and merging with POWHEG

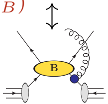
[Hamilton,Nason] arXiv:1004.1764
[Krauss,Schönherr,Siegert,SH] arXiv:1009.1127

- Increase accuracy below $Q_{\rm cut}$ to full NLO

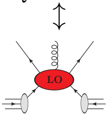
$$\langle O\rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B)\Big[\,\Delta^{(\mathrm{R})}(t_c, s_{\rm had})\, O(\Phi_B)$$




$$+ \int_{t_c}^{s_{\rm had}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\rm had})\Theta(Q_{\rm cut} - Q)\, O(\Phi_R)\Big]$$




$$+ \int \mathrm{d}\Phi_R\, \mathrm{k}^{(\mathrm{R})}(\Phi_R)\mathrm{R}(\Phi_R)\, \Delta^{(\mathrm{K})}(t(\Phi_R), \mu_Q^2)\, \Theta(Q - Q_{\rm cut})\, O(\Phi_R)$$

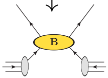



- Local $K$-factor for smooth merging

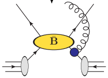
# Combined matching and merging with MC@NLO

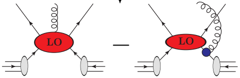
- Increase accuracy below $Q_{\text{cut}}$ to full NLO

$$\langle O\rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\Bigg[\Delta^{(\mathrm{K})}(t_c,\mu_Q^2)\,O(\Phi_B)$$

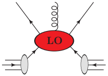



$$+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\Delta^{(\mathrm{K})}(t,\mu_Q^2)\Theta(Q_{\text{cut}}-Q)\,O(\Phi_R)\Bigg] + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\Theta(Q_{\text{cut}}-Q)O(\Phi_R)$$



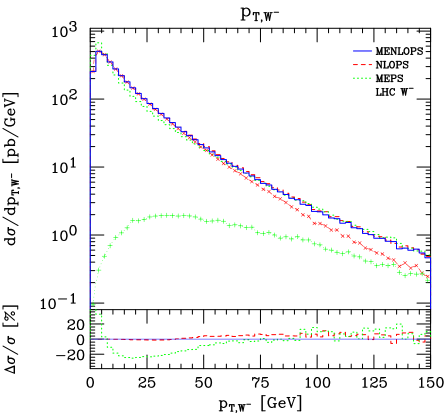

$$+\int \mathrm{d}\Phi_R\,\mathrm{k}^{(\mathrm{K})}(\Phi_R)\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t,\mu_Q^2;>Q_{\text{cut}})\,\Theta(Q-Q_{\text{cut}})\,O(\Phi_R)$$



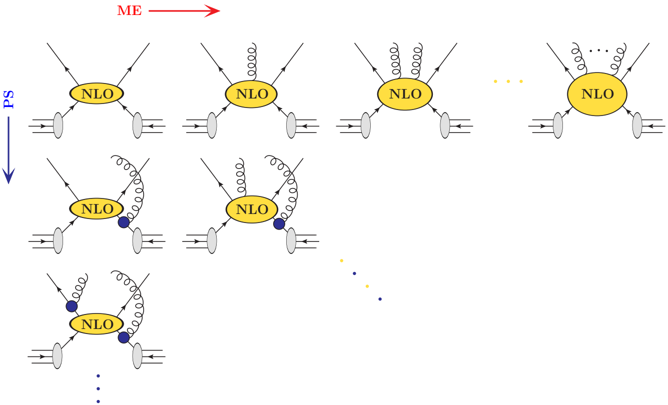

- Local $K$-factor for smooth merging

# Combining matching and merging

[Hamilton,Nason] arXiv:1004.1764



# Merging of multiple matched calculations



- ME+PS merging for 0+1-jet in MC@NLO notation

$$\langle O\rangle = \int d\Phi_B \mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B) + \int_{t_c} d\Phi_1 \mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t)\,\Theta(Q_{\mathrm{cut}} - Q)\,O(\Phi_R)\right]$$
$$+ \int d\Phi_R\,\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t(\Phi_R),\mu_Q^2)\,\Theta(Q - Q_{\mathrm{cut}})\,O(\Phi_R)$$

- Reorder by parton multiplicity $k$, change notation $\mathrm{R}_k \to \mathrm{B}_{k+1}$
- Analyze exclusive contribution from $k$ hard partons only $(t_0 = \mu_Q^2)$

$$\langle O\rangle_k^{\mathrm{excl}} = \int d\Phi_k\,\mathrm{B}_k \prod_{i=0}^{k-1} \Delta_i^{(\mathrm{K})}(t_{i+1}, t_i)\,\Theta(Q_k - Q_{\mathrm{cut}})$$
$$\times \left[\Delta_k^{(\mathrm{K})}(t_c, t_k)\,O_k + \int_{t_c}^{t_k} d\Phi_1\;\mathrm{K}_k\,\Delta_k^{(\mathrm{K})}(t_{k+1}, t_k)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})\,O_{k+1}\right]$$

# Merging of multiple matched calculations

[Lavesson,Lönnblad,Prestel] arXiv:0811.2912 arXiv:1211.7278
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031 arXiv:1207.5030
[Frederix,Frixione] arXiv:1209.6215

- Analyze exclusive contribution from $k$ hard partons

$$\langle O\rangle_k^{\rm excl} = \int {\rm d}\Phi_k\,\bar{\rm B}_k^{\rm (K)} \prod_{i=0}^{k-1}\Delta_i^{\rm (K)}(t_{i+1},t_i)\,\Theta(Q_k-Q_{\rm cut})$$
$$\times\left(1+\frac{\rm B}{\bar{\rm B}_k^{\rm (K)}}\sum_{i=0}^{k-1}\int_{t_{i+1}}^{t_i}{\rm d}\Phi_1{\rm K}_i\,\Theta(Q_i-Q_{\rm cut})+\dots\right)$$
$$\times\left[\Delta_k^{\rm (K)}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}{\rm d}\Phi_1\,{\rm K}_k\,\Delta_k^{\rm (K)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]$$
$$+\int{\rm d}\Phi_{k+1}\,{\rm H}_k^{\rm (K)}\,\Delta_k^{\rm (K)}(t_k,\mu_Q^2)\,\Theta(Q_k-Q_{\rm cut})\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}$$

- Born matrix element $\rightarrow$ NLO-weighted Born
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms from truncated vetoed PS

# A different perspective on NLO merging

SLAC

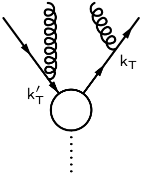
- Define compound evolution kernel

$$\tilde{\mathrm{K}}_k(\Phi_{k+1}) = \mathrm{K}_k(\Phi_{k+1})\,\Theta(t_k - t_{k+1}) + \sum_{i=n}^{k-1} \mathrm{K}_i(\Phi_i)\,\Theta(t_i - t_{k+1})\Theta(t_{k+1} - t_{i+1})$$

- Extend modified subtraction

$$\tilde{\mathrm{B}}_k^{(\mathrm{K})}(\Phi_k) = \Big[\mathrm{B}_k(\Phi_k) + \tilde{\mathrm{V}}_k(\Phi_k) + \mathrm{I}_k(\Phi_k)\Big] + \int \mathrm{d}\Phi_1 \Big[\mathrm{B}_k(\Phi_k)\tilde{\mathrm{K}}_k(\Phi_1) - \mathrm{S}_k(\Phi_{k+1})\Big]$$

$$\tilde{\mathrm{H}}_k^{(\mathrm{K})}(\Phi_{k+1}) = \mathrm{R}_k(\Phi_{k+1}) - \mathrm{B}_k(\Phi_k)\tilde{\mathrm{K}}_k(\Phi_1)$$

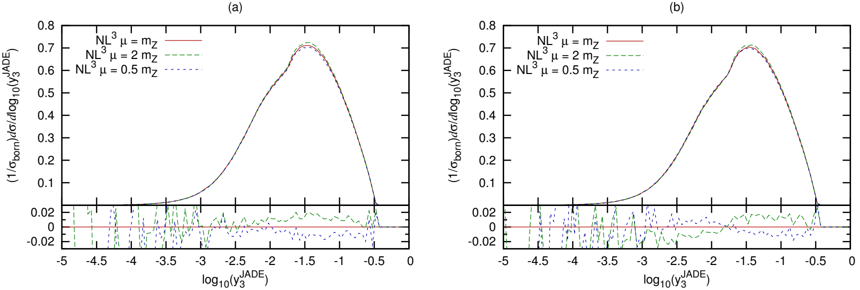



- Differential event rate for exclusive $n+k$-jet events

$$\langle O\rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\,\tilde{\mathrm{B}}_k^{(\mathrm{D})}\,\Theta(Q_k - Q_{\mathrm{cut}}) \times \Bigg[\tilde{\Delta}_k^{(\mathrm{K})}(t_c,\mu_Q^2)\,O_k + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\,\tilde{\mathrm{K}}_k\,\tilde{\Delta}_k^{(\mathrm{K})}(t,\mu_Q^2)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})\,O_{k+1}\Bigg] + \int \mathrm{d}\Phi_{k+1}\,\tilde{\mathrm{H}}_k^{(\mathrm{D})}\,\tilde{\Delta}_k^{(\mathrm{K})}(t_{k+1},\mu_Q^2)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})$$

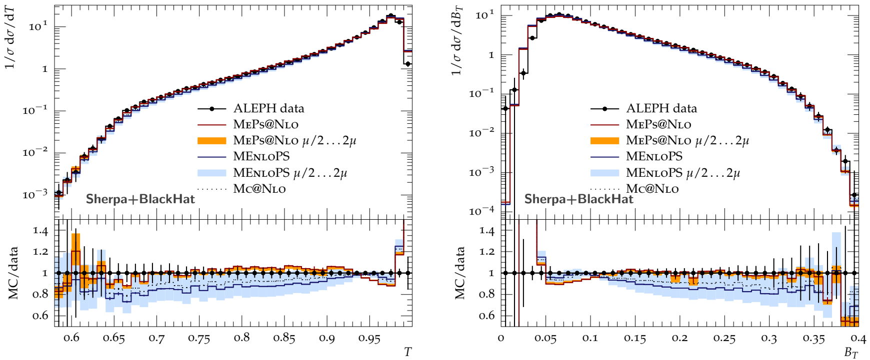
# $e^+e^-\to$hadrons at LEP

[Lavesson,Lönnblad] arXiv:0811.2912




- Scale variations around 2%
- Agreement between 1- and 2-loop
  but no further reduction of uncertainty

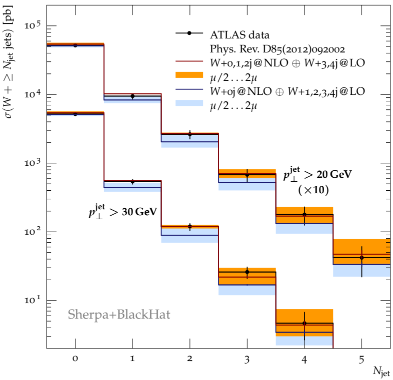
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031

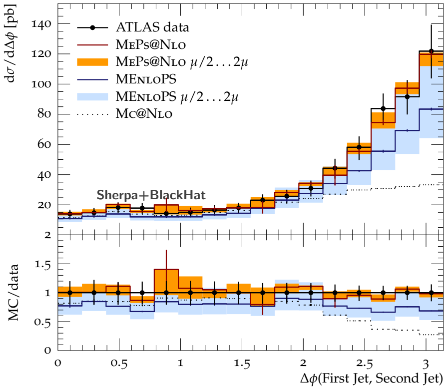



- Thrust and total jet broadening
- NLO merging of 2, 3 & 4 jets plus 5 & 6 jets at LO vs MC@NLO merged with up to 6 jets at LO

# $W$+jets production at the LHC

[ATLAS] arXiv:1201.1276
[Krauss,Schönherr,Siegert,SH] arXiv:1207.5030

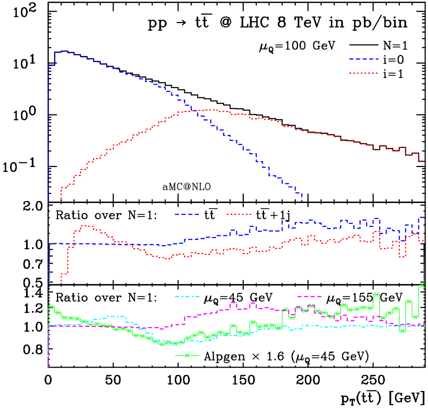


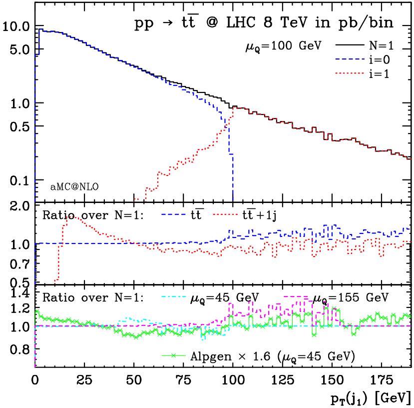




- NLO merging of 0, 1 & 2 jets plus 3 & 4 jets at LO vs MC@NLO merged with up to 4 jets at LO

# Top pair production at the LHC

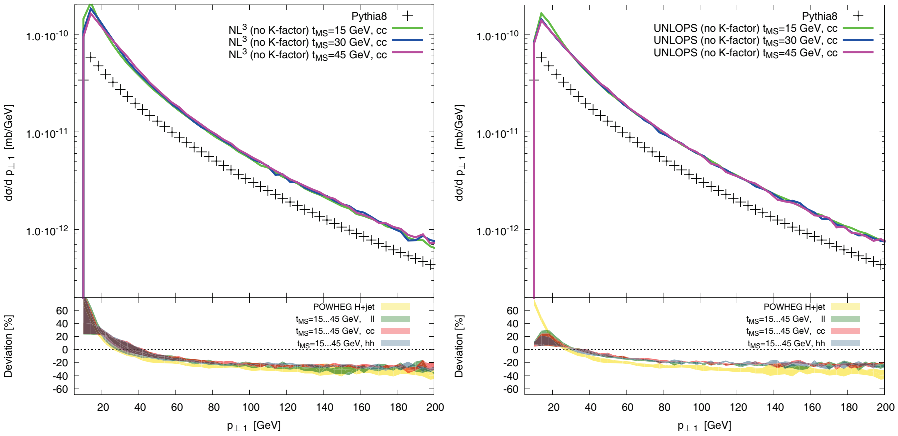
[Frederix,Frixione] JHEP12(2012)061



# Unitarized merging at NLO

[Lönnblad,Prestel] arXiv:1211.7278




- Effect on Higgs+jets production at the LHC

# Combining Matching and Merging
# NNLO Matching

# Unitary Matrix-Element Parton-Shower merging

SLAC

[Lönnblad,Prestel] arXiv:1211.4827

- PS expression for infrared safe observable, $O$

$$\langle O \rangle = \int \mathrm{d}\Phi_0 \, \mathrm{B}_0 \, \mathcal{F}_0(\mu_Q^2, O)$$

$$\mathcal{F}_n(t, O) = \Delta_n(t_c, t) \, O(\Phi_n) + \int_{t_c}^{t} \mathrm{d}\hat{\Phi}_1 \, \mathrm{K}_n \, \Delta_n(\hat{t}, t) \, \mathcal{F}_{n+1}(\hat{t}, O)$$

- Add ME correction to first emission $(\mathrm{B}_0\mathrm{K}_0 \to \mathrm{B}_1)$ & unitarize

$$+ \int_{t_c} \mathrm{d}\Phi_1 \, \Delta_0(t_1, \mu_Q^2) \, \mathrm{B}_1 \, \mathcal{F}_1(t_1, O) - \int_{t_c} \mathrm{d}\Phi_1 \, \Delta_0(t_1, \mu_Q^2) \, \mathrm{B}_1 \, O(\Phi_0)$$

- ME evaluated at fixed scales $\mu_{R/F}$ $\to$ need to adjust to PS

$$w_1 = \frac{\alpha_s(b\, t_1)}{\alpha_s(\mu_R^2)} \, \frac{f_a(x_a, t_1)}{f_a(x_a, \mu_F^2)} \, \frac{f_{a'}(x_{a'}, \mu_F^2)}{f_{a'}(x_{a'}, t_1)}$$

- Replace $\mathrm{B}_0$ by vetoed xs $\bar{\mathrm{B}}_0^{t_c} = \mathrm{B}_0 - \int_{t_c} \mathrm{d}\Phi_1 \mathrm{B}_1$

$$\langle O \rangle = \left\{ \int \mathrm{d}\Phi_0 \, \bar{\mathrm{B}}_0^{t_c} + \int_{t_c} \mathrm{d}\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) \, w_1 \right] \mathrm{B}_1 \right\} O(\Phi_0)$$

$$+ \int_{t_c} \mathrm{d}\Phi_1 \, \Delta_0(t_1, \mu_Q^2) \, w_1 \, \mathrm{B}_1 \, \mathcal{F}_1(t_1, O)$$

# Extension to NNLO

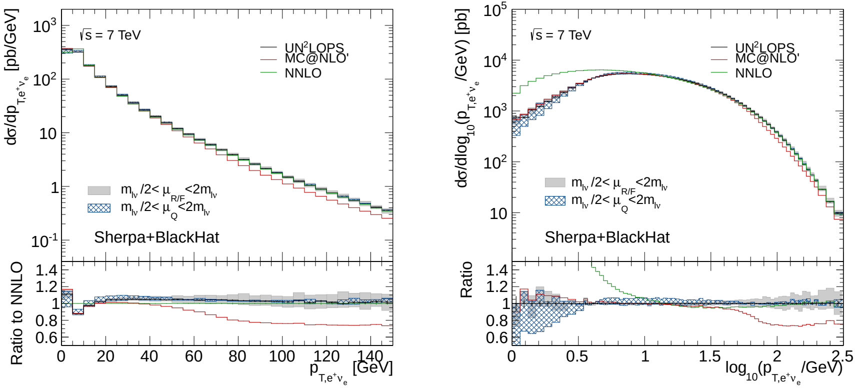
[Lönnblad,Prestel] arXiv:1211.7278
[Li,Prestel,SH] arXiv:1405.3607

- Promote vetoed cross section to NNLO
- Add NLO corrections to $\mathrm{B}_1$ using S-MC@NLO
- Subtract $\mathcal{O}(\alpha_s)$ term of $w_1$ and $\Delta_0$

$$
\begin{aligned}
\langle O\rangle = &\int \mathrm{d}\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{t_c}\, O(\Phi_0) \\
&+ \int_{t_c} \mathrm{d}\Phi_1 \left[1-\Delta_0(t_1,\mu_Q^2)\left(w_1 + w_1^{(1)} + \Delta_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1\, O(\Phi_0) \\
&+ \int_{t_c} \mathrm{d}\Phi_1\, \Delta_0(t_1,\mu_Q^2)\left(w_1 + w_1^{(1)} + \Delta_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1\, \bar{\mathcal{F}}_1(t_1,O) \\
&+ \int_{t_c} \mathrm{d}\Phi_1 \left[1-\Delta_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}\, O(\Phi_0) + \int_{t_c} \mathrm{d}\Phi_1 \Delta_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}\, \bar{\mathcal{F}}_1(t_1,O) \\
&+ \int_{t_c} \mathrm{d}\Phi_2 \left[1-\Delta_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}\, O(\Phi_0) + \int_{t_c} \mathrm{d}\Phi_2\, \Delta_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}\, \mathcal{F}_2(t_2,O) \\
&+ \int_{t_c} \mathrm{d}\Phi_2\, \mathrm{H}_1^{\mathrm{E}}\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

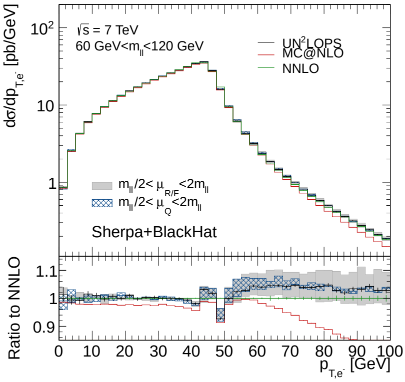
- $\tilde{\mathrm{B}}_1^{\mathrm{R}} = \bar{\mathrm{B}}_1 - \mathrm{B}_1 = \tilde{\mathrm{V}}_1 + \mathrm{I}_1 + \int \mathrm{d}\Phi_{+1}\mathrm{S}_1\Theta(t_2-t_1)$
  $\mathrm{H}_1^{\mathrm{R}}$ ($\mathrm{H}_1^{\mathrm{E}}$) $\rightarrow$ regular (exceptional) double real configurations

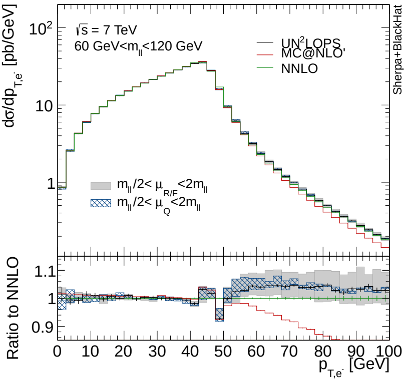
# Comparison with MC@NLO




- Good agreement at low $p_{T,W}$
- $W$+1-jet $K$-factor at high $p_{T,W}$

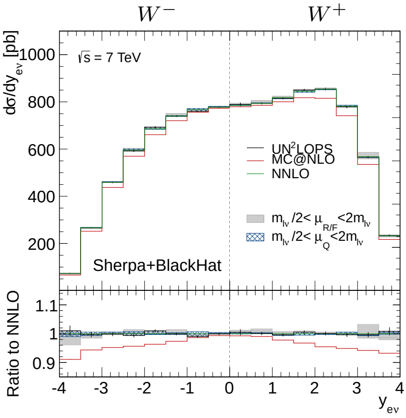
# Impact of PDFs

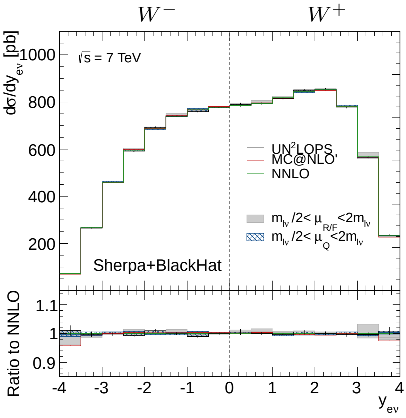



- MC@NLO with NLO PDFs
- MC@NLO with NNLO PDFs

## Impact of PDFs




- MC@NLO with NLO PDFs
- MC@NLO with NNLO PDFs

**Thank you for your attention**