Kinematical distribution of the Deck effect

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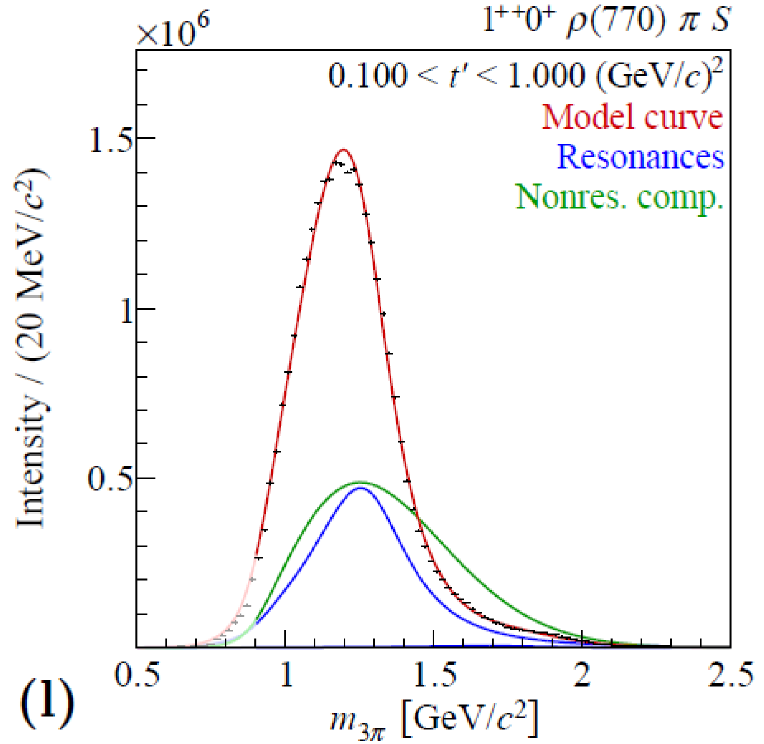
1-1. What is the problem ?

- COMPASS measured and analyzed $\pi^- p \to \pi^-\pi^-\pi^+ p$ reaction at CERN.
- They carried out mass-independent PWA and obtained the spin-density matrices at first.
- Then they fitted a resonance-model with the non-resonant amplitude:

$$D_j^{NR}(m_{3\pi}, t'; b, c_0, c_1, c_2) = \left[\frac{m_{3\pi} - m_{th}}{m_{norm}}\right]^b e^{-(c_0 + c_0 t' + c_0 t'^2)} \bar{q}^2_{\xi\pi}(m_{3\pi})$$

to the spin-density matrices.

- They found significant non-resonant contributions (again).

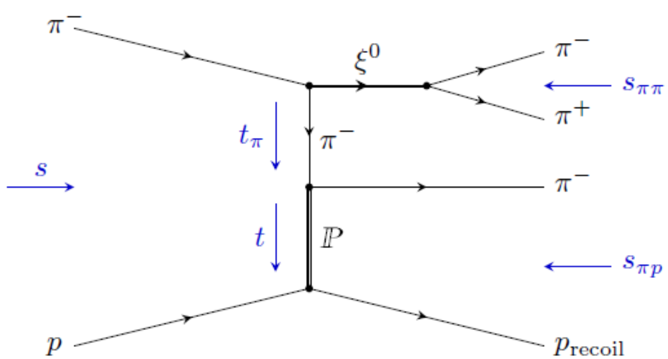


(1)

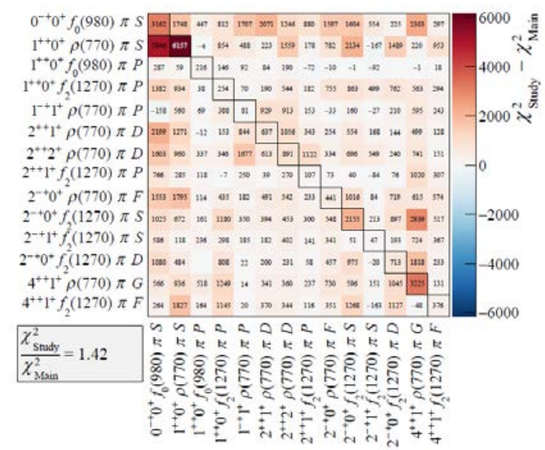
(From the Fig.30(l) of *"R. Akhunzyanov et al., "Light isovector resonances in $\pi^- p \to \pi^-\pi^-\pi^+ p$ at 190 GeV/c"*, arXiv:1802.05913[hep-ex]".)

1-2. What is the origin of the non-resonant contribution ?

- COMPASS also studied Deck-like processes as the non-resonant contributions as a check.
- They generated 10^8 Monte Carlo events of the Deck model, performed mass-independent PWA for them and obtained a non-resonant shape for each wave.
- They compared the effects of the Deck model shapes with their non-resonant parametrization.



Deck model
(From Fig. 8 of the COMPASS paper.)



Differences of χ^2 between the standard parametrization and the Deck model intensities.(From Fig. 9 of the COMPASS paper.)

2. Deck's model

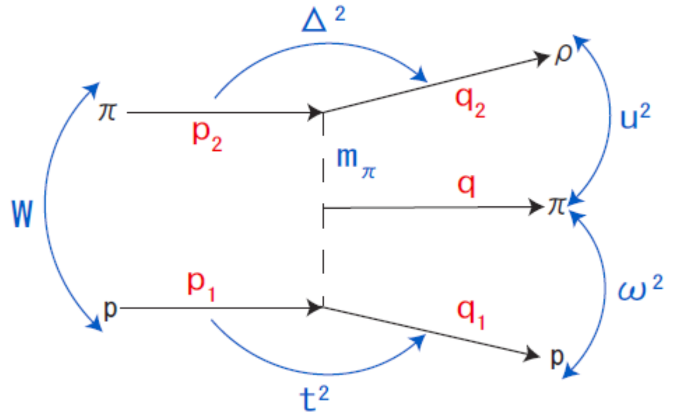
- Dr. Robert.T. Deck proposed a kinematical interpretation for the first resonance(=A_1) of the π-ρ mass spectrum in the π-p reaction in 1964. (R.T. Deck, *Phys. Rev. Lett.* 13(1964) 169–173.)
- He thought the peak was not a real resonance, but it was kinematically generated. And it might be expressed by a diffractive dissociation diagram, and the associated cross section was;

$$d\sigma = \frac{1}{(2\pi)^4}\left(\frac{g^2}{4\pi}\right)\frac{1}{16F_I}\frac{[\Delta^2-(m_2-m)^2][\Delta^2-(m_2+m)^2]}{m_2^2}\frac{F(\Delta^2)}{(\Delta^2-m^2)^2}|M_{\pi N'}|^2\delta^4(p_f-p_i)\frac{d\vec{q}_1}{q_{10}}\frac{d\vec{q}_2}{q_{20}}\frac{d\vec{q}}{q_0}.$$

- After some simplifications,

he obtained the following expression;

$$d\sigma = \frac{1}{(2\pi)^2}\left(\frac{g^2}{4\pi}\right)\frac{1}{F_I}\left(\frac{d\sigma}{d\Omega}\right)_0\frac{(m_2^2-4m^2)^2}{(\Delta^2-m^2)^2}$$
$$\times\,\omega^2\,e^{\lambda t^2}\delta^4(p_f-p_i)\frac{d\vec{q}_1}{q_{10}}\frac{d\vec{q}_2}{q_{20}}\frac{d\vec{q}}{q_0}.$$



(N.B. Variables are not same as Mandelstam variables)

Kinematical distribution of the Deck model

- In Deck's paper the 3π invariant mass distribution was sketched.
- But other kinematical distributions were not.
- When we do PWA, the other kinematical distributions are also important.
- I demonstrate other distributions following the Deck model.

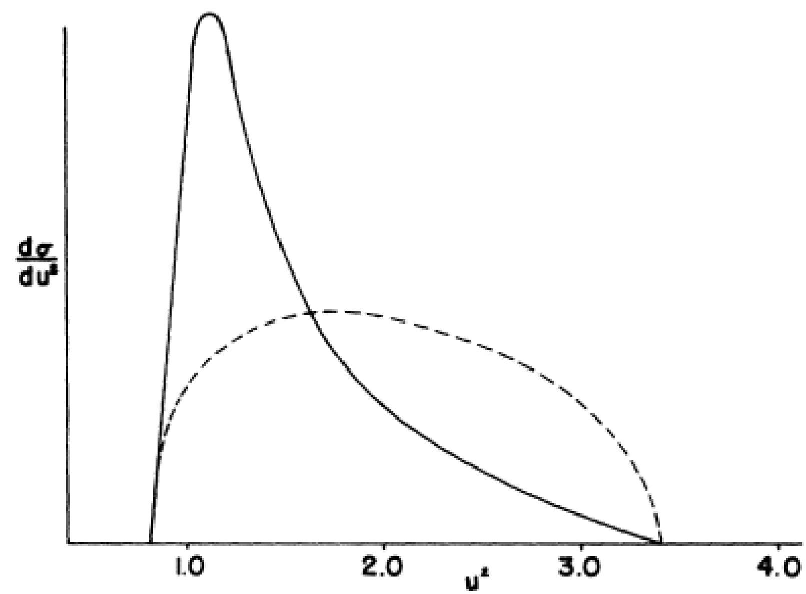


FIG. 2. Plot of the differential cross section obtained from the diagram of Fig. 1(a) as a function of the squared mass of the pi-rho system. The peak in the mass spectrum results from the assumption that the virtual exchanged pion is diffraction scattered from the nucleon. The dashed curve corresponds to phase space.

(Fig.3 of Deck's paper)

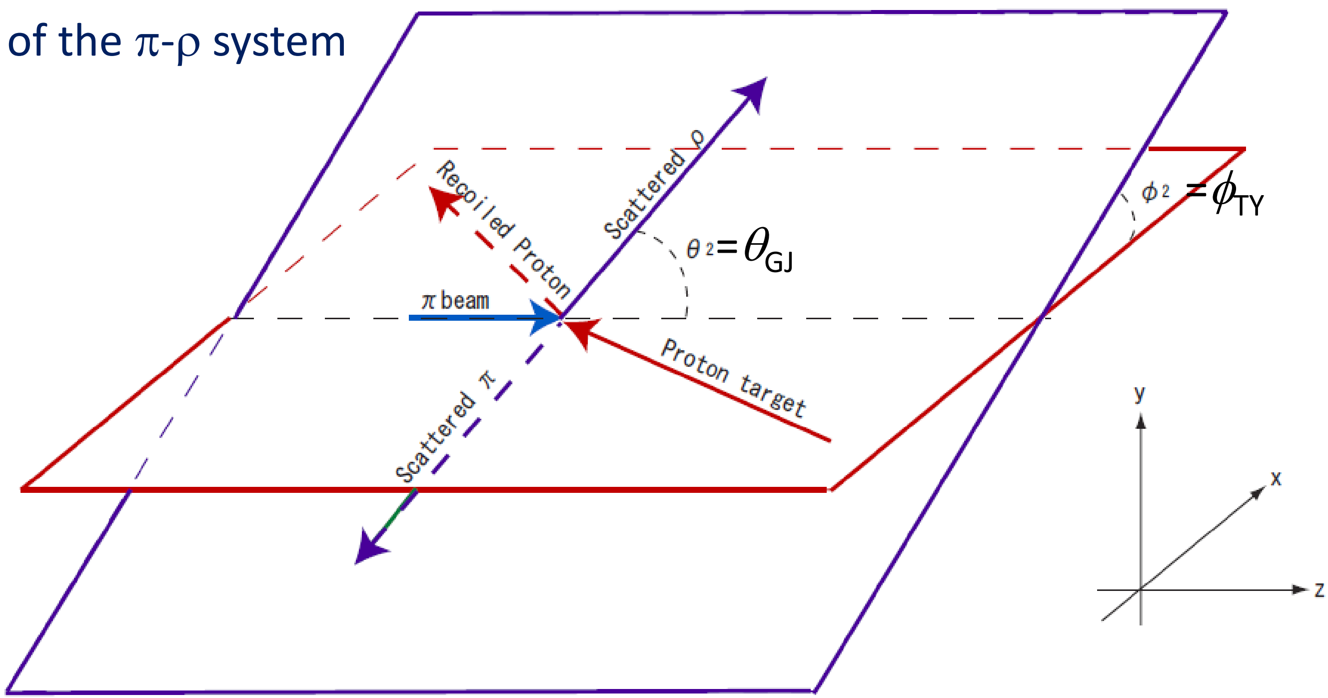
The basic of kinematic variables of the Deck model

$$d\sigma = 2\left(\frac{g^2}{4\pi}\right)\frac{\left(m_2^2-4m^2\right)^2}{4F_I}\left(\frac{d\sigma}{d\Omega}\right)_0 q\,\frac{\omega^2 e^{-\lambda(-t^2)}}{(\Delta^2-m^2)^2}d(\text{-}t^2)d\cos\theta_2\,\frac{d\varphi_2}{2\pi}\,du$$: differential cross section.

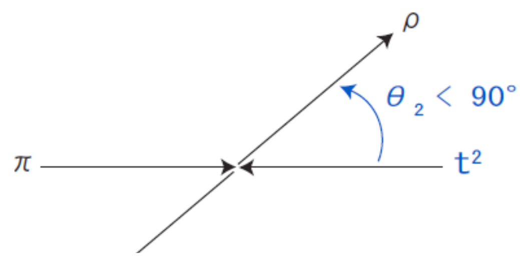
$$q = \frac{\sqrt{(u^2+m_2^2-m^2)^2-4m_2^2u^2}}{2u},\ \Delta^2 - m^2 = -a + b\cos\theta_2, \omega^2 = A - B\cos\theta_2 + C\sin\theta_2\cos\varphi_2$$

- The coefficients a, b, A, B and C are positive function of W^2, u^2, t^2, then all variables are expressed kinematically. (see the appendix of this presentation)
- λ is taken to be 6 (GeV/c)$^{-2}$ as Deck's paper.

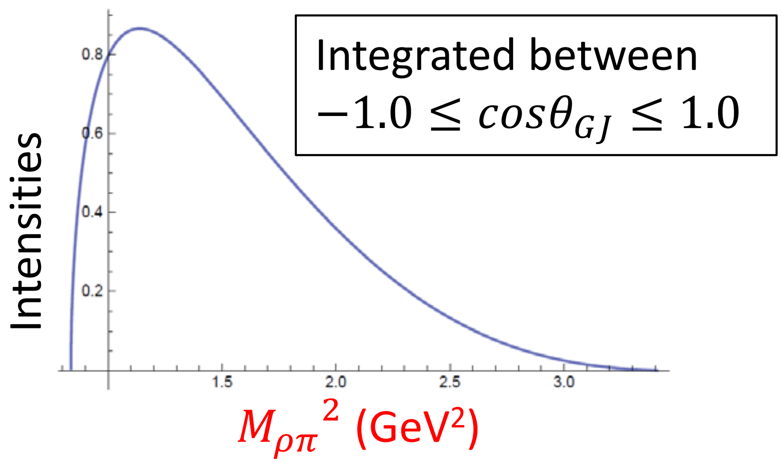
CM system of the π-ρ system

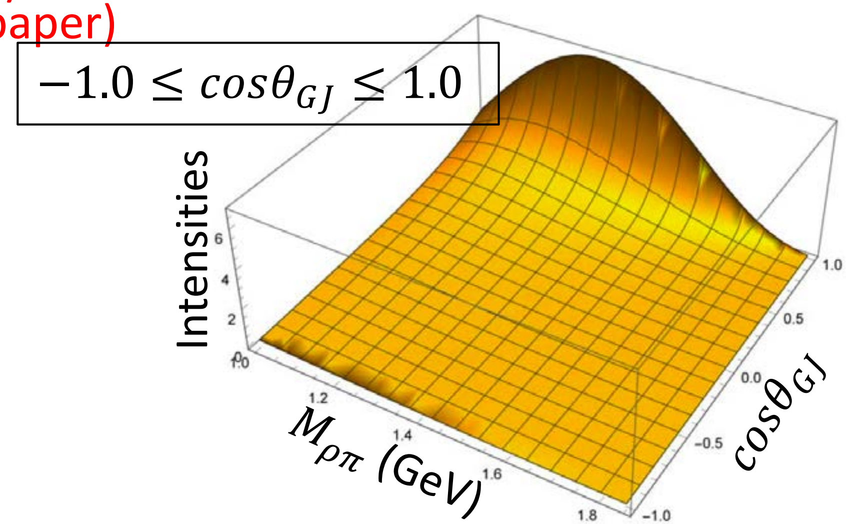


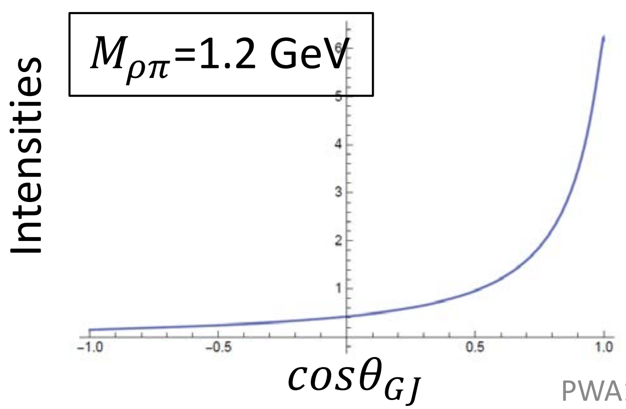
Kinematical distributions of the Deck model

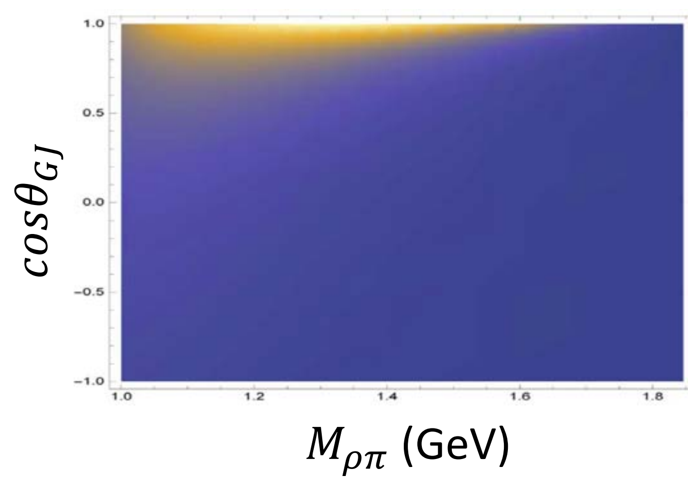


- Incident beam momentum = 3.65 GeV/c (the example beam energy of Deck's paper)
- $\pi^- p \rightarrow \pi^- \rho^0 p$
- All t^2 are integrated.



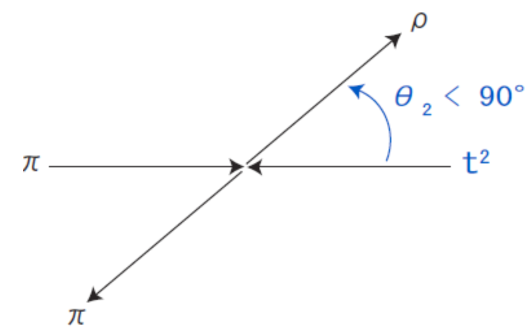




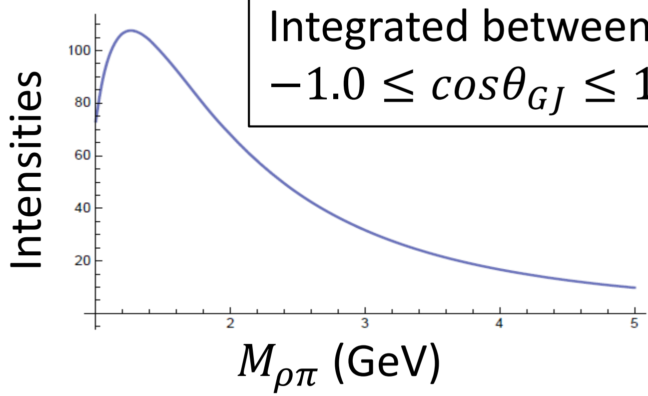


Kinematical distributions of the Deck model

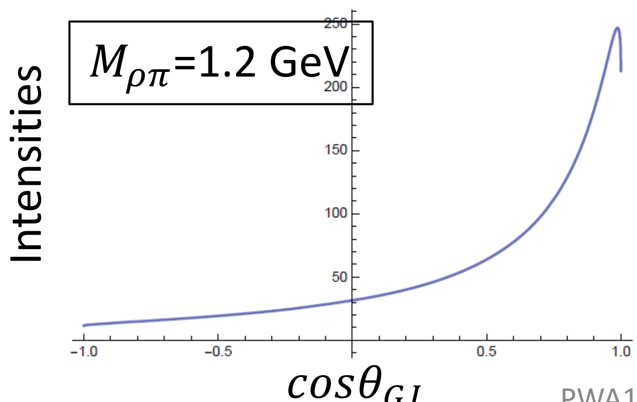
- Incident beam momentum = 190 GeV/c (COMPASS beam energy)
- $\pi^- p \rightarrow \pi^- \rho^0\, p$
- All t^2 are integrated.



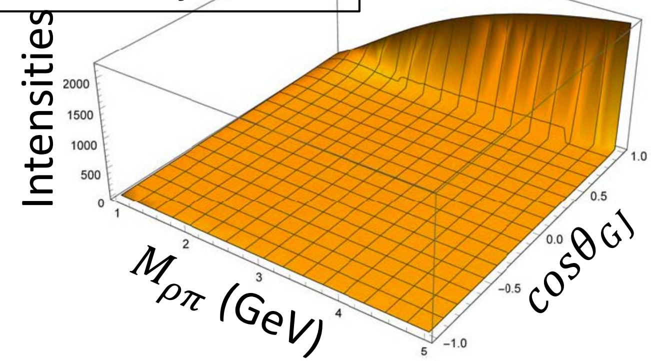
Integrated between $-1.0 \leq cos\theta_{GJ} \leq 1.0$



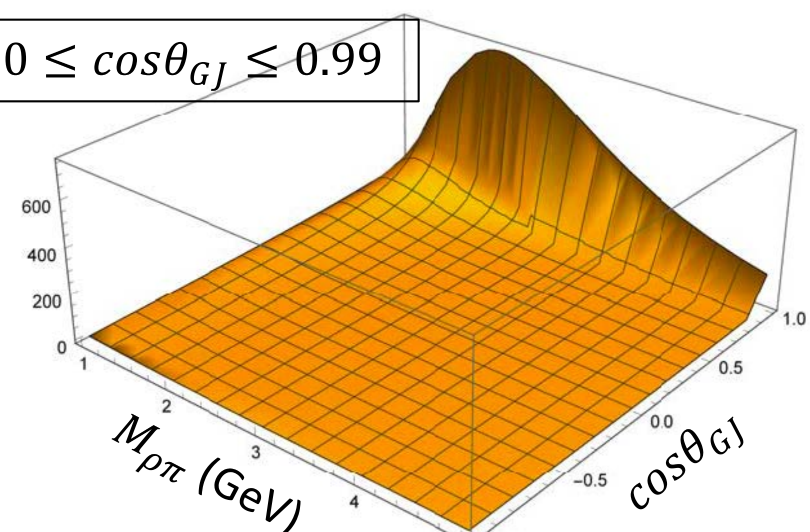
$M_{\rho\pi}$=1.2 GeV



$-1.0 \leq cos\theta_{GJ} \leq 1.0$

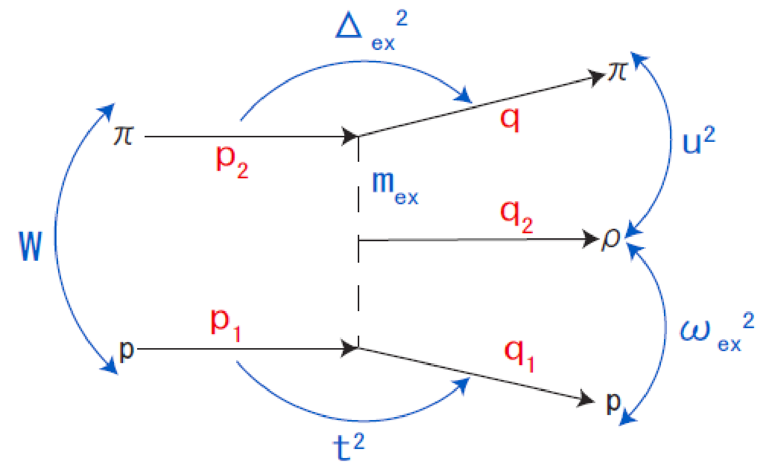


$-1.0 \leq cos\theta_{GJ} \leq 0.99$



The exchanged channel of Deck model

- In Deck's paper the exchanged reaction was also suggested, but it was not demonstrated.
- Then the variables of q and q_2 are exchanged in the previous equation.
- One ρ^0 exchange is assumed in place of one π exchange.



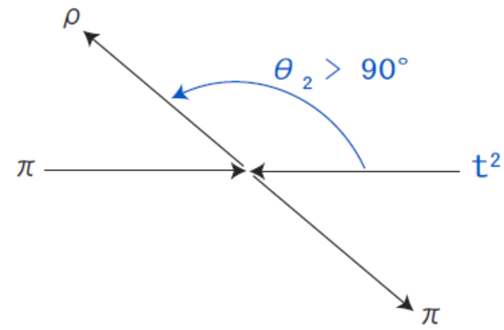
The following equations are used:

$$d\sigma = 2\left(\frac{g^2}{4\pi}\right)\frac{(m_2^2-4m^2)^2}{4F_I}\left(\frac{d\sigma}{d\Omega}\right)_0 q\,\frac{\omega_{ex}{}^2 e^{-\lambda(-t^2)}}{(\Delta_{ex}{}^2-m_{ex}{}^2)^2}d(\text{-}t^2)d\cos\theta_2\,\frac{d\varphi_2}{2\pi}du$$

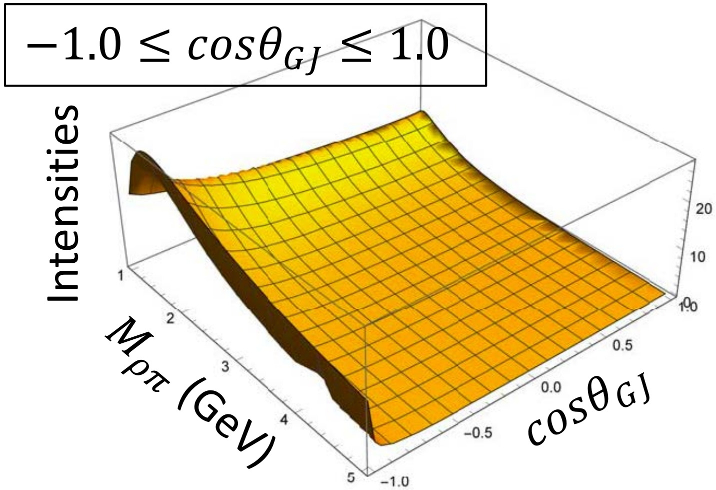
$$q = \frac{\sqrt{(u^2+m_2^2-m^2)^2-4m_2^2u^2}}{2u},\ \Delta_{ex}{}^2 - m_{ex}{}^2 = -a_{ex} + b_{ex}\cos\theta_2,$$

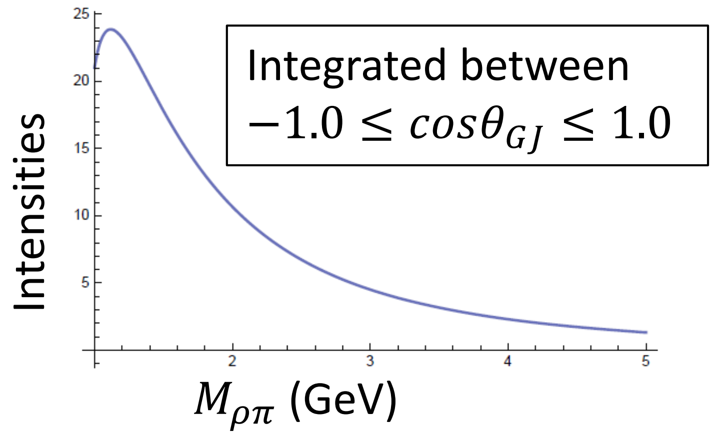
$$\omega_{ex}{}^2 = A_{ex} - B_{ex}\cos\theta_2 + C_{ex}\sin\theta_2\cos\varphi_2.$$

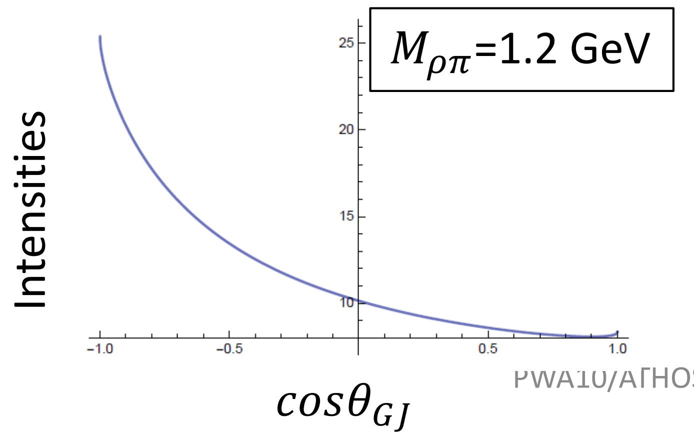
Kinematical distributions of the exchanged Deck model

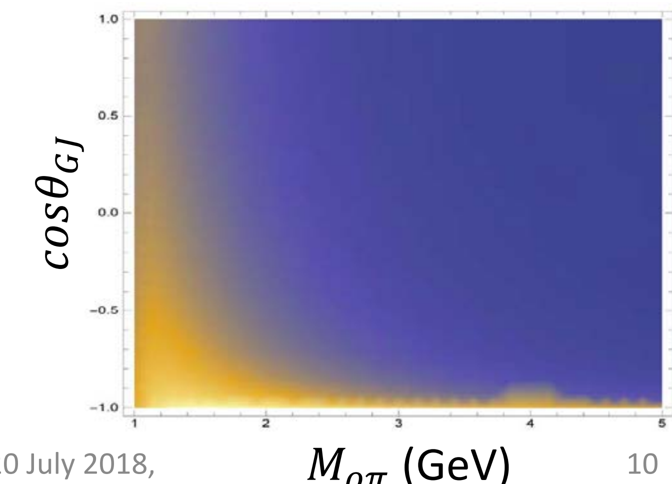


- Incident beam momentum = 190 GeV/c
- All t^2 are integrated too.
- ρ scatters backward.





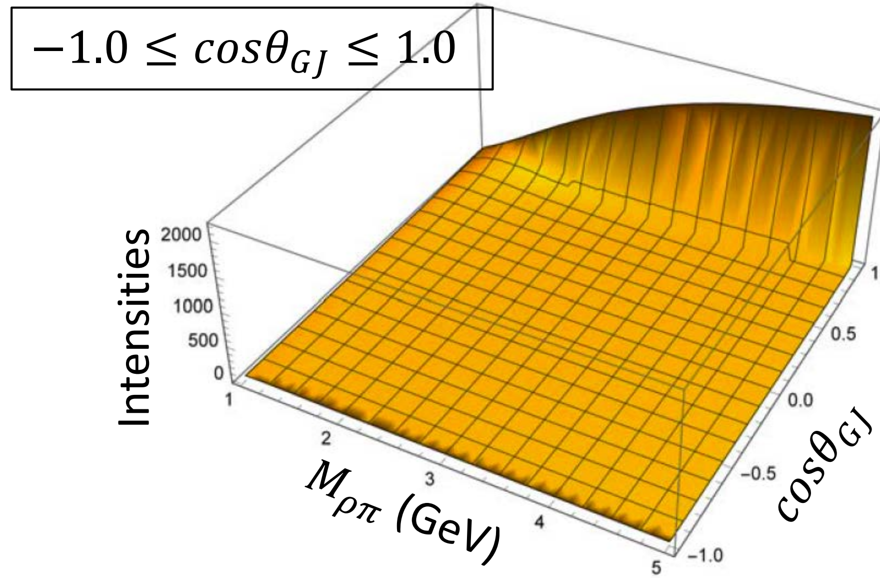


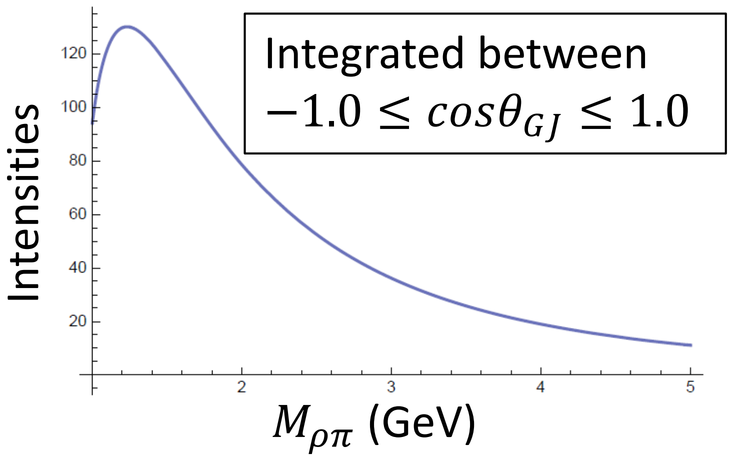


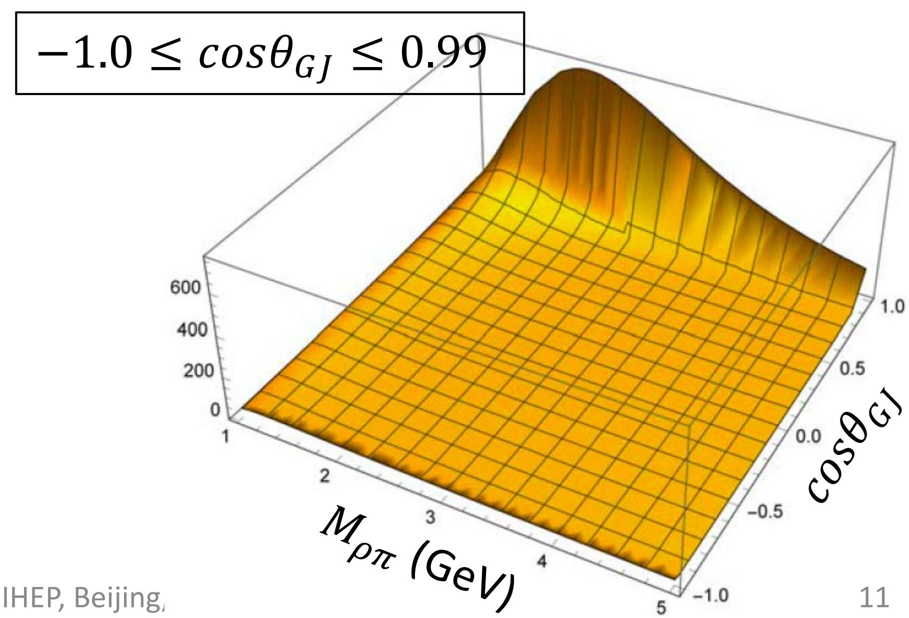
The combined distributions of both reactions

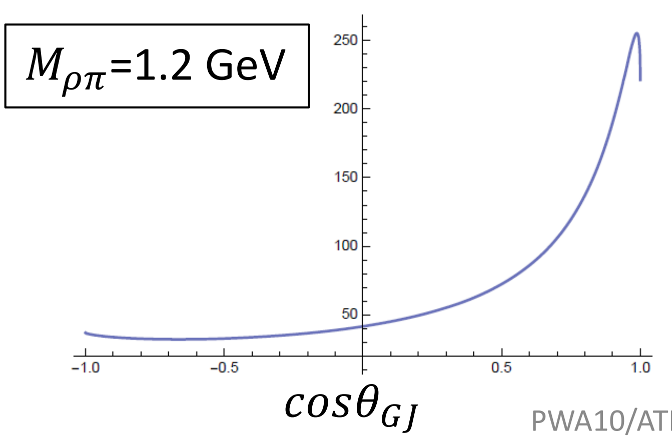
- Both previous distributions are summed.
- No adjustment between both reactions.
- The distribution depends on the exchange particle mass.

(π exchange for forward ρ scattering, ρ exchange for backward ρ scattering)





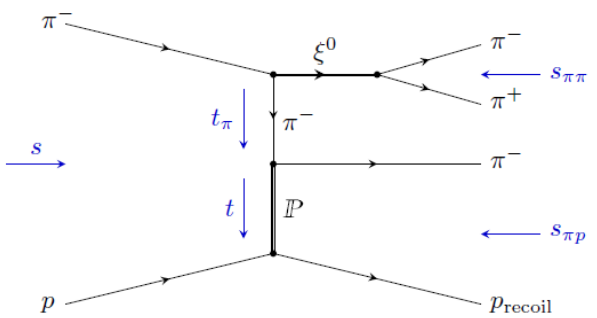




Kinematical distribution of the ACCMOR Deck model

- Deck-like reactions were studied at the ACCMOR and COMPASS experiments.
- The amplitude is $A\left(s_{\pi\pi}, s_{\pi p}, t_\pi, t\right) = A_{\pi\pi}(s_{\pi\pi})A_{\pi p}\left(s_{\pi p}, t\right)\frac{e^{-b_2(m_\pi^2 - t_\pi)}}{m_\pi^2 - t_\pi}$.
- $A_{\pi p}\left(s_{\pi p}, t\right) = i s_{\pi p}\sigma_{\pi p\to\pi p}e^{b_1 t}$
- $A_{\pi\pi}(s_{\pi\pi})$: elastic scattering amplitude, which includes $[\pi\pi]_s, \rho(770), f_0(980), f_2(1270)$, and $\rho_3(1690)$.
- In the above equations the variables correspond as

$t_\pi \to \Delta^2$, $s_{\pi p} \to \omega^2$ and $t \to t^2$ in Deck's paper's variables

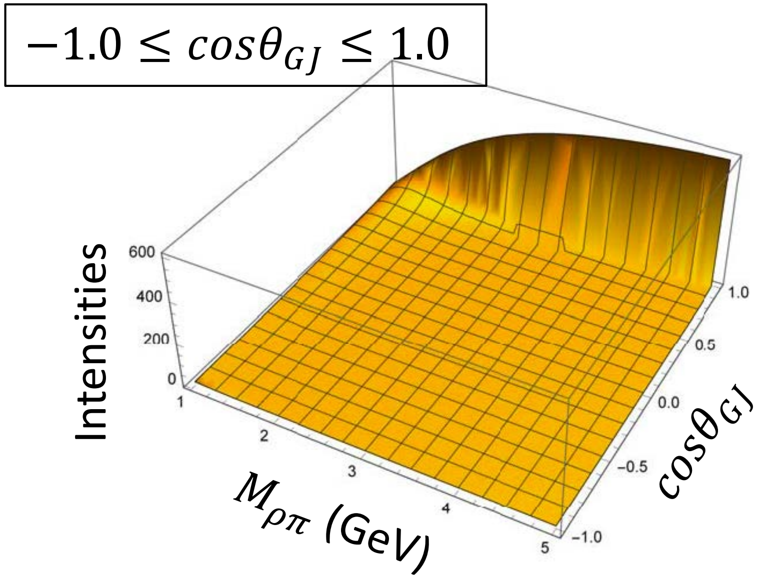


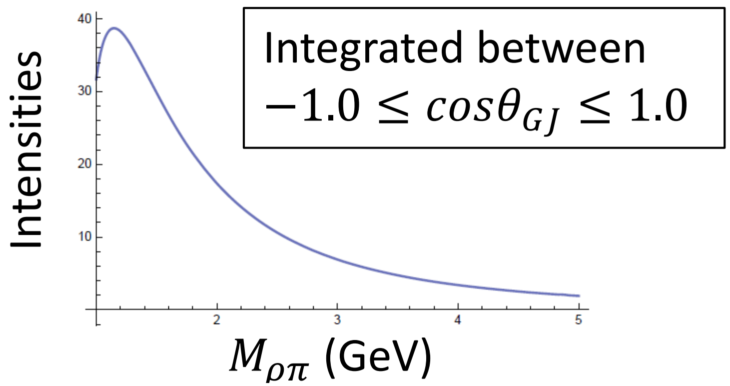
(From Fig. 8 of the COMPASS paper.)

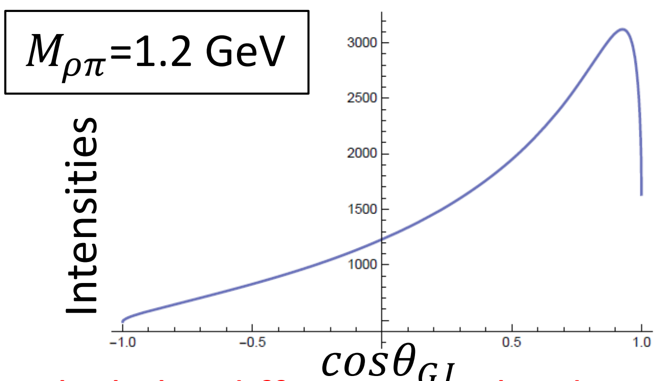
- I simplify as $\sqrt{s_{\pi\pi}} = m_\rho$ and $A_{\pi\pi}(s_{\pi\pi}) = 1$.
- The differences from the original Deck model are $e^{-b_2(m_\pi^2 - t_\pi)}$ and the number of powers of $s_{\pi p}$ except for overall constant.
- Therefore the ACCMOR Deck model set 1→ $e^{-b_2(m_\pi^2 - t_\pi)}$ and $\sqrt{s_{\pi p}} \to s_{\pi p}$.
- I take $b_1 = 8$(GeV/c)^2 and $b_2 = 0.45$(GeV/c)^2 as the COMPASS does.

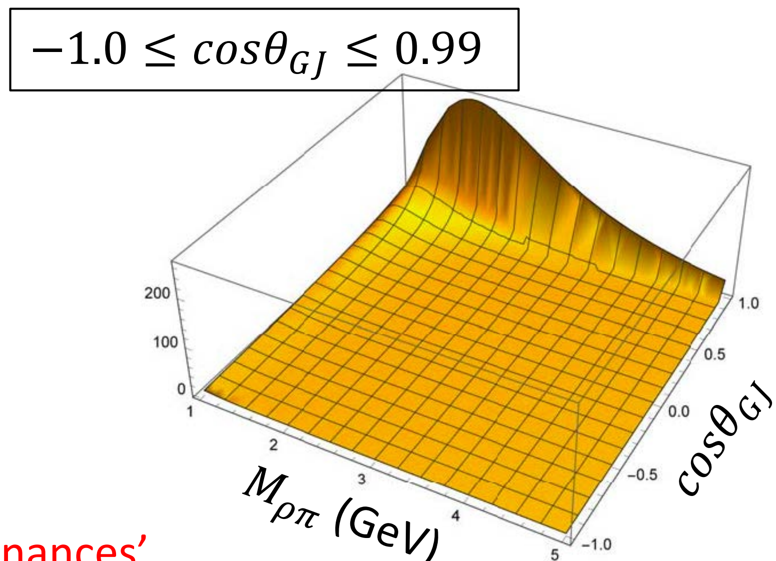
Kinematical distributions of the ACCMOR Deck model

- Incident beam momentum = 190 GeV/c
- $\pi^- p \to \pi^- \rho^0 p$
- All $t^2(t)$are integrated.
- The distribution is similar to the original Deck's one. (but slightly different)





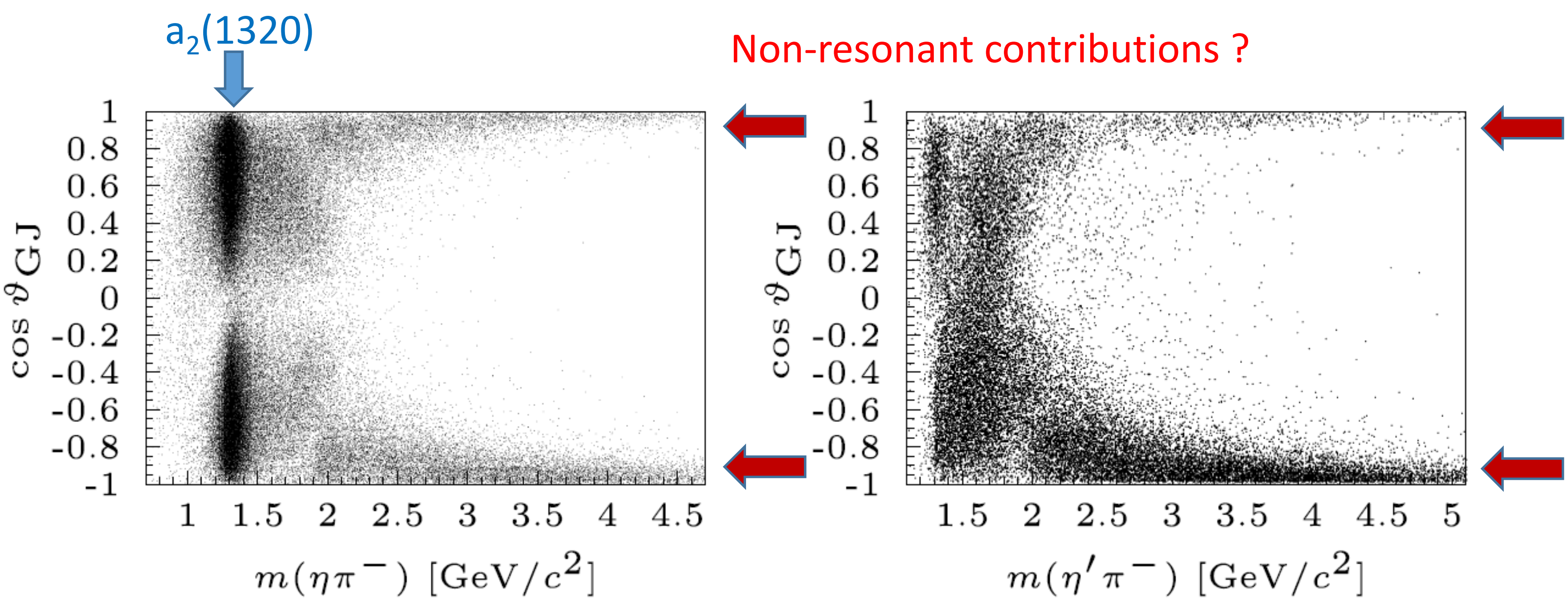




A little bit different angular distribution from resonances'

3. Non-resonant diffractive ηπ scattering

- COMPASS measured $\pi^- p \rightarrow \eta\pi^- p$ and $\pi^- p \rightarrow \eta'\pi^- p$ *at 191* GeV/c too.
- Non-resonant contributions seem to be seen clearly.
- These channels are suitable to study non-resonant reactions due to simple final states and less resonances.



(a) $m(\eta\pi^-)$ vs. $\cos\vartheta_{GJ}$ (b) $m(\eta'\pi^-)$ vs. $\cos\vartheta_{GJ}$

(COMPASS, Phys. Lett. B 740 (2015) 303-311.)

"Toy study" of non-resonant contributions

- Study the non-resonant $\pi^- p \rightarrow \eta\pi^- p$ reaction.
- The same equation of the Deck model is assumed.
- The scattered ρ is replace by η.
- The exchanged meson is ;

"ρ", a_0 or a_2

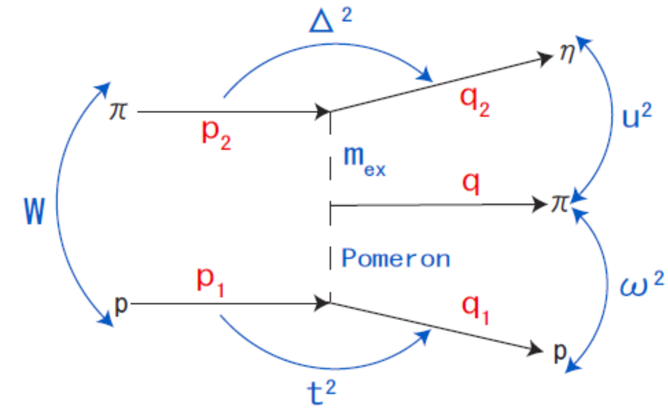
for the forward η scattering,

"ρ", f_0 or f_2

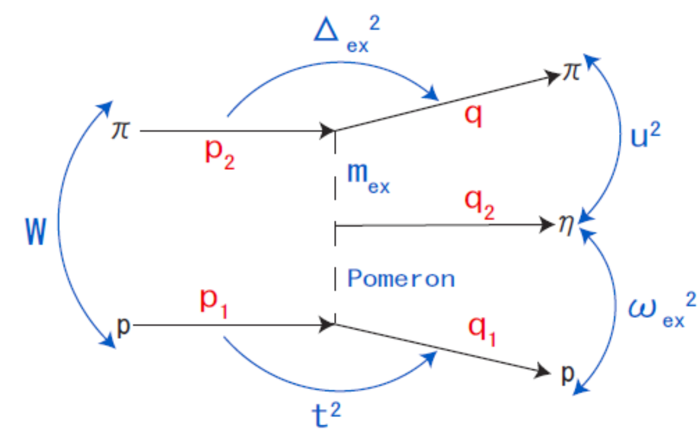
for the backward η scattering.

(Only mass is taken into account. No spin effects are considered.)

η forward scattering



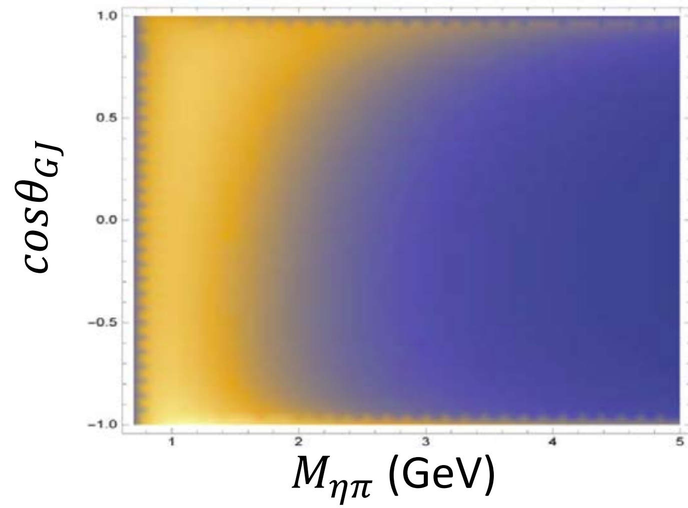
η backward scattering



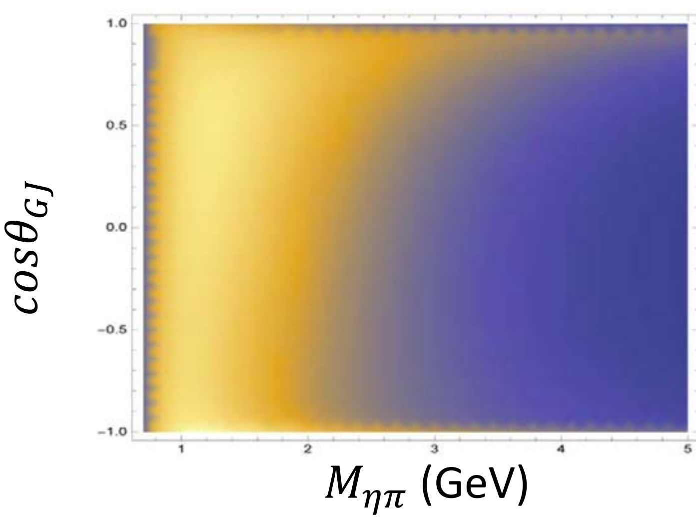
Kinematical distributions of the Deck model of ηπ scattering
(forward and backward distributions)

- $\pi^- p \rightarrow \eta\pi^- p$
- Incident beam momentum = 191 GeV/c.
- All t^2 distributions are integrated.
- Smaller mass meson exchange looks similar to the experimental result.

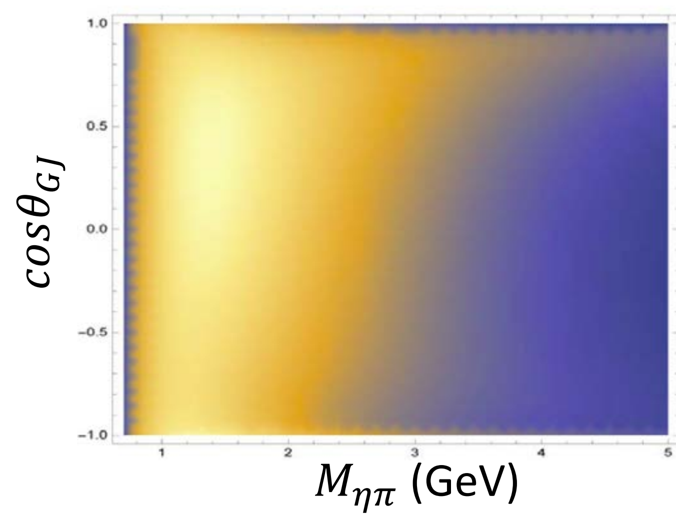
$\rho^0(770)$&$\rho^0(770)$ exchanged



$a_0(980)$&$f_0(980)$ exchanged



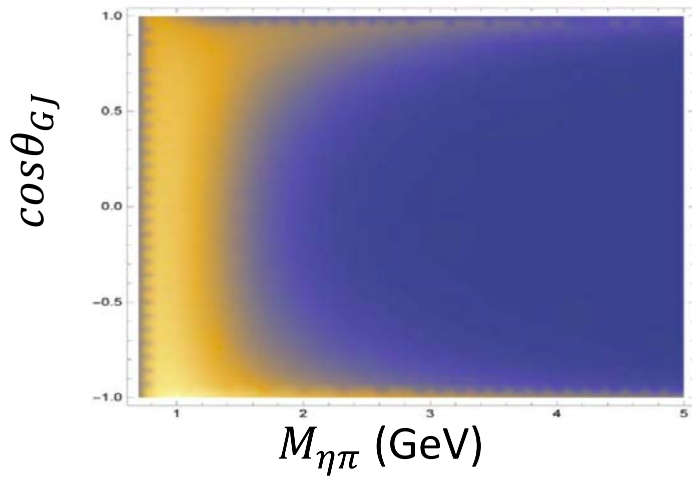
$a_2(1320)$&$f_2(1270)$ exchanged



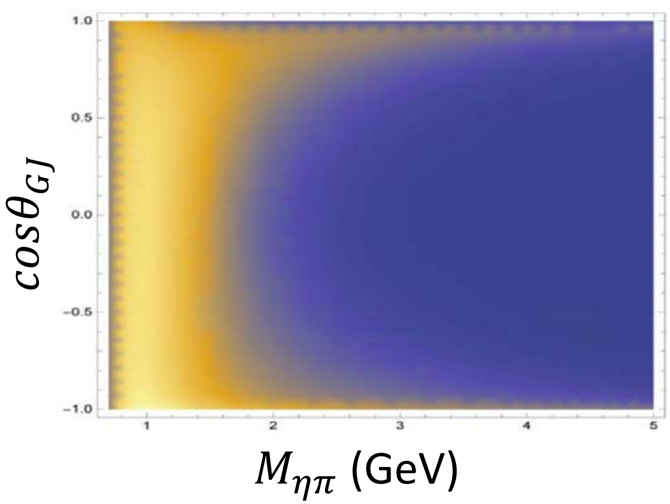
The modified ACCMOR Deck model

- Incident beam momentum = 191 GeV/c.
- Modified as $s_{\pi p} \rightarrow \sqrt{s_{\pi p}}$ from the ACCMOR Deck model. (similar to the original Deck's one)
- The term of $e^{-b_2(m_\pi^2 - t_\pi)}$ remains.
- "Higher mass exchanges" looks similar to the experimental distribution.

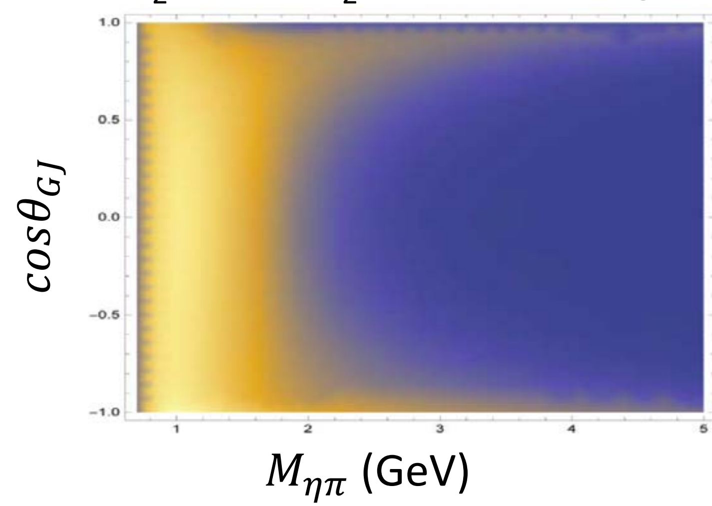
ρ^0(770)&ρ^0(770) exchanged



a_0(980)&f_0(980) exchanged

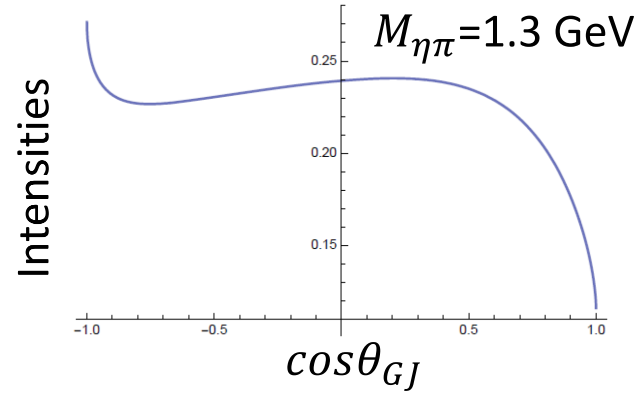


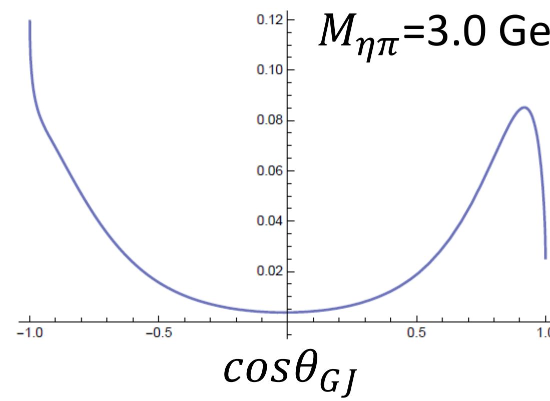
a_2(1320)&f_2(1270) exchanged

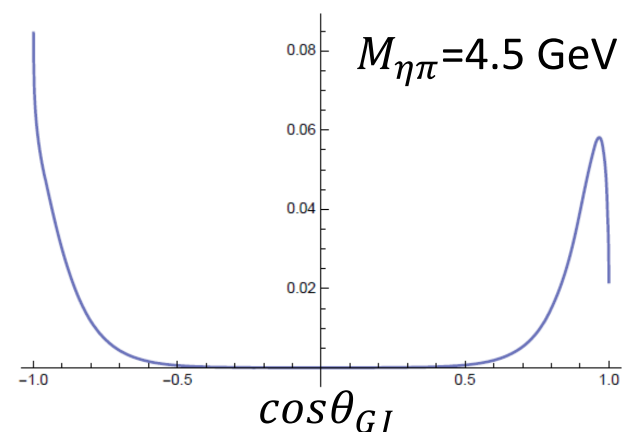


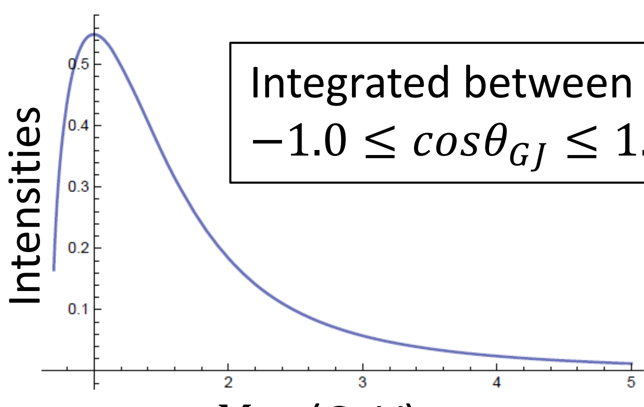
The distributions of the modified ACCMOR Deck model

- The angular distributions of the non-resonant Deck model are quite different from resonances' ones.
- If the mass-independent PWA is done for the non-resonant parts, the results behave unlike resonances. (very different at the higher invariant mass region, and similar at the lower mass region.)





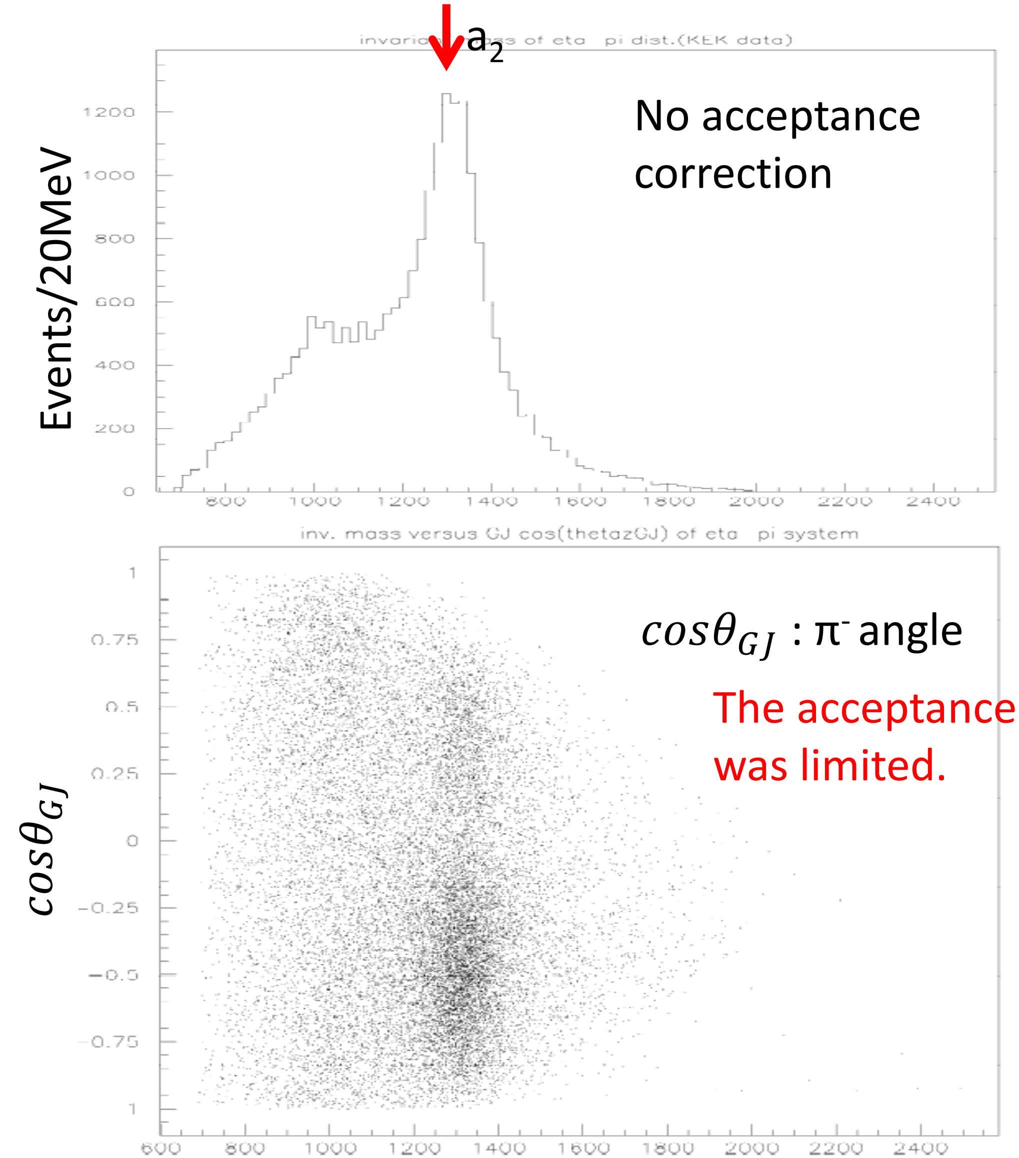




Integrated between
$-1.0 \leq cos\theta_{GJ} \leq 1.0$

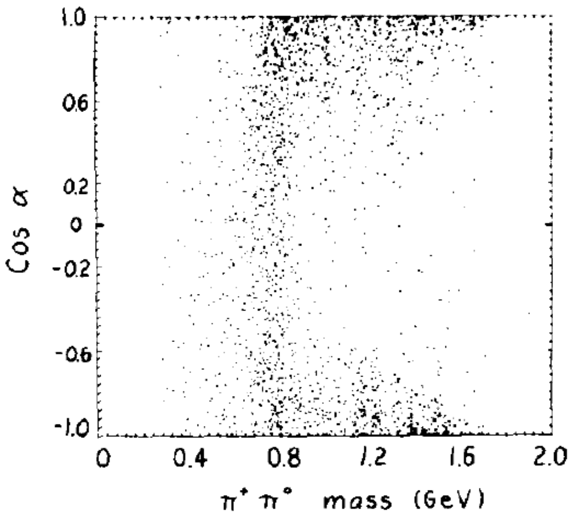
4. Non-resonant reactions at diffractive meson scatterings

- $\pi^- p \to \eta\pi^- p$ at 6.3 GeV/c.
- KEK 12 GeV PS E179 Experiment in 1991. (**Phys.Lett. B314 (1993) 246-254.**)
- We found strong forward-backward asymmetries.
- We insisted the existence of P-wave of the η-π system.
- But the acceptance was limited due to the low beam energy.
- There was no chance to find the non-resonant contribution by this experiment at that time.



4. Non-resonant reactions at diffractive meson scatterings

- The (partially) non-resonant reactions may often occur at diffractive meson scatterings, and their contributions may not be small. e.g. $\pi^+ p \to \pi^+\pi^0 p$ at 3.7 GeV/c, Nucl. Phys. 33B (1976) 75-81.
- The non-resonant kinematical distributions are quite different from (pure) resonant distributions.
- The standard mass-independent PWA may not be better for the reactions of which non-resonant contributions are significant.
- If the form of the non-resonant amplitudes would be known, it should consider before doing mass independent PWA, or should consider them at the same time.
- And the azimuthal angle ϕ_{TY} may give good information.



Summary and suggestion ?

- The kinematical distributions of the Deck model are demonstrated.
- The non-resonant kinematical distributions are sometimes similar to the resonance ones and at other times quite different, and non-resonant and resonant amplitudes will interfere with each other.
- The differences will emerge more clearly at higher masses.
- Higher beam momentum experiments will reveal non-resonant contributions clearly.
- It may be better to study non-resonant contributions before the mass independent PWA, because the mass independent PWA slices a continuous non-resonant distribution.
- The non-resonant meson scattering itself is very interesting too.
- The COMPASS $\pi^- p \to \eta^{(\prime)}\pi^- p$ data are suitable to study the non-resonant diffractive scattering physics.

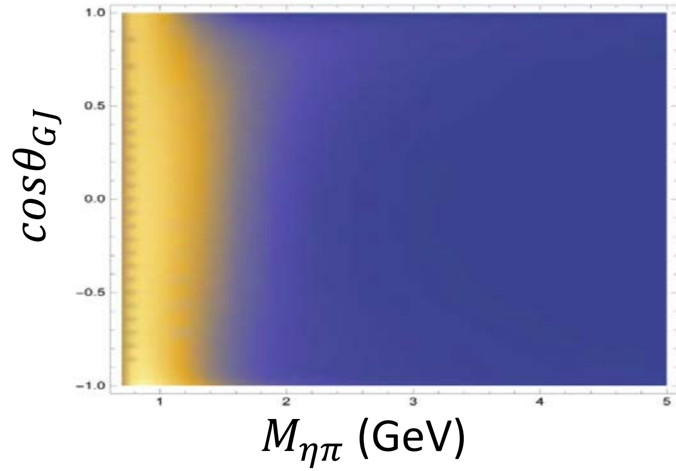
Appendix

- $a = -(m_2^2 - 2\sqrt{(q^2 + m_2^2)({p_2}^2 + m^2)})$
- $b = 2qp_2$
- $A = m^2 + M^2 + 2\sqrt{(q^2 + m^2)({q_1}^2 + M^2)}$
- $B = -2q_1q_2\cos\theta_1{}'$
- $C = 2q_1q_2\sin\theta_1{}'$
- $q = \frac{\sqrt{(u^2+m_2^2-m^2)^2-4m_2^2u^2}}{2u} = q_2$
- $p_2 = \frac{\sqrt{(u^2+m^2-t^2)^2-4m^2u^2}}{2u}$
- $p_1 = \frac{\sqrt{(W^2-m^2-M^2+t^2)^2-4M^2u^2}}{2u}$
- $q_1 = \frac{\sqrt{(W^2-u^2-M^2)^2-4M^2u^2}}{2u}$
- $p_1\cos\theta_1 = \frac{-W^2+m^2+M^2+2\sqrt{({p_2}^2+m^2)({p_1}^2+M^2)}}{2p_2}$
- $q_1\cos\theta_1' = p_1\cos\theta_1 + p_2$, $q_1\sin\theta_1' = q_1\sqrt{1 - cos^2\theta_1'}$

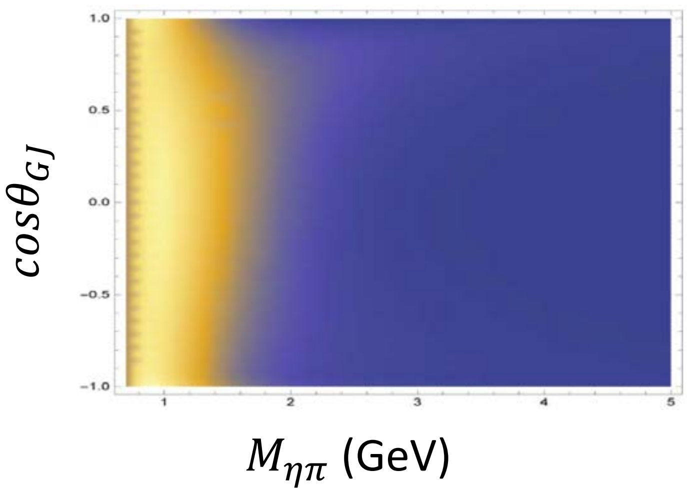
Kinematical distributions of the (not modified) ACCMOR Deck model of ηπ scattering (Combined distributions)

- Incident beam momentum = 191 GeV/c.
- The original Deck model is assumed.
- The exchange reactions are summed.
- The exchanging mesons are changed.
- All $t^2$are integrated.

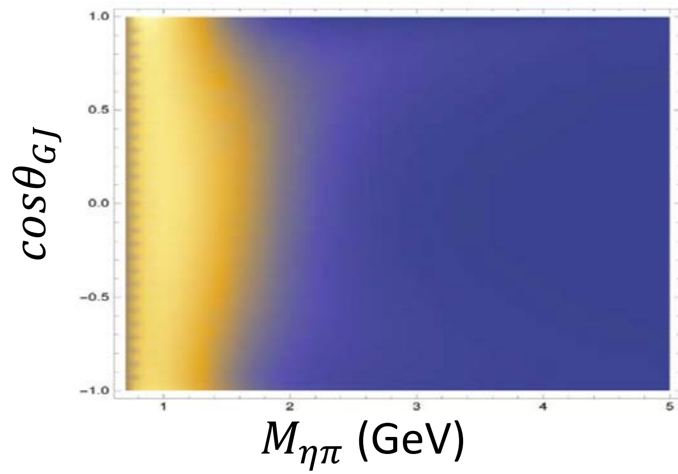
ρ^0(770)&ρ^0(770) exchanged



a_0(980)&f_0(980) exchanged



a_2(1320)&f_2(1270) exchanged



t^2 distribution of the Deck model