



Study of K_S^0 pair and $\eta_c(1S)$, $\eta_c(2S)$ and non-resonant $\eta'\pi\pi$ production in two-photon collisions at Belle

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On behalf of the Belle collaboration

IHEP & UCAS, Beijing

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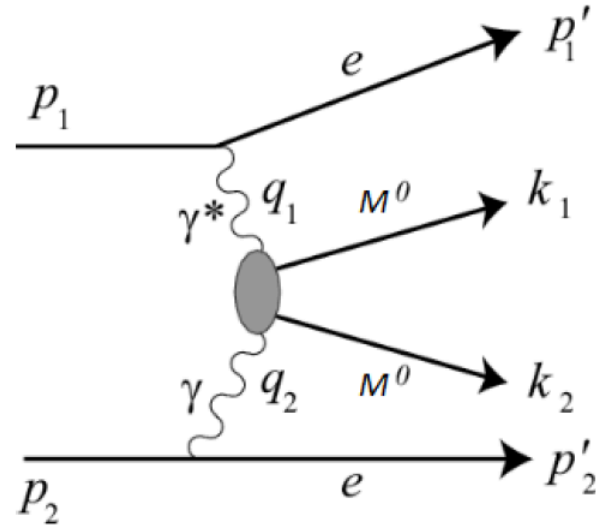
16-20 July 2018, IHEP

Motivation of single-tag two-photon process

Reaction:

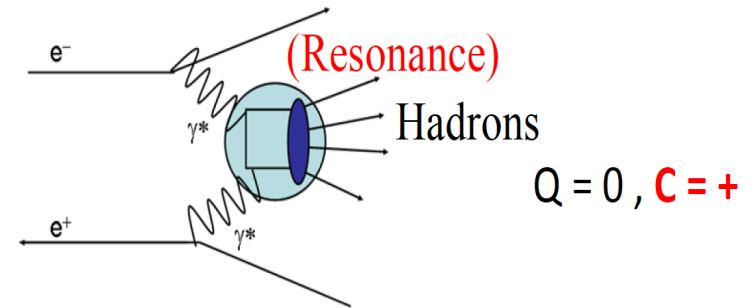
$e^+e^- \rightarrow e^\pm(\text{undetected } e^\mp) \text{ hadrons}$

- ▶ Study strong interaction in low energy region, where pQCD can't be applied;
- ▶ Measure Q^2 dependence of Transition Form Factor (TFF);
- ▶ Provide input for a data-driven estimate of the hadronic light-by-light contribution significant for the problem of muon $g-2$.



Motivation of no-tag two-photon process

- ▶ Lowest heavy-quarkonium $\eta_c(1S)$, plus J/psi, $\eta_b(1S)$ and $\Upsilon(1S)$, as benchmarks for the fine tuning of input parameters in QCD calculation.
- ▶ Attempt to measure $\Gamma_{\gamma\gamma}$ for $\eta_c(2S)$ and to address the discrepancy between data and QCD predictions.

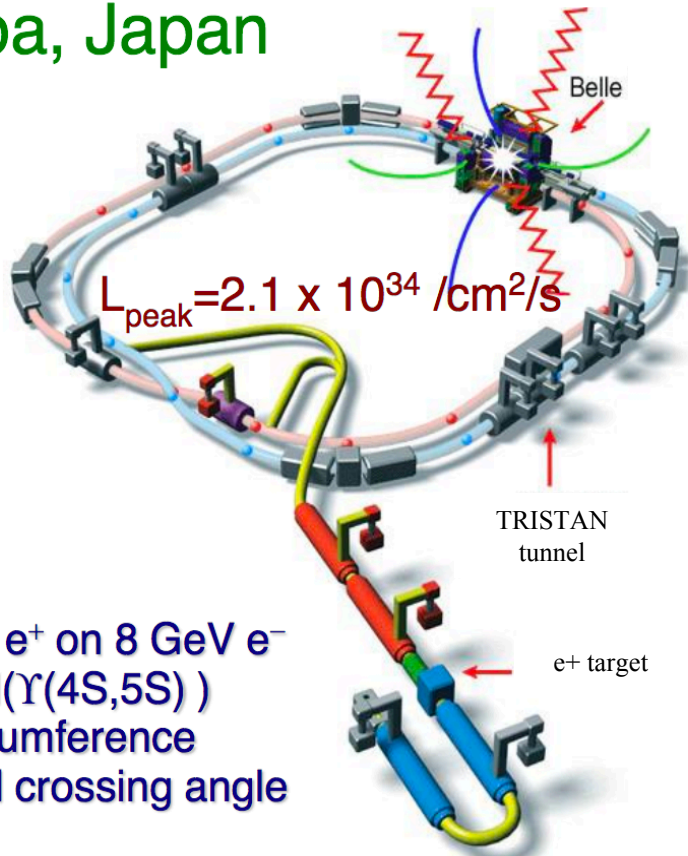


- ▶ Improved precision in both data and QCD predictions at higher W mass would provide more sensitive comparisons.
- ▶ **pseudo-scalar meson** pairs were measured by Belle [1]
Charged-meson pairs: $\pi^+\pi^-$, K^+K^- . Neutral-meson pairs: $K_S^0K_S^0$, $\pi^0\pi^0$, $\eta\pi^0$, $\eta\eta$.
- ▶ **pseudo-scalar tensor** pair $\eta'f_2(1270)$ and three-body final state $\eta'\pi\pi$ would provide new information to validate QCD models.

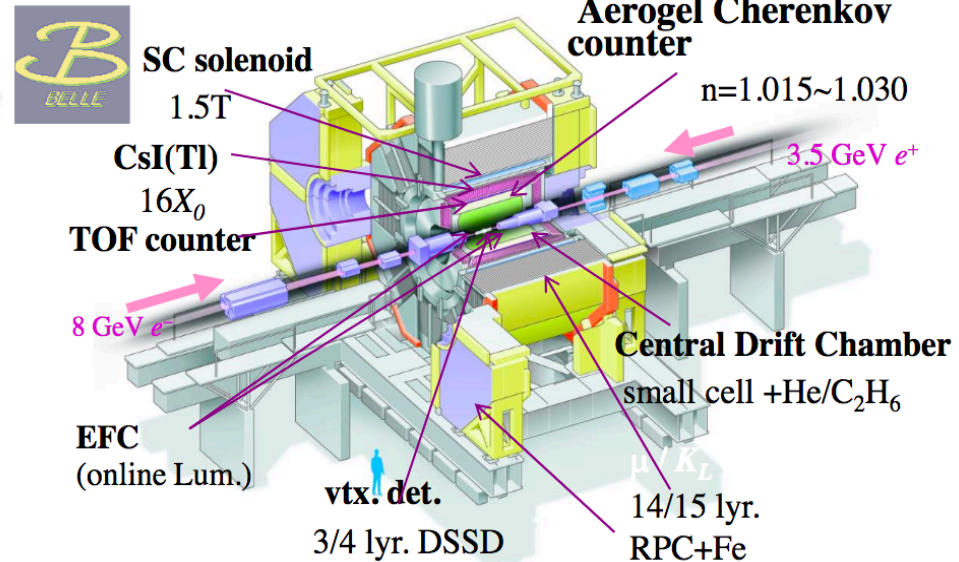
[1] Belle, Euro.Phys.Jour.C (2014) 74:3026

KEKB and Belle Detector

Tsukuba, Japan



3.5 GeV e^+ on 8 GeV e^-
 $W_{\text{CM}} = M(\Upsilon(4S, 5S))$
 3km circumference
 ~11mrad crossing angle

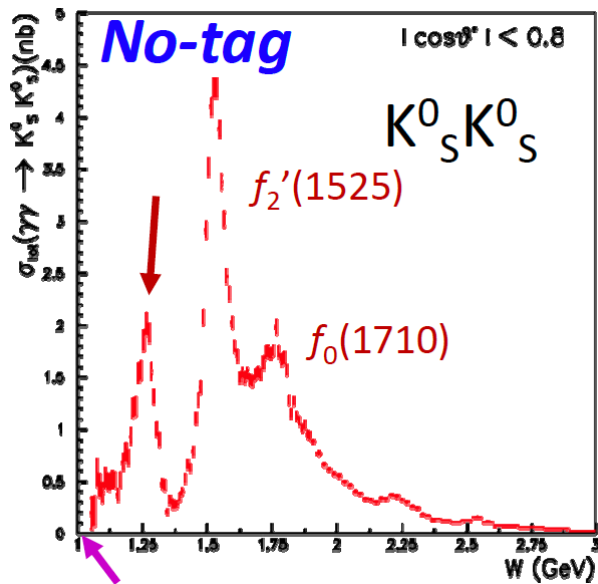


$$\Upsilon^* \Upsilon \rightarrow K_S^0 K_S^0$$

Dataset: 759 fb⁻¹

PRD 97, 052003 (2018)

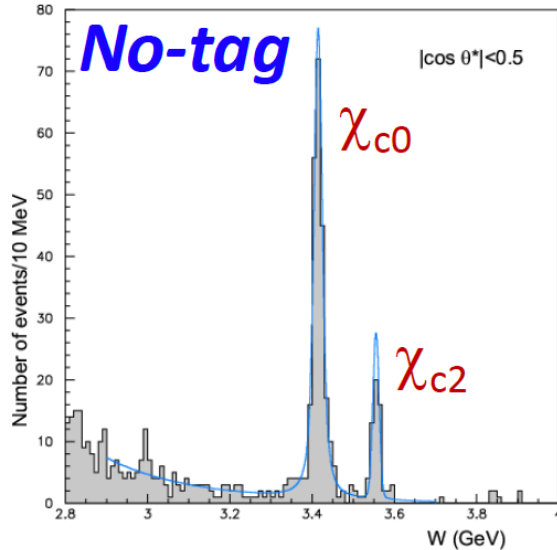
No-tag results for $K_S^0 K_S^0$ process



PTEP 2013, 123C01 (2013)

Maximum at the $f_2'(1525)$ peak
 \downarrow $f_2(1270)/a_2(1320)$ destructive interference
 Two-photon coupling of $f_0(1710)$

\nwarrow No data near the $K_S^0 K_S^0$ mass threshold



χ_{c_j} Yield

Two-photon decay width \times $B(K_S^0 K_S^0)$

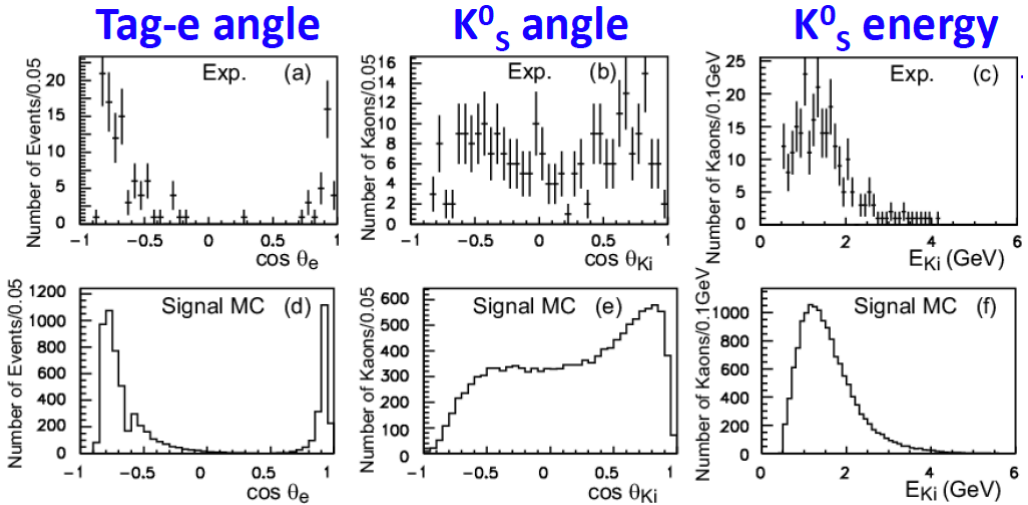
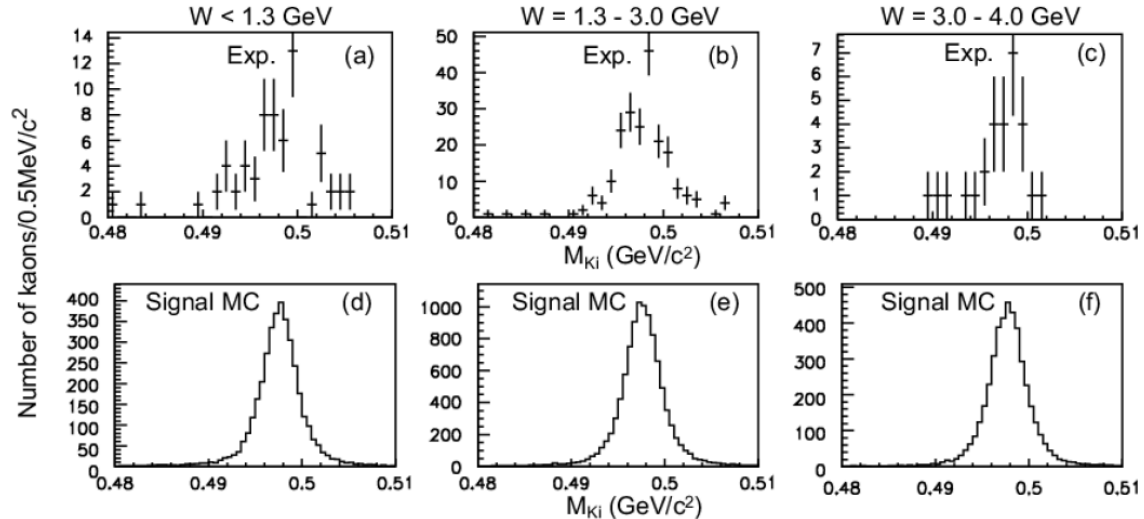
Interference	$N_{\chi_{c0}}$	$N_{\chi_{c2}}$	$-2 \ln \mathcal{L}/\text{ndf}$
not included	$248.3^{+17.9}_{-17.2}$	$53.0^{+8.1}_{-7.4}$	57.34/73
included	266 ± 53	53^{+14}_{-12}	57.22/71

Interference	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c0})$ (eV)	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c2})$ (eV)
not included	$8.09 \pm 0.58 \pm 0.83$	$0.268^{+0.041}_{-0.037} \pm 0.028$
included	$8.7 \pm 1.7 \pm 0.9$	$0.27^{+0.07}_{-0.06} \pm 0.03$
Belle 2007	$7.00 \pm 0.65 \pm 0.71$	$0.31 \pm 0.05 \pm 0.03$
PDG 2012	7.3 ± 0.5	0.297 ± 0.026

$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

Reconstructed mass, angles and Energy of the Signal candidates

Reconstructed K_S^0 mass

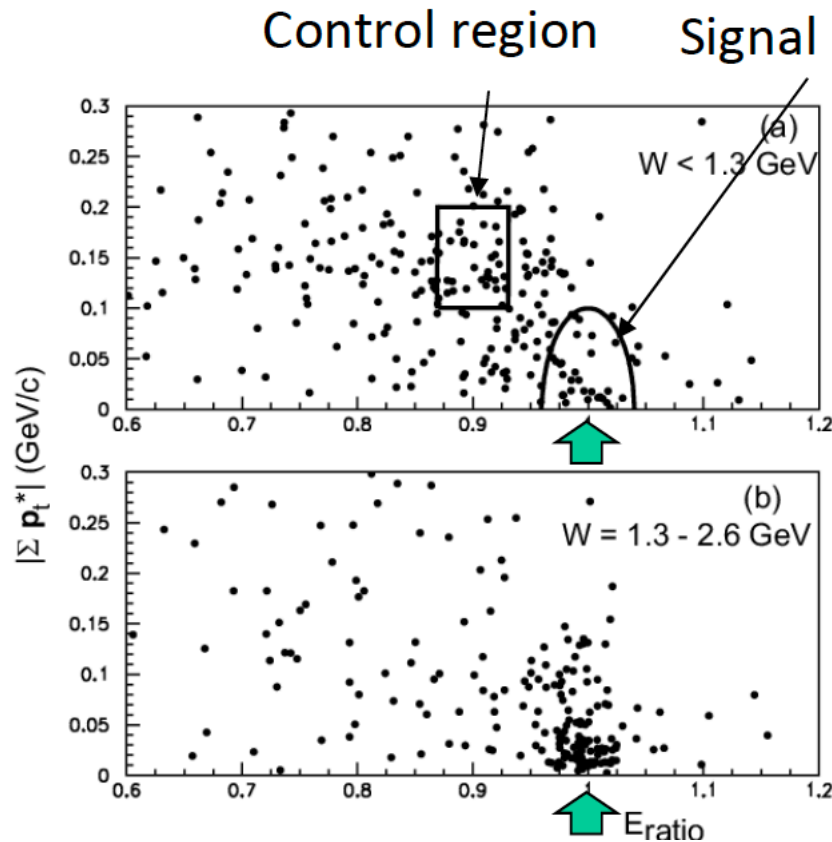


(in lab. frame,
for the signal-candidate events)

$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

Background processes

Rejection of non-exclusive background, $K_S^0 K_S^0 X$ using $|\Sigma p_t^*|$ vs. E_{ratio}



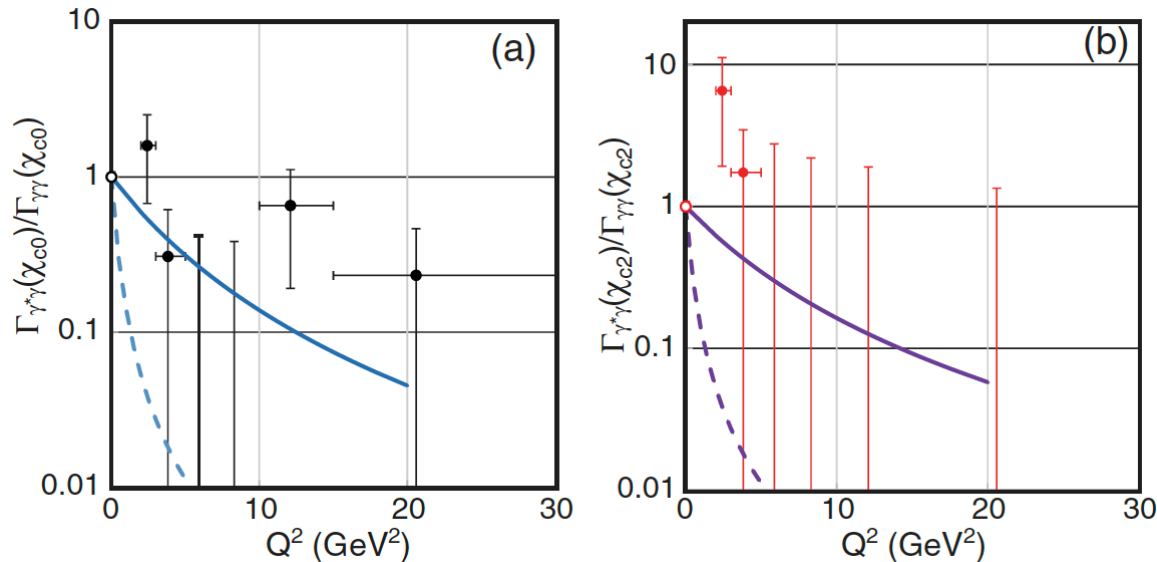
14% background
only for $W < 1.3 \text{ GeV}$

$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

Partial decay width of χ_{cJ} mesons

Assume that in total 7 events (3 events) peaking near the χ_{c0} (χ_{c2}) mass are purely from the charmonium (backgrounds are estimated <1 event in total)

Q^2 dependence $\Gamma_{\gamma^* \gamma} / \Gamma_{\gamma \gamma}$



The first measurement of χ_{cJ} production in high- Q^2 single-tag two-photon collisions.

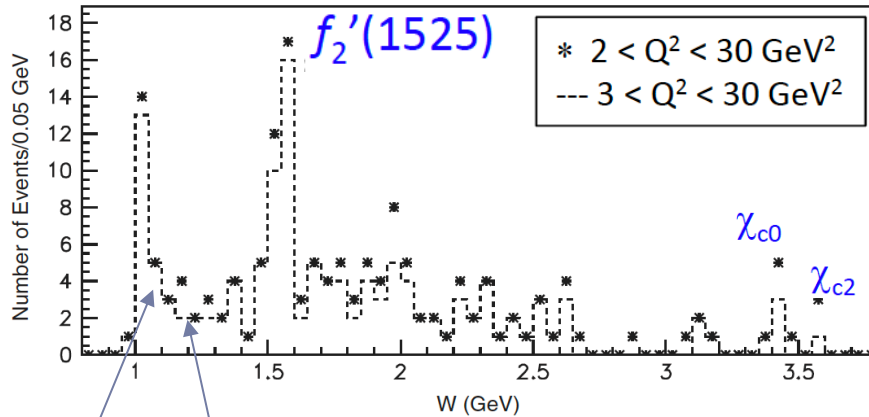
Solid curve: SBG [1] with the charmonium-mass scale (much favored).
Dashed curve: With the ρ -mass scale (VDM like)

[1] Schuler, Berends, and van Gulik, Nucl. Phys. B523, 423 (1998).

$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

W dependence and $\gamma^* \gamma$ cross section at Q^2 bins

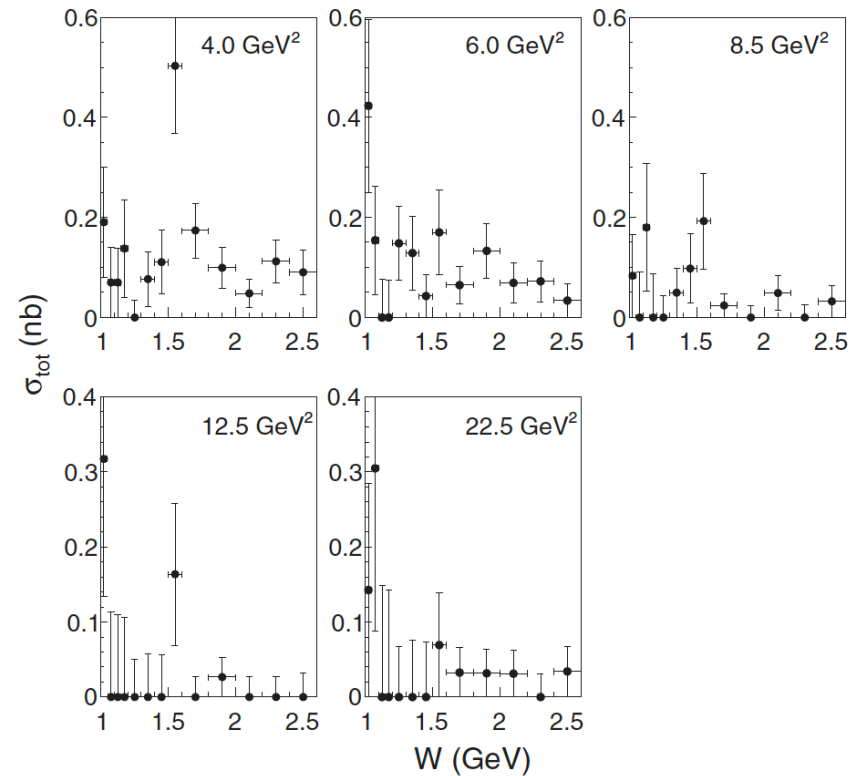
W distributions includes background



No $f_2(1270)/a_2(1320)$ is seen

Threshold enhancement, may be associated with $f_0(980)/a_0(980)$.

$$\sigma_{\text{tot}}(\gamma^* \gamma \rightarrow K_S^0 K_S^0) = \frac{1}{2} \frac{d^2 L_{\gamma^* \gamma}}{dW dQ^2} \frac{Y(W, Q^2)}{(1 + \delta) \varepsilon(W, Q^2) \Delta W \Delta Q^2 \int \mathcal{L} dt B^2}$$



$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

Partial Wave Analysis for TFF of $f_2'(1525)$

Waves: S, D_0 , D_1 , and D_2 , contribute ($W < 1.8$ GeV)

PRD 97, 052003 (2018)

Spherical Harmonics

$$\frac{d\sigma(\gamma^* \gamma \rightarrow K_S^0 K_S^0)}{d\Omega} = \sum_{n=0}^2 t_n \cos(n\varphi^*)$$

$$t_0 = |SY_0^0 + D_0 Y_2^0|^2 + |D_2 Y_2^2|^2 + 2\varepsilon_0 |D_1 Y_2^1|^2$$

$$t_1 = 2\varepsilon_1 \operatorname{Re}((D_2 |Y_2^2| - SY_0^0 - D_0 Y_2^0) D_1^* |Y_2^1|)$$

$$t_2 = -2\varepsilon_1 \operatorname{Re}(D_1^* |Y_2^1| (SY_0^0 + D_0 Y_2^0))$$

$$\varepsilon_0, \varepsilon_1 \text{ are variables that depend on } x = \frac{q_1 \cdot q_2}{p_1 \cdot p_2}$$

$$Y_0^0 = \sqrt{\frac{1}{4\pi}},$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta^* - 1),$$

$$|Y_2^1| = \sqrt{\frac{15}{8\pi}} \sin \theta^* \cos \theta^*,$$

$$|Y_2^2| = \sqrt{\frac{15}{32\pi}} \sin^2 \theta^*.$$

Parameterization of amplitudes

S and D_i amplitudes:

$$S = A_{BW} e^{i\phi_{BW}} + B_S e^{i\phi_{BS}},$$

$$D_i = \sqrt{r_{ifa}(Q^2)} (A_{f_2(1270)} - A_{a_2(1320)}) e^{i\phi_{faDi}}$$

$$+ \sqrt{r_{ifp}(Q^2)} A_{f_2'(1525)} e^{i\phi_{fpDi}}$$

$$+ B_{Di} e^{i\phi_{BDi}},$$

$r_{ifa}(Q^2)$ is fraction of $f_2'(1525)$ contribution in D wave.

$$r_{0fp} + r_{1fp} + r_{2fp} = 1$$

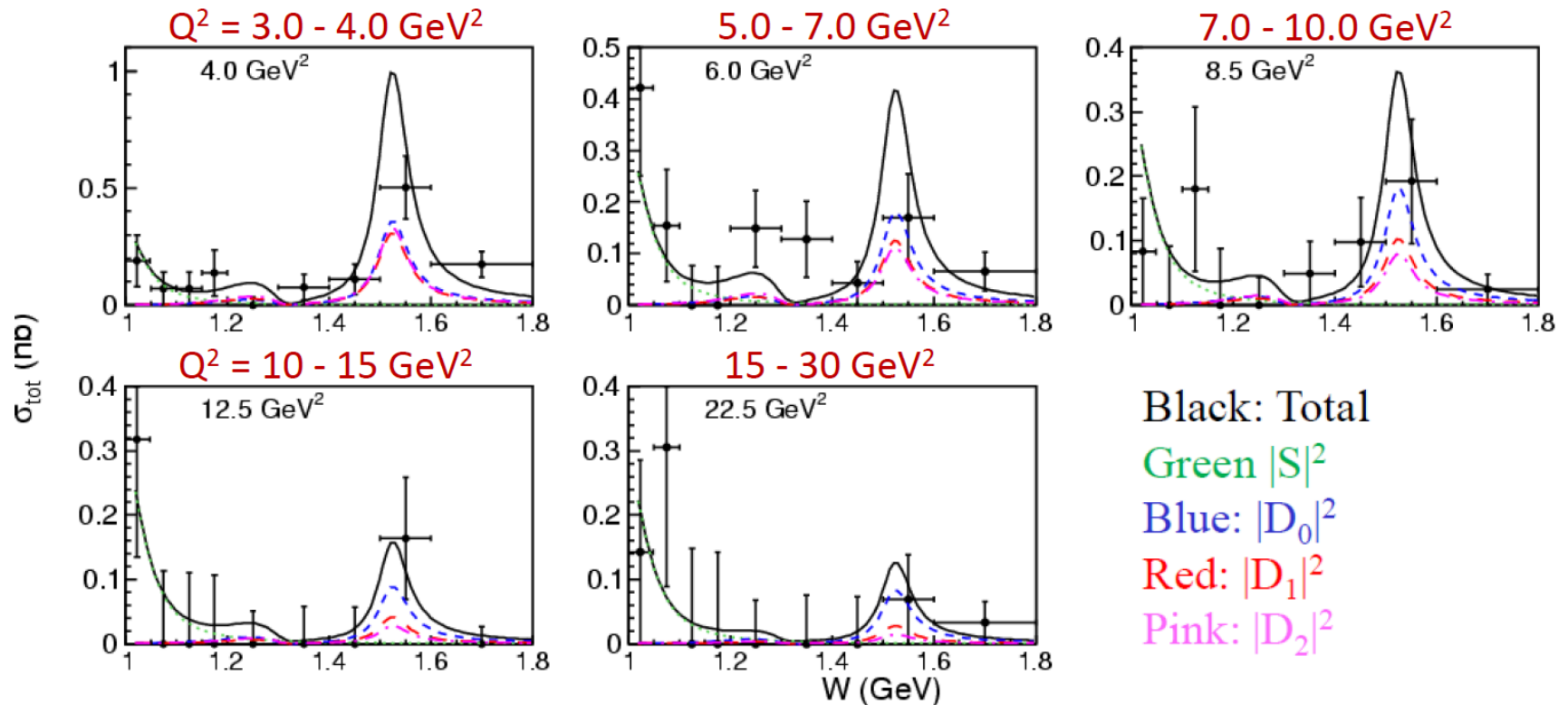
Transition Form Factors

$$A_R^J(W) = F_R(Q^2) \sqrt{1 + \frac{Q^2}{m_R^2}} \sqrt{\frac{8\pi(2J+1)m_R}{W}}$$

$$\times \frac{\sqrt{\Gamma_{\text{tot}}(W)\Gamma_{\gamma\gamma}(W)\mathcal{B}(K_S^0 K_S^0)}}{m_R^2 - W^2 - im_R \Gamma_{\text{tot}}(W)},$$

$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

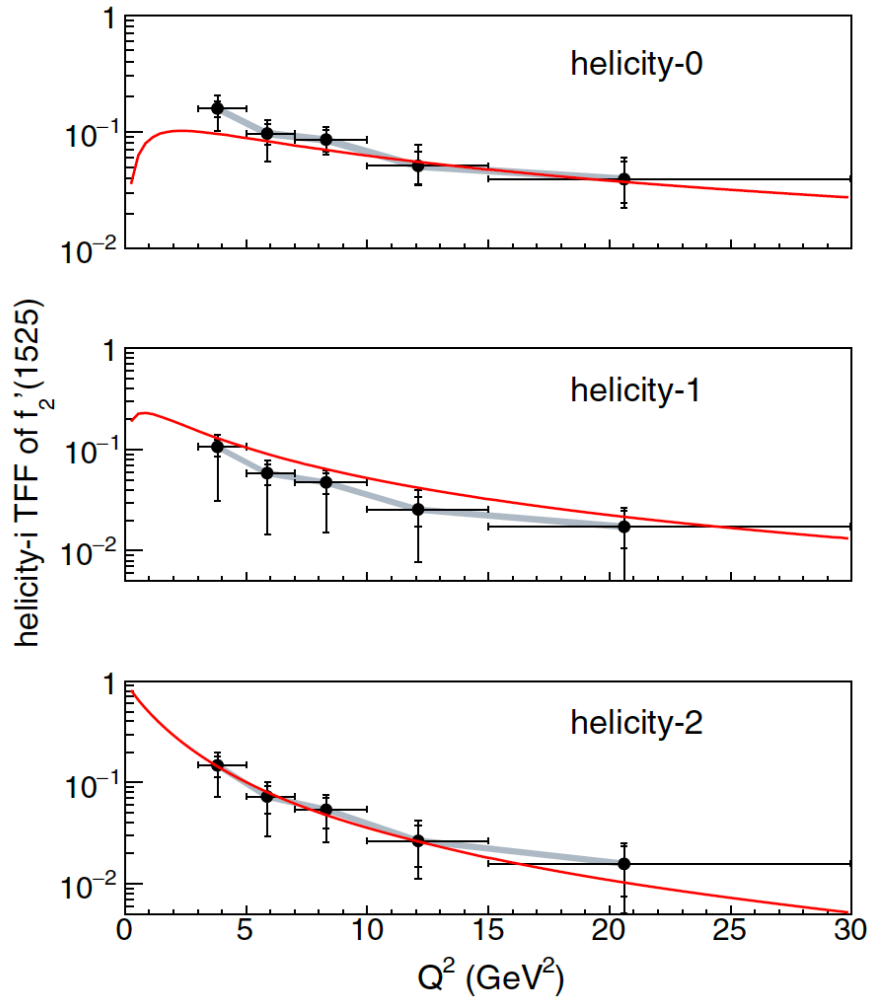
PWA results in W dependence at Q^2 bins



- Non-zero D_0 and D_1 components in the $f_2'(1525)$.
- No $f_2(1270)/a_2(1320)$ is seen.
- An enhancement near the threshold (0.995 GeV).

$$\gamma^* \gamma \rightarrow K_S^0 K_S^0$$

$f'_2(1525)$ TFF results



The obtained helicity-0, -1, and -2 TFF of the $f'_2(1525)$ meson as a function of Q^2 .

Shorter error bars: statistical

Longer error bars: statistical and systematic

Shaded areas: overall systematic on $\Gamma_{\gamma\gamma}$.

— Schuler, Berends, van Glick (SBG)
Nucl. Phys. B 523, 423, (1998).

helicity-0 and -2 agree well with SBG.
helicity-1 -- slightly smaller, but not inconsistent.

$$\gamma^* \gamma \rightarrow \pi^0 \pi^0$$

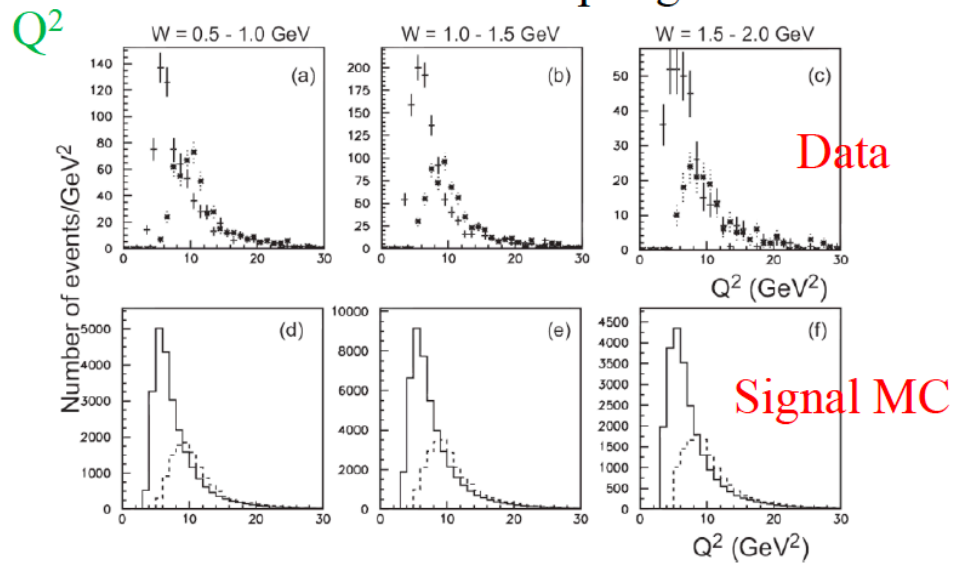
Dataset: 759 fb⁻¹

PRD 93, 032003 (2016)

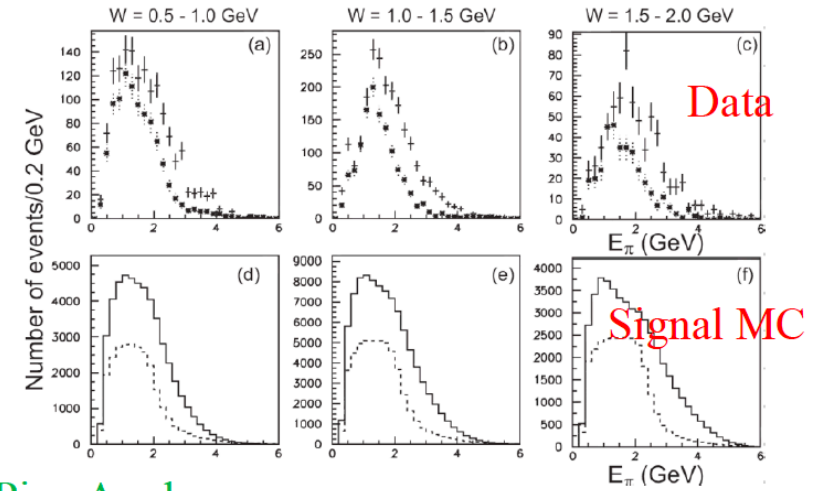
$$\gamma^* \gamma \rightarrow \pi^0 \pi^0$$

Reconstructed Q^2 , Energy and Angle of the Signal candidates

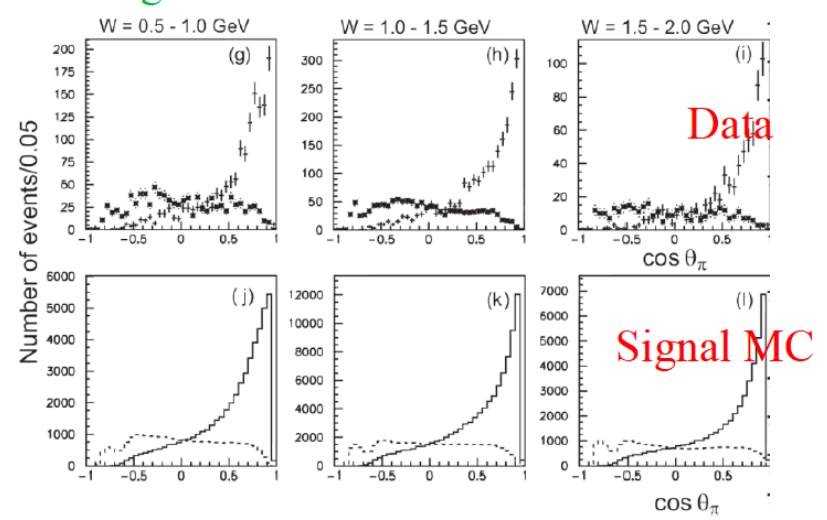
+ or solid histo : e-tag
 * or dashed histo : p-tag



Pion Energy

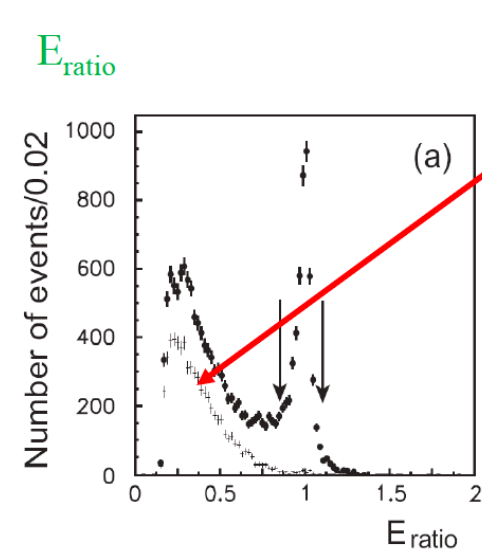


Pion Angle



$$\gamma^* \gamma \rightarrow \pi^0 \pi^0$$

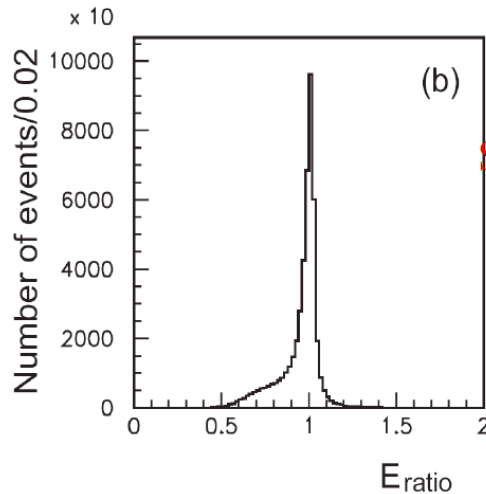
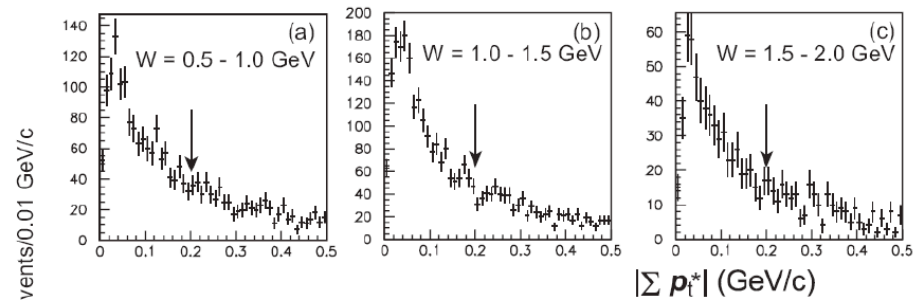
Background processes



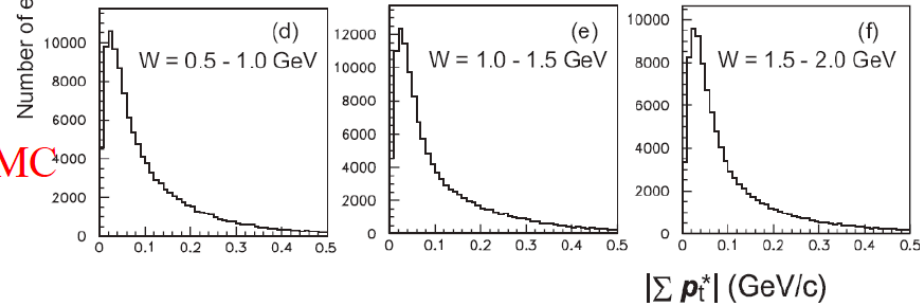
The wrong-sign events, that the tagged $e^+(e^-)$ have wrong charge sign.

P_t

Data

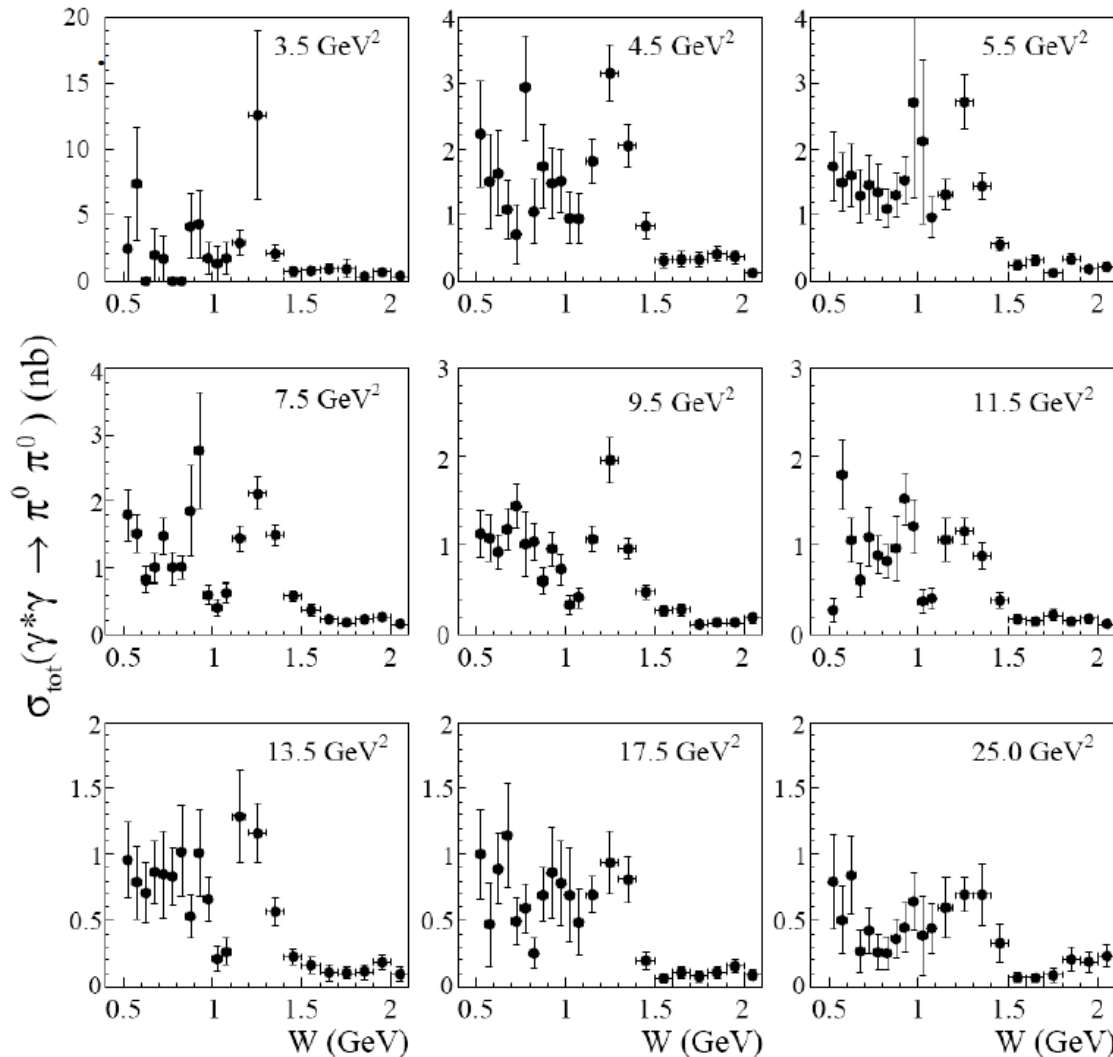


Signal MC



$$\gamma^* \gamma \rightarrow \pi^0 \pi^0$$

W dependence and $\gamma^* \gamma$ cross section at Q^2 bins



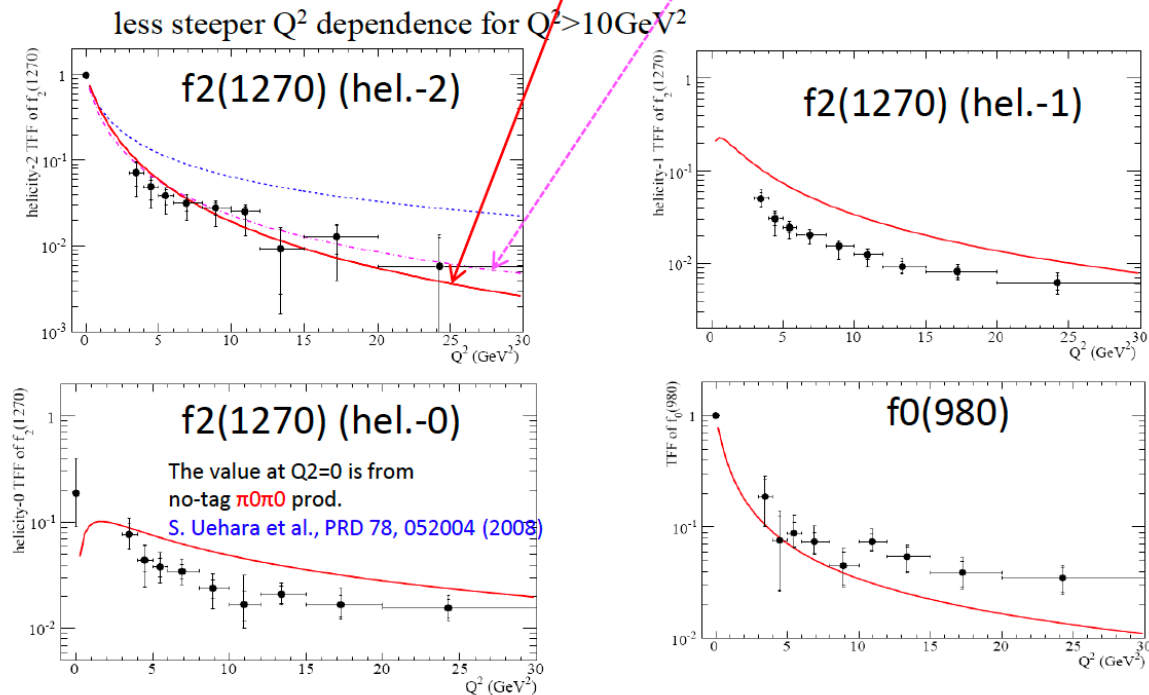
Peaks corresponding to $f_0(980)$ and $f_2(1270)$.



$$\gamma^* \gamma \rightarrow \pi^0 \pi^0$$

TFF results

- hel.-2 TFF of $f_2(1270)$ agrees with the prediction by Ref.[4] and Ref. [5].
- hel.-0 and 1 TFF, a factor of 1.5 – 2 smaller than the prediction by Ref.[4].
- TFF of $f_0(980)$: agree well with the prediction by Ref.[4] for $Q^2 < 10 \text{ GeV}^2$



Ref: [4] G.A. Schuler, F.A. Berends and R. van Gulik, Nucl. Phys. B 523, 423 (1998).
Based on application of heavy quark approximation to light quarks
[5] V. Pascalutsa, V. Pauk and M. Vanderhaeghen, Phys. Rev. D 85, 116001 (2012).
Based on sum rules

$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

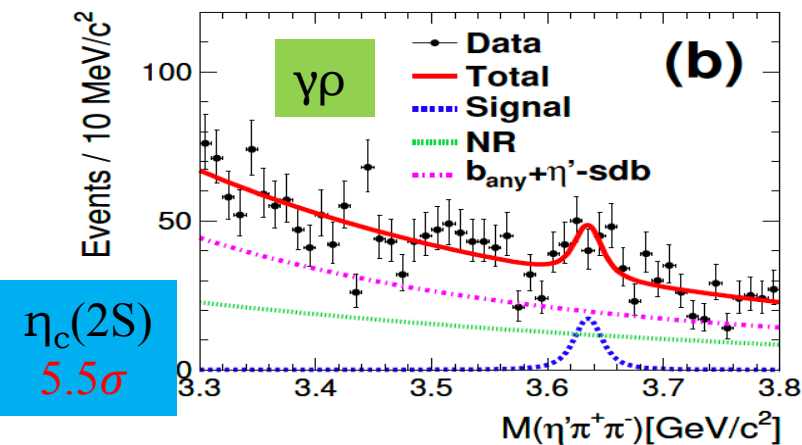
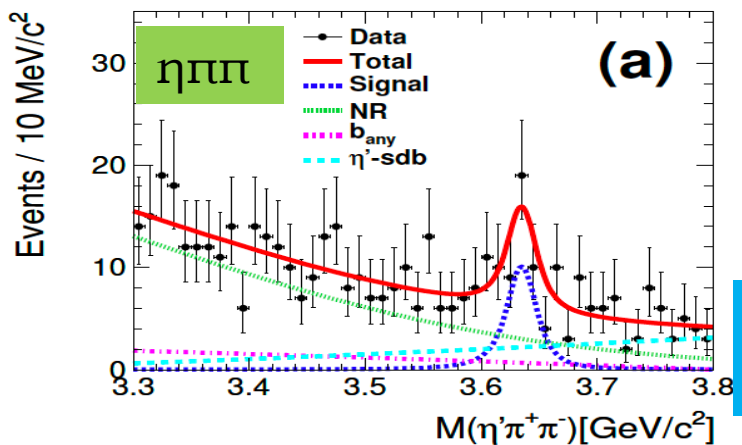
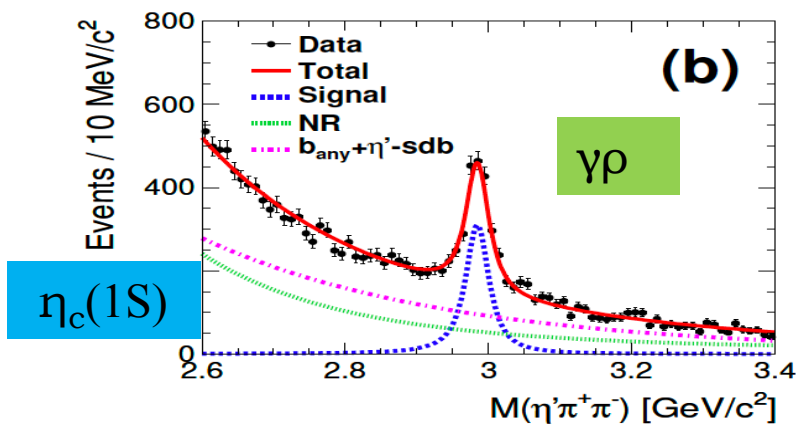
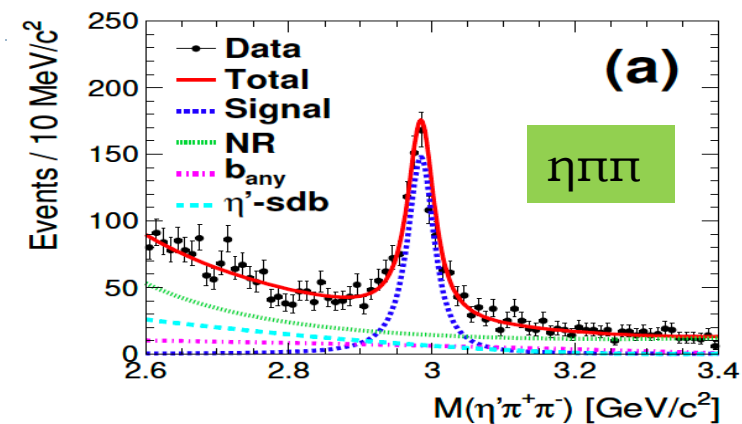
Dataset: 941 fb⁻¹

arXiv: 1805.03044

Submitted to PRD

$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Simultaneous Fit for $\eta_c(1S)$ and $\eta_c(2S)$



	$\eta_c(1S)$		$\eta_c(2S)$	
	$\gamma\rho$	$\eta\pi^+\pi^-$	$\gamma\rho$	$\eta\pi^+\pi^-$
n_s	1728^{+69}_{-68}	945^{+38}_{-37}	65^{+14}_{-13}	41^{+9}_{-8}
M (MeV/ c^2)	$2984.6 \pm 0.7 \pm 2.2$		$3635.1 \pm 3.7 \pm 2.9$	
Γ (MeV)	$30.8^{+2.3}_{-2.2} \pm 2.5$		11.3[fixed]	
$\Gamma_{\gamma\gamma}\mathcal{B}$ (eV)	$65.4 \pm 2.6 \pm 6.9$		$5.6^{+1.2}_{-1.1} \pm 1.1$	

$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Discussion on $\Gamma_{\gamma\gamma}$ of $\eta_c(2S)$

- ◆ Defining the ratio $R = \frac{\Gamma_{\gamma\gamma}(\eta_c(2S))B(\eta_c(2S))}{\Gamma_{\gamma\gamma}(\eta_c(1S))B(\eta_c(1S))}$, which is directly measured,

	This work	BaBar($K\bar{K}\pi$)[1]	CLEO[2]
R	$(8.6 \pm 2.6) \cdot 10^{-2}$	$(10.6 \pm 2.0) \cdot 10^{-2}$	$(18 \pm 5 \pm 2) \cdot 10^{-2}$

Consistent

so, we have $R_B = \frac{B(\eta_c(2S) \rightarrow \eta' \pi \pi)}{B(\eta_c(1S) \rightarrow \eta' \pi \pi)} \cong \frac{B(\eta_c(2S) \rightarrow K\bar{K}\pi)}{B(\eta_c(1S) \rightarrow K\bar{K}\pi)}$ within error.

- ◆ Assuming $R_B \cong 1$ [3] and

using the world average value $\Gamma_{\gamma\gamma}(\eta_c(1S)) = 5.1 \pm 0.4$ keV,

we obtain $\Gamma_{\gamma\gamma}(\eta_c(2S)) = 0.44 \pm 0.13$ keV for $\eta' \pi \pi$ (this) and 0.54 ± 0.11 keV for BaBar($K\bar{K}\pi$) [1].

Both $\Gamma_{\gamma\gamma}(\eta_c(2S))$ values by Belle and BaBar are lower than 0.92 ± 0.28 keV from CLEO [2]

Discrepancy between data and QCD values

- ◆ QCD predictions for two-photon decay width of $\eta_c(2S)$ are ranged from 1.4 to 5.7.

- ◆ It is essential to have **precise measurement** of either $B(\eta_c(2S) \rightarrow K_s K \pi)$ or $B(B \rightarrow K \eta_c(2S))$

[1] del Amo Sanchez, P. et al. (BaBar Collaboration) Phys.Rev. D84 (2011) 012004.

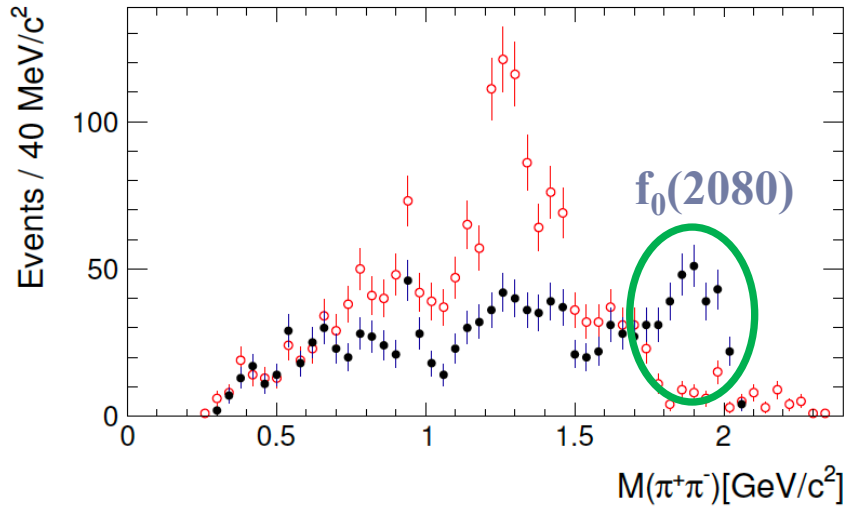
[2] D. M. Asner *et al.* CLEO Collaboration, Phys. Rev.Lett. **92** (2004) 142001.

[3] T. Barnes, T. E. Browder, and S. F. Tuan, Phys. Lett. B **385**, 391 (1996).

[4] J.P. Lansberg, T.N. Pham, AIP Conf. Proc. 1038 (2008) 259.

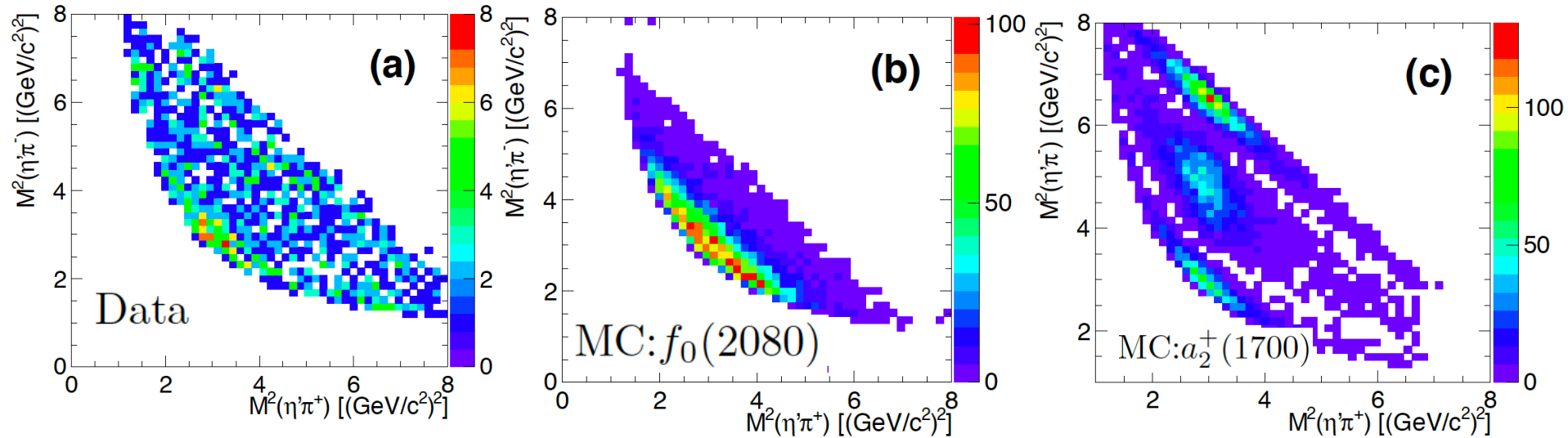
$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Study of $\eta_c(1S) \rightarrow \eta' f_0(2080) \rightarrow \pi^+ \pi^-$ decay with $f_0(2080) \rightarrow \pi^+ \pi^-$



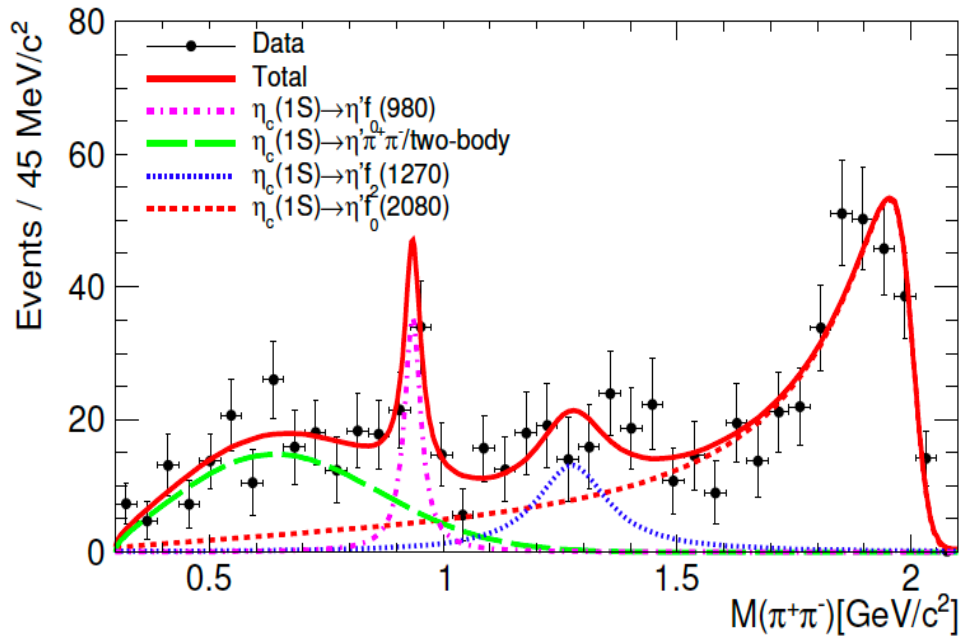
Black dots and **red circles** for events selected in $\eta_c(1S)$ **signal** and **sideband** regions.

No enhanced structure is seen in the Dalitz distributions for the $\eta_c(1S) \rightarrow a_2^\pm \pi^\mp$ with $a_2^\pm \rightarrow \eta' \pi^\pm$

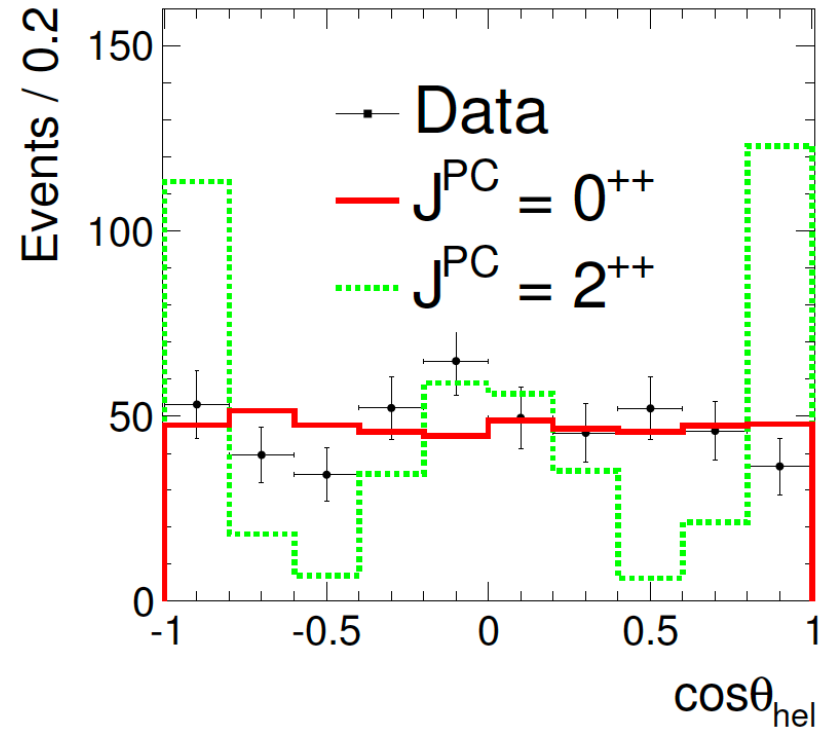


$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Study of $\eta_c(1S) \rightarrow \eta' f_0(2080)$ decay with $f_0(2080) \rightarrow \pi^+ \pi^-$



$$M = 2083_{-66}^{+63} \pm 32 \text{ MeV}, \quad \Gamma = 178_{-178}^{+60} \pm 55 \text{ MeV}$$

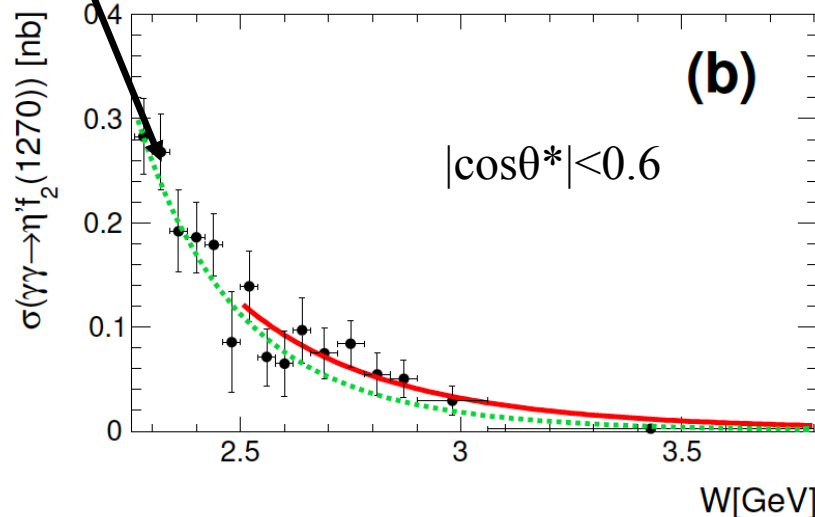
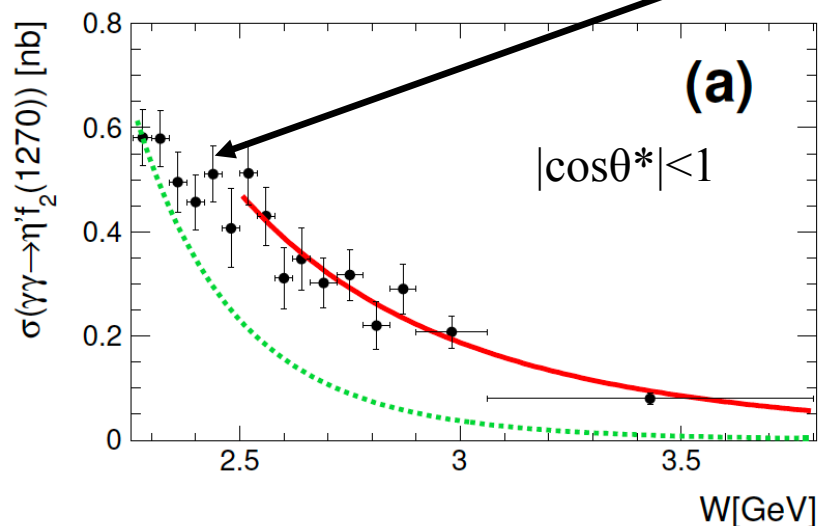


$J^{PC} = 0^{++}$ favored

$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Result of $\sigma(\gamma\gamma \rightarrow \eta' f_2(1270))$

Solid points are the measured cross section in data



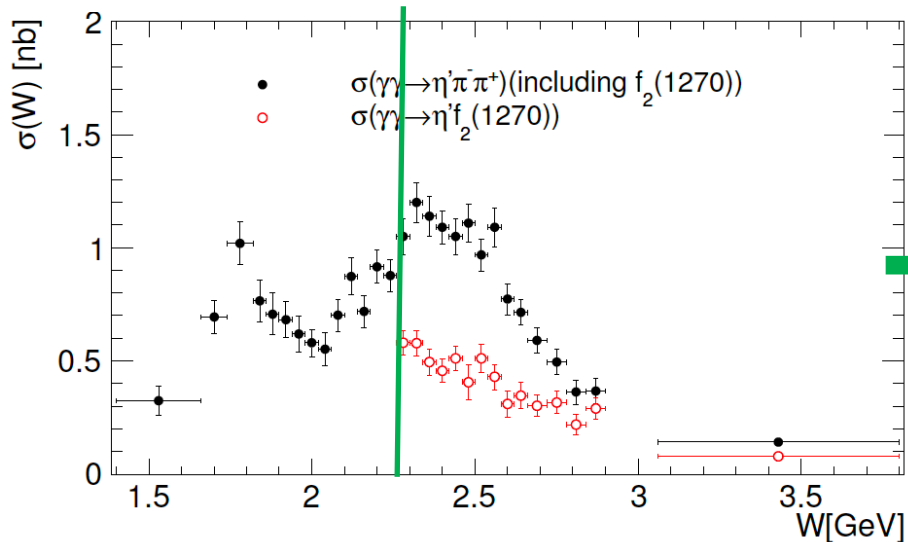
- **Green dashed** is the leading term QCD predictions for neutral meson pairs $\sim 1/W^{10}$ [1]
- No prediction for $\gamma\gamma \rightarrow \eta' f_2(1270)$.
- Assuming $\sigma \sim 1/w^n$.
- The **red solid line** is the fitted value of $n = 5.1 \pm 1.0$ for $|\cos\theta^*| < 1$ and $n = 7.5 \pm 2.0$ for $|\cos\theta^*| < 0.6$.

[1] Ed. A.J. Bevan, B. Golob, Th. Mannel, S. Prell, and B.D. Yabsley, Euro.Phys.Jour.C (2014) 74:3026.

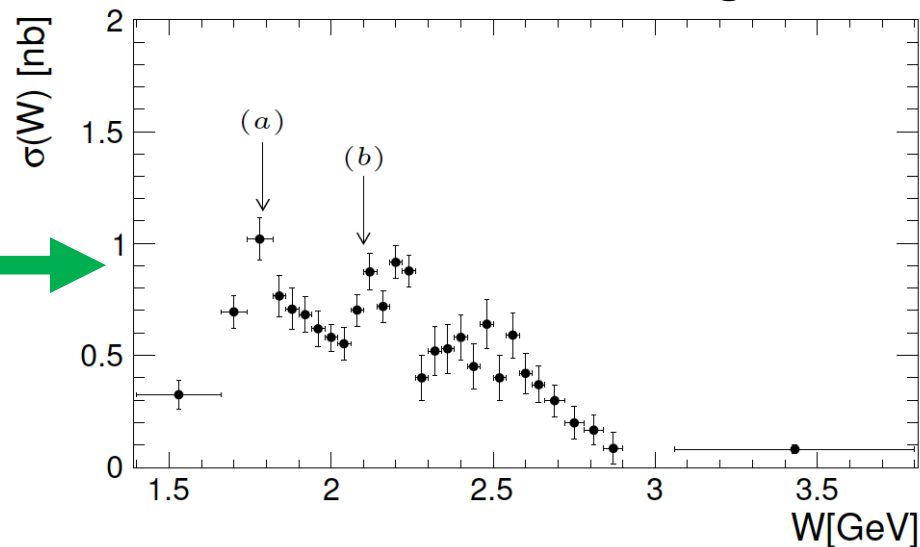
$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Result of $\sigma(\gamma\gamma \rightarrow \eta' \pi\pi)$

$\eta' f_2(1270)$ threshold



$\sigma(\gamma\gamma \rightarrow \eta' \pi\pi)$ after subtraction $\eta' f_2(1270)$ contribution in $W > 2.26 \text{ GeV}$ region.



(a). Structure near $1.8 \text{ GeV}/c^2$ is contributed from $X(1835)$ or $\eta(1760)$ [1].

(b) Enhancement at $2.1 \text{ GeV}/c^2$ is possible contribution from $\gamma\gamma \rightarrow I(2100) \rightarrow \eta' f_0(980)$.

[1] C.C. Zhang et al. Belle Collaboratin, Phys. Rev D86, 052002 (2012).

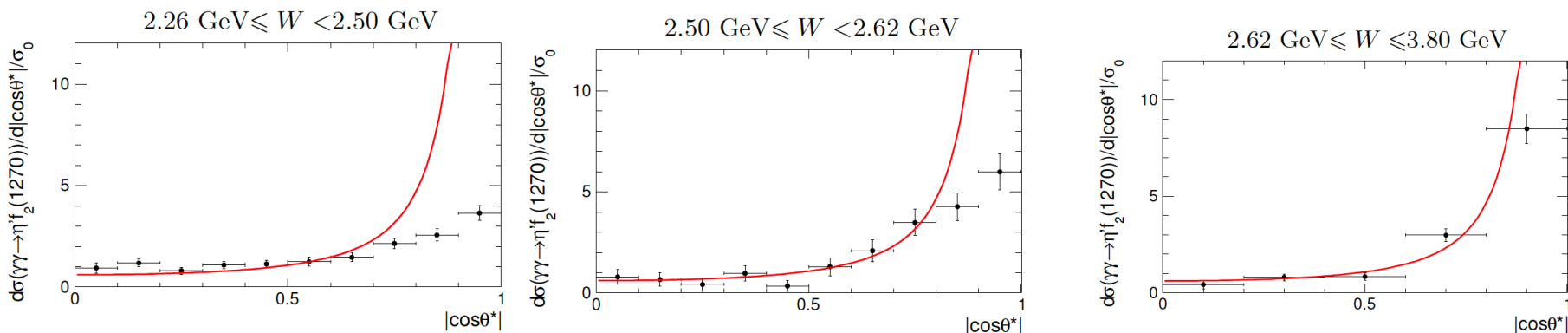
$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Cross Section in $|\cos\theta^*|$

- Black dots with error bar are the $|\cos\theta^*|$ dependent cross sections in data

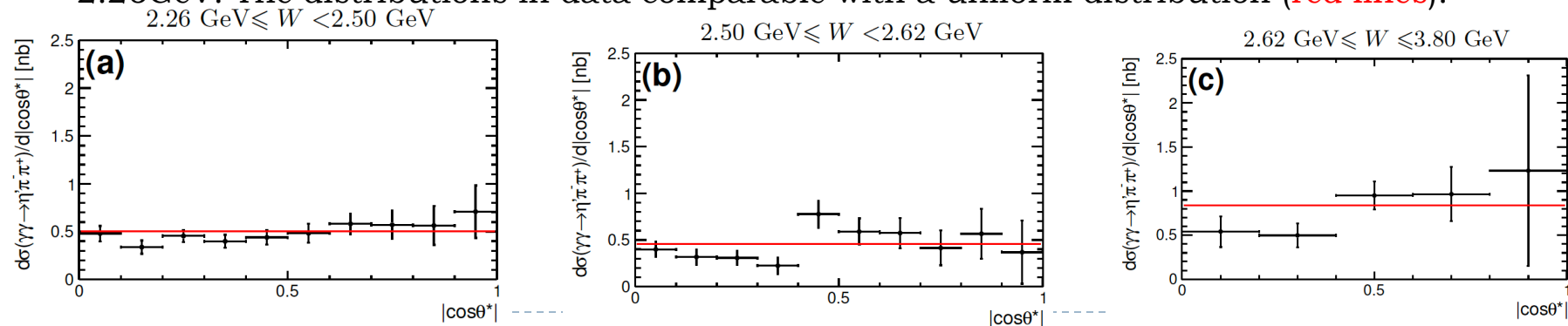
$$\gamma\gamma \rightarrow \eta' f_2(1270)$$

Red lines, normalized to the data, follows a $1/\sin^4\theta$ behavior.



$$\gamma\gamma \rightarrow \eta' \pi \pi$$

Measured cross section **after subtracting** the $\gamma\gamma \rightarrow \eta' f_2(1270)$ contribution in W region above 2.26 GeV. The distributions in data comparable with a uniform distribution (red lines).



Summary

Single-tag two-photon results

- Cross section for $\gamma^*\gamma \rightarrow K_S^0 K_S^0$ has been measured for $2M(K_S^0) < W < 2.6 \text{ GeV}$, $3 \text{ GeV}^2 < Q^2 < 30 \text{ GeV}^2$
- Q^2 dependence of $\Gamma_{\gamma^*\gamma}$ of χ_{c0} and χ_{c2} has been measured.
- Q^2 dependence of $f_2'(1525)$ TFF has been measured.
- First measurement for $\gamma^*\gamma \rightarrow \pi^0 \pi^0$ with Q^2 up to 30 GeV^2 .
- Q^2 dependence of $f_2(1270)$ TFF has been measured

No-tag two-photon results

- First observation of $\eta_c(2S) \rightarrow \eta' \pi \pi$ with a significance 5.5σ including systematic error.
- First observation of $\eta_c(1S) \rightarrow \eta' f_0(2080)$ decay with $f_0(2080) \rightarrow \pi^+ \pi^-$ with a significance 20σ
- Measurements of pseudo-scalar tensor pair $\eta' f_2(1270)$ production, as well as that of $\eta' \pi \pi$, are made for the first time.

Thanks for your attention!

Backup

Experimental analysis of Single-tag $K^0_S K^0_S$

Masuda et al. (Belle), PRD 97, 052003 (2018)

$e^+e^- \rightarrow e(e) K^0_S K^0_S, K^0_S \rightarrow \pi^+ \pi^-$ 759 fb^{-1}

Topology: 1 electron(or positron) and 4 charged pions

Event Selection Criteria:

for tracks 5 tracks satisfy $p_t > 0.1 \text{ GeV}/c$, ≥ 2 of them satisfy $p_t > 0.4 \text{ GeV}/c$,
1 of them satisfies **e-identification** and $p > 1.0 \text{ GeV}/c$

for K^0_S s Charged π/K separation

Reconstructed $K^0_S K^0_S$ masses (two-dimensional cut) :

$492.6 < \text{ave}[M(K^0_S)s] < 502.6 \text{ MeV}/c^2$ and $\text{diff}[M(K^0_S)s] < 10 \text{ MeV}/c^2$

K^0_S decay vertex: $0.3 < v_r < 8 \text{ cm}$

(a finite decay flight length in the $r\phi$ plane)

Kinematical cuts (Energy/momentum conservation and transverse-momentum balance)

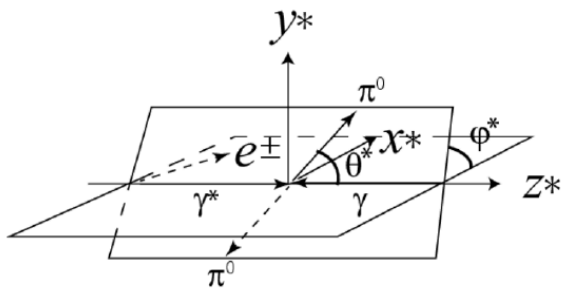
$$E_{\text{ratio}} = \frac{E_{K^0_S K^0_S}^{\text{measured}}}{E_{K^0_S K^0_S}^{\text{expected}}} \text{ and } |\Sigma p_t^*| \text{ satisfy } \sqrt{\left(\frac{E_{\text{ratio}} - 1}{0.04}\right)^2 + \left(\frac{|\Sigma p_t^*|}{0.1 \text{ GeV}/c}\right)^2} \leq 1$$

Partial Wave Analysis for TFF of $f'_2(1525)$

Applied for $W < 1.8$ GeV. We take into account partial waves up to $J=2$. $J=1$ does not couple with $K_S^0 K_S^0$ ($\rightarrow J^P = 0^+$ and 2^+)

PRD 97, 052003 (2018)

$$\frac{d\sigma(\gamma^* \gamma \rightarrow K_S^0 K_S^0)}{d\Omega} = \sum_{n=0}^2 t_n \cos(n\varphi^*),$$



$\gamma^* \gamma$ c.m. frame
 z^* axis // γ^*
 $x^* z^*$ plane includes tag-e

Resonance amplitude for f'_2 , etc.

$$A_R^J(W) = F_R(Q^2) \sqrt{1 + \frac{Q^2}{m_R^2}} \sqrt{\frac{8\pi(2J+1)m_R}{W}} \\ \times \frac{\sqrt{\Gamma_{\text{tot}}(W)\Gamma_{\gamma\gamma}(W)\mathcal{B}(K_S^0 K_S^0)}}{m_R^2 - W^2 - im_R\Gamma_{\text{tot}}(W)}$$

$$t_0 = |SY_0^0 + D_0Y_2^0|^2 + |D_2Y_2^2|^2 + 2\epsilon_0|D_1Y_2^1|^2, \\ t_1 = 2\epsilon_1 \Re [(D_2^*|Y_2^2| - S^*Y_0^0 - D_0^*Y_2^0)D_1|Y_2^1|], \\ t_2 = -2\epsilon_0 \Re [D_2^*|Y_2^2|(SY_0^0 + D_0Y_2^0)].$$

TFF of f'_2 for helicity $i = \lambda$

$$\sqrt{r_{ifp}} F_{f2p} \quad (i = 0, 1, 2) \\ r_{0fp} + r_{1fp} + r_{2fp} = 1$$

S, D_0 , etc. --- Partial-wave amplitudes

ϵ_0, ϵ_1 --- Spin-dependent flux factor ratios for the virtual photon

Y_j^m --- Spherical harmonics

Formalism of PWA and parametrizations

Problems: Low statistics

Only 3 out of S , D_0 , D_1 and D_2 are independent

Non-unique solution (multiple solutions for resonances)

→ Parametrization of the amplitudes with modelled W and Q^2 dependences

$$\begin{aligned}
 S &= A_{BW} e^{i\phi_{BW}} + B_S e^{i\phi_{BS}}, \\
 D_i &= \sqrt{r_{ifa}(Q^2)} (A_{f_2(1270)} - A_{a_2(1320)}) e^{i\phi_{faD_i}} \\
 &\quad + \sqrt{r_{ifp}(Q^2)} A_{f_2'(1525)} e^{i\phi_{fpD_i}} \\
 &\quad + B_{D_i} e^{i\phi_{BD_i}},
 \end{aligned}$$

$$\begin{aligned}
 A_{BW}(W) &= \sqrt{\frac{8\pi m_S}{W}} \frac{f_S}{m_S^2 - W^2 - im_S g_S} \\
 &\quad \times \frac{1}{(Q^2/m_0^2 + 1)^{p_S}},
 \end{aligned}$$

Nominal fit
 $B_S = 0$

$$B_S = \frac{\beta a_S (W_0/W)^{b_S}}{(Q^2/m_0^2 + 1)^{c_S}},$$

$$B_{D_0} = \frac{\beta^5 a_{D_0} (W_0/W)^{b_{D_0}}}{(Q^2/m_0^2 + 1)^{c_{D_0}}},$$

$$B_{D_1} = \frac{\beta^5 Q^2 a_{D_1} (W_0/W)^{b_{D_1}}}{(Q^2/m_0^2 + 1)^{c_{D_1}}},$$

$$B_{D_2} = \frac{\beta^5 a_{D_2} (W_0/W)^{b_{D_2}}}{(Q^2/m_0^2 + 1)^{c_{D_2}}},$$

$\beta = \sqrt{1 - 4m_{K_S^0}^2/W^2}$ is the K_S^0 velocity

$$r_{0fp} : r_{1fp} : r_{2fp} = k_0 Q^2 : k_1 \sqrt{Q^2} : 1$$

-Destructive interference between $f_2(1270)$ and $a_2(1320)$
 $-r_i(Q^2)$ and TFF for $f_2(1270)$ and $a_2(1320)$ are the same;
 use the values obtained in single-tag $\pi^0\pi^0$

Determine each component and the relative phase by a fit

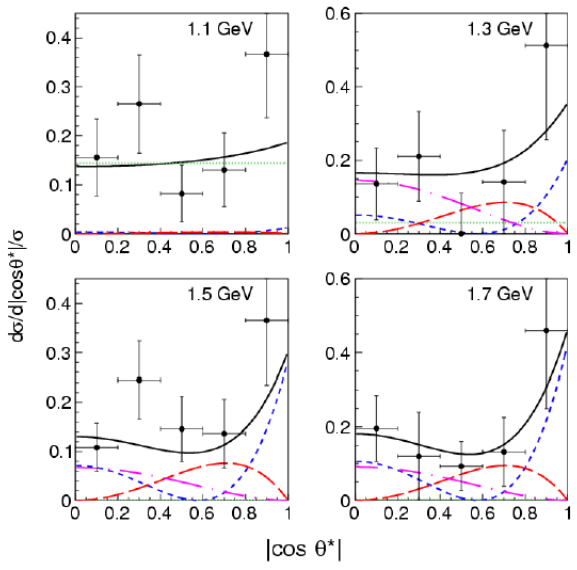
Angular dependence and the PWA fit

Due to a lack of statistics, we use **Q²-integrated angular differential cross section** derived with the following convention (MC generated isotropically)

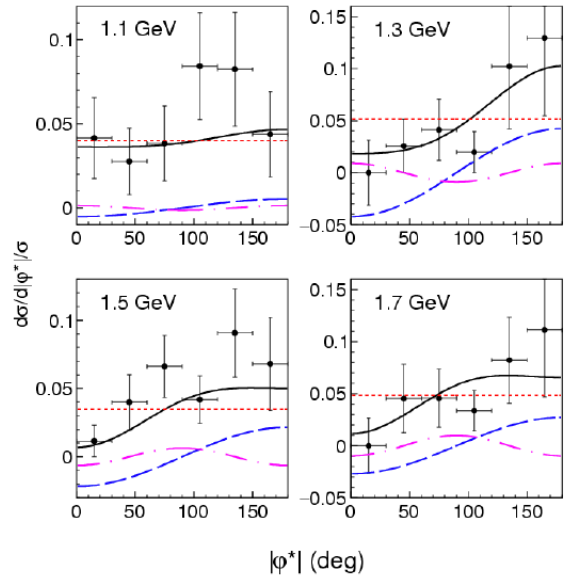
$$\frac{d^2\sigma/d|\cos\theta^*|d|\varphi^*|}{N_{\text{EXP}}(|\cos\theta^*|, |\varphi^*|)/N_{\text{MC}}(|\cos\theta^*|, |\varphi^*|)} \propto$$

Q²: integrated over the full range between 3 and 30 GeV²
 W: 4 bins

|cos θ*| dependence (|φ*| integrated)



|φ*| dependence (|cos θ*| integrated)



We regard this as the angular dependence at $\langle Q^2 \rangle = 6.5 \text{ GeV}^2$

Fit:
 Black: total
 Red: t_0
 Blue: $t_1 \cos\varphi^*$
 Magenta: $t_2 \cos 2\varphi^*$

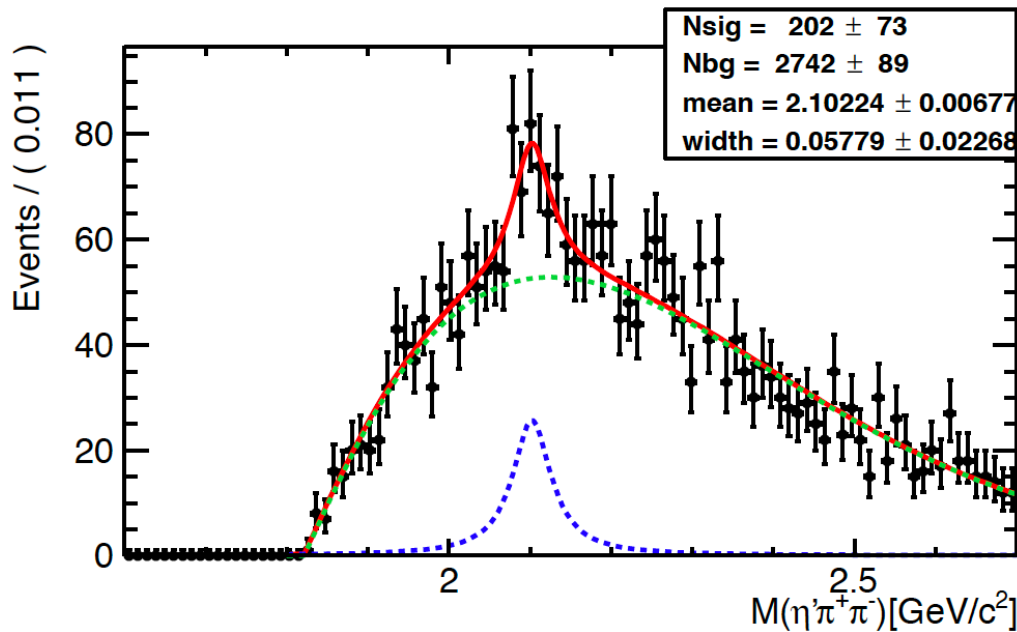
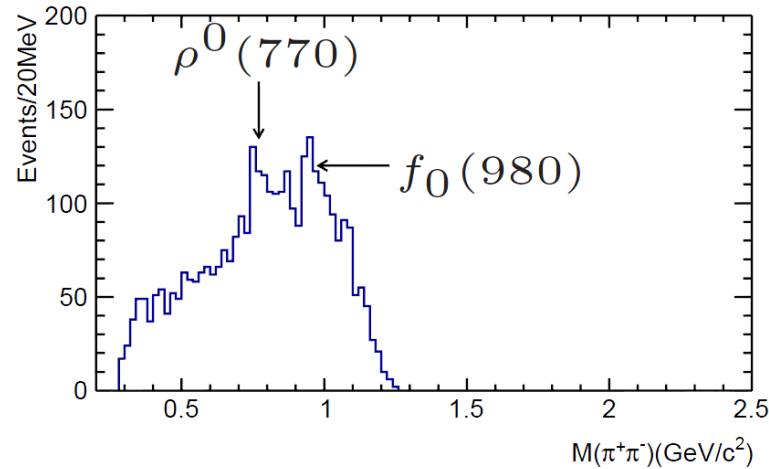
The fit is applied to the two-dimensional angular-dependence data.
 Forward enhancement is from the helicity-0 component.

From PDG 2017

	Branching fraction
$\eta_c(1S) \rightarrow K\bar{K}\pi$	$(7.3 \pm 0.5)\%$
$\eta_c(2S) \rightarrow K\bar{K}\pi$	$(1.9 \pm 1.2)\%$
$B \rightarrow K(\eta_c(1S) \rightarrow K_S K\pi)$	$(2.7 \pm 0.6) \times 10^{-5}$
$B \rightarrow K(\eta_c(2S) \rightarrow K_S K\pi)$	$(3.4^{+2.3}_{-1.6}) \times 10^{-6}$

Possible intermediate from $\gamma\gamma \rightarrow I(2100) \rightarrow \eta' f_0(980)$

$$2.0 < W < 2.2 \text{ GeV}/c^2$$



- In $f_0(980)$ signal region $0.86 < M(\pi\pi) < 1.10 \text{ GeV}/c^2$.
- $I(2100)$ with statistic significance 3.5σ .