

Amplitude analyses of charm meson decays at BESIII

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Outline

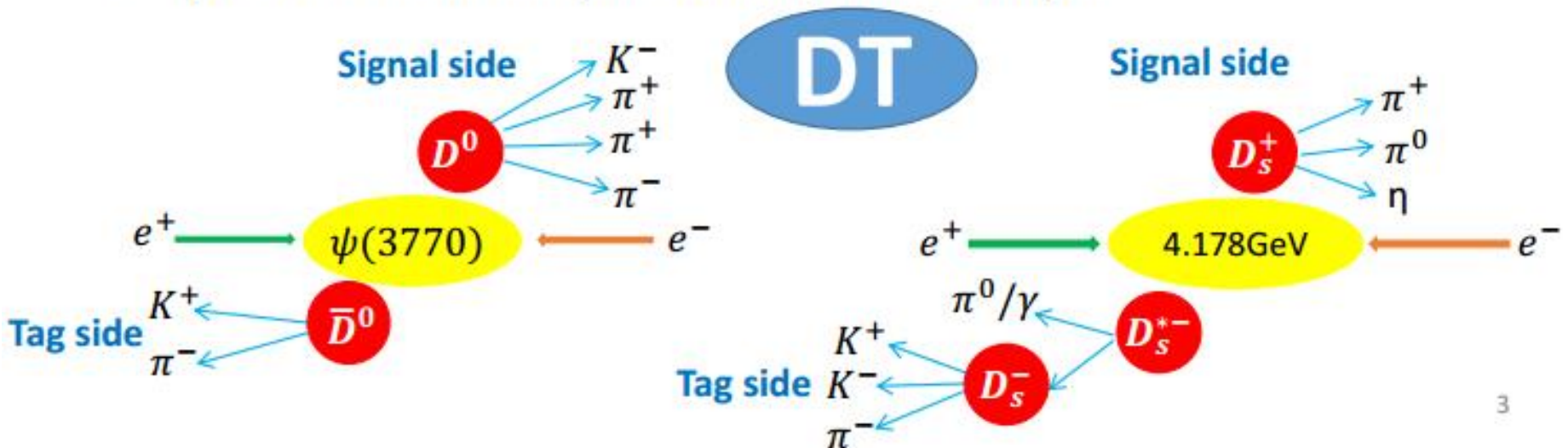
- The used dataset.
- Amplitude analysis.
- Recent results for $D_{(s)}$ hadronic decays
 1. Amplitude analysis of $D^0/D^+ \rightarrow K\pi\pi\pi$.
 2. Amplitude analysis of $D_S^+ \rightarrow \pi^+\pi^0\eta$ and the first observation of $D_S^+ \rightarrow a_0(980)\pi$
- Summary

Used Dataset

- 2.93 fb^{-1} at $E_{\text{cm}} = 3.773 \text{ GeV}$:
ten millions of $D\bar{D}$ pairs (totally 21 M D^0 and 16 M D^+).
- 3.19 fb^{-1} at $E_{\text{cm}} = 4.178 \text{ GeV}$:
hundred thousands of $D_s\bar{D}_s^*$ pairs (totally 400K D_s).

Paired production allows two different ways:

- **Single Tag (ST):** reconstruct only one of the charmed meson.
- **Double Tag (DT):** reconstruct both of two charmed meson.
 - provides access to absolute branching fractions (BFs).
 - provides clean samples for amplitude analysis.



Amplitude analysis

● Based on RooFit:

- Use RooFit framework to **construct the PDF**
- Use RooMinuit to perform the unbinned likelihood fit
- Use RooFitResult to deal with the fit results

● Using GPU to accelerate the analysis:

- C99 standard for programs in kernel
- Connect the RooFit and GPU kernel with OpenCL and C++
- Much faster than CPU version. (One or two weeks → three hours for a fit to data in Amplitude analysis of $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$)

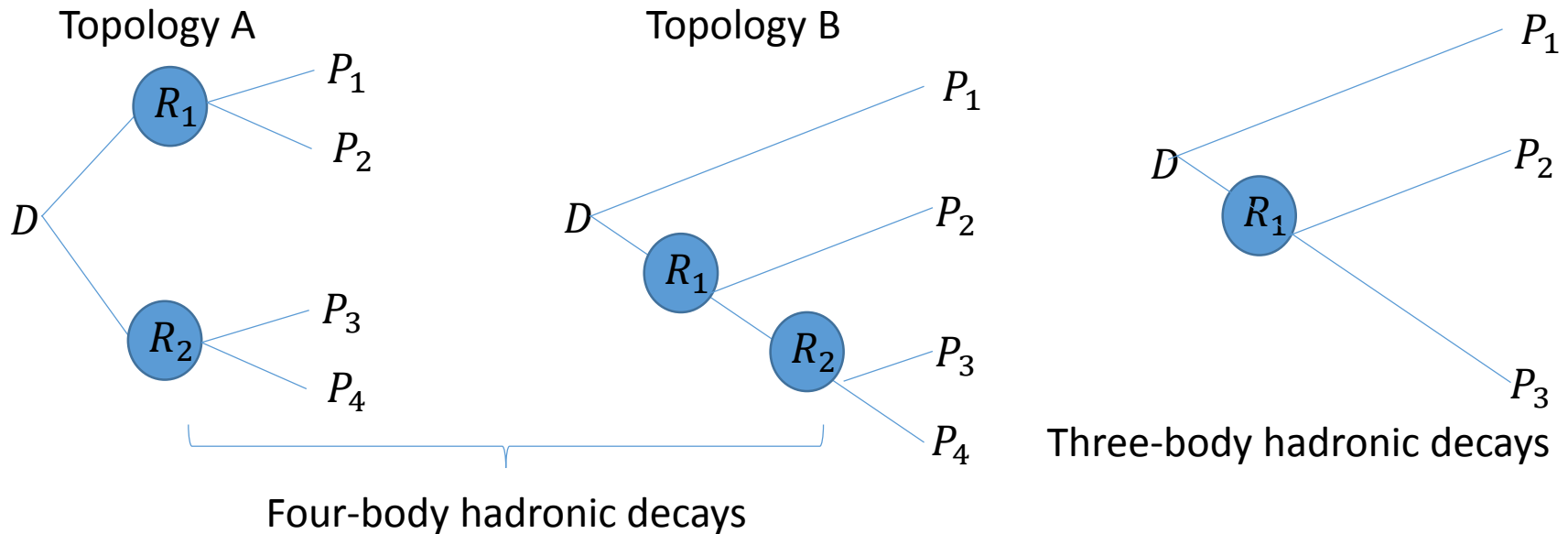


PDF construction

Isobar model is used in amplitude analysis:

the total amplitude $M(p_j)$ is the coherent sum of the sub-amplitudes $A_n(p_j)$,

$$M(p_j) = \sum_n \rho_n e^{i\phi_n} A_n(p_j)$$

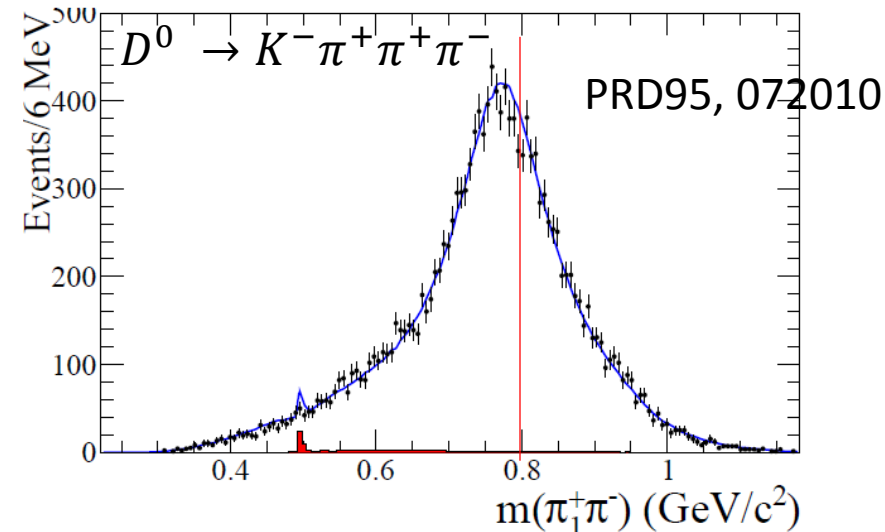


$$A_n(p_j) = P_n^1(m_1)P_n^2(m_2)S_n(p_j)F_n^1(p_j)F_n^2(p_j)F_n^D(p_j) \quad A_n(p_j) = P_n(m_R)S_n(p_j)F_n^R(p_j)F_n^D(p_j)$$

- $P_n(m_R)$: Propagator of intermediate resonance.
- $S_n(p_j)$: Angular terms, constructed with covariant tensor formalism (Zemach tensor).
- $F_n(p_j)$: Blatt-Weisskopf barriers

Propagator of intermediate resonance

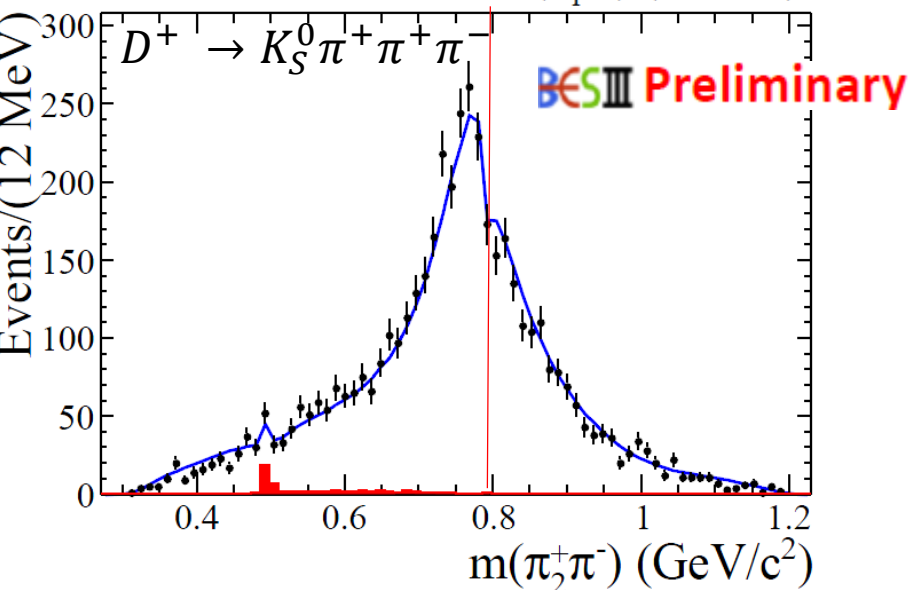
- For the resonance ρ , the GS formula (PRL 21, 244) is considered. In the decay of $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$, the obviously $\rho - \omega$ interference effect should be considered.



$$P_{\rho-\omega}(m) = P_{\rho}^{\text{GS}}(m)(1 + a_r e^{i\phi_r} P_{\omega}^{\text{RBW}}(m))$$

GS
RBW

Relative magnitude and phase.



Propagator of intermediate resonance

For resonance σ ($f_0(500)$), the Bugg formula is used (PLB 572, 1):

$$P_{f_0(500)}(m) = \frac{1}{m_0^2 - m^2 - im_0\Gamma_{\text{tot}}(m)},$$

where, $\Gamma_{\text{tot}}(m)$ is decomposed into two parts:

$$\Gamma_{\text{tot}}(m) = g_1 \frac{\rho_{\pi\pi}(m)}{\rho_{\pi\pi}(m_0)} + g_2 \frac{\rho_{4\pi}(m)}{\rho_{4\pi}(m_0)},$$

and

$$g_1 = (b_1 + b_2 m^2) \frac{m^2 - m_\pi^2/2}{m_0^2 - m_\pi^2/2} e^{(m_0^2 - m^2)/a}.$$

$$\rho_{4\pi} = \sqrt{\left(1 - \frac{16m_\pi^2}{m^2}\right) / \left(1 + e^{\frac{2.8 - m^2}{3.5}}\right)}$$

All the parameters are fixed to Ref .PLB 598, 149.

For resonance $a_0(980)$ and $f_0(980)$, the flatte formula is used,

$$P_{a_0(f_0)}(m) = \frac{1}{m_0^2 - m^2 - i(\sum_j g_j^2 \rho_j)}.$$

Here, only two channels coupling ($\pi\eta$, KK for $a_0(980)$ and $\pi\pi$, KK for $f_0(980)$) is considered.

The parameters for $a_0(980)$ are fixed to Ref. PRD 95, 032002.

The parameters for $f_0(980)$ are fixed to Ref. PLB 607, 243.

- The $\rho_{\pi\pi}$ (ρ_j) above is the two-body phase space factor, $2q/m$, q is the magnitude of the daughter momentum in the resonance rest frame.
- For other resonances, the RBW is used.

Propagator of intermediate resonance

The $(K\pi)_{S\text{-wave}}$ parameterization:

The LASS model used in the analysis of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ of BABAR (PRD 78, 034023),

$$A(m) = F \sin \delta_F e^{i\delta_F} + R \sin \delta_R e^{i\delta_R} e^{i2\delta_F},$$

Scattering term
 $K_0^*(1430)$ term

$$\delta_F = \phi_F + \cot^{-1} \left(\frac{1}{aq} + \frac{rq}{2} \right),$$

$$\delta_R = \phi_R + \tan^{-1} \left(\frac{M\Gamma(m_{K\pi})}{M^2 - m_{K\pi}^2} \right);$$

$M(\text{GeV}/c^2)$	1.463 ± 0.002
$\Gamma(\text{GeV}/c^2)$	0.233 ± 0.005
F	0.80 ± 0.09
ϕ_F	2.33 ± 0.13
R	1(fixed)
ϕ_R	-5.31 ± 0.04
a	1.07 ± 0.11
r	-1.8 ± 0.3

The parameters are fixed to BABAR results.

Angular terms

- The decays of $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ and $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ allow the decay chains shown as topologies A and B.
- The decay of $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ only allows the decay chain of topology B since the contributions from doubly Cabibbo suppressed decays are negligible with our data statistic.

Decay mode	$S(p)$
$D[S] \rightarrow V_1 V_2, V_1 \rightarrow P_1 P_2, V_2 \rightarrow P_3 P_4$	$\tilde{t}^{(1)\mu}(V_1) \tilde{t}_\mu^{(1)}(V_2)$
$D[P] \rightarrow V_1 V_2, V_1 \rightarrow P_1 P_2, V_2 \rightarrow P_3 P_4$	$\epsilon_{\mu\nu\lambda\sigma} p^\mu(D) \tilde{T}^{(1)\nu}(D) \tilde{t}^{(1)\lambda}(V_1) \tilde{t}^{(1)\sigma}(V_2)$
$D[D] \rightarrow V_1 V_2, V_1 \rightarrow P_1 P_2, V_2 \rightarrow P_3 P_4$	$\tilde{T}^{(2)\mu\nu}(D) \tilde{t}_\mu^{(1)}(V_1) \tilde{t}_\nu^{(1)}(V_2)$
$D \rightarrow AP_1, A[S] \rightarrow VP_2, V \rightarrow P_3 P_4$	$\tilde{T}_1^\mu(D) P_{\mu\nu}^{(1)}(A) \tilde{t}^{(1)\nu}(V)$
$D \rightarrow AP_1, A[D] \rightarrow VP_2, V \rightarrow P_3 P_4$	$\tilde{T}^{(1)\mu}(D) \tilde{t}_{\mu\nu}^{(2)}(A) \tilde{t}^{(1)\nu}(V)$
$D \rightarrow AP_1, A \rightarrow SP_2, S \rightarrow P_3 P_4$	$\tilde{T}^{(1)\mu}(D) \tilde{t}_\mu^{(1)}(A)$
$D \rightarrow VS, V \rightarrow P_1 P_2, S \rightarrow P_3 P_4$	$\tilde{T}^{(1)\mu}(D) \tilde{t}_\mu^{(1)}(V)$
$D \rightarrow V_1 P_1, V_1 \rightarrow V_2 P_2, V_2 \rightarrow P_3 P_4$	$\epsilon_{\mu\nu\lambda\sigma} p_V^\mu q_{V_1}^\nu p_{P_1}^\lambda q_{V_2}^\sigma$
$D \rightarrow PP_1, P \rightarrow VP_2, V \rightarrow P_3 P_4$	$p^\mu(P_2) \tilde{t}_\mu^{(1)}(V)$
$D \rightarrow TS, T \rightarrow P_1 P_2, S \rightarrow P_3 P_4$	$\tilde{T}^{(2)\mu\nu}(D) \tilde{t}_{\mu\nu}^{(2)}(T)$

Four body

Three body

$$S_n = 1 \quad (S \text{ wave}),$$

$$S_n = \tilde{T}^{(1)\mu}(D_s) \tilde{t}_\mu^{(1)}(R) \quad (P \text{ wave}),$$

$$S_n = \tilde{T}^{(2)\mu\nu}(D_s) \tilde{t}_{\mu\nu}^{(2)}(R) \quad (D \text{ wave}),$$

- The determination of \tilde{T} and \tilde{t} are the same with **ref. EPJA 16, 537**.
- The amplitudes with angular momentum > 2 are not considered.

Likelihood construction

The signal PDF $f(p_j)$ is given by:

Independent with the floated parameters

$$f(p_j) = \frac{\epsilon(p_j) |M(p_j)|^2 d\phi_n(p_j)}{\int \epsilon(p_j) |M(p_j)|^2 d\phi_n(p_j)},$$

where, $\epsilon(p_j)$ is the efficiency function; $d\phi_n(p_j)$ is the n-body phase space. Then the likelihood is:

$$\ln L = \sum_i^{N_{data}} w_i^{data} \ln f(p_j) + \sum_i^{N_{bkg}} (-w_i^{bkg}) \ln f(p_j),$$

Signal part

Background part

- Background sample can be simulated events or sideband events in data.
- The normalized integration is performed with MC integration method.

Fit Fraction and the statistical uncertainty

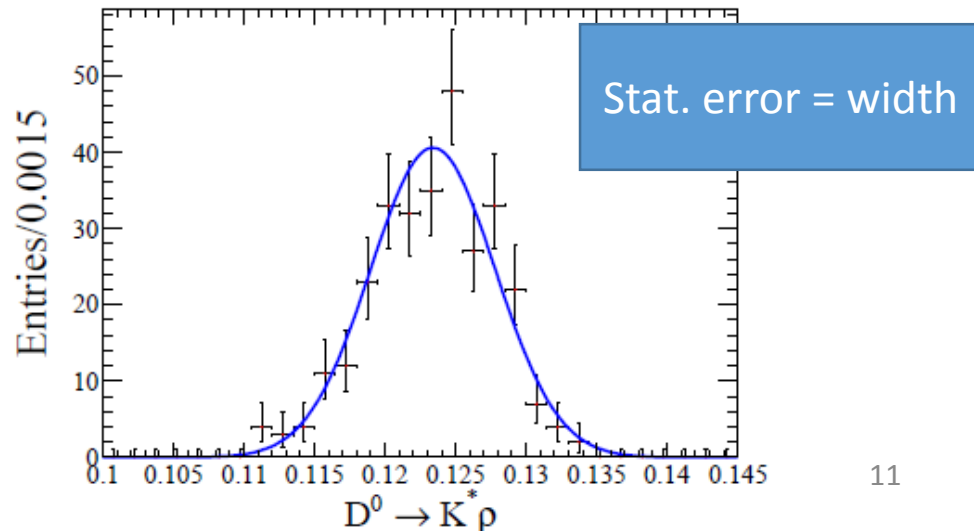
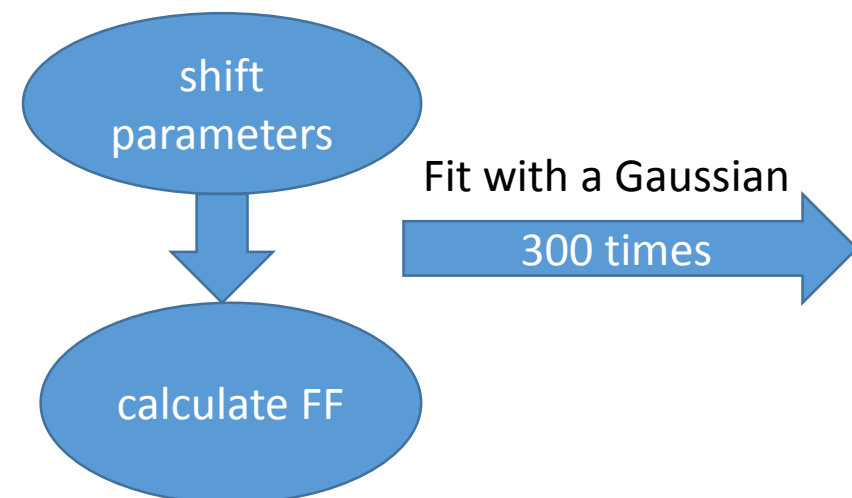
The fit fraction is calculated with MC integration method:

$$FF(n) = \frac{\sum_k^{N_{MC}} |\tilde{A}_n(p_j)|^2}{\sum_k^{N_{MC}} |M(p_j)|^2},$$

- N_{MC} is the number of MC sample events.
- $\tilde{A}_n(p_j)$ is either the n^{th} amplitude ($\tilde{A}_n(p_j) = \rho_n e^{i\phi_n} A_n(p_j)$) or the n^{th} subset (component) of coherent sum of amplitudes ($\tilde{A}_n(p_j) = \sum_{n_l} \rho_{n_l} e^{i\phi_{n_l}} A_{n_l}(p_j)$).

To obtain the statistical uncertainty, the fit results are randomly modified according to the covariance matrix of the fit result. Under the RooFit framework:

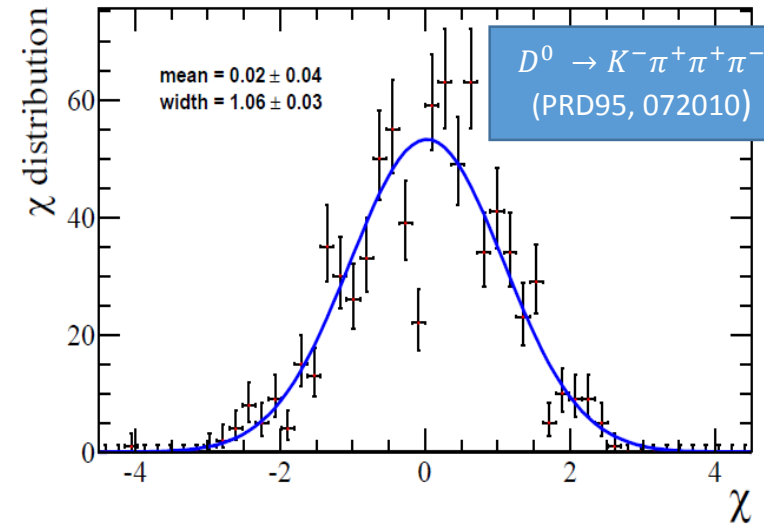
the function: `randomizePars()` returns the randomly shifted values.



Fit quality

Binned method:

- The global χ^2 is calculated in a five-dimensional (two-dimensional) phase space for four-body (three-body) decays: $\chi^2 = \sum_p \chi_p$ and $\chi_p = \frac{(N_p - N_p^{\text{exp}})^2}{N_p^{\text{exp}}}$.
- N_p and N_p^{exp} is the number of data and simulated events in p^{th} cell.
- Cells with expected events less than 20 will be merged with next cell until they satisfy the minimum number of events criterion.



Unbinned method:

A mixed-sample method is used in $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ unbinned likelihood fit to determine the fit quality (M. Williams, Journal of Instrumentation 5, P09004 (2010)).

Recent results for $D_{(S)}$ hadronic decays

Amplitude analysis of $D^0/D^+ \rightarrow K\pi\pi\pi$:

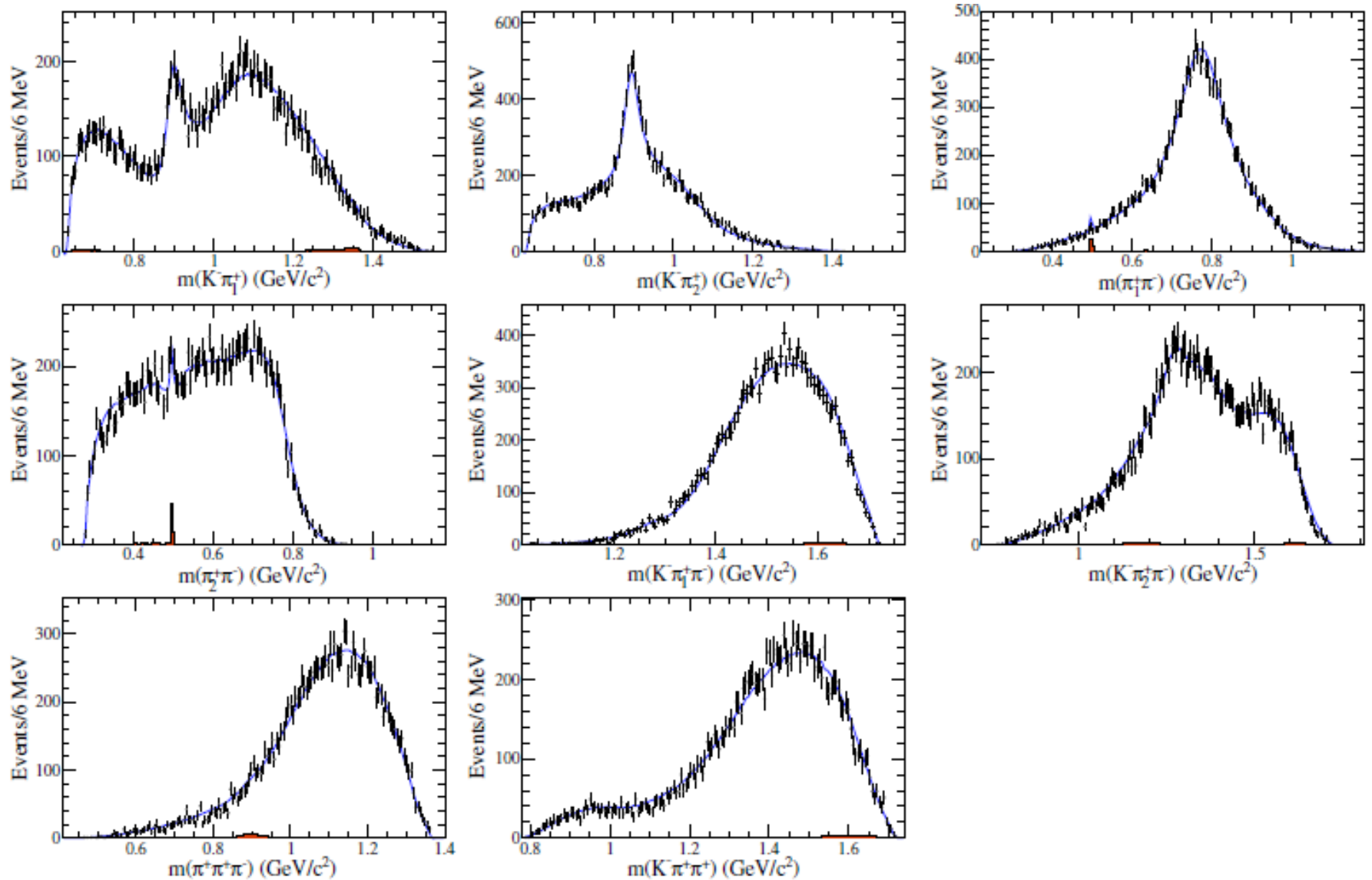
- These decays provide windows to investigate the decay modes $D \rightarrow VV$ and $D \rightarrow AP$ (P : pseudo-scalar, V : vector, A : axial-vector), which are important in searching for CPV and learning $D^0\bar{D}^0$ mixing.
- The amplitude analysis results can also be used in many other experimental measurements:
 - Branching fraction measurement.
 - Strong phase determination (Only for D^0)
 - γ angle determination (Only for D^0)
- There are seven decays of $D^0/D^+ \rightarrow K\pi\pi\pi$, previous amplitude analyses for $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$, $K_S^0\pi^+\pi^-\pi^0$, and $D^+ \rightarrow K_S^0\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^+\pi^0$ have been performed by Mark III and E691. Both measurements are affected by low statistics.
- The results of $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^0\pi^0$, and $D^+ \rightarrow K_S^0\pi^+\pi^+\pi^-$ are presented here.

Result of $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ (PRD95, 072010)

Double tag: $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ vs $\bar{D}^0 \rightarrow K^+ \pi^-$ are reconstructed. About 16K events with a purity $\sim 99.4\%$ are selected for amplitude analysis. Model is constructed with 23 amplitudes.

Amplitude	ϕ_i	Fit fraction (%)
$D^0[S] \rightarrow \bar{K}^* \rho^0$	$2.35 \pm 0.06 \pm 0.18$	$6.5 \pm 0.5 \pm 0.8$
$D^0[P] \rightarrow \bar{K}^* \rho^0$	$-2.25 \pm 0.08 \pm 0.15$	$2.3 \pm 0.2 \pm 0.1$
$D^0[D] \rightarrow \bar{K}^* \rho^0$	$2.49 \pm 0.06 \pm 0.11$	$7.9 \pm 0.4 \pm 0.7$
$D^0 \rightarrow K^- a_1^+(1260), a_1^+(1260)[S] \rightarrow \rho^0 \pi^+$	0(fixed)	$53.2 \pm 2.8 \pm 4.0$
$D^0 \rightarrow K^- a_1^+(1260), a_1^+(1260)[D] \rightarrow \rho^0 \pi^+$	$-2.11 \pm 0.15 \pm 0.21$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \rightarrow K_1^-(1270) \pi^+, K_1^-(1270)[S] \rightarrow \bar{K}^{*0} \pi^-$	$1.48 \pm 0.21 \pm 0.24$	$0.1 \pm 0.1 \pm 0.1$
$D^0 \rightarrow K_1^-(1270) \pi^+, K_1^-(1270)[D] \rightarrow \bar{K}^{*0} \pi^-$	$3.00 \pm 0.09 \pm 0.15$	$0.7 \pm 0.2 \pm 0.2$
$D^0 \rightarrow K_1^-(1270) \pi^+, K_1^-(1270) \rightarrow K^- \rho^0$	$-2.46 \pm 0.06 \pm 0.21$	$3.4 \pm 0.3 \pm 0.5$
$D^0 \rightarrow (\rho^0 K^-)_A \pi^+, (\rho^0 K^-)_A [D] \rightarrow K^- \rho^0$	$-0.43 \pm 0.09 \pm 0.12$	$1.1 \pm 0.2 \pm 0.3$
$D^0 \rightarrow (K^- \rho^0)_P \pi^+$	$-0.14 \pm 0.11 \pm 0.10$	$7.4 \pm 1.6 \pm 5.7$
$D^0 \rightarrow (K^- \pi^+)_{S\text{-wave}} \rho^0$	$-2.45 \pm 0.19 \pm 0.47$	$2.0 \pm 0.7 \pm 1.9$
$D^0 \rightarrow (K^- \rho^0)_V \pi^+$	$-1.34 \pm 0.12 \pm 0.09$	$0.4 \pm 0.1 \pm 0.1$
$D^0 \rightarrow (\bar{K}^{*0} \pi^-)_P \pi^+$	$-2.09 \pm 0.12 \pm 0.22$	$2.4 \pm 0.5 \pm 0.5$
$D^0 \rightarrow \bar{K}^{*0} (\pi^+ \pi^-)_S$	$-0.17 \pm 0.11 \pm 0.12$	$2.6 \pm 0.6 \pm 0.6$
$D^0 \rightarrow (\bar{K}^{*0} \pi^-)_V \pi^+$	$-2.13 \pm 0.10 \pm 0.11$	$0.8 \pm 0.1 \pm 0.1$
$D^0 \rightarrow ((K^- \pi^+)_{S\text{-wave}} \pi^-)_A \pi^+$	$-1.36 \pm 0.08 \pm 0.37$	$5.6 \pm 0.9 \pm 2.7$
$D^0 \rightarrow K^- ((\pi^+ \pi^-)_S \pi^+)_A$	$-2.23 \pm 0.08 \pm 0.22$	$13.1 \pm 1.9 \pm 2.2$
$D^0 \rightarrow (K^- \pi^+)_{S\text{-wave}} (\pi^+ \pi^-)_S$	$-1.40 \pm 0.04 \pm 0.22$	$16.3 \pm 0.5 \pm 0.6$
$D^0[S] \rightarrow (K^- \pi^+)_V (\pi^+ \pi^-)_V$	$1.59 \pm 0.13 \pm 0.41$	$5.4 \pm 1.2 \pm 1.9$
$D^0 \rightarrow (K^- \pi^+)_{S\text{-wave}} (\pi^+ \pi^-)_V$	$-0.16 \pm 0.17 \pm 0.43$	$1.9 \pm 0.6 \pm 1.2$
$D^0 \rightarrow (K^- \pi^+)_V (\pi^+ \pi^-)_S$	$2.58 \pm 0.08 \pm 0.25$	$2.9 \pm 0.5 \pm 1.7$
$D^0 \rightarrow (K^- \pi^+)_T (\pi^+ \pi^-)_S$	$-2.92 \pm 0.14 \pm 0.12$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \rightarrow (K^- \pi^+)_{S\text{-wave}} (\pi^+ \pi^-)_T$	$2.45 \pm 0.12 \pm 0.37$	$0.5 \pm 0.1 \pm 0.1$

Projections of $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ (PRD95, 072010)

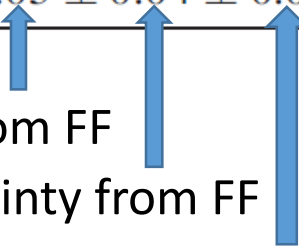


Points with error bars: data, curves: fit, red histograms: background.

The two identical π^+ are required with: $m(\pi_1^+\pi^-) > m(\pi_2^+\pi^-)$

Branching fractions for different components (PRD95, 072010)

Component	Branching fraction (%)	PDG value (%)
$D^0 \rightarrow \bar{K}^{*0} \rho^0$	$0.99 \pm 0.04 \pm 0.04 \pm 0.03$	1.05 ± 0.23
$D^0 \rightarrow K^- a_1^+(1260) (\rho^0 \pi^+)$	$4.41 \pm 0.22 \pm 0.30 \pm 0.13$	3.6 ± 0.6
$D^0 \rightarrow K_1^-(1270) (\bar{K}^{*0} \pi^-) \pi^+$	$0.07 \pm 0.01 \pm 0.02 \pm 0.00$	0.29 ± 0.03
$D^0 \rightarrow K_1^-(1270) (K^- \rho^0) \pi^+$	$0.27 \pm 0.02 \pm 0.04 \pm 0.01$	
$D^0 \rightarrow K^- \pi^+ \rho^0$	$0.68 \pm 0.09 \pm 0.20 \pm 0.02$	0.51 ± 0.23
$D^0 \rightarrow \bar{K}^{*0} \pi^+ \pi^-$	$0.57 \pm 0.03 \pm 0.04 \pm 0.02$	0.99 ± 0.23
$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	$1.77 \pm 0.05 \pm 0.04 \pm 0.05$	1.88 ± 0.26



 Stat. uncertainty from FF

 Sys. uncertainty from FF

 uncertainties related to $B(D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-)$ in PDG

Significantly improved than previous PDG values.

Result of $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ (BESIII Preliminary)

Double tag: $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ vs $\bar{D}^0 \rightarrow K^+ \pi^-$ are reconstructed. About 5.95K events with a purity $\sim 99\%$ are selected for amplitude analysis. Model is constructed with 26 amplitudes.

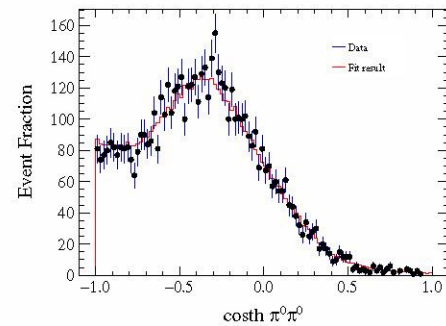
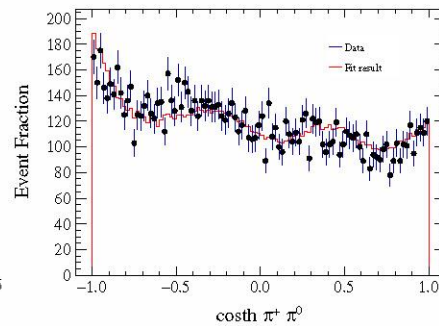
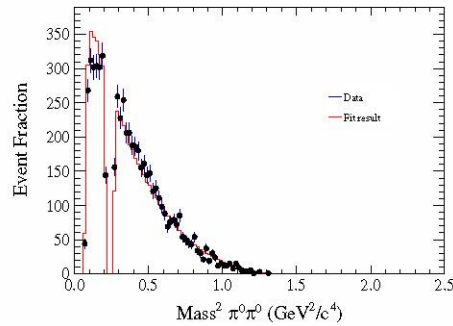
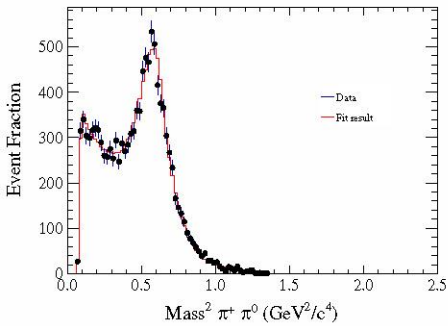
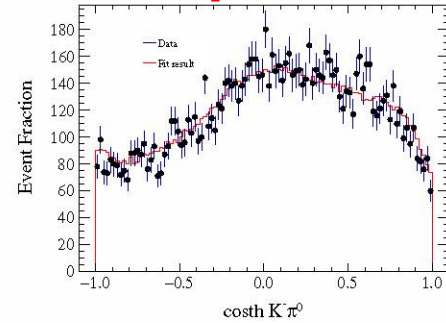
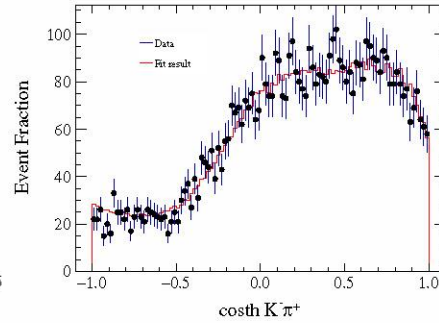
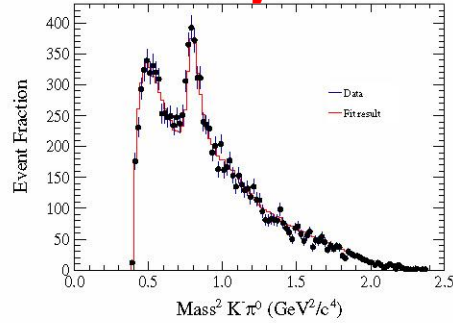
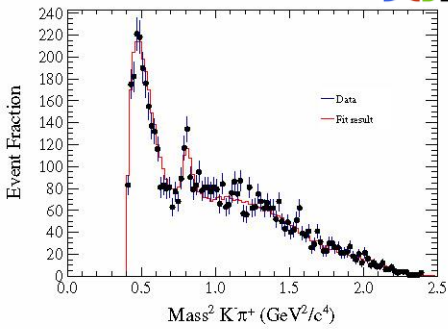
Amplitude mode	FF(%)	Phase (ϕ)
$D \rightarrow SS$		
$D \rightarrow (K^- \pi^+)_{S\text{-wave}}(\pi^0 \pi^0)_S$	$6.92 \pm 1.44 \pm 2.86$	$-0.75 \pm 0.15 \pm 0.47$
$D \rightarrow (K^- \pi^0)_{S\text{-wave}}(\pi^+ \pi^0)_S$	$4.18 \pm 1.02 \pm 1.77$	$-2.90 \pm 0.19 \pm 0.47$
$D \rightarrow AP, A \rightarrow VP$		
$D \rightarrow K^- a_1(1260)^+, \rho^+ \pi^0[S]$	$28.36 \pm 2.50 \pm 3.53$	0 (fixed)
$D \rightarrow K^- a_1(1260)^+, \rho^+ \pi^0[D]$	$0.68 \pm 0.29 \pm 0.30$	$-2.05 \pm 0.17 \pm 0.25$
$D \rightarrow K_1(1270)^- \pi^+, K^{*-} \pi^0[S]$	$0.15 \pm 0.09 \pm 0.18$	$1.84 \pm 0.34 \pm 0.43$
$D \rightarrow K_1(1270)^0 \pi^0, K^{*0} \pi^0[S]$	$0.39 \pm 0.18 \pm 0.30$	$-1.55 \pm 0.20 \pm 0.26$
$D \rightarrow K_1(1270)^0 \pi^0, K^{*0} \pi^0[D]$	$0.11 \pm 0.11 \pm 0.13$	$-1.35 \pm 0.43 \pm 0.48$
$D \rightarrow K_1(1270)^0 \pi^0, K^- \rho^+[S]$	$2.71 \pm 0.38 \pm 0.29$	$-2.07 \pm 0.09 \pm 0.20$
$D \rightarrow (K^{*-} \pi^0)_A \pi^+, K^{*-} \pi^0[S]$	$1.85 \pm 0.62 \pm 1.11$	$1.93 \pm 0.10 \pm 0.15$
$D \rightarrow (K^{*0} \pi^0)_A \pi^0, K^{*0} \pi^0[S]$	$3.13 \pm 0.45 \pm 0.58$	$0.44 \pm 0.12 \pm 0.21$
$D \rightarrow (K^{*0} \pi^0)_A \pi^0, K^{*0} \pi^0[D]$	$0.46 \pm 0.17 \pm 0.29$	$-1.84 \pm 0.26 \pm 0.42$
$D \rightarrow (\rho^+ K^-)_A \pi^0, K^- \rho^+[D]$	$0.75 \pm 0.40 \pm 0.60$	$0.64 \pm 0.36 \pm 0.53$
$D \rightarrow AP, A \rightarrow SP$		
$D \rightarrow ((K^- \pi^+)_{S\text{-wave}} \pi^0)_A \pi^0$	$1.99 \pm 1.08 \pm 1.55$	$-0.02 \pm 0.25 \pm 0.53$
$D \rightarrow VS$		
$D \rightarrow (K^- \pi^0)_{S\text{-wave}} \rho^+$	$14.63 \pm 1.70 \pm 2.41$	$-2.39 \pm 0.11 \pm 0.35$
$D \rightarrow K^{*-}(\pi^+ \pi^0)_S$	$0.80 \pm 0.38 \pm 0.26$	$1.59 \pm 0.19 \pm 0.24$
$D \rightarrow K^{*0}(\pi^0 \pi^0)_S$	$0.12 \pm 0.27 \pm 0.27$	$1.45 \pm 0.48 \pm 0.51$
$D \rightarrow VP, V \rightarrow VP$		
$D \rightarrow (K^{*-} \pi^+)_V \pi^0$	$2.25 \pm 0.43 \pm 0.45$	$0.52 \pm 0.12 \pm 0.17$
$D \rightarrow VV$		
$D[S] \rightarrow K^{*-} \rho^+$	$5.15 \pm 0.75 \pm 1.28$	$1.24 \pm 0.11 \pm 0.23$
$D[P] \rightarrow K^{*-} \rho^+$	$3.25 \pm 0.55 \pm 0.41$	$-2.89 \pm 0.10 \pm 0.18$
$D[D] \rightarrow K^{*-} \rho^+$	$10.90 \pm 1.53 \pm 2.36$	$2.41 \pm 0.08 \pm 0.16$
$D[P] \rightarrow (K^- \pi^0)_V \rho^+$	$0.36 \pm 0.19 \pm 0.27$	$-0.94 \pm 0.19 \pm 0.28$
$D[D] \rightarrow (K^- \pi^0)_V \rho^+$	$2.13 \pm 0.56 \pm 0.92$	$-1.93 \pm 0.22 \pm 0.25$
$D[D] \rightarrow K^{*-}(\pi^+ \pi^0)_V$	$1.66 \pm 0.52 \pm 0.61$	$-1.17 \pm 0.20 \pm 0.39$
$D[S] \rightarrow (K^- \pi^0)_V(\pi^+ \pi^0)_V$	$5.17 \pm 1.91 \pm 1.82$	$-1.74 \pm 0.20 \pm 0.31$
$D \rightarrow TS$		
$D \rightarrow (K^- \pi^+)_{S\text{-wave}}(\pi^0 \pi^0)_T$	$0.30 \pm 0.21 \pm 0.32$	$-2.93 \pm 0.31 \pm 0.82$
$D \rightarrow (K^- \pi^0)_{S\text{-wave}}(\pi^+ \pi^0)_T$	$0.14 \pm 0.12 \pm 0.10$	$2.23 \pm 0.38 \pm 0.65$

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Projections of $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ (BESIII Preliminary)

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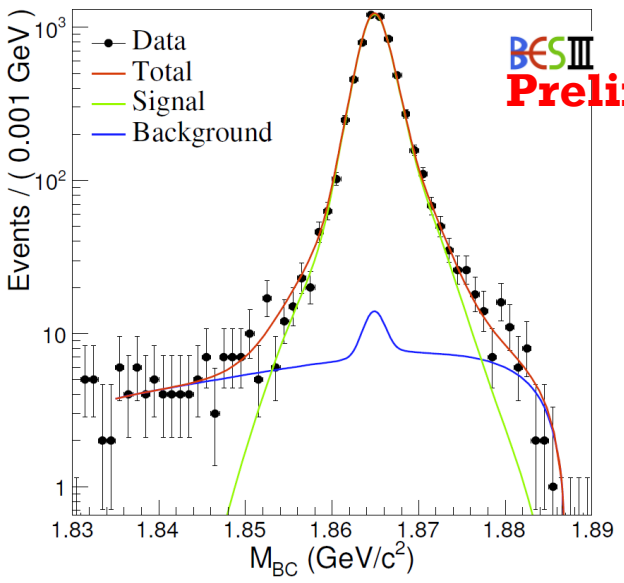
Points with error bars: data, red histograms: fit.

Branching fractions for $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ (BESIII Preliminary)

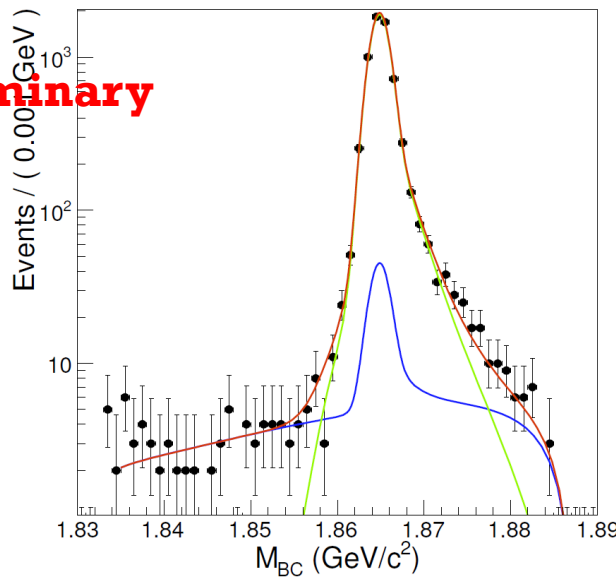
Fits to M_{BC} distributions of DT and ST data:

Beam-Constrained Mass: $M_{BC} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_D|^2}$

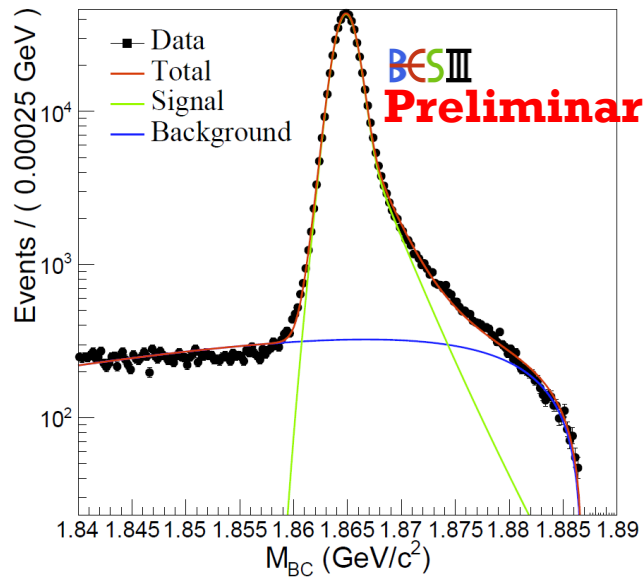
Signal: M_{BC} peaks at D mass



DT ($K^- \pi^+ \pi^0 \pi^0$)



DT ($K^+ \pi^-$)



ST

Yields: DT, 6101 ± 83 ; ST, 534581 ± 769 .

The amplitude analysis results are used to determine the reconstructed efficiency.

First measurement!

$$\mathcal{B}(D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0) = (8.98 \pm 0.13(\text{stat}) \pm 0.40(\text{syst}))\%$$

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Result of $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ (BESIII Preliminary)

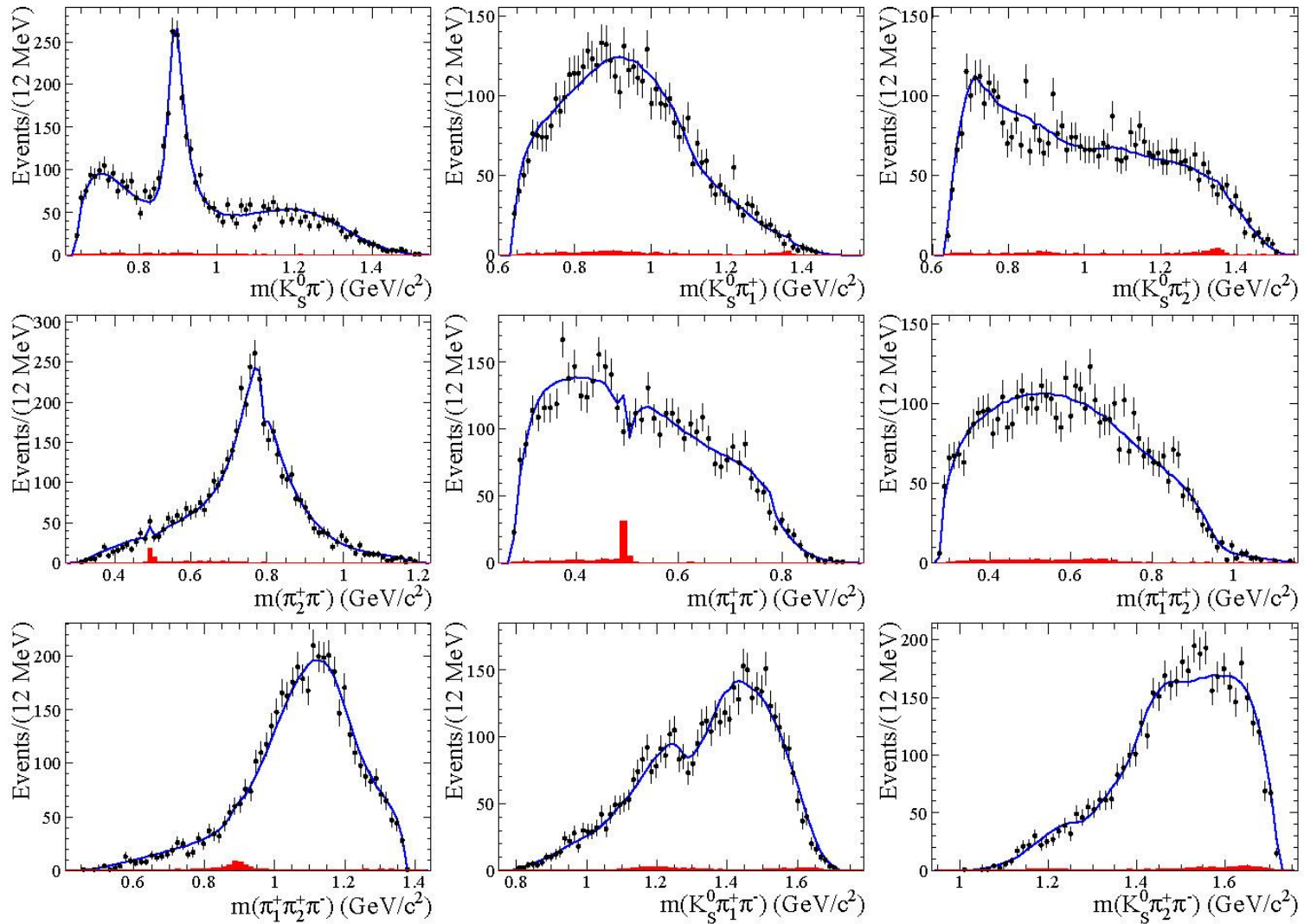
Double tag: $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ vs $D^- \rightarrow K^+ \pi^- \pi^-$ are reconstructed. About 4.56K events with a purity $\sim 99\%$ are selected for amplitude analysis. Model is constructed with 12 amplitudes.

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Amplitude	ϕ	Fit fraction
$D^+ \rightarrow K_S^0 a_1(1260)^+, a_1(1260)^+ \rightarrow \rho^0 \pi^+[S]$	0.000(fixed)	$0.567 \pm 0.020 \pm 0.044$
$D^+ \rightarrow K_S^0 a_1(1260)^+, a_1(1260)^+ \rightarrow f_0(500)\pi^+$	$-2.023 \pm 0.068 \pm 0.113$	$0.050 \pm 0.006 \pm 0.007$
$D^+ \rightarrow \bar{K}_1(1400)^0 \pi^+, \bar{K}_1(1400)^0 \rightarrow K^{*-} \pi^+[S]$	$-2.714 \pm 0.038 \pm 0.051$	$0.380 \pm 0.013 \pm 0.014$
$D^+ \rightarrow \bar{K}_1(1400)^0 \pi^+, \bar{K}_1(1400)^0 \rightarrow K^{*-} \pi^+[D]$	$3.431 \pm 0.137 \pm 0.117$	$0.015 \pm 0.004 \pm 0.005$
$D^+ \rightarrow \bar{K}_1(1270)^0 \pi^+, \bar{K}_1(1270)^0 \rightarrow K_S^0 \rho^0[S]$	$-0.418 \pm 0.070 \pm 0.087$	$0.036 \pm 0.004 \pm 0.002$
$D^+ \rightarrow \bar{K}(1460)^0 \pi^+, \bar{K}(1460)^0 \rightarrow K_S^0 \rho^0$	$-1.850 \pm 0.120 \pm 0.223$	$0.014 \pm 0.004 \pm 0.003$
$D^+ \rightarrow (K_S^0 \rho^0)_A [D] \pi^+$	$2.328 \pm 0.097 \pm 0.068$	$0.011 \pm 0.003 \pm 0.002$
$D^+ \rightarrow K_S^0 (\rho^0 \pi^+)_P$	$1.656 \pm 0.083 \pm 0.056$	$0.031 \pm 0.004 \pm 0.010$
$D^+ \rightarrow (K^{*-} \pi^+)_A [S] \pi^+$	$1.962 \pm 0.047 \pm 0.073$	$0.132 \pm 0.011 \pm 0.011$
$D^+ \rightarrow (K^{*-} \pi^+)_A [D] \pi^+$	$0.989 \pm 0.158 \pm 0.229$	$0.013 \pm 0.004 \pm 0.004$
$D^+ \rightarrow (K_S^0 (\pi^+ \pi^-)_S)_A \pi^+$	$-2.935 \pm 0.060 \pm 0.125$	$0.051 \pm 0.004 \pm 0.003$
$D^+ \rightarrow ((K_S^0 \pi^-)_S \pi^+)_P \pi^+$	$1.864 \pm 0.069 \pm 0.288$	$0.022 \pm 0.003 \pm 0.003$

Projections of $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ (BESIII Preliminary)

BESIII Preliminary



Points with error bars: data, red histograms: fit, green histograms: background estimated from MC.

The two identical π^+ are required with: $m(\pi_1^+ \pi^-) < m(\pi_2^+ \pi^-)$.

Branching fractions for different components (BESIII Preliminary)

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Component	Branching fraction (%)
$D^+ \rightarrow K_S^0 a_1(1260)^+ (\rho^0 \pi^+)$	$1.684 \pm 0.059 \pm 0.131 \pm 0.062$
$D^+ \rightarrow K_S^0 a_1(1260)^+ (f_0(500) \pi^+)$	$0.149 \pm 0.018 \pm 0.021 \pm 0.006$
$D^+ \rightarrow \bar{K}_1(1400)^0 (K^{*-} \pi^+) \pi^+$	$1.105 \pm 0.045 \pm 0.048 \pm 0.041$
$D^+ \rightarrow \bar{K}_1(1270)^0 (K_S^0 \rho^0) \pi^+$	$0.107 \pm 0.012 \pm 0.006 \pm 0.004$
$D^+ \rightarrow \bar{K}(1460)^0 (K_S^0 \rho^0) \pi^+$	$0.042 \pm 0.012 \pm 0.009 \pm 0.002$
$D^+ \rightarrow K_S^0 \pi^+ \rho^0$	$0.131 \pm 0.015 \pm 0.015 \pm 0.005$
$D^+ \rightarrow K^{*-} \pi^+ \pi^+$	$0.413 \pm 0.036 \pm 0.059 \pm 0.015$
$D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$	$0.220 \pm 0.015 \pm 0.024 \pm 0.008$

Stat. uncertainty from FF

Sys. uncertainty from FF

uncertainties related to $B(D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-)$ in PDG

Amplitude analysis of $D_S^+ \rightarrow \pi^+ \pi^0 \eta$:

- Extract the branching fraction of the W -annihilation process involved decay $D_S^+ \rightarrow \rho^+ \eta$.
- Improve the precise of $B(D_S^+ \rightarrow \pi^+ \pi^0 \eta)$.
- A Cabibbo favored decay with large branching fraction but poor precision (PDG: $(9.2 \pm 1.2)\%$)
- Search for the pure W -annihilation decay $D_S^+ \rightarrow a_0(980)\pi$.
- The pure W -annihilation decay $D_S^+ \rightarrow \omega \pi^+$ has been observed with BF $\sim 0.1\%$.
- Theoretical analyses based on previous measurement show the size of W -annihilation amplitude is about 0.1-0.2 of the size of tree-emission amplitude in the decay modes of $D \rightarrow PP$ and VP .
- No experimental or theoretical studies on W -annihilation processes are performed for the decay mode $D \rightarrow SP$ (S: scalar)

(The W -annihilation processes, induced by FSI in charmed decays, can not be calculated with perturbative theory. Determination W -annihilation amplitude with the experimental input is the only method.)

Result of $D_S^+ \rightarrow \pi^+ \pi^0 \eta$ (BESIII Preliminary)

Events are selected with double tag:

- Tag modes: $D_S^- \rightarrow K_S^0 K^-$, $D_S^- \rightarrow K^+ K^- \pi^-$, $D_S^- \rightarrow K_S^0 K^- \pi^0$, $D_S^- \rightarrow K^+ K^- \pi^- \pi^0$, $D_S^- \rightarrow K_S^0 K^+ \pi^- \pi^-$, $D_S^- \rightarrow \pi^- \eta_{\gamma\gamma}$, and $D_S^- \rightarrow \pi^- \eta'_{\pi^+ \pi^- \eta}$.
- Signal mode: $D_S^+ \rightarrow \pi^+ \pi^0 \eta$.
- Multi-variate analysis is performed to suppress the background from fake η .

1239 events are selected with a purity of $(97.7 \pm 0.5)\%$ for amplitude analysis.

The significances, phases, and FFs for intermediate processes obtained from the amplitude analysis.

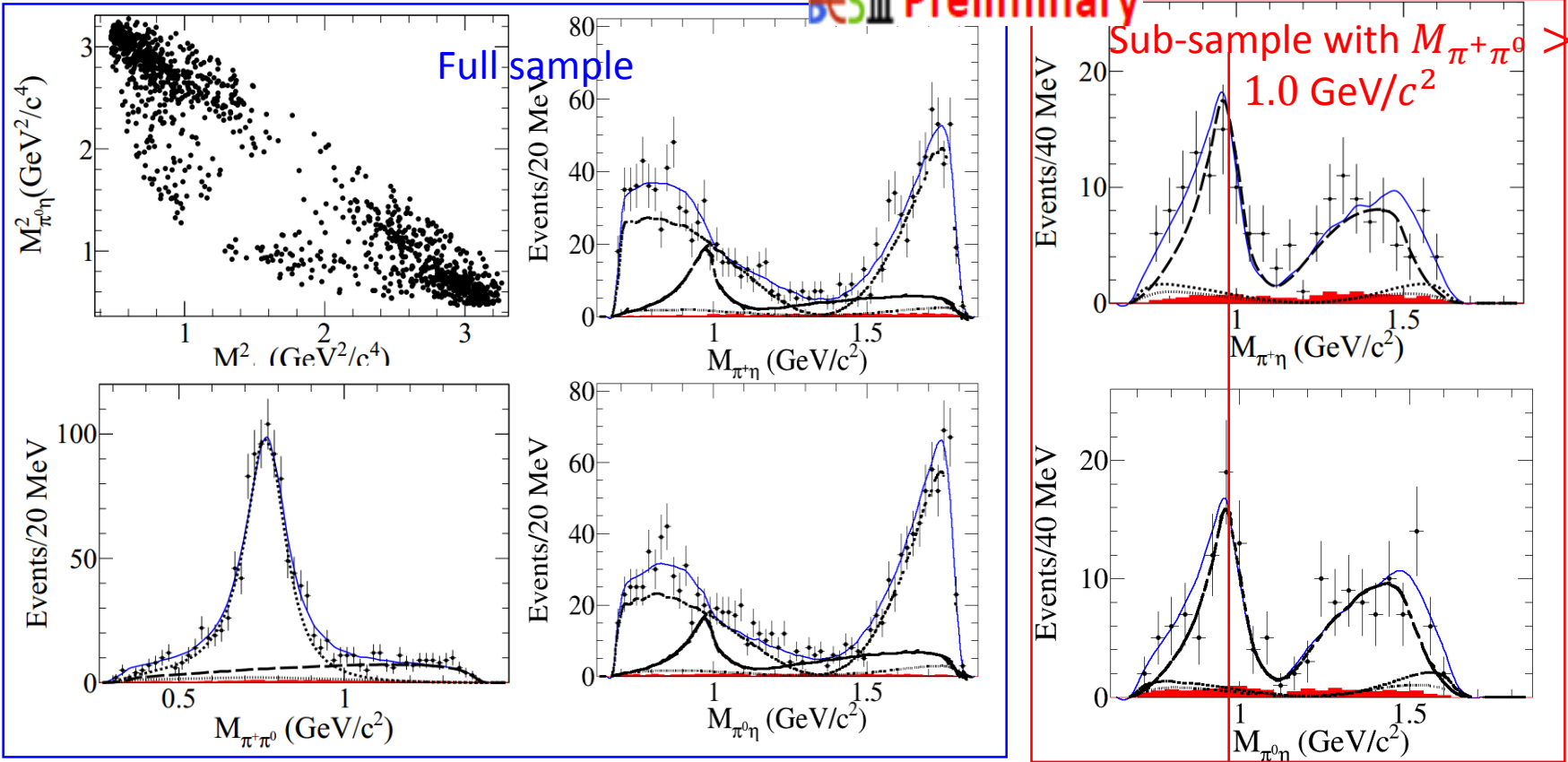
Amplitude	Significance (σ)	Phase	FF
$D_S^+ \rightarrow \rho^+ \eta$	> 20	0.0 (fixed)	$0.783 \pm 0.050 \pm 0.021$
$D_S^+ \rightarrow (\pi^+ \pi^0)_V \eta$	5.7	$0.612 \pm 0.172 \pm 0.342$	$0.054 \pm 0.021 \pm 0.026$
$D_S^+ \rightarrow a_0(980) \pi$	16.2	$2.794 \pm 0.087 \pm 0.041$	$0.232 \pm 0.023 \pm 0.034$

The amplitudes agree with the isospin conserved expectation ($A(D_S^+ \rightarrow a_0(980)^+ \pi^0) = -A(D_S^+ \rightarrow a_0(980)^0 \pi^+)$) within statistical uncertainty, thus we set the magnitudes of these two amplitudes to be the same and the phase difference to be π .

Projections of $D_s^+ \rightarrow \pi^+ \pi^0 \eta$ (BESIII Preliminary)

Dalitz plot and projections

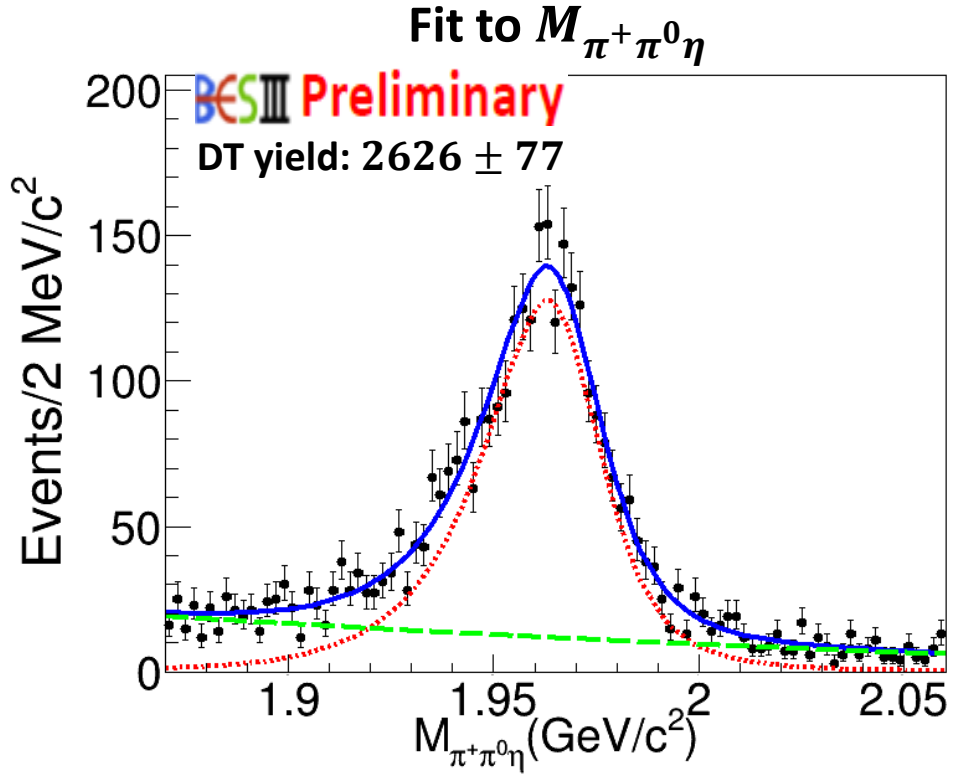
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Dots with error bar: data; **solid**: total fit; dashed: $D_s^+ \rightarrow \rho^+ \eta$; dotted: $D_s^+ \rightarrow (\pi^+ \pi^0)_V \eta$; long dashed: $D_s^+ \rightarrow a_0(980) \pi$.

Obvious peaks for two $a_0(980)$ mesons!

Branching fraction measurement (BESIII Preliminary)



- Dots with error bars: data.
- Total fit.
- Signal: MC shape convoluted with a Gaussian.
- Background: second-order Chebychev.

Efficiency is determined with the amplitude analysis result.

$$BF(D_s^+ \rightarrow \pi^+\pi^0\eta) = (9.50 \pm 0.28_{stat.} \pm 0.41_{sys.})\% \text{ BESIII Preliminary}$$

The systematic uncertainty on $BF(D_s^+ \rightarrow \pi^+\pi^0\eta)$ is dominated by π^0 and η reconstruction (4%).

Branching fraction measurement (BESIII Preliminary)

For the n^{th} amplitude, the BF can be calculated with:

$$BF(n) = BF(D_s^+ \rightarrow \pi^+ \pi^0 \eta) FF(n)$$

Branching fraction	This measurement (%)	PDG value (%)
$BF(D_s^+ \rightarrow \rho^+ \eta)$	$7.44 \pm 0.48_{stat.} \pm 0.44_{sys.}$	8.9 ± 0.9
$BF(D_s^+ \rightarrow a_0(980)\pi)^*$	$2.20 \pm 0.22_{stat.} \pm 0.34_{sys.}$	-
$BF(D_s^+ \rightarrow a_0(980)^+ \pi^0)^*$	$1.46 \pm 0.15_{stat.} \pm 0.22_{sys.}$	-
$BF(D_s^+ \rightarrow a_0(980)^0 \pi^+)^*$		-

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*Here, $a_0(980) \rightarrow \pi\eta$.

- The $BF(D_s^+ \rightarrow a_0(980)^{+/-0} \pi^{0/+})$ is larger than the PDG values of $BF(D_s^+ \rightarrow \omega \pi^+)$ ($(2.4 \pm 0.6) \times 10^{-3}$) and $BF(D_s^+ \rightarrow p \bar{n})$ ($(1.3 \pm 0.4) \times 10^{-3}$) by one order of magnitude.
- The magnitude for the ratio of the W -annihilation amplitude over tree-emission amplitude $|A/T|$ is obtained to be 0.84 ± 0.23 in the decay mode of $D \rightarrow SP$, which is much larger than the level of 0.1-0.2 in the decay mode of $D \rightarrow VP$ (PRD 93, 114010).
- The contribution from W -annihilation processes should be taken into considered in investigating the $D \rightarrow SP$ decays.
- This provides theoretical challenge in understanding such a large W -annihilation contribution in $D \rightarrow SP$ decays.

Summary

- BESIII provides large data samples close to charm related threshold to study the $D_{(s)}$ multi-body hadronic decays.
- Amplitude analysis is a powerful method to investigate the multi-body decays and to extract the physical information.
- Three amplitude analysis results for $D \rightarrow K\pi\pi\pi$ and one for D_s^+ decay are presented.
- Amplitude analyses for $D^0 \rightarrow K^-\pi^+\pi^0\pi^0$ and $D_s^+ \rightarrow \pi^+\pi^0\eta$ are firstly performed.
 - With the amplitude analysis result, the $BF(D^0 \rightarrow K^-\pi^+\pi^0\pi^0)$ is firstly measured to be $(8.98 \pm 0.13_{stat.} \pm 0.40_{sys.})\%$.
 - The precision of $BF(D_s^+ \rightarrow \pi^+\pi^0\eta)$ is improved with a factor of 2.5 to the PDG values.
 - The decays $BF(D_s^+ \rightarrow a_0(980)^+\pi^0)$ and $BF(D_s^+ \rightarrow a_0(980)^0\pi^+)$ via $a_0(980) \rightarrow \pi\eta$ are firstly observed and measured to be $(1.46 \pm 0.15_{stat.} \pm 0.22_{sys.})\%$
- More results for $D_{(s)}$ multi-body hadronic decays are coming.

Thank you!