Glueballs on Lattice

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Outline

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II. A brief introduction to lattice QCD

III. Glueballs spectrum from Lattice QCD

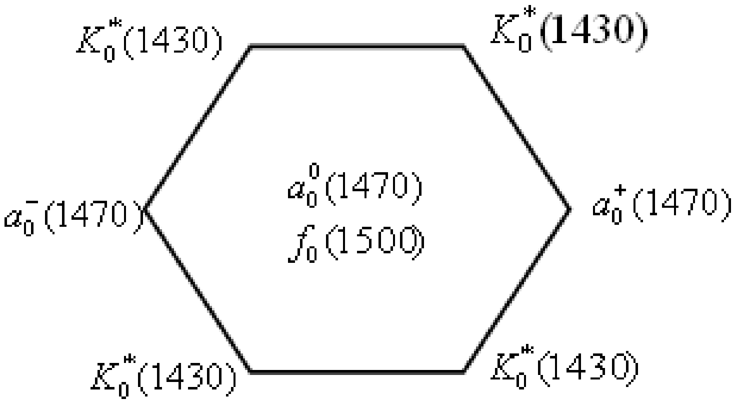
IV. Prodcution rates of glueballs in J/psi radiative decays

V. Summary

I. Experimental status of glueballs

1. Scalar mesons

$q\bar{q}$??



$+$ $f_0(1370)$

$f_0(1710)$

These ten scalar mesons can be assigned as a q-barq meson nonet plus a possible scalar glueball which can be either one of the three isoscalars or an admixture of them. There are Many mixing models, the details are out of the scope of this talk.

2. Pseudoscalar glueball candidate

$$\eta(1295)/\eta(1405)/\eta(1475)???$$

If only two states, there is unnecessarily a glueball candidate here.

3. Tensor glueball candidate ???

- **Previous $\xi(2230)$ observed by BES, but not confirmed by BESII and BESIII**

- **BESIII new results for** $\boxed{J/\psi \to \gamma\eta\eta}$

(M. Ablikim et al. (BES Collaboration), Phys. Rev. D 87, 092009 (2013) (arXiv:1301.0053)

Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	$8.2\ \sigma$
$f_0(1710)$	$1759\pm6^{+14}_{-25}$	$172\pm10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	$25.0\ \sigma$
$f_0(2100)$	$2081\pm13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	$13.9\ \sigma$
$f_2'(1525)$	$1513\pm5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	$11.0\ \sigma$
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	$6.4\ \sigma$
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	$7.6\ \sigma$

- **BESIII new results for** $J/\psi \to \gamma\phi\phi$

(M. Ablikim et al. (BES Collaboration), Phys. Rev. D 93, 112011 (2016) (arXiv:1602.01523)

TABLE I. Mass, width, $\mathcal{B}(J/\psi \to \gamma X \to \gamma\phi\phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.(×10^{-4})	Sig.
$\eta(2225)$	$2216^{+4}_{-5}{}^{+21}_{-11}$	$185^{+12}_{-14}{}^{+43}_{-17}$	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	$2050^{+30}_{-24}{}^{+75}_{-26}$	$250^{+36}_{-30}{}^{+181}_{-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15}_{-19}{}^{+101}_{-23}$	$230^{+64}_{-35}{}^{+56}_{-33}$	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

II. A brief introduction to lattice QCD

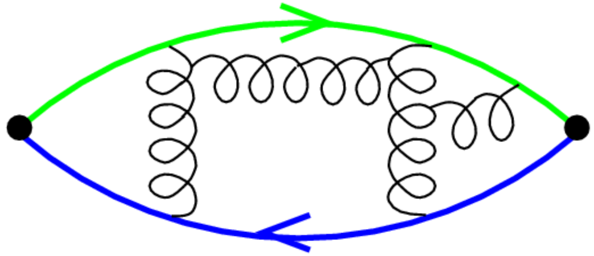
The lattice formulation of QCD---Lattice QCD

$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, e^{-S}$$

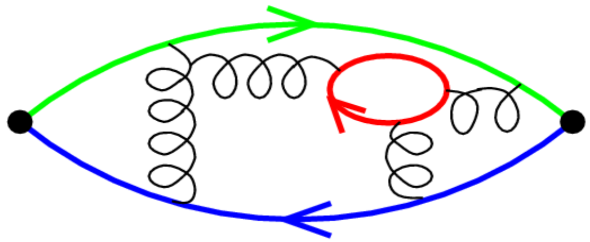
$$S = S_{gauge} + S_{quarks} = \int d^4x \, \left(\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det}\, M_i)$$

$$Z = \int \mathcal{D}A_\mu \, \det M \; e^{\int d^4x \, \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)}.$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S} \, .$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

The methods for the hadron spectroscoapy in lattice QCD

- Interpolation field operators --- starting point
  for a meson (-like) system with given J^{PC} and flavor quantrum numbers:

$$\mathcal{O}_i: \quad \bar{q}_1\Gamma q_2 \quad [\bar{q}_1\Gamma_1 q\,][\bar{q}\Gamma_2 q_2\,] \quad [q_1^T\Gamma_1 q][\bar{q}\Gamma_2\bar{q}_2^T\,], \ldots\ldots$$

- Two-point functions --- Observables

$$\mathcal{C}_{ij}(t) = \left\langle 0\middle|\mathcal{O}_i(t)\mathcal{O}_j^+(0)\middle|0\right\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle \left\langle n\middle|\mathcal{O}_j^+\middle|0\right\rangle e^{-E_n t}$$

In principle, all the physical states with the same quantum numbers $|n\rangle$ contribute to the two point functions $\mathcal{C}_{ij}(t)$ as the eigenstates of the QCD Hamiltonian with the energy eigenvalue E_n:

- " one-particle state" : $E_n = m_n$
- " two-particle state" : $E_n = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + \vec{p}^2} + \Delta E, \; \vec{p} = \frac{2\pi}{L}\vec{n}$
- ……

Comparison of the hadron spectra

Euclidean spacetime lattice		Minkowski continuum spacetime
One particle states	⟷	Stable particles
Multiple particle states with discrete relative spatial Momentum (scattering States in a finite volume	→	Bound states of hadrons Resonances
		Continuum scattering states

All the energies are Discretized.

Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan \delta(p) = \frac{\sqrt{\pi}\; p\; L}{2\; \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

Resonances

$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}, \quad \frac{p^{2l+1}}{\sqrt{s}} \cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$

Bound states

$$p \cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2, \quad -|p_B| = \frac{1}{a_0} - \frac{1}{2} r_0 |p_B|^2$$

$$T = \frac{}{\cot(\delta_l(p_B)) - i} = \infty$$

$$m_B = E_{H_1}(p_B) + E_{H_2}(p_B), \; p_B = i|p_B|$$

The situation will be much more complicated if the multi-channel coupling is considered and if there are three particle interactions.

QCD in quenched approximation (QQCD) vs. Full-QCD (for glueball relevant studies)

QQCD:

- **not a unitary (physical) theory**
- **the systematical uncertainties due to the neglect of sea quarks are not under control**

- **glueballs are well-defined objects**
- **large statistics can be easily achieved**

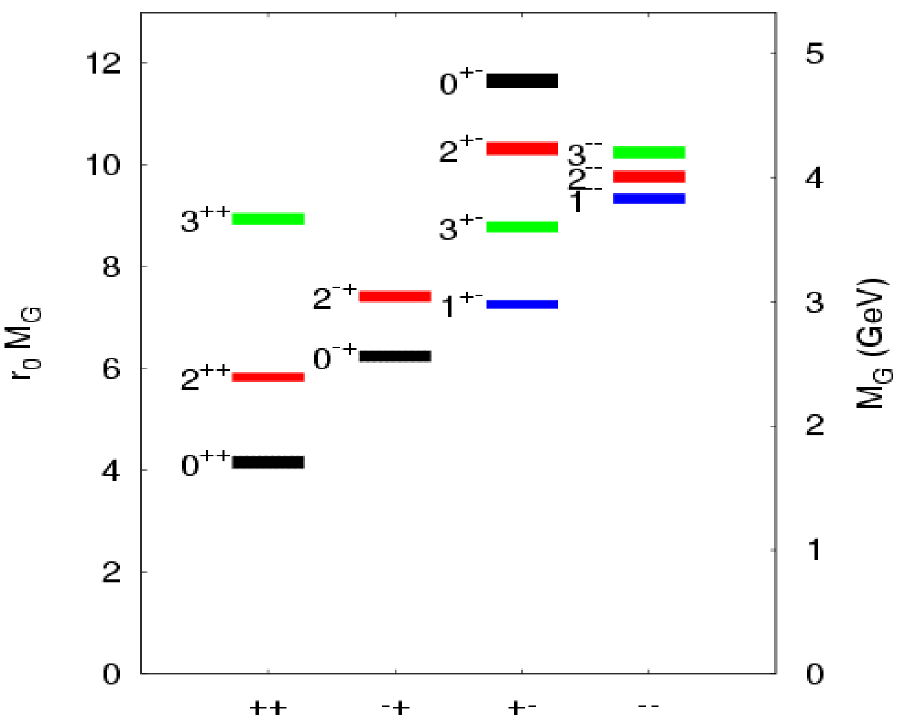
Full-QCD:

- **most of hadrons are observed as resonances**
- **how to define a glueball state, even a qqbar meson?**
- **if it can be defined in some theoretical picture**
- **then the mixing between glueballs and conventional mesons should not be neglected**
- **A more rigorous treatment should be done in the framework of hadron-hadron scattering.**
- **Far beyond the capability of present lattice QCD calculation**

III. Glueball spectrum from lattice QCD

I). Quenched LQCD predicts glueball spectrum

Lowest-lying glueballs have masses in the range 1~3GeV



J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al, Phys. Rev. D 73, 014516 (2006)

II). Lattice QCD calculations with dynamical quarks (full QCD)

- **Recently, we generated gauge ensembles with Nf=2 clover Wison fermions on anisotropic lattices**

  [W. Sun et al (CLQCD), Chin. Phys. C (in press) ， arXiv:1702.08174(hep-lat)]

Table 1. Parameters of configurations

m_π	β	$L^3 \times T$	ξ	a_s	N_{conf}
~ 650MeV	2.5	$12^3 \times 128$	5	$0.114fm$	4800
~ 938MeV	2.5	$12^3 \times 128$	5	$0.118fm$	10400

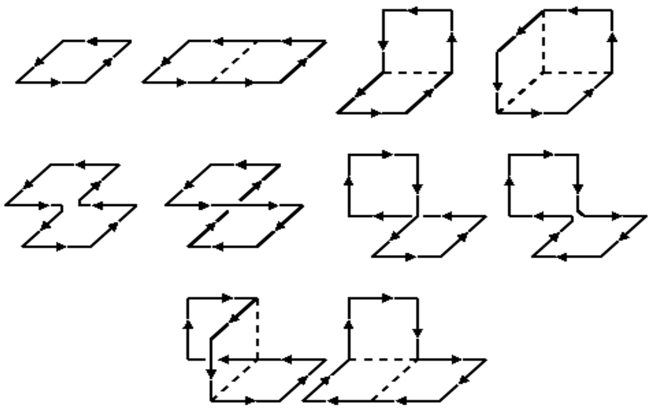
Gauge action: Tadpole improved Symanzik's action
Fermion action: Wilson clover action

The pion masses are still very large.
The physical volumes are very small.
The statistics is relatively large.

The lattice spacings have explicit quark mass dependences.

- **Gluonic operators in the scalar, tensor, and pseudoscalar channels.**

	Continuum limit	Finite lattice
Symmetry Group	$SO(3) \otimes P \otimes T$	$O \otimes P \otimes T$
Irreducible Representation (R)	J^{PC}, $J = 0,1,2,\ldots$	R^{PC}, $R = A_1, A_2, E, T_1, T_2$



C.Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999

- **Use these Wilson loops as prototypes**
- **They can be contructed through smeared gauge links**
- **Different irreps can be realized through proper linear conbinations of the different spatial orientation of these prototypes.**
- **Finally, one can build a set of operators for a specific quantum number** R^{PC} **.**

Solving the generalized eigenvalue problem (GEVP)

The essence of the VM is to find a set of combinational coefficients

$$\{v_\alpha, \alpha = 1,2,...24\}$$

such that the operator

$$\Phi = \sum_\alpha v_\alpha \phi_\alpha$$

couples mostly to a specific state.

$$\tilde{C}(t_D)\mathbf{v}^{(R)} = e^{-t_D \tilde{m}(t_D)} \tilde{C}(0)\mathbf{v}^{(R)}$$

$$\tilde{C}_{\alpha\beta}(t) = \sum_\tau \langle 0|\phi_\alpha(t+\tau)\phi_\beta(\tau)|0\rangle$$

$$\tilde{m}(t_D) = -\frac{1}{t_D} \ln \frac{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(t_D)}{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(0)}$$

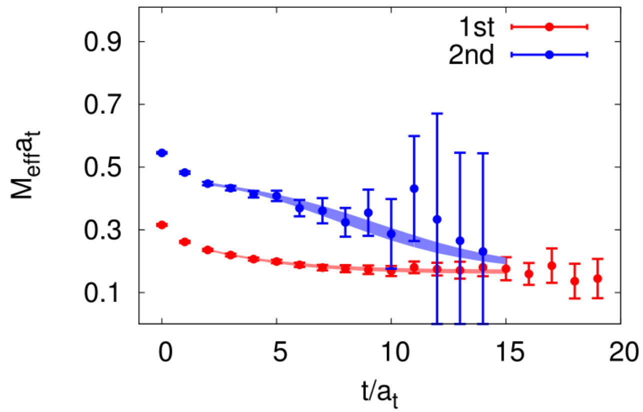
These techniques have been successfully implemeted previous quenched studies.

C.Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999
Y. Chen et al, Phys. Rev. D 73, 014516, 2006

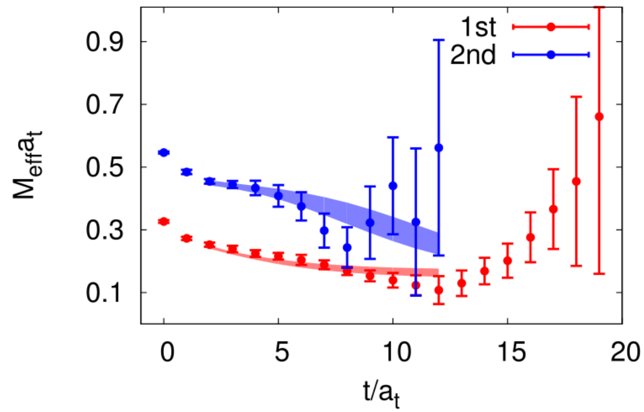
$$\tilde{C}_1^{(R)}(t) = W_{11}^{(R)} e^{-m_1 t} + W_{12}^{(R)} e^{-m_2 t},$$
$$\tilde{C}_2^{(R)}(t) = W_{21}^{(R)} e^{-m_1 t} + W_{22}^{(R)} e^{-m_2 t},$$

A_1^{++}

0^{++}



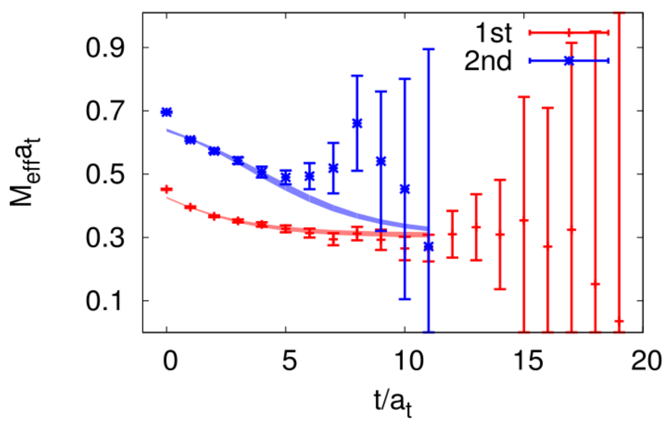
(a) $m_\pi \sim 938$ MeV



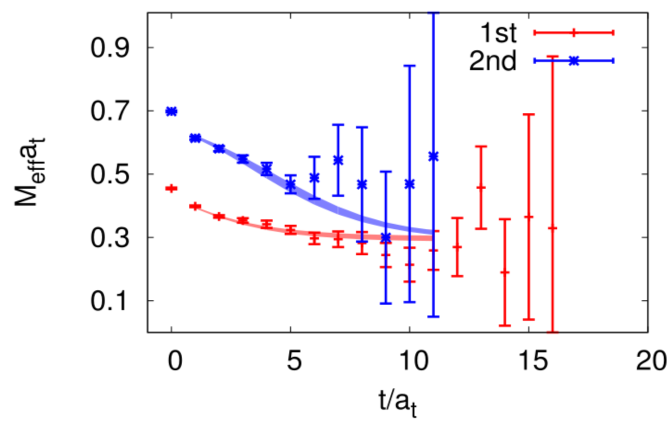
(b) $m_\pi \sim 650$ MeV

A_1^{-+}

0^{-+}

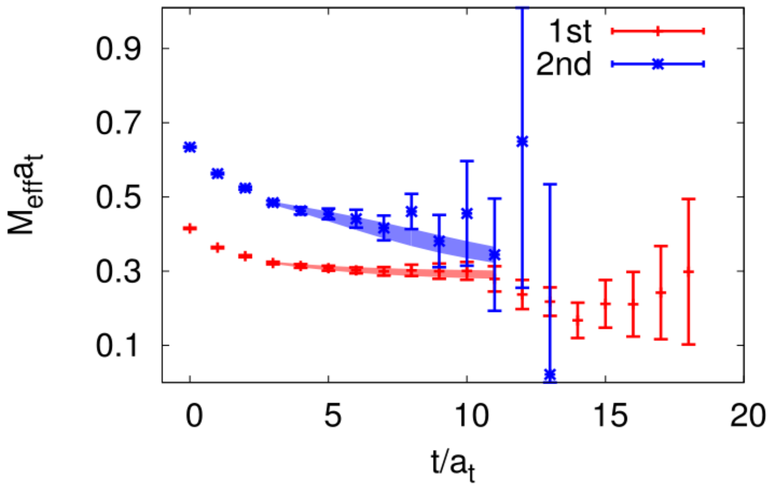


(a) $m_\pi \sim 920 MeV$



(b) $m_\pi \sim 580 Mev$

E^{++} T_2^{++} 2^{++}



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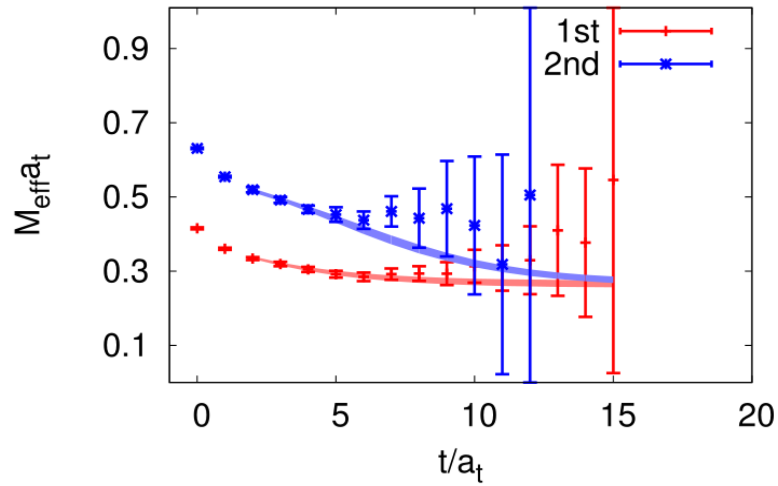

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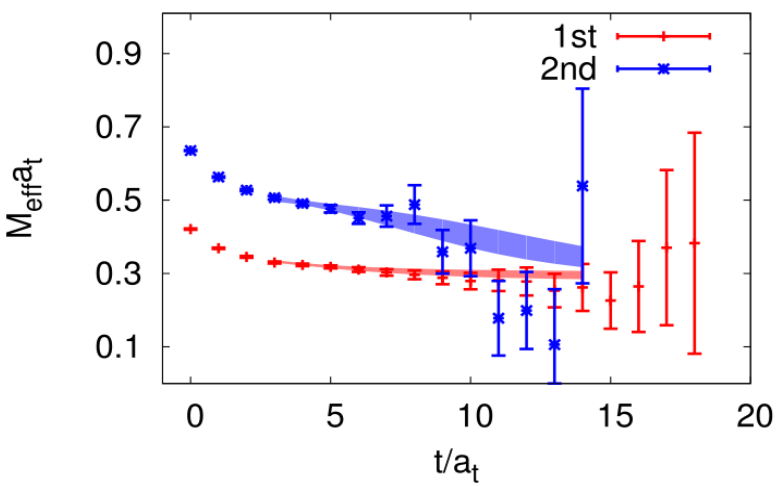

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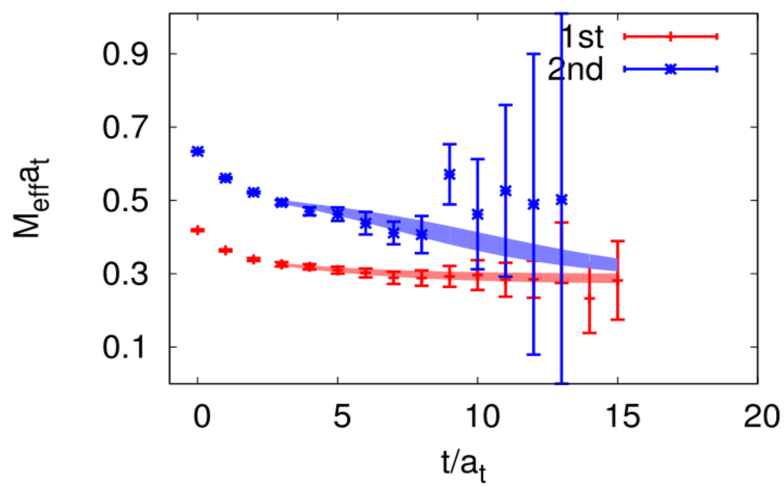


(b) $m_\pi \sim 580Mev$

- ## Comparison with previous results

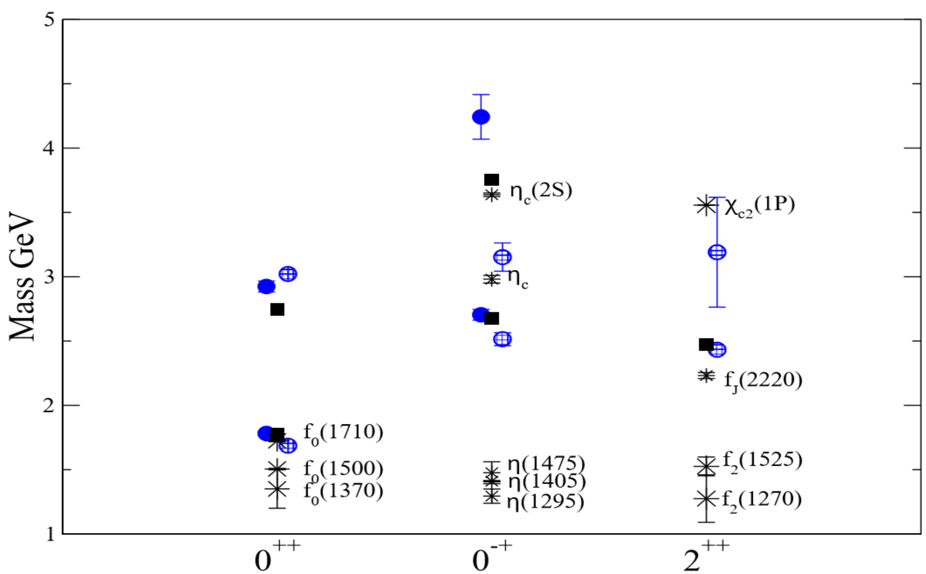
	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1417(30)	2363(39)	2573(55)
	650	1498(58)	2384(67)	2585(65)
$N_f = 2+1$ [22]	360	1795(60)	2620(50)	—
quenched [13]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [14]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

Nf=2: W. Sun et al (CLQCD), arXiv:1702.08174(hep-lat)

[14] C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999

[13] Y. Chen et al, Phys. Rev. D 73, 014516, 2006

[22] E. Gregory et al., JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)



Filled Squares： QQCD
Open circles: full QCD, coarse lattice
Closed circles: full QCD, fine lattice

C.M. Richards et al., [UKQCD Collab.], Phys. Rev. D82, 034501 (2010).

- **More discussion on flavor singlet pseudoscalars**

$\eta'(\eta_2)$ *v.s.* pseudoscalar glueball

The pseudoscalar operators

- $U(1)_A$ anomaly gives

$$\partial_\mu A^\mu(x) = 2mP(x) - N_f q(x)$$

- P(x):flavor singlet pseudoscalar density

$$P(x) = \psi\bar{(x)}\gamma_5\psi(x)$$

- q(x):topological charge density

$$q(x) = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} TrF_{\mu\nu}F_{\rho\sigma}$$

- correlation functions of $P(x)$, $q(x)$(this work) and conventional glueball operator O_G(this work)

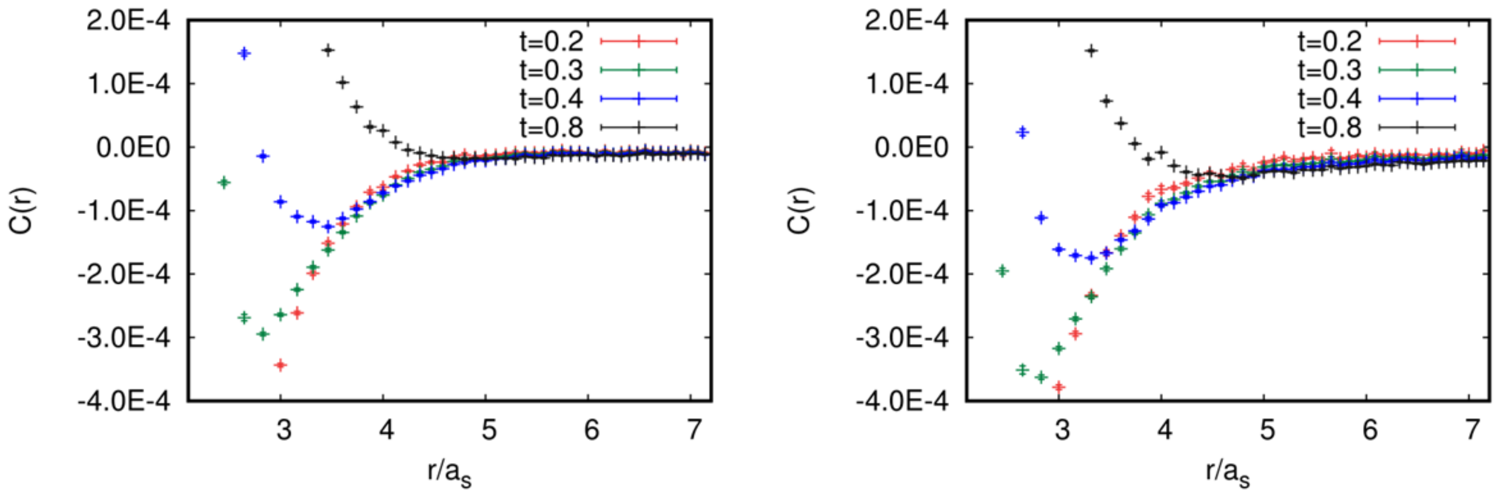
- **More discussions on the flavoer singlet pseudoscalars**

$U_A(1)$ **Anomaly** $$\partial_\mu A^\mu(x) = 2mP(x) - \frac{N_f}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma} TrF_{\mu\nu}F_{\rho\sigma}$$

Topological charge density $$q(x) = -\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma} TrF^{\mu\nu}(x)F^{\rho\sigma}(x)$$

$$C_q(x-y) = \langle q(x)q(y)\rangle$$

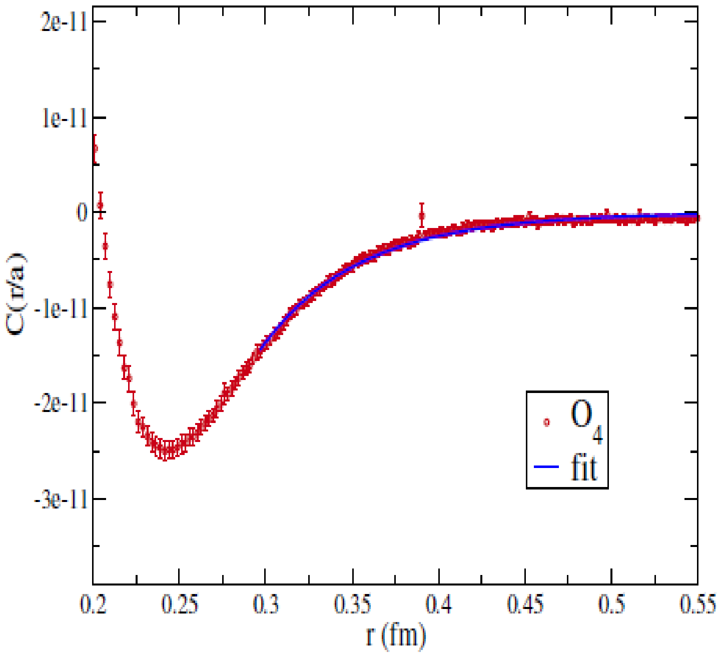
$$C_q(x-y) = A\delta^4(x-y) + \bar{C}_q(x-y), \qquad \bar{C}_q(r) = N\frac{m_{\rm PS}}{4\pi^2 r}K_1(m_{\rm PS}r)$$



(a) $m_\pi \sim 938$ MeV (b) $m_\pi \sim 650$ MeV

In the quenched approximation

(A. Chowdhury et al., Phys. Rev. D 91 (2015)074507, [arXiv:1409.6459])



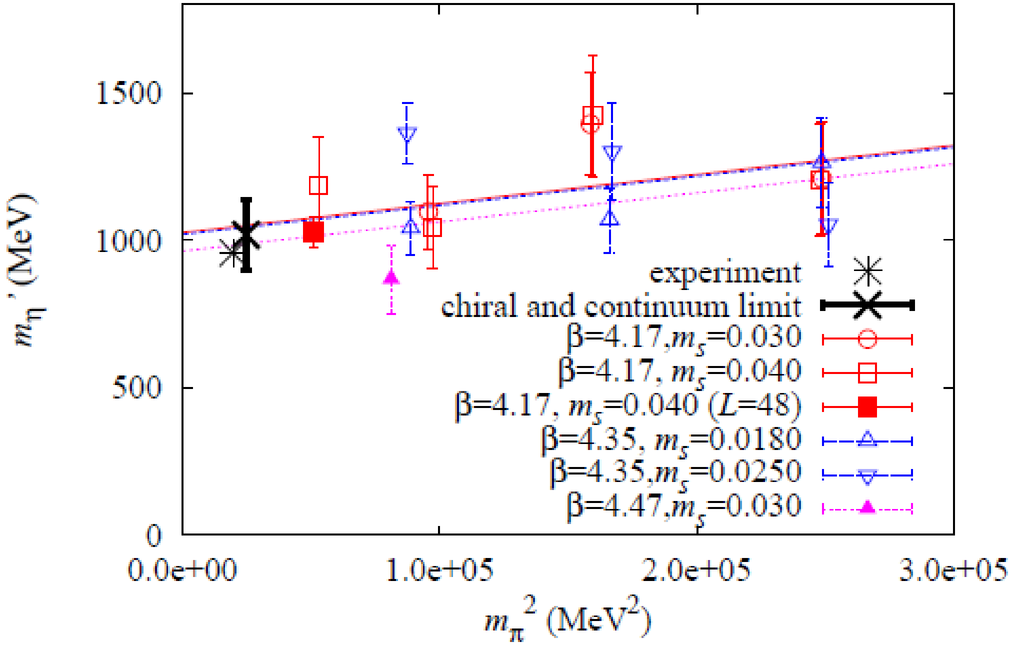
Lattice	$r_{\min}$ (fm)	am	m (MeV)
O_1	0.31	0.887(39)	2624(114)
P_1	0.30	0.831(36)	2459(108)
O_2	0.29	0.648(18)	2590(78)
P_2	0.33	0.648(25)	2560(100)
O_3	0.28	0.535(29)	2625(140)
P_3	0.27	0.524(17)	2573(81)
O_4	0.31	0.445(11)	2545(63)

The mass of the lowest pseudoscalar is in good agreement with the pseudoscalar glueball mass.

In a full-QCD lattice study

Eta' mass from Nf=2+1 full QCD calculation using the topological charge density as the operator for pseudoscalars

(H. Fukaya et al, Phy. Rev. D92 (R), 111501 (2015), arXiv: 1509.00944)



$$m_{\eta'} = 1019(119)(^{+97}_{-86}) \quad \mathrm{MeV}$$

summary of pseudoscalar results

	$P(x)$	$q(x)$	O_G
$N_f=0$	——	2563(34)MeV A.Chowdhury, PRD91(2015)	2590(40)(130)MeV Y.Chen, PRD73(2006)
$N_f=2$	768(24)MeV C.Urbach, Lattice2017	890(38)MeV this work $m_\pi=650$MeV	2605(52)MeV this work $m_\pi=650$MeV
$N_f=2+1$	947(142)MeV N.Christ, PRL105(2010)	1019(119)MeV JLQCD, PRD92(2015)	——
$N_f=2+1+1$	1006(54)(38)MeV C.Michael, PRL111(2013)	——	——

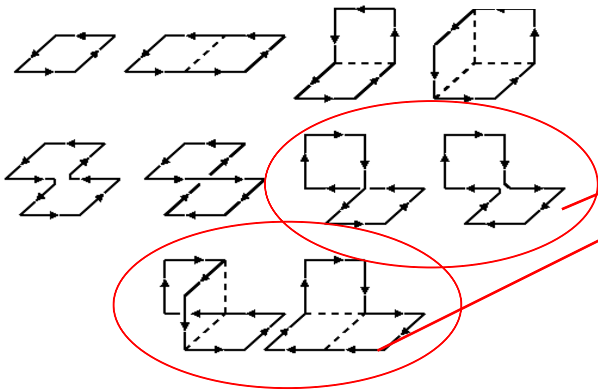
- Lattice operators for pseudoscalar: O_G (gluonic operators) and

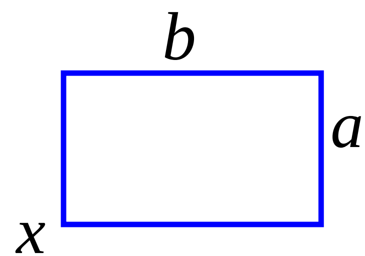
$$P(x)=\bar{\psi(x)}\gamma_5\psi(x) \qquad q(x)=\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}TrF_{\mu\nu}F_{\rho\sigma}$$

- **In the quenched approximation** ($N_f=0$), $q(x)$ and O_G operators couple to the same state with a mass of roughly 2.6 GeV. This state is nothing else but the pseudoscalar glueball in the pure gauge theory.
- **In the presence of sea quarks**, $P(x)$ and $q(x)$ couple to a state of a mass around 1 GeV. This state is of course the flavor singlet conventional qqbar meson, say, $\eta'(\eta_2)$. In contrast, O_G operator still couples exclusively to a heavier state of a mass roughly 2.6 GeV.
- Both $\eta'(\eta_2)$ and pseudoscalar glueball exist in the spectrum of full QCD.

In the calculation of the glueball spectrum, the gluonic operator for the pseudoscalar is defined as

$$\phi_\alpha^{A_1^{-+}}(\mathbf{x},t) = \sum_{R\in O} c_{A_1} ReTr\left[R \circ W_\alpha(\mathbf{x},t) - \mathcal{P}R \circ W_\alpha(\mathbf{x},t)\mathcal{P}^{-1}\right]$$





$$P_{\mu\nu}^{a\times b}(x) = \mathbf{1} + ab(F_{\mu\nu}(x) + \frac{1}{2}(aD_\mu + bD_\nu)F_{\mu\nu}(x) + \ldots)$$

Thus one can easily verify that the continuum form of the operator is

$$\boxed{\phi_\alpha^{A_1^{-+}}(\mathbf{x},t) \propto \epsilon_{ijk} TrB_i(\mathbf{x},t)D_jB_k(\mathbf{x},t) + O(a_s^2)}$$

which is in sharp constrast with the topological charge density

$$q(x) \propto \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}(x)F^{\rho\sigma}(x) \propto \mathbf{E}(x)\cdot\mathbf{B}(x)$$

IV. The production rates of glueballs in the J/psi radiative decays

- **Radiative decay width:**

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i+1} \times \sum_{r_i, r_j, r_\gamma} \left|M_{r_i, r_j, r_\gamma}\right|^2,$$

- **Transition amplitudes:**

$$M_{r_i, r_f, r_\gamma} = \epsilon_\mu^*(\vec{q}, r_\gamma)\langle f(\vec{p}_f, r_f)|j^\mu_{\mathrm{em}}(0)|i(\vec{p}_i, r_i)\rangle$$

- **Multipole decomposition:**

$$\langle f(\vec{p}_f, r_f)|j^\mu_{\mathrm{em}}(0)|i(\vec{p}_i, r_i)\rangle = \sum_k \alpha_k^\mu(p_i, p_f) F_k(Q^2).$$

- **Decay width expressed in terms of the form factors**

$$\Gamma(i \to \gamma f) \propto \sum_k F_k^2(0).$$

- **So the major task is to calculate the matrix elements, which can be derived from the three-point functions**

$$\Gamma^{(3)\mu i}(\vec{p}_f, \vec{q}; t_f, t) = \frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}} e^{+i\vec{q}\cdot\vec{y}} \left\langle O_G(\vec{p}_f, t_f+\tau)\, j^\mu(\vec{y}, t+\tau) O_{J/\psi}^{i,+}(\tau)\right\rangle$$

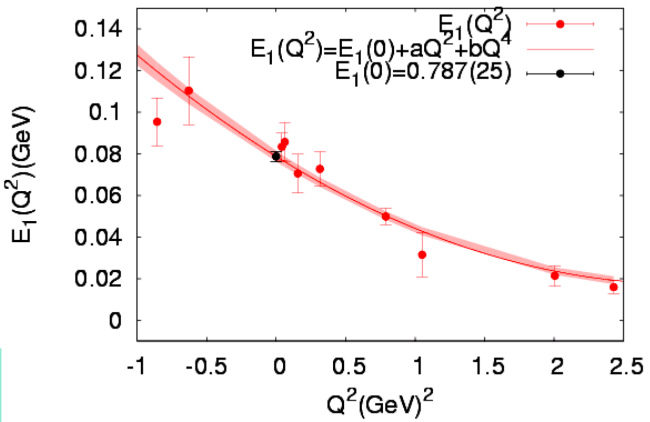
A). J/psi radiatively decaying to the scalar glueball

(L.Gui, et al. (CLQCD Collaboration), Phys. Rev. Lett. 110, 021601 (2013))

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2$$

Interpolated on-shell form factor E1(0) and its continuum limit

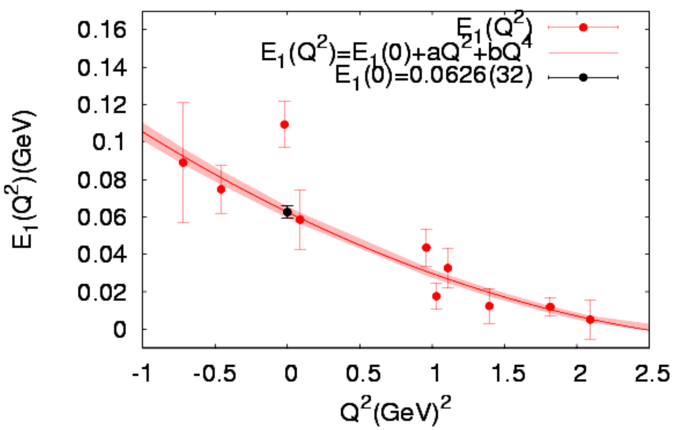
β	M_G(GeV)	$Z_V^{(s)}(a)$	$E_1(0,a)$ (GeV)	Γ(keV)
2.4	1.360(9)	1.39(2)	0.0787(25)	-
2.8	1.537(7)	1.11(1)	0.0626(32)	-
∞	1.710(90) [3]	-	0.0536(57)	0.35(8)



The predicted width and the branch ratio

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$



Lattice prediction:

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$

Experimental results for J/psi radiatively decaying to scalars

C. Amsler et al. (Particle Data Group), Phy. Rev. D 86, 010001 (2012)

$$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi \qquad (1.01 \pm 0.32)\times 10^{-4}$$

$$Br(f_0(1500) \to \pi\pi) = (34.9 \pm 2.3)\% \quad \Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta \qquad (2.35^{+1.27}_{-0.77})\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta \qquad (1.65^{+0.57}_{-1.50})\times 10^{-4}$$

BESIII results (PRD87, 092009)

$$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K} \qquad (8.5^{+1.2}_{-0.9})\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi \qquad (4.0 \pm 1.0)\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega \qquad (3.1 \pm 1.0)\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \qquad > (1.5 \pm 0.3)\times 10^{-3}$$

Using $Br(f_0(1710) \to K\underline{K}) = 0.36 \quad \Rightarrow \quad Br(J/\psi \to \gamma f_0(1710)) = 2.4\times 10^{-3}$

$Br(f_0(1710) \to \pi\pi) = 0.15 \quad \Rightarrow \quad Br(J/\psi \to \gamma f_0(1710)) = 2.7\times 10^{-3}$

Our result support f0(1710) as the candidate for the scalar glueball

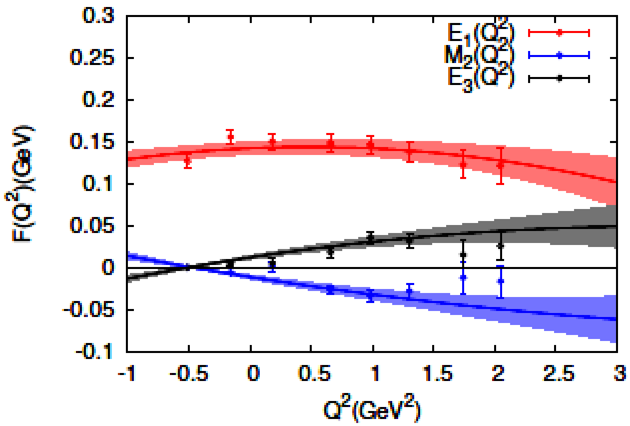
B). J/psi radiatively decaying to the tensor glueball

(Y.B. Yang ,et al .(CLQCD Collaboration), Phys. Rev. Lett. 111, 091601 (2013))

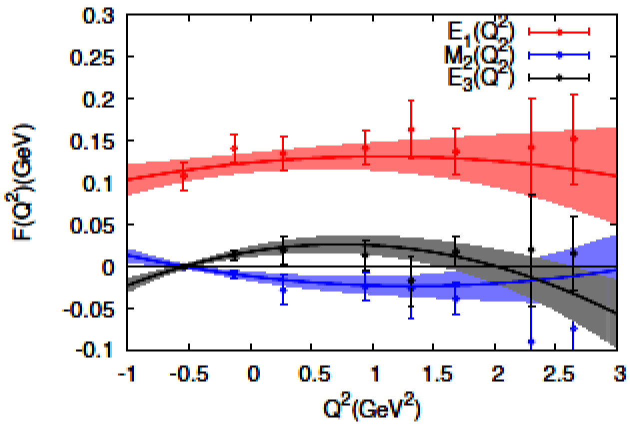
$$\boxed{\Gamma(J/\psi \to \gamma G_{2^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left[\left|E_1(0)\right|^2 + \left|M_2(0)\right|^2 + \left|E_3(0)\right|^2\right]}$$

- The form factors we obtained from the lattice QCD

β	M_T (GeV)	E_1 (GeV)	M_2 (GeV)	E_3 (GeV)
2.4	2.360(20)	0.142(07)	−0.012(2)	0.012(2)
2.8	2.367(25)	0.125(10)	−0.011(4)	0.019(6)
∞	2.372(28)	0.114(12)	−0.011(5)	0.023(8)



- We also carry out a similar lattice study on the tensor glueball production rate in J/psi radiative decay.



$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV$$

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$

- **Flavor-blindness of glueball decays**

$$\boxed{\frac{1}{P.S.}\Gamma(G\to\pi\pi:K\overline{K}:\eta\eta:\eta\eta':\eta'\eta')=3:4:1:0:1}$$

As such, one can estimate,

$$\Gamma(G\to\eta\eta)/\Gamma(G\to PP)\sim O(10\%)$$

which can be compared with that of f0(1710):

$J/\psi\to\gamma f_0(1710)\to\gamma\eta\eta$	$\left(2.35^{+1.27}_{-0.77}\right)\times10^{-4}$
$J/\psi\to\gamma f_0(1500)\to\gamma\eta\eta$	$\left(1.65^{+0.57}_{-1.50}\right)\times10^{-4}$

$Br(J/\psi\to\gamma f_0(1710)) = 2.4\times10^{-3}$

- **PP final states in the tensor glueball decays should be in D-wave, considering the centrifugal barrier effects,**

$$\Gamma(G\to M\overline{M})=\eta\alpha\frac{k^{2L+1}}{m_G{}^{2L}}=\frac{\eta\alpha}{m_G}\left(\frac{k}{m_G}\right)^{2L+1}$$

$$\frac{k}{m_G}=\frac{1}{2}\sqrt{1-\left(\frac{2m_M}{m_G}\right)^2}\sim0.5-0.3$$

So the partial width of the tensor glueball decaying into two pseudoscalars can be suppressed by an order of magnitude, so intutitively one has,

$$Br(G_{2^+} \to PP) \sim O(10\%)$$

- With the BESIII result,

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = 5.6(2.3)\times 10^{-5}$$

the production rate of f_2(2340) in the J/psi radiative decay can be 100 times larger, and consistent with our prediction

$$Br(J/\psi \to \gamma f_2(2340)) \sim 10^{-2}$$

with the new result of BES

- It is desirable to do a systematic analysis of decay modes $J/\psi \to \gamma VV$

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\phi\phi) = (1.91 \pm 0.14)\times 10^{-4}$$

C). J/psi radiatively decaying to the tensor glueball (preliminary)

L.-C. Gui, Y. Chen, Y.-B. Yang et al, in preparation

β	ξ	$a_s(\mathrm{fm})$	$La_s(\mathrm{fm})$	$L^3 \times T$	$N_{conf.}$
2.4	5	0.222(2)	1.78	$8^3 \times 96$	20000
2.8	5	0.138(1)	1.66	$12^3 \times 144$	20000
3.0	5	0.110(1)	1.76	$16^3 \times 160$	10000

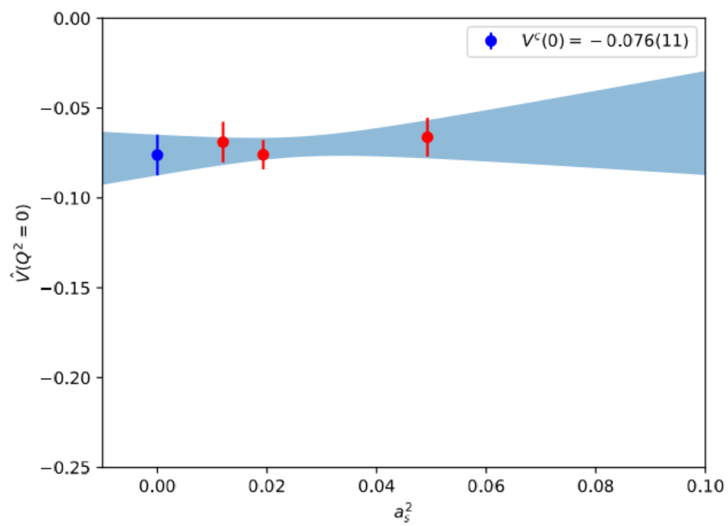
Quenched approximation
Anisotropic lattices
Large statistics

$$\boxed{\left\langle G_{0^{-+}}(\vec{p}_G)\right| j^\mu(Q^2)\left|V(\vec{p}_V,\lambda)\right\rangle = \frac{M(Q^2)}{\sqrt{\Omega}}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^*{}_\nu(\vec{p}_V,\lambda)p_{G,\rho}p_{V,\sigma}}$$

$$\hat{V}(0) = -0.076$$

$$\begin{aligned}\Gamma(J/\psi \to \gamma G_{0-+}) &= \frac{16}{27}\alpha\frac{|p|^3}{(m_{0-+}+m_{J/\psi})^2}|\hat{V}(0)|^2 \\ &= 0.094(28)\ \ \mathrm{keV}\end{aligned}$$

$$\Gamma(J/\psi \to \gamma G_{0-+})/\Gamma = 1.0(3)\times 10^{-3}$$



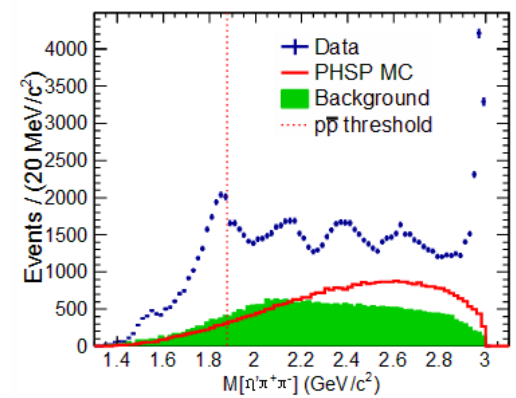
- **BESIII new results for** $J/\psi \to \gamma\phi\phi$

TABLE I. Mass, width, $\mathcal{B}(J/\psi \to \gamma X \to \gamma\phi\phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

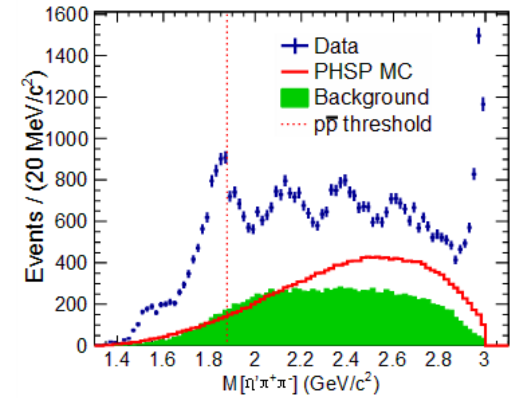
Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times 10^{-4}$)	Sig.
$\eta(2225)$	2216^{+4+21}_{-5-11}	185^{+12+43}_{-14-17}	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

BESIII, PRD93(2016)112011

$J/\psi \to \gamma\eta'\pi^+\pi^-$



$\eta' \to \gamma\pi^+\pi^-$



$\eta' \to \eta(\to \gamma\gamma)\pi^+\pi^-$

BESIII, PRL117(2016)042002
arXiv:1603.09653

V. Summary

1. Glueball spectrum from quenched and full QCD lattice calculation.

2. It seems that the unqenched effects on glueball masses are small.

3. Especially, the mass of the pseudoscalar glueball is confirmed to be around 2.6 GeV.

4. Scalar, tensor and pseudscalar glueballs have large branching fraction in J/psi radiative decays.

Thanks!