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Coupled Channel Analysis with PAWIAN

Beijing

PWA10/ATHOS5 2018

July 16 – 20, 2018

Outline

- Introduction to the PWA package PAWIAN
- Activities in the past
  - coupled channel analysis with $\bar{p}p$ data
- Current activities
  - unitarity and analyticity
  - Chew-Mandelstam functions for unstable particles
  - coupled channel analysis with e^+e^- and scattering data
- Summary and outlook

PAWIAN

- Development of a PWA software package with the aim
  - to provide a user-friendly and generic software package
  - to support various physics cases to be studied at hadron spectroscopy experiments

Software package PAWIAN (**PA**rtial **W**ave **I**nteractive **An**alysis)
already in a good shape,
and several analyses with $\bar{p}p$ and e^+e^- data
have been performed

PAWIAN

- Full hypothesis and other settings defined via configuration files
  - formalisms: canonical, helicity and partly Rarita-Schwinger
  - dynamics: Breit-Wigner, Flatté, K-matrix, . . .
- Event based maximum likelihood fit, minimization by MINUIT2
- Multithreading and network support
- Possible to analyze channels with arbitrary number of final state particles
- Support for various reactions: $\bar{p}p$- and e^+e^-- annihilation, $\gamma\gamma$-fusion, $\pi\pi$- scattering process, decays of isolated resonances
- Analysis tools: extraction of spin density matrices, pole positions, ...
- Event generator, histogramming, ...

Coupled Channel Analysis

- Advantages compared to single channel fits
  - more constraints due to common amplitudes
  - common and unique description of the dynamics (K-matrix)
  - better fulfillment of the conservation of unitarity
  - better description of threshold effects
  - can solve possible ambiguities and more model independent

- PAWIAN
  - works generically with any number of channels
  - easy use of K-matrix with P-vector approach via configuration files

K-Matrix with P-Vector Approach

S.U. Chung, E.Klempt „A Primer on K-matrix Formalism“, BNL Preprint (1995)

Aitchison: „The K-Matrix formalism for overlapping resonances“, Nucl Phys A189 (1972) 417

- A two body scattering process can be fully described by the S-matrix

$$\mathbf{S} = \mathbf{I} + \mathbf{2i}\,\sqrt{\rho}\,\mathbf{T}\,\sqrt{\rho}$$

- ρ diagonal phase space matrix $\rho_\mathbf{i} = \sqrt{[\mathbf{1} - (\frac{\mathbf{m_a}+\mathbf{m_b}}{\mathbf{m}})^\mathbf{2}]\;[\mathbf{1} - (\frac{\mathbf{m_a}-\mathbf{m_b}}{\mathbf{m}})^\mathbf{2}]}$
- T-matrix can be expressed by K-matrix: $\mathbf{T} = (\mathbf{I} - \mathbf{i}\,\mathbf{K}\,\rho)^{-\mathbf{1}}\,\mathbf{K}$
- Elements of the K-matrix: $\mathbf{K_{ij}} = \sum_\alpha \frac{\mathbf{g_{\alpha i}}\,\mathbf{g_{\alpha j}}}{\mathbf{m}_\alpha^\mathbf{2} - \mathbf{s}} + \sum_\mathbf{k} \mathbf{c_{kij}}\,\mathbf{s}^\mathbf{k}$
- Dynamical function for P-vector approach: $\mathbf{F} = (\mathbf{I} - \mathbf{i}\,\mathbf{K}\,\rho)^{-\mathbf{1}}\,\mathbf{P}$

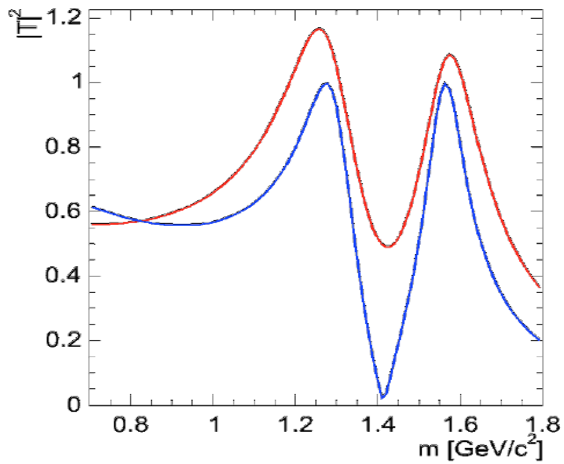
  with: $\mathbf{P_i} = \sum_\alpha \frac{\beta_\alpha\,\mathbf{g_{\alpha i}}}{\mathbf{m}_\alpha^\mathbf{2} - \mathbf{s}} + \sum_\mathbf{k} \mathbf{c_{ki}}\,\mathbf{s}^\mathbf{k}$
- Unique pole parameters can be derived from the 1st unphysical Riemann sheet of the T-matrix

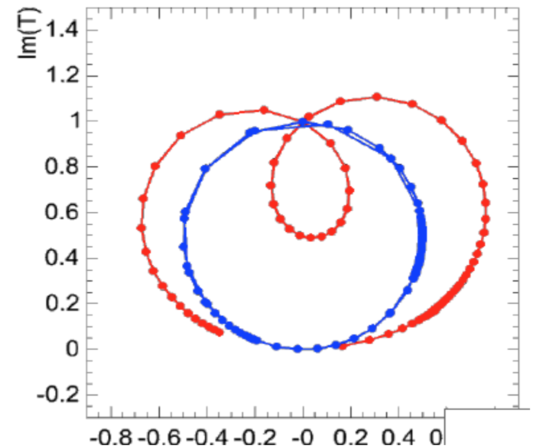
K-Matrix vs. Breit Wigner

K. Peters (GSI and Frankfurt)
School on Concepts
of Modern Amplitude Analysis Techiques
Flecken-Zechlin, 2013

2 nearby Poles 1 Channel

- $m_a = 1.27\ \mathrm{GeV}/c^{2,}$
- $m_b = 1.50\ \mathrm{GeV}/c^{2,}$

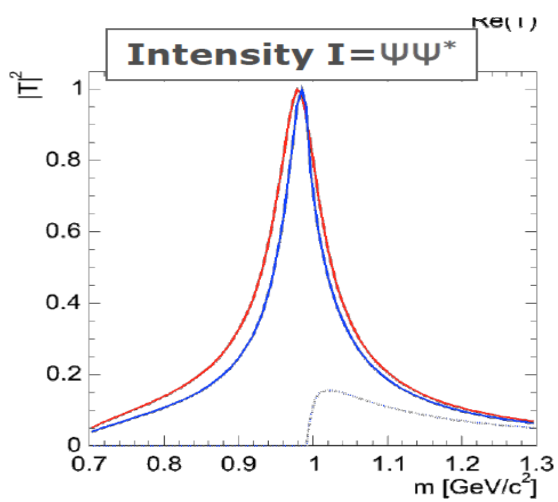


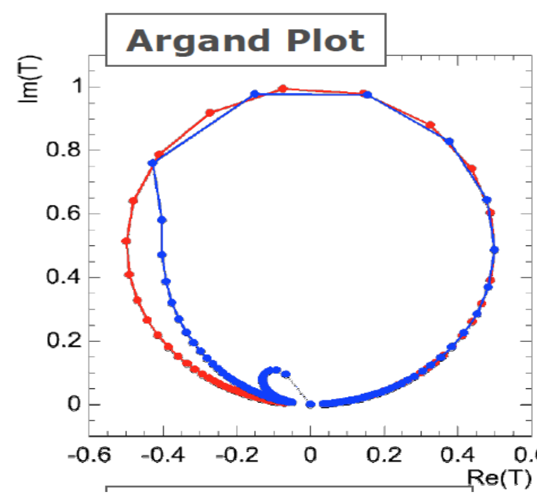


K-Matrix
2 Breit-Wigner

1 Pole 2 Channels

- $a_0(980)$ with coupling to $\eta\ \pi$ and K $\overline{\mathrm{K}}$





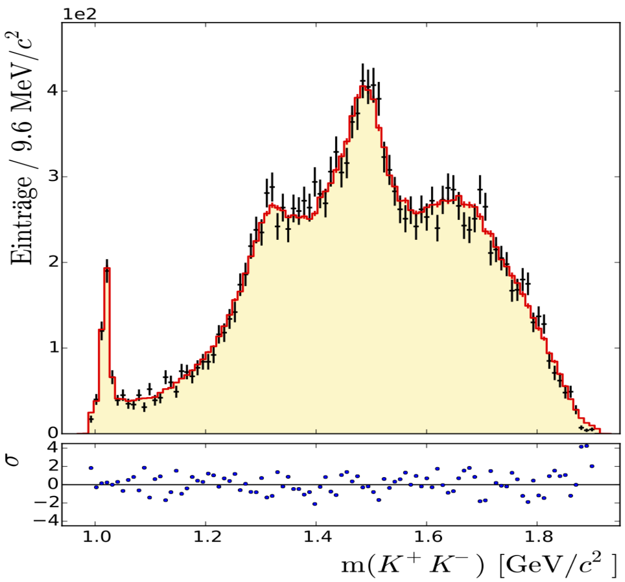
K-Matrix
Breit-Wigner

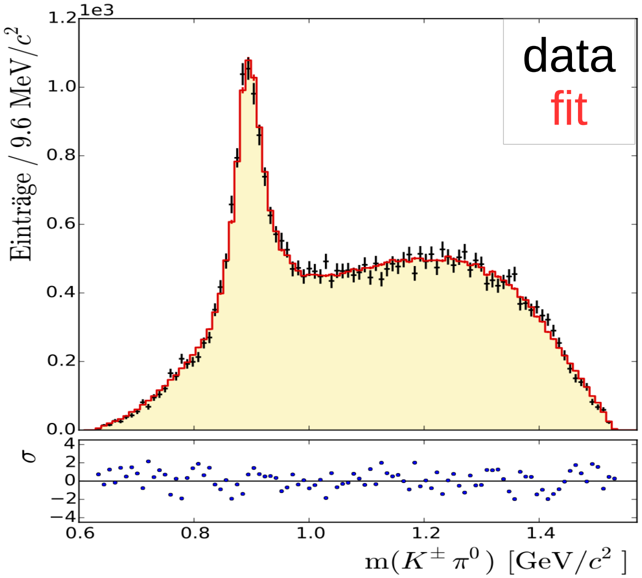
$\bar{p}p \rightarrow K^+ K^- \pi^0,\ \pi^0 \pi^0 \eta,\ \pi^0 \eta\ \eta$

Julian Pychy, PhD thesis 2016 (RUB)

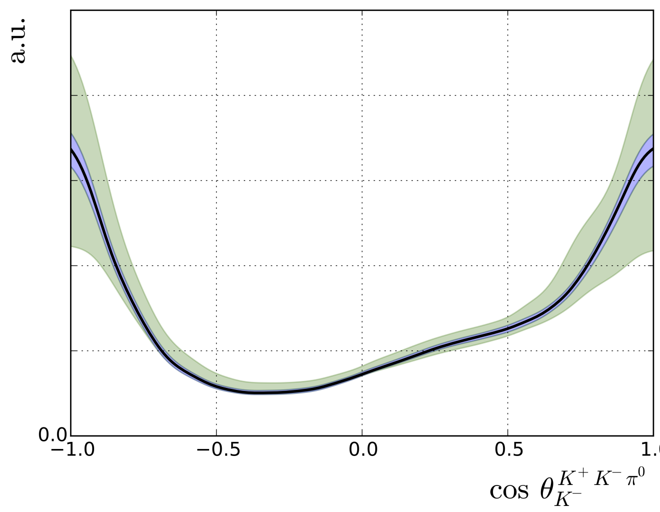
- Crystal Barrel @ LEAR data: $\bar{p}$-momentum at 0.9 GeV/c
- Free K-matrix parameters for a_0, a_2 and f_2 contributions
- Fixed parametrization for
  - $(\pi\pi)_S$ -wave by Anisovich and Sarantsev with 5 poles, 5 channels below 1.9 GeV/c^2 *Eur. Phys J A16 229(2003)*
  - $(K\pi)_S$ (I=1/2)-wave used by FOCUS with 1 pole and 2 channels below 1.5 GeV/c^2 *Phys Lett B653 (2007)*
- Breit-Wigner parameterization for isolated resonances
- Outcome: good description of the data with reasonable physics results

Coupled Channel Fit Result for $\bar{p}p \rightarrow K^+ K^- \pi^0$



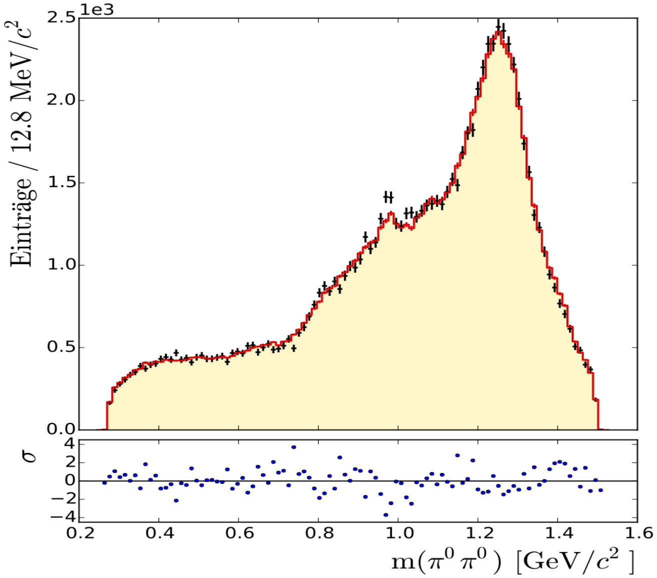


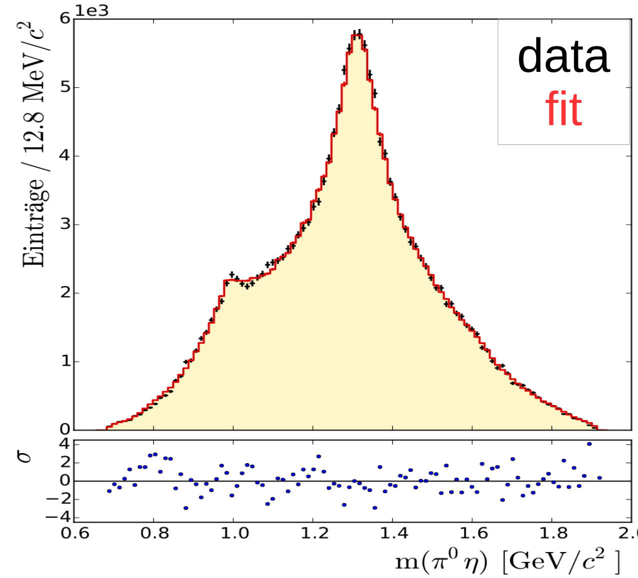
- Fit probability: p = 0.06
- Also all production and decay angular distributions are in agreement with the data
- Asymmetric distribution of the K^{*-} production angle observed



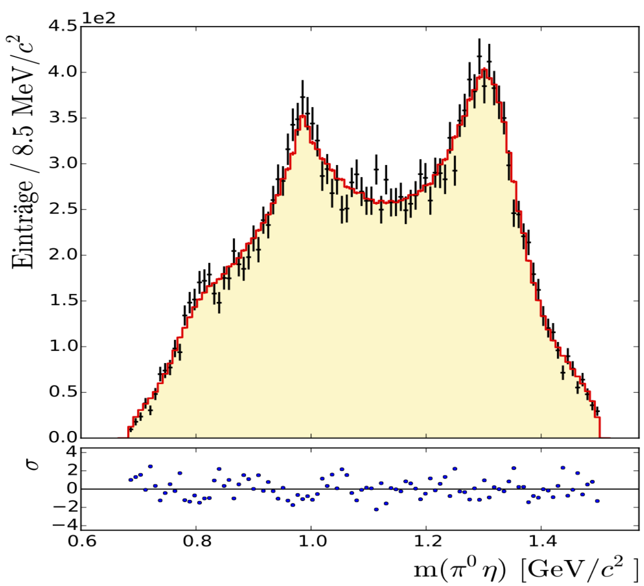
Coupled Channel Fit Result for $\bar{p}p \rightarrow \pi^0\pi^0\eta$ and $\pi^0\eta\eta$

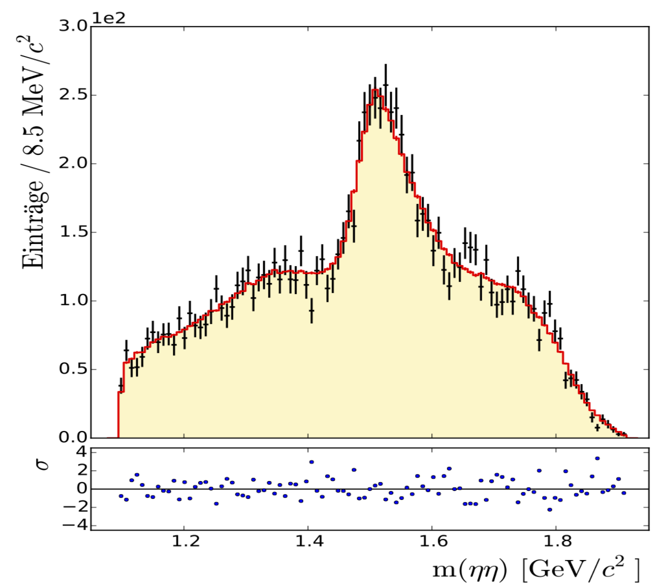
$\pi^0\pi^0\eta$:
$p = 0.017$





$\pi^0\eta\eta$:
$p = 0.3$





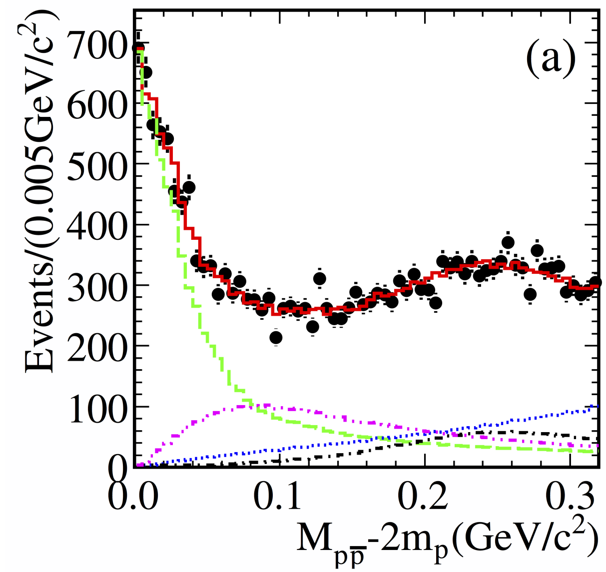
More details presented by M. Albrecht (RUB) on Friday:
„Hadron Spectroscopy from $\bar{p}p$ Experiments“

Coupled Channel Fit: $J/\psi \to \omega\bar{p}p$, $\omega\pi^0\pi^0$ and ωK^+K^-

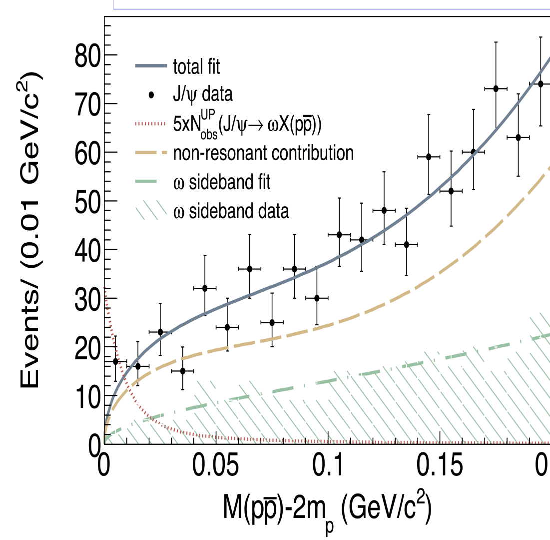
Motivation for $J/\psi \to \omega\bar{p}p$

- $\bar{p}p$ spectrum near threshold has been studied but w/o detailed PWA
- Obvious differences to $\gamma\bar{p}p$ with a dominant 0^{-+} contribution
- Better description of the production of f_0 and f_2 resonances by coupling it with the channels $\omega\pi^0\pi^0$ and ωK^+K^-
- Search for baryons decaying to ωp

BES: Phys Rev Lett 91 (2003)



BESIII: Phys Rev D87 112004 (2013)



Coupled Channel Fit: $J/\psi \rightarrow \omega\bar{p}p$, $\omega\pi^0\pi^0$ and ωK^+K^-

Motivation for adding $J/\psi \rightarrow \omega\pi^0\pi^0$, ωK^+K^- and scattering data

- $J/\psi \rightarrow \omega\pi^0\pi^0$ and $J/\psi \rightarrow \omega K^+K^-$
  - investigation of b and ρ resonances coupling to $\omega\pi$
  - investigation of K and K* resonances decaying to ωK
- Scattering data
  - mass range up to 2.3 GeV/c^2 $\rightarrow$ K-matrix parameterization for the $(\pi\pi)_S$-wave utilized for the $\bar{p}p$ data are not useful here
  - processes only characterized by elasticity and phase motion $\rightarrow$ good and easy access to resonance properties
  - available for I=0 S- and D-wave and I=1 P- and F-wave

Unitarity and Analyticity

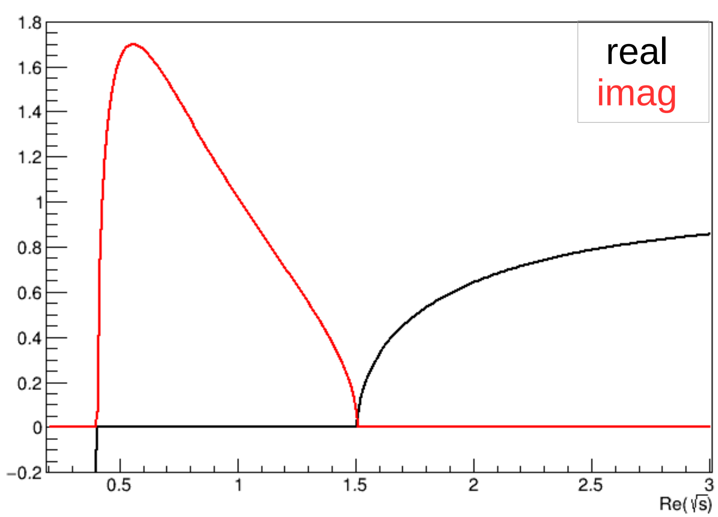
- So far: K-matrix description with standard phase space factors

$$\rho = \sqrt{[1-(\frac{\mathbf{m_a+m_b}}{\mathbf{m}})^2]\,[1-(\frac{\mathbf{m_a-m_b}}{\mathbf{m}})^2]}$$

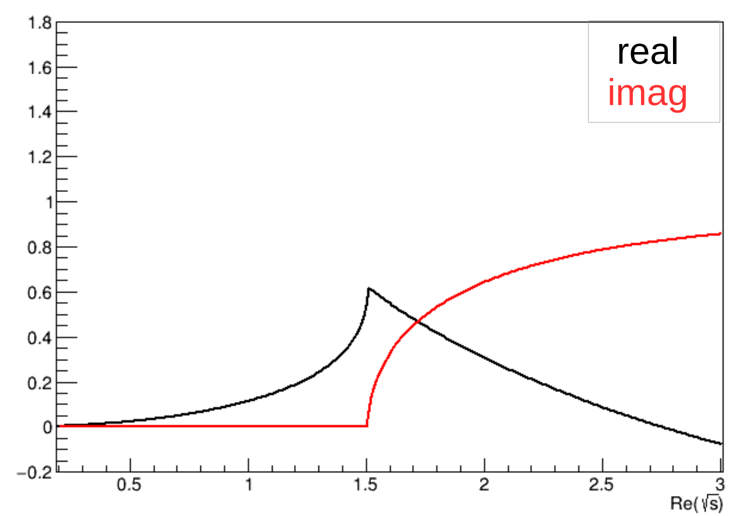
  - violates constraints from analyticity: pole at s=0 and unphysical cuts in case of unequal masses

- Proper description with Chew-Mandelstam function from Basdevant and Berger *Phys Rev D19 239(1979)*
  - above threshold: $\rho(s)$ = Im(CM(s))
  - $\mathbf{T} = (\mathbf{I} - \mathbf{i}\,\mathbf{K}\,\rho)^{-1}\,\mathbf{K}$ replaced by $\mathbf{T} = (\mathbf{I} - \mathbf{K}\,\mathbf{CM(s)})^{-1}\,\mathbf{K}$

standard ρ for $\eta\eta'$



CM function for $\eta\eta'$



CM Function with unstable Particles

- Chew-Mandelstam function only working for decay channels with stable particles → channels with unstable particles: ωπ, ωK, ρπ and ρρ
- Modified CM function for channels with unstable particles like ρπ, K*π proposed by Basdevant and Berger *Phys Rev D19 239(1979)*
  - correct analyticity properties
  - satisfies quasi-two-body unitarity
  - calculations are very time-consuming and realized with lookup-tables during the fit procedure

$$\tilde{C}(s; m^*, \mu) = \frac{1}{\pi} \int_{(m_1+m_2)^2}^{\infty} ds' \frac{f^2 \, \mathrm{Im}\Sigma(s')}{|d(s')|^2} \times C(s; \sqrt{s'}, \mu) .$$

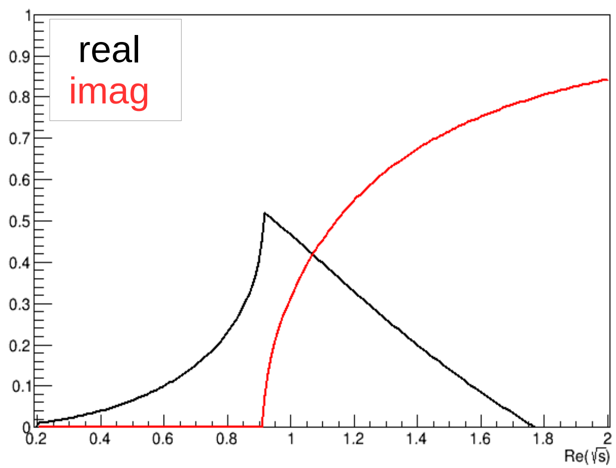
CM Function with unstable Particles

- Example: $\rho\pi$
- Obtained Chew-Mandelstam function on the real axis in full agreement with Basdevant and Berger
- For extracting pole positions an expansion into the complex energy plane is needed

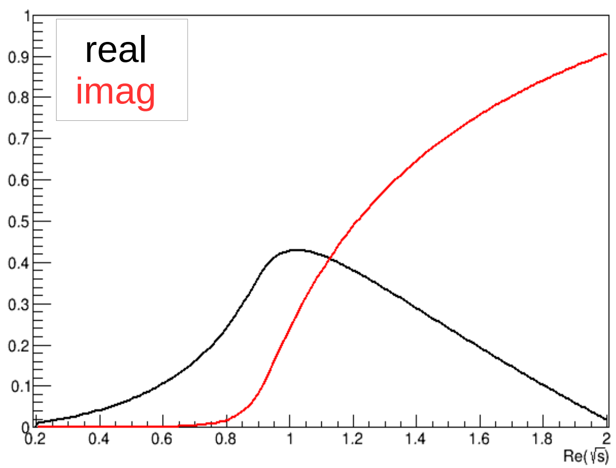
Basdevant, Berger: Phys. Rev. D19 (1979) 239

M. Kuhlmann (RUB)

CM for $\rho\pi$ (stable)



CM for $\rho\pi$ (unstable)



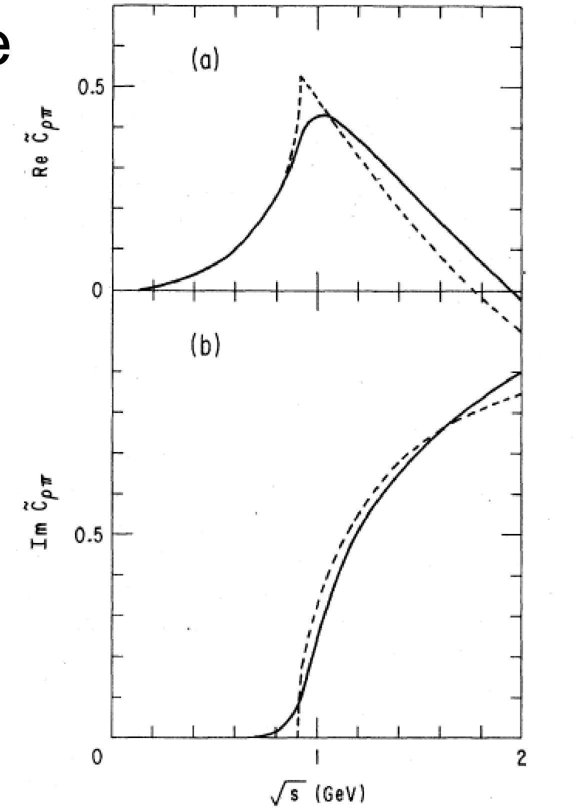


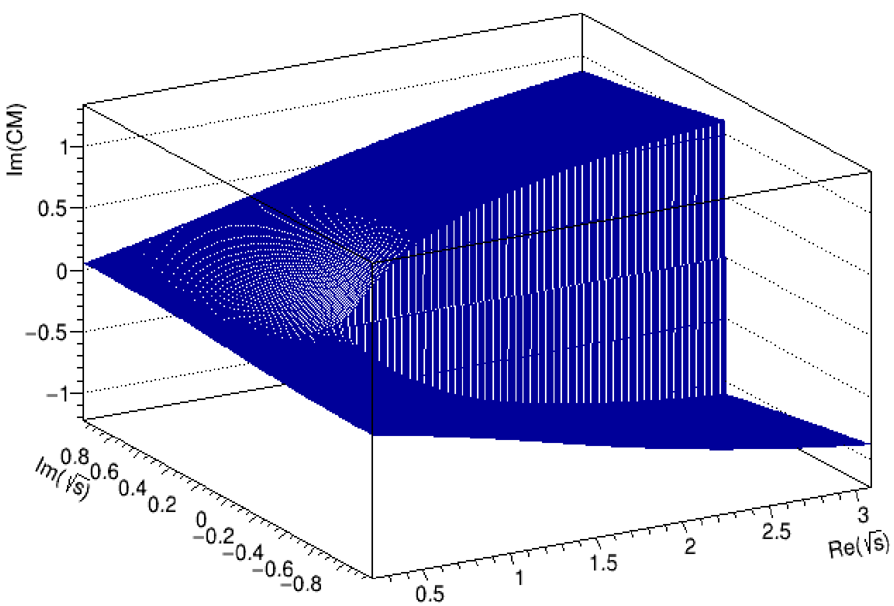
FIG. 4. The function $\tilde{C}_{\rho\pi}(s)$ (solid line) is compared with the stable particle Chew-Mandelstam function (dashed line): (a) real parts, (b) imaginary parts.

CM Function with unstable Particles

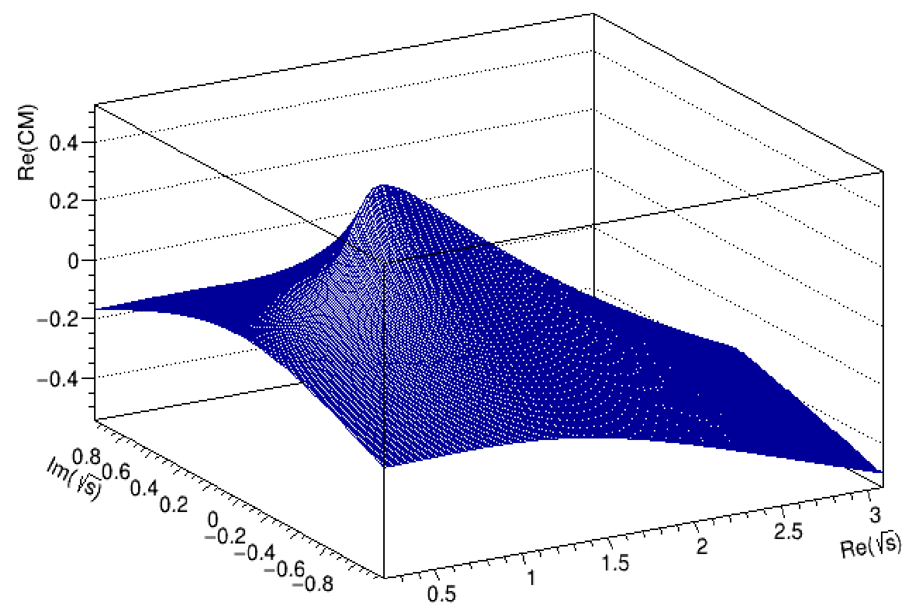
- Reasonable shapes within the complete complex energy plane
- Fulfillment of the Hermitian analyticity: CM(s)* = CM(s*)

M. Kuhlmann (RUB)

Im(CM(s)) for $\rho\pi$



Re(CM(s)) for $\rho\pi$



Coupled Channel Fit: $J/\psi \to \omega\bar{p}p$, $\omega\pi^0\pi^0$ and ωK^+K^-

Data Samples

Xiaoshuai Qin (RUB)

- BESIII data samples consist of roughly 30.000 ωK^+K^-, 160.000 $\omega\bar{p}p$ and 850.000 $\omega\pi^0\pi^0$ events after applying all selection criteria
- Here toy data with
  - ωK^+K^- : 20k events with f_0, f_2 and K_1 resonances
  - $\omega\bar{p}p$: 50k events with f_0, f_2, X(1835) and $\eta(2225)$ resonances
  - $\omega\pi^0\pi^0$: 62k events with f_0, f_2, b_1 , ρ and ρ_3 resonances
- Scattering data
  - I=0 S-wave $\pi\pi \to \pi\pi$
  - I=0 S-wave $\pi\pi \to \eta\eta, \eta\eta'$
  - I=0 D-wave $\pi\pi \to \pi\pi, \eta\eta$
  - I=1 P-wave $\pi\pi \to \pi\pi$
  - I=1 F-wave $\pi\pi \to \pi\pi$

Phys. Rev. D83(2011) 074004 *Nucl. Phys B64 (1973) 134-162*

Nucl. Phys B64 (1973) 134-162

Nucl. Phys B269 (1986) 485

Nouvo Cimento A80 (1984) 363

- Toy data are not consistent with BESIII data but in agreement with the phase shifts and elasticities from the scattering data

I=0 S-Wave $\pi\pi \to \pi\pi$

- Simple and ready to use parameterization
- Precise and model independent description up to 1.4 GeV/c^2
- Fulfills crossing symmetry
- Describes the existing data with rather small errors
- Consistent with dispersion relation within uncertainties

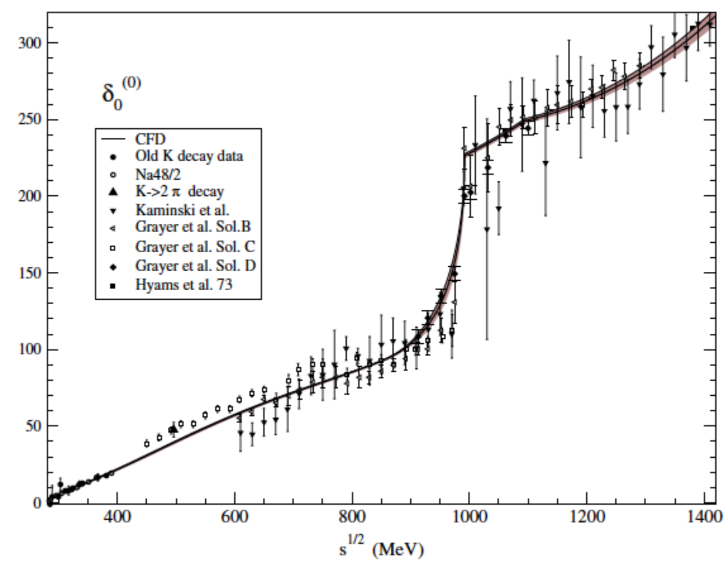


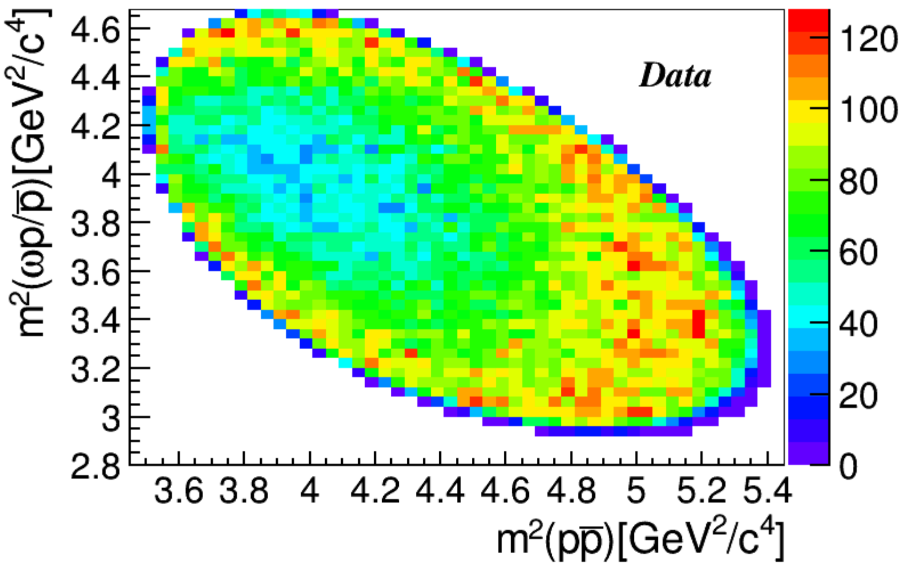
FIG. 15 (color online). The new CFD for the S0 wave versus the existing phase-shift data from [29,30]. The dark band covers the uncertainties.

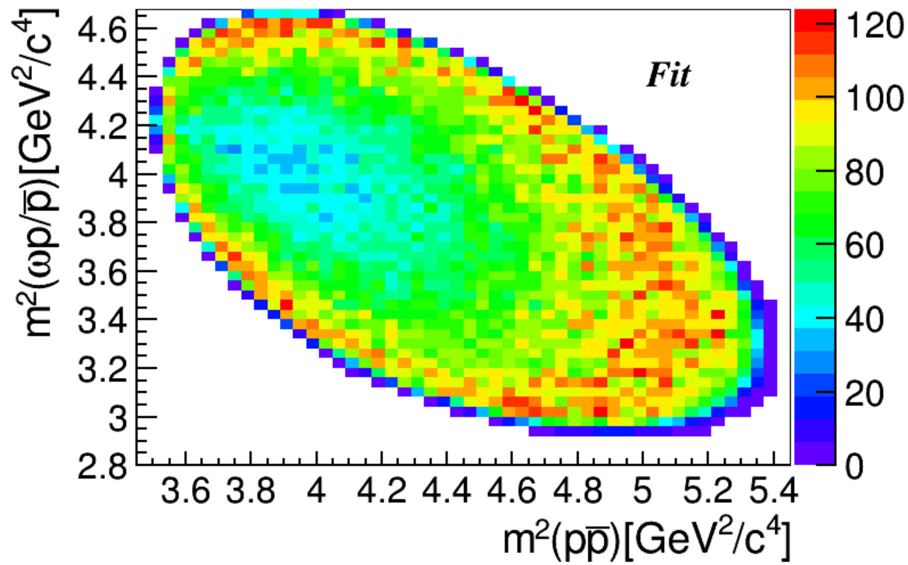
Garcia-Martin, Kaminski, Palaez, Ruiz del Elvira, Yndurain: Phys. Rev. D83(2011) 074004

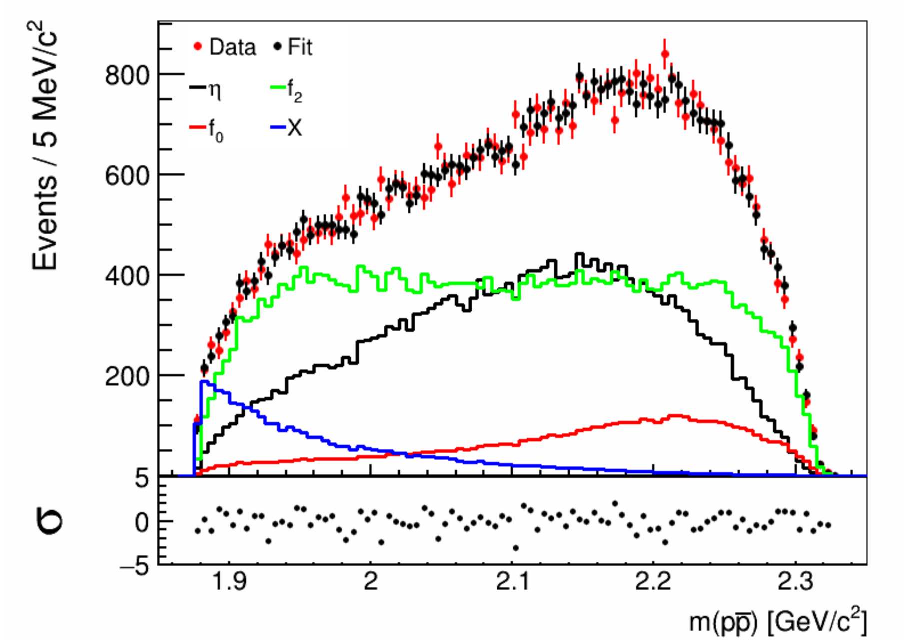
PWA Strategy

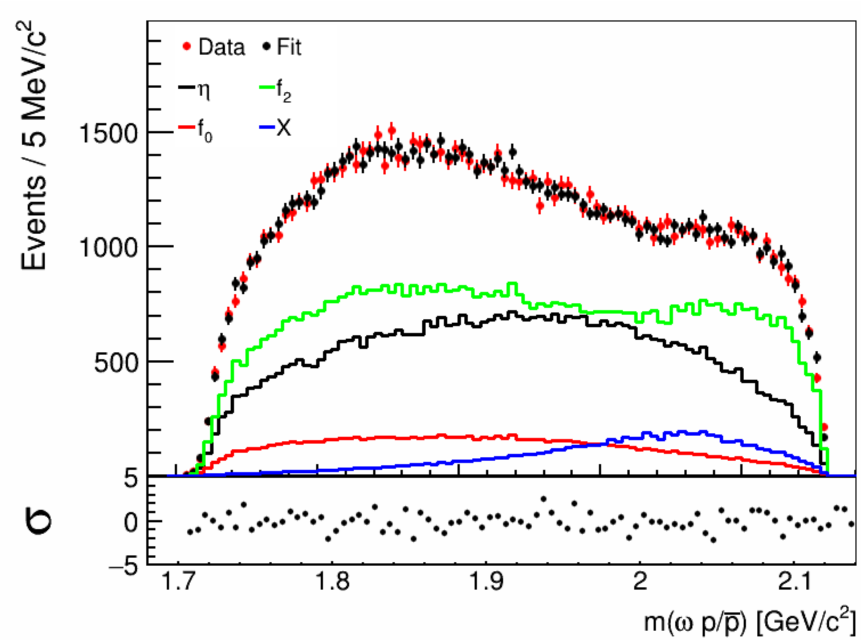
- K-matrix description with 0th order background terms, each for
  - f_0 with 7 poles, 6 channels ($\pi\pi$, $\rho\rho$, $\bar{K}K$, $\eta\eta$, $\eta\eta'$, $\bar{p}p$)
  - f_2 with 6 poles, 5 channels ($\pi\pi$, 4π, $\bar{K}K$, $\eta\eta$, $\bar{p}p$)
  - ρ with 4 poles, 3 channels ($\pi\pi$, 4π, $\omega\pi$)
  - ρ_3 with 2 poles, 3 channels ($\pi\pi$, 4π, $\omega\pi$)
  - b_1 with 2 poles, 2 channels ($\pi\pi$, $\omega\pi$)
  - K_1 with 3 poles, 2 channels ($K^*\pi$, $K\omega$)
- Breit-Wigner description for
  - $\eta(2225) \to \bar{p}p$
  - $X(1835) \to \bar{p}p$

Fit result for J/ψ → $\omega\bar{p}p$ (Toy Data)

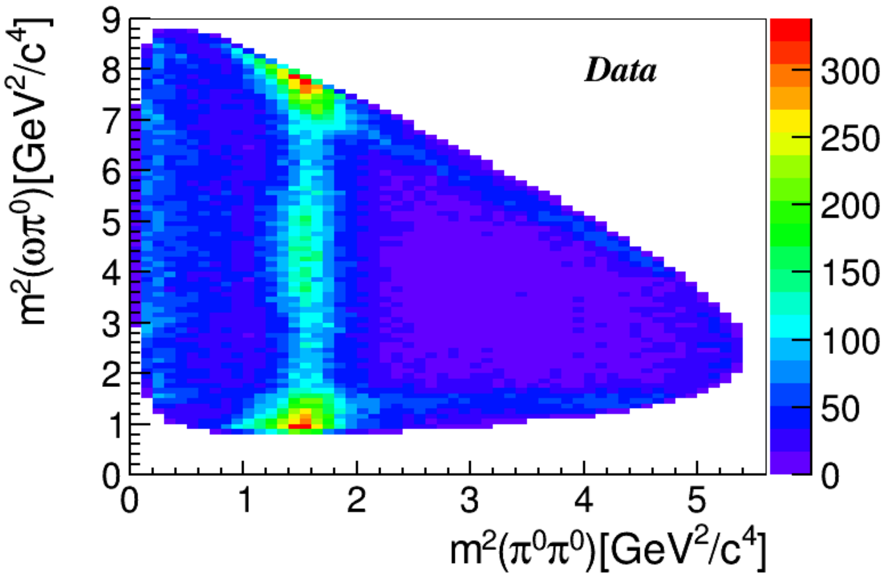


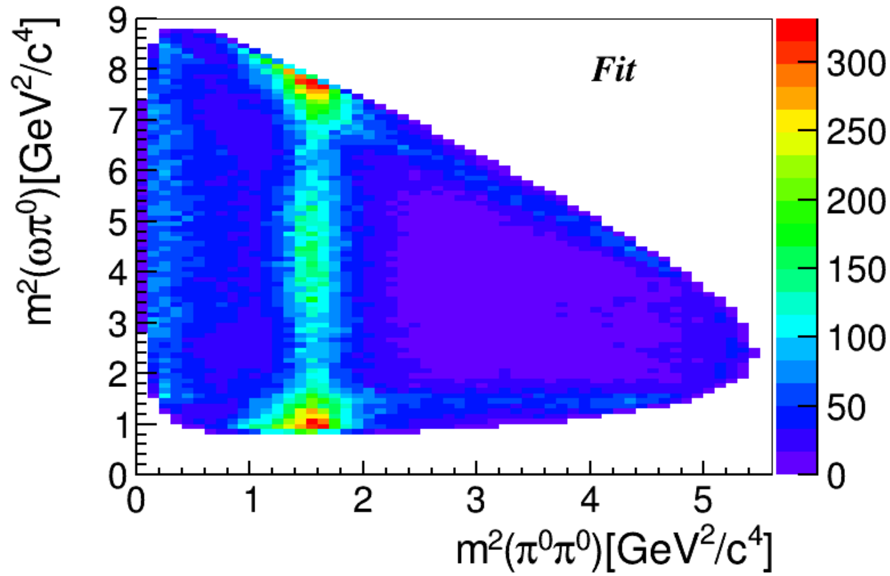


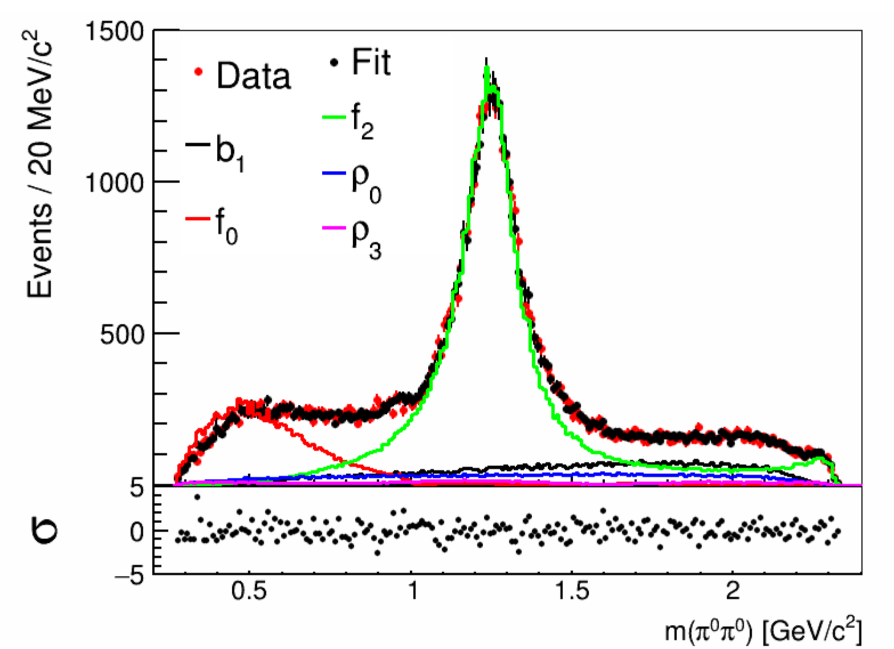


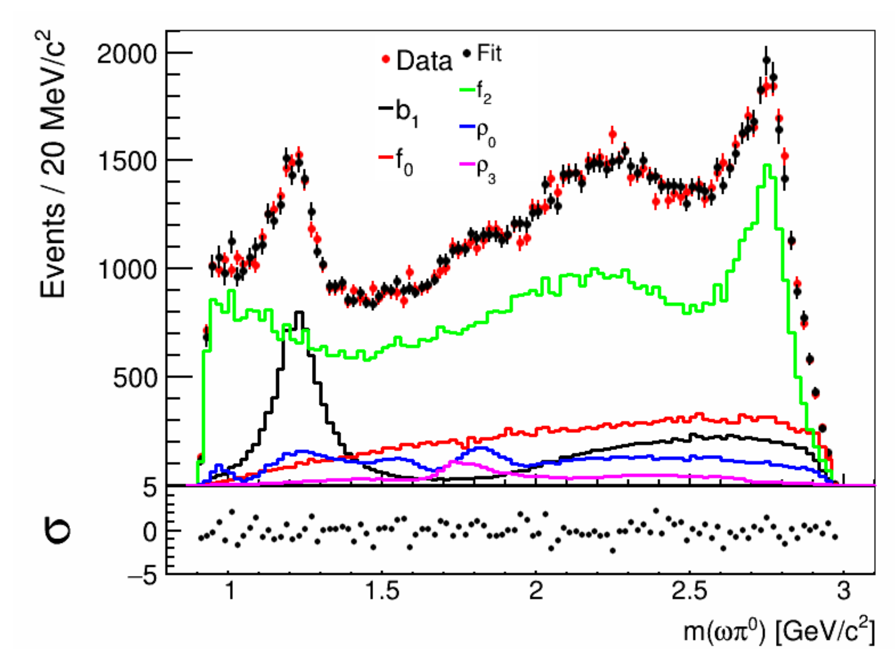


Fit result for J/ψ → $\omega\pi^0\pi^0$ (Toy Data)

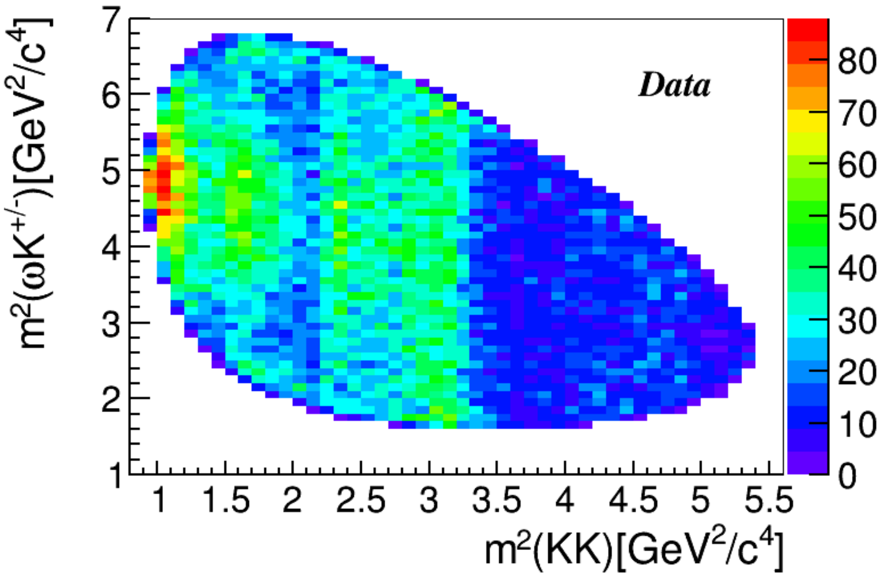


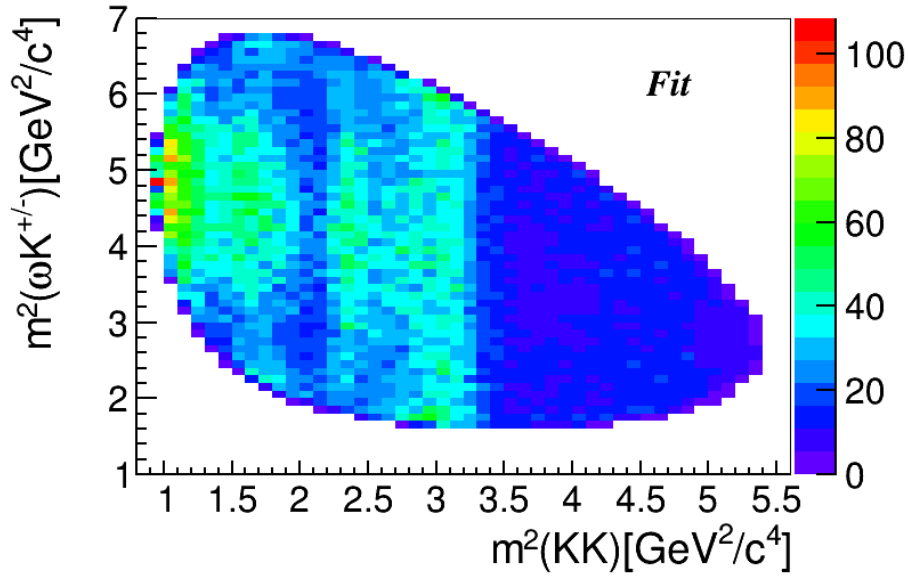


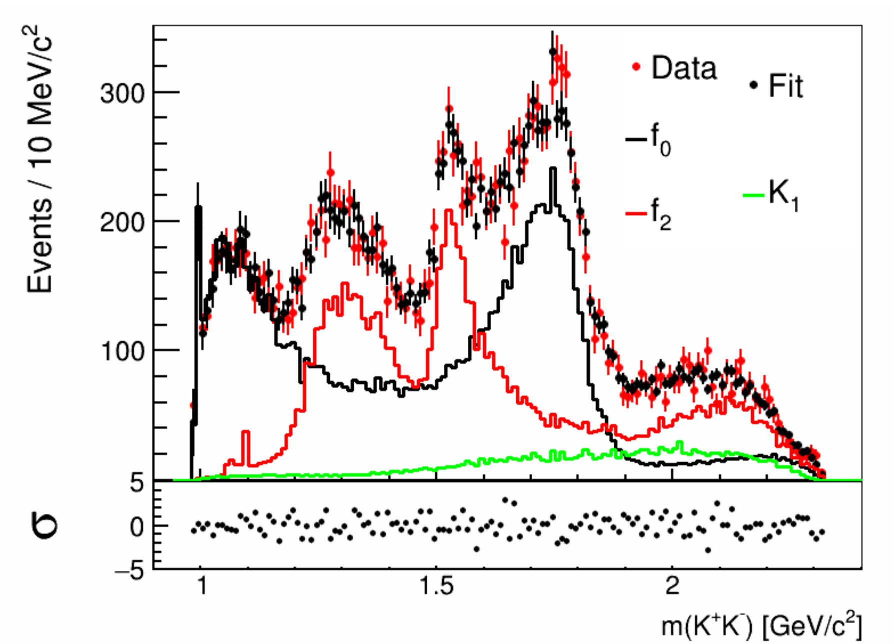


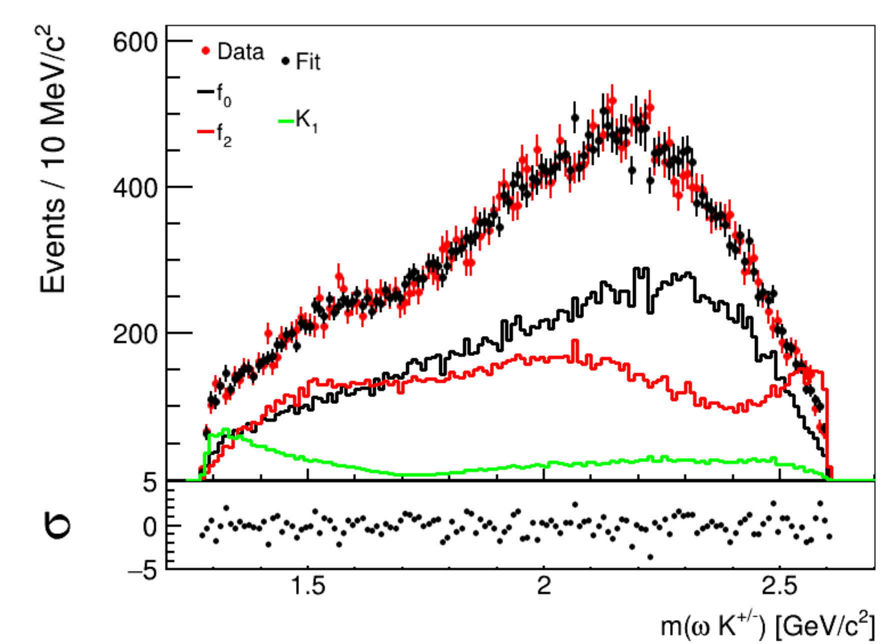


Fit result for J/ψ → ωK⁺K⁻ (Toy Data)

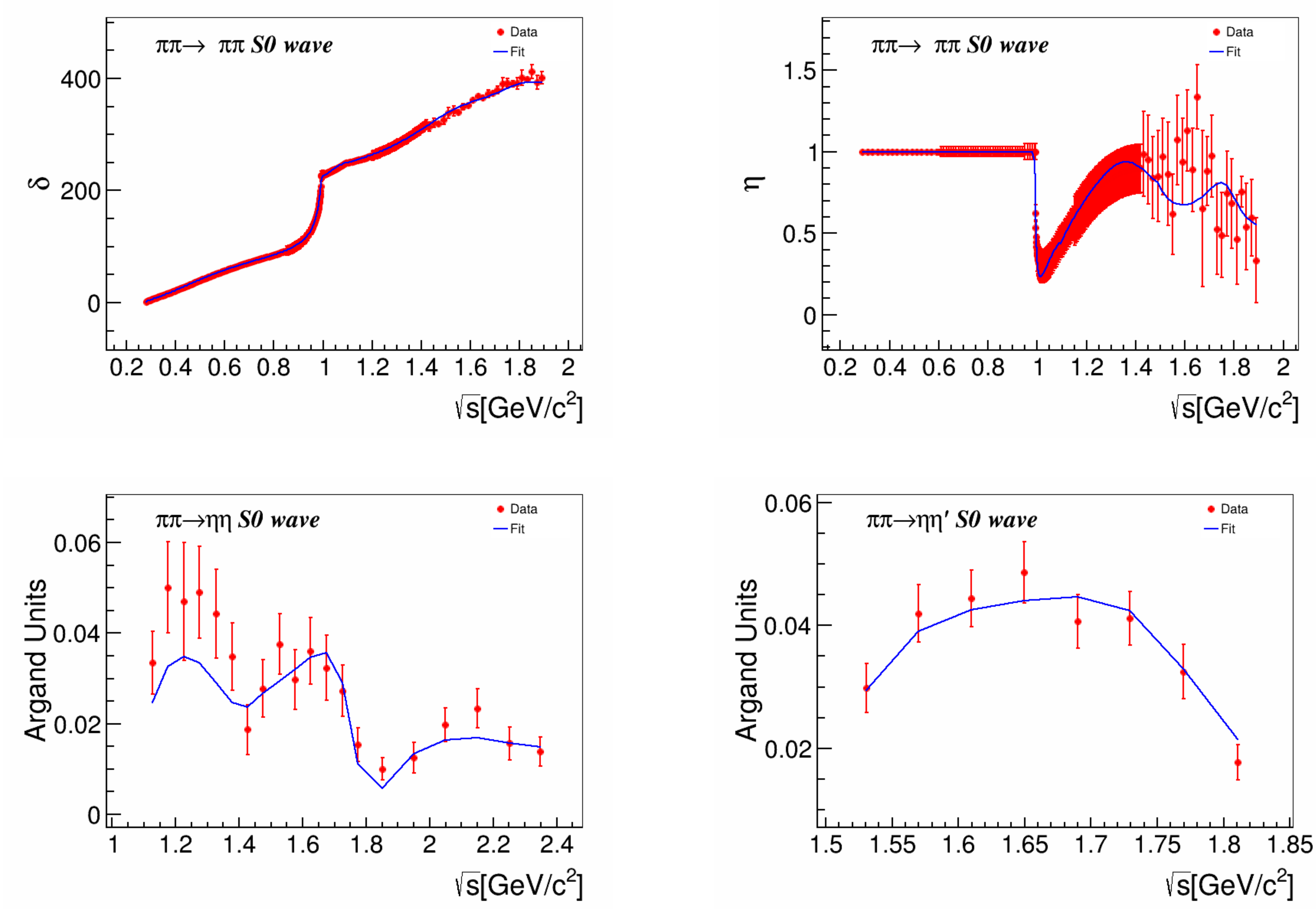




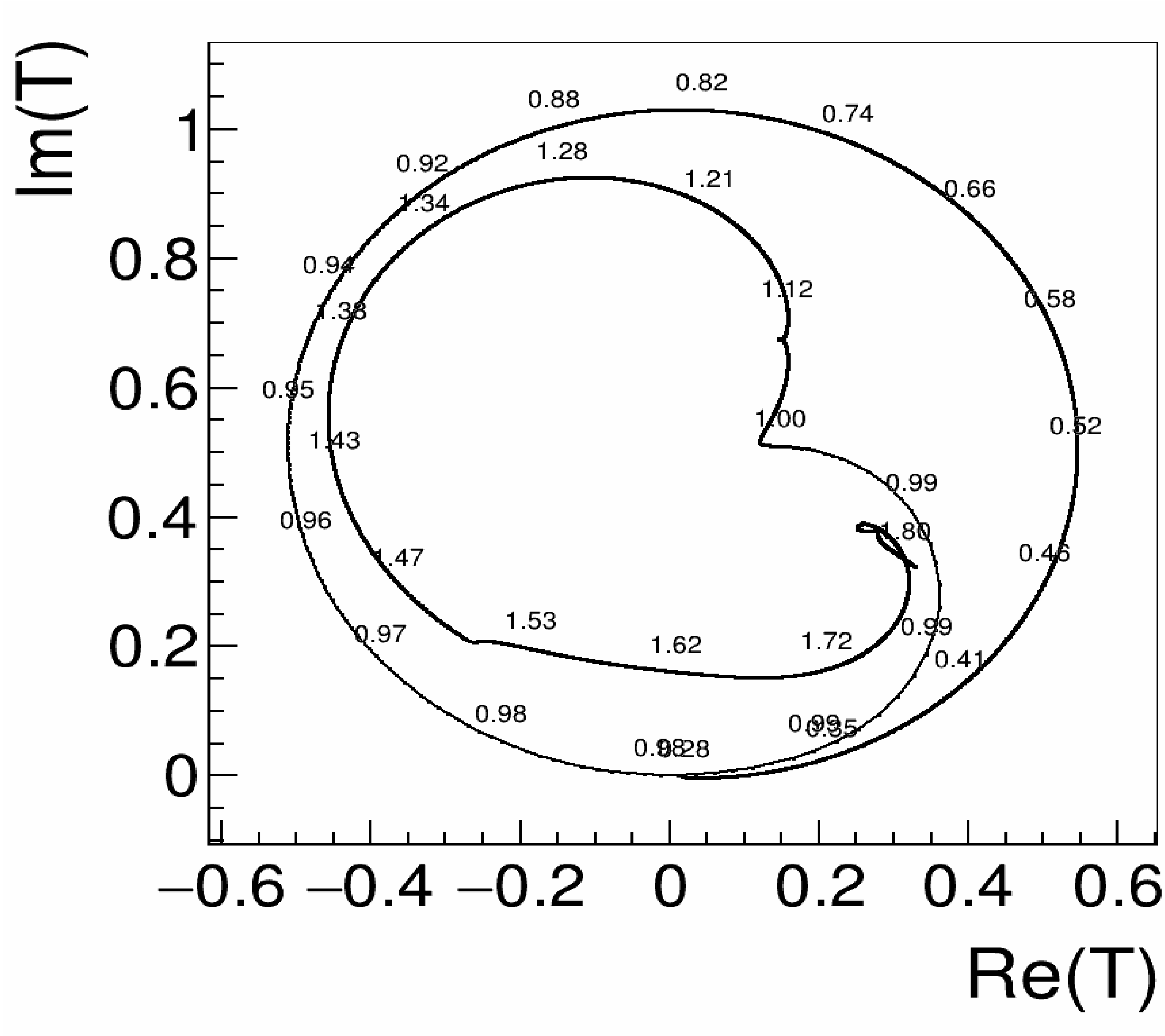




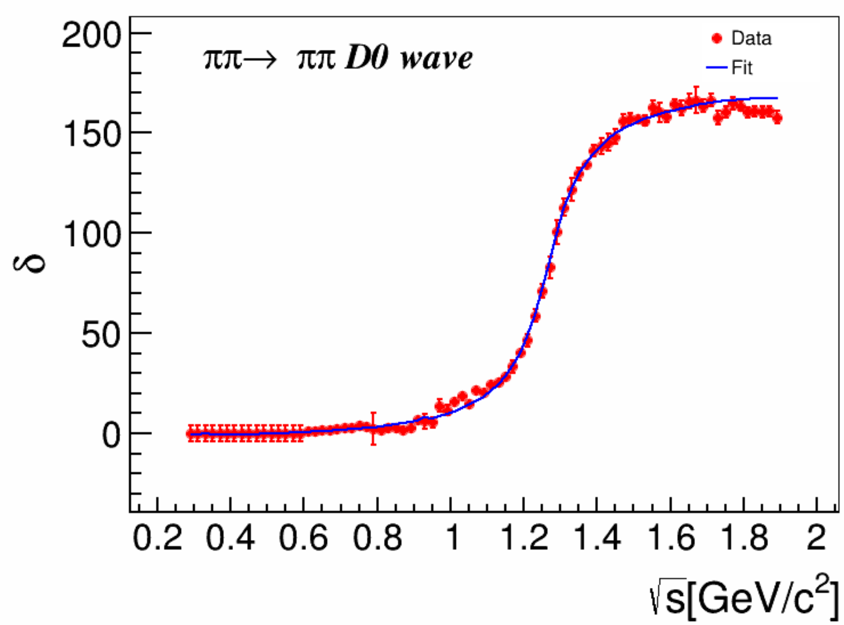
Fit result for Scattering Data: ππ S-Wave

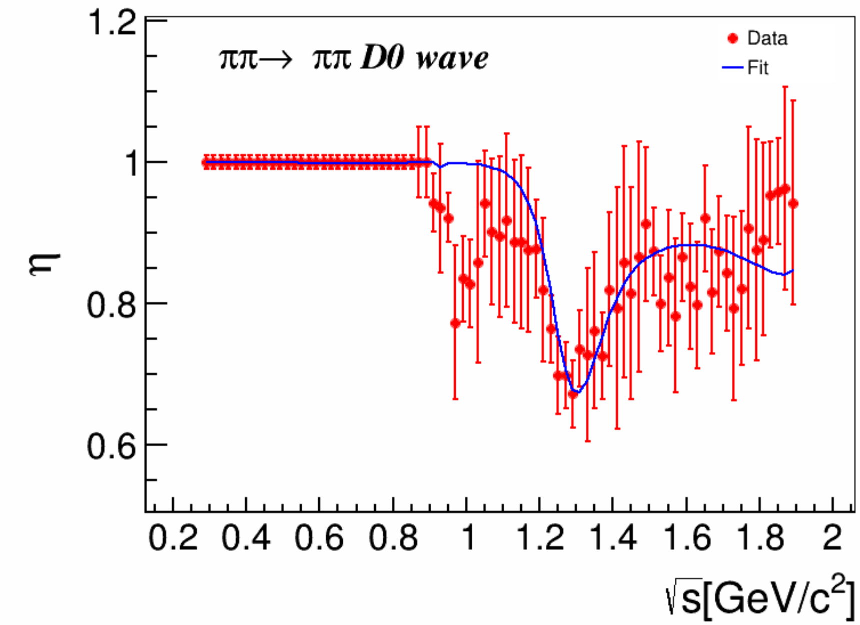


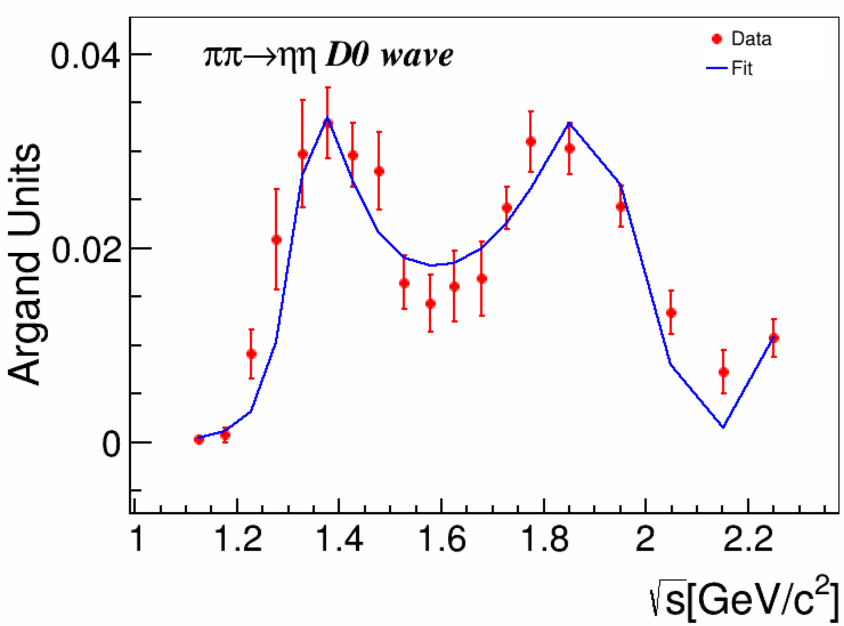
Fit result for Scattering Data: ππ S-Wave

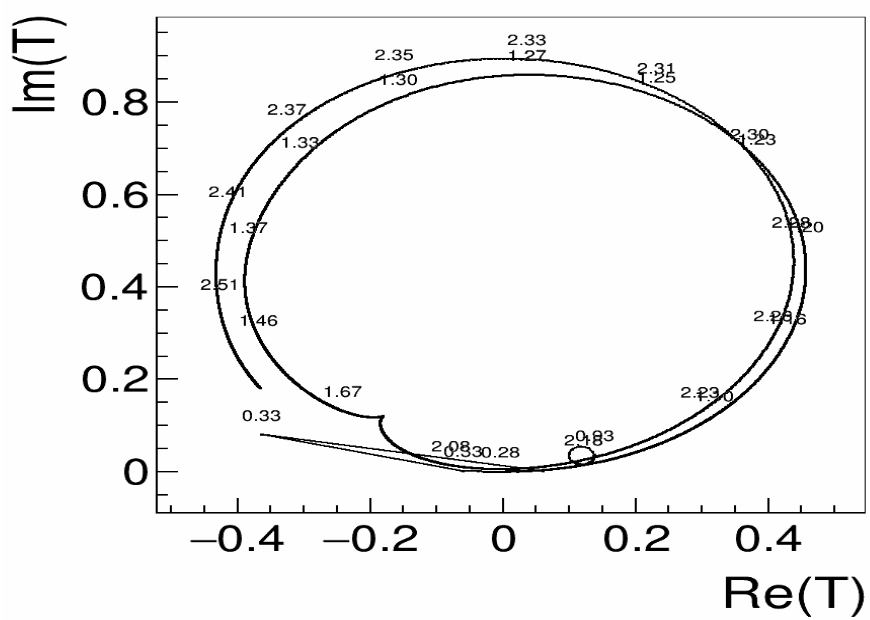


Fit result for Scattering Data: ππ D-Wave

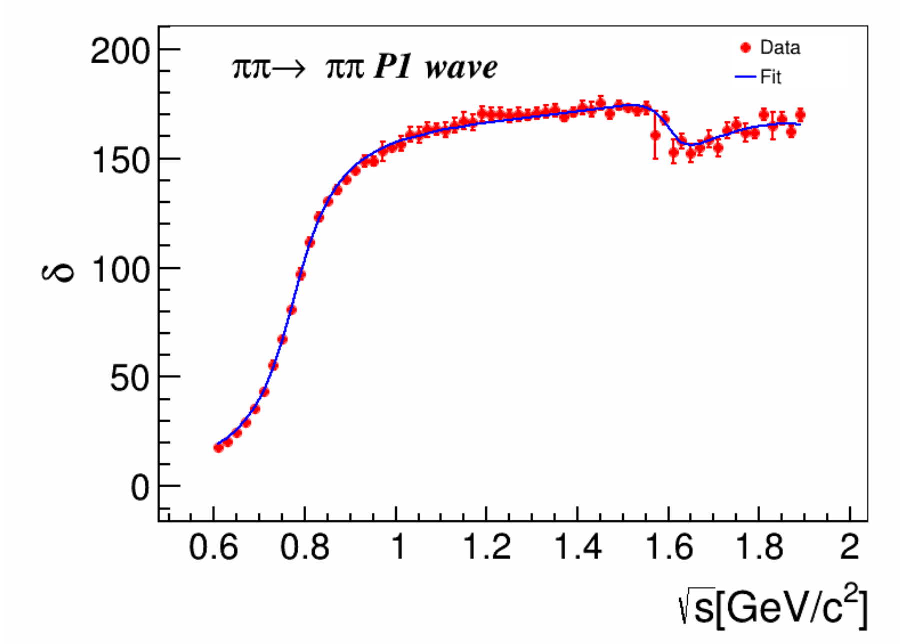


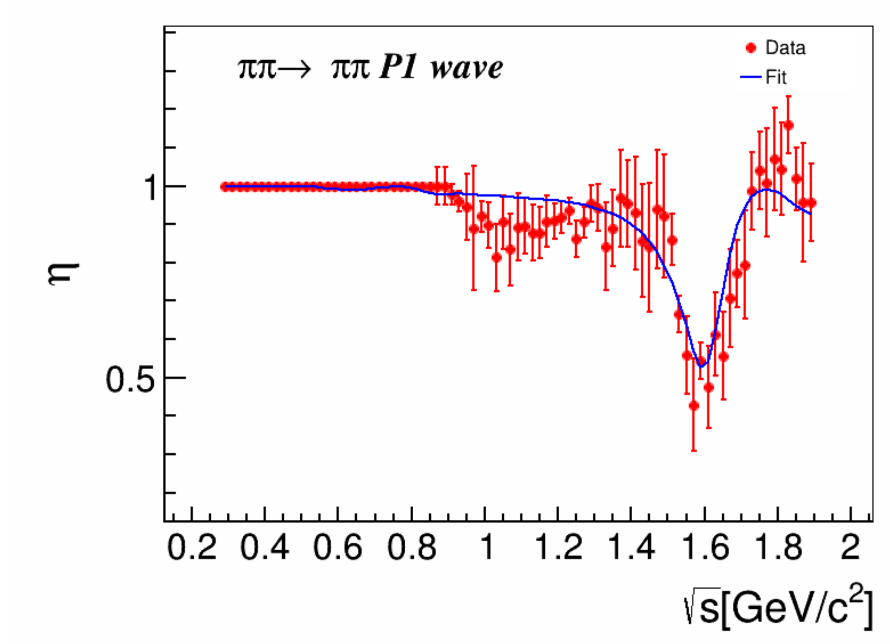


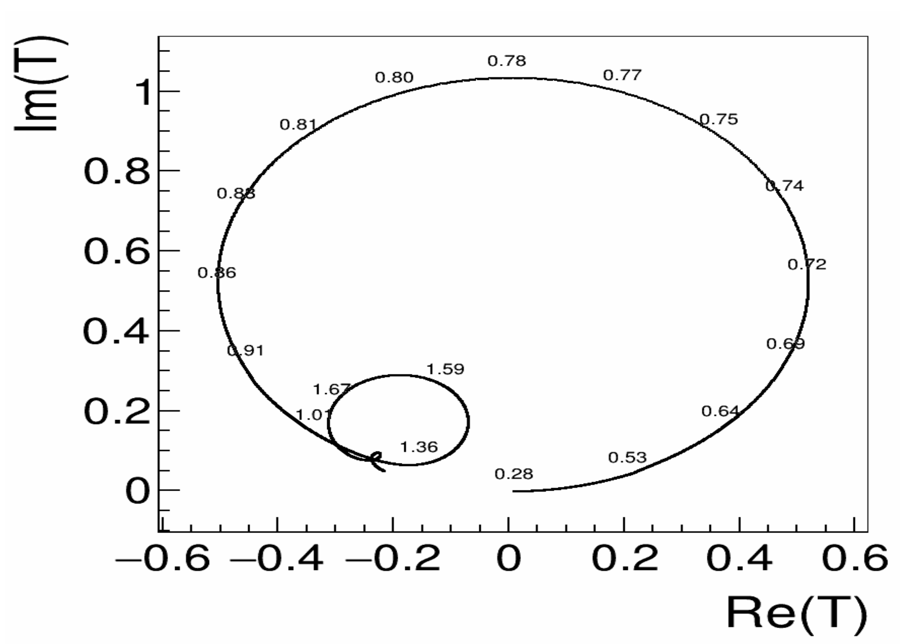




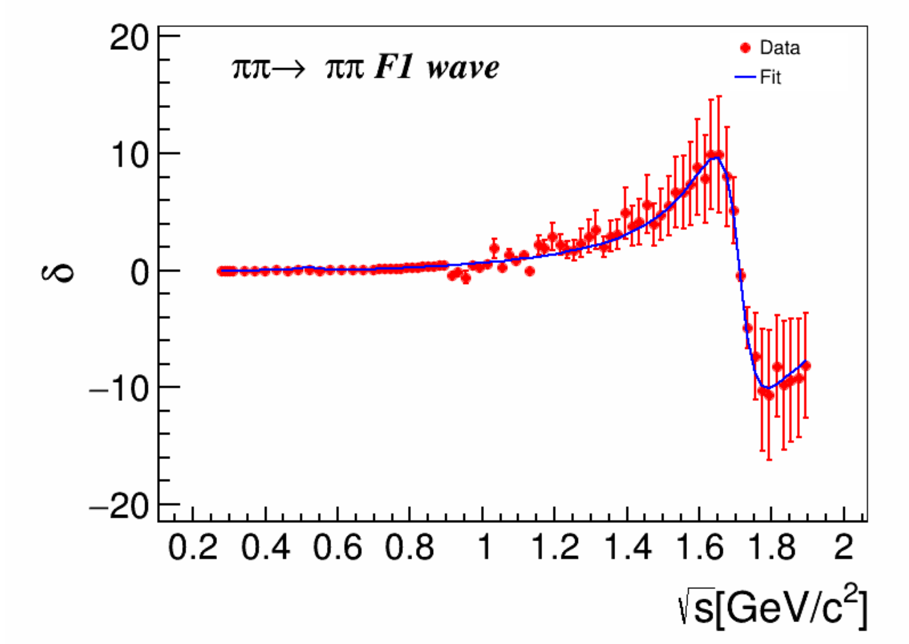
Fit result for Scattering Data: ππ P-Wave

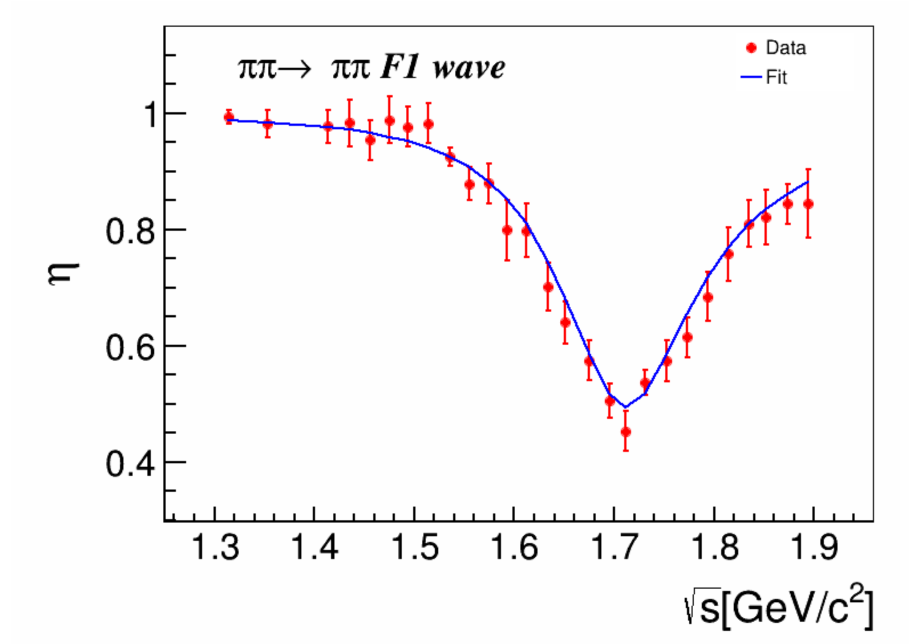


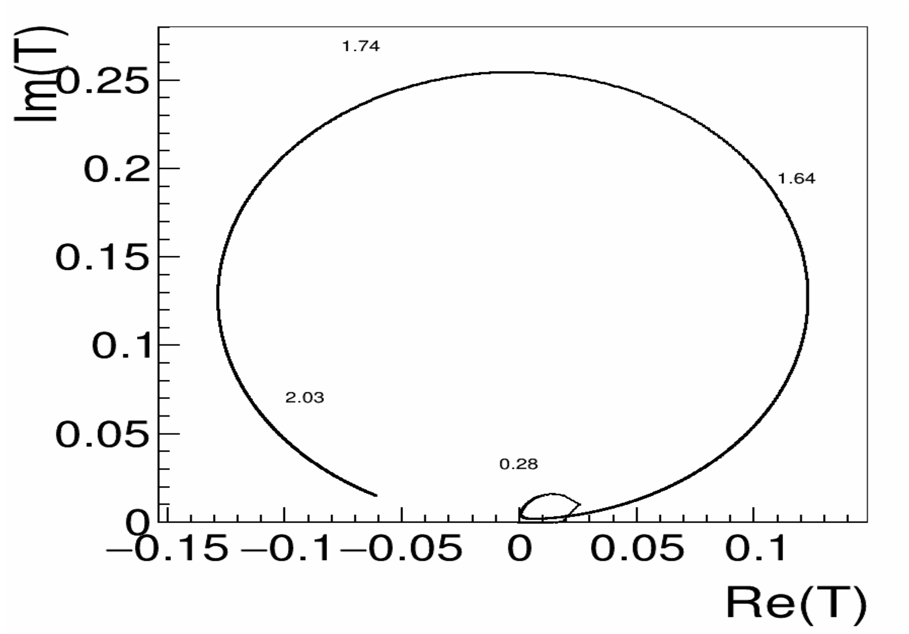




Fit result for Scattering Data: ππ F-Wave

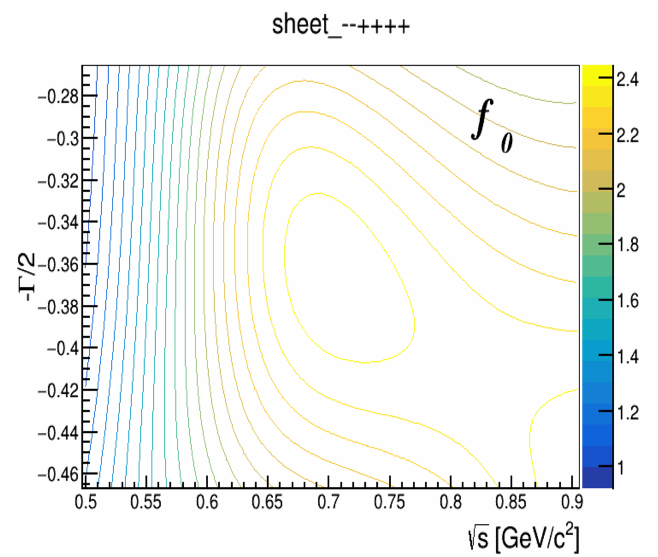


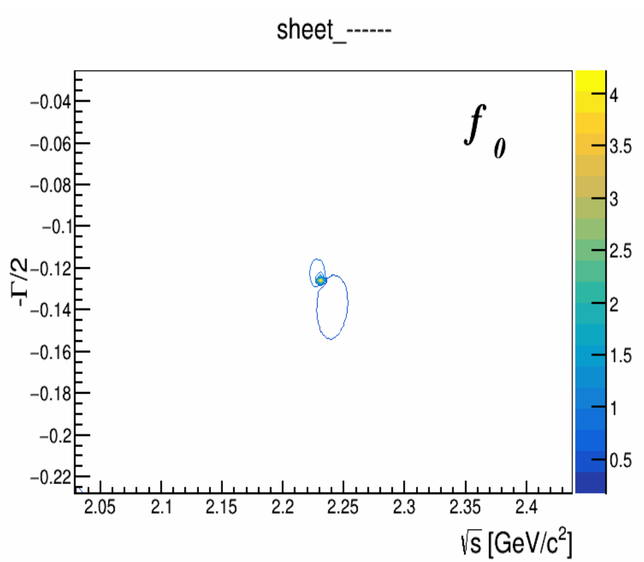


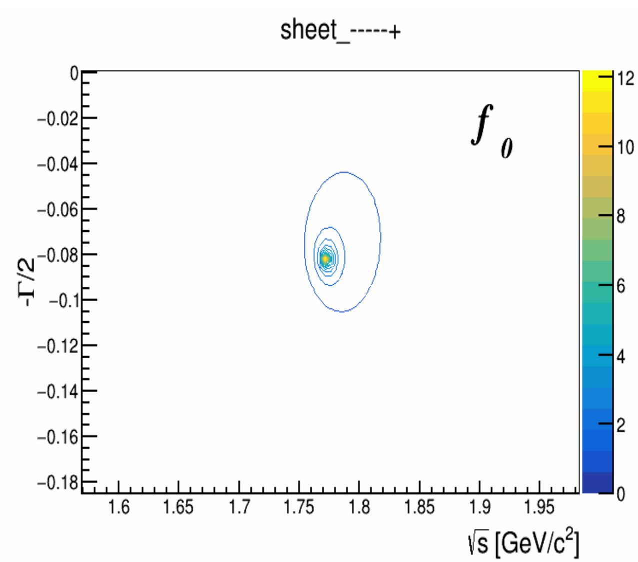


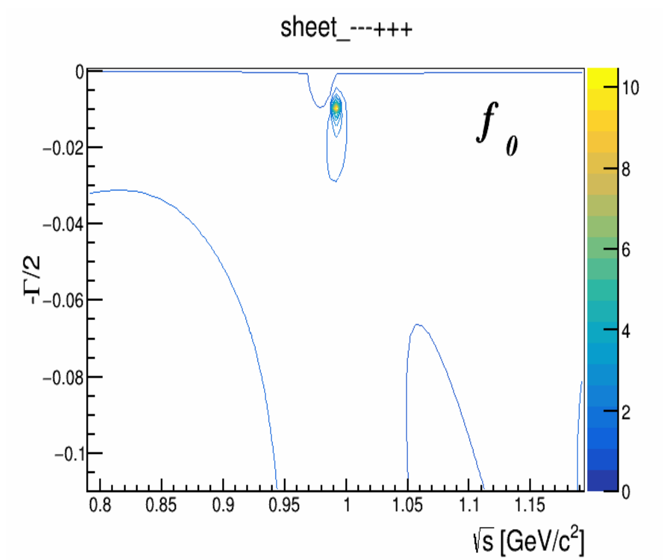
Obtained Pole Positions for f_0 Resonances

Fit Result [MeV/c^2]		PDG [MeV/c^2]		
Mass	Width	PDG State	Mass	Width
683	742	$f_0(550)$	400 - 550	400 - 700
998	46	$f_0(980)$	990 ± 20	10 - 100
1335	319	$f_0(1370)$	1200 - 1500	200 - 500
1771	163	$f_0(1710)$	1723 ± 6	139 ± 8
2231	251	$f_0(2200)$	2189 ± 13	238 ± 50



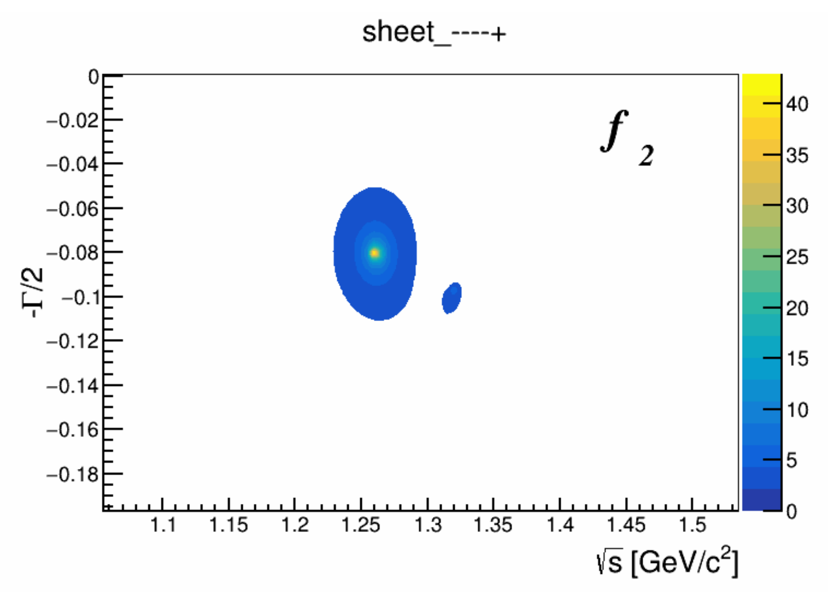


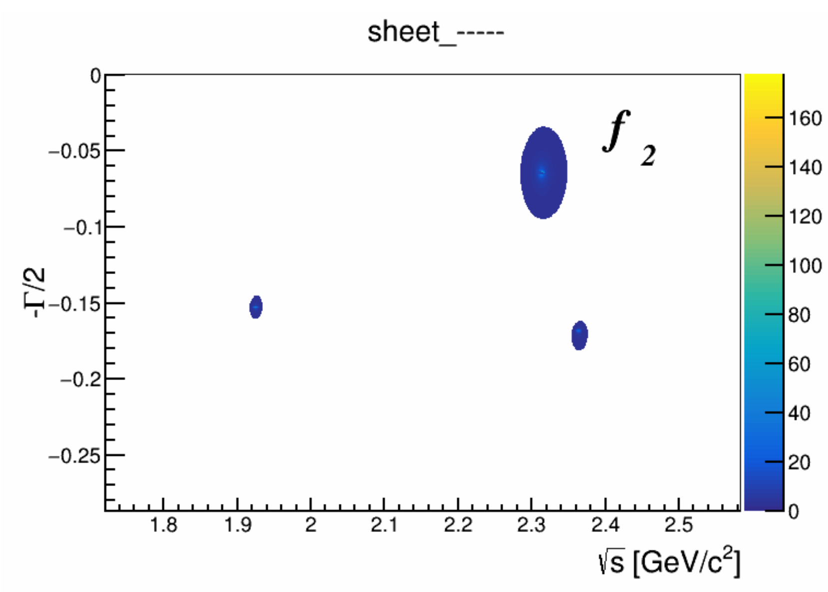




Obtained Pole Positions for f_2 Resonances

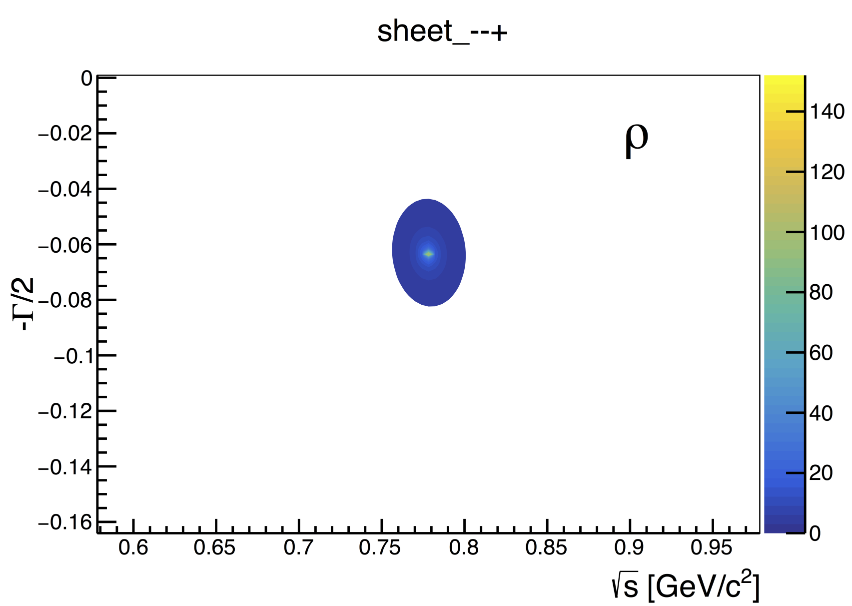
Fit Result [MeV/c²]		PDG [MeV/c²]		
Mass	Width	PDG State	Mass	Width
1260	161	$f_2(1270)$	1275.5 ± 0.8	186.7 ± 2.5
1320	193	$f_2(1430)$	~ 1430	?
1925	305	$f_2(1950)$	1944 ± 12	472 ± 18
2313	129	$f_2(2300)$	2297 ± 28	149 ± 40
2363	337	$f_2(2340)$	2345 ± 50	322 ± 70

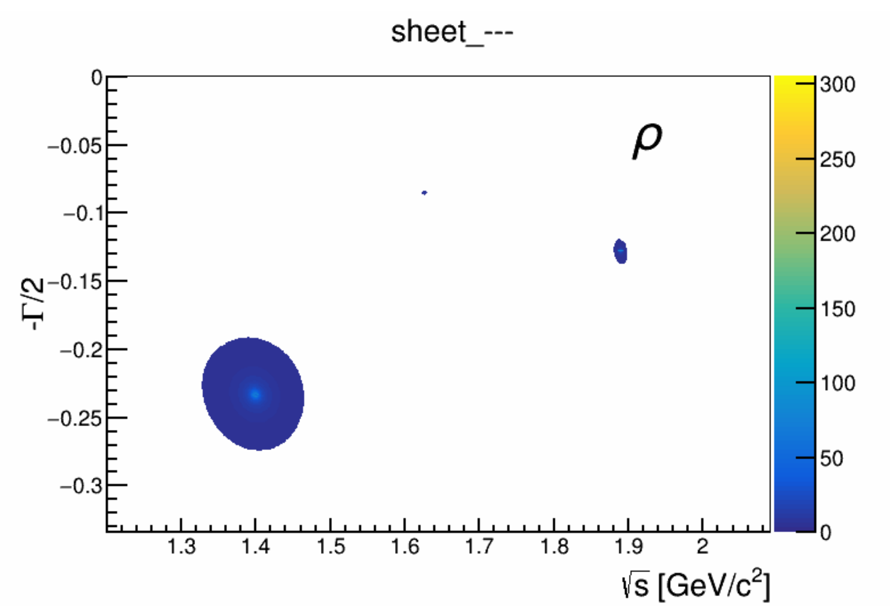




Obtained Pole Positions for ρ Resonances

Fit Result [MeV/c²]		PDG [MeV/c²]		
Mass	Width	PDG State	Mass	Width
778	127	ρ(770)	775.26 ± 0.25	147.8 ± 0.9
1401	467	ρ(1450)	1465 ± 25	400 ± 60
1627	169	ρ(1570)	1570 ± 70	144 ± 90
1890	255	ρ(1900)	1840-1930	15 - 180





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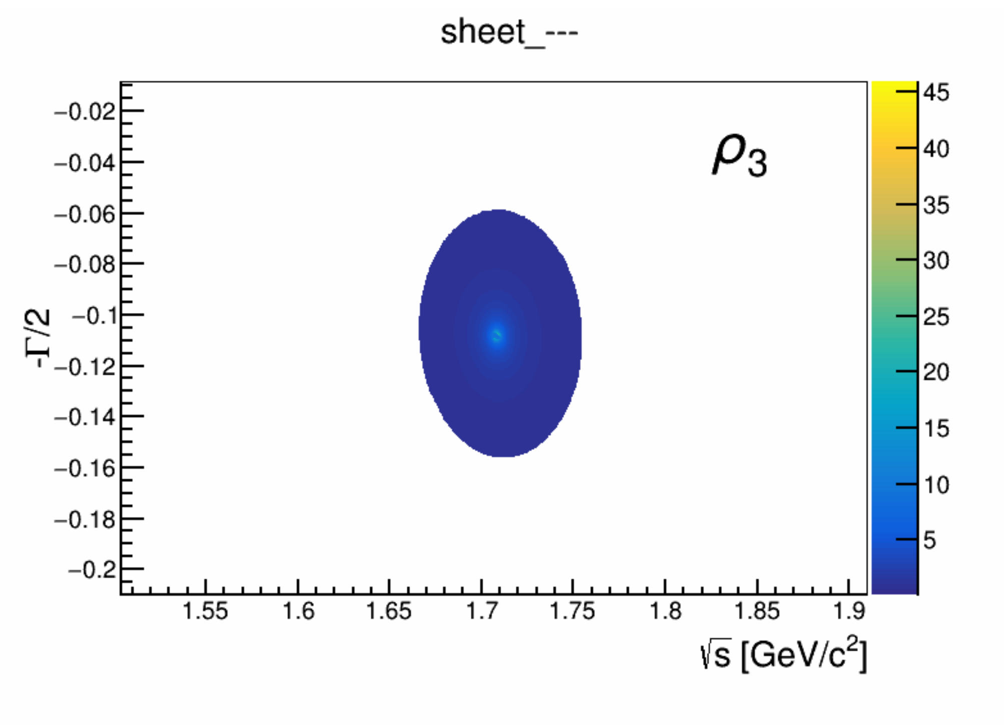
Obtained Pole Positions for ρ_3 Resonances

Fit Result [MeV/c^2]		PDG [MeV/c^2]		
Mass	Width	PDG State	Mass	Width
1708	217	$\rho_3(1690)$	1688.8 ± 2.1	161 ± 10



Summary

- PAWIAN is a user-friendly and generic PWA software package
- Coupled channel analysis with K-Matrix description
  - advantages to fits performed with only Breit-Wigner descriptions
  - can solve ambiguities and is more model independent
- Analysis in the past: $\bar{p}p \rightarrow K^+ K^- \pi^0$, $\pi^0 \pi^0 \eta$, $\pi^0 \eta \eta$
  - fixed K-Matrix parameterization for I=0 $(\pi\pi)_S$ -wave and I=1/2 $(K\pi)_S$ -wave
- Current Analysis: $J/\psi \rightarrow \omega\pi^0\pi^0$, ωK^+K^-, $\omega\bar{p}p$ with scattering data
  - strong constraints to phase shifts and pole positions
  - replacement of normal phase space factor by Chew-Mandelstam function
  - Chew-Mandelstam function for unstable particles
  - reasonable results for the obtained pole positions for f_0, f_2, ρ and ρ_3 states

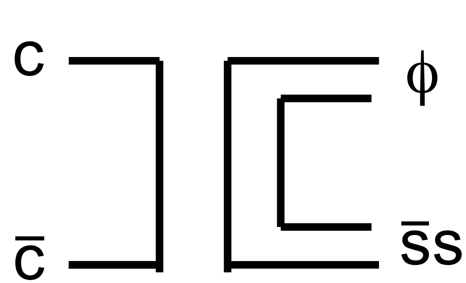
Outlook

- Adding corresponding channels where the ω is replaced be a ϕ and radiative γ, respectively

- Determination of the coupling constants for the different production processes may shed light on the structure of the observed resonances
  - radiative processes are gluon rich processes
  - resonances with s-quark contents are expected to be produced with ϕ recoil particle

J/ψ decay with ω recoil

J/ψ decay with ϕ recoil



Radiative J/ψ decay