Amplitude analyses of multibody hadronic decays at Belle

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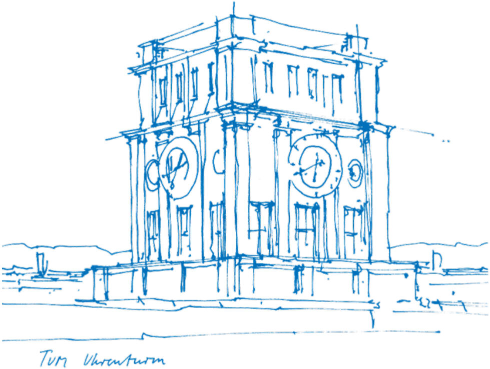

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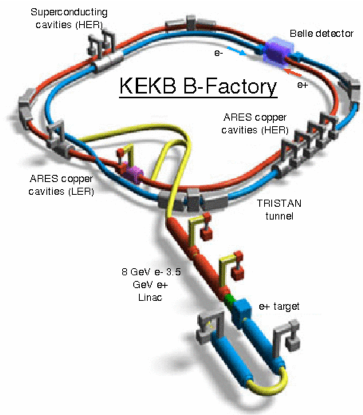

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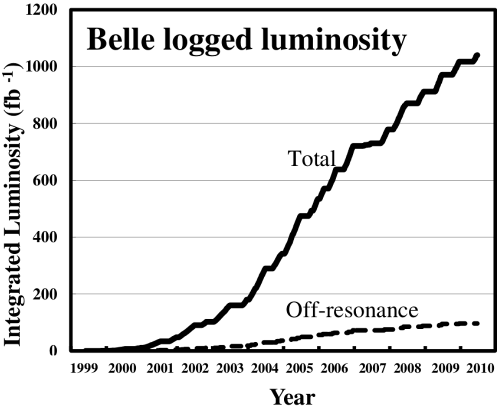
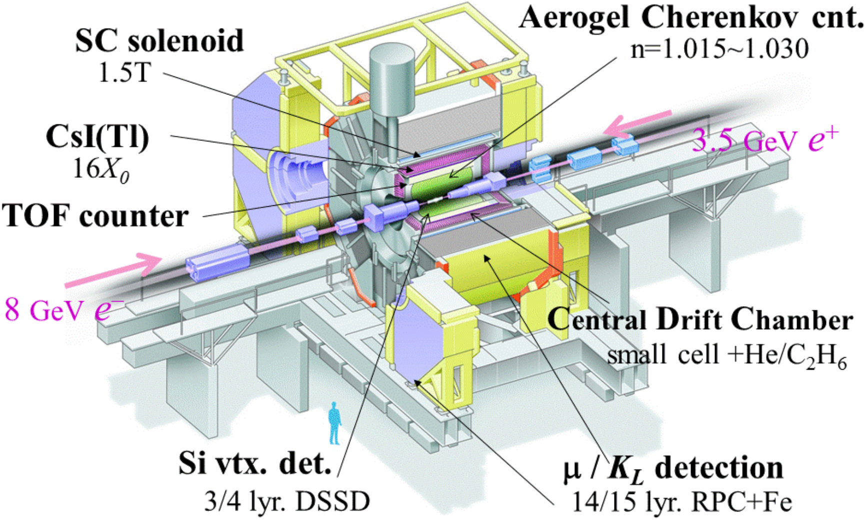
Over a decade of operation, Belle collected approx. $1\,\text{ab}^{-1}$ of integrated luminonsity.



$\Upsilon(4S)$	$711.0\,\text{fb}^{-1}$
Off resonance	$122.4\,\text{fb}^{-1}$
$\Upsilon(5S)$	$121.4\,\text{fb}^{-1}$
$\Upsilon(2S)$	$24.9\,\text{fb}^{-1}$
$\Upsilon(1S)$	$5.7\,\text{fb}^{-1}$
$\Upsilon(3S)$	$2.9\,\text{fb}^{-1}$


SC solenoid
1.5T
CsI(Tl)
$16X_0$
TOF counter
8 GeV e^-
Aerogel Cherenkov cnt.
n=1.015~1.030
3.5 GeV e^+
Central Drift Chamber
small cell +He/C_2H_6
Si vtx. det.
3/4 lyr. DSSD
μ / K_L detection
14/15 lyr. RPC+Fe

I will focus on techniques used rather than specific results:

- "Classic" Dalitz-plot analysis of B/D decays to spinless final-state particles

  $D^0 \rightarrow K^0_S\pi^+\pi^-$, $B^0 \rightarrow K^0_S\pi^+\pi^-$, $\overline{D}^0\pi^+\pi^-$; $B^+ \rightarrow K^+\pi^+\pi^-$, $K^+K^+K^-$

- Variable-initial-mass Dalitz analysis:

  $B^+ \rightarrow (c\bar{c}) + K^+\pi^+\pi^-$

- Dalitz plot analysis of B decays to spinfull final-state particles

  $\overline{B}^0 \rightarrow J/\psi K^-\pi^+$, $\psi' K^-\pi^+$, $\chi_{c1}K^-\pi^+$, $D^{*+}\omega\pi^-$

- Amplitude analysis of

  $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$, $\Upsilon(nS)\pi^0\pi^0$, $J/\psi\pi^+\pi^-$

- τ decay

  $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$

arXiv:1804.06152 / 1804.06152 ($924\,\mathrm{fb}^{-1}$ on and off resonance)

For study of $\cos 2\beta$ in $B^0 \to D^{(*)} h^0$, amplitude analysis of

$$D^0 \to K^0_S \pi^+ \pi^-$$

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Events reconstructed by detection of "good" K^0_S and pions consistent with D^0

and D^0 and $\pi^\pm$ consistent with being from $D^{*\pm} \longrightarrow$ tags D^0/ $\overline{D}^0$

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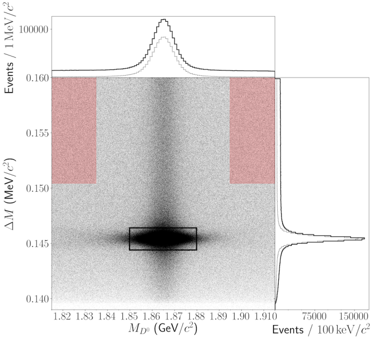

1.3×10^6 events in signal region (black box)

with 94 % purity
background includes wrongly tagged D^0

$D^0 \rightarrow K_S^0 \pi^+ \pi^-$

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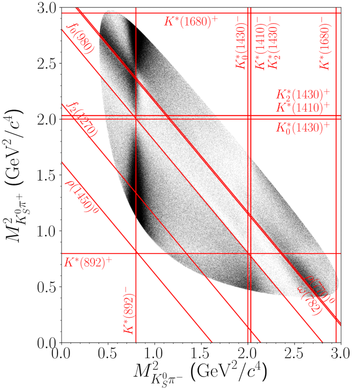

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For study of $\cos 2\beta$ in $B^0 \to D^{(*)} h^0$, amplitude analysis of

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with decay model combining conventional isobar with K matrix and LASS for S waves

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Unbinnned maximum-likelihood fit to data of:

$$P(\mathrm{data}|\vec{\lambda}) = \prod_i f_{\mathrm{sig}} P_{\mathrm{sig}}(\vec{\tau}_i) + (1 - f_{\mathrm{sig}}) f_{\not{D}^*} P_{\not{D}^*}(\vec{\tau}) + (1 - f_{\mathrm{sig}})(1 - f_{\not{D}^*}) P_{\mathrm{bg}}(\vec{\tau})$$

with f_{sig} and $f_{\not{D}^*}$ fixed from fit to M_D–ΔM fit. (All P normalized.)

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$$P_{\not{D}^*} = (1 - f_{\text{w.t.}}) P_{\text{sig}}(\vec{\tau}) + f_{\text{w.t}} P_{\text{sig}}(\vec{\tau}')$$

with $f_{\text{w.t.}} = (49.2 \pm 7.5)\,\%$ the wrong-tag fraction—fixed from fit to ΔM side band.

$\vec{\tau}' =$ CP conjugated $\vec{\tau} \Rightarrow M(K^0_S\pi^+) \leftrightarrow M(K^0_S\pi^-)$.

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$$P_{\rm bg} = \mathsf{Pol}_6(\vec{\tau}) + \sum_r a_r \left| A_r(m_r^2) \right|^2$$

with A_r Breit-Wigner line-shapes for $K^*(892)$, $K^*(1410)$, $K^*(1680)$, $\rho(770)$.

$D^0 \to K_S^0 \pi^+ \pi^-$: Signal

$$P_{\text{sig}}(\vec{\tau}) \propto \text{acceptance}(\vec{\tau}) \times \left| \sum_R \alpha_R A_R^{L\neq 0}(\vec{\tau}) + A_S^{\pi\pi}(\vec{\tau}) + A_S^{K\pi}(\vec{\tau}) \right|^2 \quad \text{(normalized)}$$

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acceptance is parameterized in $\{m^2_{K\pi^-},\ \cos\theta_K\}$, with

$$\cos\theta_K \equiv -\hat{p}_D \cdot \hat{p}_K \quad \text{in } K_S^0\pi^- \text{ r.f.}$$

from large MC sample

$D^0 \rightarrow K^0_S\pi^+\pi^-$: $L\neq0$ Isobar Signal Model

$$A_R^{L\neq0} = F_{\mathsf{D}}^{(L)}(\vec{\tau}) \cdot F_R^{(L)}(\vec{\tau}) \cdot \Omega^{(L)}(\vec{\tau}) \cdot T_R(m_2^2)$$

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Resonances:

$D^0 \to K^0_S\pi^+\pi^-$: $L\neq 0$ Isobar Signal Model

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$\pi^+\pi^-$ states:

$$\rho(770),\ \omega(782),\ \mathrm{f}_2(1270),\ \rho(1450)$$

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$D^0 \rightarrow K^0_S\pi^+\pi^-$: $L\neq0$ Isobar Signal Model

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All masses and widths fixed, except for $K^*(892)$

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$$A^{\pi\pi}_{\mathrm{S}}(\vec{\tau}) = \left[\mathbb{1} - iK(m^2_{\pi\pi})\,\rho(m^2_{\pi\pi})\right]^{-1}_{(\pi\pi),X} P_X(m^2_{\pi\pi})$$

where $X = \pi\pi,\ \mathrm{K\overline{K}^0},\ \pi\pi\pi\pi,\ \eta\eta,\ \eta\eta'$

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$$K_{ij}(s) = \left(f_{ij}^{\text{scat}} \frac{1\,\text{GeV} - s_0^{\text{scat}}}{s - s_0^{\text{scat}}} + \sum_\alpha \frac{g_i^\alpha g_j^\alpha}{m_\alpha^2 - s}\right) f_{A0}(s)$$

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first term describes slowly varying smooth part of amplitude.

$D^0 \to K^0_S \pi^+ \pi^-$: $\pi\pi$ S-wave Model

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first term describes slowly varying smooth part of amplitude.
second term describes physical poles at m_α with couplings to the channels, g_i^α

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first term describes slowly varying smooth part of amplitude.
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all multiplied by Adler zero factor

$$F_{\mathrm{A0}}(s) \equiv \frac{1\,\mathrm{GeV} - s_{\mathrm{A0}}}{s - s_{\mathrm{A0}}}\left(s - s_{\mathrm{A}}\frac{m_\pi^2}{2}\right)$$

to suppress kinematic singularity at $\pi\pi$ threshold.

A K matrix is used for the $\pi\pi$ S wave:

$$A_{\mathrm{S}}^{\pi\pi}(\vec{\tau}) = \left[\mathbb{1} - iK(m_{\pi\pi}^2)\,\rho(m_{\pi\pi}^2)\right]^{-1}_{(\pi\pi),X} P_X(m_{\pi\pi}^2)$$

where $X = \pi\pi,\ \mathrm{K}\overline{\mathrm{K}}^0,\ \pi\pi\pi\pi,\ \eta\eta,\ \eta\eta'$

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and P vector mimicks K-matrix structure

$$P_j(s) = f_{1j}^{\mathrm{prod}}\frac{1\,\mathrm{GeV} - s_0^{\mathrm{prod}}}{s - s_0^{\mathrm{prod}}} + \sum_\alpha \frac{\beta_\alpha g_j^\alpha}{m_\alpha^2 - s}$$

A K matrix is used for the $\pi\pi$ S wave:

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first term describes slowly varying production
second term describes production of channels via complex couplings, β_α

$D^0 \to K^0_S\pi^+\pi^-$: $\pi\pi$ S-wave Model

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All K-matrix parameters are fixed to results of
Aubert (BaBar) PRD78, 034023 (2008) and Anisovich & Sarantsev EPJA16 229 (2003)

P-vector parameters (f_{1j}^{prod}, β_α) free in fit

The LASS parameterization (NuclPhys B296 394, 1988) is used for the $K^0_S\pi$ S wave:

$D^0 \to K^0_S\pi^+\pi^-$: Kπ S-wave Model

The LASS parameterization (NuclPhys B296 394, 1988) is used for the $K^0_S\pi$ S wave:

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$D^0 \to K^0_S\pi^+\pi^-$: $K\pi$ S-wave Model

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$$A_{\mathrm{S}}^{K\pi}(\vec{\tau}) = |\alpha_R| \sin\delta_R(m_{K\pi}^2)\, e^{i\delta_R(m_{K\pi}^2)}\, e^{i2\delta_{\mathrm{nr}}(m_{K\pi}^2)} + |\alpha_{\mathrm{nr}}| \sin\delta_{\mathrm{nr}}(m_{K\pi}^2)\, e^{i\delta_{\mathrm{nr}}(m_{K\pi}^2)}$$

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α_R and α_{nr} are complex-valued amplitudes

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α_R and α_{nr} are complex-valued amplitudes

$$\delta_R \equiv \arg(\alpha_R) + \tan^{-1}\left(\frac{M_R\,\Gamma_R(m^2_{K\pi})}{M_R^2 - m^2_{K\pi}}\right)$$

$$\delta_{nr} \equiv \arg(\alpha_{nr}) + \cot^{-1}\left(\frac{1}{aq} + \frac{rq}{2}\right)$$

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$$A^{K\pi}_{S}(\vec{\tau}) = |\alpha_R| \sin\delta_R(m^2_{K\pi})\, e^{i\delta_R(m^2_{K\pi})}\, e^{i2\delta_{nr}(m^2_{K\pi})} + |\alpha_{nr}| \sin\delta_{nr}(m^2_{K\pi})\, e^{i\delta_{nr}(m^2_{K\pi})}$$

α_R and α_{nr} are complex-valued amplitudes

$$\delta_R \equiv \arg(\alpha_R) + \tan^{-1}\left(\frac{M_R\,\Gamma_R(m^2_{K\pi})}{M^2_R - m^2_{K\pi}}\right)$$

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$D^0 \to K^0_S\pi^+\pi^-$: $K\pi$ S-wave Model

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M_R and $\Gamma_R(m^2_{K\pi})$ describe the $K^*_0(1430)$ resonance
(q is momentum of spectator pion in the $K\pi$ r.f.)

All parameters are free in fit; But only one set of parameters is used for both Cabibbo-favored $K^*_0(1430)^-$ and Cabibbo-suppressed $K^*_0(1430)^+$

An unbinned maximum-likelihood fit is performed,
but the goodness of fit is calculated by binning the data:

$$\chi^2/\mathsf{ndf} = 1.05$$

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result is worsened by

- adding more resonances
- replacing K-matrix or LASS by isobars
- freeing massses and widths
- using more complicated line shapes (e.g. Gounaris-Sakurai)

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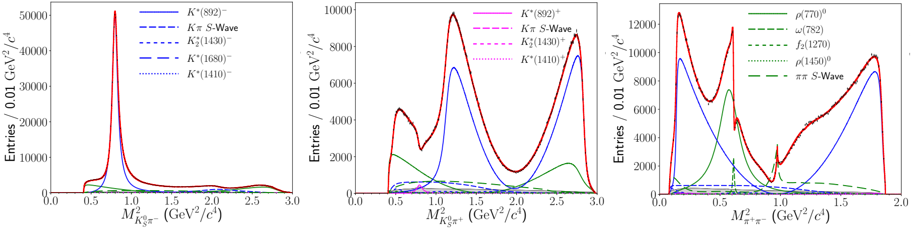

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Resonance	Amplitude	Phase (deg)	Fit Fraction (%)
$K^0_S\rho(770)^0$	1 (fixed)	0 (fixed)	20.4
$K^0_S\omega(782)$	0.0388 ± 0.0005	120.7 ± 0.7	0.5
$K^0_S f_2(1270)$	1.43 ± 0.03	-36.3 ± 1.1	0.8
$K^0_S\rho(1450)^0$	2.85 ± 0.10	102.1 ± 1.9	0.6
$K^*(892)^-\pi^+$	1.720 ± 0.006	136.8 ± 0.2	59.9
$K^*_2(1430)^-\pi^+$	1.27 ± 0.02	-44.1 ± 0.8	1.3
$K^*(1680)^-\pi^+$	3.31 ± 0.20	-118.2 ± 3.1	0.5
$K^*(1410)^-\pi^+$	0.29 ± 0.03	99.4 ± 5.5	0.1
$K^*(892)^+\pi^-$	0.164 ± 0.003	-42.2 ± 0.9	0.6
$K^*_2(1430)^+\pi^-$	0.10 ± 0.01	-89.6 ± 7.6	< 0.1
$K^*(1410)^+\pi^-$	0.21 ± 0.02	150.2 ± 5.3	< 0.1

Phys. Rev. D 83, 032005 (2011) (492 fb^{-1} on-resonance)

Amplitude analysis of $K^+\pi^+\pi^-$ produced in

$$B^+ \to J/\psi K^+\pi^+\pi^- \quad (\text{and } B^+ \to \psi' K^+\pi^+\pi^-)$$

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Events reconstructed by detection of "good" charmonium and hadrons with

$$M_{\rm bc} \equiv \sqrt{\frac{s}{4} - \left(\sum_i \vec{p}_i\right)^2} > 5.27\,\text{GeV} \quad \text{and} \quad |\Delta E| \equiv \left|\frac{\sqrt{s}}{2} - \sum_i E_i\right| < 0.2\,\text{Gev}$$

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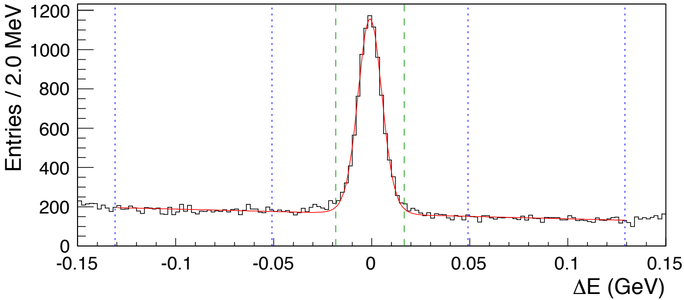

10 594 signal-region events, 12 913 side-band events

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with an isobar model for three-body resonances R_3 and two-body resonances R_2:

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$$P(\text{data}|\vec{\lambda}) = \prod_i f_{\text{bg}} P_{\text{bg}}(\vec{\tau}_i) + (1 - f_{\text{bg}}) P_{\text{sig}}(\vec{\tau}_i|\vec{\lambda})$$

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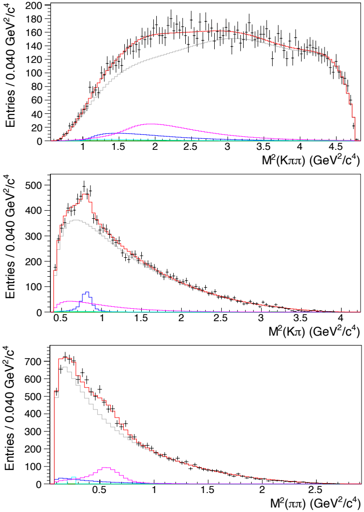
and

r	$\lvert A_r \rvert^2$
$K^*(892)$	$\left\lvert \text{Breit-Wigner}(m^2_{K\pi}) \right\rvert^2$
$\rho(770)$	$\left\lvert \text{Breit-Wigner}(m^2_{\pi\pi}) \right\rvert^2$
D^0	$\text{Gaus}(m^2_{K\pi})$
K^0_S	$\text{Gaus}(m^2_{\pi\pi})$

all normalized to kinematically allowed phsp

$B^+ \to J/\psi + K^+\pi^+\pi^-$: Background



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acceptance & phase-space density taken from MC
acceptance in $(0.15\,\text{GeV}^2)^3$ bins
density in $(0.02\,\text{GeV}^2)^3$ bins

$$P_{\text{sig}}(\vec{\tau}|\vec{\lambda}) = \text{acceptance}(\vec{\tau}) \times \text{phsp-density}(\vec{\tau}) \times s(\vec{\tau}|\vec{\lambda})$$

$$s(\vec{\tau}|\vec{\lambda}) = |\alpha_3^{\text{nr}}|^2 + \sum_{J_3^P} \left| \sum_{R_3 \text{ with } J_3^P} \sum_{R_2} \alpha_{R_3 \to R_2} \cdot \Omega_{J_3 \to J_2}(\vec{\tau}) \cdot T_{R_3}(m_3^2) \cdot T_{R_2}(m_2^2) \right|^2$$

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Two-body resonances: $\rho(770)$, ω, $\mathrm{f}_0(980)$, $\mathrm{f}_2(1270)$, $\mathrm{K}^*(892)$, $\mathrm{K}^*(1430)$

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Two-body resonances: $\rho(770)$, ω, $f_0(980)$, $f_2(1270)$, $K^*(892)$, $K^*(1430)$

All with fixed masses and widths. (Varied within uncertainties for syst. unc.)

Data features prominent $K_1(1270)$ peak—

start with basic model of $K_1(1270) \rightarrow K^*(892)\pi$ and $K_1(1270) \rightarrow K\rho$

then add channels successively until reasonably good fit achieved:

$B^+ \to J/\psi + K^+\pi^+\pi^-$: Results

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TABLE V. Fitted parameters of the signal function for $B^+ \to J/\psi K^+\pi^+\pi^-$, along with the corresponding decay fractions.

J_1	Submode	Modulus	Phase (radians)	Decay fraction
	Nonresonant $K^+\pi^+\pi^-$	1.0 (fixed)	0 (fixed)	$0.152 \pm 0.013 \pm 0.028$
1^+	$K_1(1270) \to K^*(892)\pi$	$0.962 \pm 0.058 \pm 0.176$	0 (fixed)	$0.232 \pm 0.017 \pm 0.058$
	$K_1(1270) \to K\rho$	$1.813 \pm 0.090 \pm 0.243$	$-0.764 \pm 0.069 \pm 0.127$	$0.383 \pm 0.016 \pm 0.036$
	$K_1(1270) \to K\omega$	$0.198 \pm 0.036 \pm 0.041$	$1.09 \pm 0.18 \pm 0.18$	$0.0045 \pm 0.0017 \pm 0.0014$
	$K_1(1270) \to K_0^*(1430)\pi$	$0.95 \pm 0.16 \pm 0.24$	$2.83 \pm 0.18 \pm 0.18$	$0.0157 \pm 0.0052 \pm 0.0049$
	$K_1(1400) \to K^*(892)\pi$	$0.894 \pm 0.066 \pm 0.125$	$-2.300 \pm 0.044 \pm 0.078$	$0.223 \pm 0.026 \pm 0.036$
1^-	$K^*(1410) \to K^*(892)\pi$	$0.516 \pm 0.090 \pm 0.103$	0 (fixed)	$0.047 \pm 0.016 \pm 0.015$
2^+	$K_2^*(1430) \to K^*(892)\pi$	$0.663 \pm 0.051 \pm 0.085$	0 (fixed)	$0.088 \pm 0.011 \pm 0.011$
	$K_2^*(1430) \to K\rho$	0.371 (fixed)	$-1.12 \pm 0.22 \pm 0.29$	0.0233 (fixed)
	$K_2^*(1430) \to K\omega$	0.040 (fixed)	$0.58 \pm 0.51 \pm 0.27$	0.00036 (fixed)
	$K_2^*(1980) \to K^*(892)\pi$	$0.775 \pm 0.054 \pm 0.118$	$-1.59 \pm 0.15 \pm 0.14$	$0.0739 \pm 0.0073 \pm 0.0095$
	$K_2^*(1980) \to K\rho$	$0.660 \pm 0.048 \pm 0.101$	$0.86 \pm 0.22 \pm 0.21$	$0.0613 \pm 0.0058 \pm 0.0059$
2^-	$K(1600) \to K^*(892)\pi$	$0.131 \pm 0.021 \pm 0.024$	0 (fixed)	$0.0187 \pm 0.0058 \pm 0.0050$
	$K(1600) \to K\rho$	$0.193 \pm 0.017 \pm 0.029$	$-0.27 \pm 0.27 \pm 0.18$	$0.0424 \pm 0.0062 \pm 0.0110$
	$K_2(1770) \to K^*(892)\pi$	$0.122 \pm 0.021 \pm 0.026$	$2.22 \pm 0.49 \pm 0.37$	$0.0164 \pm 0.0055 \pm 0.0061$
	$K_2(1770) \to K_2^*(1430)\pi$	$0.286 \pm 0.043 \pm 0.044$	$1.78 \pm 0.39 \pm 0.24$	$0.0100 \pm 0.0028 \pm 0.0020$
	$K_2(1770) \to K f_2(1270)$	$0.444 \pm 0.069 \pm 0.077$	$2.30 \pm 0.37 \pm 0.32$	$0.0124 \pm 0.0033 \pm 0.0022$
	$K_2(1770) \to K f_0(980)$	$0.113 \pm 0.029 \pm 0.024$	$1.83 \pm 0.45 \pm 0.53$	$0.0034 \pm 0.0017 \pm 0.0011$

All masses and widths fixed. Unbinned fit; binned g.o.f. check: $\chi^2/\text{ndf} = 1.26$

2nd fit with freed mass and width for $K_1(1270)$:

$$M_{K_1(1270)} = (1248.1 \pm 3.3 \pm 1.4)\ \text{MeV} \quad \text{and} \quad \Gamma_{K_1(1270)} = (119.5 \pm 5.2 \pm 6.7)\ \text{MeV}$$

$B^+ \to J/\psi + K^+\pi^+\pi^-$: Results

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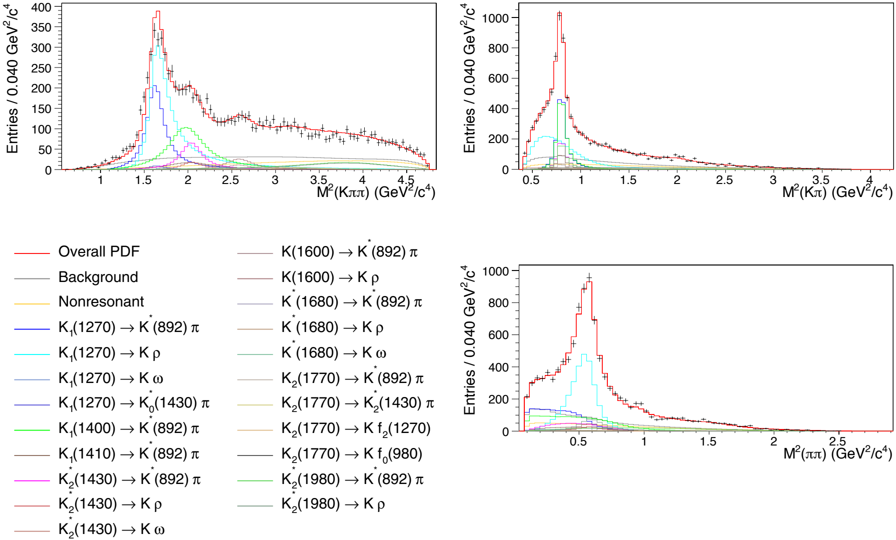

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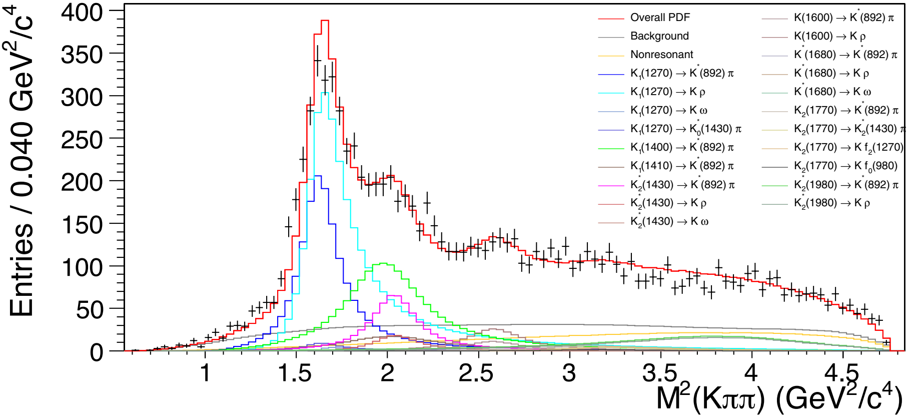

Phys. Rev. D 90, 112009 (2014) ($711\,\mathrm{fb}^{-1}$ on resonance)

Amplitude analysis of

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for Z_c spectroscopy.

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Again, events reconstructed by detection of "good" J/ψ and hadrons with

M_{bc} within 7 MeV of B

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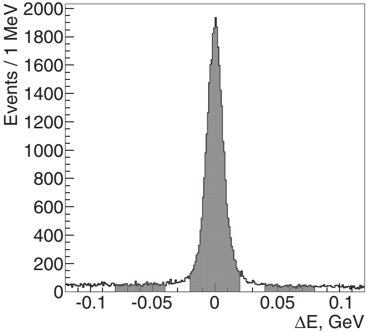

31 774 signal-region events, $(94.4 \pm 0.6)\,\%$ purity

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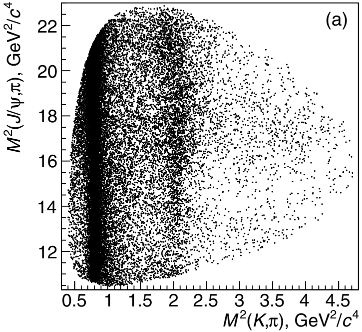
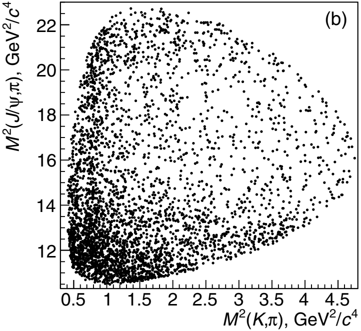
signal



sideband



Phys. Rev. D 90, 112009 (2014) (711 fb^{-1} on resonance)

Amplitude analysis of

$$\overline{B}^0 \rightarrow J/\psi K^- \pi^+$$

for Z_c spectroscopy.

with an isobar-model analysis & freed-isobar (model-independent) check

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$$\vec{\tau} = \{m^2_{\text{K}\pi}, m^2_{\text{J}/\psi\pi}, \theta_{\text{J}/\psi}, \phi\}$$

$$\theta_{\text{J}/\psi} \equiv \text{angle}(\vec{p}_{\ell^+}, \vec{p}_{\text{K}\pi}) \text{ in J}/\psi \text{ r.f.} \quad \text{and} \quad \phi \equiv \text{angle}(\hat{n}_{\ell^+\ell^-}, \hat{n}_{\text{K}\pi}) \text{ in } \overline{\text{B}}^0 \text{ r.f.}$$

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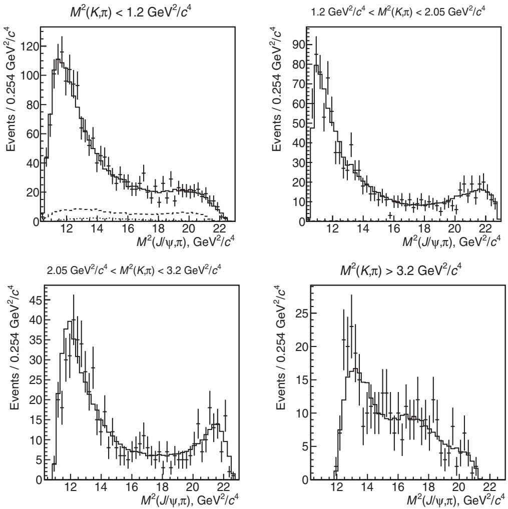
and

r	$\lvert A_r \rvert^2$
$\text{K}^*(892)$	$\left\lvert \text{Breit-Wigner}(m^2_{\text{K}\pi}) \right\rvert^2$
K^0_S with π seen as K	$\text{Gaus}\left(m^2_{\text{K}\pi} \lvert \mu(m^2_{\text{J}/\psi\pi})\right)$


M²(K,π) < 1.2 GeV²/c⁴
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2.05 GeV²/c⁴ < M²(K,π) < 3.2 GeV²/c⁴
M²(K,π) > 3.2 GeV²/c⁴
Events / 0.254 GeV²/c⁴
M²(J/ψ,π), GeV²/c⁴

$$P_{\text{sig}}(\vec{\tau}) = \sum_{\zeta=-1,1} \left| \sum_R \sum_\lambda \alpha_\lambda^R \cdot \Omega_{\lambda\zeta}^{j_R}(\vec{\tau}) \cdot T_R(m_R^2) \cdot F_{\mathrm{B}}^{(L_{\mathrm{B}})} F_R^{(L_R)} \left(\frac{q_{\mathrm{B}}}{m_{\mathrm{B}}}\right)^{L_{\mathrm{B}}} \left(\frac{q_R}{m_{\mathrm{B}}}\right)^{L_R} \right|^2$$

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spin amplitudes given in helicity formalism

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for resonances in $\mathrm{J}/\psi\pi$. $d_{0\lambda}^{j}(\theta_{\mathrm{K}\pi}^{(\mathrm{J}/\psi\pi)}) \to d_{\lambda 0}^{j}(\theta_{\mathrm{J}/\psi\pi}^{(\mathrm{K}\pi)})$ for resonances in $\mathrm{K}\pi$.

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T are all relativistic Breit-Wigner lineshapes with mass-dependent widths.

$\overline{B}^0 \to J/\psi K^-\pi^+$: Signal Model

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The model includes resonances in $K\pi$

$$K_0^*(800),\ K^*(892),\ K^*(1410),\ K_0^*(1430),\ K_2^*(1430),$$
$$K^*(1680),\ K_3^*(1780),\ K_0^*(1950),\ K_2^*(1980),\ K_4^*(2045)$$

all with fixed masses and widths.
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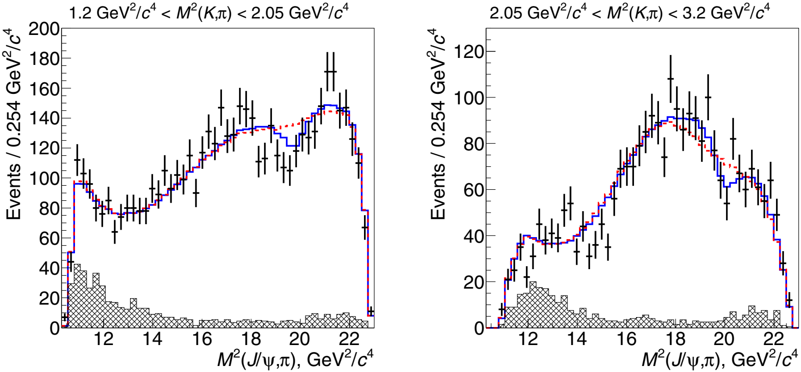
The masses and widths are left free,
but Gaussian priors are placed on those of the $\mathsf{Z_c}(4430)^+$ from previous measurement

$$M = 4485^{+36}_{-25}\ \text{MeV} \quad \text{and} \quad \Gamma = 200^{+49}_{-58}\ \text{MeV}$$

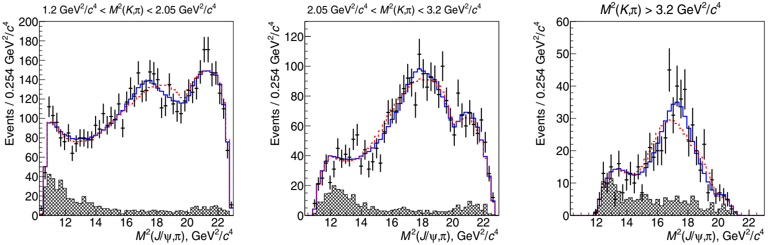


with and without the $Z_c(4430)^+$ (both without the $Z_c(4200)^+$)

Seen with stat. significance of 5.1 σ (4.0 σ with syst.) $\to$ new decay channel

Several J^P hypotheses were tried for the $\mathrm{Z_c}(4200)^+$

J^P	0^-	1^-	1^+	2^-	2^+
Mass, MeV/c^2	4318 ± 48	4315 ± 40	4196^{+31}_{-29}	4209 ± 14	4203 ± 24
Width, MeV	720 ± 254	220 ± 80	370 ± 70	64 ± 18	121 ± 53
Significance (Wilks)	3.9σ	2.3σ	8.2σ	3.9σ	1.9σ



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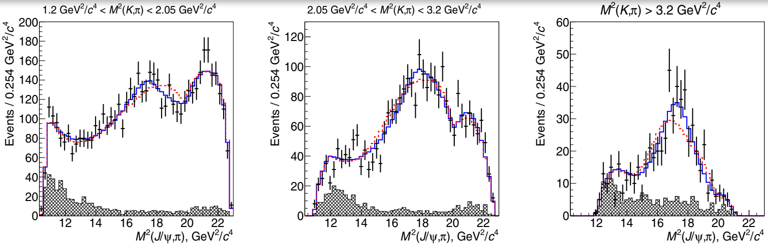
$\overline{B}^0 \rightarrow J/\psi K^- \pi^+$: Results



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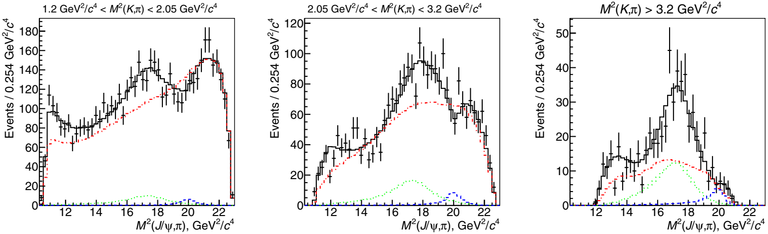

$K\pi$ resonances, $Z_c(4200)$, $Z_c(4430)$

Full model altered:
Breit Wigner of $J^P = 1^+$ $Z_c(4200)^+ \longrightarrow$ complex-valued step functions

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$\overline{B}^0 \rightarrow J/\psi K^- \pi^+$: Model-Independent Fit

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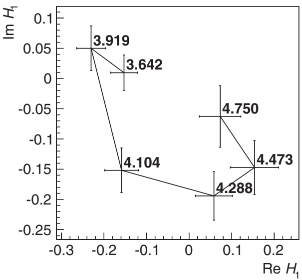

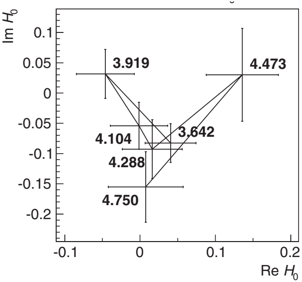

Phys. Rev. D 91, 072003 (2015) ($121.4\,\mathrm{fb}^{-1}$ on $\Upsilon(5S)$ resonance)

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Events reconstructed via $e^+e^- \to \mu^+\mu^-\pi^+\pi^-$

$$m^2_{\mu\mu} \sim m^2_{\Upsilon} \quad \text{and} \quad m^2_{\text{miss}} \equiv (\sqrt{s} - E_{\pi\pi})^2 - |\vec{p}_{\pi\pi}|^2 \sim m^2_{\Upsilon}$$

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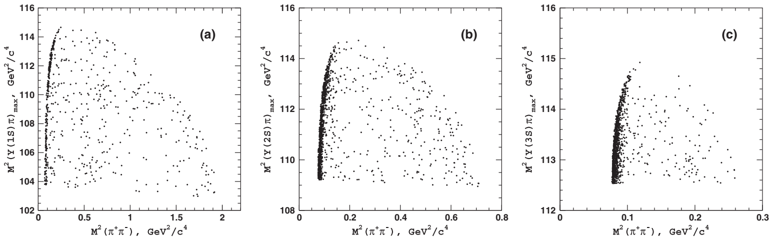

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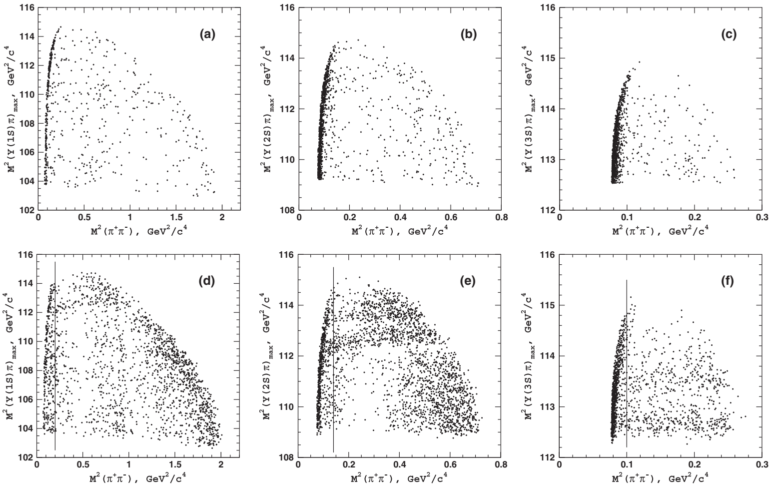

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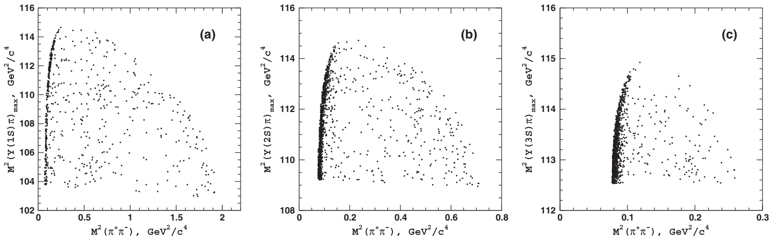

Remaining background is parameterized as flat in phase space with additional component exponential in $m^2_{\pi\pi}$

$e^+e^- \to \Upsilon(nS)\pi^+\pi^-$: Signal

$$P_{\text{sig}}(\vec{\tau}) \propto \left| \alpha^{(1)}_{\text{n.r.}} + \alpha^{(2)}_{\text{n.r.}} m^2_{\pi\pi} + \sum_R \alpha_R \cdot \Omega_R(\vec{\tau}) \cdot T_R(\vec{\tau}) \right|^2$$

convolved with resolution

normalized with detector-simulated MC → acceptance accounted for

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All Breit-Wigner, but f_0 as Flatte   all shape parameters fixed

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In $\Upsilon(nS)\pi$:

$$Z_b(10610)^\pm,\ Z_b(10650)^\pm$$

Both Breit-Wigner

all parameters free

Several J^P hypotheses are tested for the $Z_b^\pm$:

$Z_b(10610)$	$Z_b(10650)$ 1^+	1^-	2^+	2^-
1^+	0(0)	60(33)	42(33)	77(63)
1^-	226(47)	264(73)	224(68)	277(106)
2^+	205(33)	235(104)	207(87)	223(128)
2^-	289(99)	319(111)	321(110)	304(125)

$$\Delta \equiv \log P(\text{both } J^P = 1^+) - \log P \text{ for } \Upsilon(2S)\pi\pi\ (\Upsilon(3S)\pi\pi)$$

For $\Upsilon(1S)\pi\pi$, J^P assumed the same for both Z_b:

$$\Delta(1^-) = 64 \quad \Delta(2^+) = 41 \quad \Delta(2^-) = 59$$

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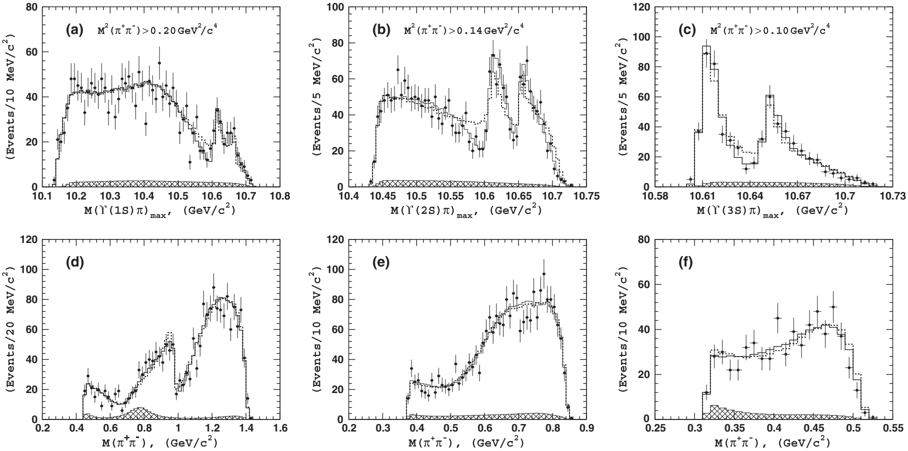
$J^P = 1^+$ favored over other configurations by over 6σ



solid = both $J^P = 1^+$ dotted = both $J^P = 2^+$

$e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$: Results

Parameter	$\Upsilon(1S)\pi^+\pi^-$	$\Upsilon(2S)\pi^+\pi^-$	$\Upsilon(3S)\pi^+\pi^-$
$f_{Z_b^\mp(10610)\pi^\pm}$, %	$4.8 \pm 1.2^{+1.5}_{-0.3}$	$18.1 \pm 3.1^{+4.2}_{-0.3}$	$30.0 \pm 6.3^{+5.4}_{-7.1}$
$Z_b(10610)$ mass, MeV/c^2	$10608.5 \pm 3.4^{+3.7}_{-1.4}$	$10608.1 \pm 1.2^{+1.5}_{-0.2}$	$10607.4 \pm 1.5^{+0.8}_{-0.2}$
$Z_b(10610)$ width, MeV	$18.5 \pm 5.3^{+6.1}_{-2.3}$	$20.8 \pm 2.5^{+0.3}_{-2.1}$	$18.7 \pm 3.4^{+2.5}_{-1.3}$
$f_{Z_b^\mp(10650)\pi^\pm}$, %	$0.87 \pm 0.32^{+0.16}_{-0.12}$	$4.05 \pm 1.2^{+0.95}_{-0.15}$	$13.3 \pm 3.6^{+2.6}_{-1.4}$
$Z_b(10650)$ mass, MeV/c^2	$10656.7 \pm 5.0^{+1.1}_{-3.1}$	$10650.7 \pm 1.5^{+0.5}_{-0.2}$	$10651.2 \pm 1.0^{+0.4}_{-0.3}$
$Z_b(10650)$ width, MeV	$12.1^{+11.3+2.7}_{-4.8-0.6}$	$14.2 \pm 3.7^{+0.9}_{-0.4}$	$9.3 \pm 2.2^{+0.3}_{-0.5}$
ϕ_Z, degrees	$67 \pm 36^{+24}_{-52}$	$-10 \pm 13^{+34}_{-12}$	$-5 \pm 22^{+15}_{-33}$
$c_{Z_b(10650)}/c_{Z_b(10610)}$	$0.40 \pm 0.12^{+0.05}_{-0.11}$	$0.53 \pm 0.07^{+0.32}_{-0.11}$	$0.69 \pm 0.09^{+0.18}_{-0.07}$
$f_{\Upsilon(nS)f_2(1270)}$, %	$14.6 \pm 1.5^{+6.3}_{-0.7}$	$4.09 \pm 1.0^{+0.33}_{-1.0}$	–
$f_{\Upsilon(nS)(\pi^+\pi^-)_S}$, %	$86.5 \pm 3.2^{+3.3}_{-4.9}$	$101.0 \pm 4.2^{+6.5}_{-3.5}$	$44.0 \pm 6.2^{+1.8}_{-4.3}$
$f_{\Upsilon(nS)f_0(980)}$, %	$6.9 \pm 1.6^{+0.8}_{-2.8}$	–	–

for both $J^P = 1^+$

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- in B and D decays to light pseudoscalar mesons
- in B decays to flavorless mesons and open and closed charm states
- in $e^+e^- \rightarrow$ quarkonia $+\pi\pi$ production
- in hadronic τ decays
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$\pi^+\pi^-$ S-wave Parameters			10.0
β_1	8.5 ± 0.5	68.5 ± 3.4	
β_2	12.2 ± 0.3	24.0 ± 1.4	
β_3	29.2 ± 1.6	-0.1 ± 2.5	
β_4	10.8 ± 0.5	-51.9 ± 2.4	
f_{11}^{prod}	8.0 ± 0.4	-126.0 ± 2.5	
f_{12}^{prod}	26.3 ± 1.6	-152.3 ± 3.0	
f_{13}^{prod}	33.0 ± 1.8	-93.2 ± 3.1	
f_{14}^{prod}	26.2 ± 1.3	-121.4 ± 2.7	
s_0^{prod}	-0.07 (fixed)		
$K\pi$ S-wave Parameters			
$K_0^*(1430)^-\pi^+$	2.36 ± 0.06	99.4 ± 1.7	7.0
$K_0^*(1430)^+\pi^-$	0.11 ± 0.01	162.3 ± 6.6	< 0.1
$\text{M}_{K_0^*(1430)^\pm}$ (GeV/c^2)	1.441 ± 0.002		
$\Gamma_{K_0^*(1430)^\pm}$ (GeV)	0.193 ± 0.004		
F	$+0.96 \pm 0.07$		
R	1 (fixed)		
a	$+0.113 \pm 0.006$		
r	-33.8 ± 1.8		
ϕ_F (deg)	0.1 ± 0.3		
ϕ_R (deg)	-109.7 ± 2.6		
$K^*(892)^\pm$ Parameters			
$\text{M}_{K^*(892)^\pm}$ (GeV/c^2)	0.8937 ± 0.0001		
$\Gamma_{K^*(892)^\pm}$ (GeV)	0.0472 ± 0.0001		

TABLE II. The K-matrix parameters estimated by a global analysis of available $\pi\pi$ scattering data (taken from Refs. [57, 61]). The units of the pole masses m_α and the coupling constants g_i^α are in GeV/c^2. The units of s_0^{scatt} and s_{A0} are GeV$^2/c^4$, while s_A is dimensionless.

m_α	$g^\alpha_{\pi^+\pi^-}$	$g^\alpha_{K\bar{K}}$	$g^\alpha_{4\pi}$	$g^\alpha_{\eta\eta}$	$g^\alpha_{\eta\eta'}$
0.65100	0.22889	−0.55377	0.00000	−0.39899	−0.34639
1.20360	0.94128	0.55095	0.00000	0.39065	0.31503
1.55817	0.36856	0.23888	0.55639	0.18340	0.18681
1.21000	0.33650	0.40907	0.85679	0.19906	−0.00984
1.82206	0.18171	−0.17558	−0.79658	−0.00355	0.22358
	f_{11}^{scatt}	f_{12}^{scatt}	f_{13}^{scatt}	f_{14}^{scatt}	f_{15}^{scatt}
	0.23399	0.15044	−0.20545	0.32825	0.35412
s_0^{scatt}	s_{A0}	s_A			
−3.92637	−0.15	1			

TABLE II. The absolute values and phases of the helicity amplitudes in the default model for the 1^+ spin-parity of the $Z_c(4200)^+$. Errors are statistical only.

Resonance	$\lvert H_0 \rvert$	arg H_0	$\lvert H_1 \rvert$	arg H_1	$\lvert H_{-1} \rvert$	arg H_{-1}
$K_0^*(800)$	1.12 ± 0.04	2.30 ± 0.04	...	...	...	...
$K^*(892)$	1.0 (fixed)	0.0 (fixed)	$(8.44 \pm 0.10) \times 10^{-1}$	3.14 ± 0.03	$(1.96 \pm 0.14) \times 10^{-1}$	-1.70 ± 0.07
$K^*(1410)$	$(1.19 \pm 0.27) \times 10^{-1}$	0.81 ± 0.26	$(1.23 \pm 0.38) \times 10^{-1}$	-1.04 ± 0.26	$(0.36 \pm 0.39) \times 10^{-1}$	0.67 ± 1.06
$K_0^*(1430)$	$(8.90 \pm 0.28) \times 10^{-1}$	-2.17 ± 0.05	...	...	...	...
$K_2^*(1430)$	4.66 ± 0.18	-0.32 ± 0.05	4.65 ± 0.18	-3.05 ± 0.08	1.26 ± 0.23	-1.92 ± 0.20
$K^*(1680)$	$(1.39 \pm 0.43) \times 10^{-1}$	-2.46 ± 0.31	$(0.82 \pm 0.48) \times 10^{-1}$	-2.85 ± 0.49	$(1.61 \pm 0.56) \times 10^{-1}$	1.88 ± 0.28
$K_3^*(1780)$	16.8 ± 3.6	-1.43 ± 0.24	19.1 ± 4.5	2.03 ± 0.31	10.2 ± 5.2	1.55 ± 0.62
$K_0^*(1950)$	$(2.41 \pm 0.60) \times 10^{-1}$	-2.39 ± 0.25	...	...	...	...
$K_2^*(1980)$	4.53 ± 0.74	-0.26 ± 0.16	3.78 ± 0.98	3.08 ± 0.28	3.51 ± 1.03	2.63 ± 0.34
$K_4^*(2045)$	590 ± 136	-2.66 ± 0.23	676 ± 164	0.06 ± 0.25	103 ± 174	-1.03 ± 1.62
$Z_c(4430)^+$	1.12 ± 0.32	-0.31 ± 0.26	1.17 ± 0.46	0.77 ± 0.25	$H_{-1} = H_1$	
$Z_c(4200)^+$	0.71 ± 0.37	2.14 ± 0.40	3.23 ± 0.79	3.00 ± 0.15	$H_{-1} = H_1$	