FSI in hadronic three-body decay

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PAATHOS - 18/07/2018 - Beijing

International workshop on Partial Wave Analyses and Advanced Tools for Hadron Spectroscopy

outline

- TRR110 Workshop - Amplitudes for Three-Body Final States

  $\longrightarrow$ theory and experiments on 3-body : interplay, challenges and achievements

  - why study three-body decays?

    $\longrightarrow$ dynamics of heavy/light meson decays

    $\longrightarrow$ Final State Interaction

  - highlights of the works concerning 3-body hadronic decay

    $\longrightarrow$ main issues and tasks list

- example : $D^+ \to K^- K^+ K^-$ my work!

  $\longrightarrow$ Full model fitted to LHCb data

  $\longrightarrow$ based in chiral symmetry

  $\longrightarrow$ prediction for $K\overline{K}$ scattering amplitude

was discussed in Munich WS

TRR110 Workshop - Amplitudes for Three-Body Final States

CRC Bochum/Bonn/Beijing/Munich

Munich 11 -13 July 2018

https://indico.ph.tum.de/event/3988/

how to improve data analyses?
where theory is crucial in data analyses?
how experimentalists can help theoreticians?

Meson-Meson scattering

Jacobo Ruiz de Elvira (Bern)
Dispersive analysis for πK

Bachir Moussallam (Paris)
Isospin and chiral symmetries in D → K ππ amplitudes in the Khuri-Treiman formalism

Miguel Albaladejo (Murcia)
Khuri-Treiman for ππ scattering

Stefan Ropertz (Bonn)
Extensions of pion form factors beyond 1 GeV

Manoel Robilotta (Sao Paulo) this talk
KK scattering predictions from D+ → K-K+K+ decay amplitude

Bertram Kopf (Bochum - BESS II) Tuesday
Coupled Channel Analysis with e+e- and Scattering data

Three-body final states interactions

Tobias Isken (Bonn) A. Jackura
Khuri--Treiman for η' decays and D+ → K-π+π+

Bruno El-Bennich (Sao Paulo)
Parametrisations of hadronic three-body decay amplitudes

Feng-Kun Guo (Beijing) Monday
What we learn about the charmed meson spectrum from B → Dππ

Keri Vos (Siegen)
CP Violation in Multi-body B-decays from QCD Factorization

Emilie Passemar (Indiana)
Hadronic tau decays

Experimental Three-body final states interactions

M. Mikhasenko (Bonn - COMPASS)
3π production in COMPASS

Fabian Krinner (TUM - COMPASS) Dima R., B. Grube
Model-independent analysis in COMPASS

Jonas Rademacker (Bristol - LHCb)
LHCb owerview on FSI of many-body decays

Jeremy Dalseno (Santiago de Compostela - LHCb)
B → 3h amplitude analysis in LHCb

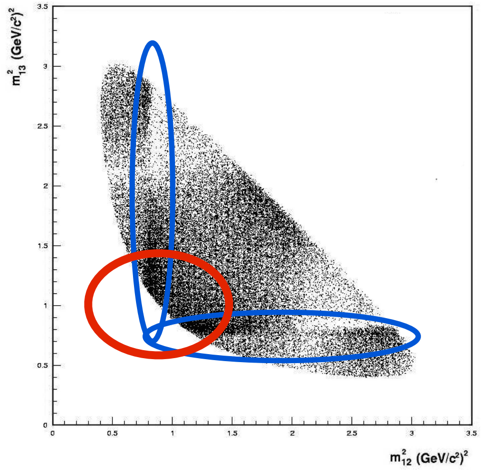
Alexey Garmash (Novosibirsk - Belle)
Belle results in three-body B decays

Alexander Austregesilo (Jlab - Gluex)
Strategy and foreseeable issues in GlueX 3-body analysis

Patricia Magalhães
Bastian Kubis
Christoph Hanhart
Norbert Kaiser
Stephan Paul

Motivation

- three-body HADRONIC decays are dominated by resonances
  - spectroscopy



$D^+ \to K^- \pi^+ \pi^+$

↳ also B, τ and light mesons

κ(800)

K*(892)

  - information of MM interactions ⟶ no $K\bar{K}$ data available

↳ investigate MM up to higher energies in different context

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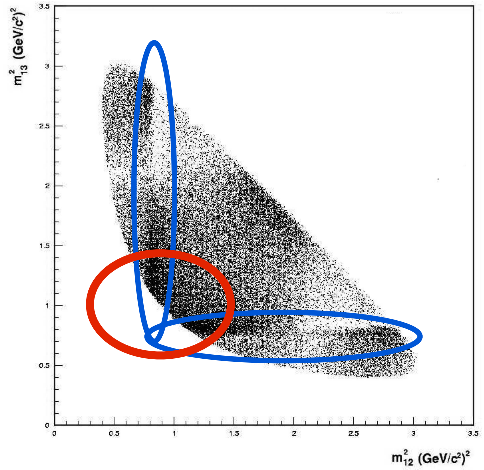

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    ↳ $B^\pm \to h^+ h^- h^\pm$: FSI can explain CP violation at low mass

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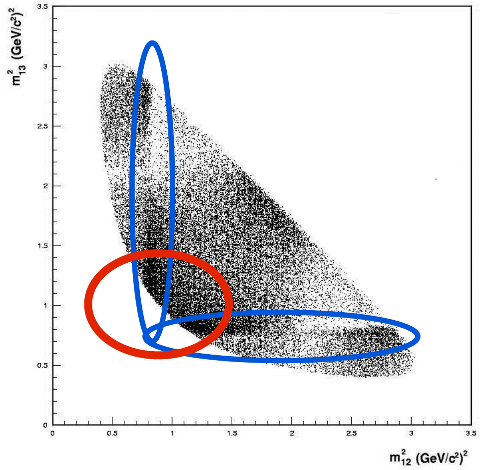

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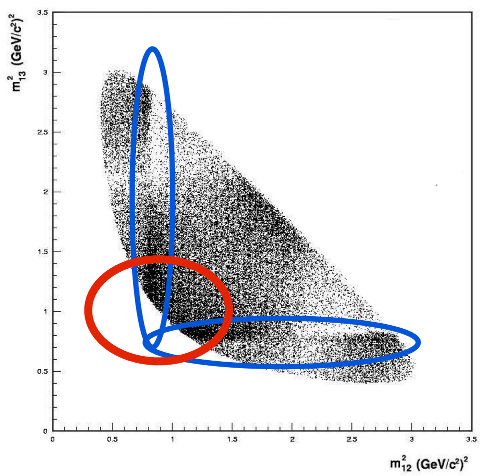

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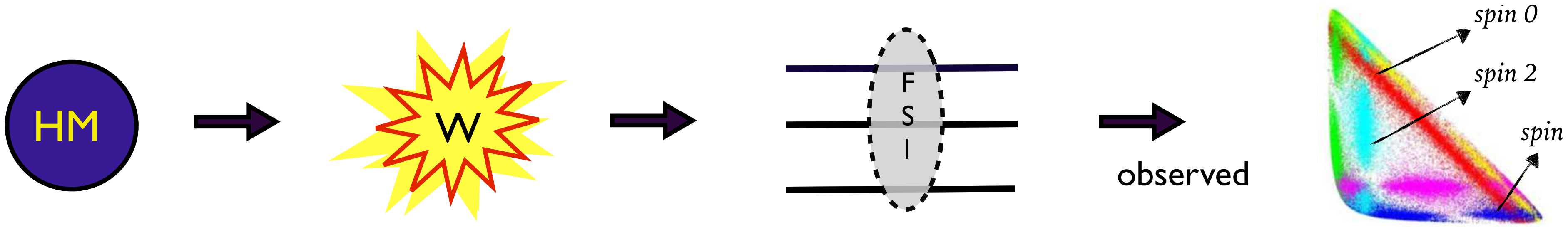
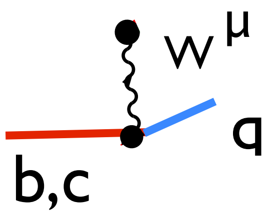
- new high data sample from LHCb, Belle II (soon), Compass, … more to came

  ⟶ need better models !!

- dynamics



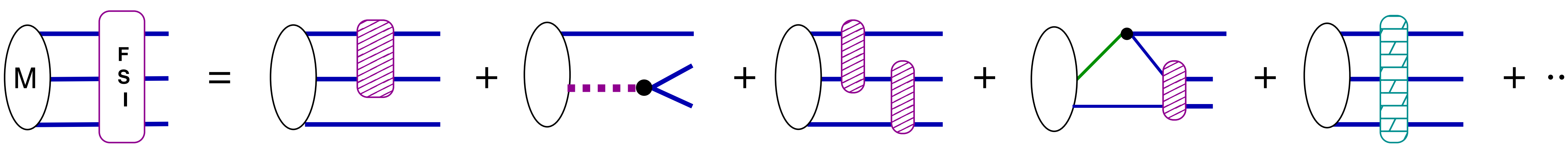
- weak primary vertex



quark → hadron

→ light mesons diffractive production do not have a well defined decay model…

- Final State Interactions (FSI)



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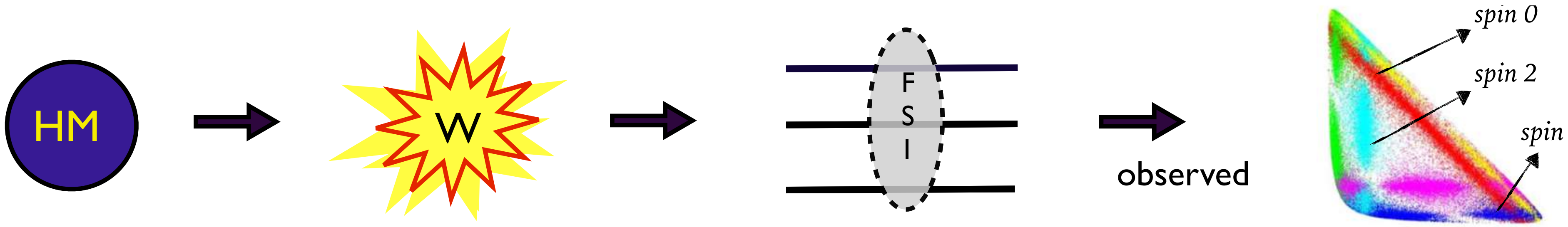

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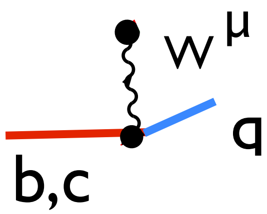

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- QCD factorization approach
  - → not precise for 3-body: no FSI and 3-body
    - ↳ ok for B decays
    - ↳ NOT for D decays

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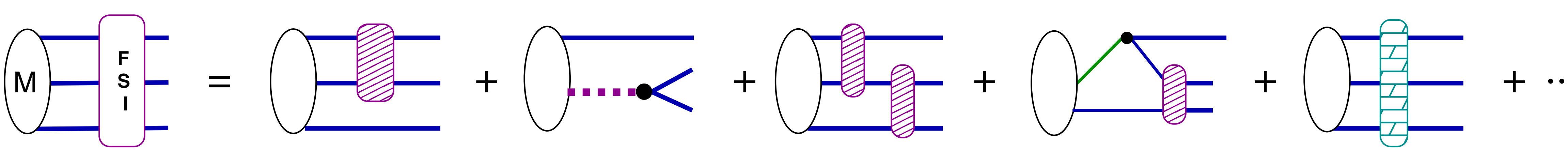

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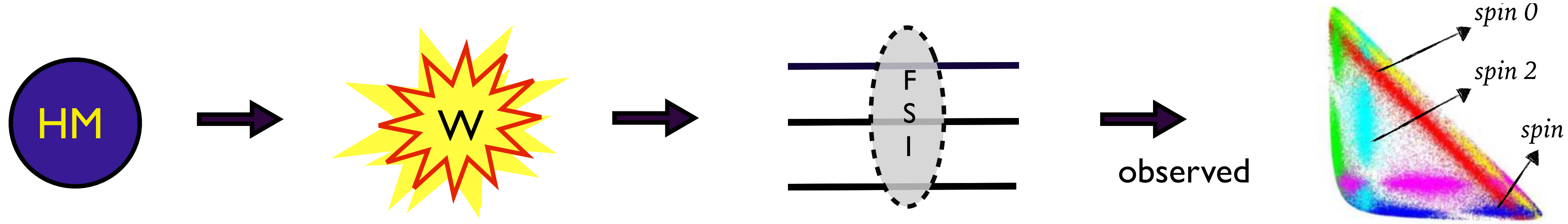

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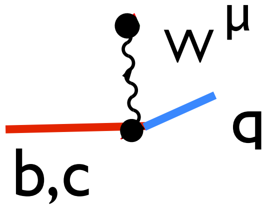

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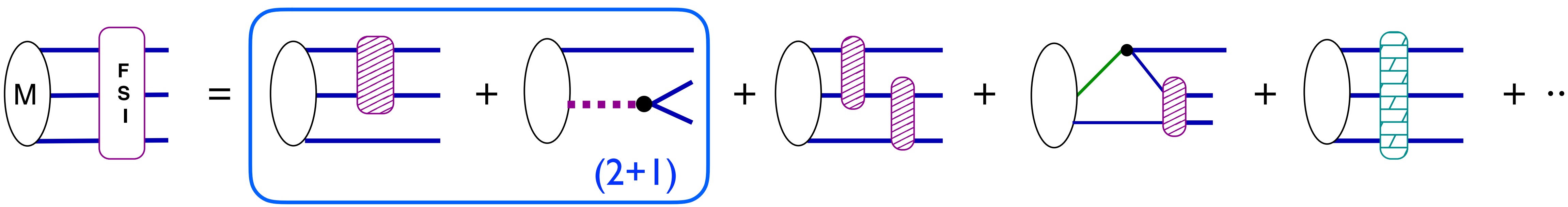
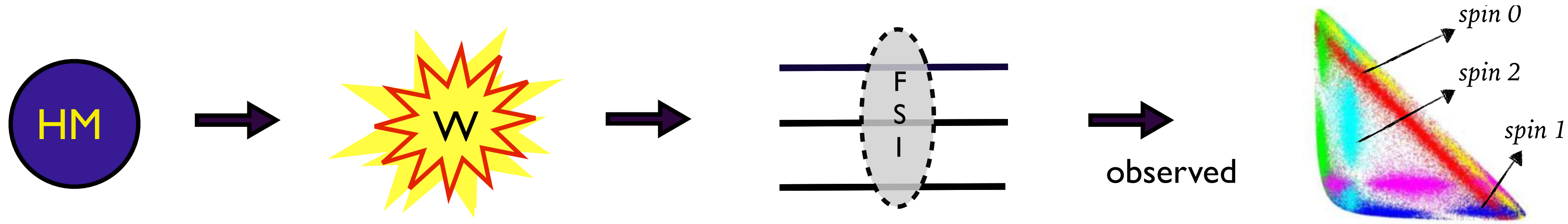


- 2-body is crucial
- full unitarity: Faddeev, Khury-Trieman, triangles

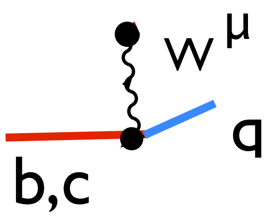
heavy meson decay

- dynamics



to extract information from data we need an amplitude MODEL

- weak primary vertex



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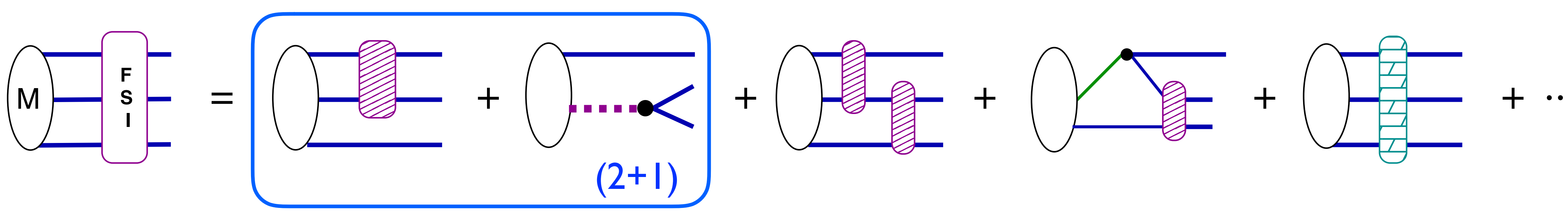

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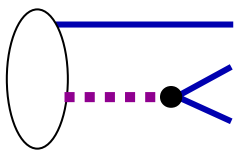
standard isobar model...

- $$A_{decay} = A^{NR} + \sum c_k A_k \qquad A_k = [FF] \times [spin] \times [A_{BW}]$$



$$A_{BW}(s) = \frac{1}{M_R^2 - s - i\, M_R \Gamma_R}$$

good for narrow and isolated resonance

- violates two-body unitarity (2 res in the same channel);
- does NOT include rescattering and coupled-channels;
- free parameters are not connected with theory !

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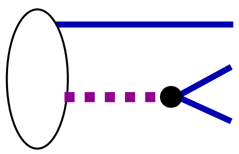

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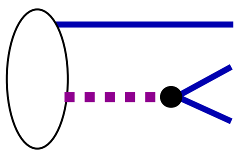

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experimentalist and theorist agreement

- pole position: universal for each resonances

→ light scalars can be deep in complex plane (phase not at 90 degrees)

could be hidden… modified by production

↓

different in different processes

?● meaning?

data summary

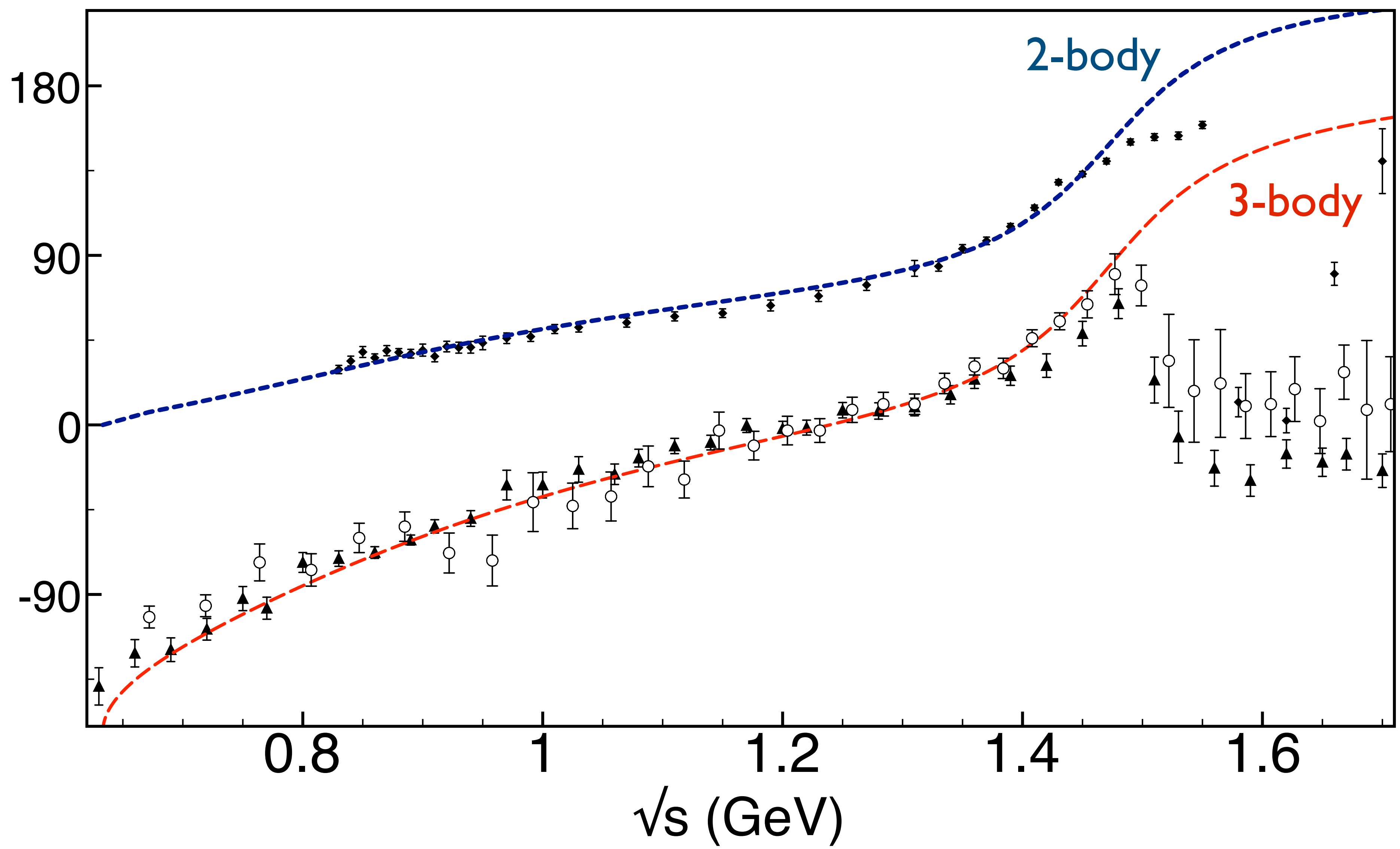
Resonance	Magnitude	Phase (°)	Fraction (%)
$\rho(770)$	1 [fixed]	0 [fixed]	24.1±0.3
$f_0(980)$	3.9±0.02	-157.9±0.5	8.1±0.2
$f_2(1270)$	1.1±0.01	89±0.5	14.5±0.2
$\rho(1450)$	0.7±0.03	-80.2±2.5	0.4±0.1
$f_0(1370)$	0.9±0.05	184.4±3.3	0.4±0.1
$\sigma(500)$	23.2±0.2	-88.1±0.4	58.2±1.5
NR	10.1±0.2	-148.3±1.1	7.5±0.6
$f_0(1500)$	2.1±0.04	180.5±1.1	2.9±0.2
Total			116.02

M Robilotta seminar

2-body x 3-body phases

- COMMON believe: 2 body phase is enough for 3-body….
- $D^+ \to K^- \pi^+ \pi^+$ → different S- wave phase from $K^- \pi^+$scattering

PC Magalhães et.al: PRD84 094001 (2011), PRD92 094005 (2015)



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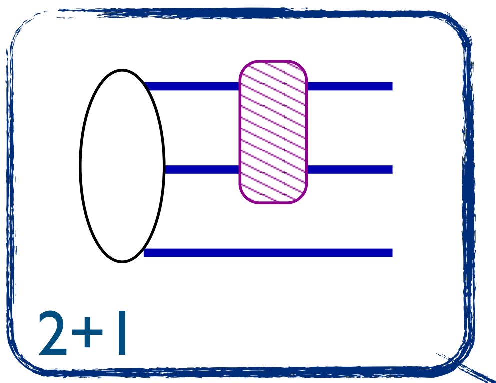

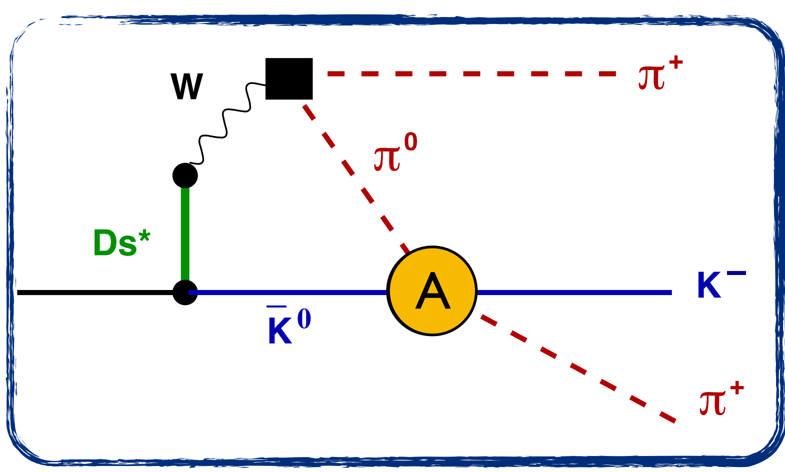

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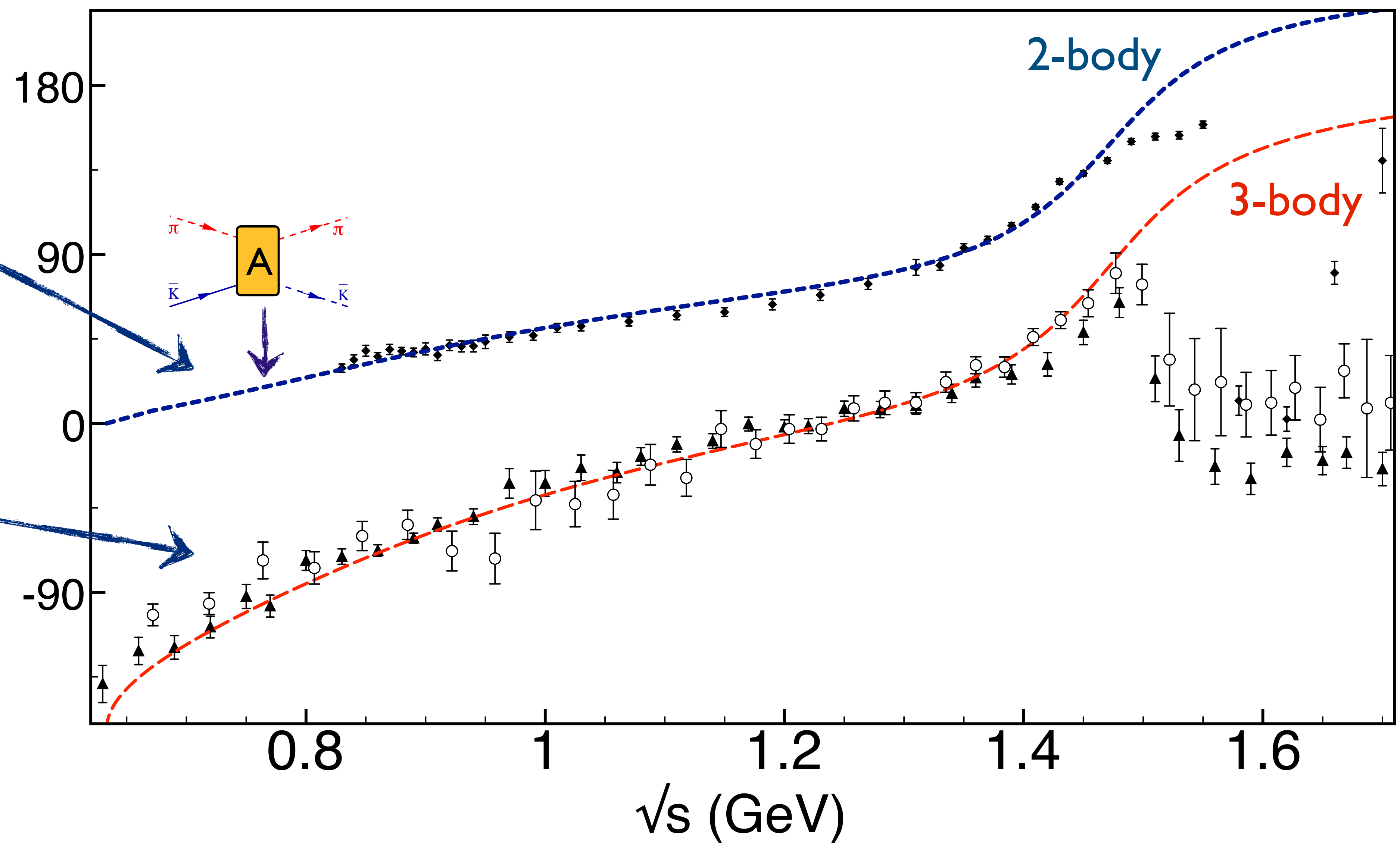

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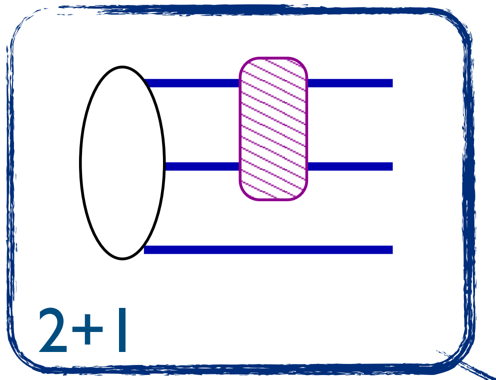

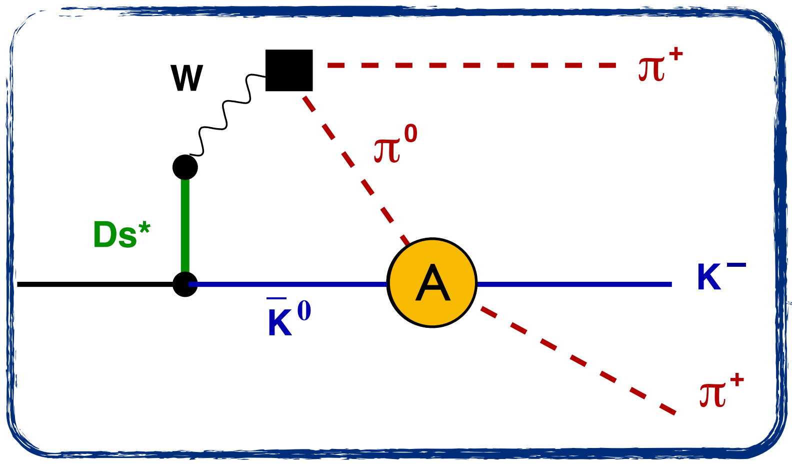

different dynamic!

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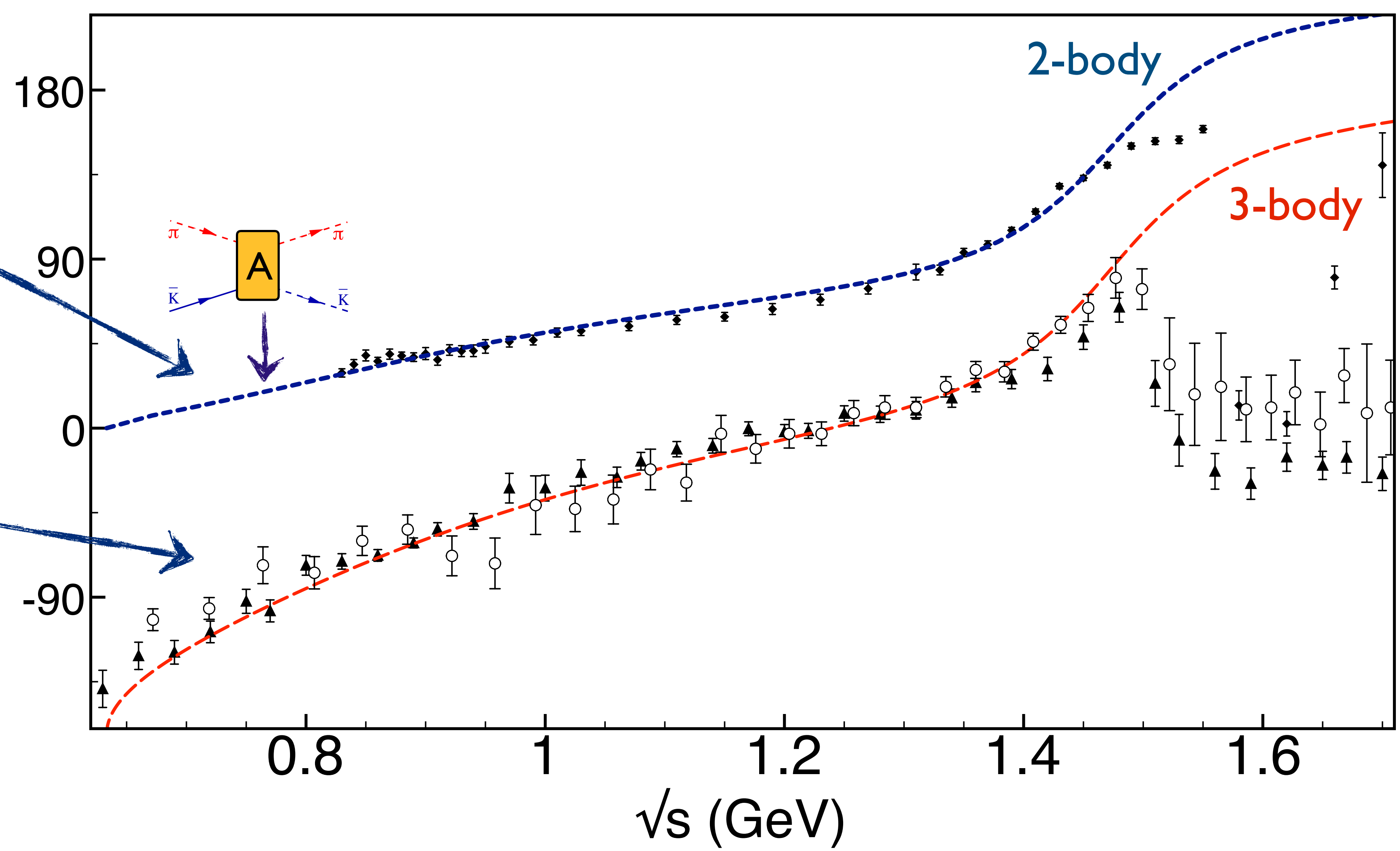
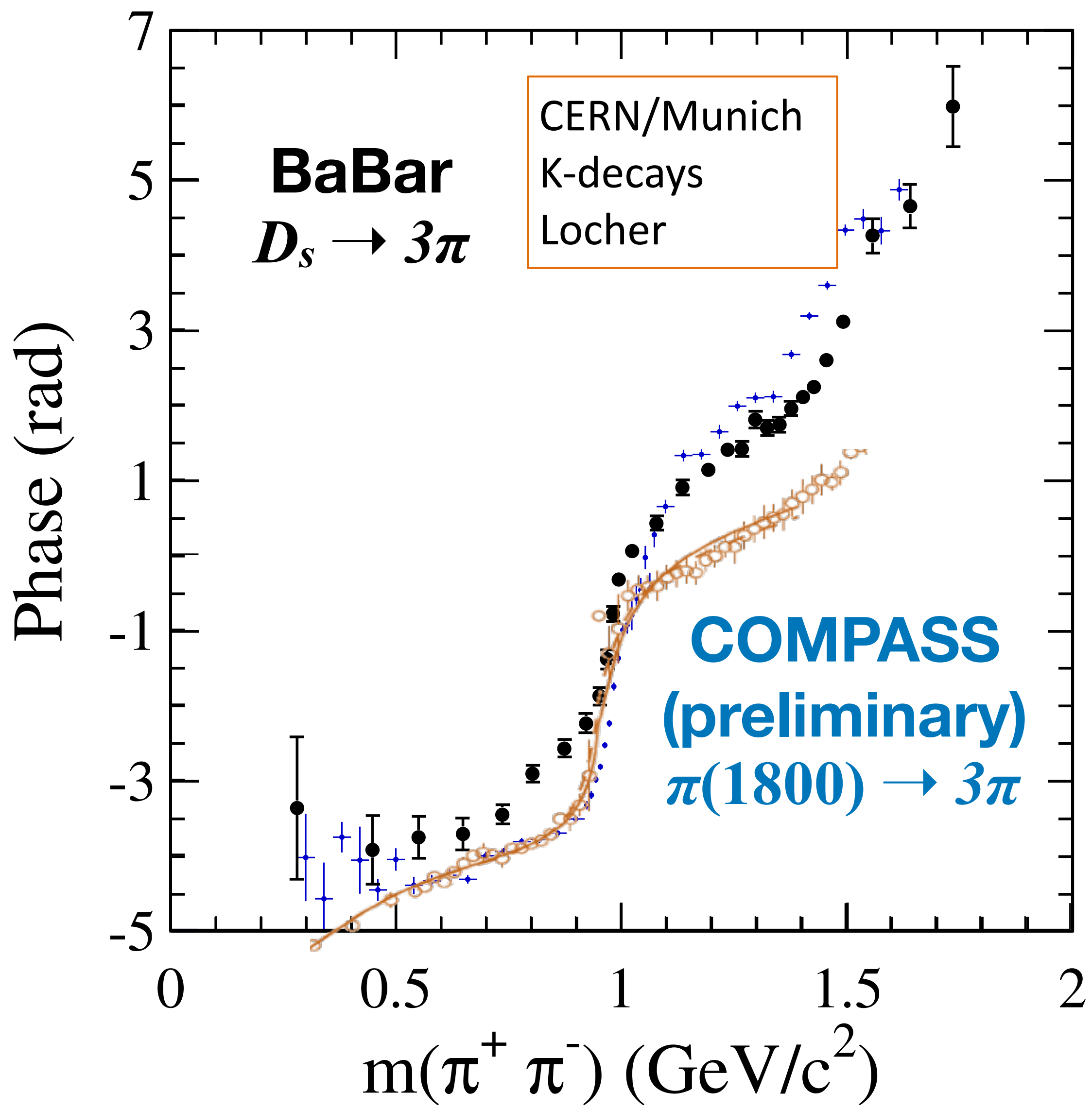


- **Heavy Flavour and Light Meson Decays: Similarities**

S. Paul seminar



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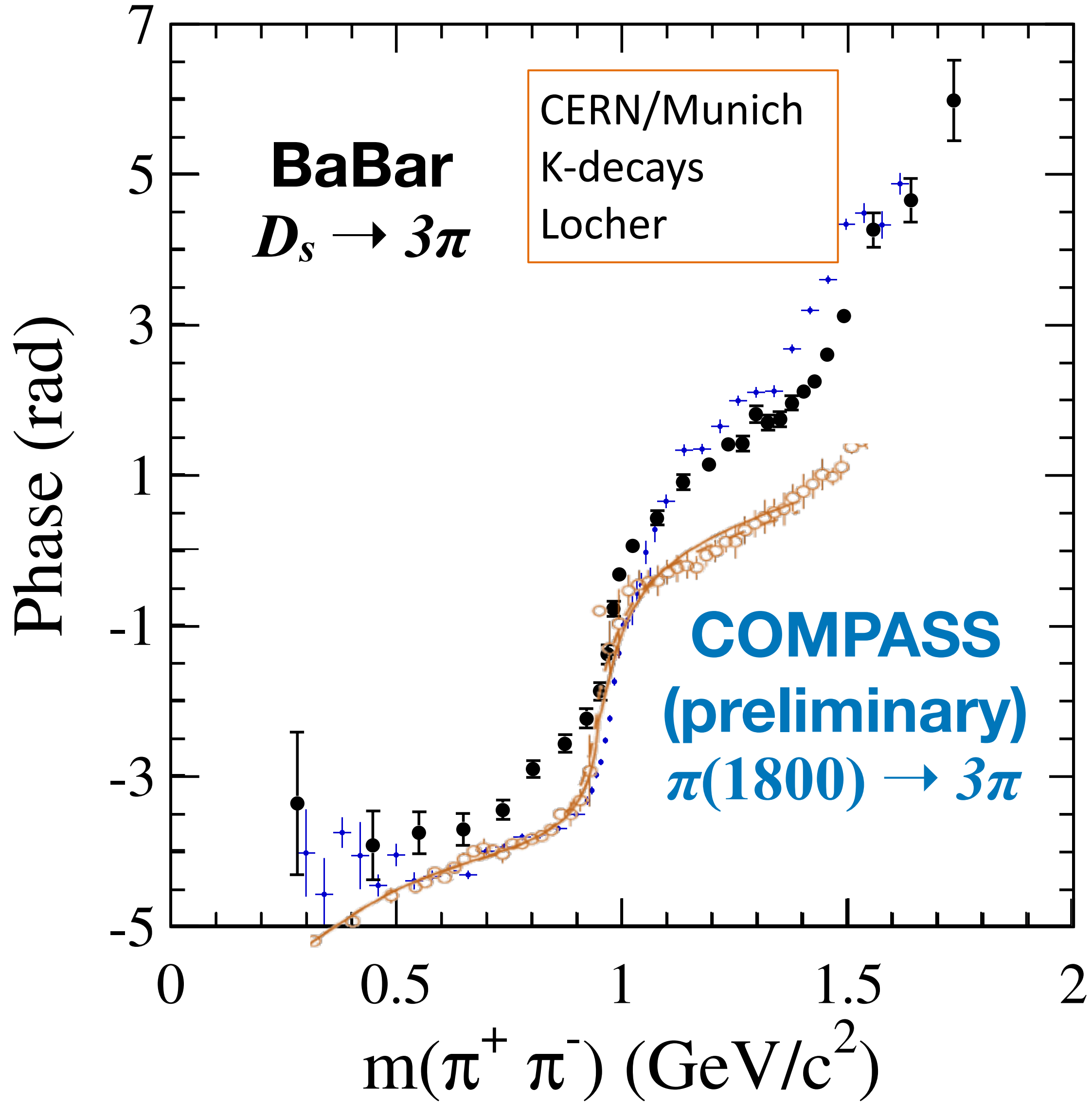

S. Paul seminar

no direct connection
between 2 and 3-body phases

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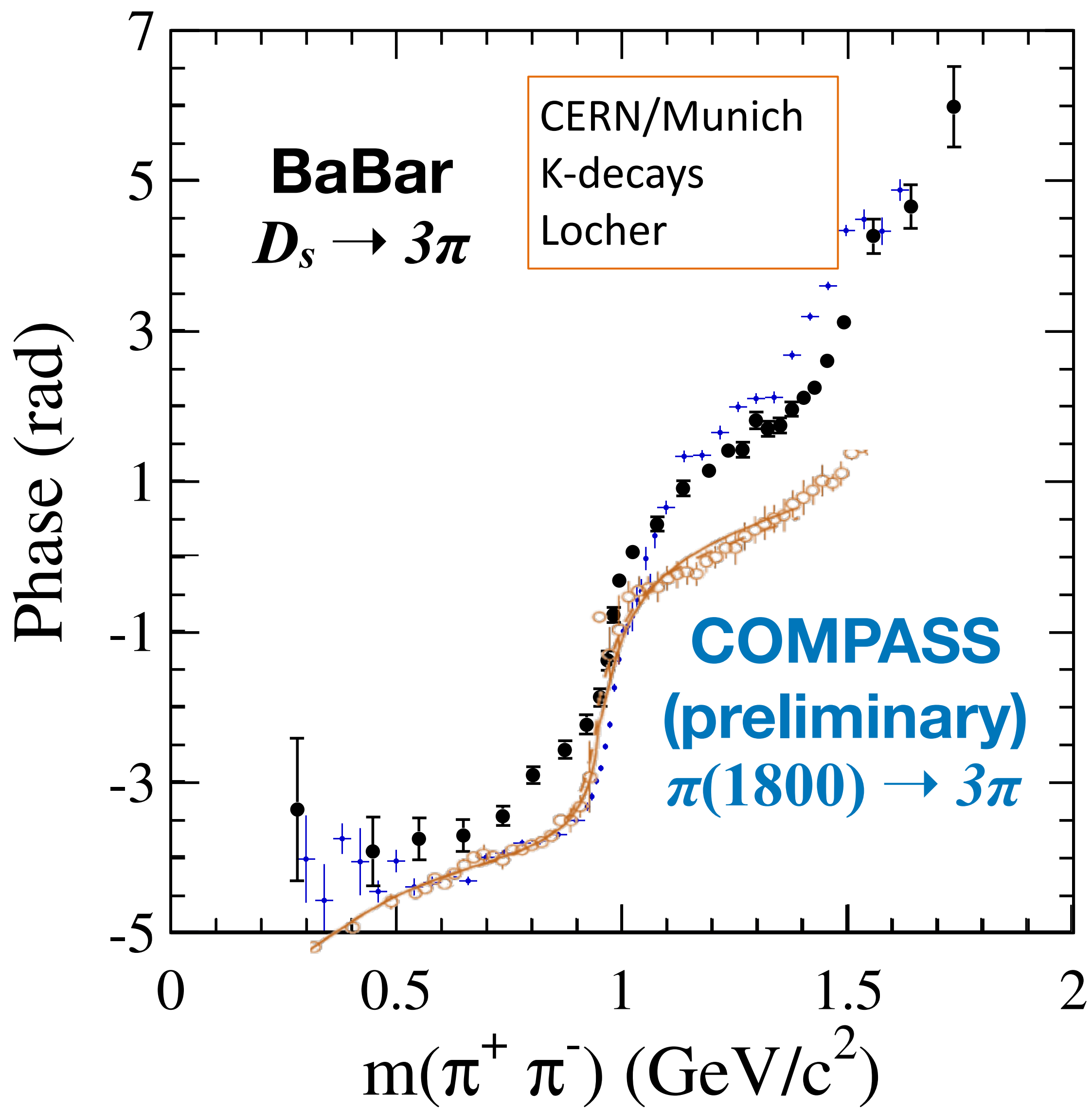

no direct connection
between 2 and 3-body phases

- 2-body amplitude: spin and isospin well defined!
- 3-body data: only spin and $\neq$ dynamics (weak vertex, FSI, 3rd particle, …)

- FSI
  - low energy MM rescattering, coupled-channels and resonances
  - 3-body

W unknown FF; critical in D decays…

how theorist can help improving PWA? $\rightleftharpoons$ what can we learn from data?

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how theorist can help improving PWA? $\rightleftharpoons$ what can we learn from data?

- experimental list of wishes…
  - how to implement 2-body unitarity in data?
  - need 2-body amplitude up to high energy !
  - theorist to work on diffractive physics ! (COMPASS and GlueX)
  - differences between triangle singularities and resonances
  - spin formalism agreement: no NR corrections. But Tensor or Helicity?
- theoretical list of issues…
  - how to describe simultaneously full energy range?
  - how to join weak and strong interactions?? (scale problem)
  - how to identify/include the importance of 3-body FSI?
  - access to data in a "nice and understandable way"

MM dispersion relations and chiral symmetry

- dispersive and analytic approach to 2-body

Advantages of dispersion relations

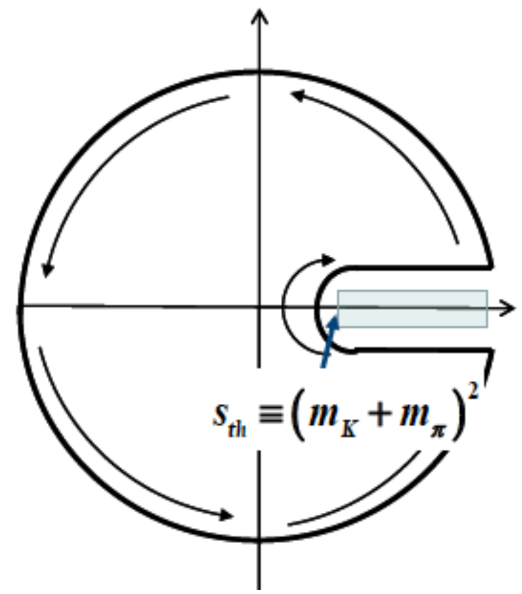
- based on **fundamental properties:**
  - analyticity (causality)
  - unitarity (probability conservation)
  - crossing symmetry

    implies left cut (could be an issue)

$\Rightarrow$ model independence

T. Isken

- Omnès representation: $\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$



$\phi_{+,0}(s)$: phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

  Kπ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

  $\Rightarrow$ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi$ $\left(\bar{f}_{+,0}(s) \to 1/s\right)$

  Brodsky & Lepage

E. Passamar

↳ calculate T where there is not data

↳ model independent extrapolation to complex s-plane

MM dispersion relations and chiral symmetry

- dispersive and analytic approach to 2-body

Advantages of d

- based on **funda**
  - analyticity (
  - unitarity (pr
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implies

⇒ model indepe

T. Is

Why Roy-Steiner equations?

Roy(-Steiner) eqs. = Partial-Wave (Hyperbolic) Dispersion Relations coupled by unitarity and crossing symmetry

- **Respect all symmetries**: analyticity, unitarity, crossing
- **Model independent** ⇒ the actual parametrization of the data is irrelevant once it is used in the integral.
- Framework allows for systematic improvements (subtractions, higher partial waves, ...)
- **PW(H)DRs** help to study processes with high precision:
  - $\pi\pi$-scattering: [Ananthanarayan et al. (2001), García-Martín et al. (2011)]
  - πK-scattering: [Büttiker et al. (2004)]
  - $\gamma\gamma \to \pi\pi$ scattering: [Hoferichter et al. (2011)]
  - πN scattering: [Hoferichter et al. (2015)]

J Ruiz Elvira

J. Ruiz de Elvira (ITP) Pion-kaon scattering TRR110 workshop 9

factor

s)

π scattering phase

wn

$\pi\pm\pi$ $(\bar{f}_{+,0}(s)\to 1/s)$

Brodsky & Lepage

calculate T w

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to complex

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J Ruiz Elvira

J. Ruiz de Elvira (ITP) | Pion-kaon scattering | TRR110 workshop | 9

ε

factor

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π scattering phase

$\pi\pm\pi$ $(\bar{f}_{+,0}(s)\to 1/s)$

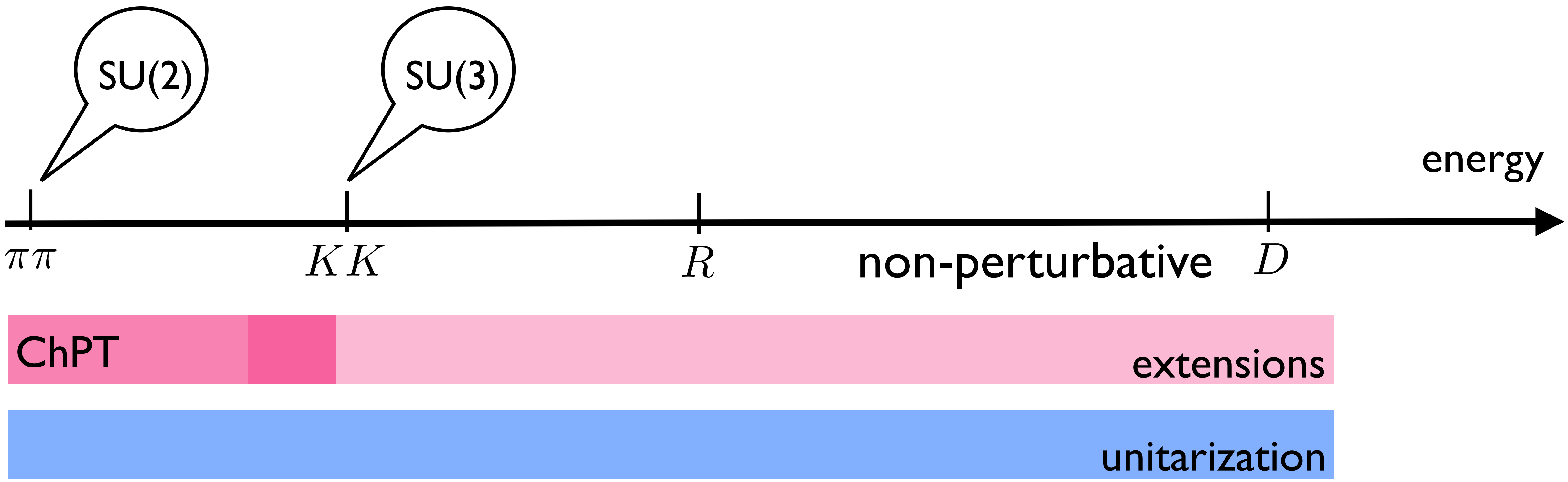
Brodsky & Lepage

↳ calculate T w

↳ model indepe
to complex

→ works well in the elastic region

→ limited to ~1 GeV


SU(2)
SU(3)
energy
$\pi\pi$
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D
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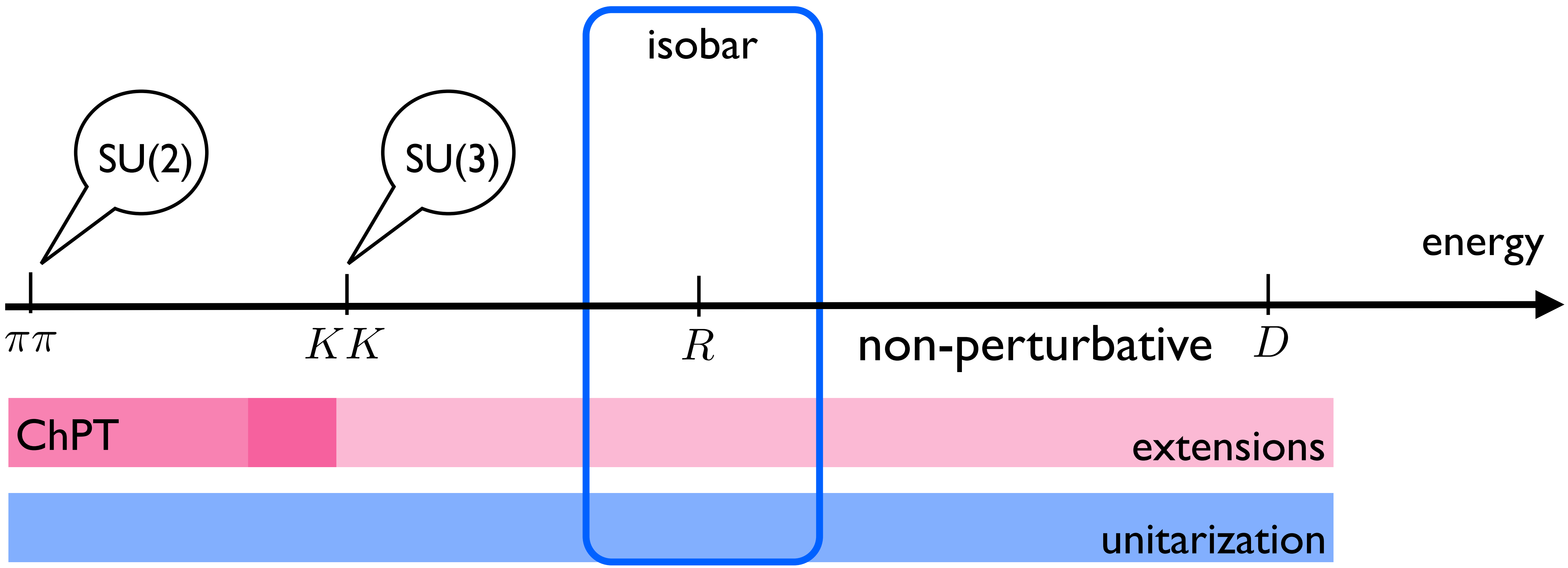
unitarization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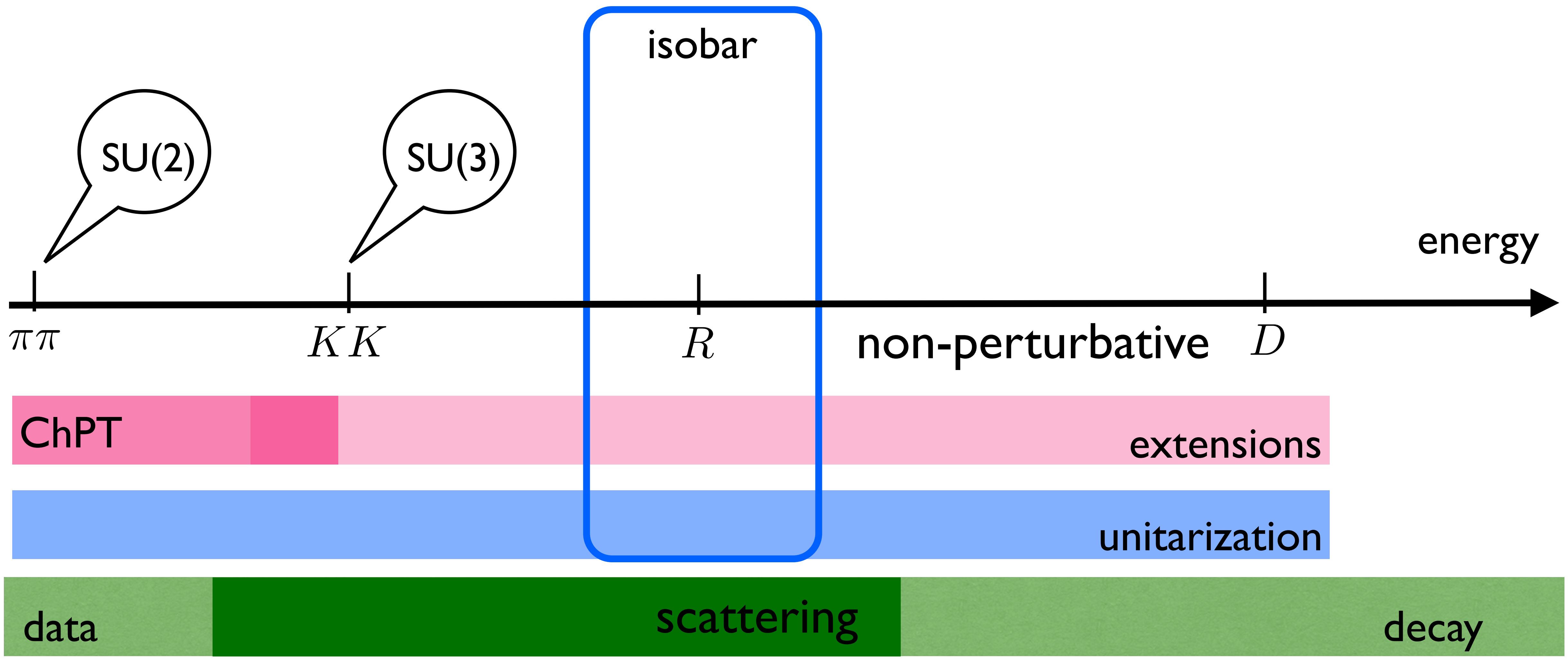
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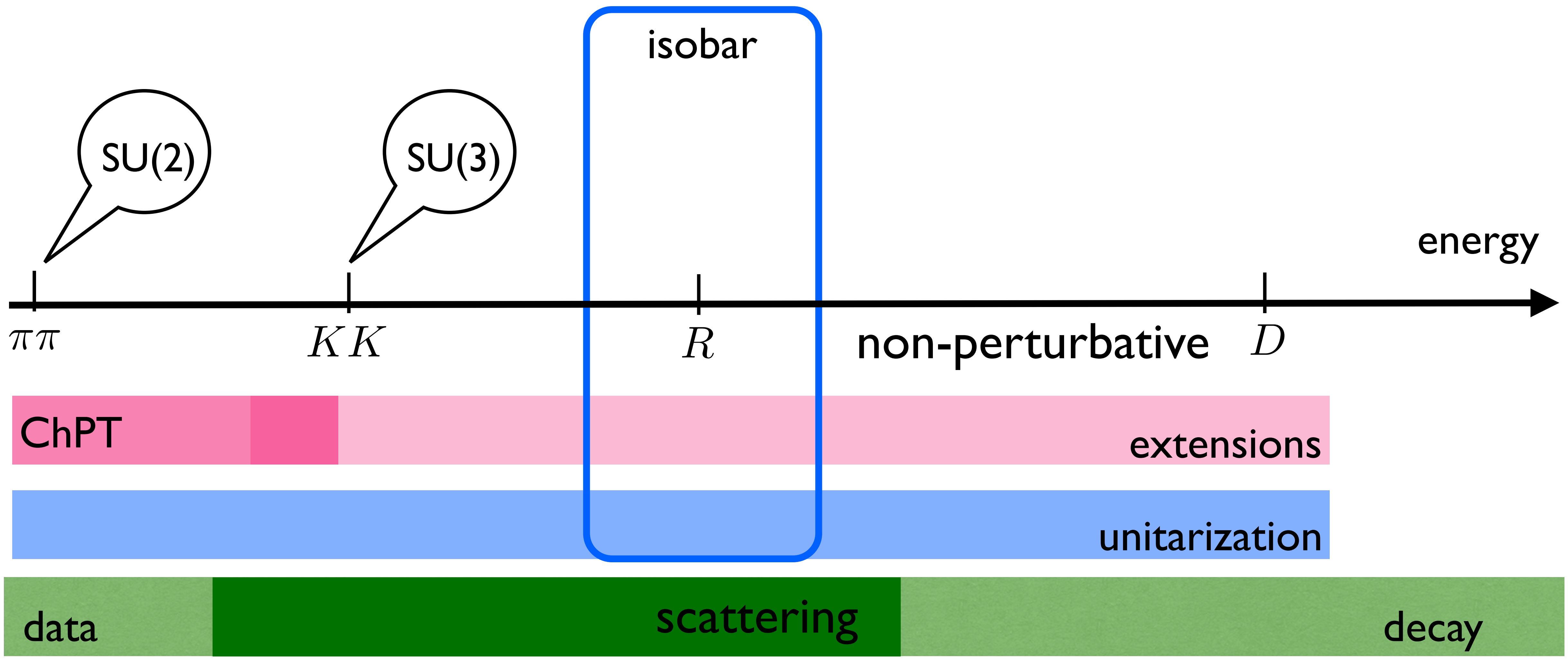
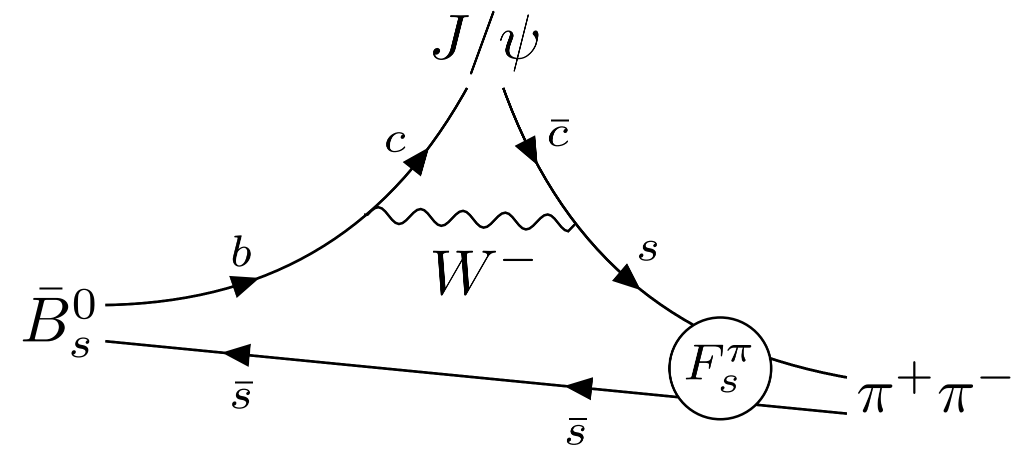


we need non-perturbative MM interactions…

S. Ropertz

Extensions of pion form factors beyond 1 GeV

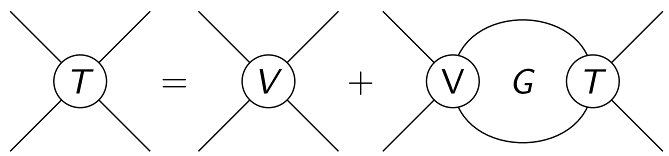
Decay $\bar{B}_s^0 \to J/\psi\pi^+\pi^-(K^+K^-)$



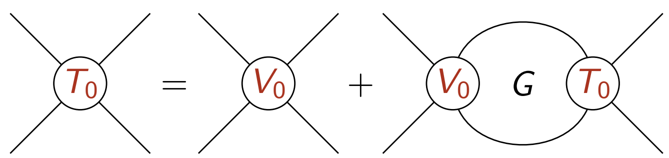
- All dynamics contained in $\pi^+\pi^-/K^+K^-$ subchannel
- For $\bar{B}_s^0 \to J/\psi\pi^+\pi^-$ only S- and D-waves (for K^+K^- also P-waves)
- $\bar{B}_s^0 \to J/\psi\pi^+\pi^-$ is S-wave dominated

Parametrization for the partial wave amplitude

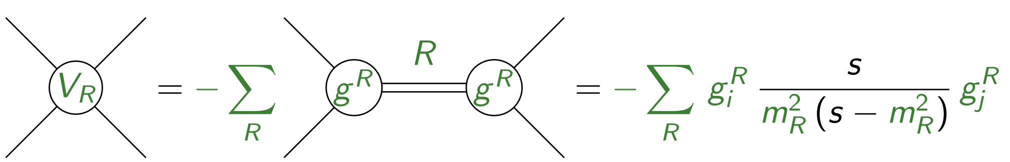
- Consider Bethe-Salpeter equation for the partial wave amplitude T



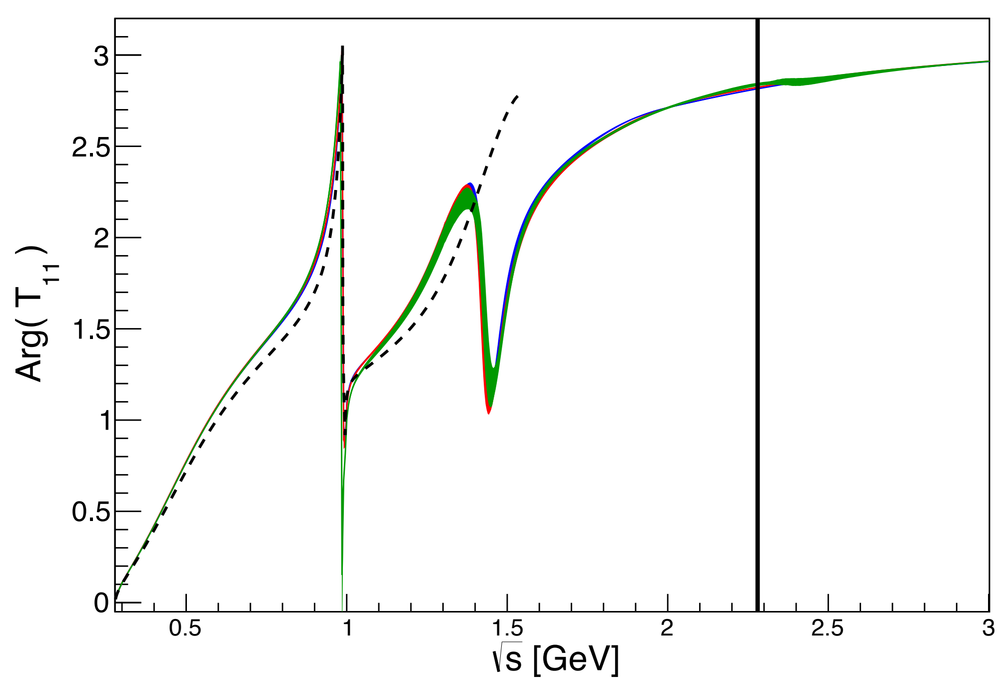
- Split scattering kernel $V = V_0 + V_R \Rightarrow T = T_0 + T_R$
- Unitary scattering amplitude T_0 (given by phases and inelasticities)



- Resonance exchange potential V_R



fit the S-wave



New Form Factor parametrization

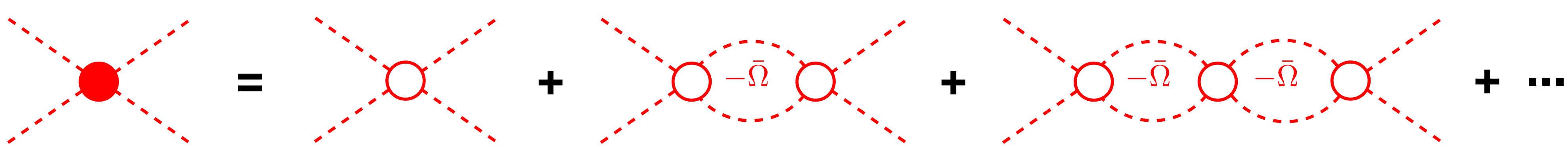
$$F = \Omega\,[\mathbb{1} - V_R\,\Sigma]^{-1}\,M$$

M. Robilotta

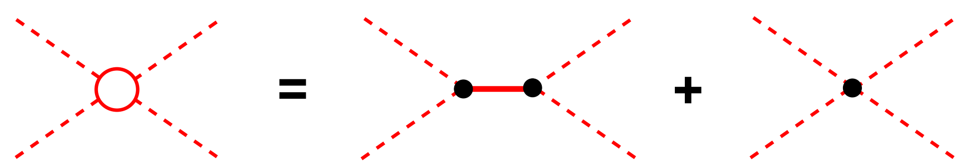
- chiral Lagrangian and Unitarized ChPT
  - → LO chiral (Gasser&Leutwyler)
  - → NLO: include resonances as a field (Ecker, Gasser, Pich and De Rafael)

$$\mathcal{L}_S^{(2)} = \frac{2\,\tilde{c}_d}{F^2}\, R_0\, \partial_\mu\phi_i\, \partial^\mu\phi_i - \frac{4\,\tilde{c}_m}{F^2}\, B\, R_0\, (\sigma_0\, \delta_{ij} + \sigma_8\, d_{8ij})\, \phi_i\, \phi_j$$
$$+ \frac{2\,c_d}{\sqrt{2}F^2}\, d_{ijk}\, R_k\, \partial_\mu\phi_i\, \partial^\mu\phi_i - \frac{4Bc_m}{\sqrt{2}F^2}\left[\sigma_0\, d_{ijk} + \sigma_8\left(\frac{2}{3}\,\delta_{ik}\,\delta_{j8} + d_{i8s}\, d_{jsk}\right)\right]\, \phi_i\, \phi_j R_k$$

- unitarize amplitude by Bethe-Salpeter eq. [Oller and Oset PRD 60 (1999)]

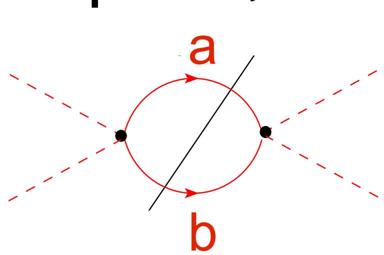


- kernel $\mathcal{K}^{(J,I)}_{ab\to cd}$



resonance (NLO) + contact (LO)

- loops → K-matrix approximation: only on-shell



$$\{I_{ab};\ I^{\mu\nu}_{ab}\} = \int \frac{d^4\ell}{(2\pi)^4}\, \frac{\{1;\ \ell^\mu\, \ell^\nu\}}{D_a\, D_b}$$

$$D_a = (\ell + p/2)^2 - M_a^2 \qquad D_b = (\ell - p/2)^2 - M_b^2$$

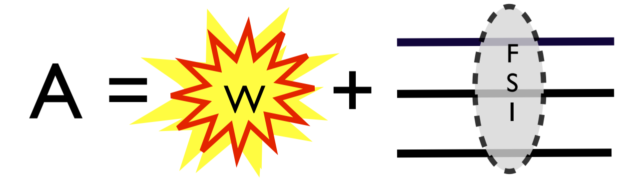
→

$$\bar{\Omega}^S_{ab} = -\frac{i}{8\pi}\, \frac{Q_{ab}}{\sqrt{s}}\, \theta(s - (M_a + M_b)^2)$$

$$\bar{\Omega}^P_{aa} = -\frac{i}{6\pi}\, \frac{Q^3_{aa}}{\sqrt{s}}\, \theta(s - 4\, M_a^2)$$

$$Q_{ab} = \frac{1}{2}\sqrt{s - 2\,(M_a^2 + M_b^2) + (M_a^2 - M_b^2)^2/s}$$

- what do we have on the market for three-body PWA?



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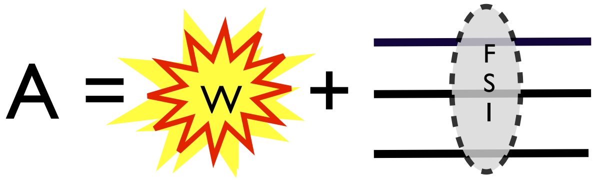
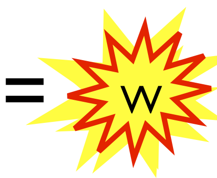
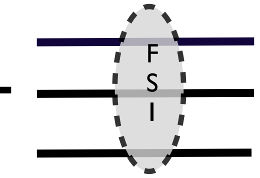


  - scalar and vector ππ, Kπ form factors up to ~1.4 GeV → (2+1)
    ↳ important tool for $\tau \to hhh\,\nu_\tau$ decay E Passamar

- what do we have on the market for three-body PWA? A = 

+ 

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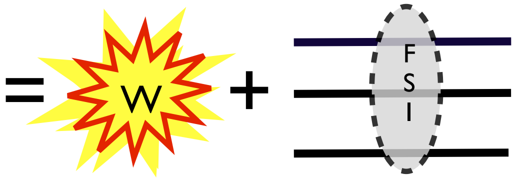
Boito et al. PRD96 (2017) [Paris,Kracov, SP] B. El-Bennich

- what do we have on the market for three-body PWA? A = 

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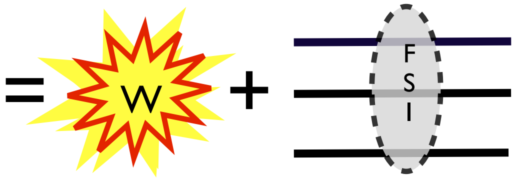
    ↳ $D^+ \to (\bar{K}\pi)\pi^+$ B. Moussallam, T. Isken - Kubis et al.[Bonn] and M. Albaladejo

- what do we have on the market for three-body PWA? A = 

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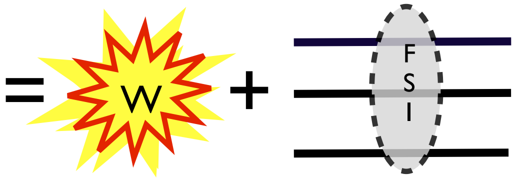
  - “K-matrix" : ππ S-wave with 5 coupled-channel (pole + polynomial) modulated by a production vector amplitude - up to 1.9 GeV 
    - ↳ (2+1) Anisovich PLB653(2007) used by Babar, LHCb, BESS11 B.Kopf (talk)
    - ↳ LHCb $B^\pm \to \pi^+\pi^-\pi^\pm$ decay analysis J. Dalceno

- what do we have on the market for three-body PWA? A = 

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  - multi-meson-model for $D^+ \to K^-K^+K^+$
    - ↳ MM: chiral Lagrangian + resonances + unitarized amplitudes
    - ↳ (2+1) + 3-body NR - fitted to LHCb M. Robilotta

my work!

- e.g. tau decay….

E. Passamar

1.3 Exclusive hadronic processes

Experimental situation :

- $\tau \to PP\nu_\tau$

$$\begin{cases} \pi^-\pi^0, K^-K^0 \\ K^-\pi^0, \bar{K}^0\pi^- \\ \eta \text{ modes} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

ALEPH, CLEOIII, OPAL
Belle, BaBar

- $\tau \to PPP\nu_\tau$

$$\begin{cases} \pi\ \pi\ \pi \\ KK\pi \\ K\pi\pi \\ \eta \text{ modes} \\ \text{KKK} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

ALEPH, CLEOIII, OPAL
Belle, BaBar

Theoretical situation

Parametrization using

ChPT + Analiticity + Unitarity
Dispersion relations on the market

⇒ Reasonably good control

Parametrization using
ChPT + Analiticity + Unitarity+ Resonances

⇒ Much more difficult and model dependent

- Working in the improvement of the theory of QDCF

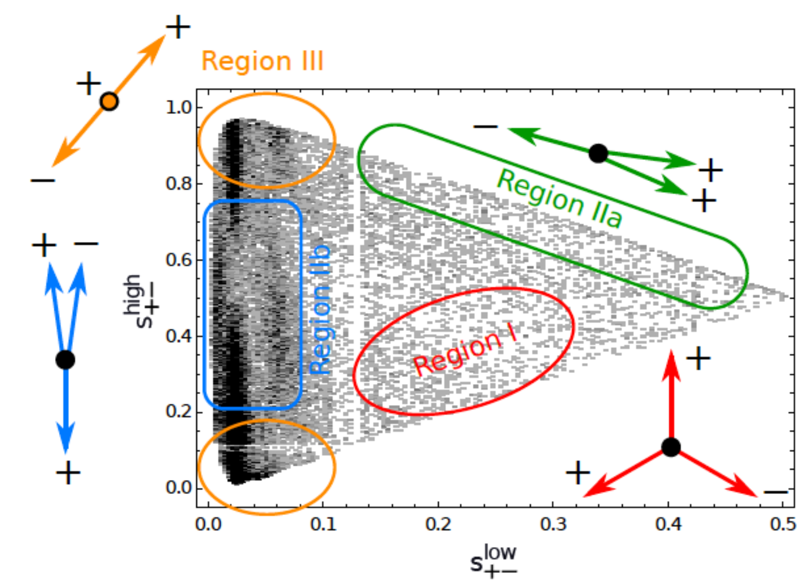
K. Vos

Factorization in three-body B decays

Kraenkl, Mannel, Virto [2015]; Klein, Mannel, Virto, KKV [2018]

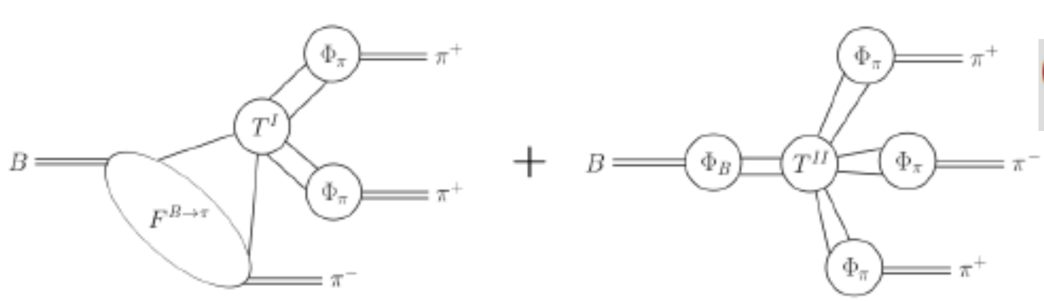
Data-driven model-independent factorization approach

- Improvement over quasi-two body interpretation
- Introduces new non-perturbative strong phases
- Focuss here on $B \to \pi\pi\pi$ but similar for $B \to hhh$



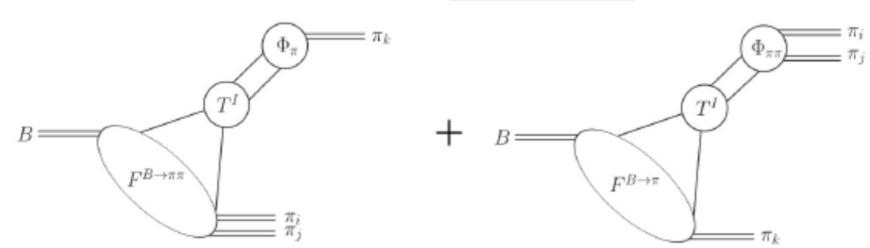
(First) Challenge: Reach the same level as two-body QCDF

→ diff factorization in each regions…



Central Region

Edges



→ working on non-perturbative sector… LCDA

QCD Factorizations with form factors

Weak effective hamiltonian

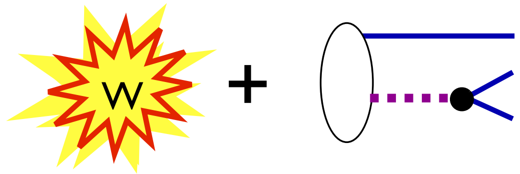
Sum of local operators Q_i multiplied by short-range Wilson coefficients $C_i(\mu)$ and CKM matrix elements:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{us}^*(C_1(\mu)O_1^u + C_2(\mu)O_2^u) - V_{tb}V_{ts}^*\sum_{i=3}^{10} C_i(\mu)O_i\right]$$

O_1 and O_2 are left-handed current-current operators, for example:

$$O_1^u = \bar{s}_\alpha\gamma_\mu(1-\gamma_5)u_\alpha\bar{u}_\beta\gamma_\mu(1-\gamma_5)b_\beta$$

B. El-Bennich



→ naive factorization

- quasi-two-body appox.

  always mediated by resonances

→ $\langle M^*|J_\mu^i|0\rangle \propto \langle M_1 M_2|J_\mu^i|0\rangle$: form factor

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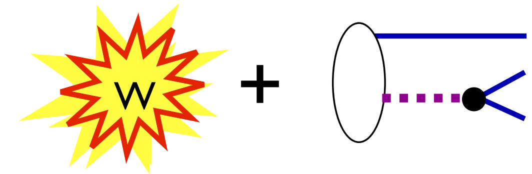

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Alternatives to Isobar model

"Parametrizations of three-body hadronic B- and D-decay amplitudes in terms of analytic and unitary meson-meson form factors"

D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escribano, R. Kamiski, L. Lesniak, B. Loiseau, Phys. Rev. D 96, 113003 (2017)

QCD Factorizations with form factors

- LHCb used such parametrization..

J. Rademacker

$B^0 \to K_S\pi\pi$ Dalitz Plot Analysis

PRL 120 (2018) no.26, 261801

	Component	Mass model
Decaying to $K^0_S\pi^\pm$	$K^*(892)^-$	Relativistic Breit-Wigner
	$(K\pi)^-_0$	EFKLLM ⟵ QCD-factorisation*
	$K^*_2(1430)^-$	Relativistic Breit-Wigner
	$K^*(1680)^-$	Flatté
Decaying to $\pi^+\pi^-$	$f_0(500)$	Relativistic Breit-Wigner
	$\rho(770)^0$	Gounaris-Sakurai
	$f_0(980)$	Flatté
	$f_0(1500)$	Relativistic Breit-Wigner
	χ_{c0}	Relativistic Breit-Wigner

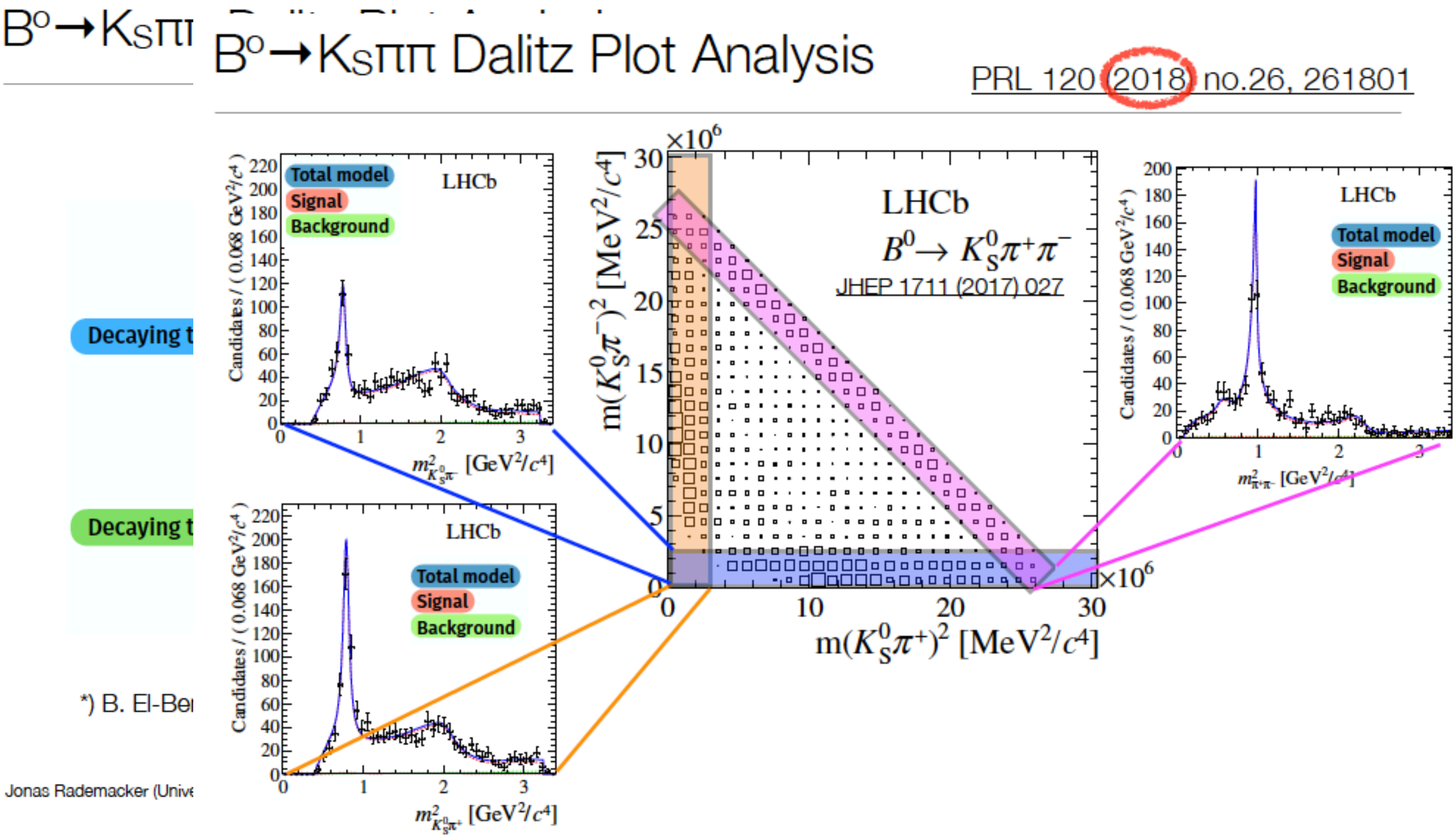
Plus a flat non-resonant contribution

*) B. El-Bennich, A. Furman, R. Kaminski, L. Lesniak, B. Loiseau, B. Moussallam
PRD79 (2009) 094005, PRD83 (2011) 039903

Jonas Rademacker (University of Bristol) FSI @ LHCb TRR110 Workshop - Amplitudes for 3-Body Final States, Munich, 11/07/2018 8

- LHCb used such parametrization..

J. Rademacker



- Khuri–Treiman (KT) equations for 3-body decays: final state interaction (FSI) among all three decay products are fully taken into account [Khuri and Treiman (1960)]

T. Isken

- $D \to \bar{K}\pi\pi$ nice laboratory for $\boxed{\text{3-body}}$ final-state interactions
  Previous work: [Franz Niecknig, Bastian Kubis, JHEP 1510,142 (2015), P.L. B780,471 (2018)]:Khuri-Treiman
  Also: [S.X. Nakamura, P.R. D93,014005 (2015), P.C. Magalhães et al. P.R. D84,094001 (2011)]

B. Moussallam

- Khuri-Treiman [PR 119,1115 (1960)] renewed interest for $\eta \to 3\pi$

- Experimentally: evidence for exotic κ meson in $D^+ \to K^-\pi^+\pi^+$ [E791, PRL 89,121801 (2001)]

  → But: κ not seen in D^0 decays ?

- 2-body systems (Omnès fcts.) obey universal (hadr.) phase relations

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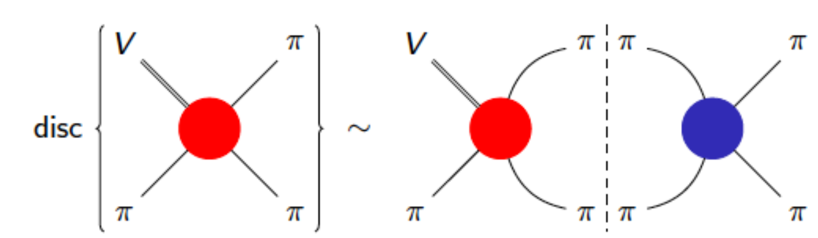
- Experimentally: evidence for exotic κ meso
  $D^+ \to K^-\pi^+\pi^+$ [E791, PRL 89,121801 (
  - → But: κ not seen in D^0 decays ?

Unitarity condition

$$\text{disc}\,\mathcal{F}(s) = 2i\,\theta(s-4M_\pi^2)\,\mathcal{F}(s)\sin\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

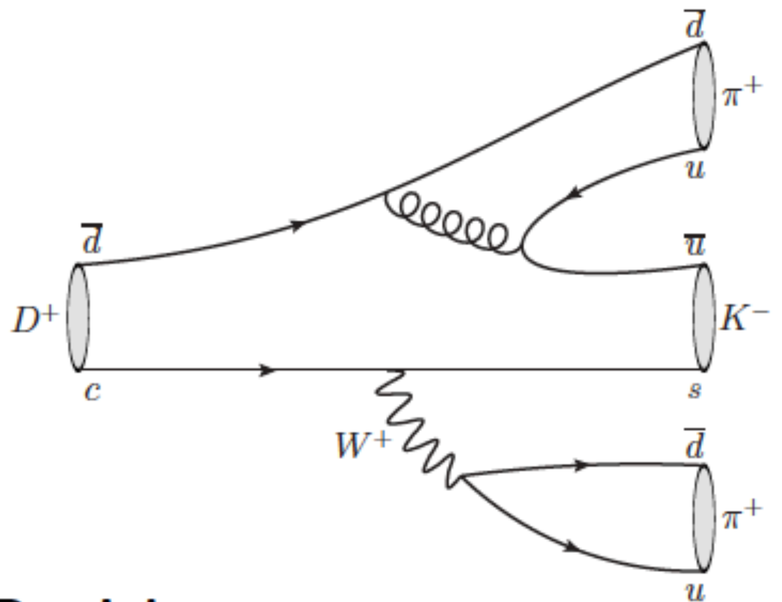


departs from two-body Uni. and by crossing impose 3-body Unitarity

- 2-body systems (Omnès fcts.) obey universal (hadr.) phase relations

Heavier decays: $D \to K\pi\pi$

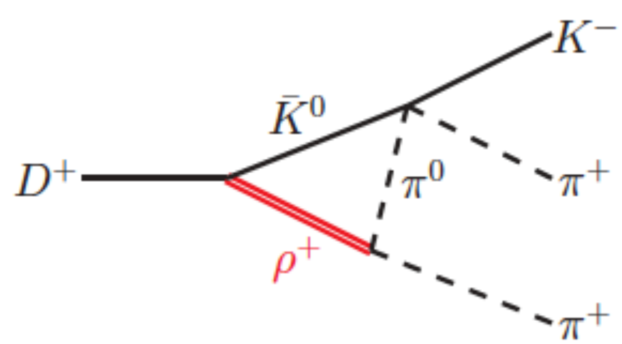
T. Isken



- Cabbibo-favoured decays, several experiments with good statistics for $D^+ \to K^-\pi^+\pi^+$ [E791 (2006), CLEO (2008), FOCUS (2009)]
- coupled to $D^+ \to \bar{K}^0\pi^0\pi^+$ [BESIII (2014)]

Partial waves:

$\pi\pi$-system	$P^1_{\pi\pi}$	$S^2_{\pi\pi}$
πK-system	$S^{1/2}_{\pi K}$	$S^{3/2}_{\pi K}$
	$P^{1/2}_{\pi K}$	$P^{3/2}_{\pi K}$
	$(D^{1/2}_{\pi K})$	



$\Rightarrow$ $\pi\pi$ P-wave only couples indirectly via $D^+ \to \bar{K}^0\pi^0\pi^+$

- $\omega/\phi \to 3\pi$ decays [Niecknig, Kubis and Schneider (2012)]
- $\eta' \to \eta\pi\pi/3\pi$ decays [Isken, Kubis, Schneider and Stoffer (2017), in preparation]

Niecknig&Kubis PLB 780

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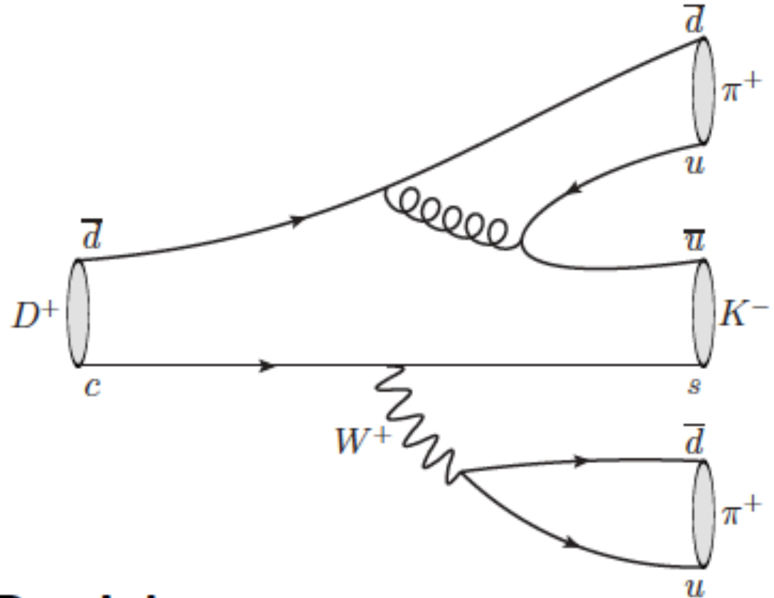

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limited to low energy knowledge of ππ and Kπ FF

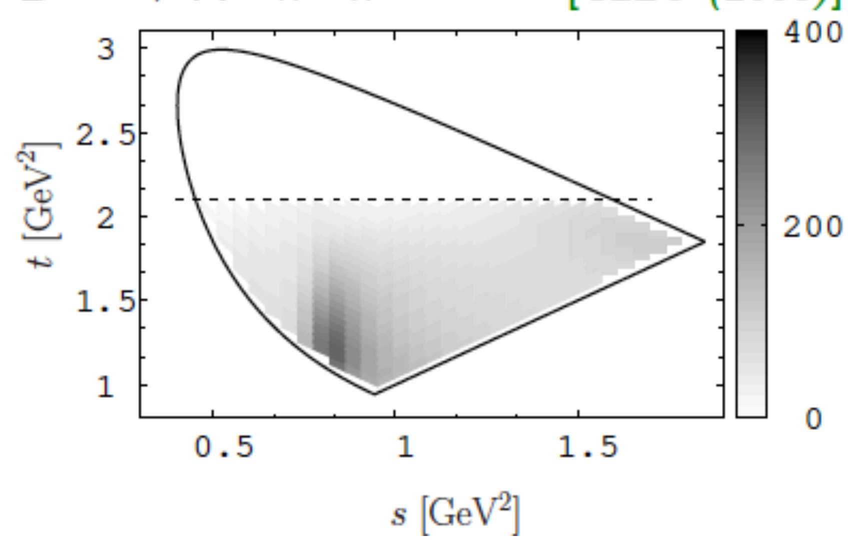
Partial waves:

$\pi\pi$-system	$P^1_{\pi\pi}$	$S^2_{\pi\pi}$
πK-system	$S^{1/2}_{\pi K}$	$S^{3/2}_{\pi K}$
	$P^{1/2}_{\pi K}$	$P^{3/2}_{\pi K}$
	$(D^{1/2}_{\pi K})$	

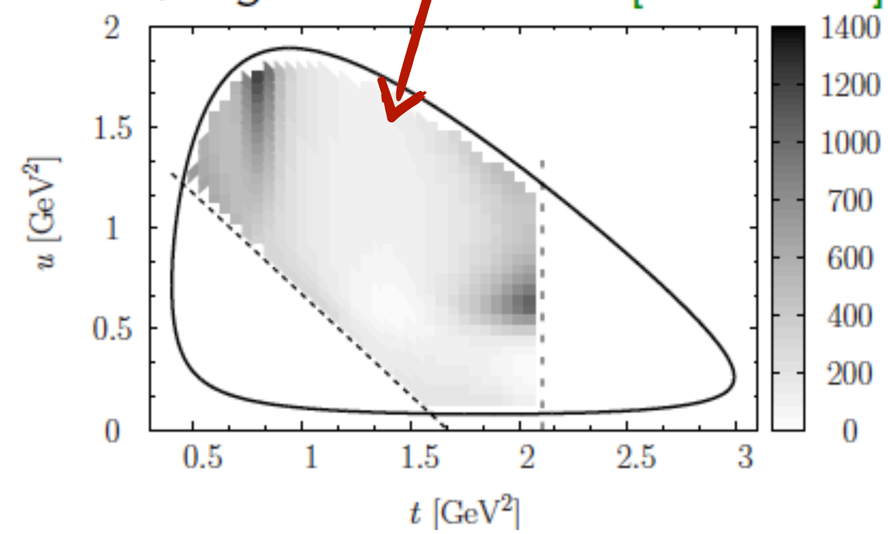
$\Rightarrow$ $\pi\pi$ P-wave only couples indirect

Dalitz plots: $D \to K\pi\pi$

$D^+ \to K^-\pi^+\pi^+$ [CLEO (2008)]



$D^+ \to \bar{K}_S\pi^0\pi^+$ [BESIII 2014]



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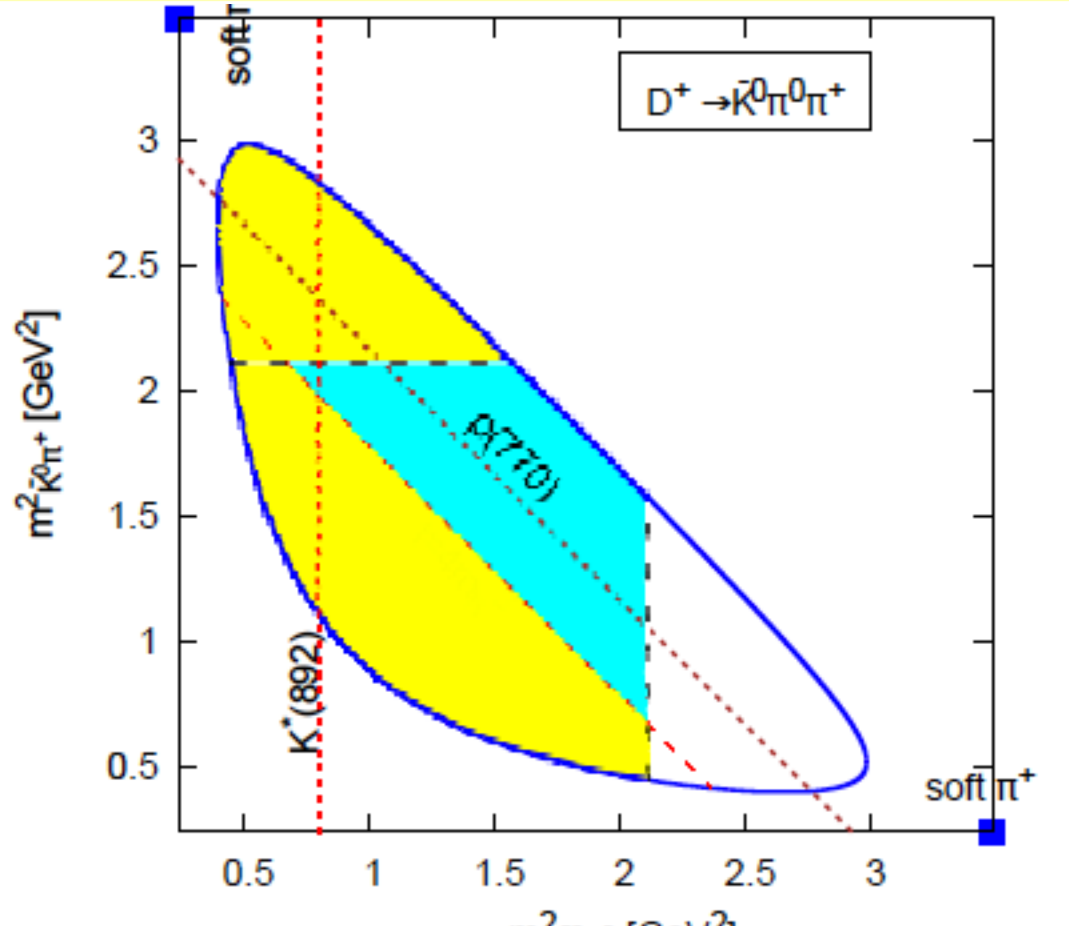
Niecknig&Kubis PLB 780

Elastic/inelastic rescattering in $D^+ \to \overline{K}^0 \pi^0 \pi^+$:

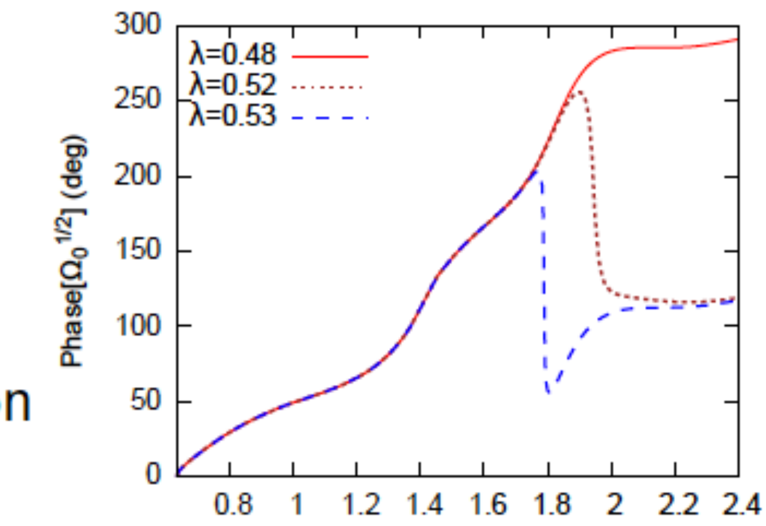
B. Moussallam



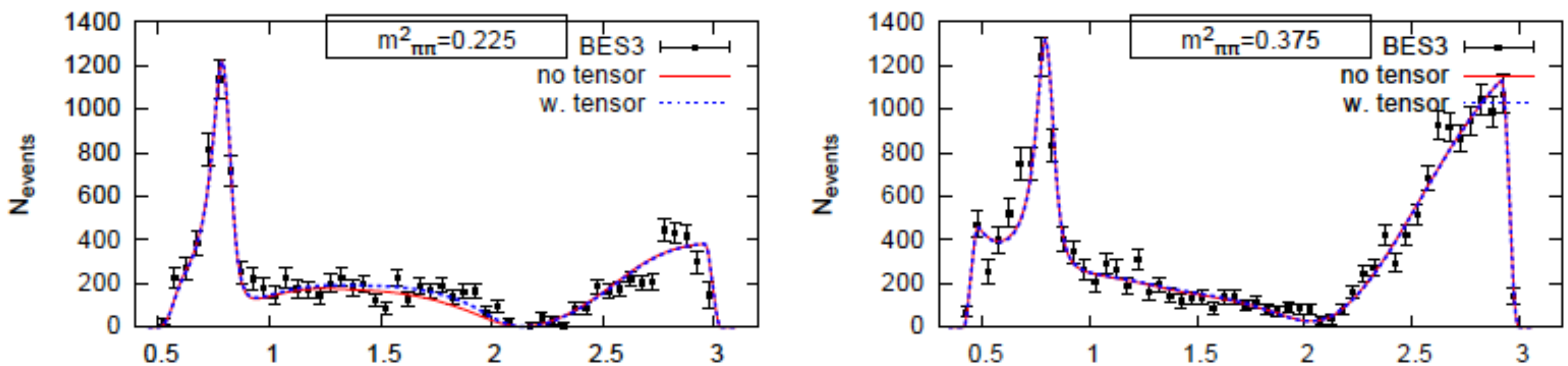
→ Effective one-channel Omnès function:
different way to include isospin breaking

$$\Omega^{eff}(s) = \Omega_{11}(s) + \lambda\,\Omega_{12}(s)$$

→ πK phase as a function of λ

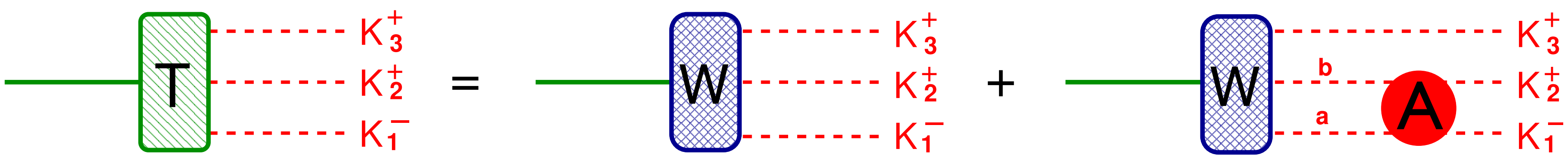


→ Probe this with soft pion relations ?



- fit to data looks nice! in agreement with PLB780

- alternative to isobar model in $D^+ \to K^- K^+ K^+$ amplitude analysis
  R. Aoude, P. C. Magalhaes, A dos Reis, M. Robilotta arXiv: 1805.11764
  ↳ better fit to LHCb data ! (non-disclose)



- FSI: (2+1) + 3-body non-resonant based on chiral Lagrangian
  - no $K\bar{K}$ scattering data ⟶ use 3-body data to obtain information from $K\bar{K}$

- alternative to isobar model in $D^+ \to K^- K^+ K^+$ amplitude analysis
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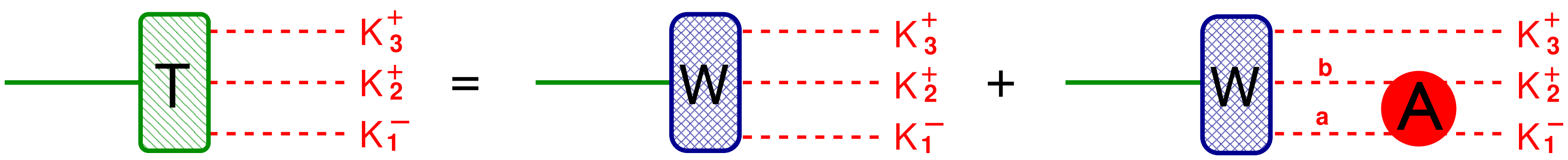

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  - $A_{ab}^{JI} \longrightarrow$ unitary scattering amplitude for $ab \to K^+K^-$

$$\boxed{\mathcal{A}_{ab}^{JI} = \frac{\mathcal{K}_{ab\to cd}^{(JI)}}{1 + \bar{\Omega}_{ab}\, \mathcal{K}_{ab\to cd}^{(JI)}}}$$

multi meson model - $D^+ \to K^- K^+ K^+$

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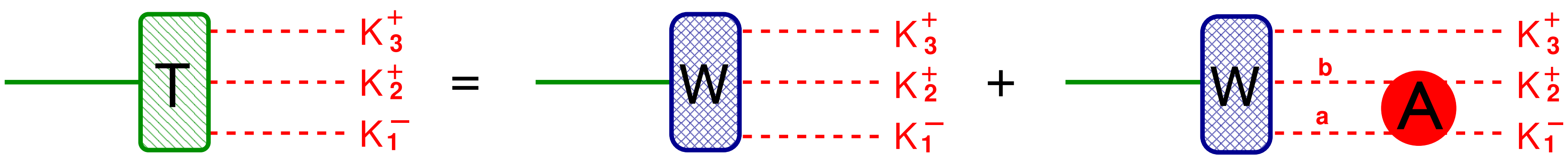

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  - A^{JI}_{ab} ⟶ unitary scattering amplitude for $ab \to K^+ K^-$

$$\boxed{\mathcal{A}^{JI}_{ab} = \frac{\mathcal{K}^{(JI)}_{ab\to cd}}{1 + \bar{\Omega}_{ab}\,\mathcal{K}^{(JI)}_{ab\to cd}}}$$

↳ parameters have physical meaning (different from LECs)

- masses:
  $m_\rho\,,\; m_{a_0}\,,\; \boxed{m_{s0}\,,\; m_{s1}}$ $SU(3)$ singlet and octet

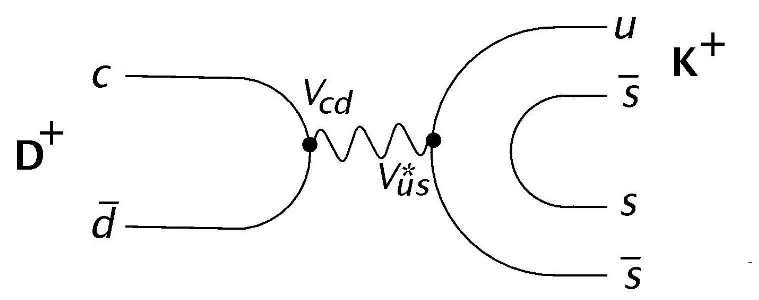
→ physical f_0 states are linear combination of $m_{s0}\,,\; m_{s1}$

- coupling constants:
  $g_\rho\,,\; g_\phi$ — vector
  $c_d\,,\; c_m\,,\; \tilde{c}_d\,,\; \tilde{c}_m$ — scalar

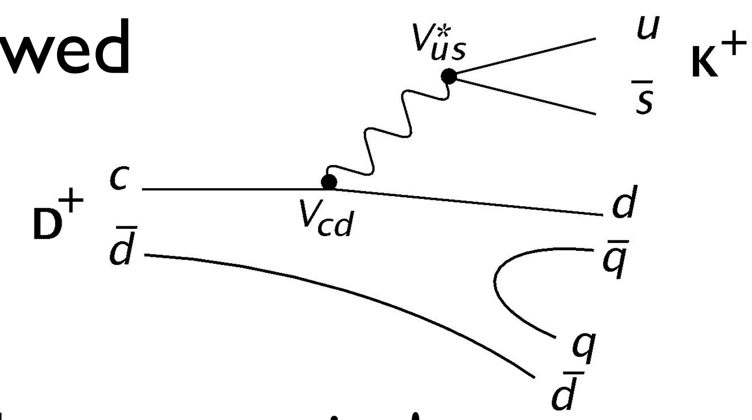
Chau [Phys. Rep. 95,1(1983)]

- tree level

annihilation

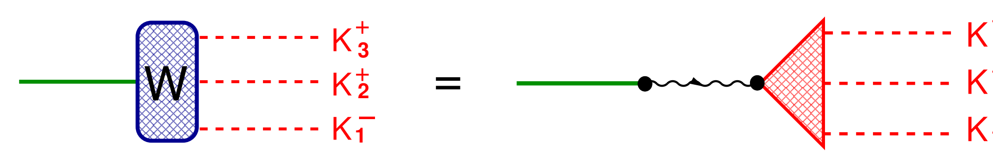


color allowed



- need a rescattering!
- unknown FF

- both are doubly Cabibbo-suppressed
- hypotheses that annihilation is dominant



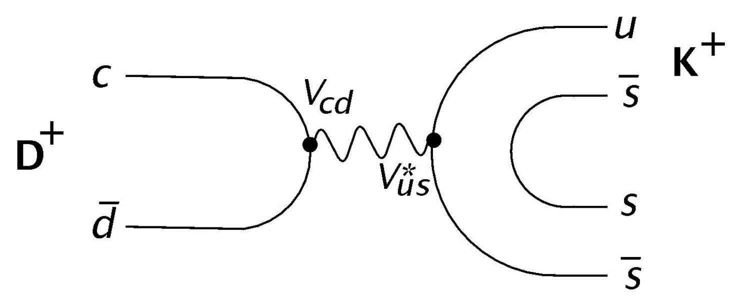
weak topologies

$c\,\bar{d} \longrightarrow \bar{u}\,s \;\; u\,\bar{s} \;\; u\,\bar{s}$

Chau [Phys. Rep. 95,1(1983)]

- tree level W

annihilation



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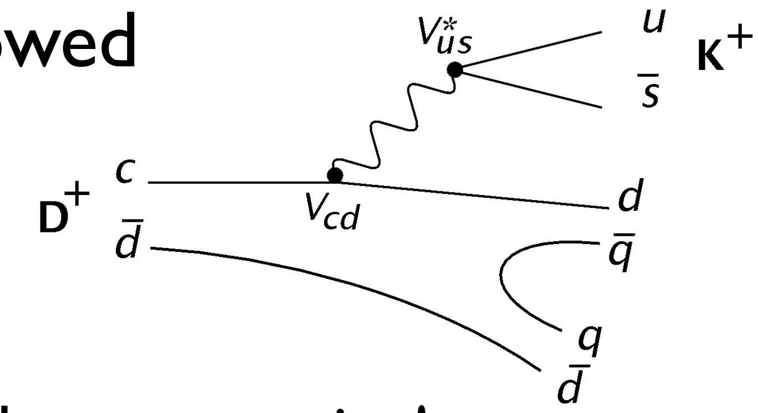

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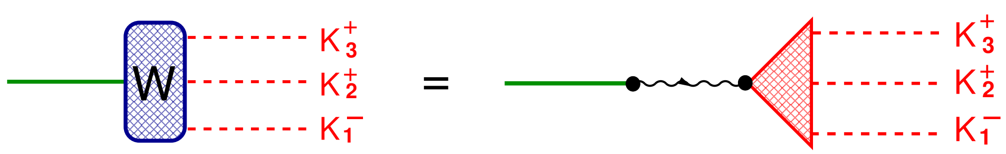


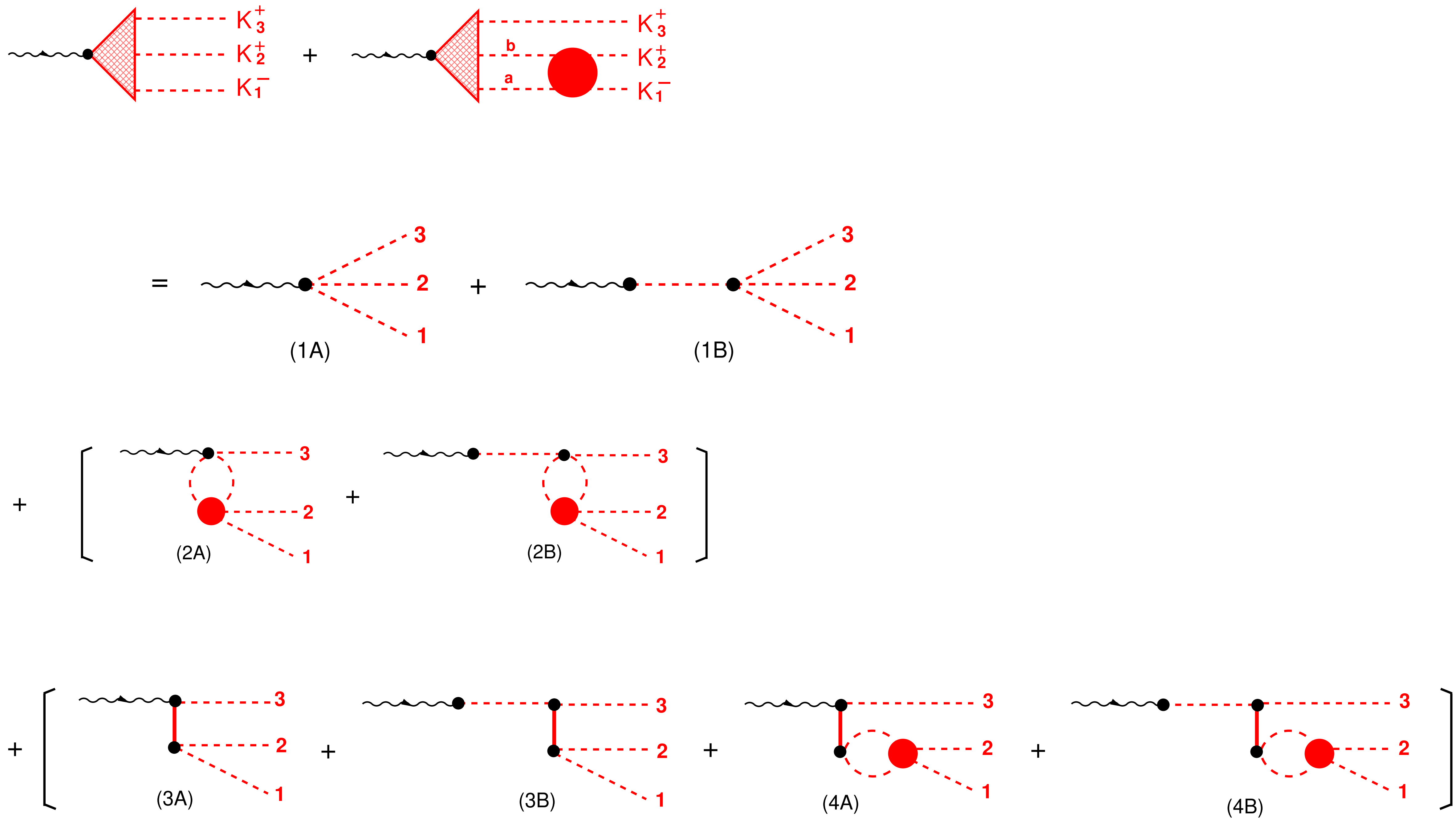
  - separate the different energy scales:

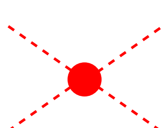
$$\mathcal{T} = \langle (KKK)^+ | T | D^+ \rangle = \underbrace{\langle (KKK)^+ | A_\mu | 0 \rangle}_{\text{chiral}} \; \langle 0 | A^\mu | D^+ \rangle .$$

$\hookrightarrow \; -i\, G_F\, \sin^2\theta_C\, F_D\, P^\mu$

$\longrightarrow$ know how to calculate everything

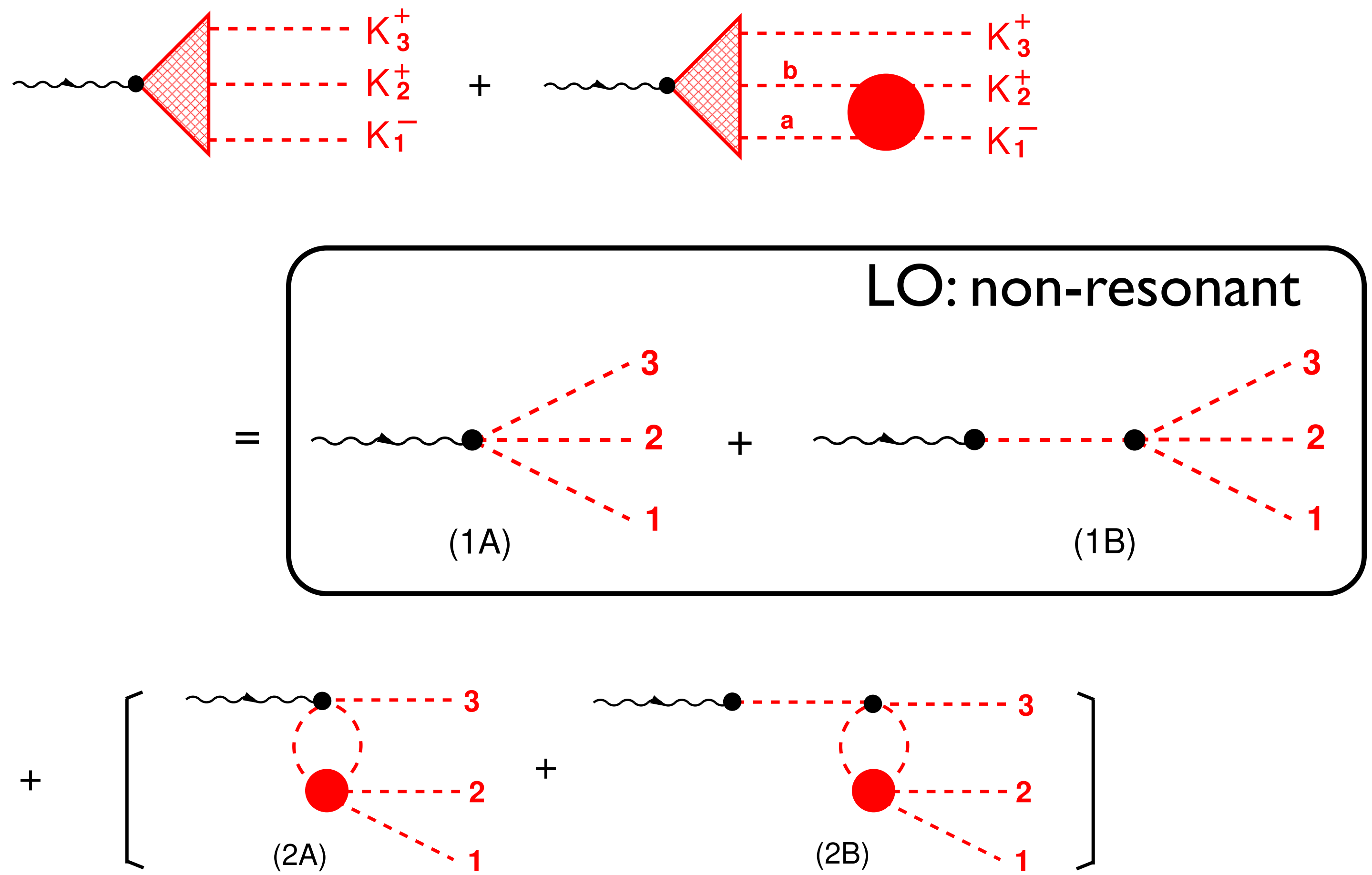
$D^+ \to K^- K^+ K^+$

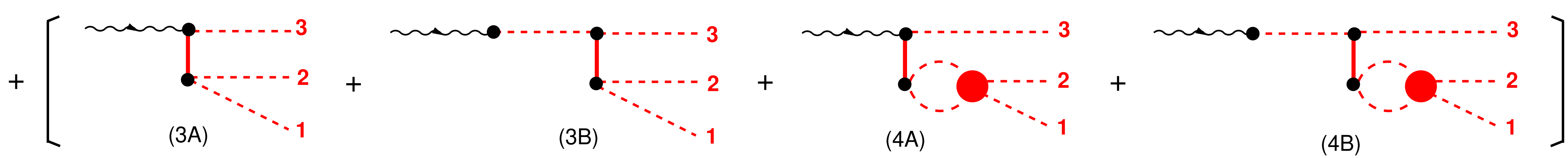


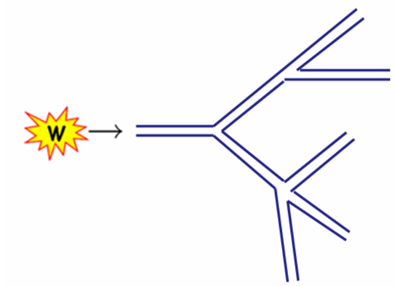

$K\bar{K}$ scattering amplitude

isospin decomposition $[J, I = (0,1), (0,1)]$

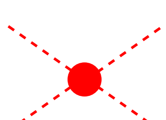
$D^+ \to K^- K^+ K^+$







Chiral symmetry

 $K\bar{K}$ scattering amplitude

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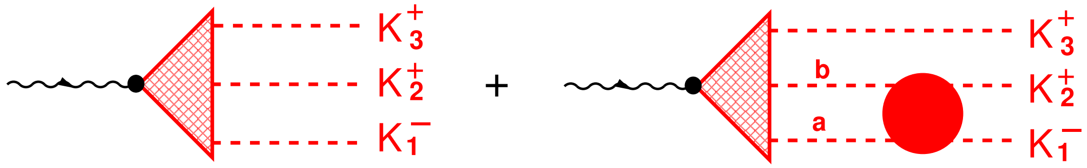
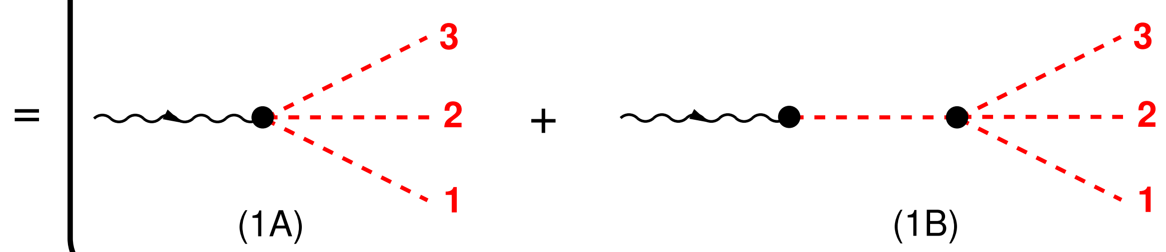
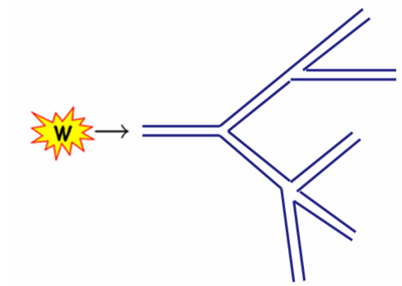


LO: non-resonant

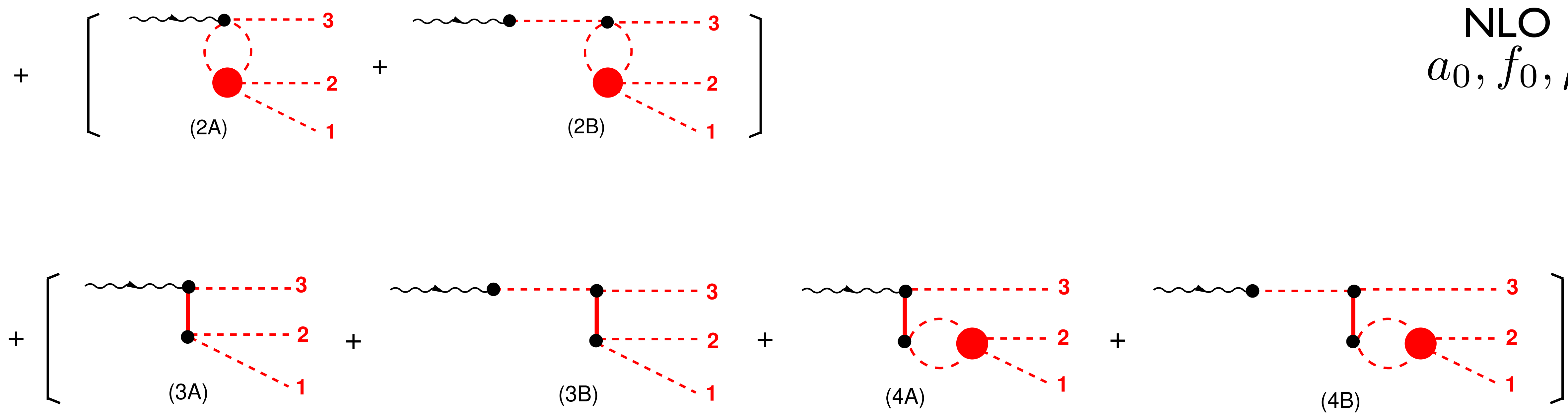


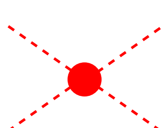


Chiral symmetry

NLO

a_0, f_0, ρ, ϕ



 $K\bar{K}$ scattering amplitude

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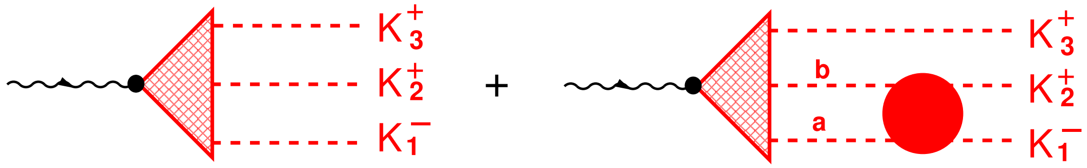
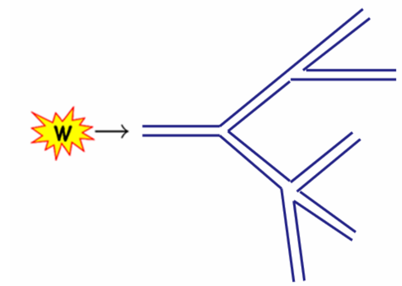
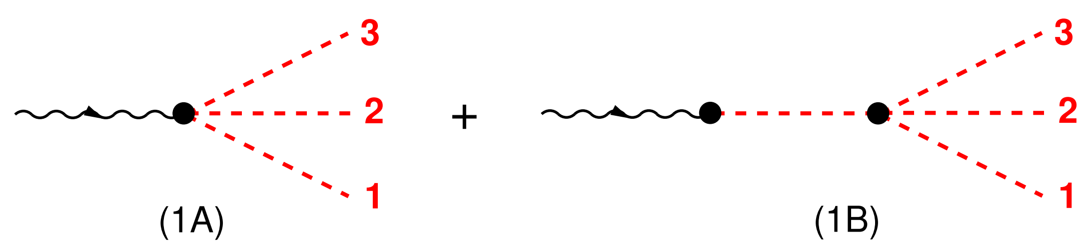




Chiral symmetry

LO: non-resonant

=



NLO

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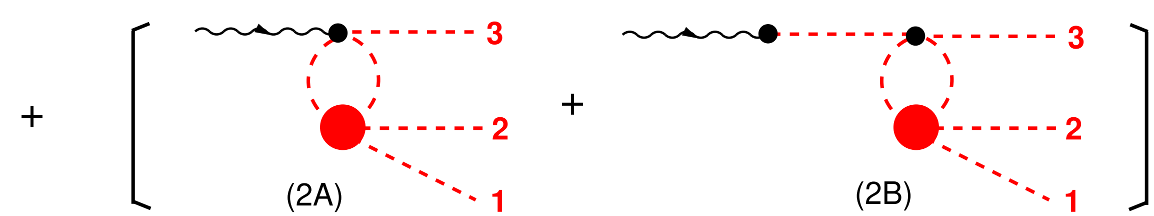
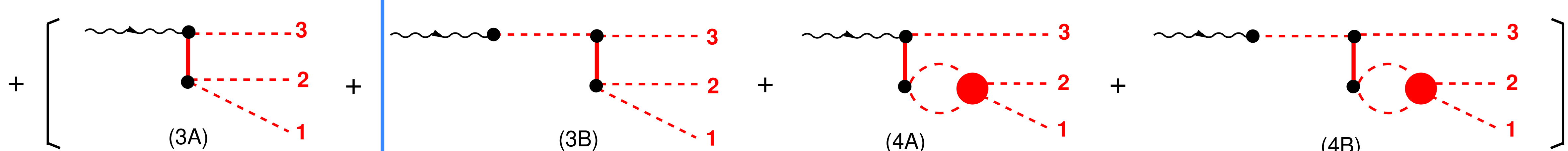
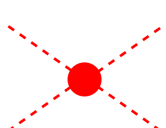


isobar



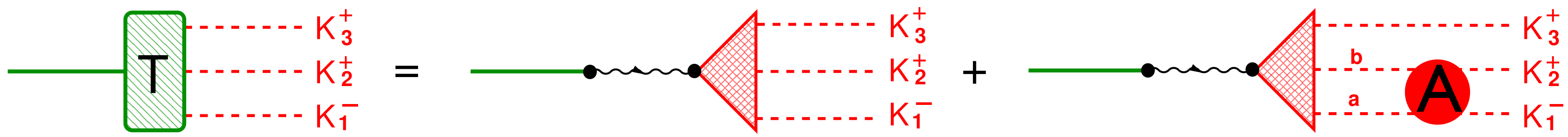
dynamical width



$K\bar{K}$ scattering amplitude

isospin decomposition $[J, I = (0,1), (0,1)]$

$D^+ \to K^-K^+K^+$ Triple- M



- $$T = \left[T^S + T^P + (2 \leftrightarrow 3)\right]$$

$$T^S = \left[\frac{C}{4}\left(M_D^2 - M_K^2 + m_{12}^2\right) + T^{(0,1)} + T^{(0,0)}\right]$$

$$T^P = \left[\frac{C}{4}\left(m_{13}^2 - m_{23}^2\right) + T^{(1,1)} + T^{(1,0)}\right] .$$

- A^{IJ} $\longrightarrow$ prediction
  - extend chiral to non perturbative region $\longrightarrow$ parameter have different meaning
    $m_{S1} = m_{So}$

- parameter for Toy studies :

masses from PDG (GeV)

$m_\rho = 0.776,\ m_\phi = 1.019,$
$m_{a0} = 0.960,\ m_{So} = 0.980$

$\longrightarrow$ $m_{S1} = 1.370\, GeV$

low energy couplings (GeV)

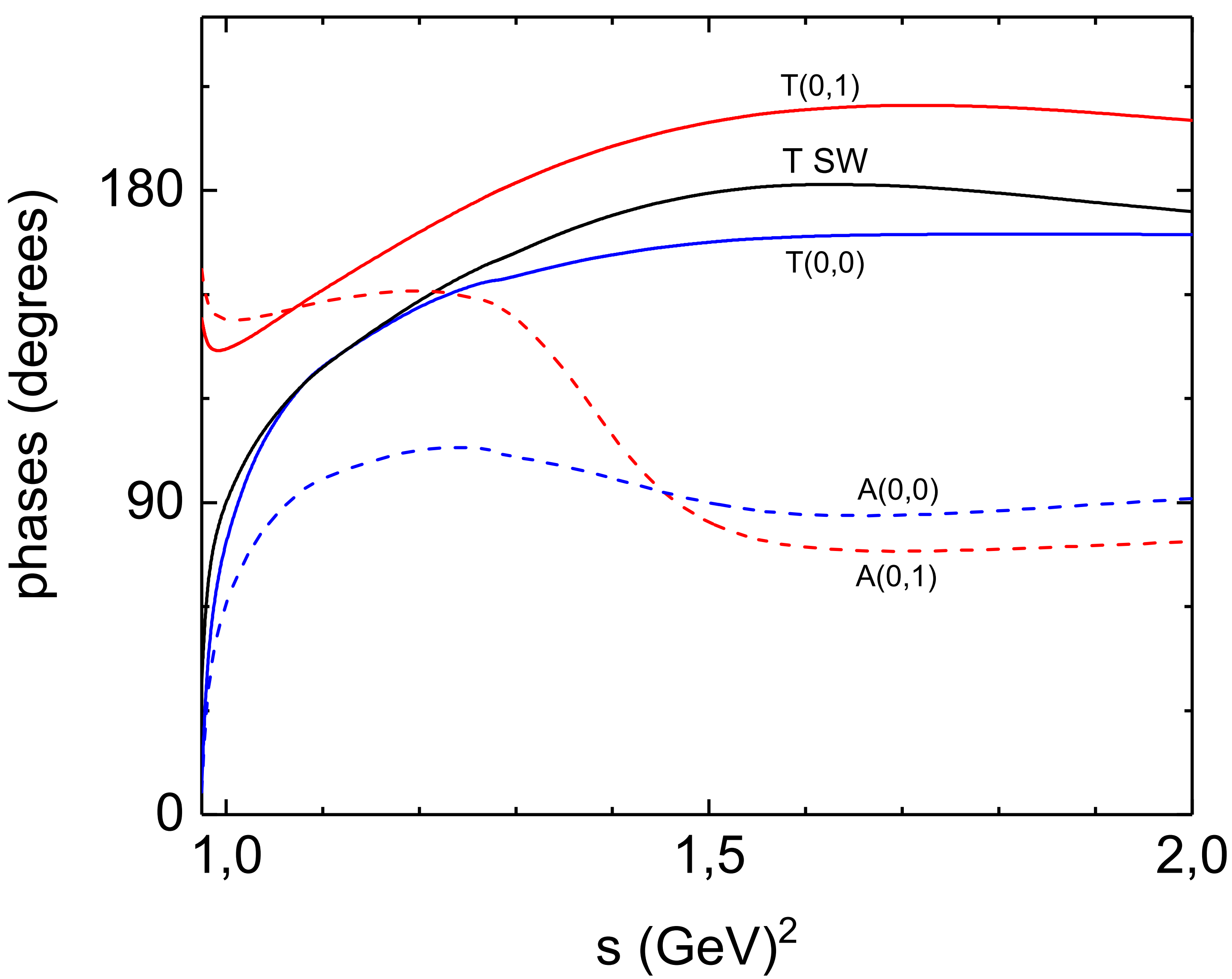
$[F, G_V] = [0.093, 0.067]$ vectors
$[c_d, c_m] = [0.032, 0.042]$ scalar octet
$[\tilde{c_d}, \tilde{c_m}] = [0.018, 0.025]$ scalar singlet

additional PDG

$\Gamma_{\phi \to K\bar{K}} \sim 3.54\, MeV$
$\sin\theta = 0.605$ $(\phi - \omega)$ mixing

$\longrightarrow$ all (13) were free in the fit to data

- non-resonant: beyond (2+1) is a 3-body amplitude
- FSI: coupled-channel meson-meson departs from chiral Lagrangian
- intensity of each component is predict by theory → $\neq$ isobar model
- Toy studies



→ can disentangle a_0 and f_0

→ couple channel structure: cannot be ignored

arXiv: 1805.11764

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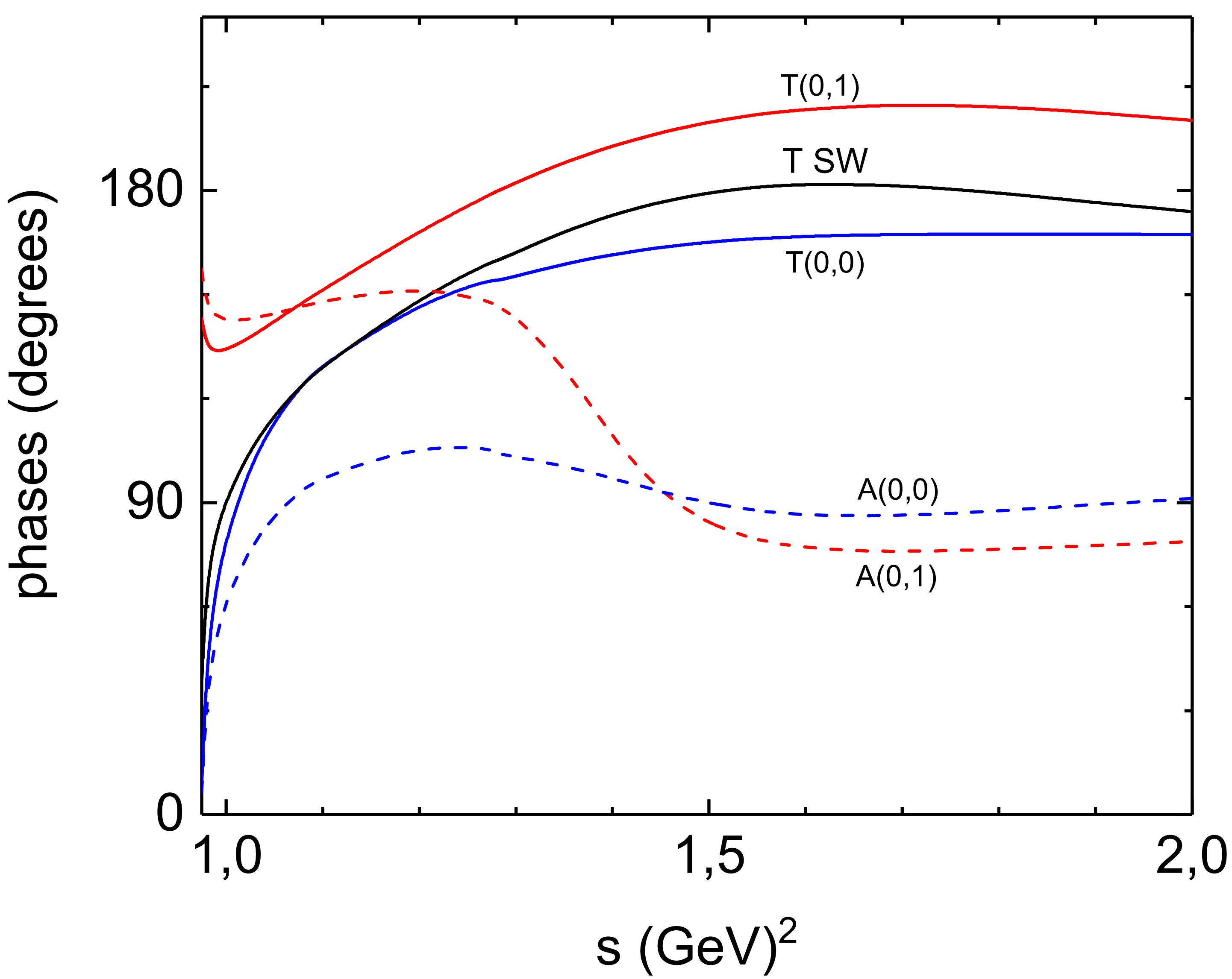


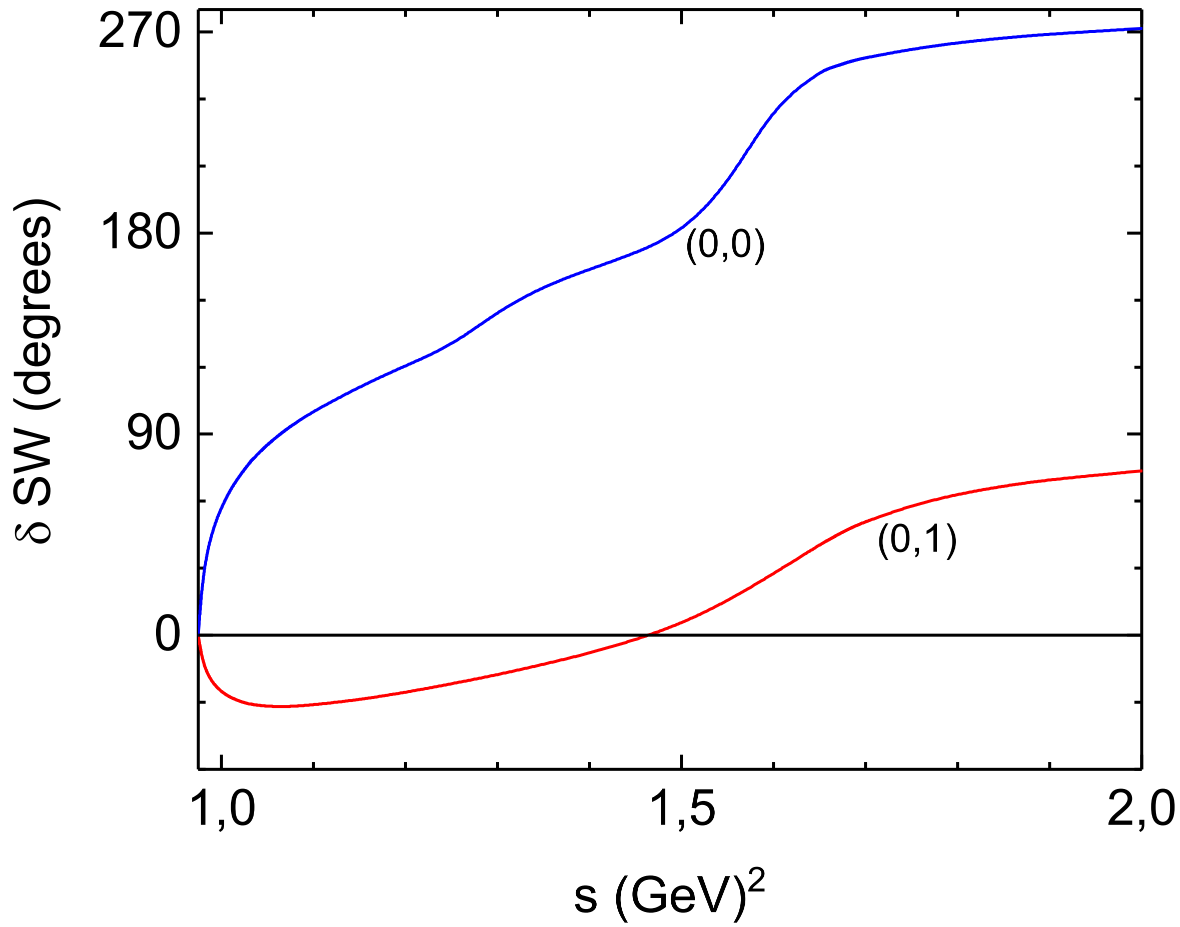
→ can disentangle a_0 and f_0

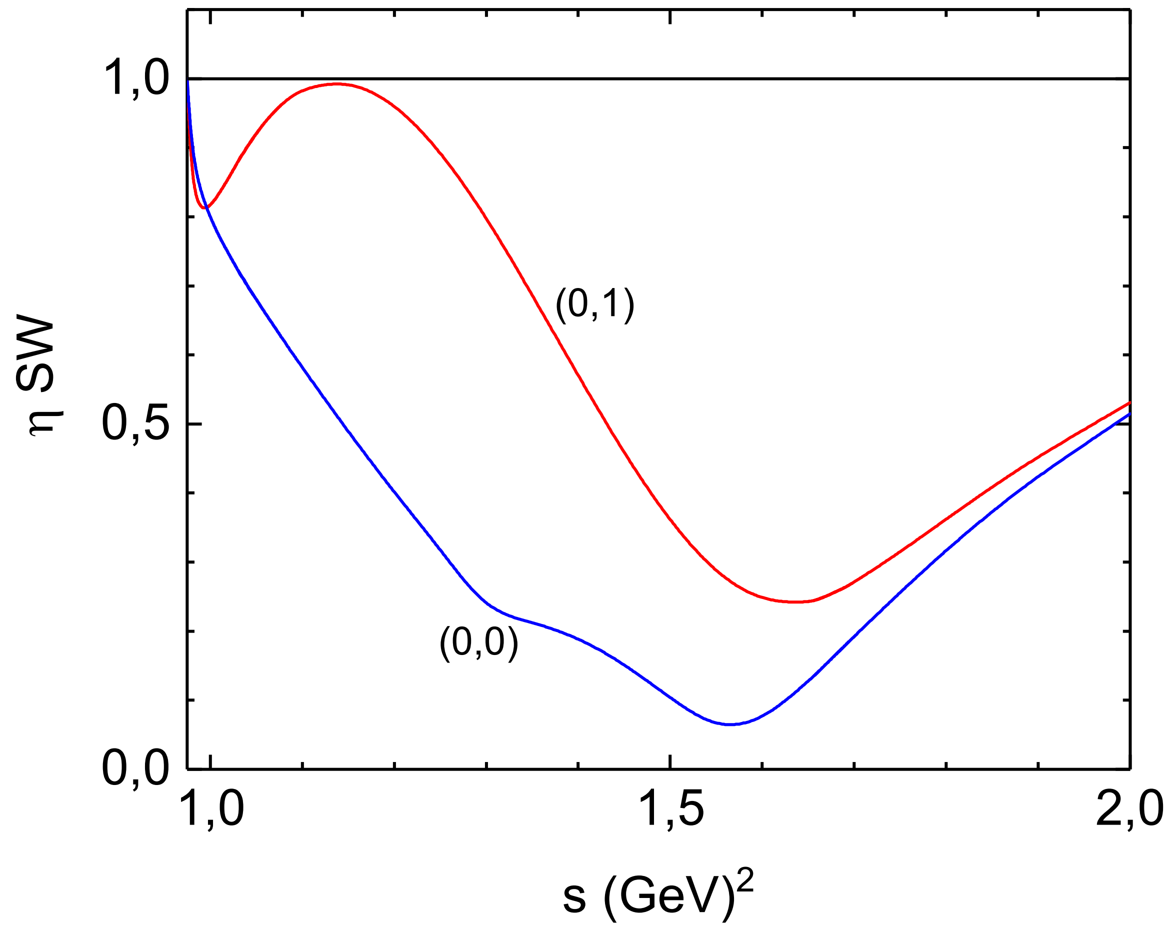
→ couple channel structure: cannot be ignored

Fitting data we can predict KK scattering phase

arXiv: 1805.11764

- predictions for KK S-wave
  - phase
  - inelasticity





- needs improvement....

↳ first model: simplicity to make the bridge

→ K-matrix approximation!

→ couple higher resonances

→ add other topologies

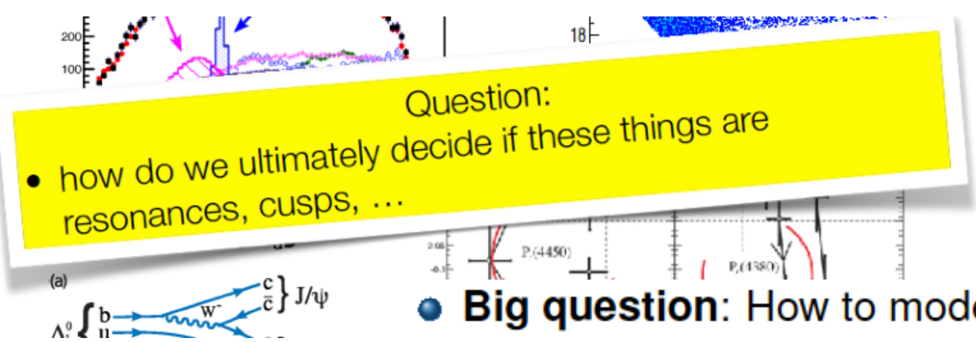
→ compare denominators and FF

final remarks

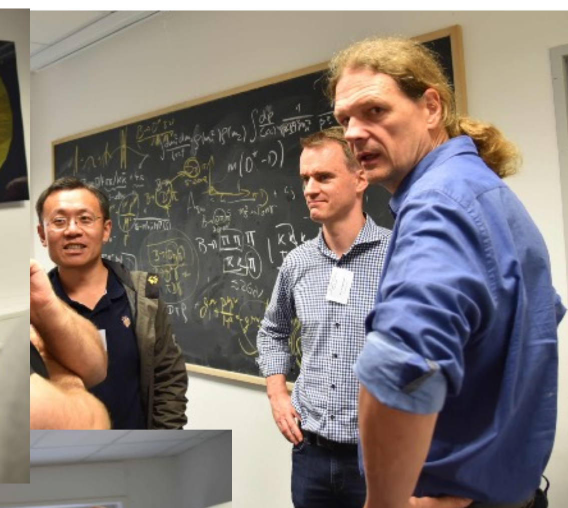
TRR110 Workshop - Amplitudes for Three-Body Final States

3 days of intense discussion!

→ speakers shared problems and questions



- **Big question**: How to model dynamic isobar amplitudes
- Requirements
  - Physical constraints: Unitarity & Analyticity
  - Simplicity to implement
  - Fit stability
  - Interpretability of results



→ But: κ not seen in D^0 decays ?

final remarks: results of everybody highlights

- FSI is crucial to all processes with Hadrons
  - all start with good 2-body amplitude → still need for high E (non-perturbative)

  → how far we really need 2-body amplitude?! all B phase-space ?

- Khuri-Treiman is very limited (besides complex): find a way to diagnose where 3-body rescattering is indeed needed?

  → start fitting with GOOD 2-body, e.g.: $B_s^0 \to K_s^0 \pi^+ \pi^-$, $D^+ \to K^- K^+ K^+$

- polynom singularities and the zero modes observed in free isobar (multiple solutions) in the PWA analysis are somehow related.

- strong correlation between FSI in heavy mesons decay (mainly D), diffractive production and tau decays.
  - In $\omega/\Phi \to 3\pi$ Khuri-Treiman, the shape of the rho change with 3π mass (mother).

    → COMPASS observed this as well!

    → how to parametrise this?

    → how to transfer knowledge?

final remarks: results of everybody highlights

- can we use free isobar to constrain models?

→ how do we quantify if we have a good model?

we have good models on the market (even with simplifications and limitations)
experimental should start using it more seriously!!

Thank you!!!

Extra slides

FSI in three-body decay :

references

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Falk et al. Phys. Rev. D 57,4290(1998);
Blok, Gronau, Rosner, *Phys. Rev Letters* 78, 3999 (1997).

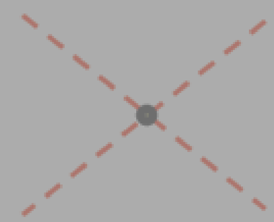
many more ...

D—>KKK

chiral Lagrangians

- solid theory to describe MM interactions at low energy

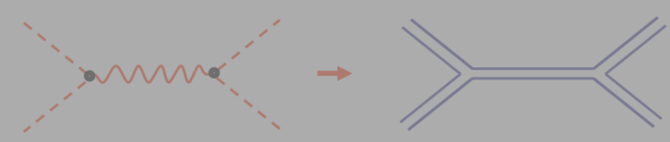
- LO:



$$\mathcal{L}_M^{(2)} = -\frac{1}{6F^2} f_{ijs}\, f_{kls}\, \phi_i\, \partial_\mu \phi_j\, \phi_k\, \partial^\mu \phi_l + \frac{B}{24F^2}\left[\sigma_0 \left(\frac{4}{3}\delta_{ij}\,\delta_{kl} + 2\, d_{ijs}\, d_{kls}\right) + \sigma_8 \left(\frac{4}{3}\delta_{ij}\, d_{kl8} + \frac{4}{3} d_{ij8}\,\delta_{kl} + 2\, d_{ijm}\, d_{kln}\, d_{8mn}\right)\right] \phi_i\, \phi_j\, \phi_k\, \phi_l\,.$$

Gasser & Leutwyler
[Nucl. Phys. B250(1985)]

- NLO: include resonances as a field



Ecker, Gasser, Pich and De Rafael
[Nucl. Phys. B321(1989)]

scalars

$$\mathcal{L}_S^{(2)} = \frac{2\,\tilde{c}_d}{F^2} R_0\, \partial_\mu \phi_i\, \partial^\mu \phi_i - \frac{4\,\tilde{c}_m}{F^2} B\, R_0 \left(\sigma_0\, \delta_{ij} + \sigma_8\, d_{8ij}\right) \phi_i\, \phi_j$$

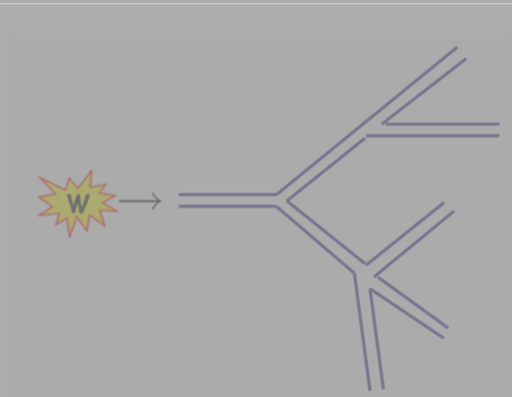
$$+ \frac{2\, c_d}{\sqrt{2}F^2} d_{ijk}\, R_k\, \partial_\mu \phi_i\, \partial^\mu \phi_i - \frac{4Bc_m}{\sqrt{2}F^2}\left[\sigma_0\, d_{ijk} + \sigma_8 \left(\frac{2}{3}\delta_{ik}\,\delta_{j8} + d_{i8s}\, d_{jsk}\right)\right] \phi_i\, \phi_j R_k$$

vectors

$$\mathcal{L}_V^{(2)} = \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$

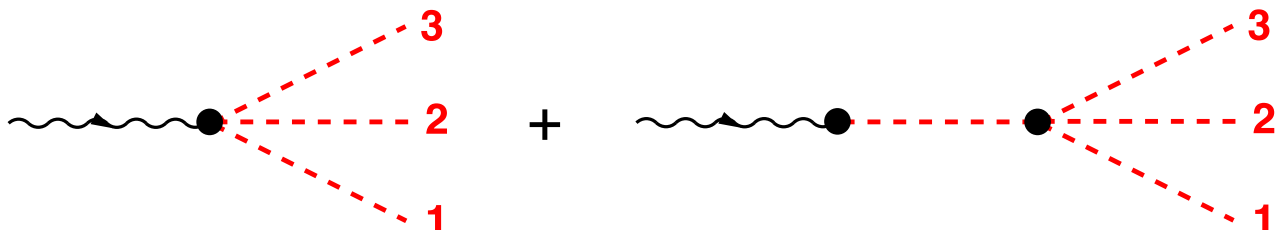
$$\langle V_{\mu\nu} u^\mu u^\nu \rangle = \frac{1}{F^2} V_a^{\mu\nu}\, \partial_\mu \phi_i\, \partial_\nu \phi_j\, (i f_{aij} + d_{aij})$$

- hadronization of Weak current

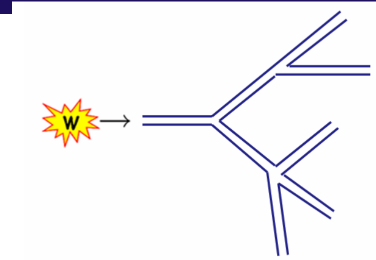


Gasser & Leutwyler
[Nucl. Phys. B250(1985)]

non-resonant



(1A) (1B)



Chiral symmetry

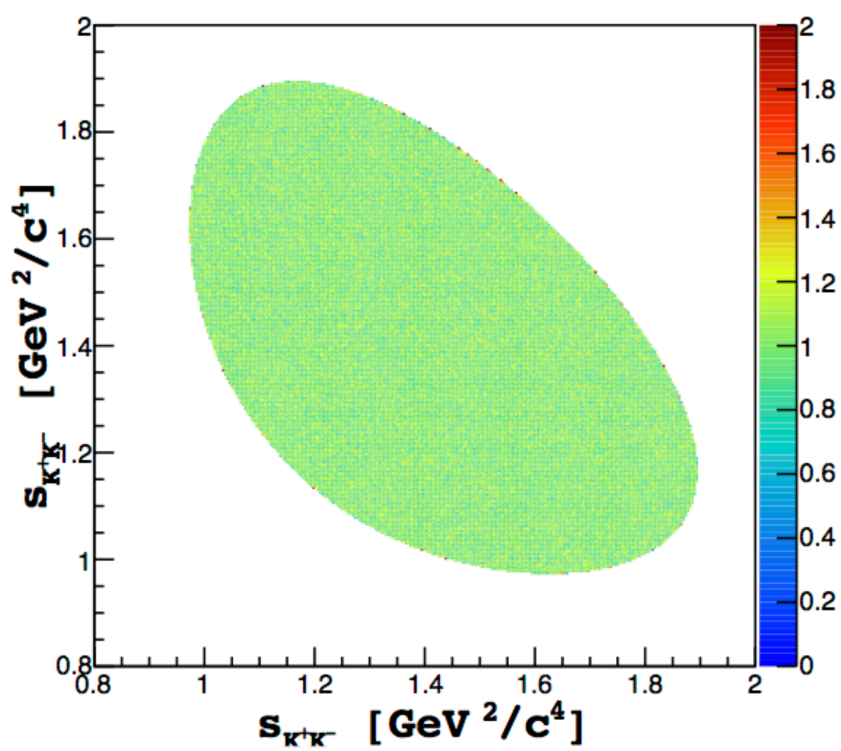
$$T_{NR} = \left[\frac{C}{4} \left(M^2 - M_K^2 + m_{12}^2 \right) + \frac{C}{4} \left(m_{13}^2 - m_{23}^2 \right) + (2 \leftrightarrow 3) \right]$$

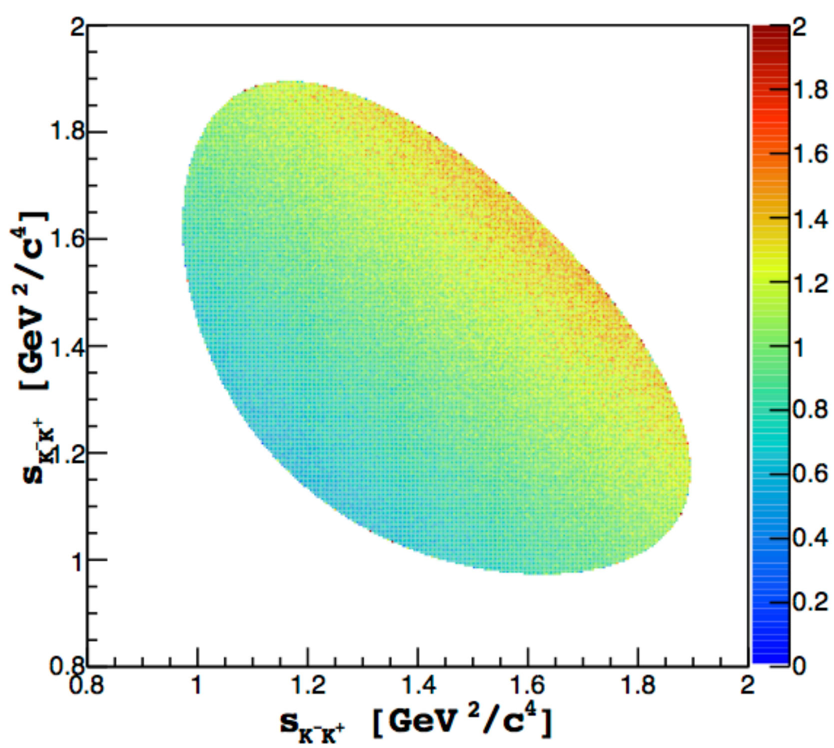
$$C = \left\{ \left[\frac{G_F}{\sqrt{2}} \sin^2 \theta_C \right] \frac{2 F_D}{F} \frac{M_K^2}{M_D^2 - M_K^2} \right\}$$

project into S- and P- wave

3-body effect predicted by Chiral symmetry

- comparing with isobar (constant)





real polynomial

no possible free parameter

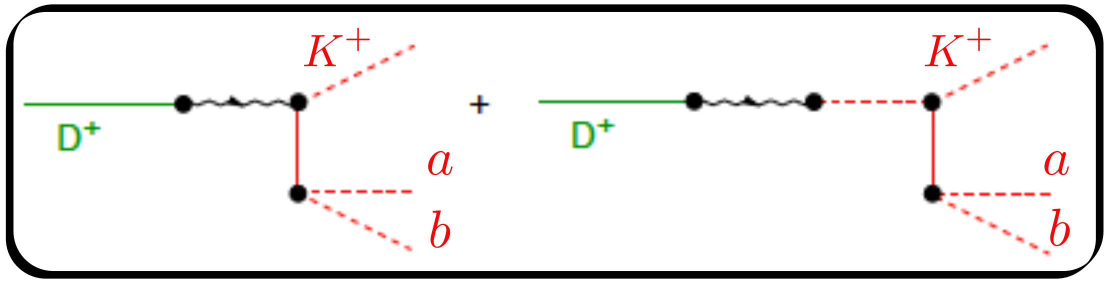
resonance channels

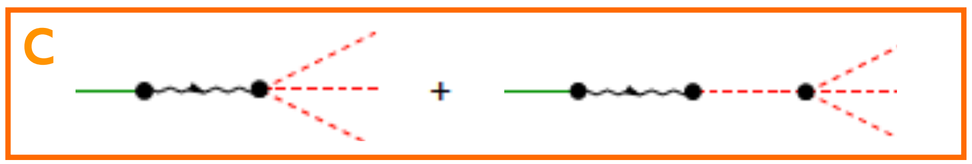
a_0 example

$[J, I = 0, 1] \rightarrow \eta\pi,\ KK$

- **tree** $D \to a\, b\, K^+$

$$\langle U_3\,(K^+)\,|\,T^{(0,1)}_{(0)}\,|\,D\,\rangle = \left\{\Gamma^{(0,1)}_{(0)\,\pi 8}\,\langle\, U_3^{\pi 8}| + \Gamma^{(0,1)}_{(0)\,KK}\,\langle\, U_3^{KK}|\right\}$$



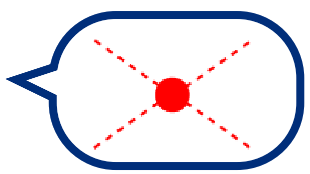


to avoid double counting!

$$\Gamma^{(0,1)}_{(0)\,\pi 8} = C\left\{\left[\frac{2\,\sqrt{2}}{\sqrt{3}\,F^2}\right]\frac{[-c_d\,P\cdot p_3 + c_m\,M_D^2]}{m_{12}^2 - m_{a_0}^2}\left[c_d\,\left(m_{12}^2 - M_\pi^2 - M_8^2\right) + 2\,c_m\,M_\pi^2\right] + \left[-\frac{\sqrt{3}}{\sqrt{2}}\,\left[M_D^2/3 - P\cdot p_3\right]\right]_c^{C}\right\}$$

$$\Gamma^{(0,1)}_{(0)\,KK} = C\left\{\left[\frac{2}{F^2}\right]\frac{[-c_d\,P\cdot p_3 + c_m\,M_D^2]}{m_{12}^2 - m_{a_0}^2}\left[c_d\,\left(m_{12}^2 - 2M_K^2\right) + 2\,c_m\,M_K^2\right] + \left[-\frac{1}{2}\,\left[M_D^2 - P\cdot p_3\right]\right]_c^{C}\right\}$$

- FSI



one interaction

$$\Gamma^{(0,1)}_{(1)\,\pi 8} = -\mathcal{K}^{(0,1)}_{\pi 8|\pi 8}\,\left[\bar{\Omega}^S_{\pi 8}\right]\,\Gamma^{(0,1)}_{(0)\,\pi 8} - \mathcal{K}^{(0,1)}_{\pi 8|KK}\left[\frac{1}{2}\,\bar{\Omega}^S_{KK}\right]\,\Gamma^{(0,1)}_{(0)\,KK}$$

$$\Gamma^{(0,1)}_{(1)\,KK} = -\mathcal{K}^{(0,1)}_{\pi 8|KK}\,\left[\bar{\Omega}^S_{\pi 8}\right]\,\Gamma^{(0,1)}_{(0)\,\pi 8} - \mathcal{K}^{(0,1)}_{KK|KK}\left[\frac{1}{2}\,\bar{\Omega}^S_{KK}\right]\,\Gamma^{(0,1)}_{(0)\,KK}$$

$$\Gamma^{(0,1)}_{(1)} = \begin{bmatrix}\Gamma^{(0,1)}_{(1)\,\pi 8}\\ \Gamma^{(0,1)}_{(1)\,KK}\end{bmatrix} = \begin{bmatrix} M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}\Gamma^{(0,1)}_{(0)\,\pi 8}\\ \Gamma^{(0,1)}_{(0)\,KK}\end{bmatrix} = M^{(0,1)}\,\Gamma^{(0,1)}_{(0)}$$

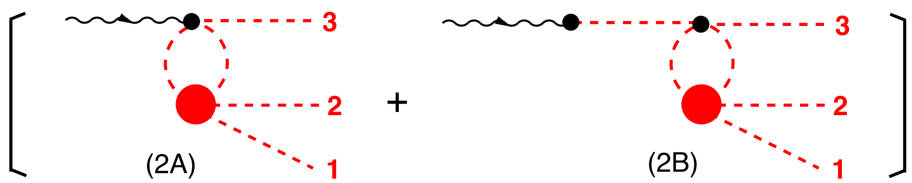
infinity interactions

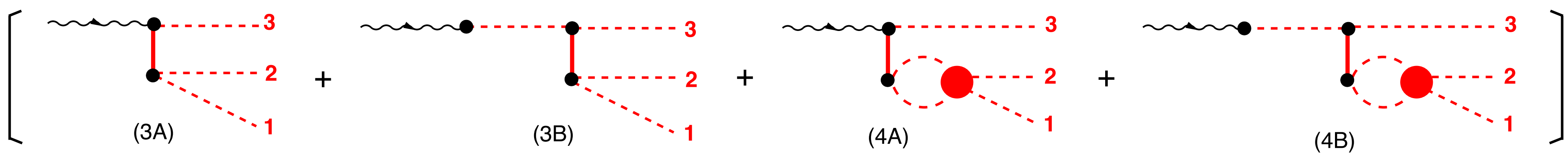
$$\Gamma^{(0,1)} = \{1 + M^{(0,1)} + [M^{(0,1)}]^2 + \cdots\}\,\Gamma^{(0,1)}_{(0)} \;\rightarrow\; \Gamma^{(0,1)} = \left[1 - M^{(0,1)}\right]^{-1}\,\Gamma^{(0,1)}_{(0)}$$

a_0 example

$[J, I = 0, 1] \rightarrow \eta\pi,\ KK$

- full FSI!





- $$T^{(0,1)} = -\frac{1}{2}\left[\bar{\Gamma}^{(0,1)}_{KK} - \Gamma^{(0,1)}_{c|KK}\right]$$

$$\rightarrow \quad \bar{\Gamma}^{(0,1)}_{KK} = \frac{(m_{12}^2 - m_{a_0}^2)}{D_{a_0}(m_{12}^2)}\left[M_{21}\,\Gamma^{(0,1)}_{(0)\,\pi 8} + (1 - M_{11})\,\Gamma^{(0,1)}_{(0)\,KK}\right]$$

$$\boxed{D_{a_0} = (m_{12}^2 - m_{a_0}^2)\ \left[(1 - M_{11})\,(1 - M_{22}) - M_{12}\,M_{21}\right]}$$

$$M_{11} = -\mathcal{K}^{(0,1)}_{\pi 8|\pi 8}\,[\bar{\Omega}^S_{\pi 8}]$$

$$M_{12} = -\mathcal{K}^{(0,1)}_{\pi 8|KK}\,[(1/2)\,\bar{\Omega}^S_{KK}]$$

$$M_{21} = -\mathcal{K}^{(0,1)}_{\pi 8|KK}\,[\bar{\Omega}^S_{\pi 8}]$$

$$M_{22} = -\mathcal{K}^{(0,1)}_{KK|KK}\,[(1/2)\,\bar{\Omega}^S_{KK}]$$

- only resonance

$$\bar{\Gamma}^{(0,1)}_{KK} = \frac{(m_{12}^2 - m_{a_0}^2)}{D_{a_0}(m_{12}^2)}\,\Gamma^{(0,1)}_{(0)\,KK}$$

Flatté

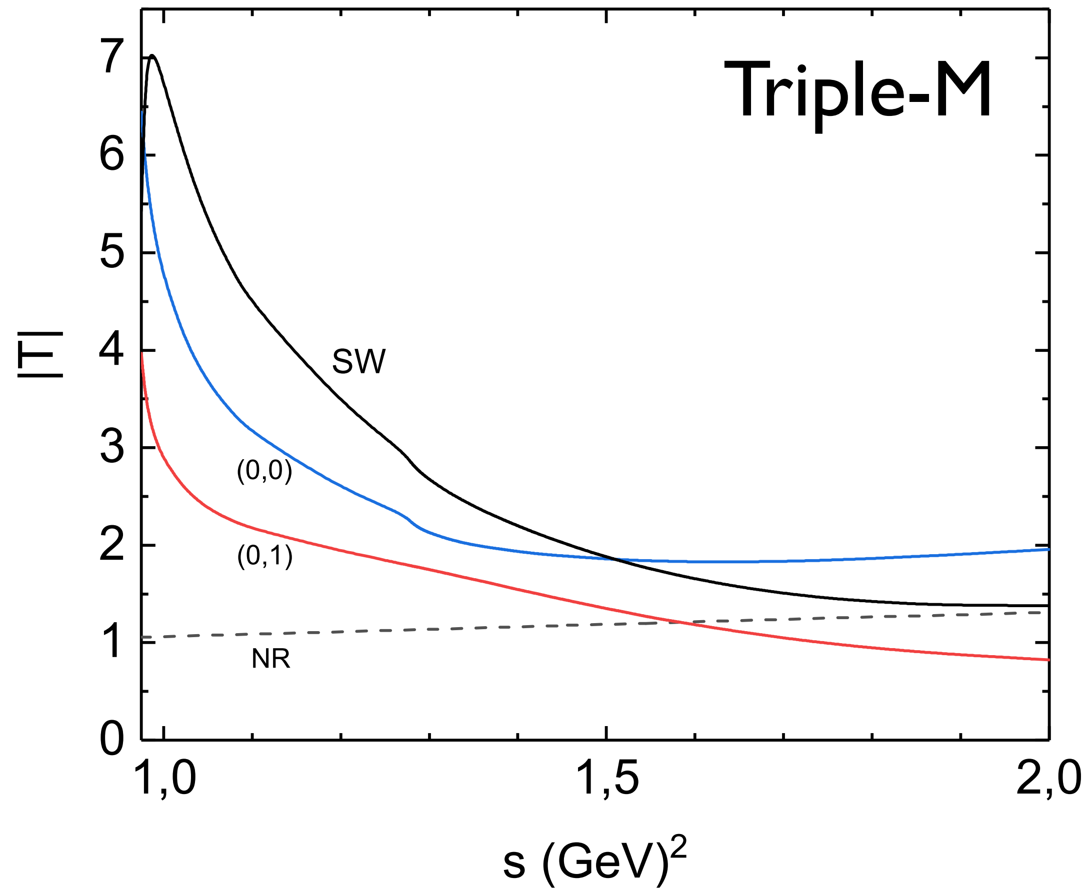
$$D_{a_0}(s) = (s - m_{a_0}^2) + i\,m_{a_0}\,\Gamma_{a_0}(s)$$

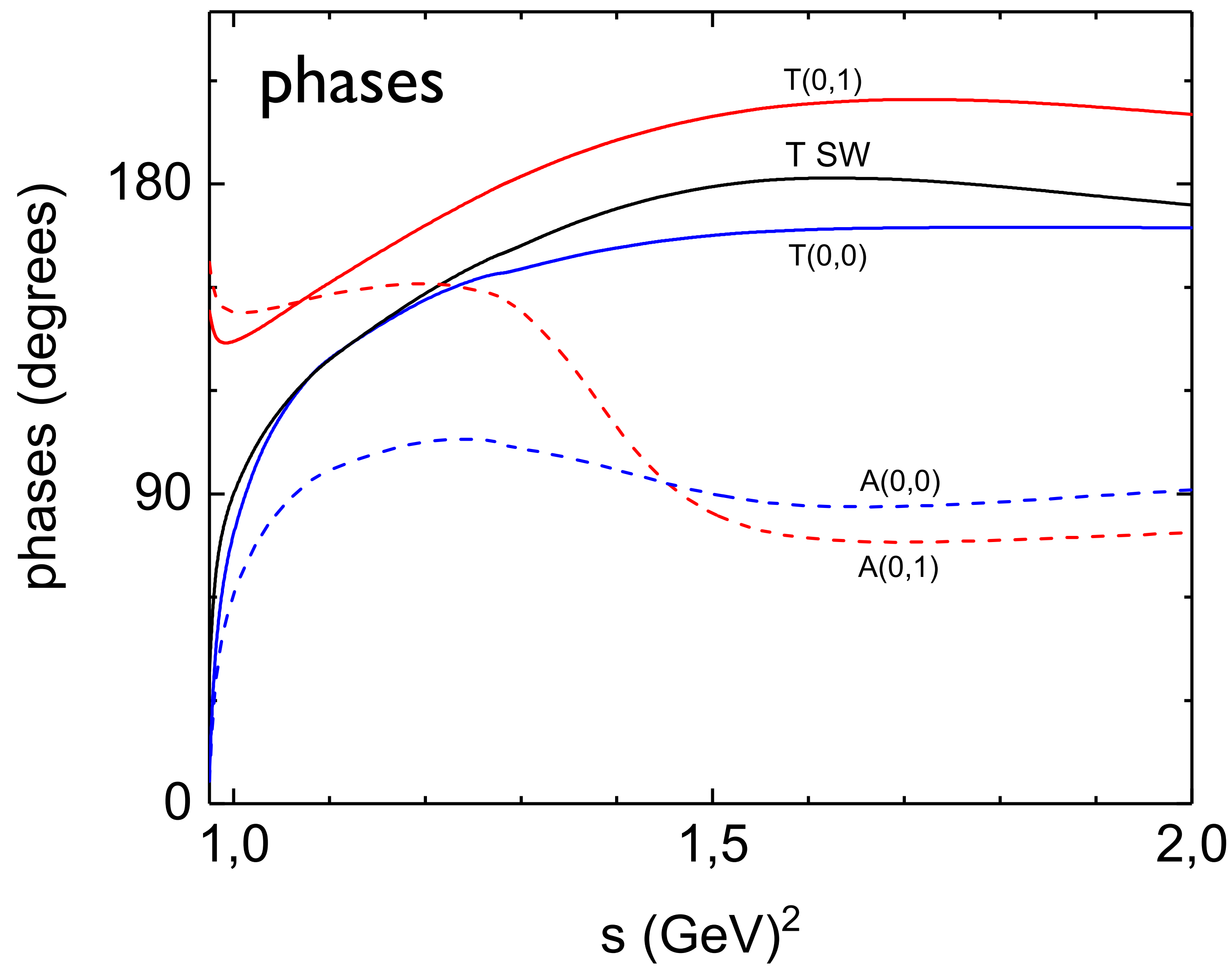
$$m_{a_0}\Gamma_{a_0}(s) = \frac{1}{8\pi\sqrt{s}}\left\{\left[\frac{4}{3\,F^4}\right]\boxed{\left[c_d\,(s - M_\pi^2 - M_8^2) + 2\,c_m\,M_\pi^2\right]^2}\,Q_{\pi 8} + \left[\frac{1}{F^4}\right]\boxed{\left[c_d\,(s - 2\,M_K^2) + 2\,c_m\,M_K^2\right]^2}\,Q_{KK}\right\}$$

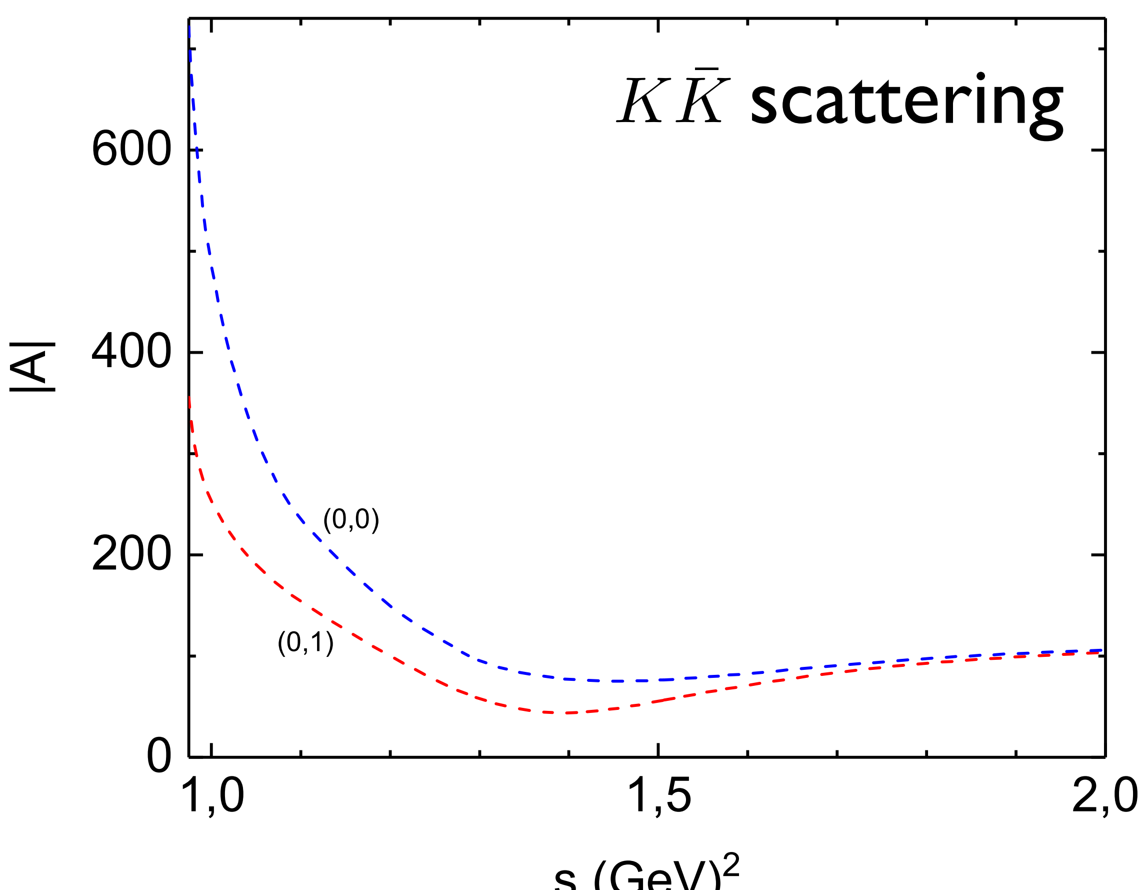
$\rightarrow$ parameter: $c_d,\ c_m\ m_{a_0}$

access two-body dynamics !

Toy results S-wave

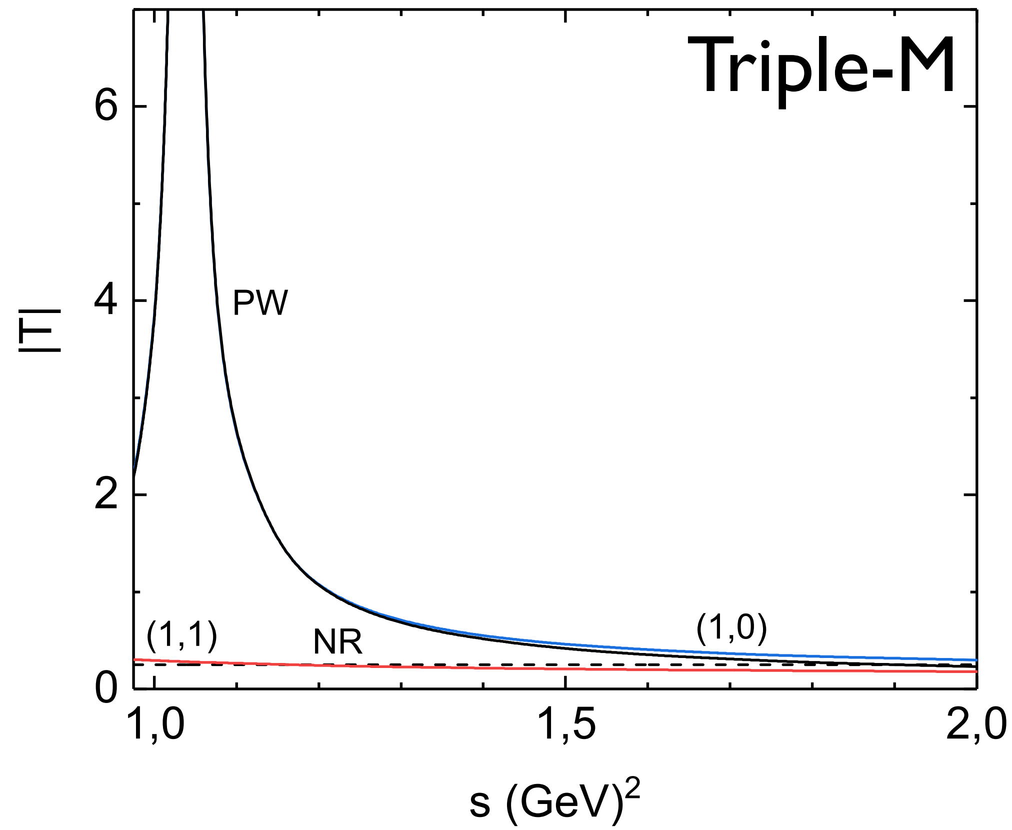


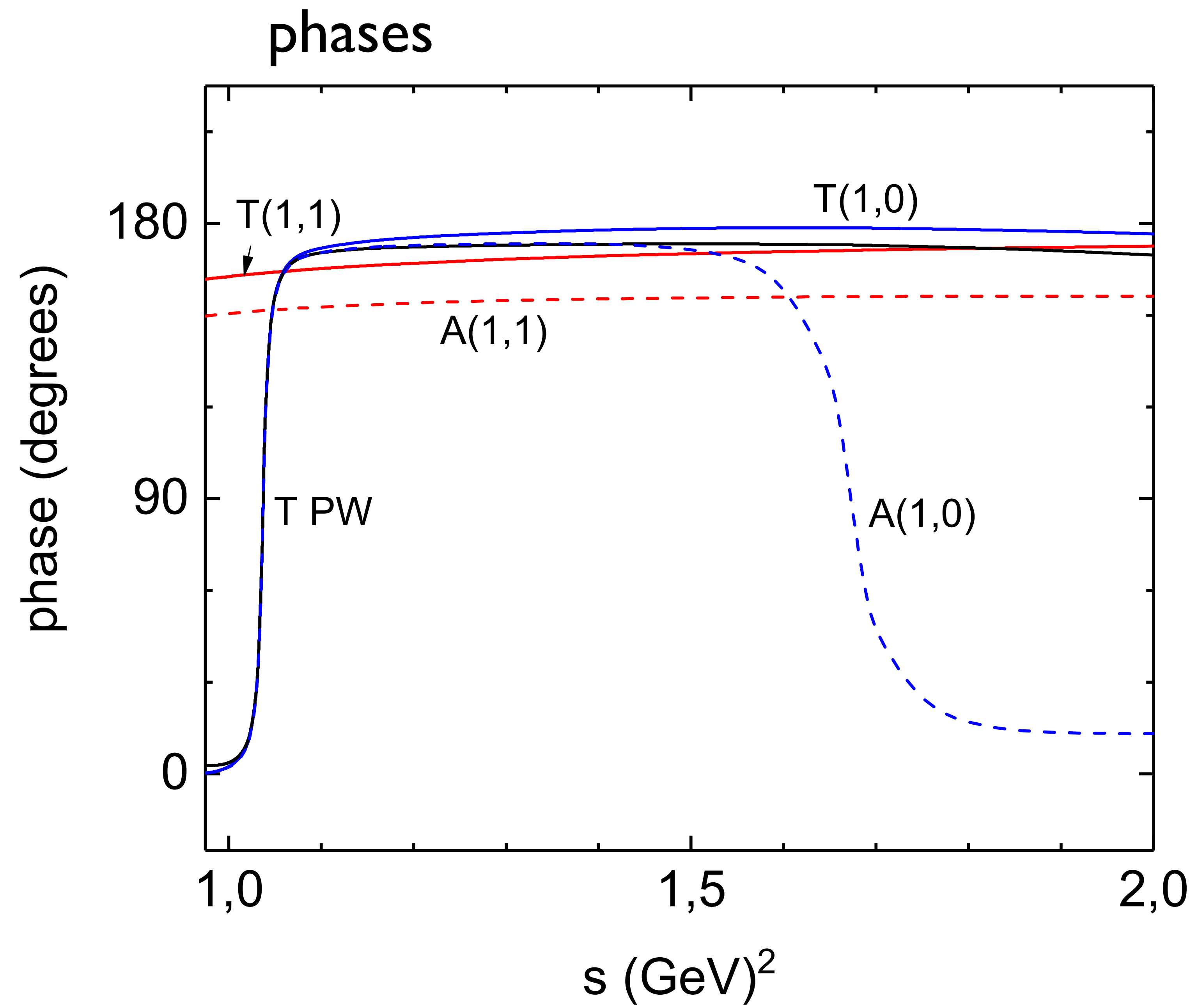


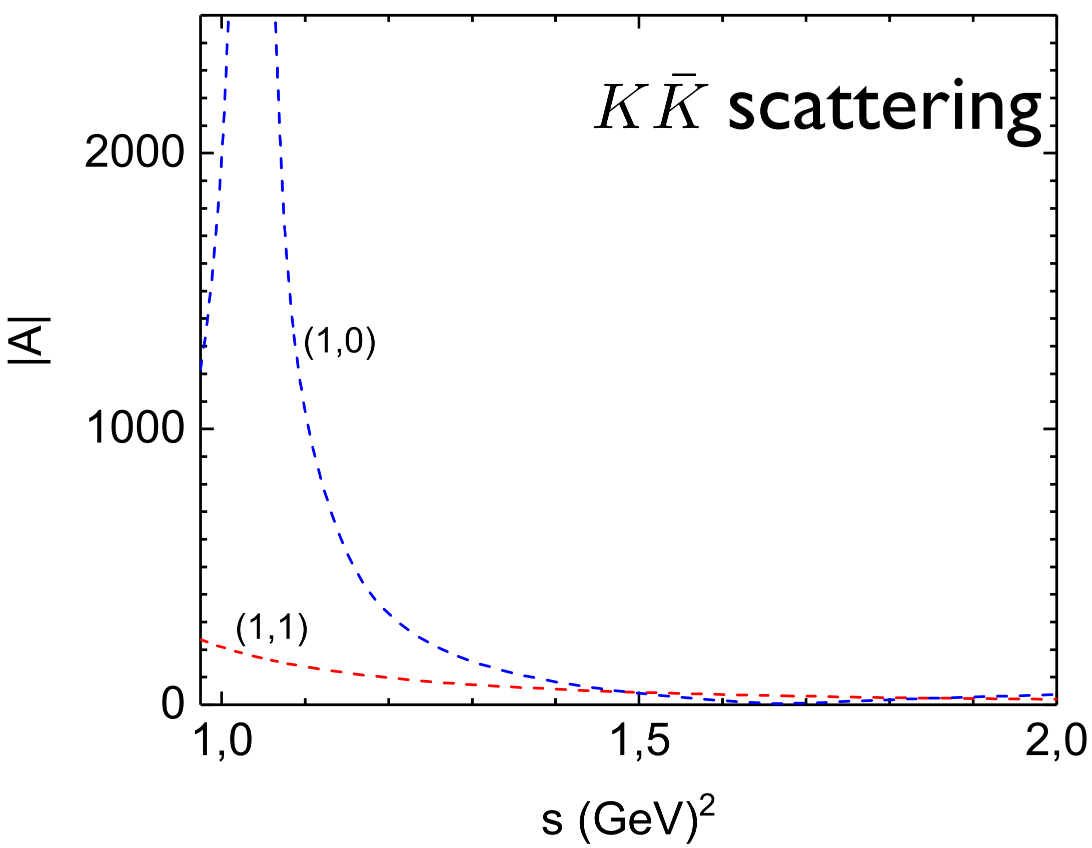


- amplitude with similar behave $\neq$ phase
- not possible to extract A(J,I) from data
- Triple-M disentangle $\neq$ isospins in data

Toy results P-wave



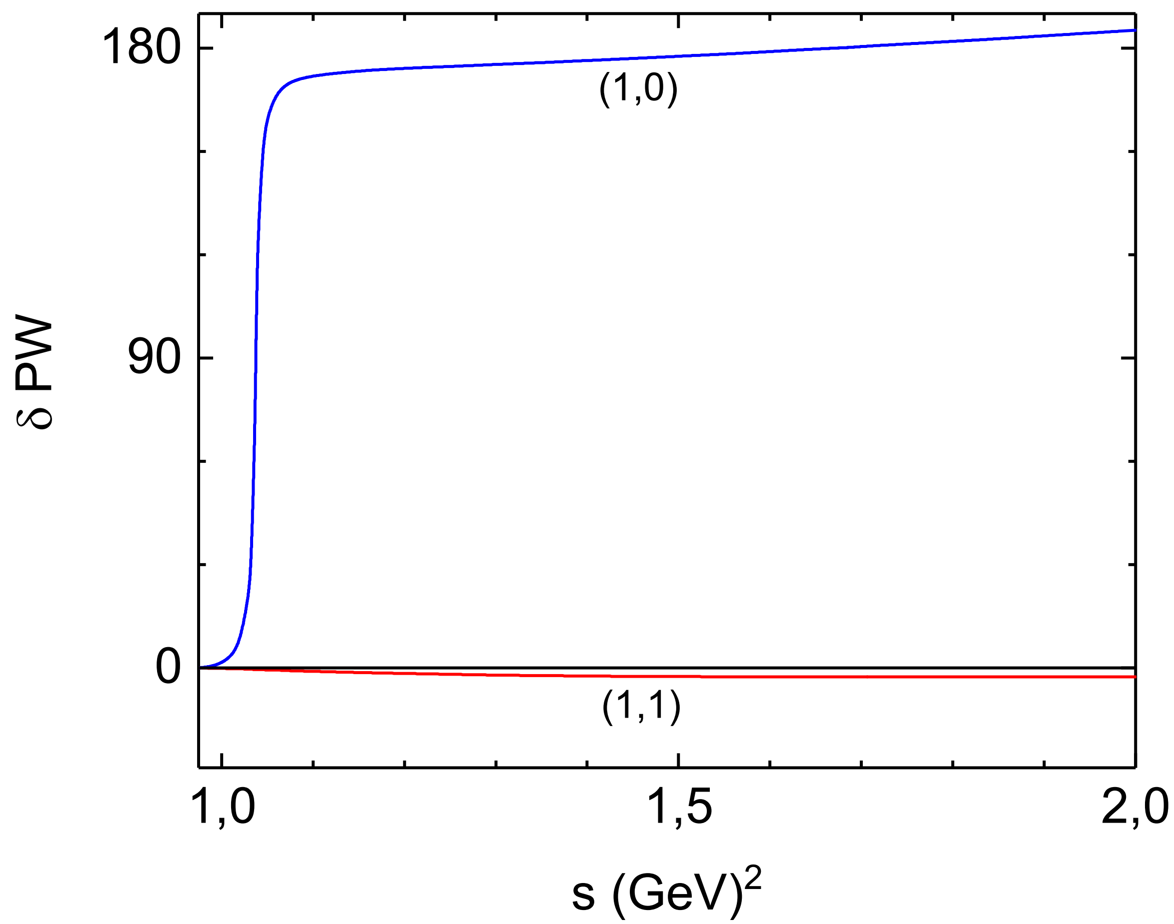


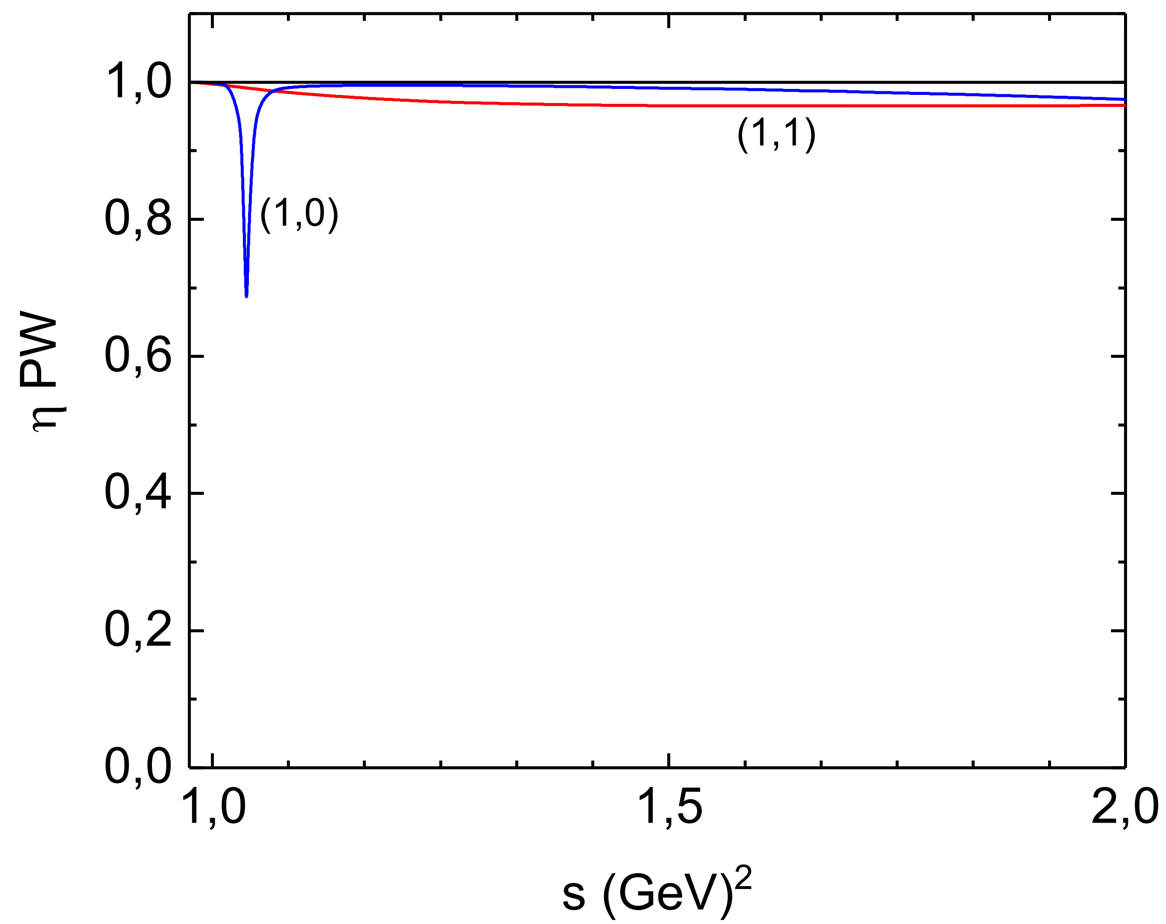


- ρ and NR contributions are tiny in TM $\longrightarrow$ small in KK
- ϕ phases are $\neq$ for $s > 1.5$

Toy results P-wave

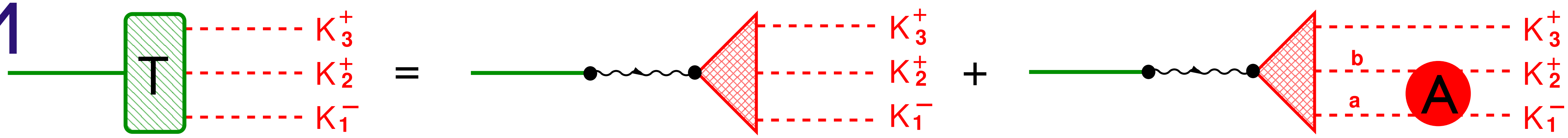
- Triple-M predictions for A
  - phase
  - inelasticity



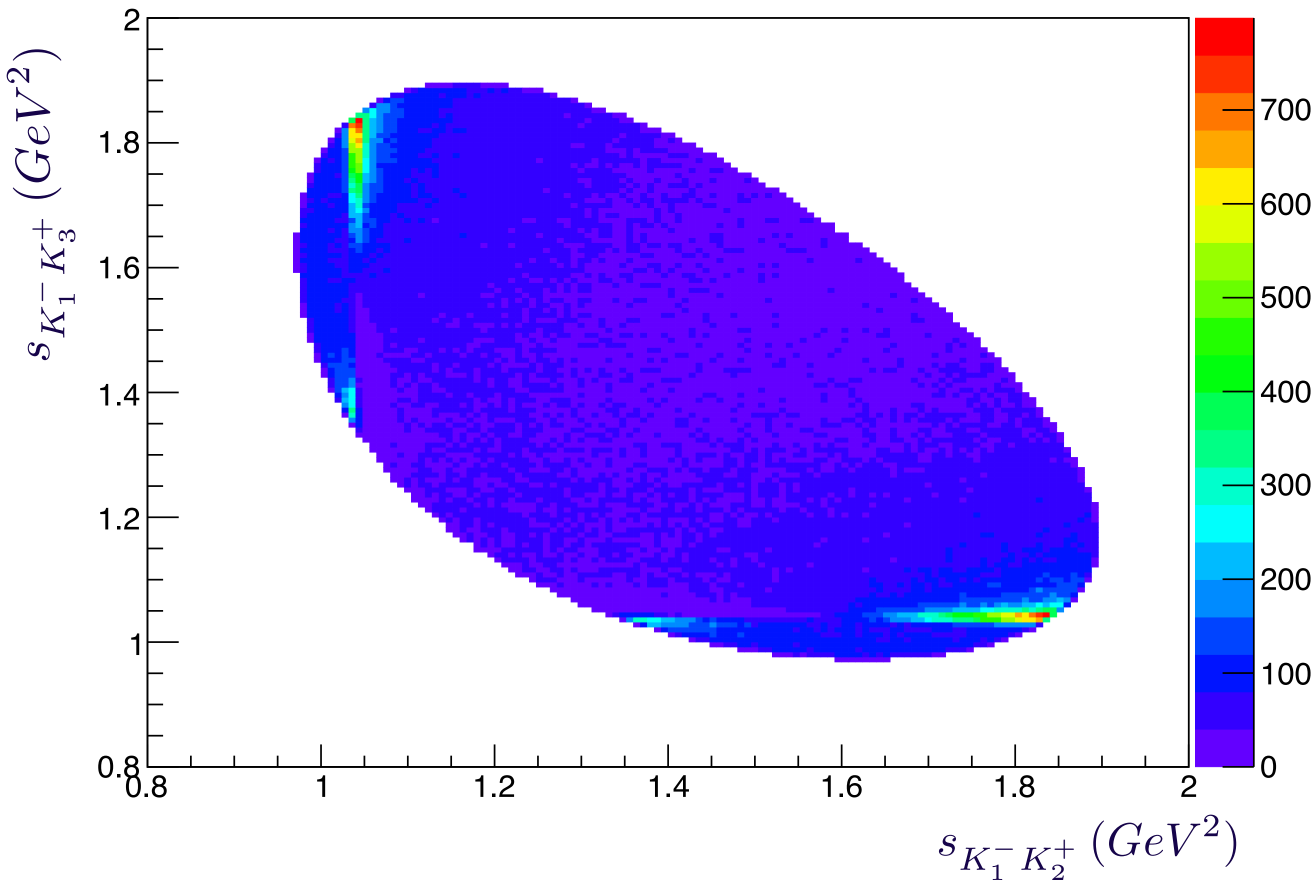


- ϕ is the dominant channel
- $\phi \to \rho\pi$ inelasticity ⟶ 15% of the life-time
- $\rho \to \pi\pi$ ⟶ constant inelasticity

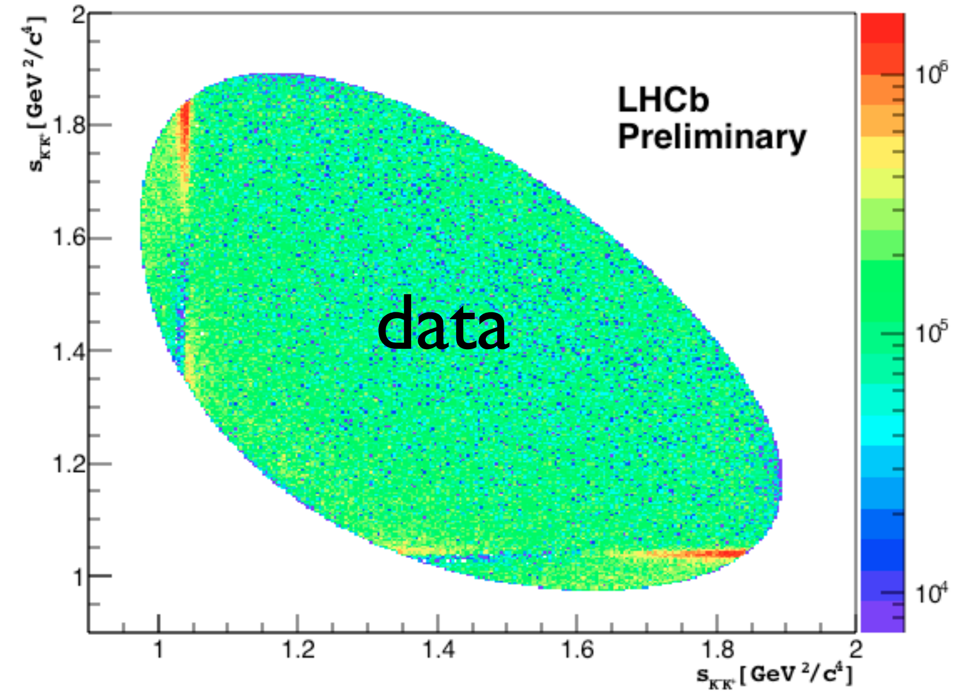
- Triple-M



Triple-M Toy Dalitz plot



$D^+ \to K^- K^+ K^-$



→ Fit data with Triple-M

→ powerful tool to extract KK scattering S-wave