Spectroscopy on the lattice

Raúl Briceño

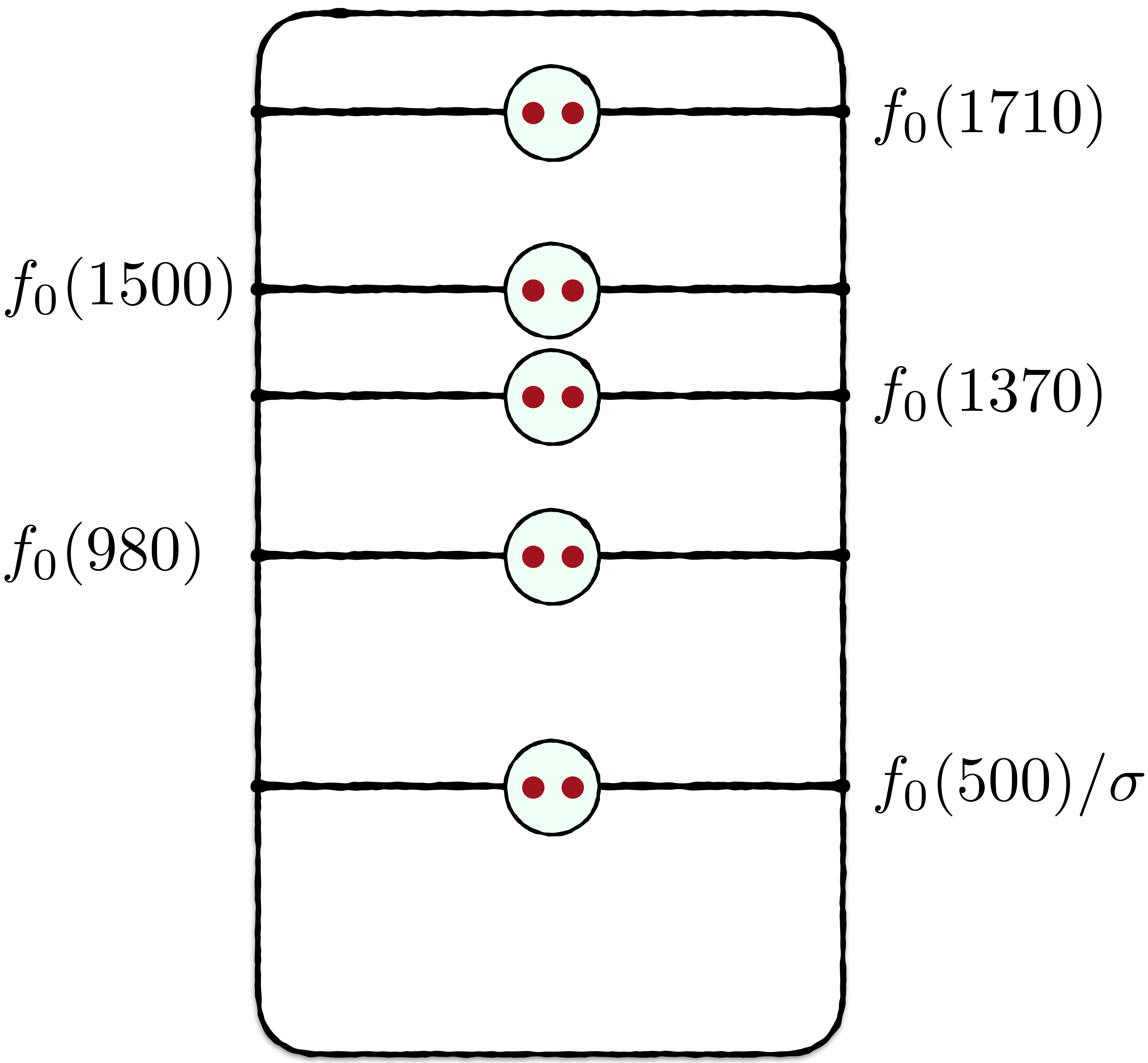
Jefferson Lab

OLD DOMINION UNIVERSITY

hadspec

PWA, 2018

The isoscalar, scalar sector



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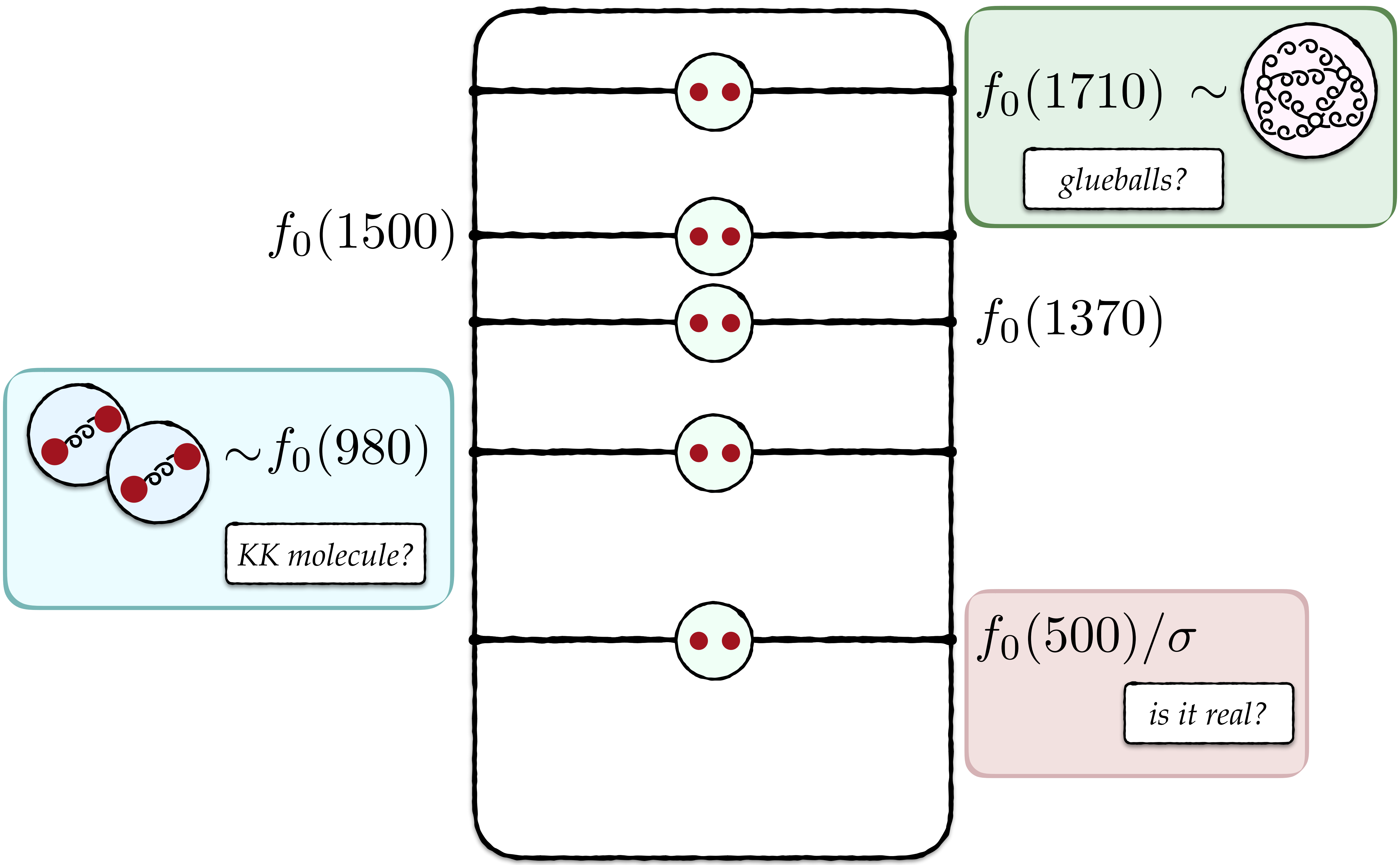

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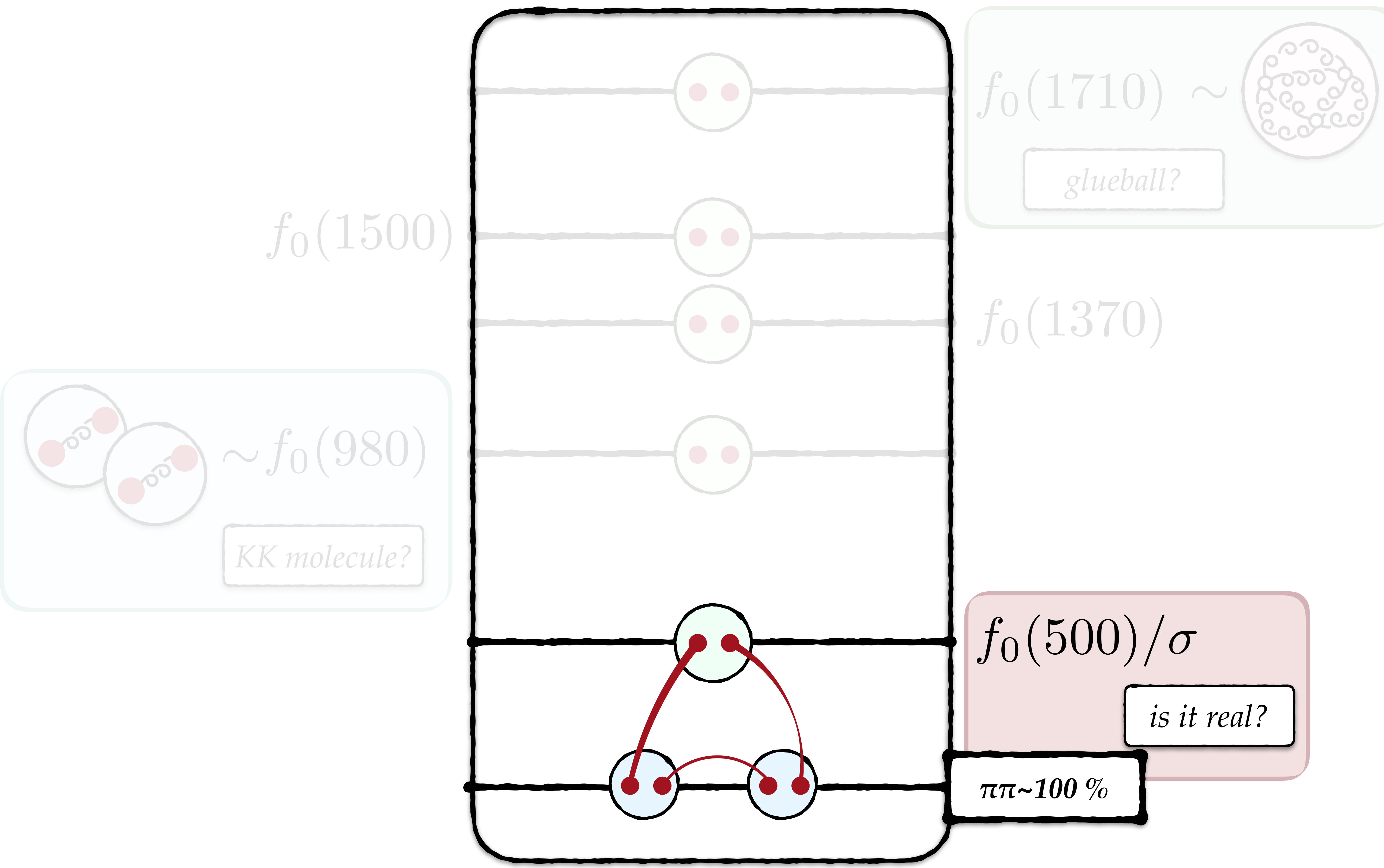

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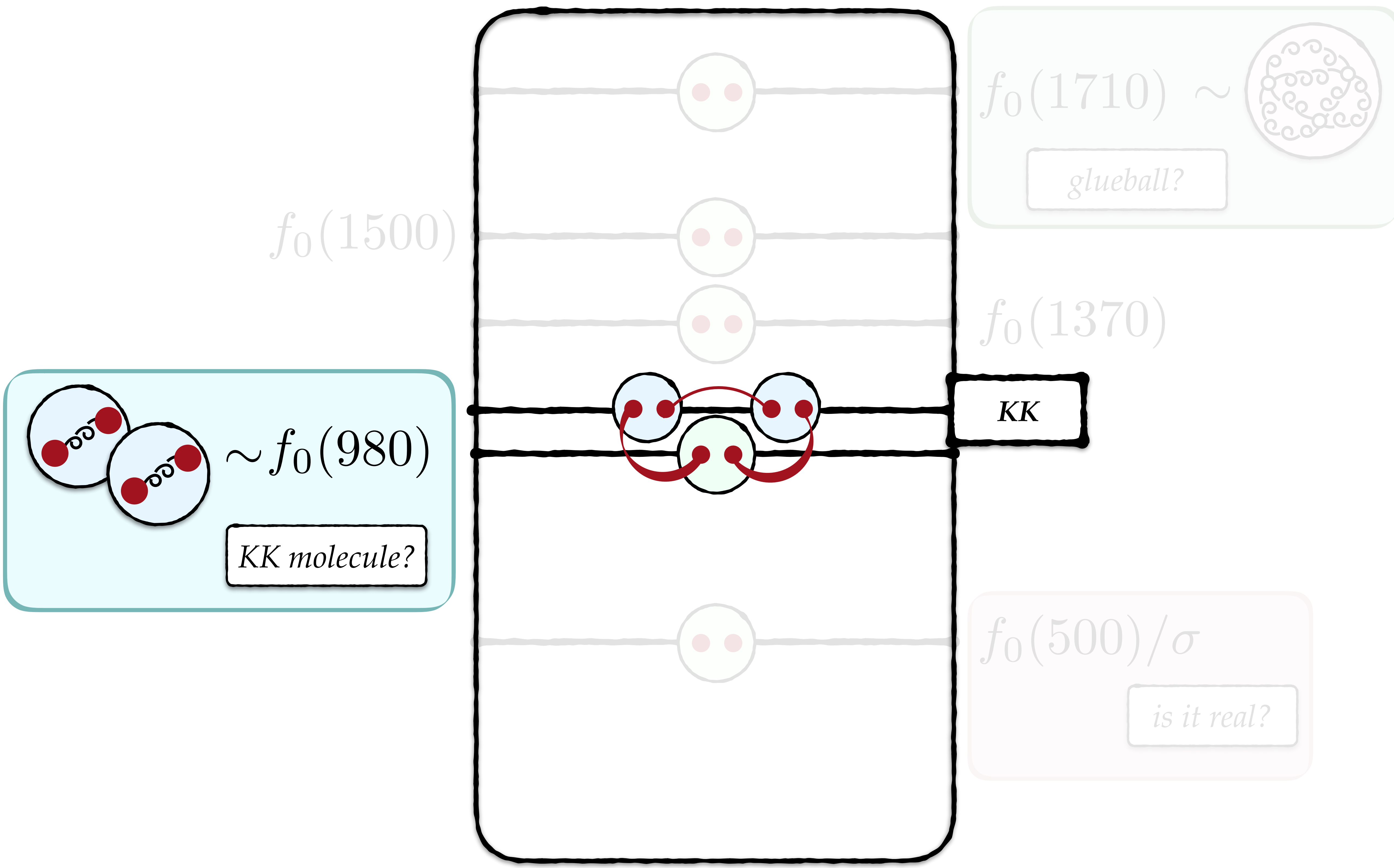

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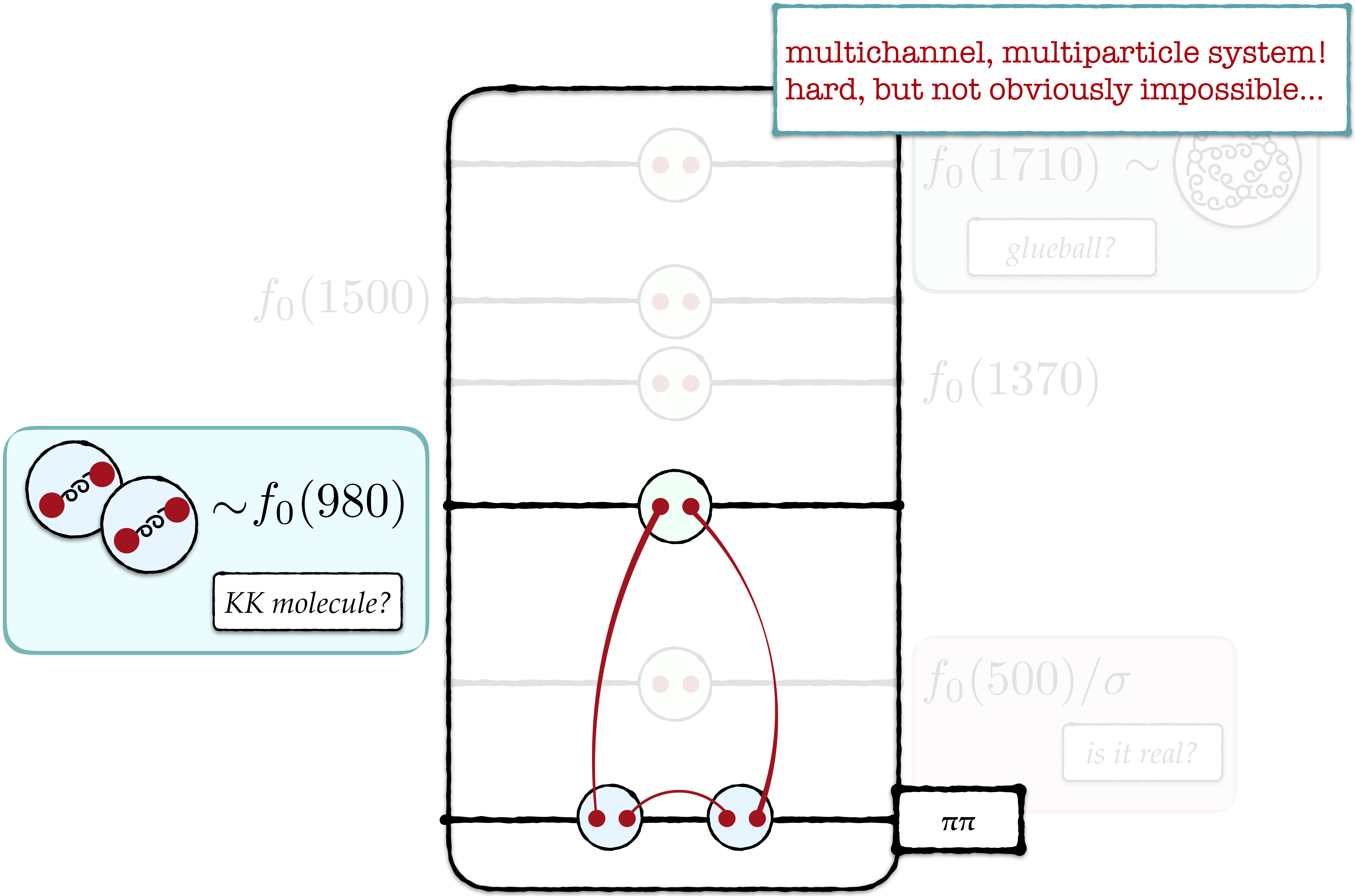


QCD spectroscopy

Amplitude analysis

Experiments

QCD

QCD spectroscopy

Amplitude analysis

Experiments

QCD

GOAL:

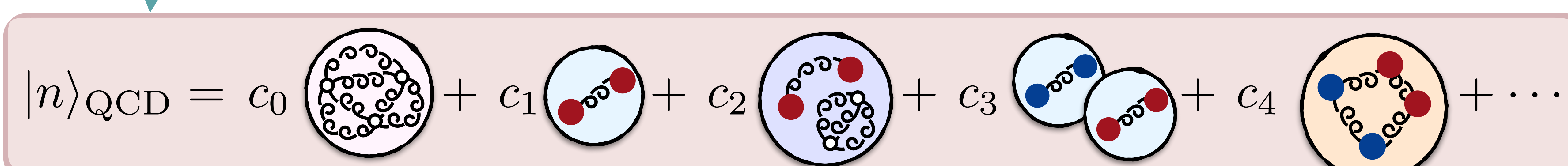
Get insights to the governing patterns and rules of QCD from emergent phenomena

Observables to test our understanding:

- Production and decay
- Exotic states
- …

Possible outcomes:

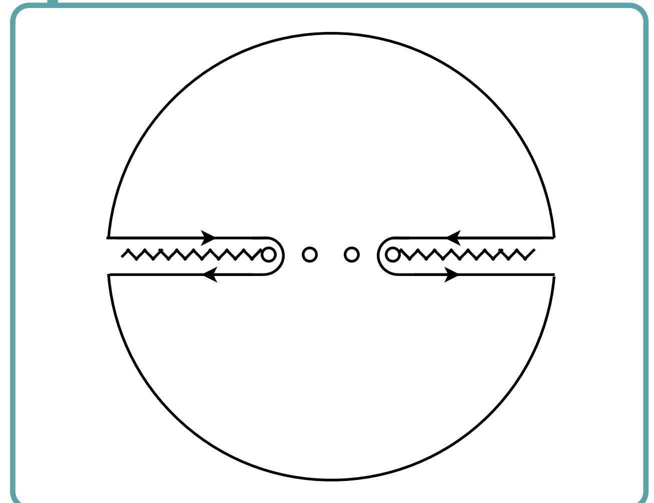
- Source of masses
- Role of glue
- Structure of excited states;
- …



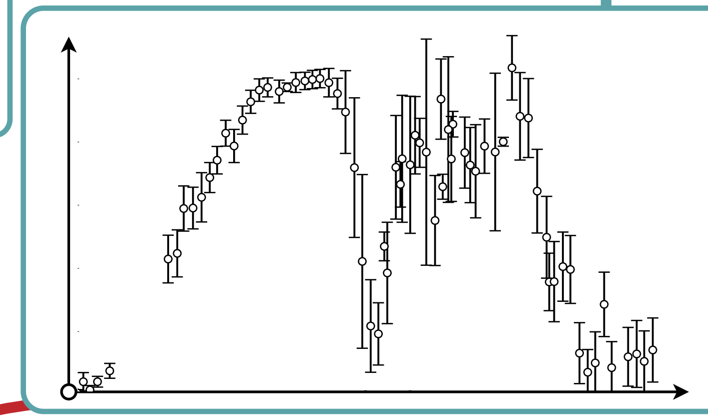
… perhaps there is a hierarchy [e.g. $c_0 > c_1 > c_2 > c_3 > c_4$]

QCD spectroscopy

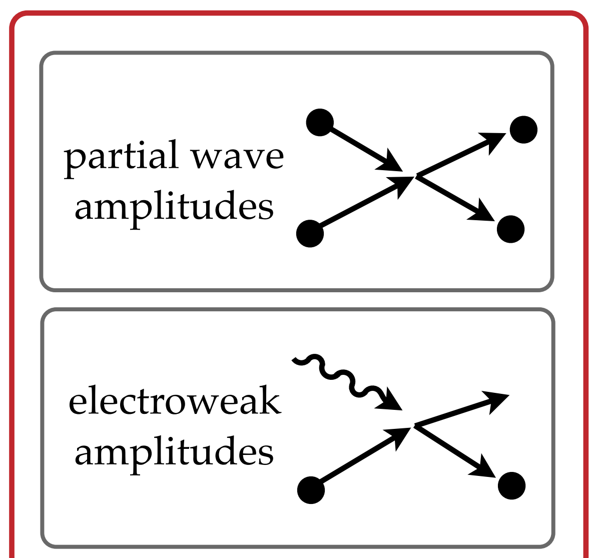
Amplitude analysis



Experiments



QCD



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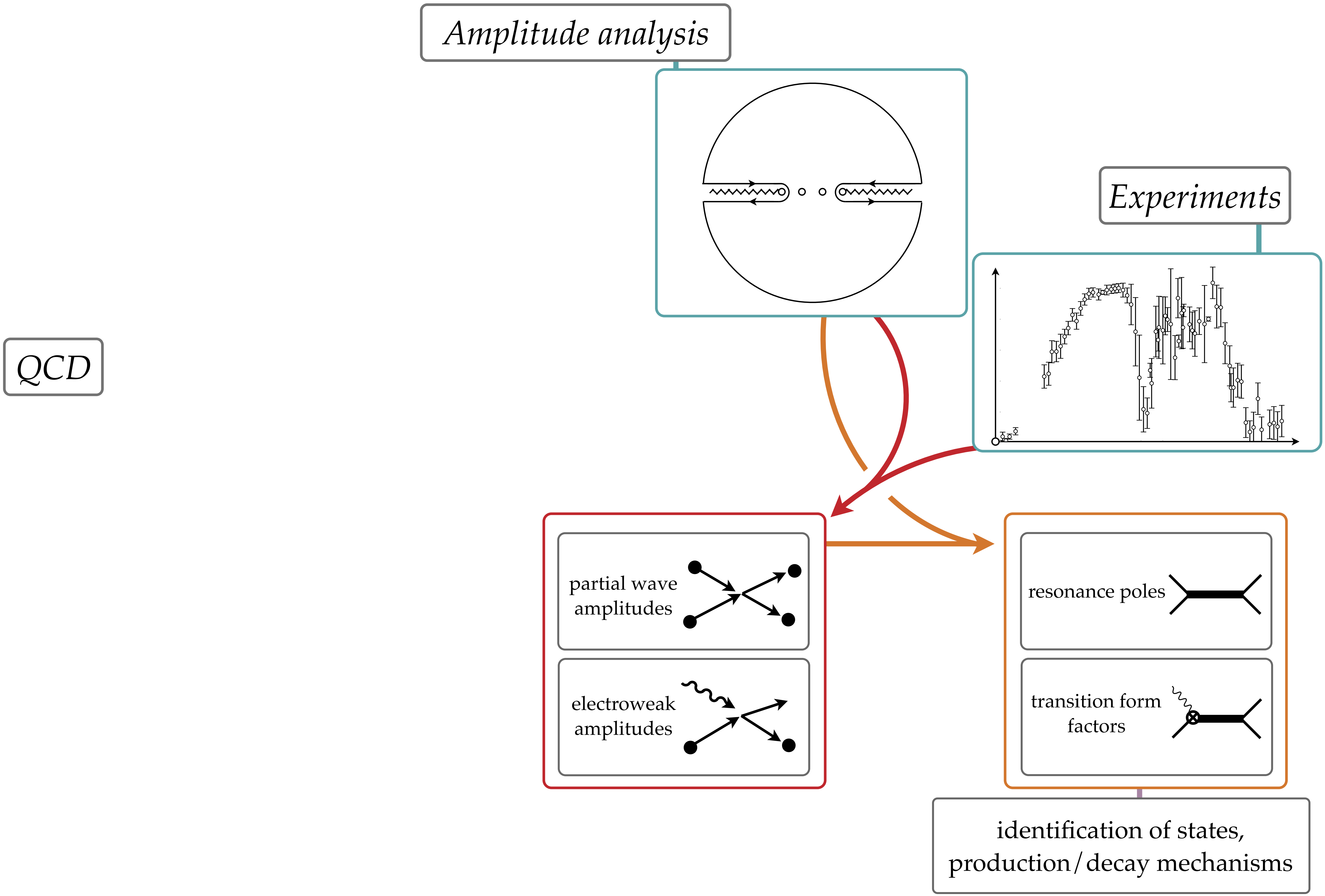

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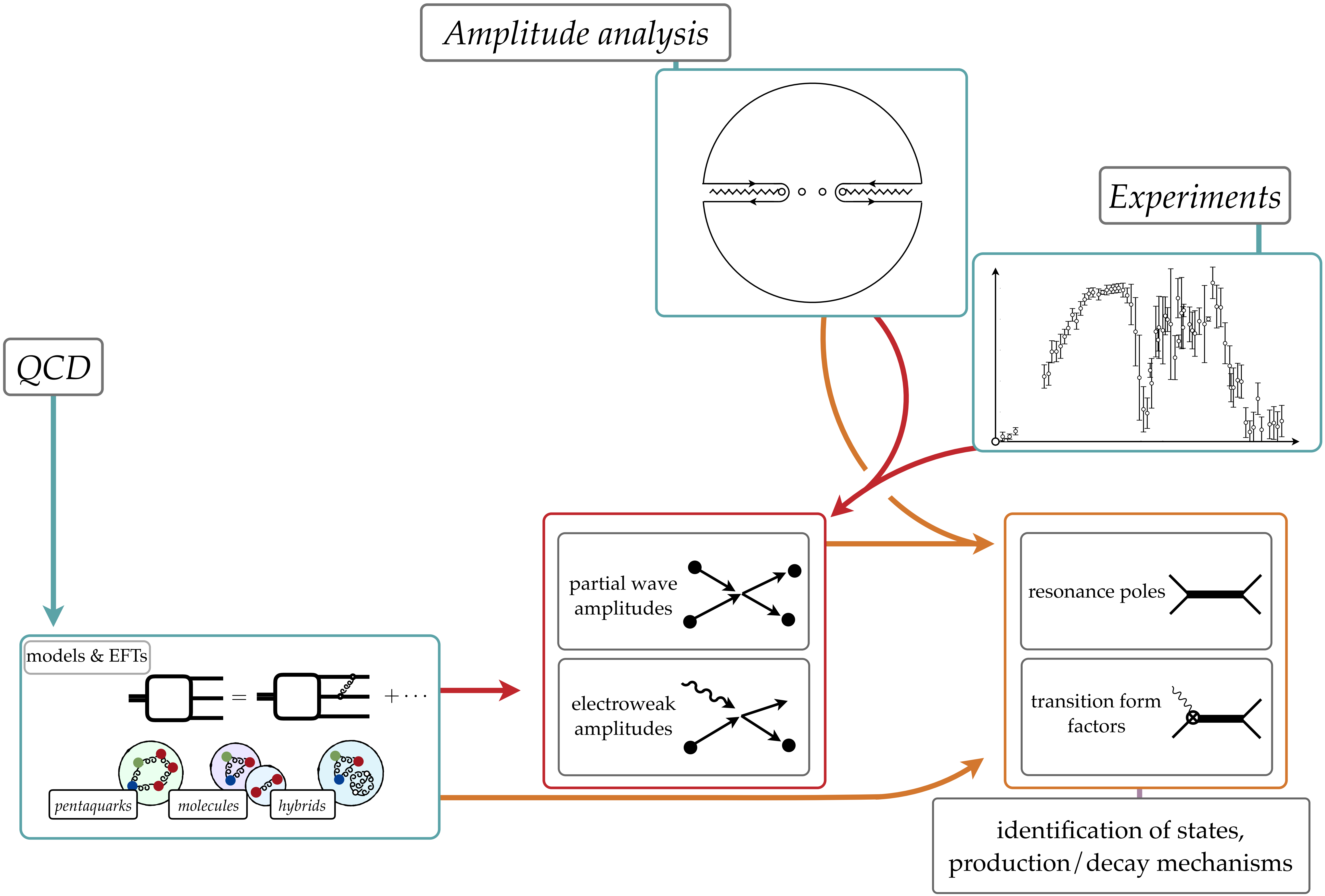

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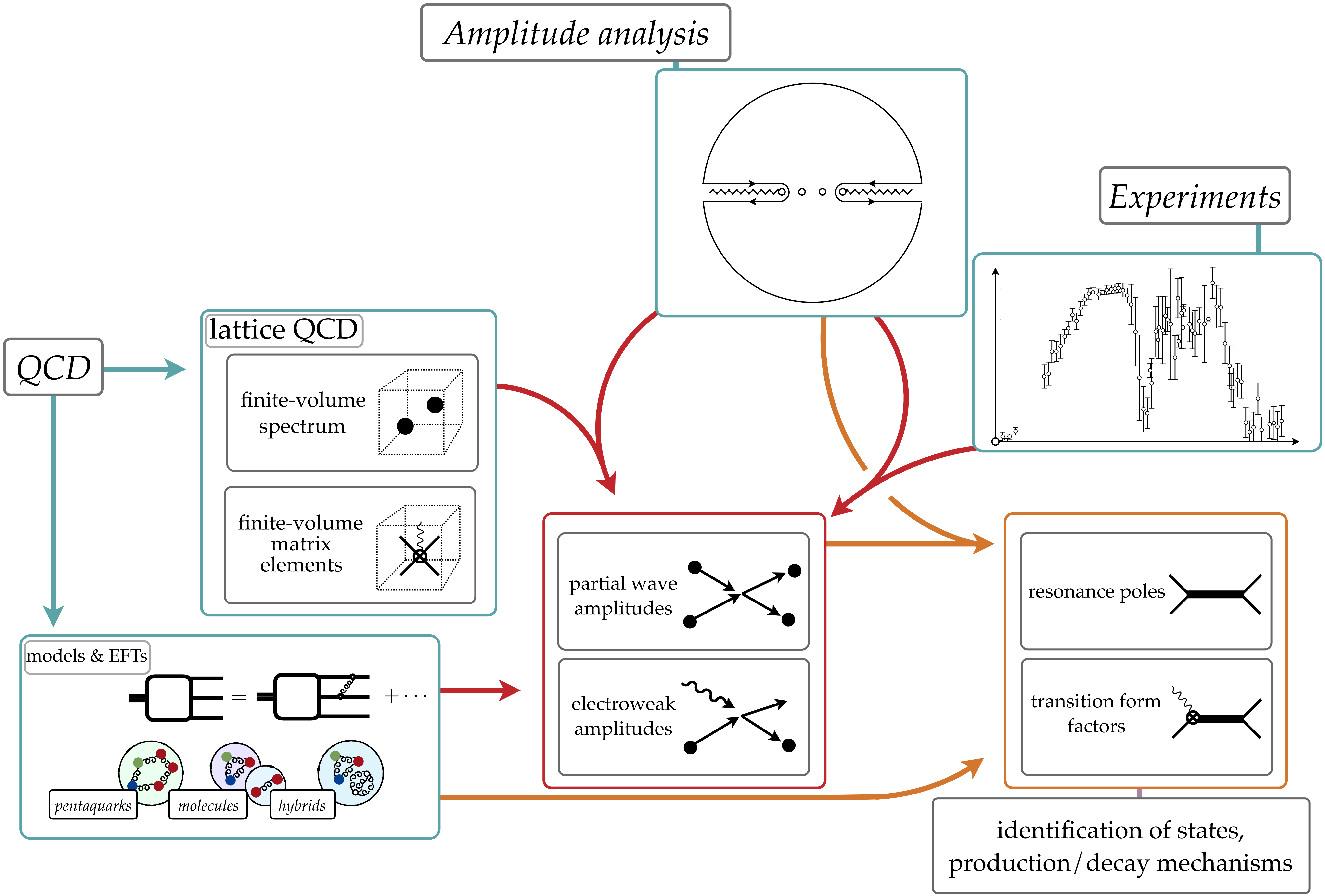
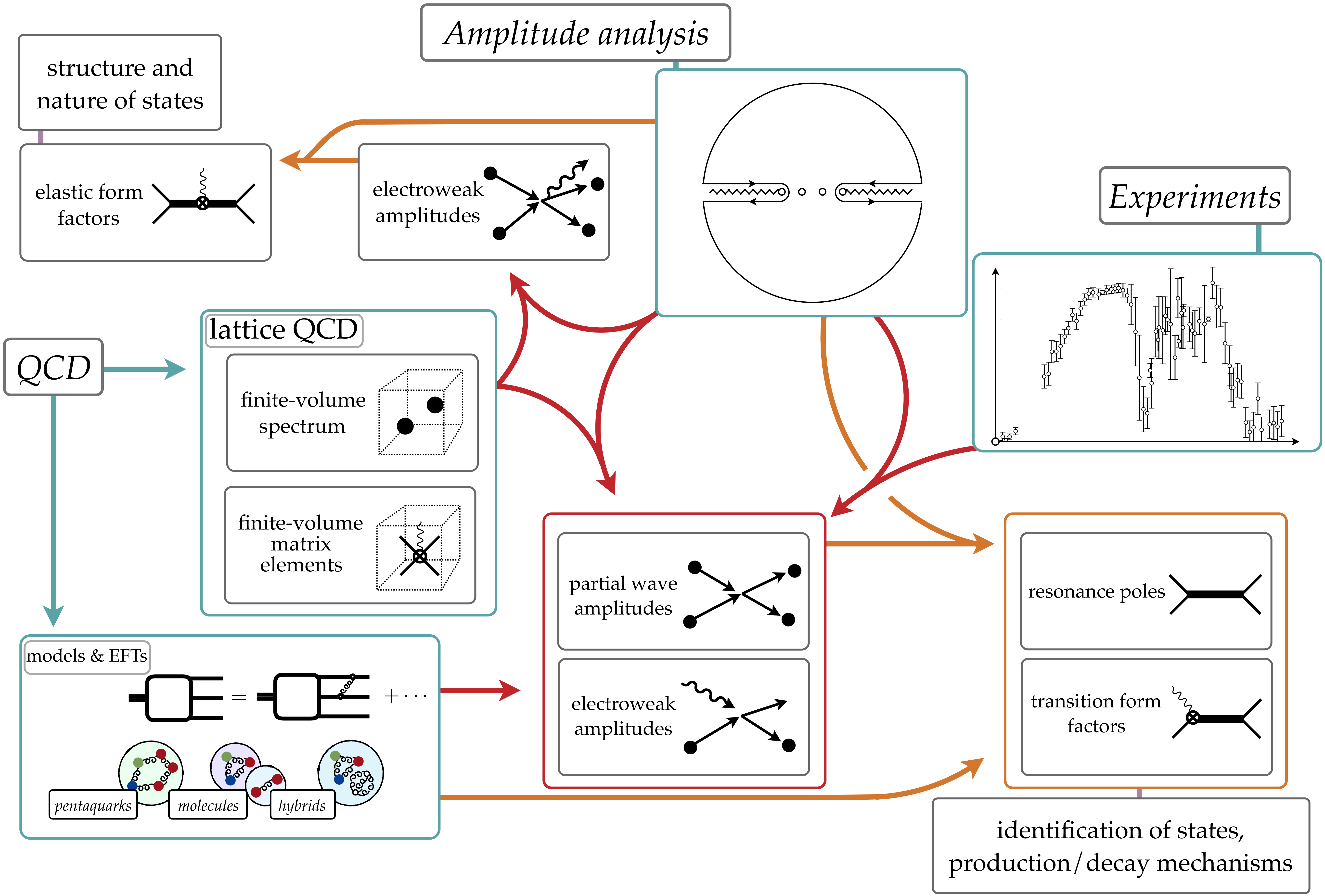


QCD spectroscopy



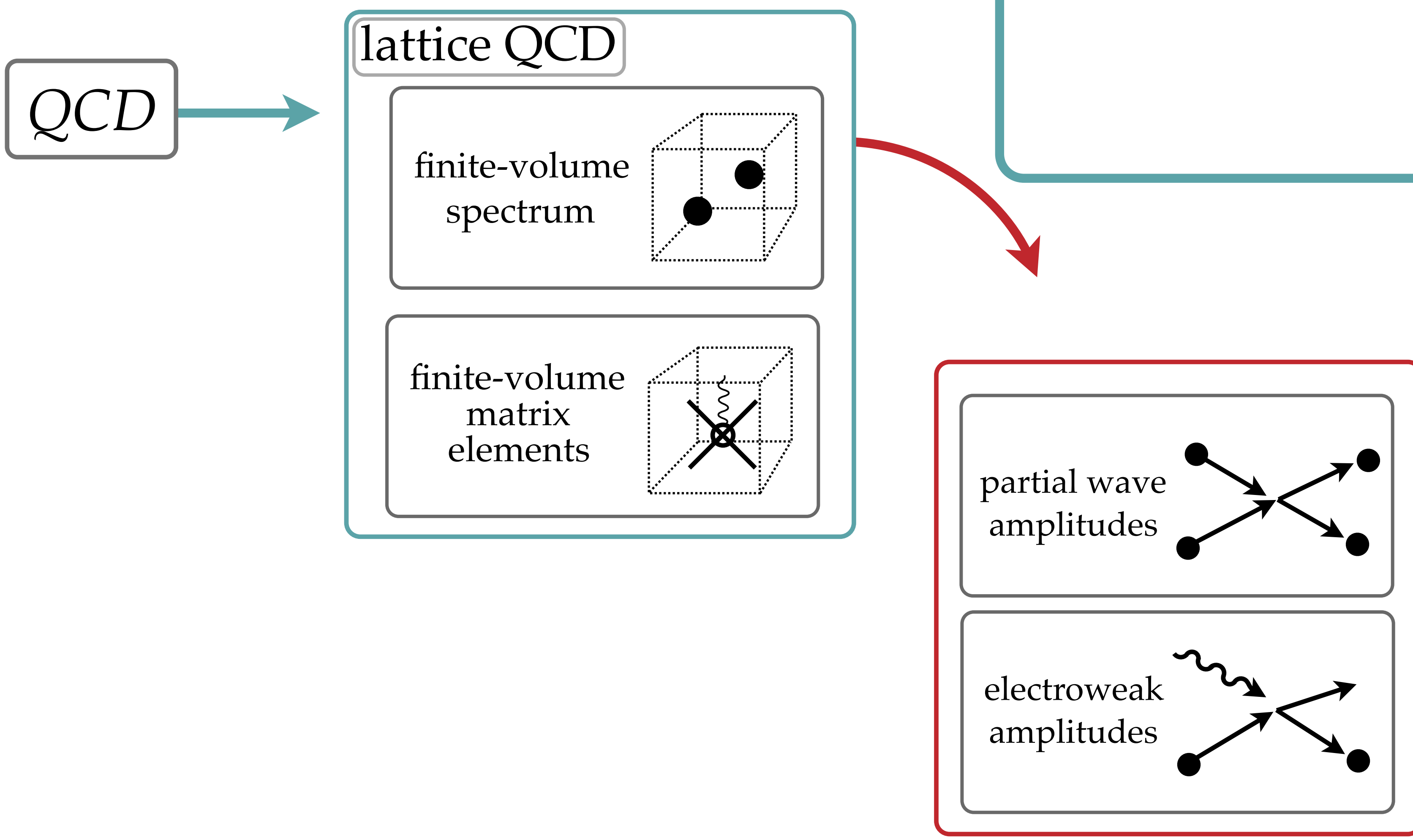
QCD spectroscopy

Part 1: Two-body physics

- formalism
- some results

Part 2: Three-body physics

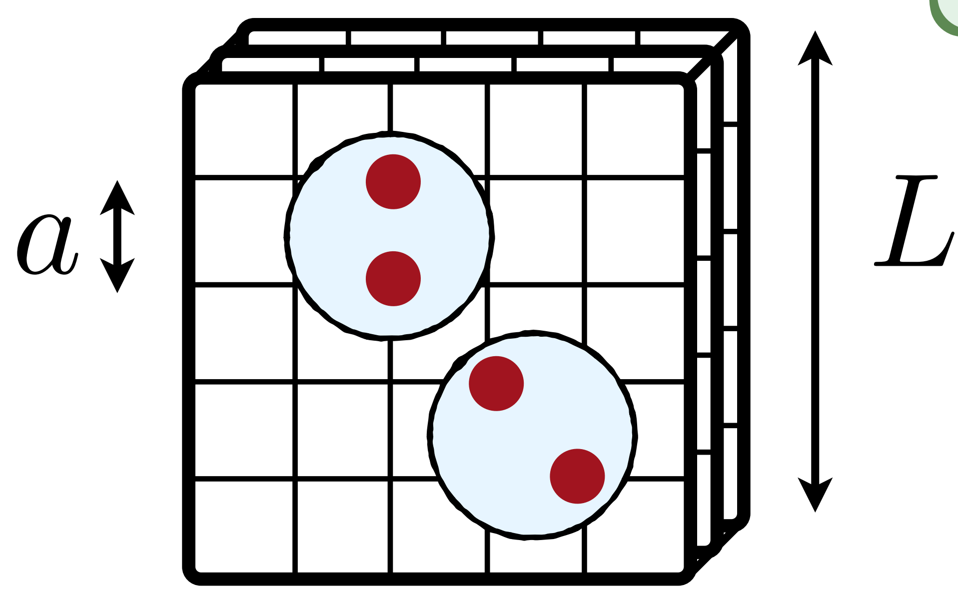
- Maxim Mai



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- quark masses: $m_q \to m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume

$$D_\mu = \Big(\qquad \Big) \updownarrow (L/a)^3 \times (T/a)$$



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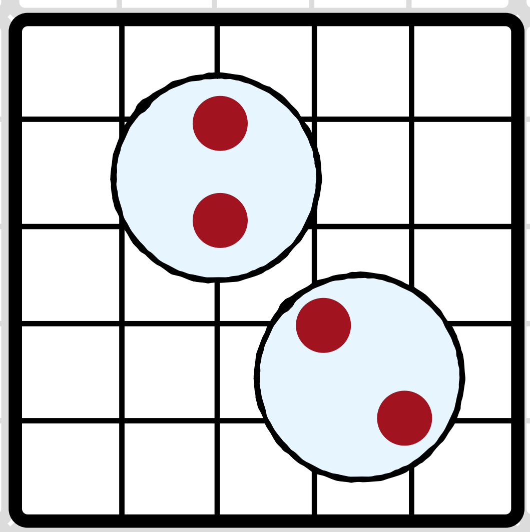

Never free!
No asymptotic states!
No scattering!

Lattice QCD

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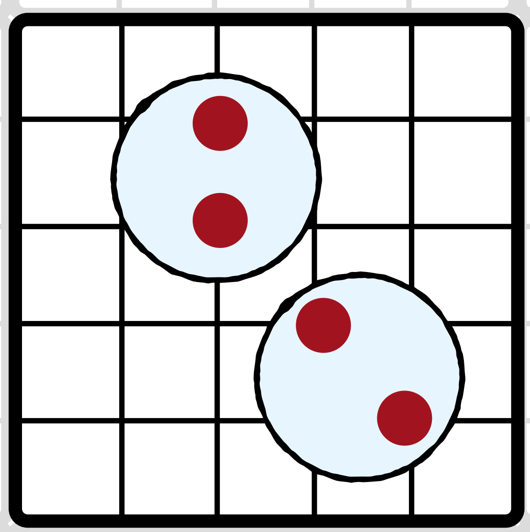
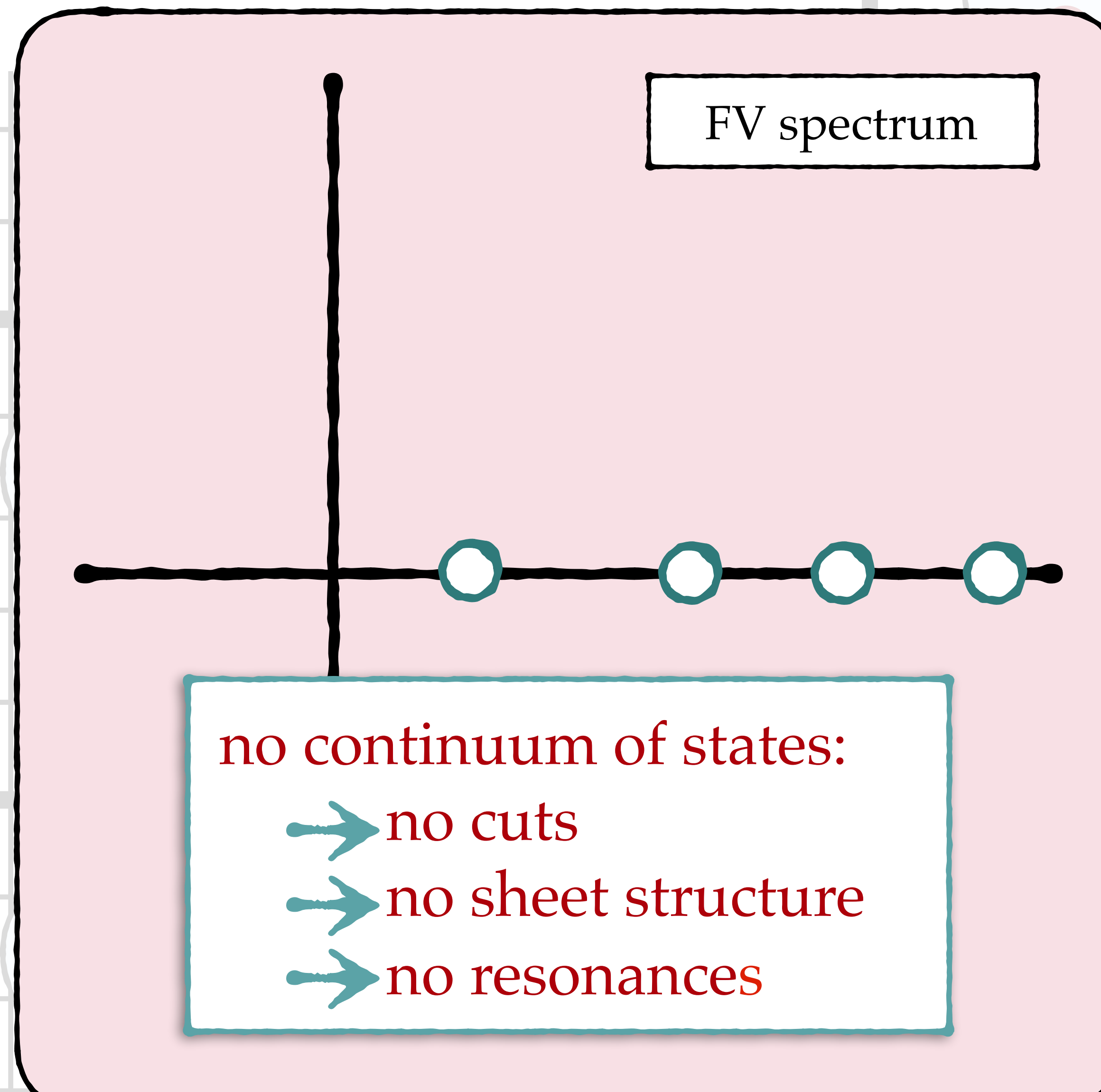
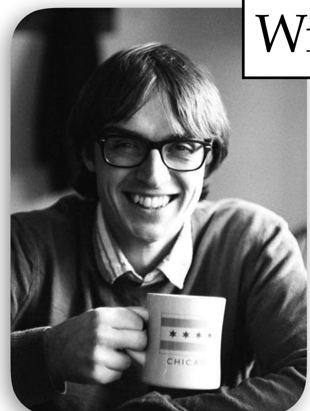




The isoscalar, scalar and tensor sectors



Wilson (Royal fellow / Trinity)

Dudek (W&M / JLab)

Edwards (JLab)

PRL **118,** 022002 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

(for the Hadron Spectrum Collaboration)

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA*
[3] *Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences,*

JLAB-THY-17-2534

Isoscalar $\pi\pi, K\overline{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

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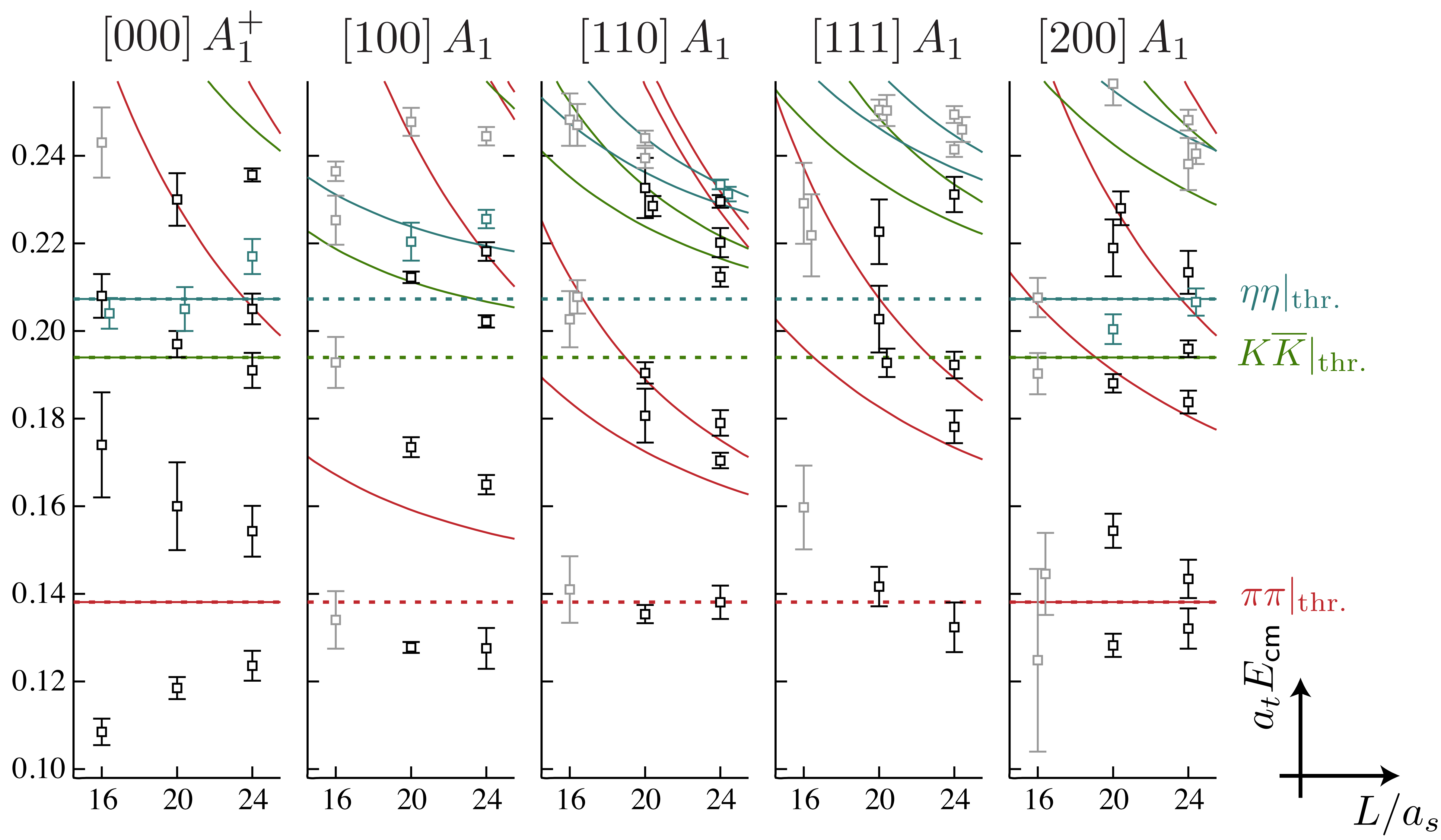
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[2] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[3] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
[4] *School of Mathematics, Trinity College, Dublin 2, Ireland*
(Dated: August 23, 2017)

We present the first lattice QCD study of coupled isoscalar $\pi\pi, K\overline{K}, \eta\eta$ S- and D-wave scattering extracted from discrete finite-volume spectra computed on lattices which have a value of the quark mass corresponding to $m_\pi \sim 391$ MeV. In the $J^P = 0^+$ sector we find analogues of the experimental σ and $f_0(980)$ states, where the σ appears as a stable bound-state below $\pi\pi$ threshold, and, similar to what is seen in experiment, the $f_0(980)$ manifests itself as a dip in the $\pi\pi$ cross section in the vicinity of the $K\overline{K}$ threshold. For $J^P = 2^+$ we find two states resembling the $f_2(1270)$ and $f_2'(1525)$, observed as narrow peaks, with the lighter state dominantly decaying to $\pi\pi$ and the heavier state to $K\overline{K}$. The presence of all these states is determined rigorously by finding the pole singularity content of scattering amplitudes, and their couplings to decay channels are established using the residues of

Isoscalar spectra: S-wave dominant

- Multi-meson ops. are crucial
- Spectrum including a larger basis: $\{\pi\pi, K\overline{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



m_π=391 MeV

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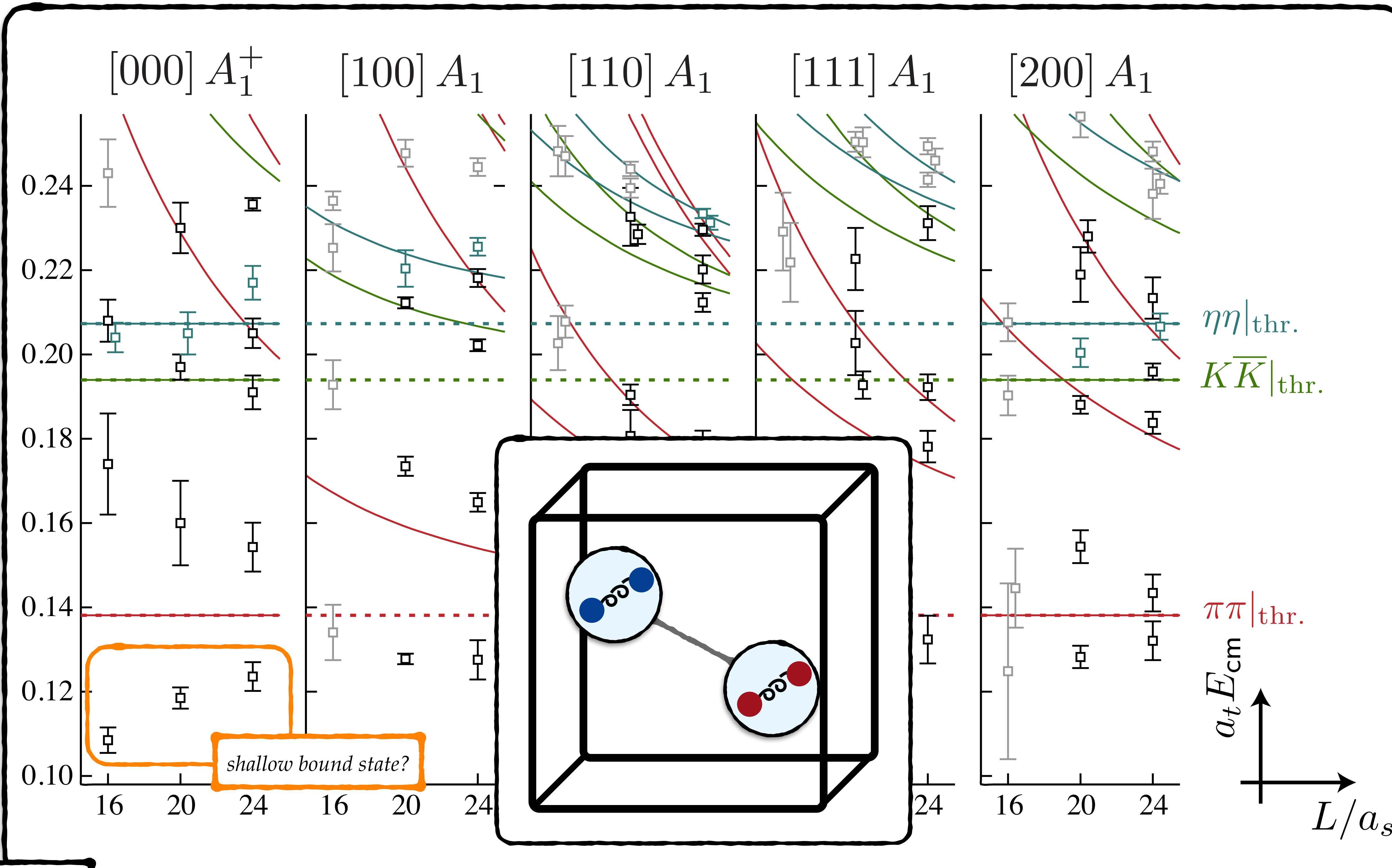

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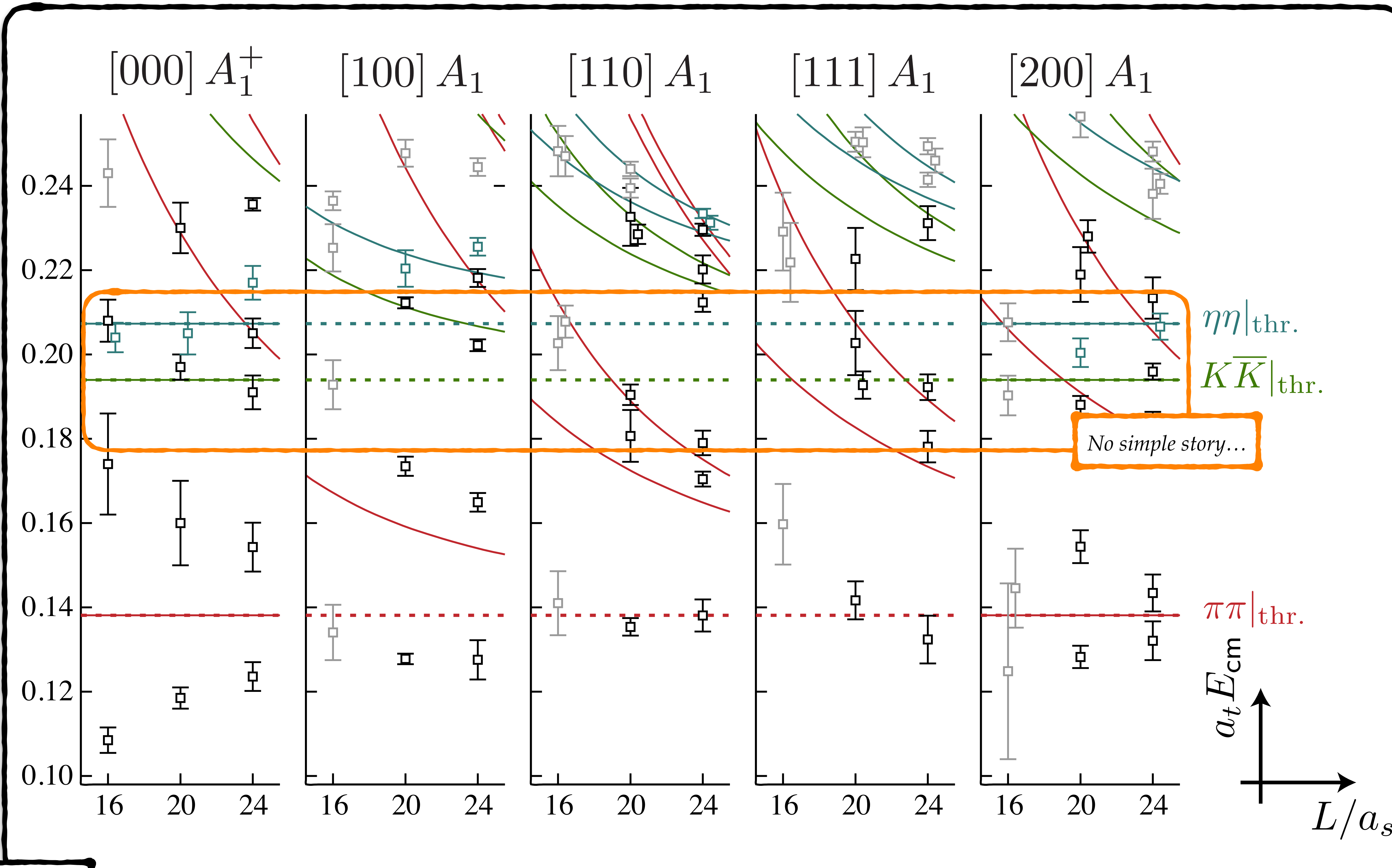

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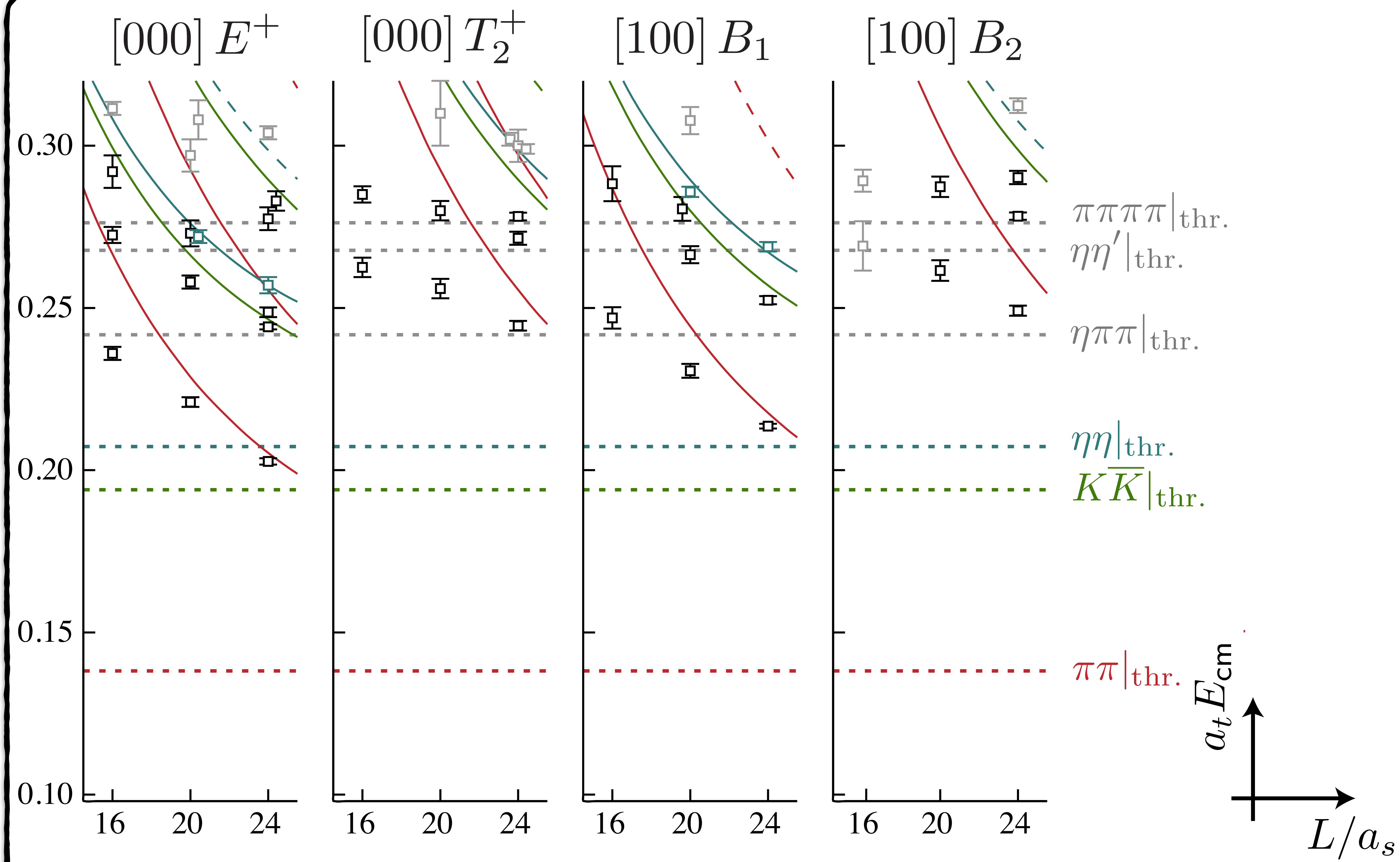

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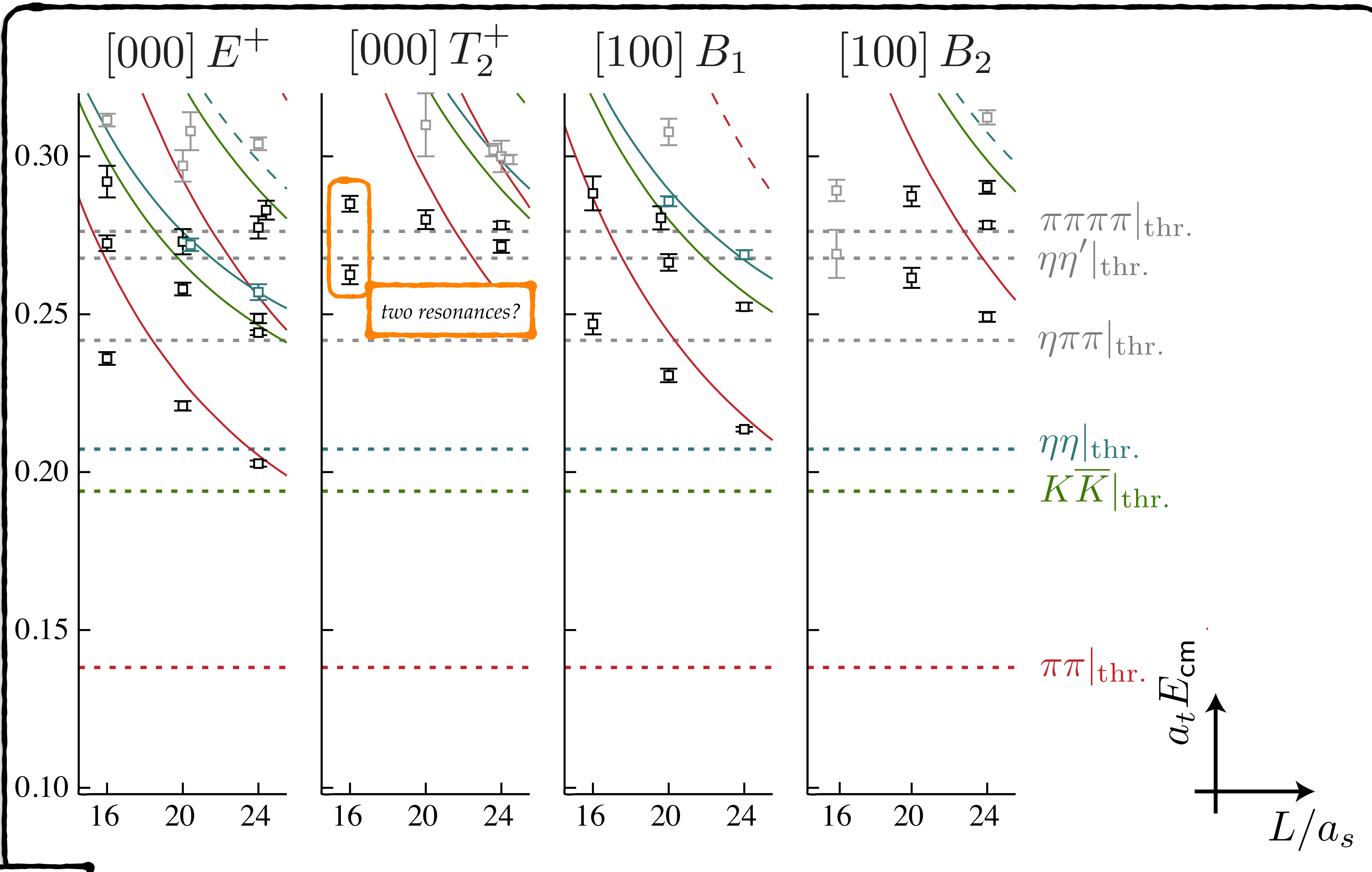
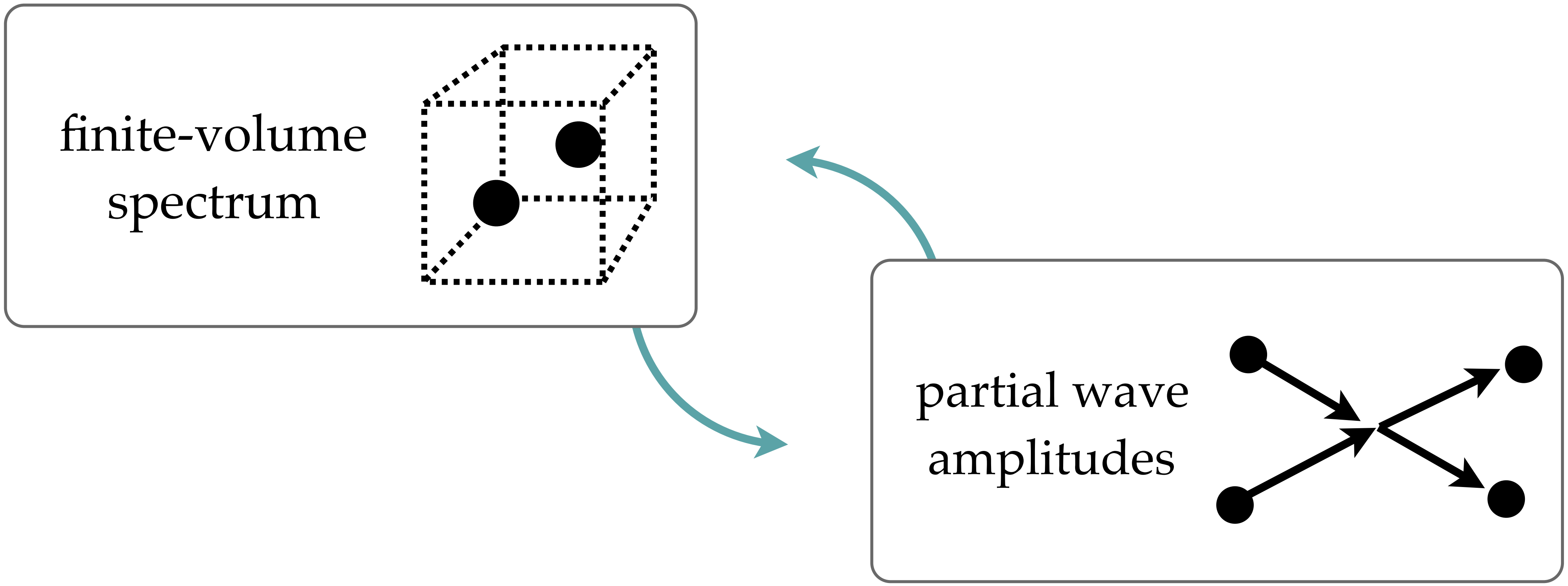


m_π=391 MeV

Finite volume physics

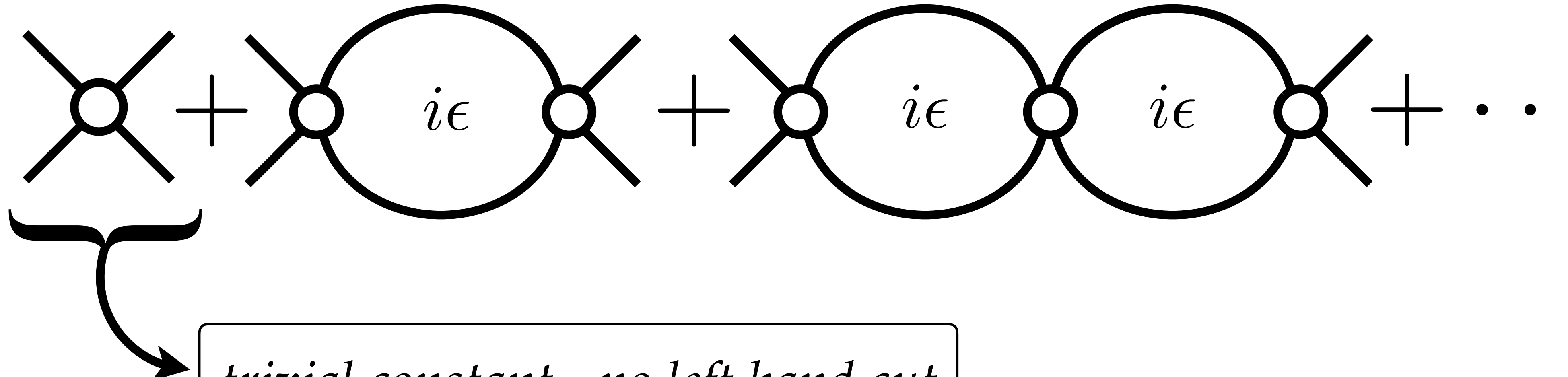


- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

Toy theory

Consider a two-body scattering [infinite volume]

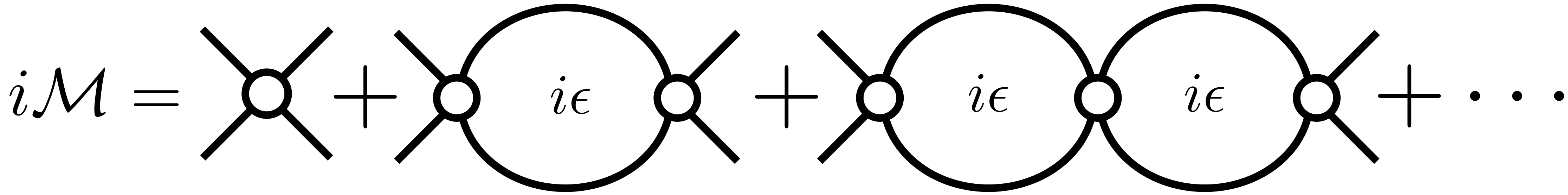
$$i\mathcal{M} =$$

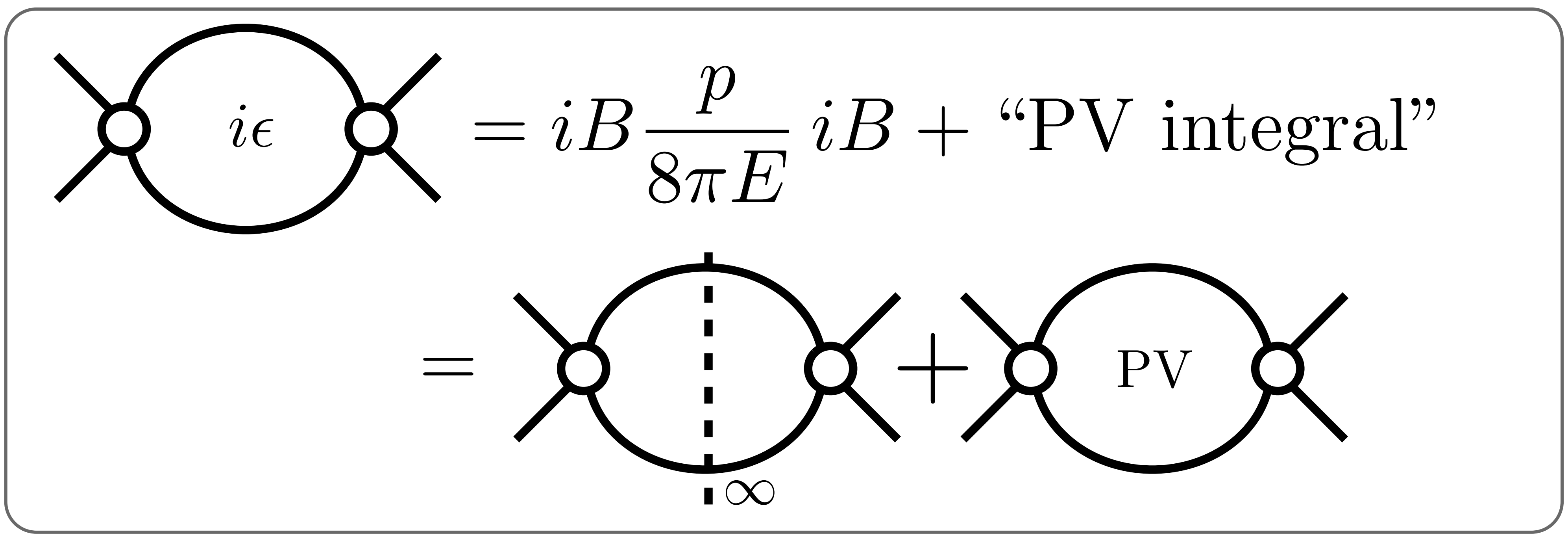


trivial constant - no left hand cut

Toy theory

Consider a two-body scattering [infinite volume]





$$p = \frac{1}{2}\sqrt{s - s_{th}}$$

Toy theory

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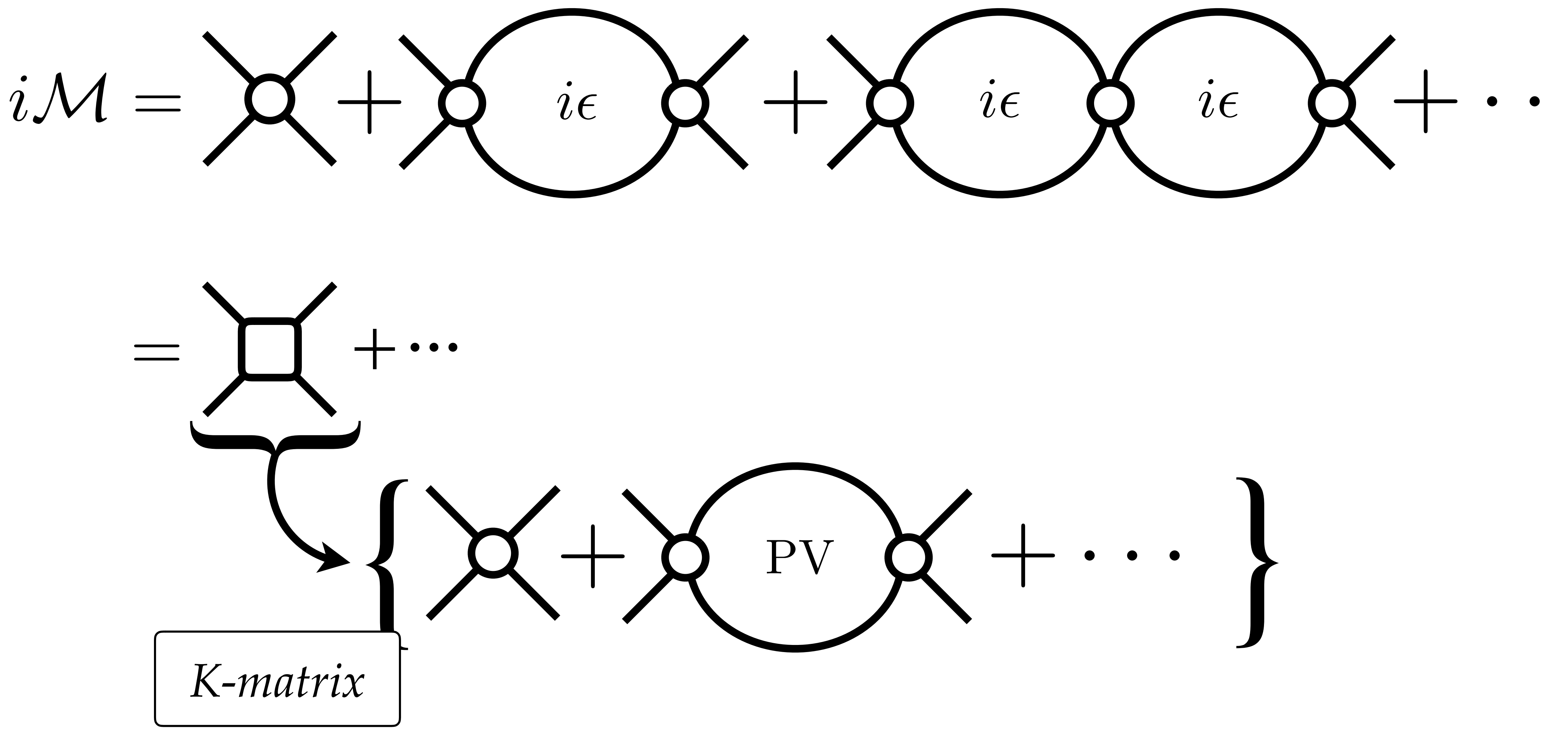

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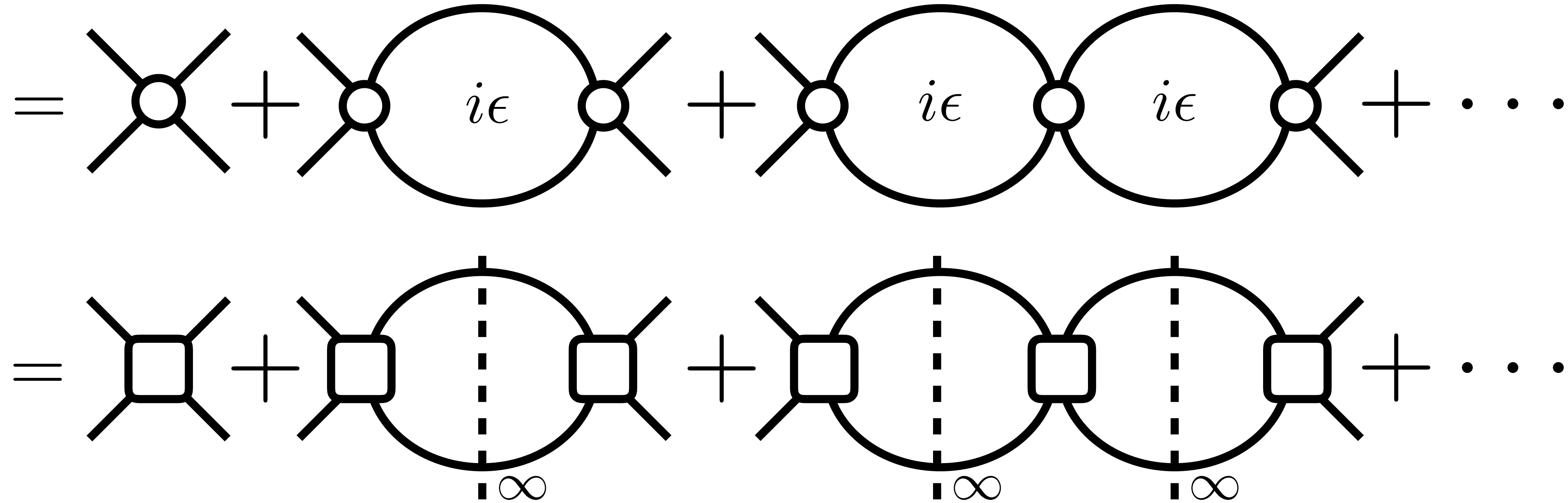


$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$

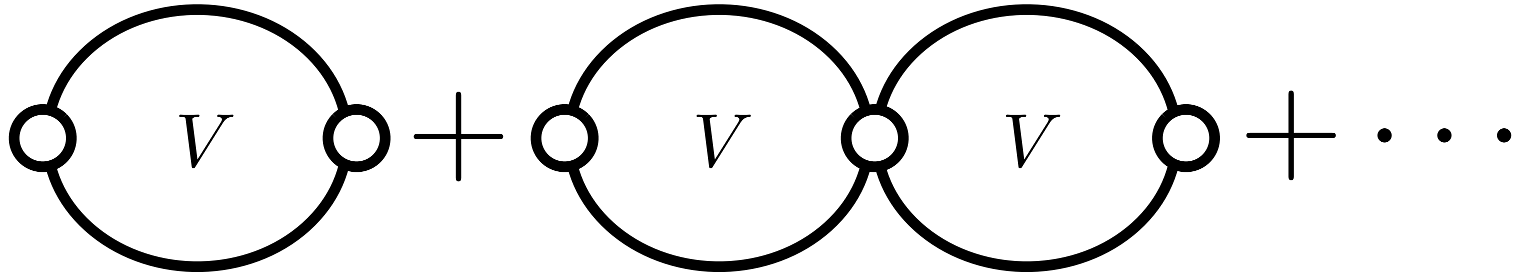
$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity. the iε assures that scattering takes place on the first sheet.

Toy theory in a finite-volume

Consider the finite-volume two-particle correlator ($E\sim 2m$).
Its poles coincide with the finite-volume spectrum

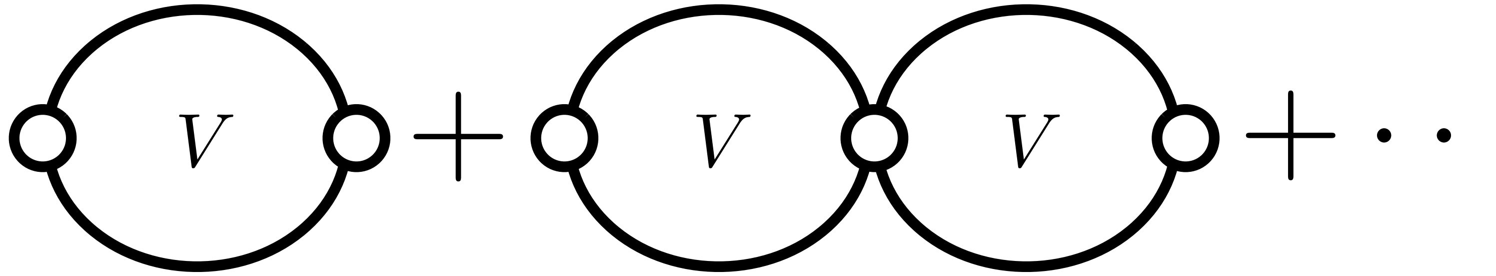
$C_L(P) =$

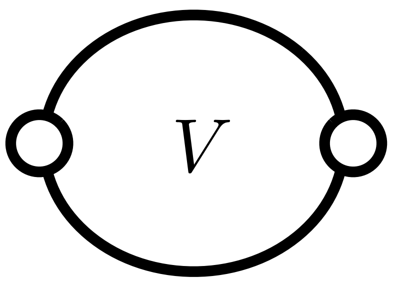


Toy theory in a finite-volume

Consider the finite-volume two-particle correlator (*E*~2*m*).
Its poles coincide with the finite-volume spectrum

$C_L(P) =$



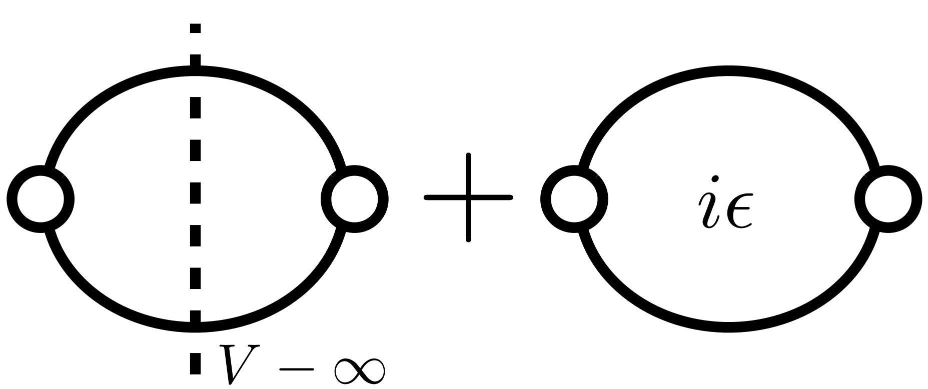


$$= \frac{1}{L^3}\sum_{\mathbf{k}} \frac{iB^2}{(2\omega_k)^2}\frac{i}{E-2\omega_k} + \text{“smooth”}$$

$$= (iB)\left(\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3}\right]\frac{1}{(2\omega_k)^2}\frac{i}{E-2\omega_k+i\epsilon}\right)(iB) + \text{“}i\epsilon\text{ integral”}$$

$$\equiv [iB]\, iF\, [iB] + \text{“}i\epsilon\text{ integral”}$$

$=$



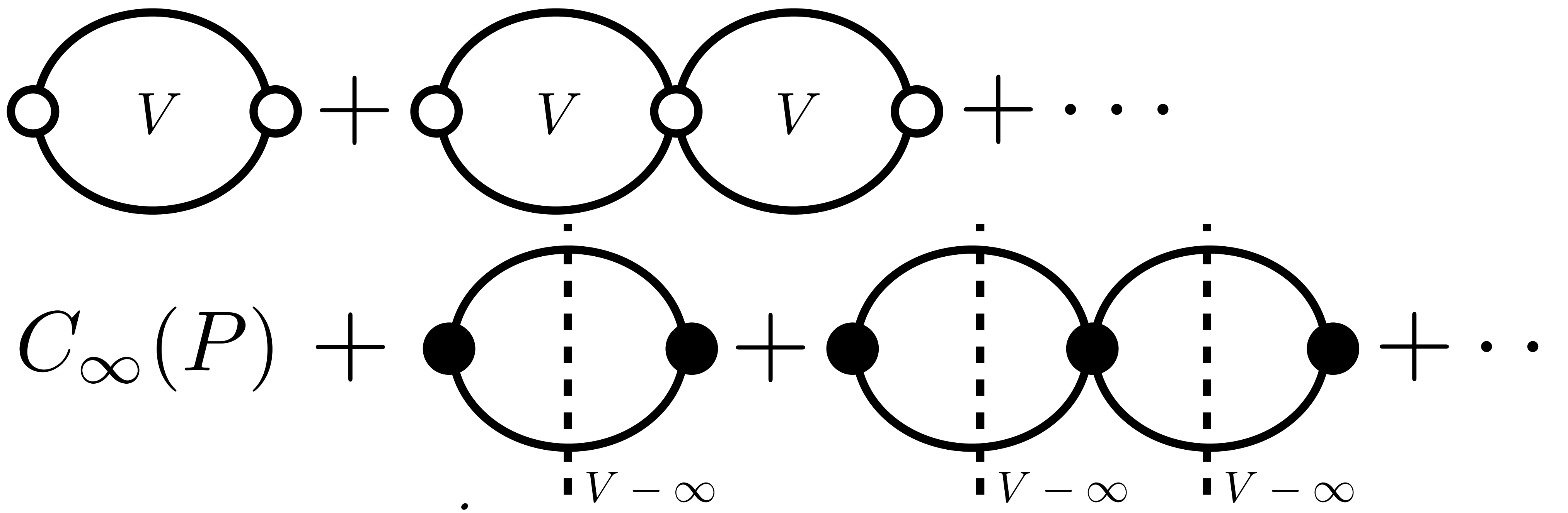
Toy theory in a finite-volume

Consider the finite-volume two-particle correlator ($E \sim 2m$).

Its poles coincide with the finite-volume spectrum

$C_L(P) =$

$= C_\infty(P) +$



$$= \text{“smooth”} + A \frac{i}{F^{-1} + \mathcal{M}} B^\dagger$$

poles satisfy: $\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
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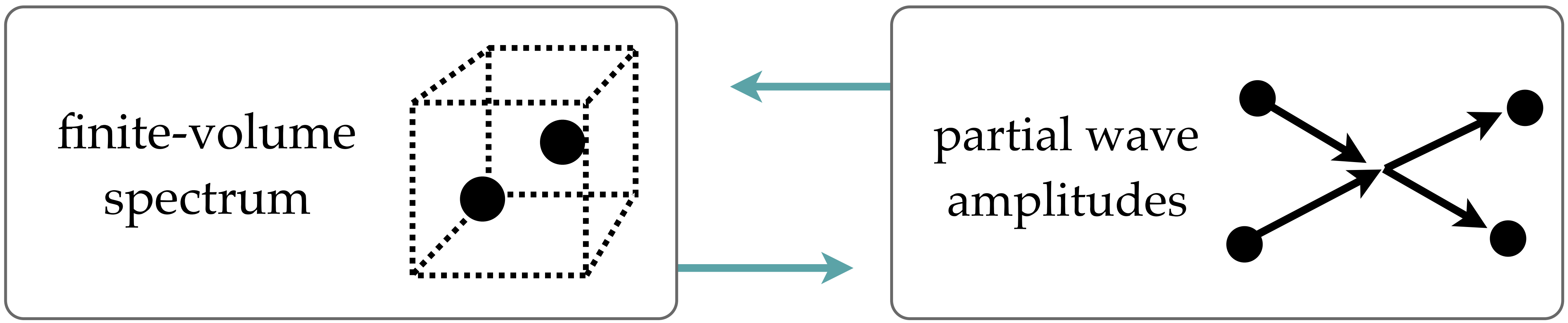
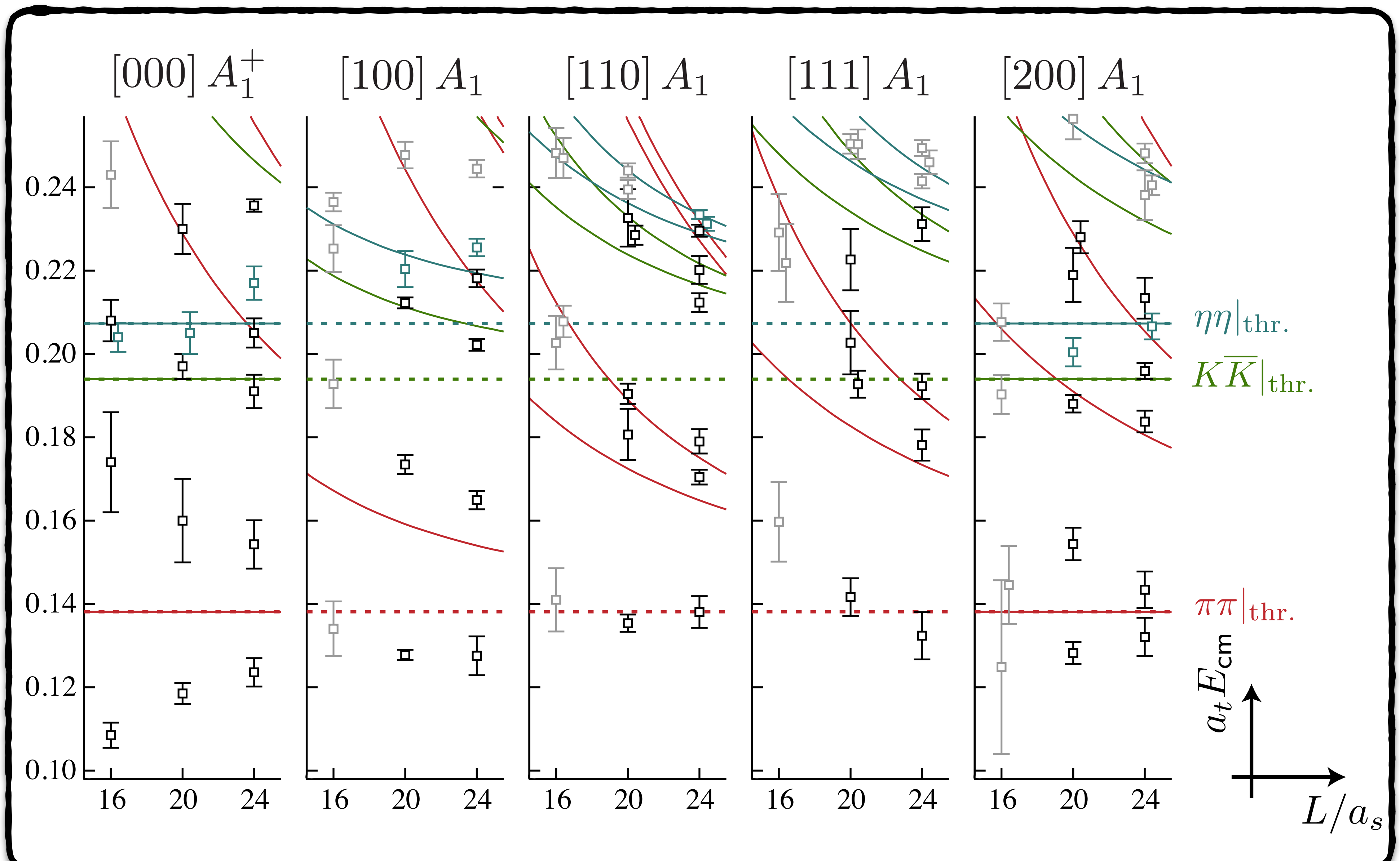
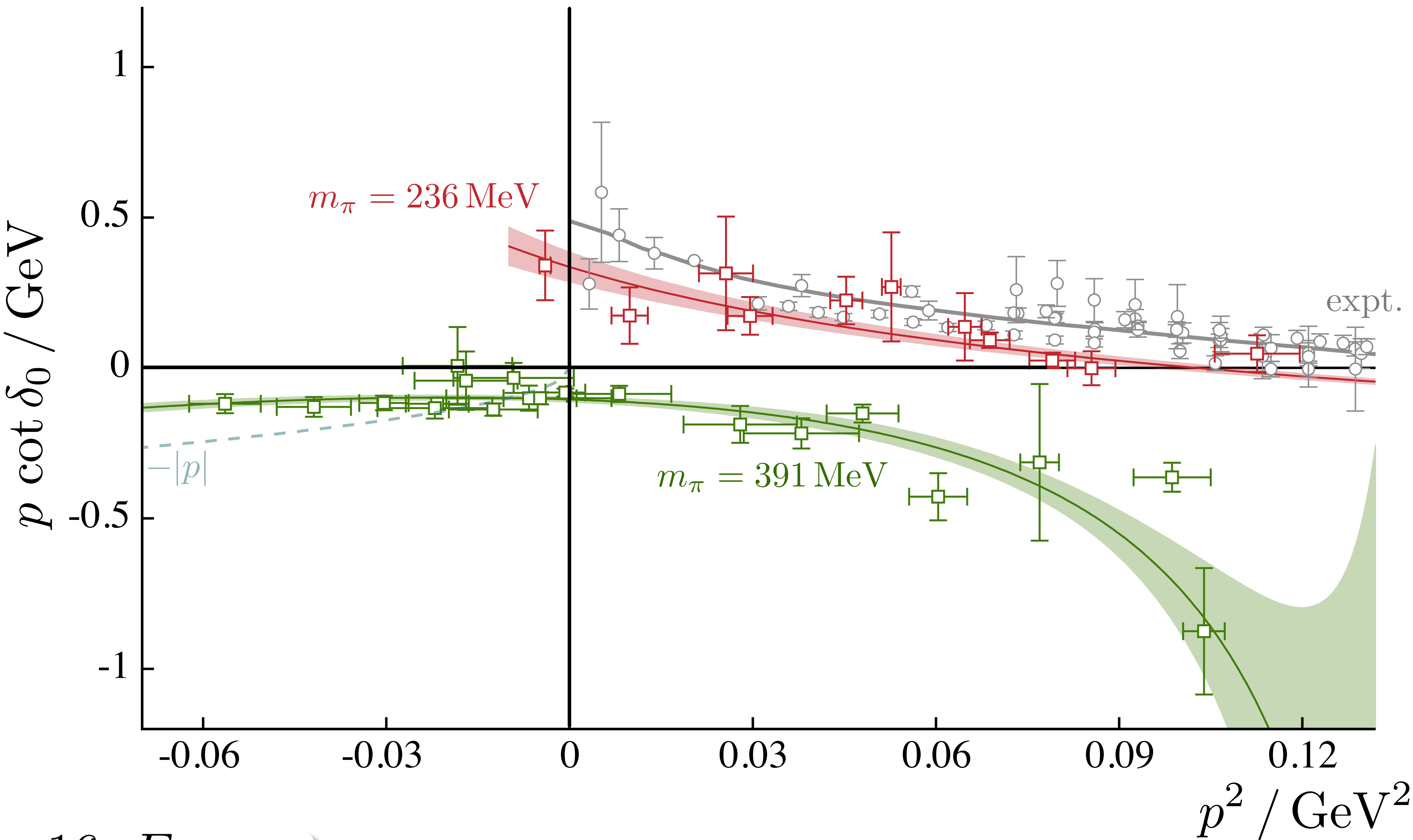
Isoscalar spectra: S-wave dominant





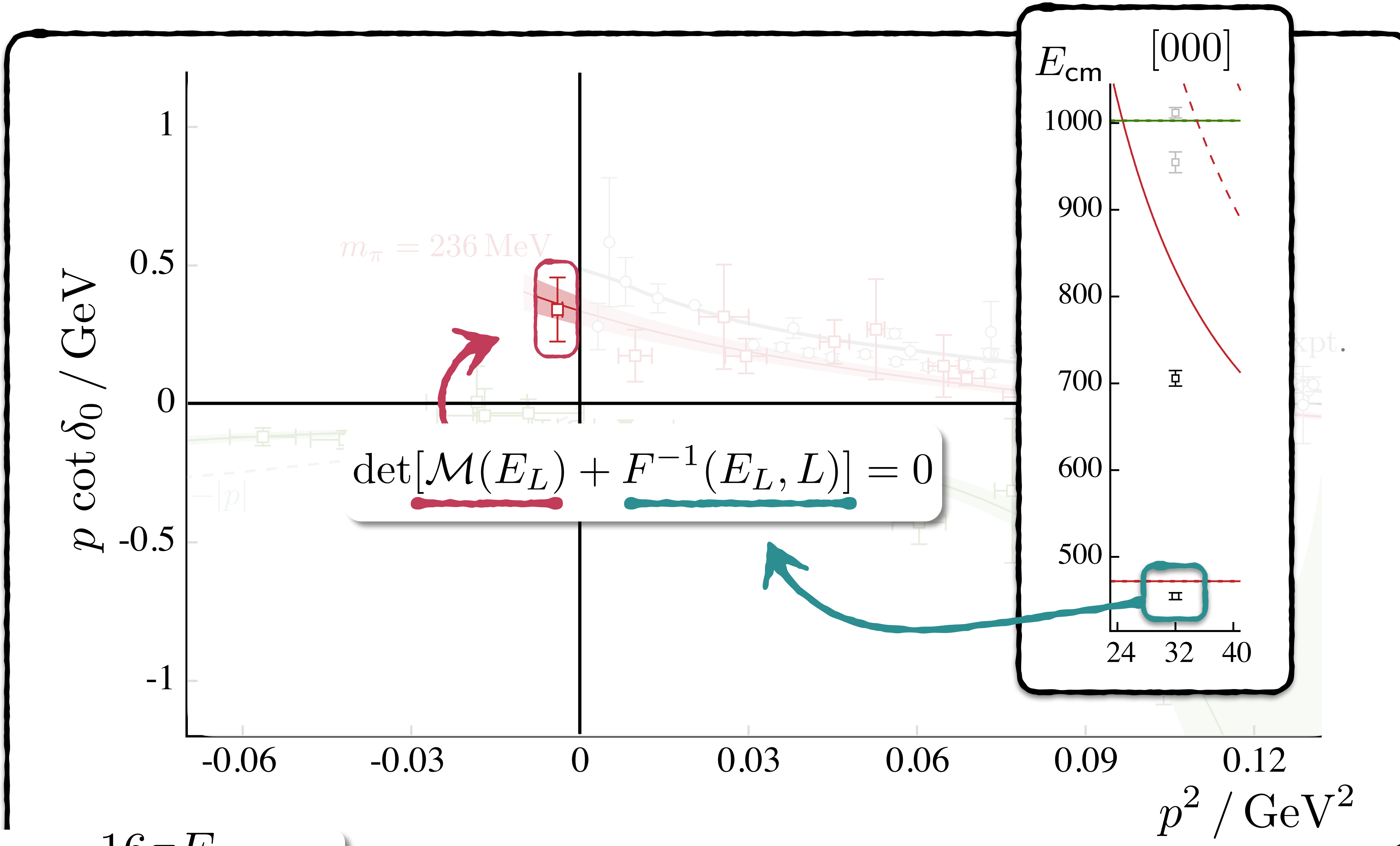
Isoscalar $\pi\pi$ scattering: elastic region



$$\mathcal{M}_0 = \frac{16\pi E_{\rm cm}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

Isoscalar $\pi\pi$ scattering: elastic region

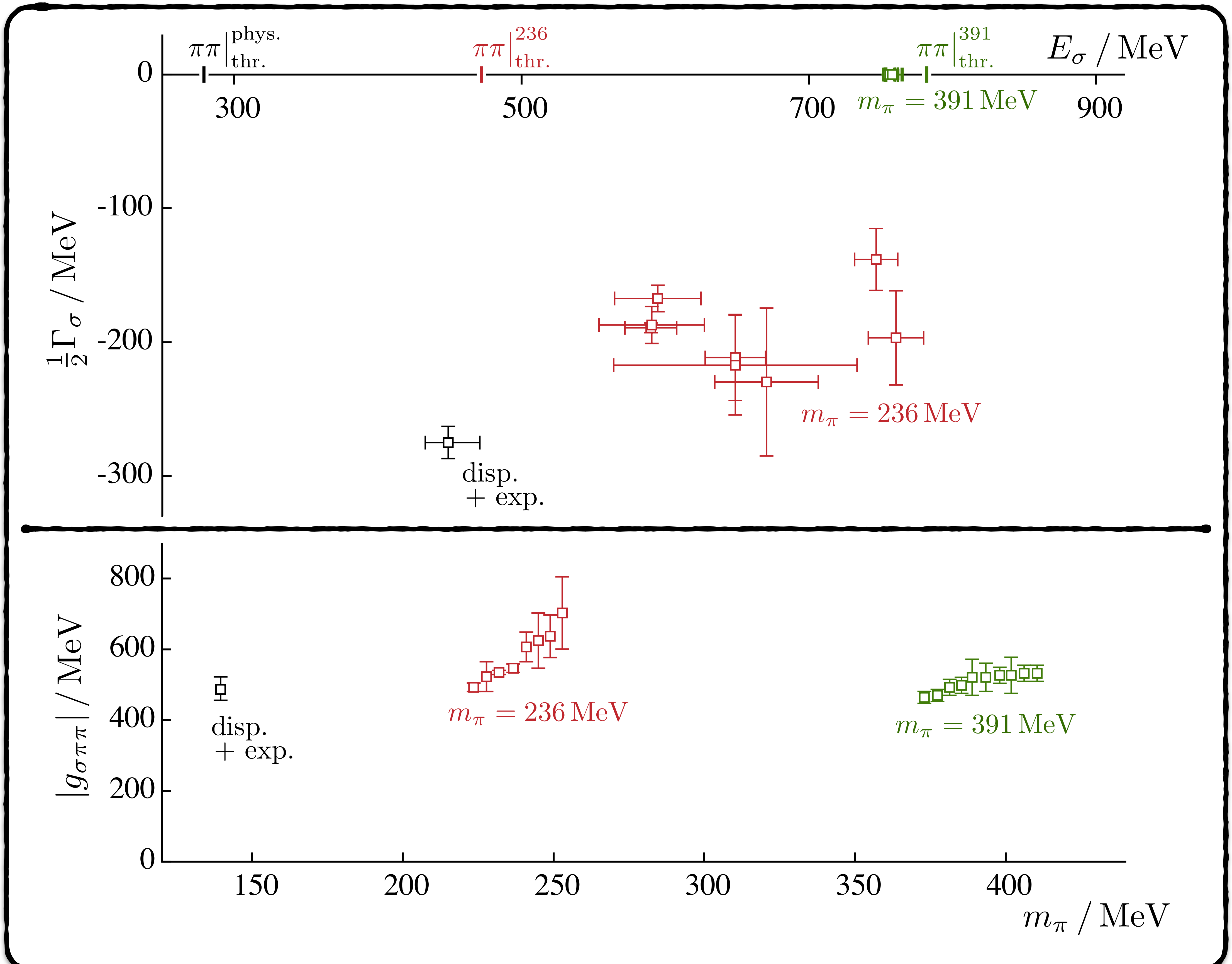


$$\det[\mathcal{M}(E_L) + F^{-1}(E_L, L)] = 0$$

$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p \cot \delta_0 - ip}$$

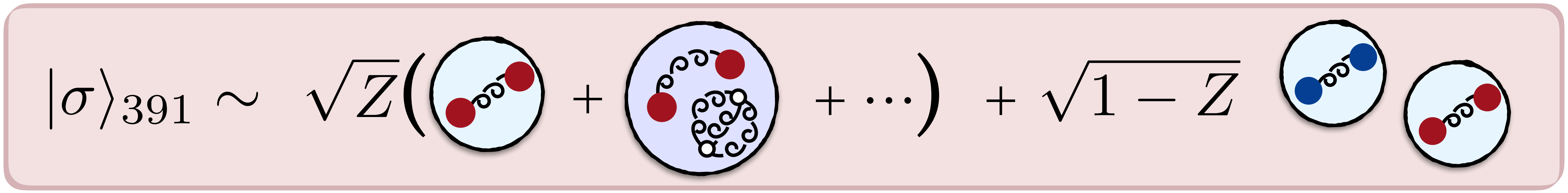
RB, Dudek, Edwards, Wilson - PRL (2017)

The σ / $f_0(500)$ vs m_π



Weinberg compositeness criterion for the σ

- For the heavier ensemble, the σ is a bound state, so we can apply Weinberg's criterion

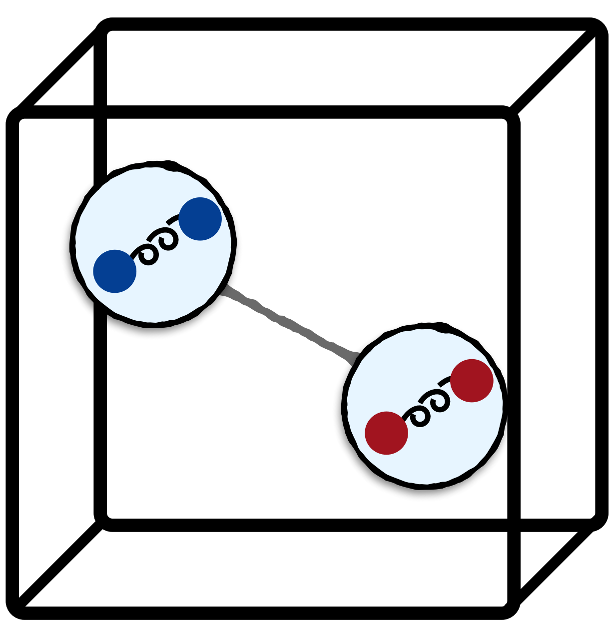


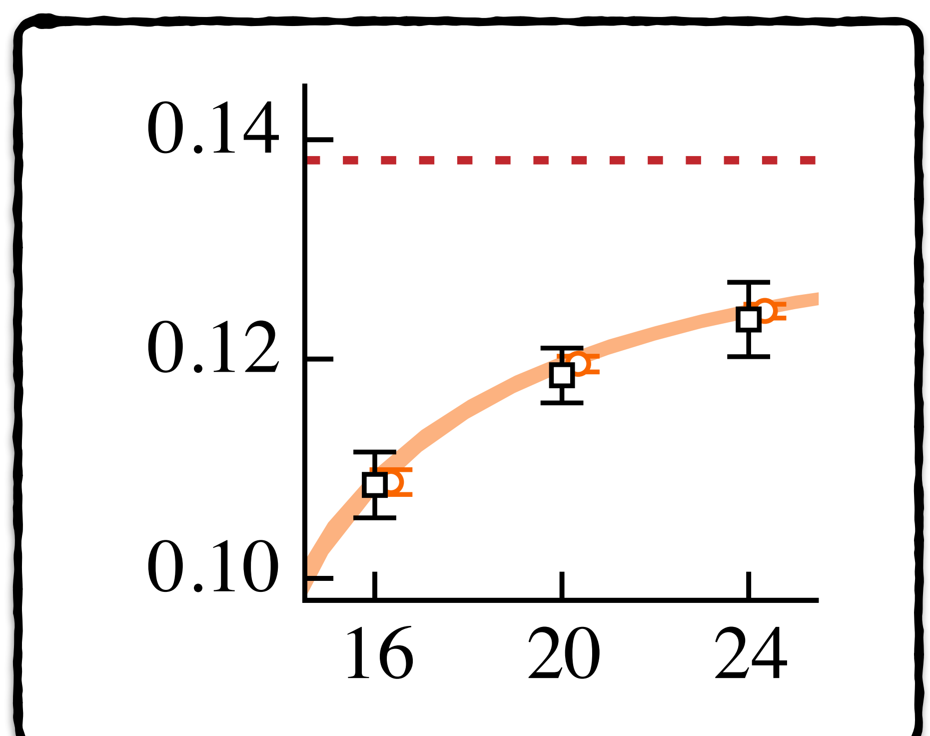
- Can relate Z to scattering information

$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}, \qquad r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}$$

- To obtain: $Z \sim 0.3(1)$

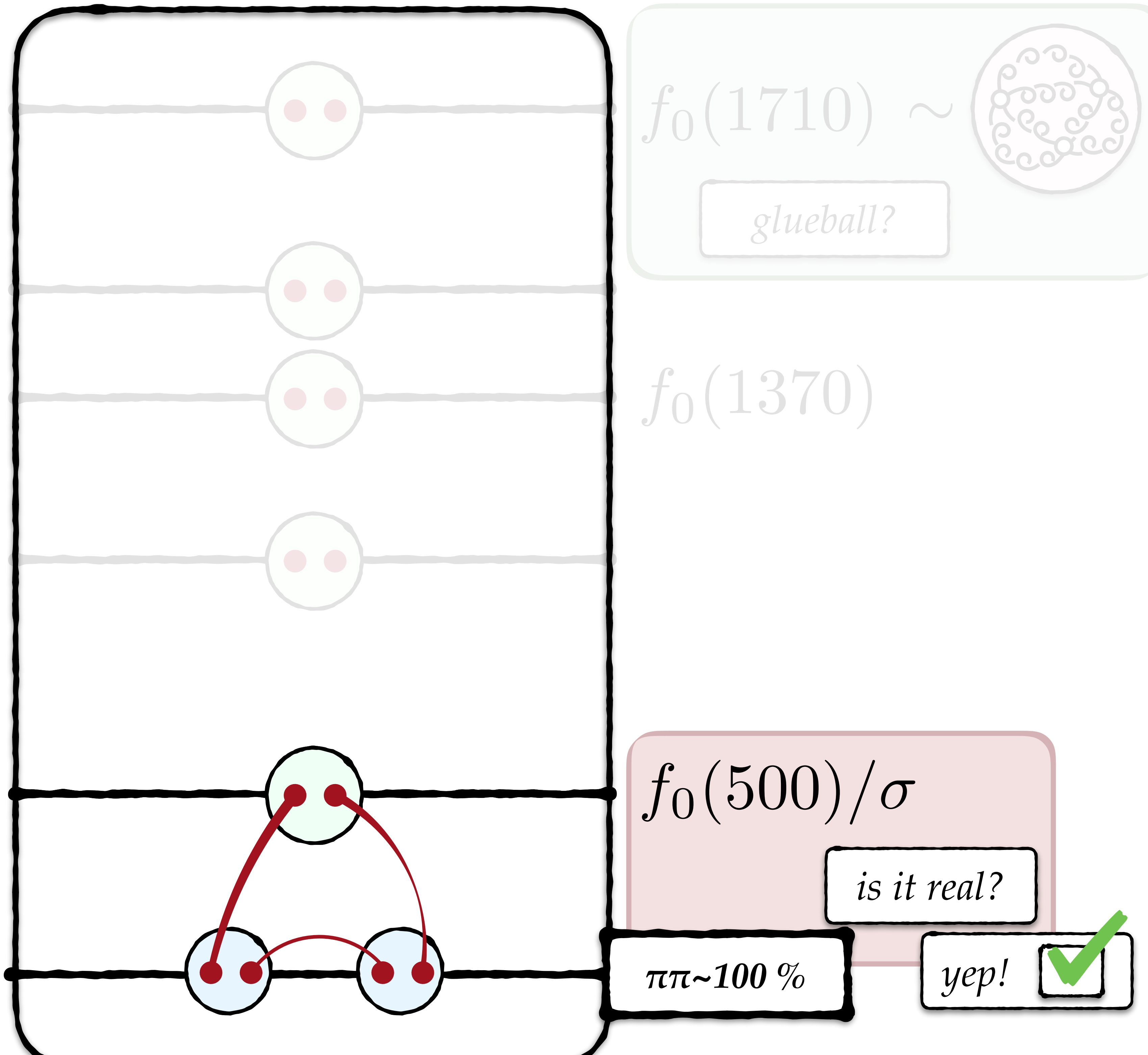
- Consistent with the large FV effects



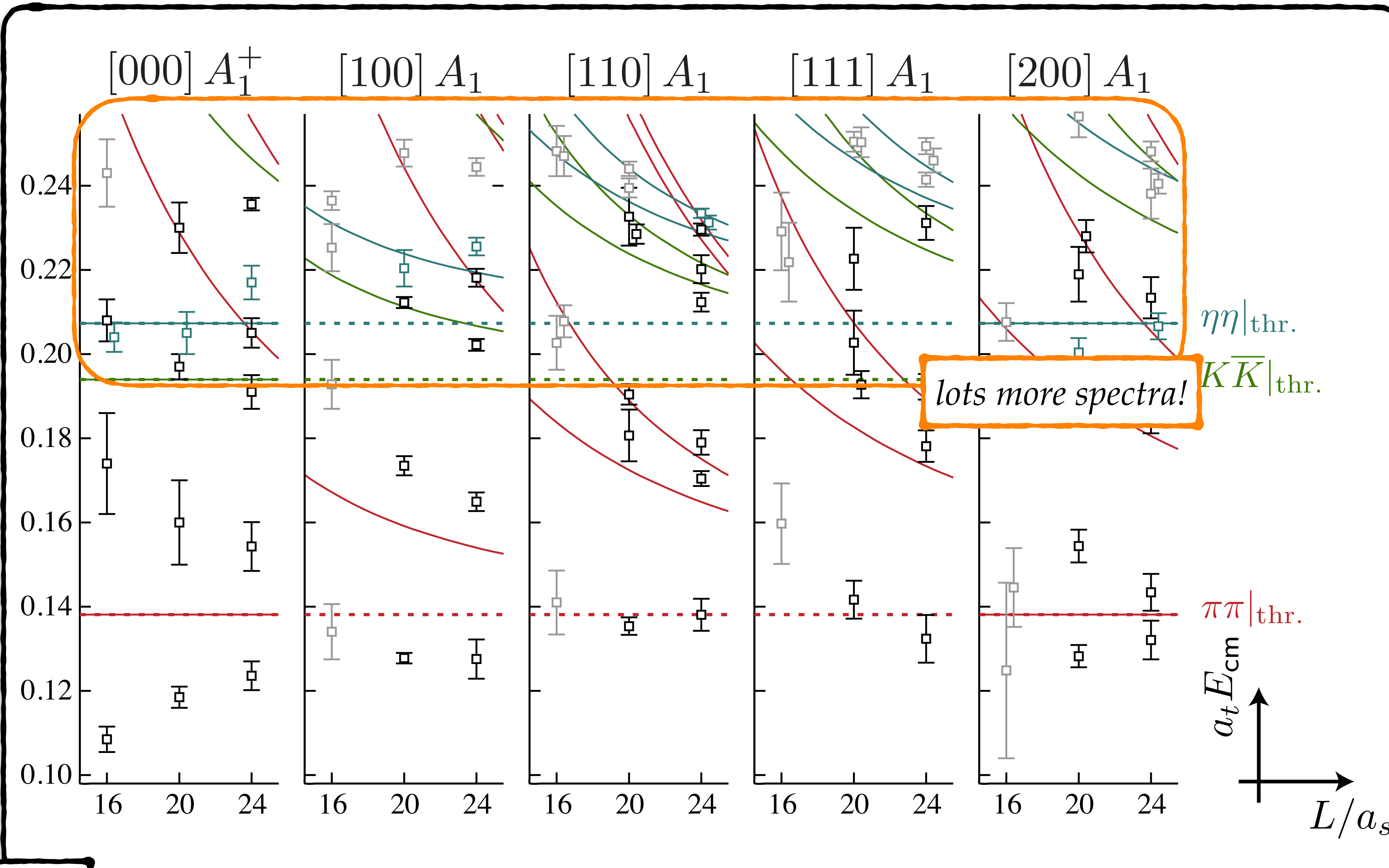


The isoscalar, scalar sector

GWU efforts will be reviewed by Maxim



Multi-channel systems - the cutting edge!



m_π=391 MeV

Multi-channel systems - the cutting edge!

- the *necessary* formalism for doing coupled-channel scattering of

Feng, Li, & Liu (2004) [inelastic scalar bosons]
Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
RB (2014) [general 2-body result]

- to date, the ***Hadron Spectrum* collaboration** is the only one to have extracted coupled-channel scattering amplitude information from QCD

$\pi\pi$, KK, $\eta\eta$ [isoscalar]:	RB, Dudek, Edwards, Wilson - PRL (2017)
	RB, Dudek, Edwards, Wilson - PRD (2018)
Kπ, Kη:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
$\pi\eta$, KK:	Dudek, Edwards, Wilson - PRD (2016)
Dπ, Dη, D_sK:	Moir, Peardon, Ryan, Thomas, Wilson - JHEP (2016)
$\pi\pi$, KK [isovector]:	Wilson, RB, Dudek, Edwards, Thomas - PRD (2015)

Coupled-channels analysis

- Above $2m_K$, there is not a one-to-one correspondence

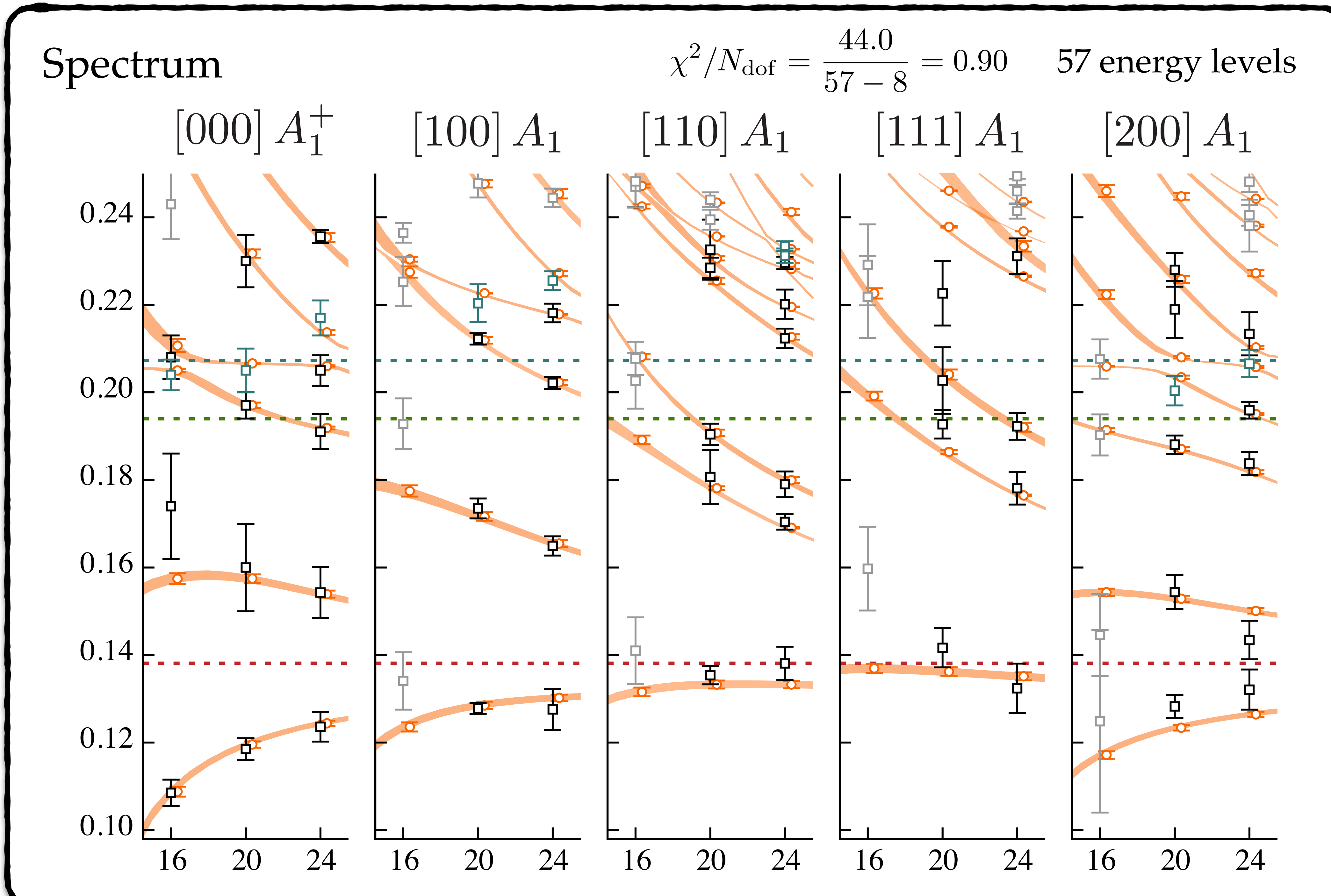
$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)

- In general, must constrain *(1/2) [N^2 + N]* functions of energy
- Need that many energy levels at the same energy
- Alternatively, parametrize scattering amplitude and do a global fit

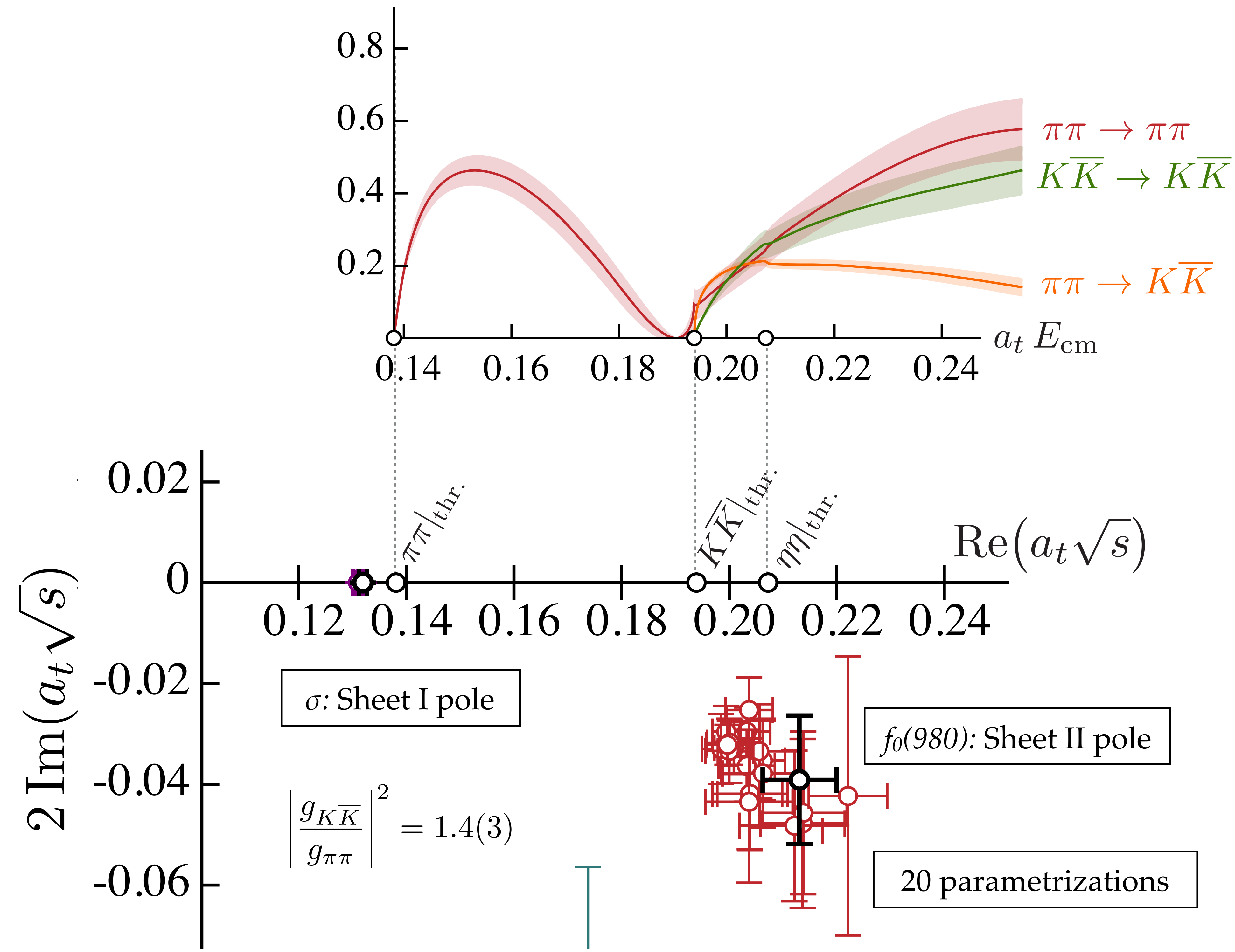
Coupled-channels analysis

- S-wave above $2m_\pi$, $2m_K$, and $2m_\eta$
- Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$



Scalar poles: σ and $f_0(980)$

- Near poles: $\mathcal{M} \sim \dfrac{g^2}{s_0 - s}$



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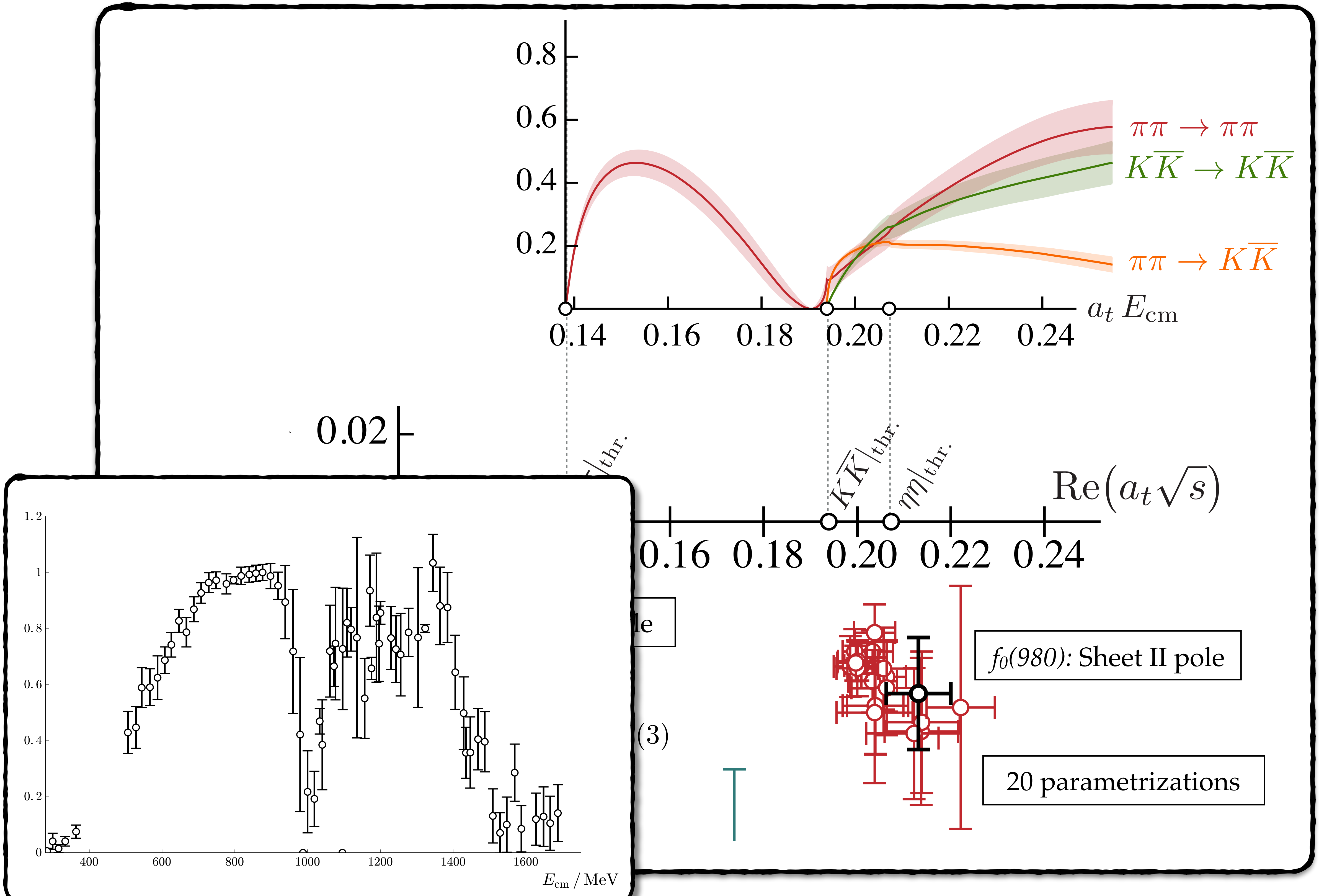
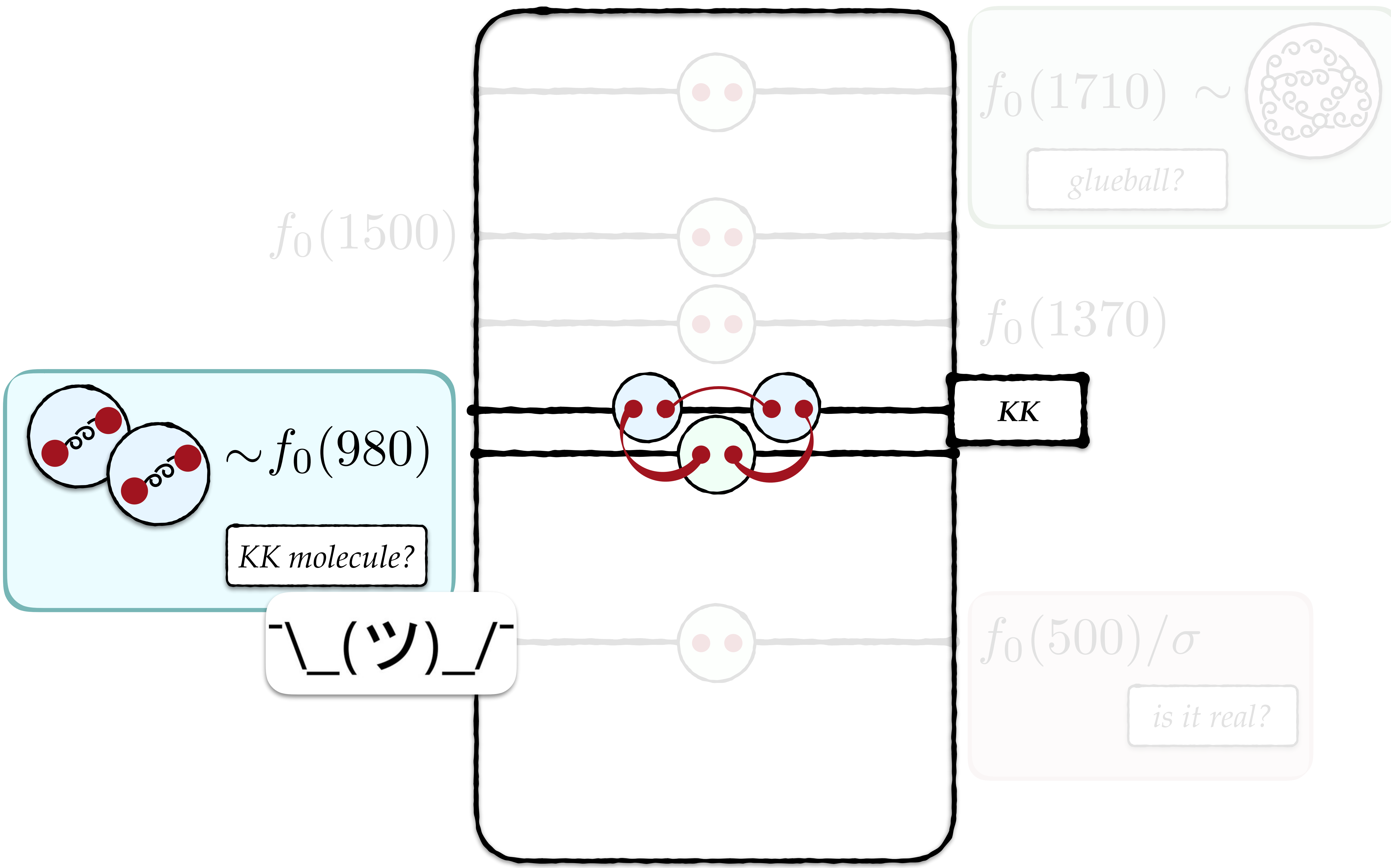


The isoscalar, scalar sector

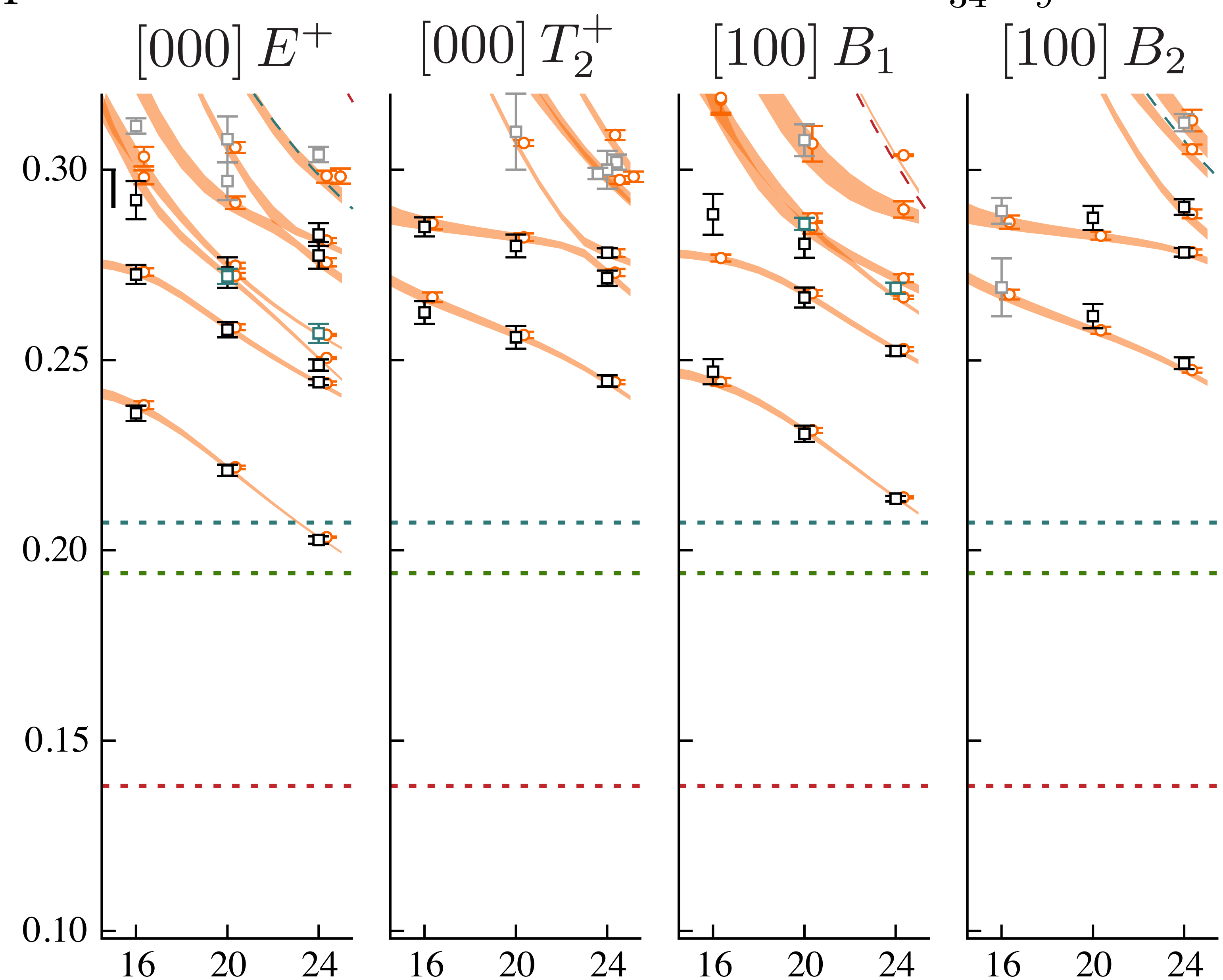


Coupled-channels analysis

- D-wave above $2m_\pi$, $2m_K$, and $2m_\eta$
- Ansatz $$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$ $\gamma_{\eta\eta} \neq 0$, $\gamma_{ij} = 0$ otherwise

Spectrum

$\chi^2/N_{\text{dof}} = \frac{28.9}{34-9} = 1.15$ 34 energy levels



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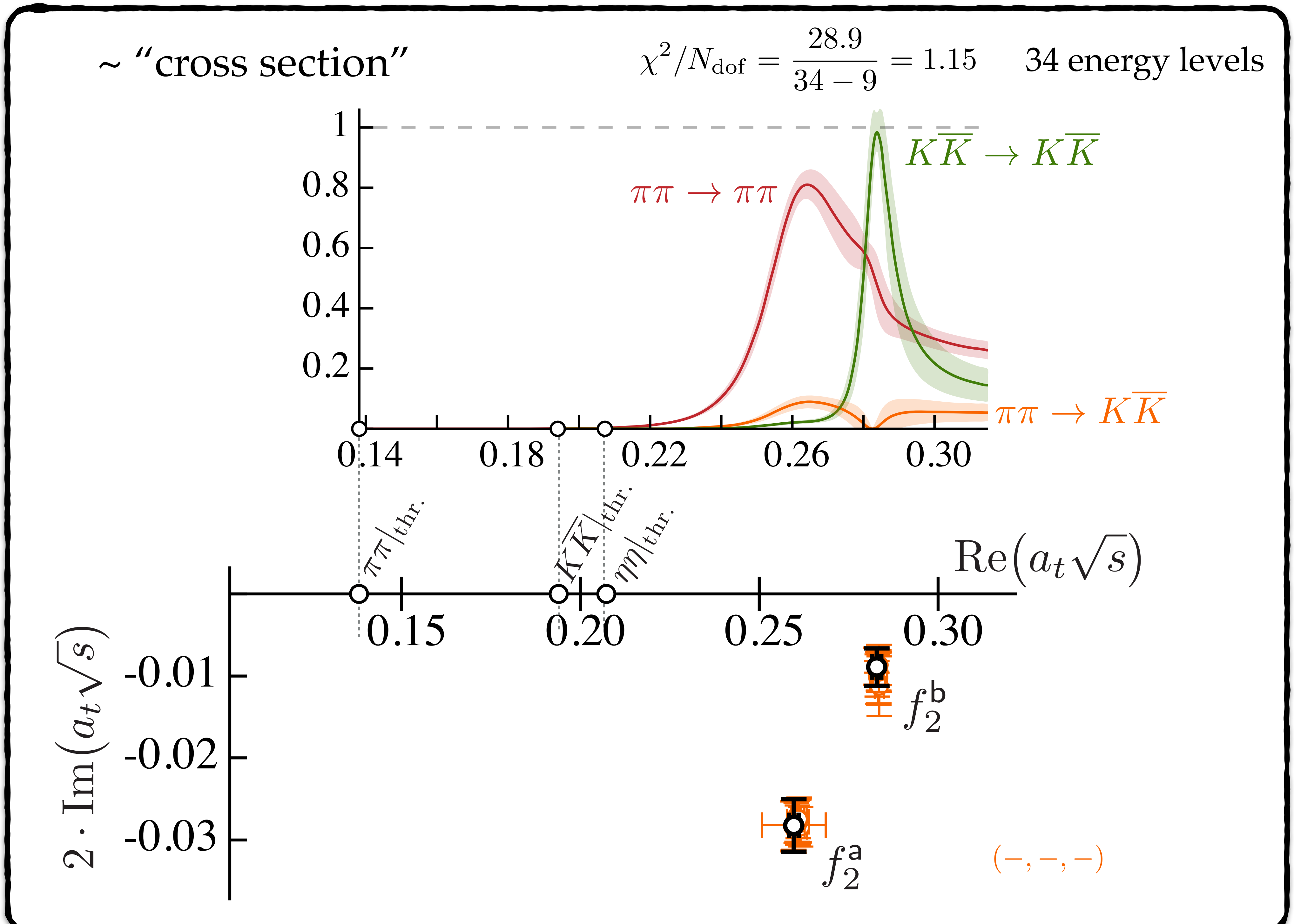

Coupled-channels analysis

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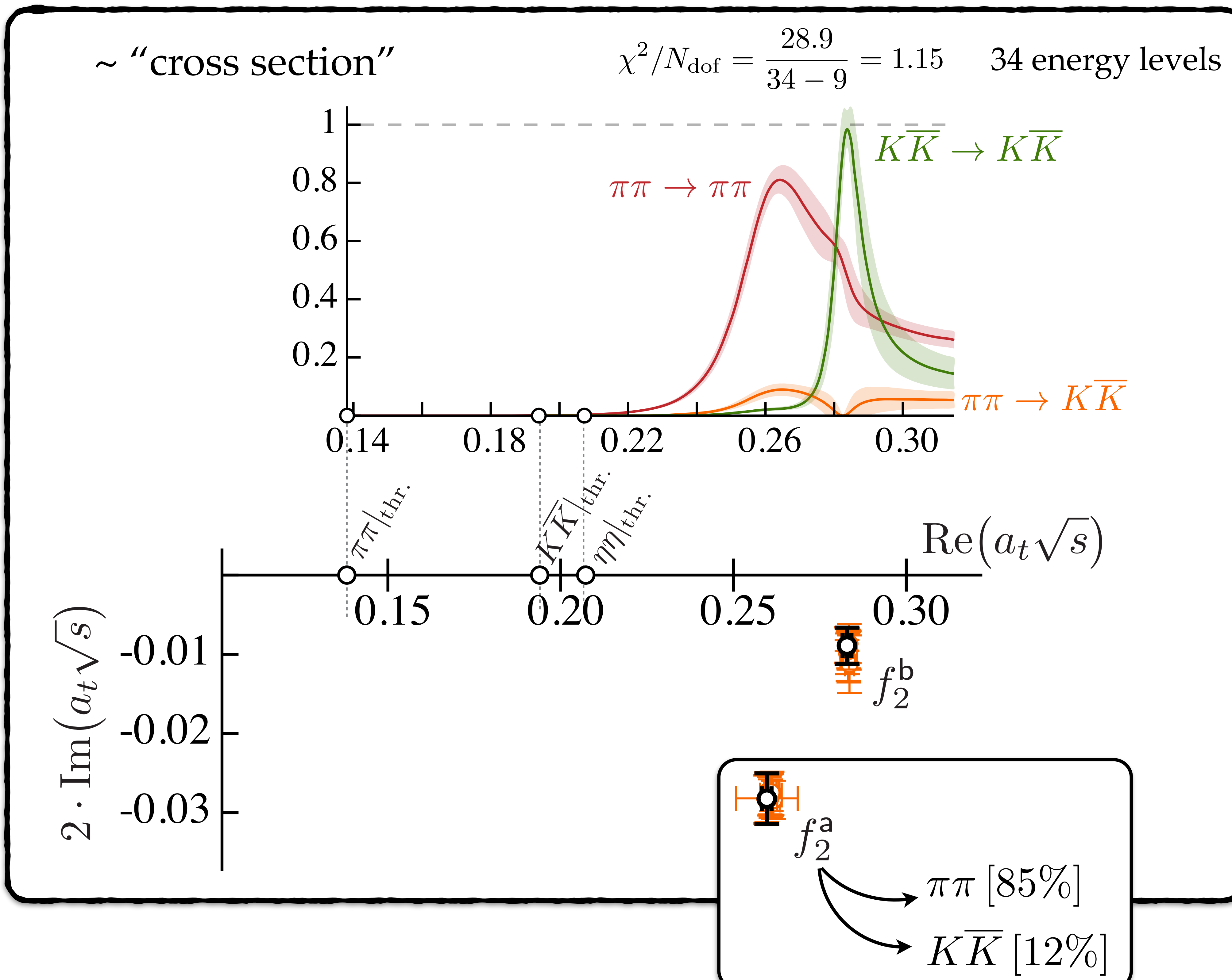

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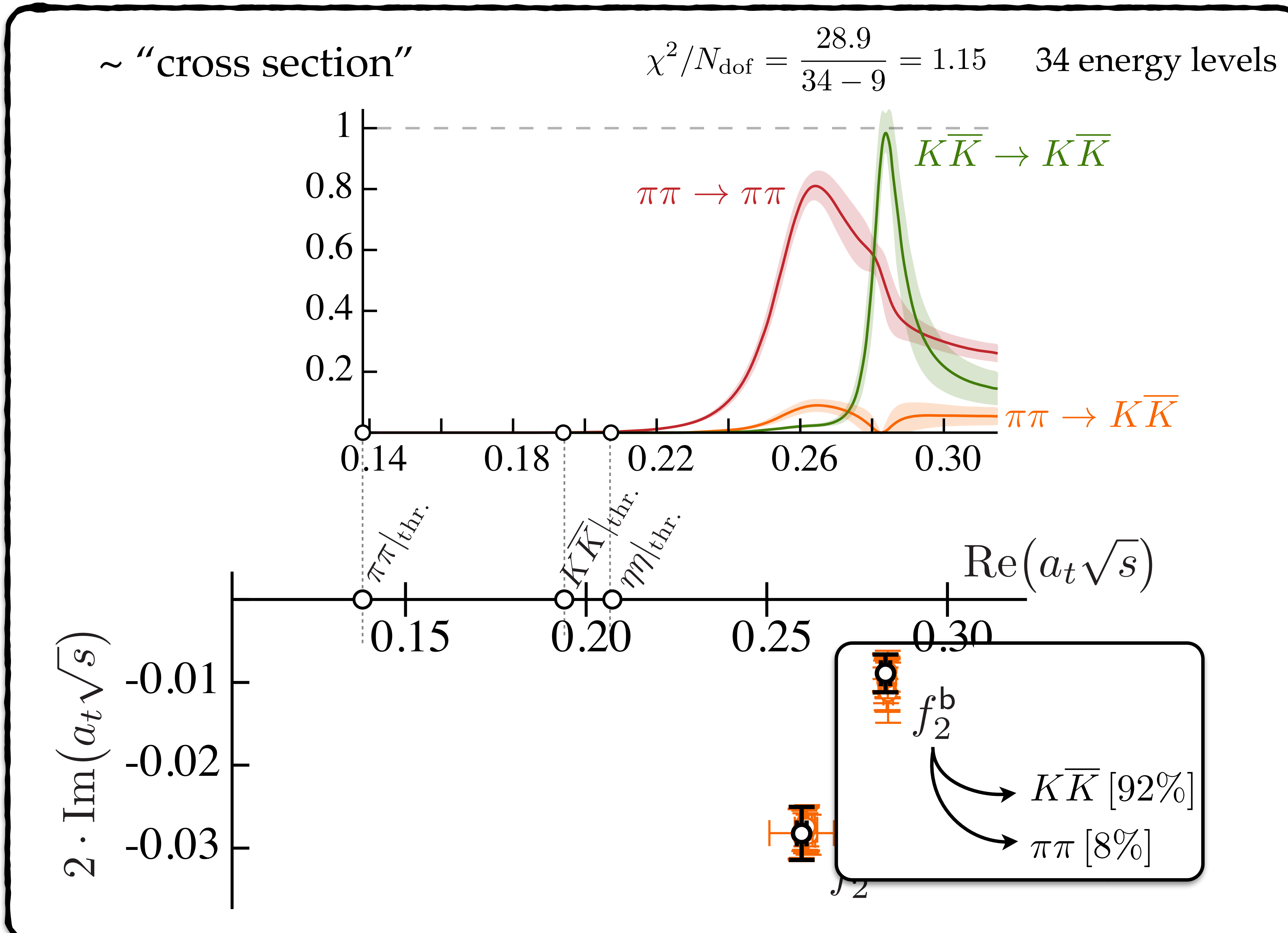

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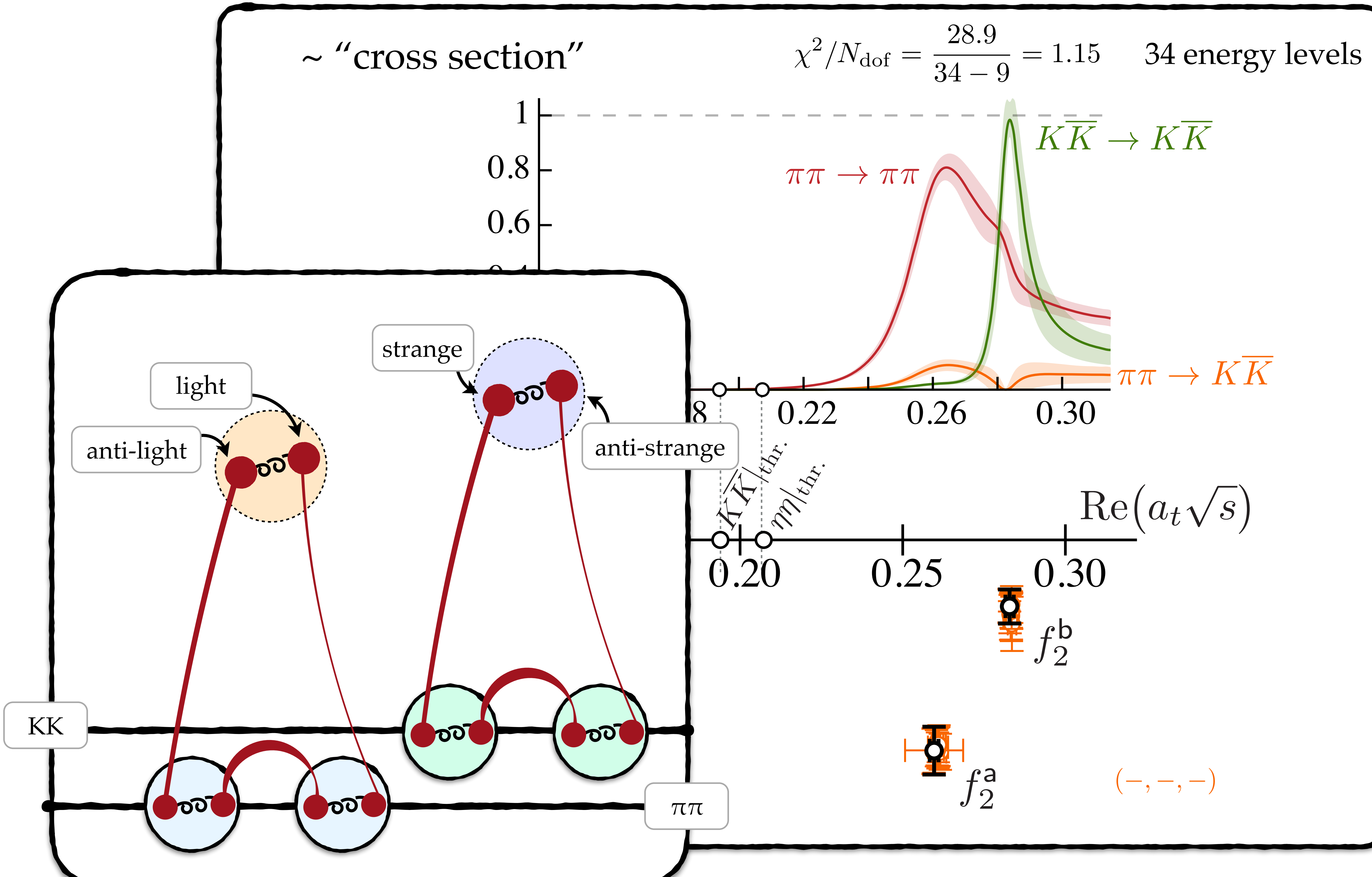
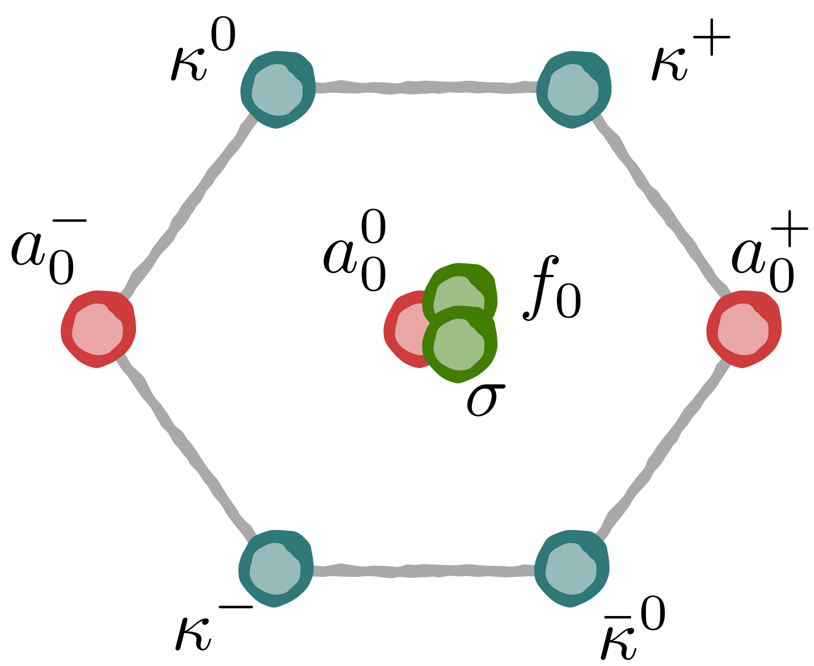
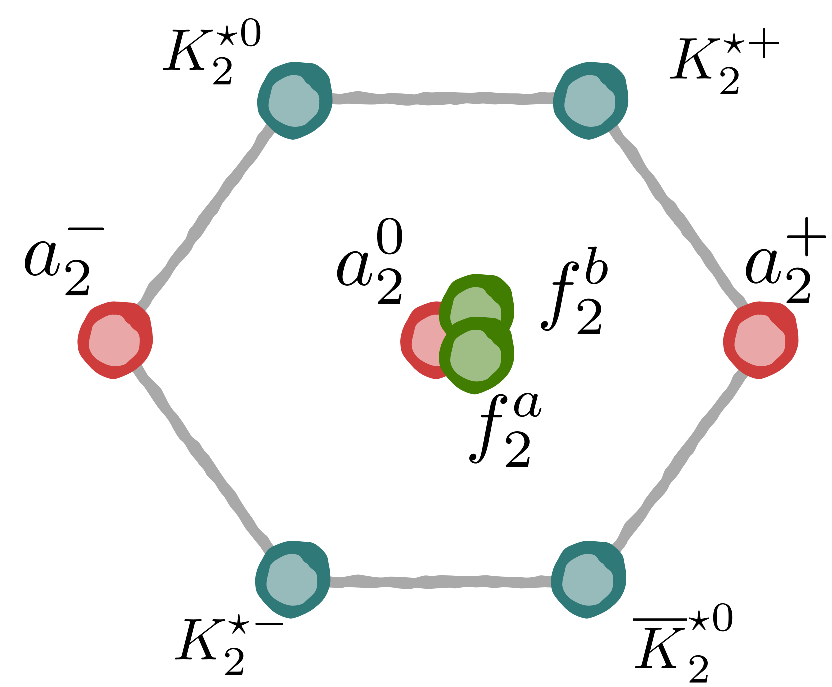


Tensor and scalar nonets

- First complete determination of the scalar and tensor nonets from LQCD :





ππ, KK, ηη:	RB, Dudek, Edwards - PRL (2017)
	RB, Dudek, Edwards - PRD (2017)
Kπ, Kη:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
πη, KK:	Dudek, Edwards, Wilson - PRD (2016)

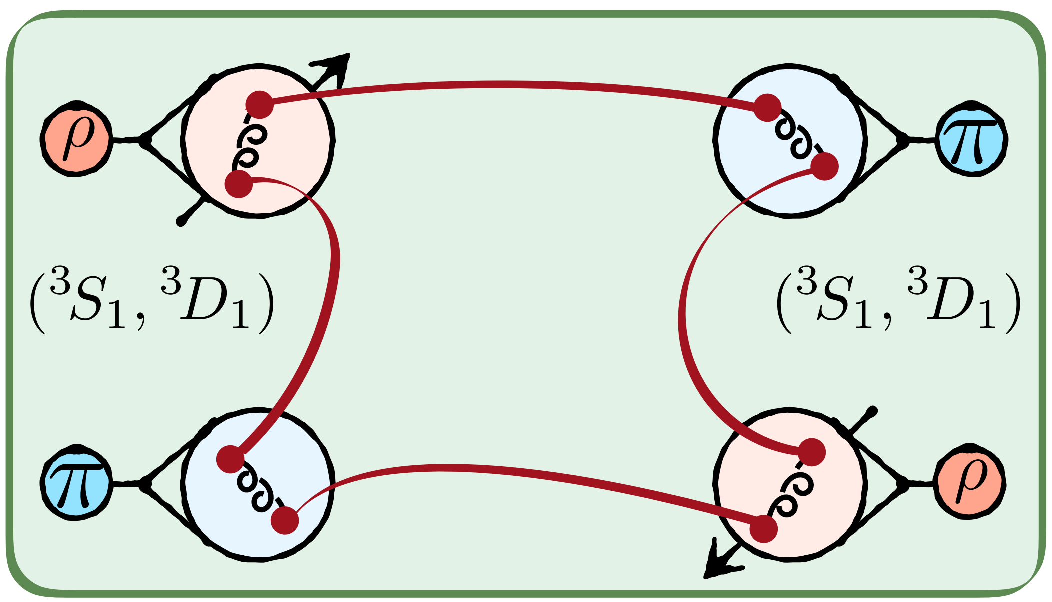
hadspec

Scattering of spinning particles - $\rho\pi$ scattering

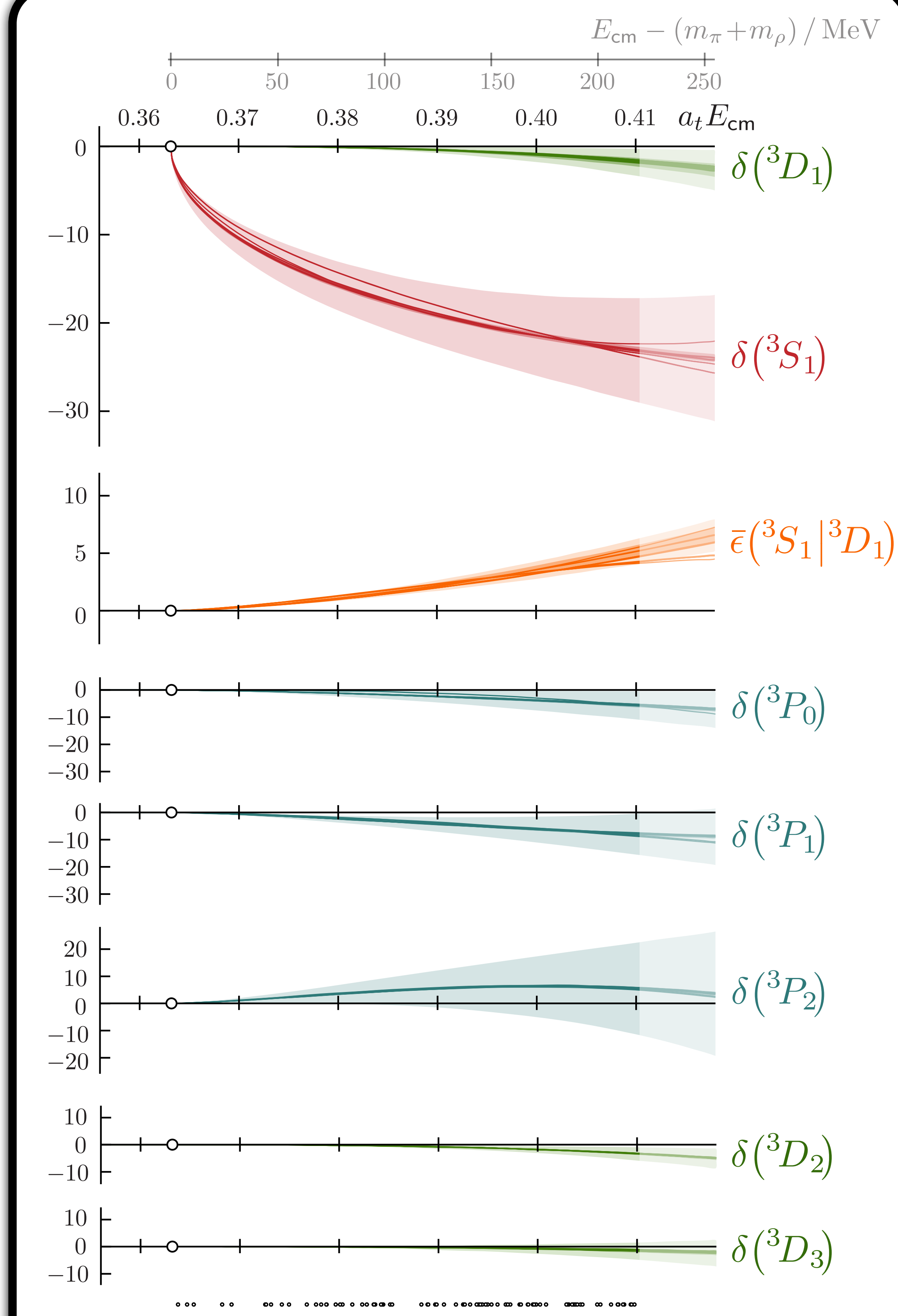
- the formalism: RB (2014)
- first and only calculation:

  $\rho\pi$ scattering in I=2

  Woss, Thomas, Dudek, Edwards, Wilson (2018)

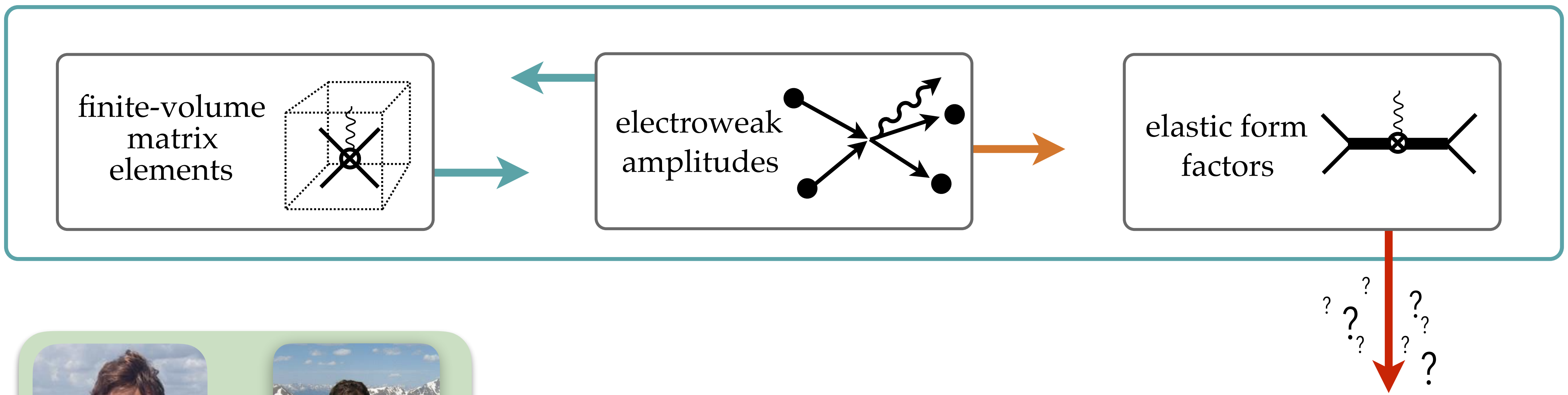


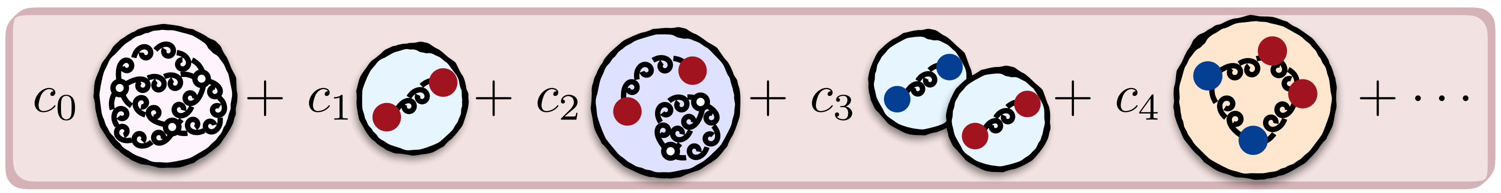
Woss



Woss, Thomas, Dudek, Edwards, Wilson (2018)

Structure of states:





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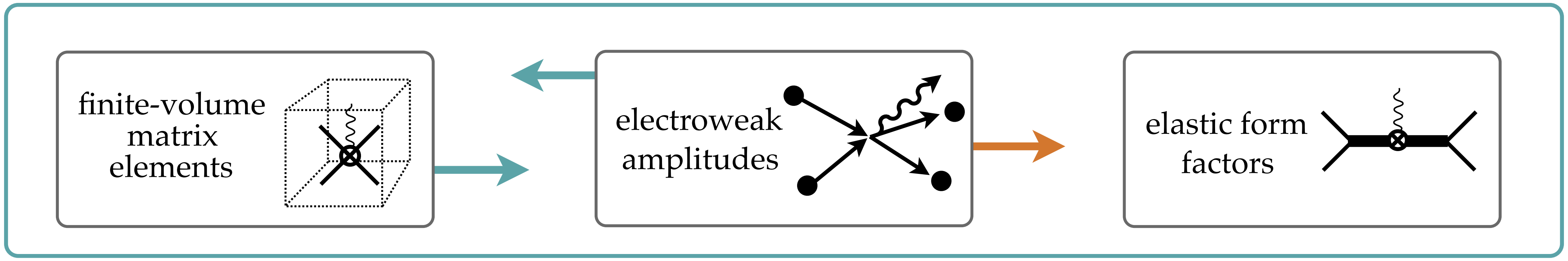
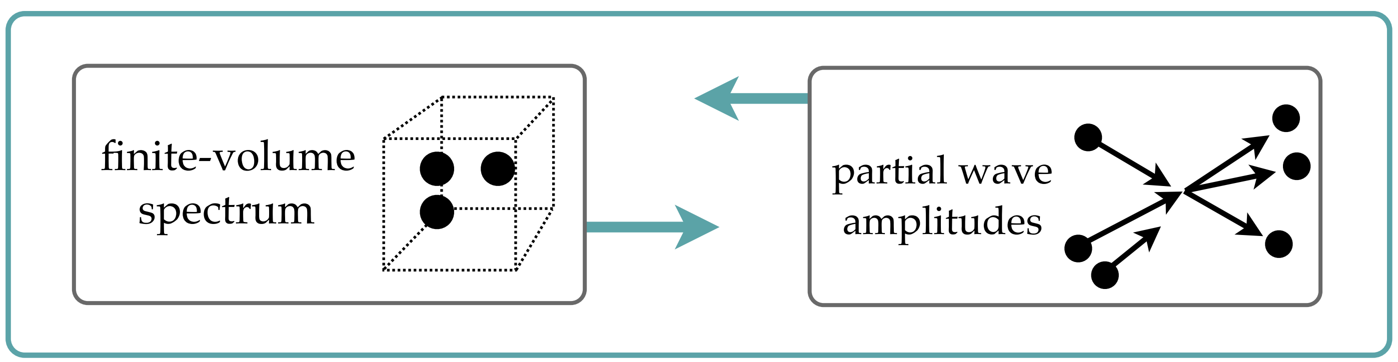


Beyond two particles



- RB & Davoudi (2013)
- Hansen & Sharpe (2014, '15)
- Hammer, Pang, Rusetsky (2017)
- Mai, Doring (2017,'18)
- RB, Hansen & Sharpe (2016,'18)

see Maxim's talk

The team and some references

more numerical - JLab

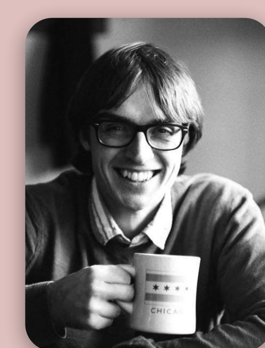

Dudek


Edwards


Winter


Joó

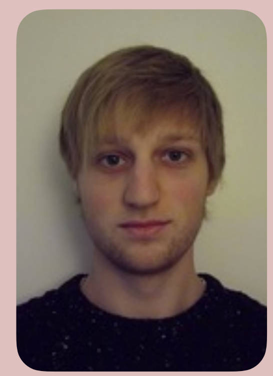
more numerical - Europe


Wilson


Peardon


Ryan


Thomas


Woss

more formal


Baroni


Hansen


Sharpe

Meson Spectrum

JHEP05 021 (2013)
PRD88 094505 (2013)
JHEP07 126 (2011)
PRD83 111502 (2011)
PRD82 034508 (2010)
PRL103 262001 (2009)

Baryon Spectrum

PRD91 094502 (2015)
PRD90 074504 (2014)
PRD87 054506 (2013)
PRD85 054016 (2012)
PRD84 074508 (2011)

Scattering

arXiv:1802.05580
arXiv:1708.06667
PRL118 022002 (2017)
JHEP011 1610 (2016)
PRD93 094506 (2016)
PRD92 094502 (2015)
PRD91 054008 (2015)
PRL113 182001 (2014)
PRD87 034505 (2013)
PRD86 034031 (2012)
PRD83 071504 (2011)

Electroweak

PRD93 114508 (2016)
PRL115 242001 (2015)
PRD91 114501 (2015)
PRD90 014511 (2014)

Techniques

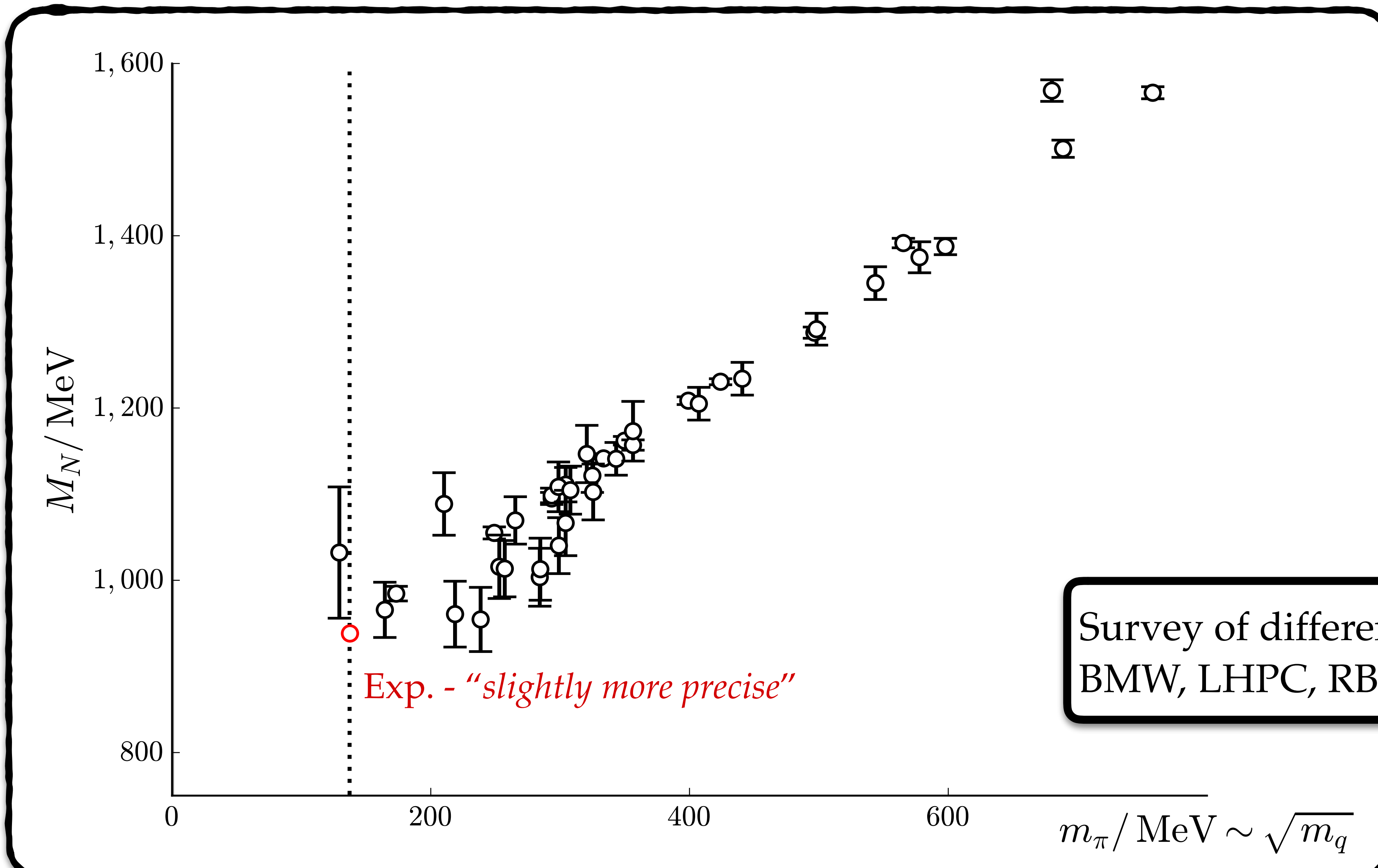
JHEP 1711 (2017)
PRD85 014507 (2012)
PRD80 054506 (2009)
PRD79 034502 (2009)

Formalism

arXiv:1803.04169 (2018)
PRD95 074510 (2017)
PRD94 013008 (2016)
PRD92 074509 (2015)
PRD91 034501 (2015)
PRD89 074507 (2014)

Status of the field

- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓



Survey of different groups:
BMW, LHPC, RBC, & χQCD

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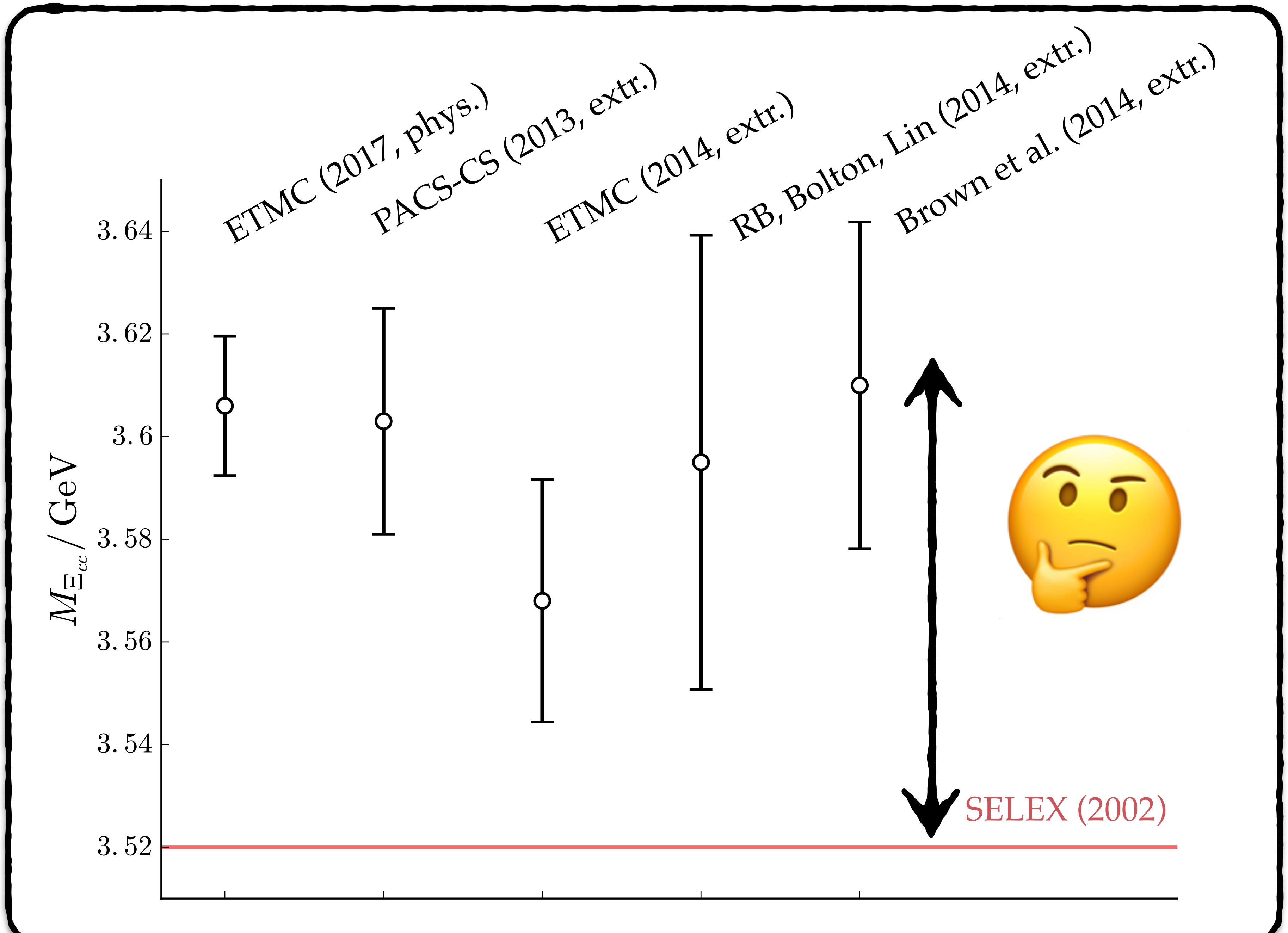

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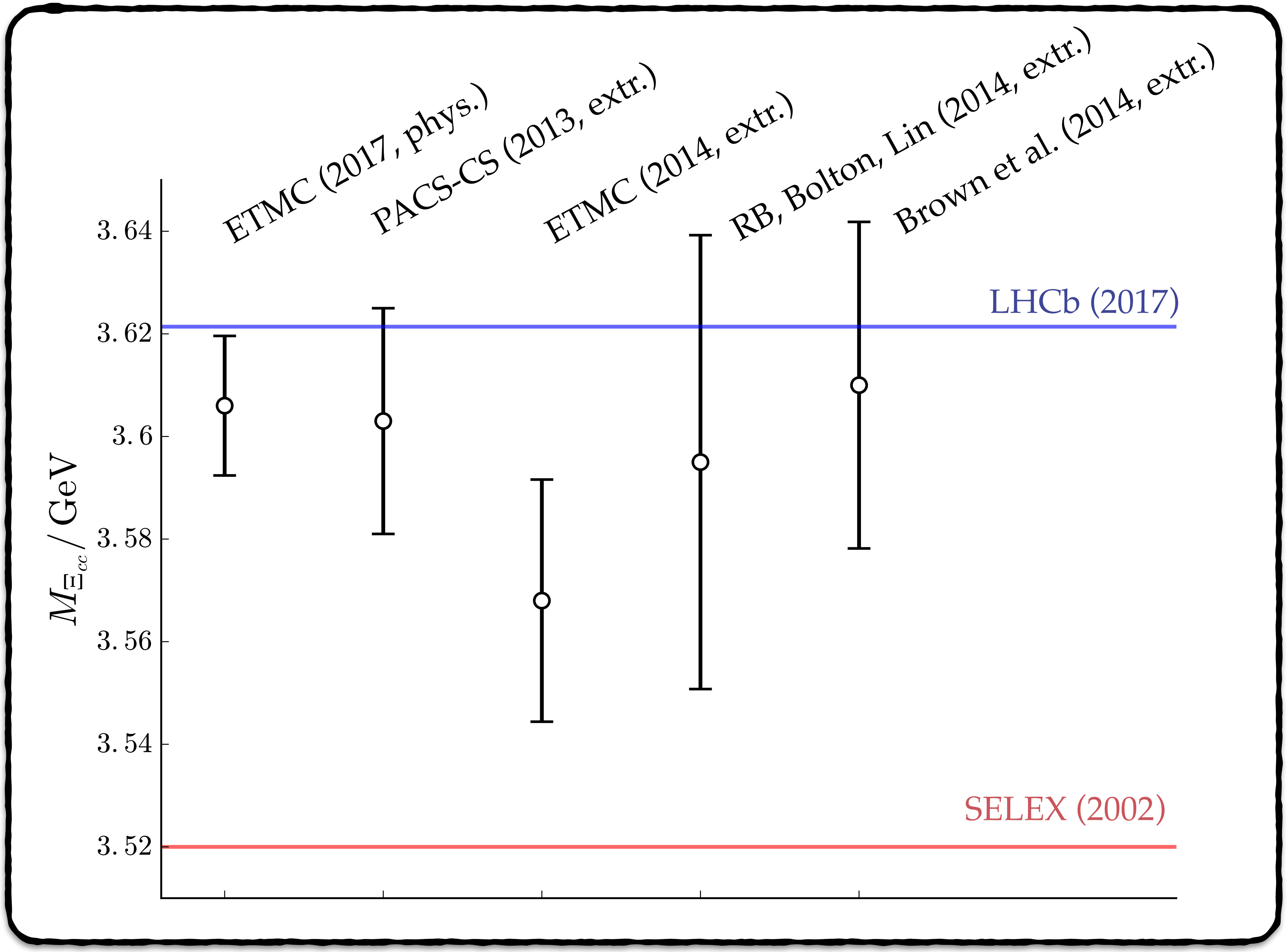
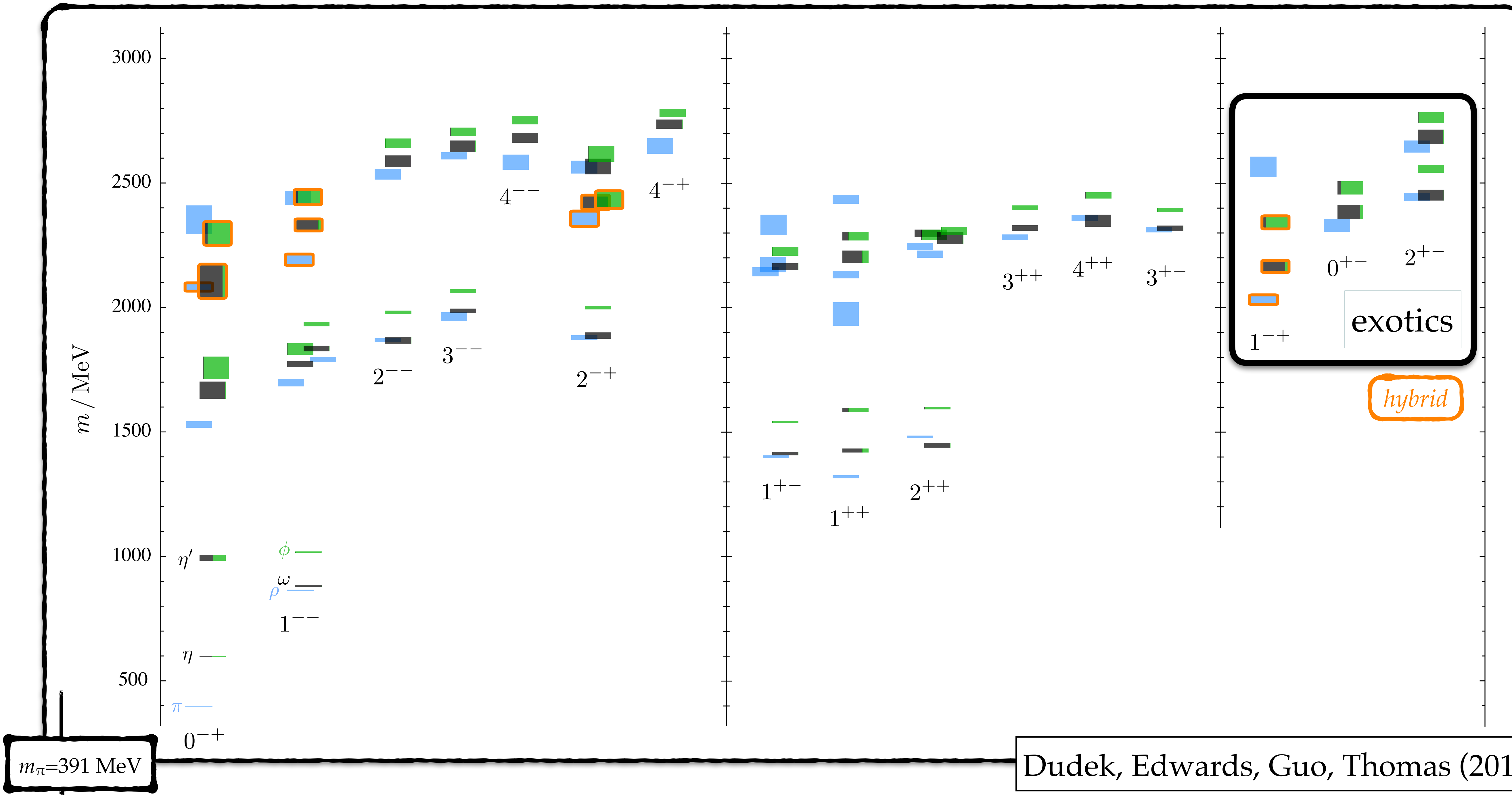


New "old-school spectroscopy"

Evaluate: $C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t, \mathbf{P})\mathcal{O}_a^\dagger(0, \mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$

…using distillation and a large number [10-30] of local ops, $\mathcal{O}_b \sim \bar{q}\,\Gamma_b\, q$



Dudek, Edwards, Guo, Thomas (2013)

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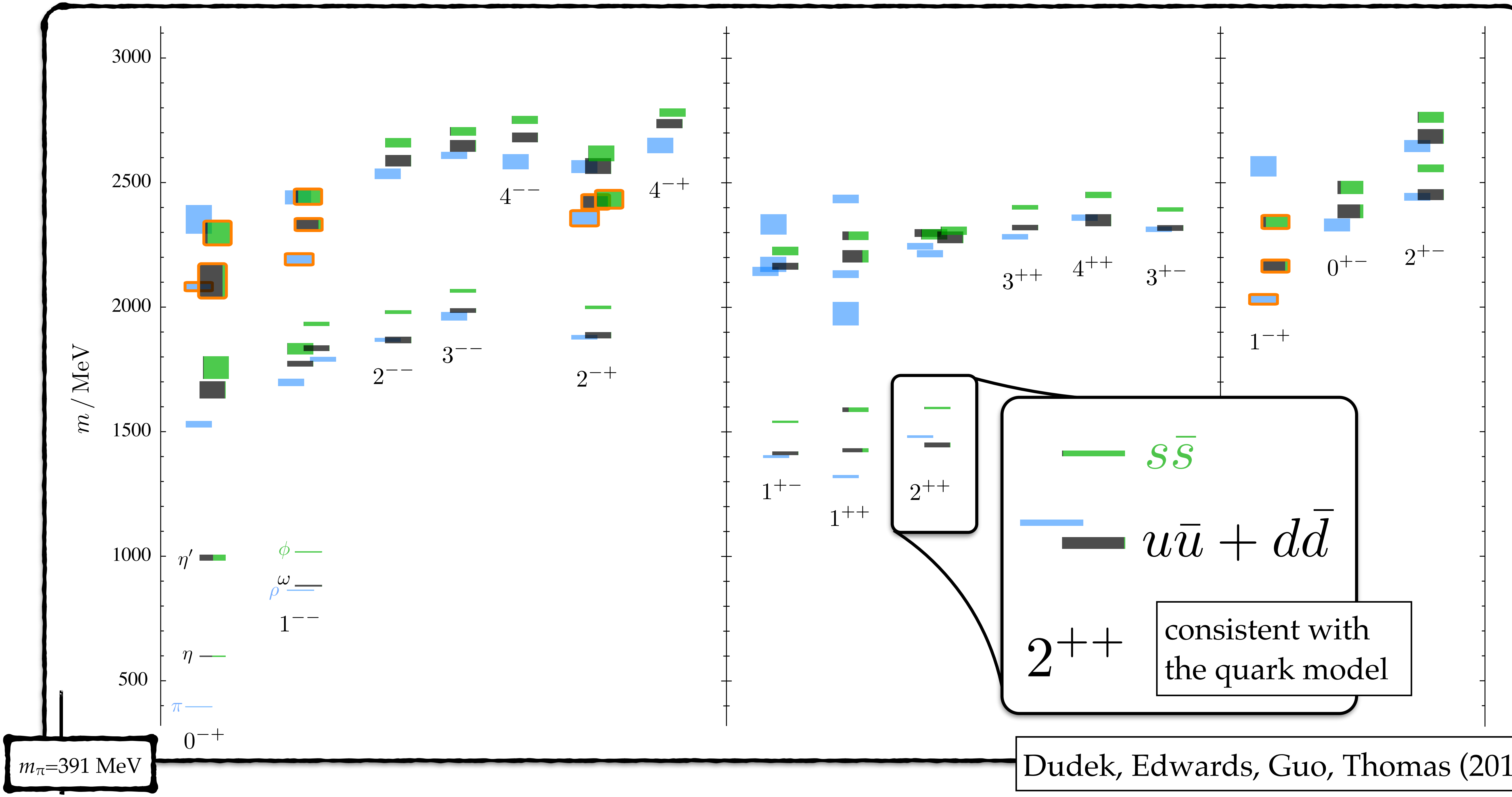

Dudek, Edwards, Guo, Thomas (2013)

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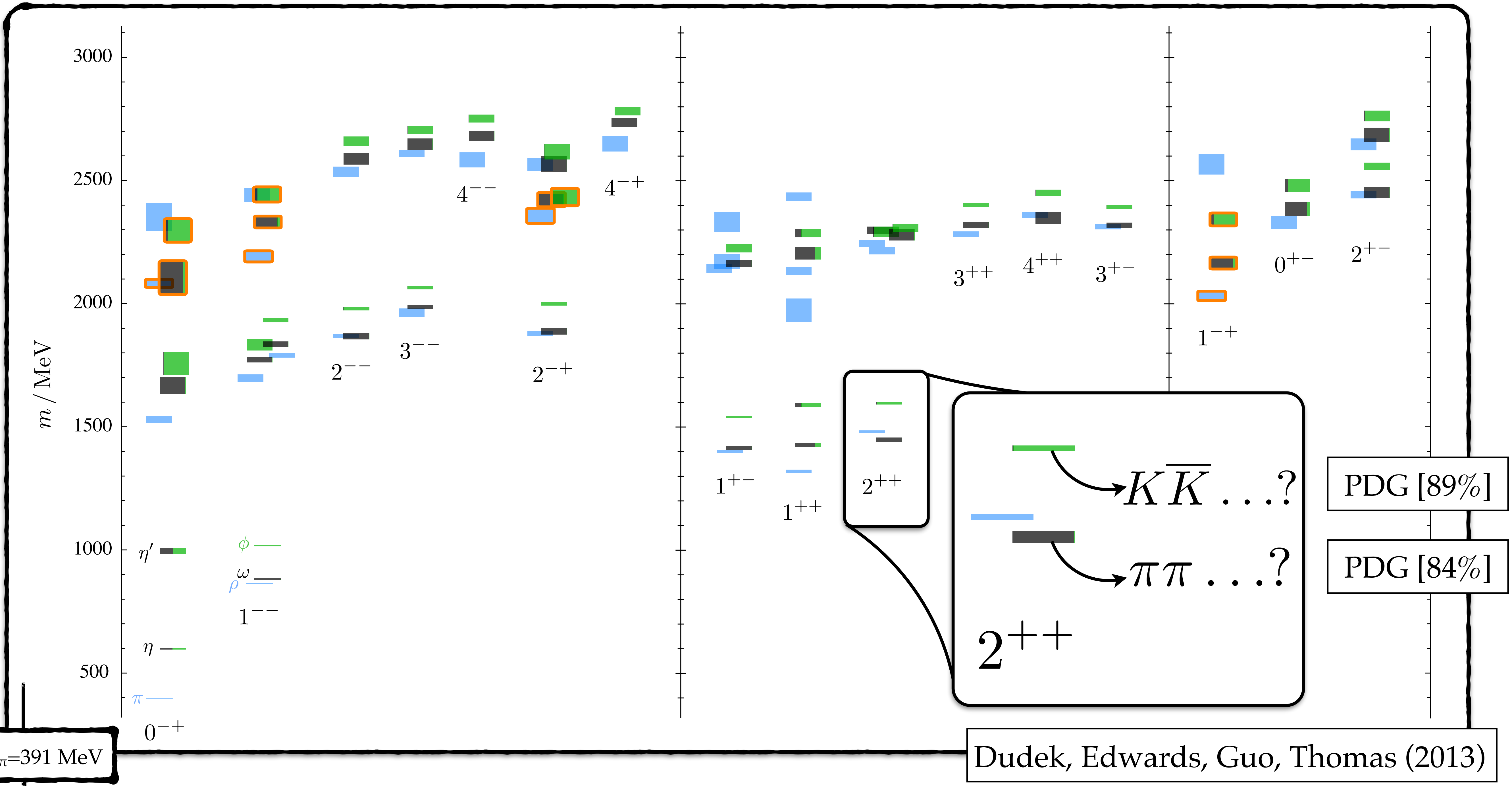

Dudek, Edwards, Guo, Thomas (2013)