北京航空航天大學
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Heavy meson and baryon

—some new mechanisms for forming molecules

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Mario Sanchez Sanchez (IPNO), Tetsuo Hyodo (YITP)

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Contents

- Where all these started–the pentaquark states
- A two-channel study of the $P_c(4450)$
  - An Efimov like effect
  - A new (long range) binding mechanism
- Two recent extensions
  - Near-threshold Coulomb-like Baryonia: $\Lambda_c(2595)\Sigma_c(\overline{\Sigma_c})$
  - Exotic doubly charmed $D_{s0}^*(2317)D$ and $D_{s1}(2460)D^*$ molecules
- Summary and outlook

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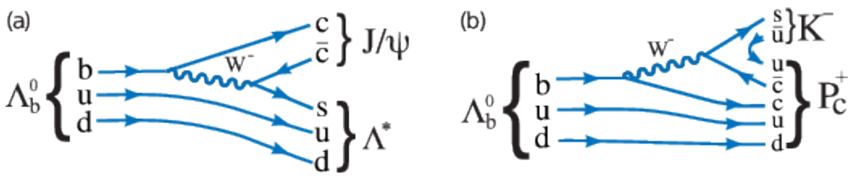
The pentaquark states

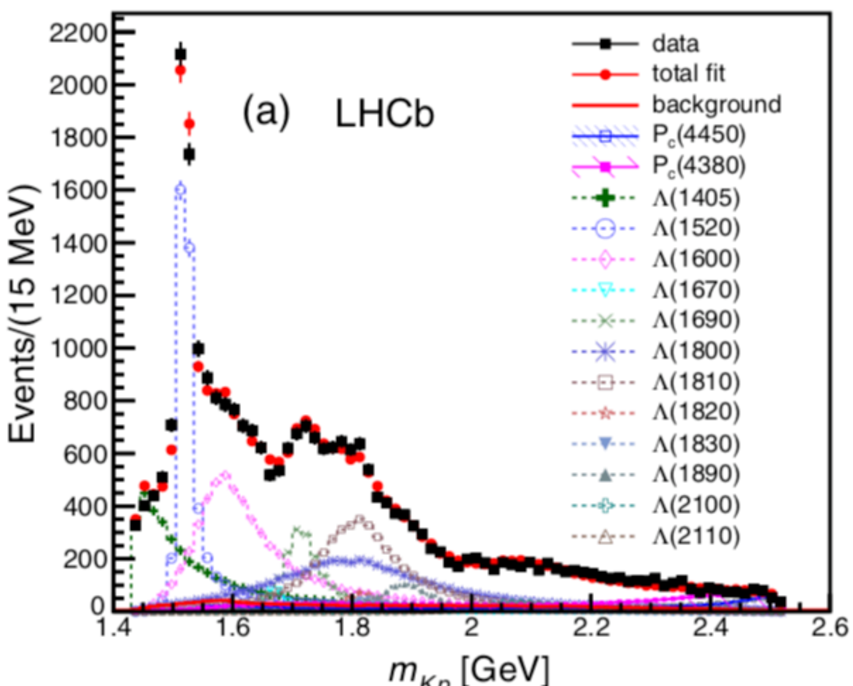


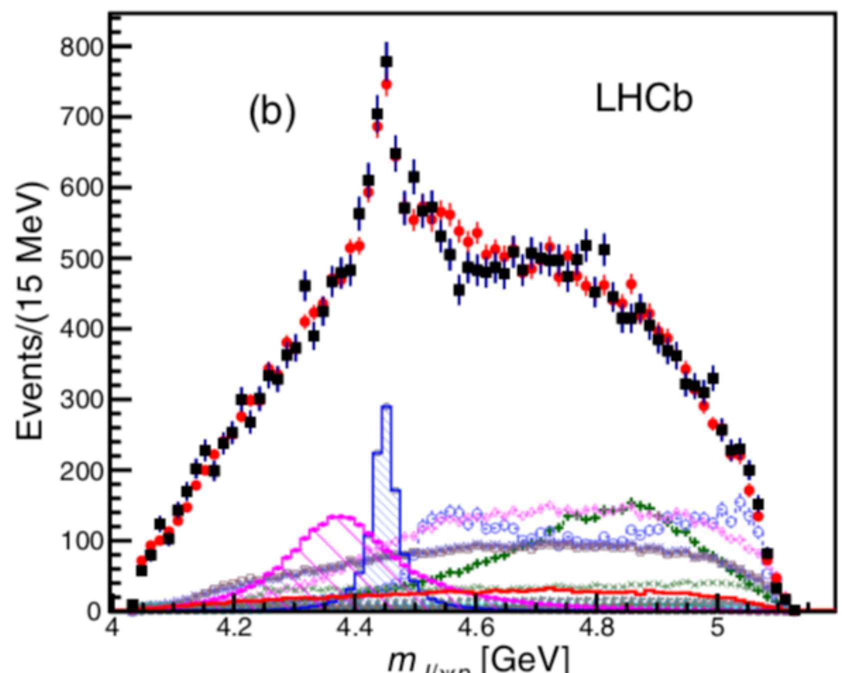
Two Breit-Wigner states

Pc(4380): $M = 4380 \pm 8 \pm 29$ MeV ;
$\Gamma = 205 \pm 18 \pm 86$ MeV

Pc(4450): $M = 4449.8 \pm 1.7 \pm 2.5$ MeV ;
$\Gamma = 39 \pm 5 \pm 19$ MeV

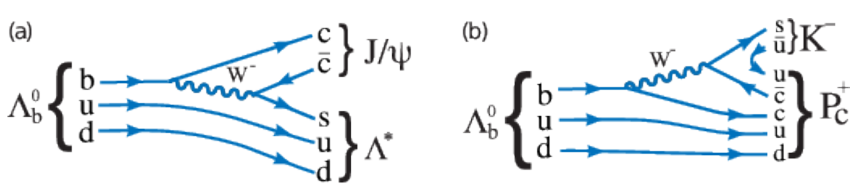
The preferred JP assignments are of opposite parity, with one state having spin 3/2 and the other 5/2: $(3/2^-,\ 5/2^+),\ (3/2^+,\ 5/2^-),\ (5/2^+,\ 3/2^-)$





Phys.Rev.Lett. 115 (2015) 072001

The pentaquark states



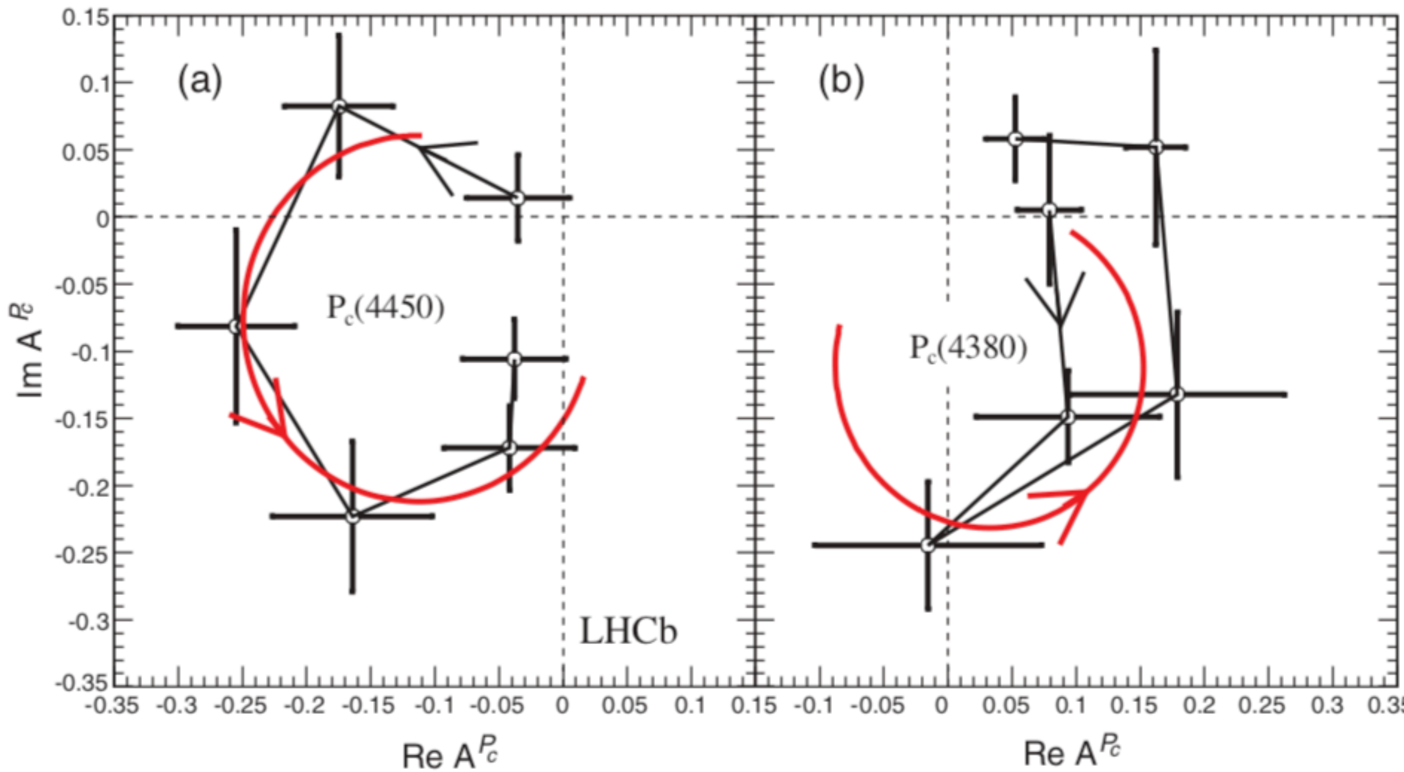
Two Breit-Wigner states

Pc(4380): $M = 4380 \pm 8 \pm 29$ MeV ;

$\Gamma = 205 \pm 18 \pm 86$ MeV

Pc(44450): $M = 4449.8 \pm 1.7 \pm 2.5$ MeV ;

$\Gamma = 39 \pm 5 \pm 19$ MeV



Argand plot

$3/2^-$, $5/2^+$

Phys.Rev.Lett. 115 (2015) 072001

The pentaquark states

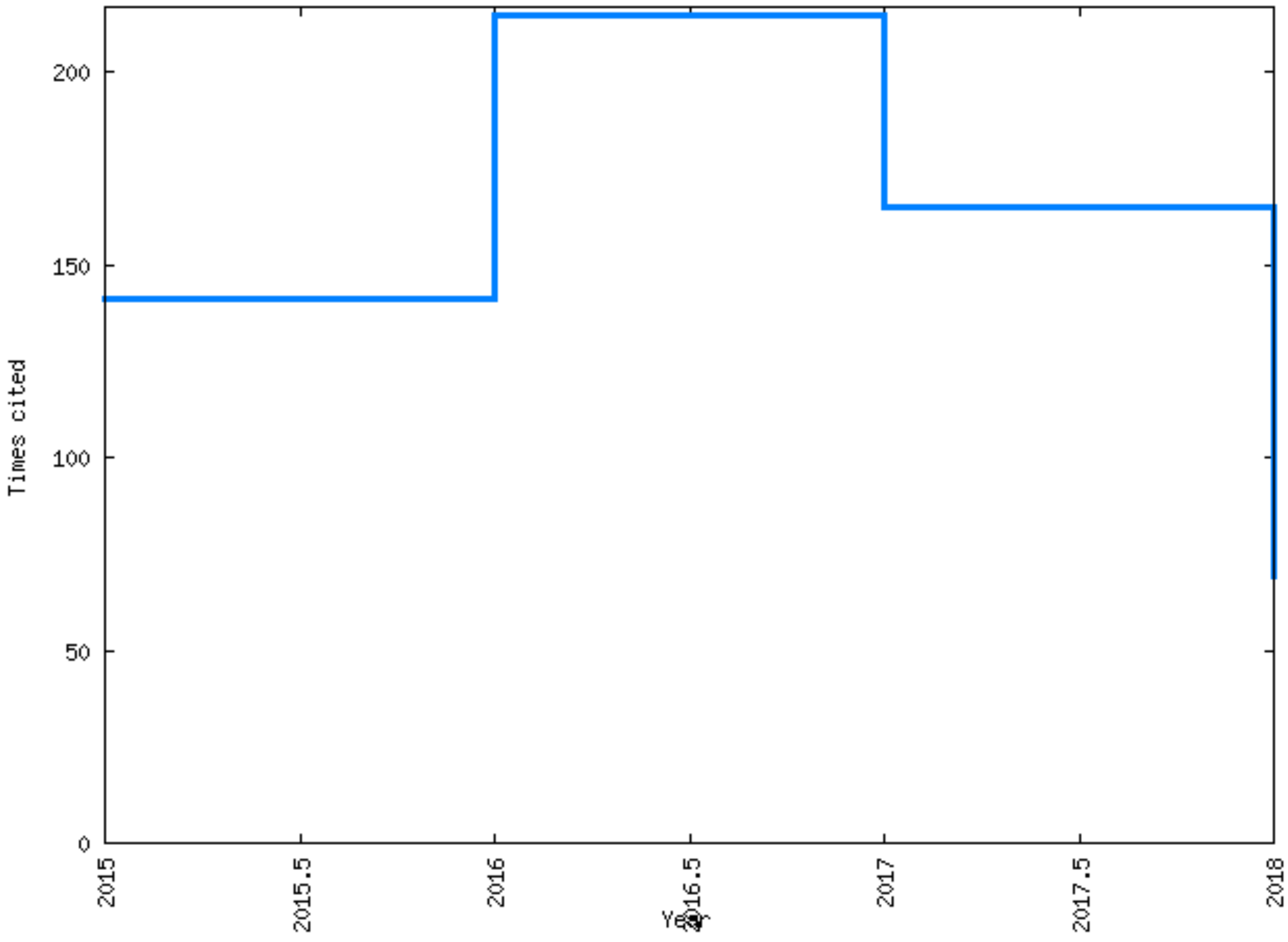
Even before the experimental discovery

- **Heavy-light diquarks.**
  - L. Maiani et al., Phys. Rev. D 71, 014028 (2005).
- **Diquark-diquark-antiquark**
  - R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003).
  - A. Chandra et al., Mod. Phys. Lett. A 27, 1250006 (2012).
- **Diquark- triquark**
  - M. Karliner and H. J. Lipkin, Phys. Lett. B 575, 249 (2003).
- **Coupled channel**
  - J.-J. Wu et al., Phys. Rev. Lett. 105, 232001 (2010).
- **Weakly bound "molecules" of a baryon plus a meson**
  - M. B. Voloshin and L. B. Okun, JETP Lett. 23, 333 (1976)
  - A. De Rujula et al., Phys. Rev. Lett. 38, 317 (1977)
  - N. A. Törnqvist, Phys. Rev. Lett. 67, 556 (1991); N.A. Törnqvist, Z. Phys. C 61, 525 (1994)
  - Z.-C. Yang et al., Chin. Phys. C 36, 6 (2012)
  - W. L. Wang et al., Phys. Rev. C 84, 015203 (2011)
  - M. Karliner and J. L. Rosner, Phys.Rev.Lett. 115, 122001 (2015)

Phys.Rev.Lett. 115 (2015) 072001

The pentaquark states

Phys.Rev.Lett. 115 (2015) 072001



Citation history from Inspire: a total of 593 as of July 18, 2018

The pentaquark states

- Tightly bound multiquark states
- Loosely bound multiquark sates—molecules
- Kinematical effects—triangle singularities

- One pion exchange or one boson exchange
- Unitary chiral coupled channels and their extensions
- Contact interactions formulated in an EFT language

Not easy to discriminate different scenarios, see, e.g., the debate on X(3872)

The pentaquark states

T.J. Burns
Eur. Phys. J. A (2015) 51: 152

	$P_c(4380)^+$	$P_c(4450)^+$
Mass	$4380 \pm 8 \pm 29$	$4449.8 \pm 1.7 \pm 2.5$
Width	$205 \pm 18 \pm 86$	$35 \pm 5 \pm 19$
Assignment 1	$3/2^-$	$5/2^+$
Assignment 2	$3/2^+$	$5/2^-$
Assignment 3	$5/2^+$	$3/2^-$
$\Sigma_c^{*+}\bar{D}^0$	4382.3 ± 2.4	
$\chi_{c1}p$		4448.93 ± 0.07
$\Lambda_c^{*+}\bar{D}^0$		4457.09 ± 0.35
$\Sigma_c^+\bar{D}^{*0}$		4459.9 ± 0.5
$\Sigma_c^+\bar{D}^0\pi^0$		4452.7 ± 0.5

Analogy

X(3872) ⟺ Pc(4450)

$$D\bar{D}^* + D^*\bar{D} \quad \Longleftrightarrow \quad \Sigma_c\bar{D}^* - \Lambda_{c1}\bar{D}$$

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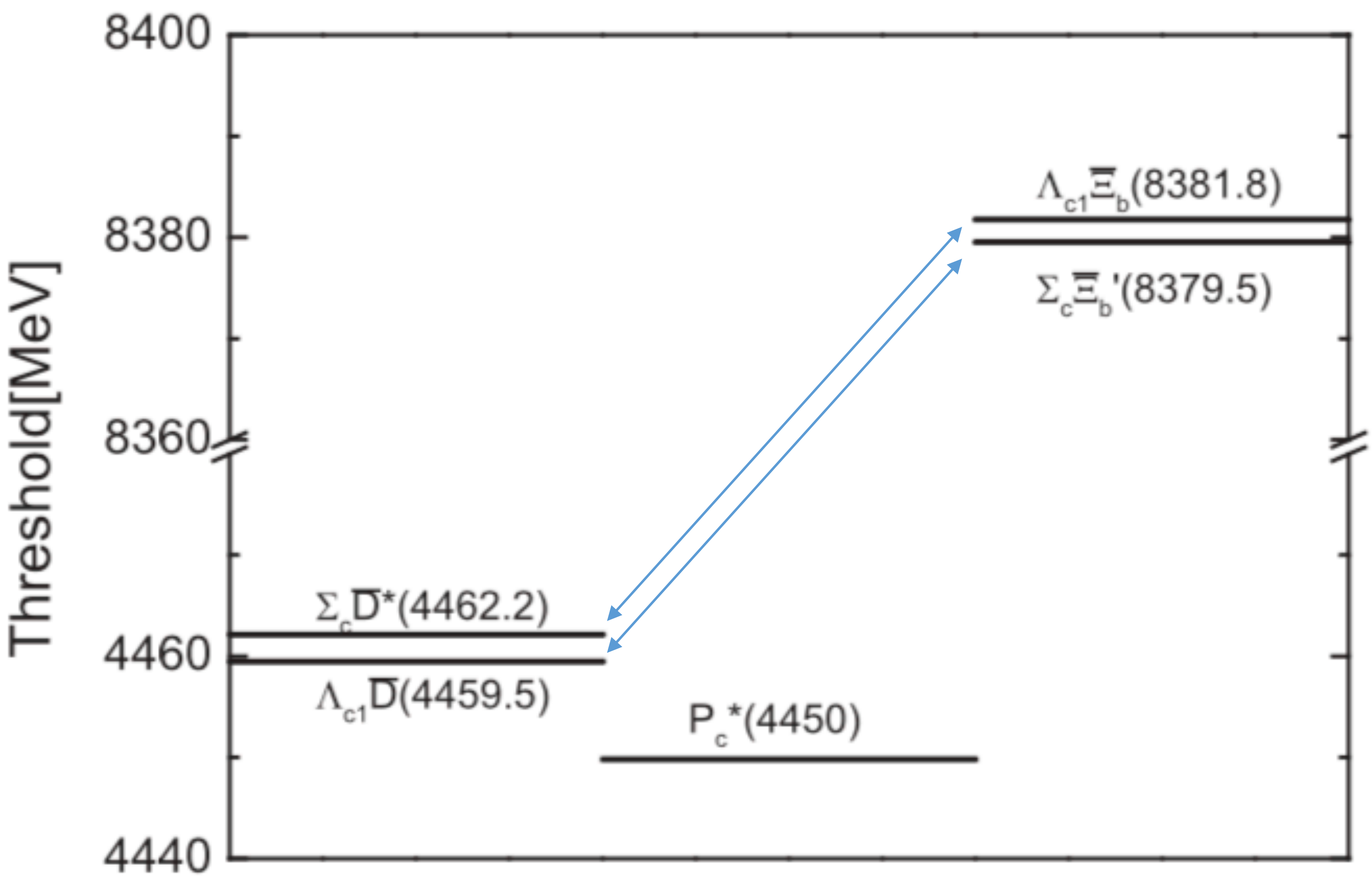
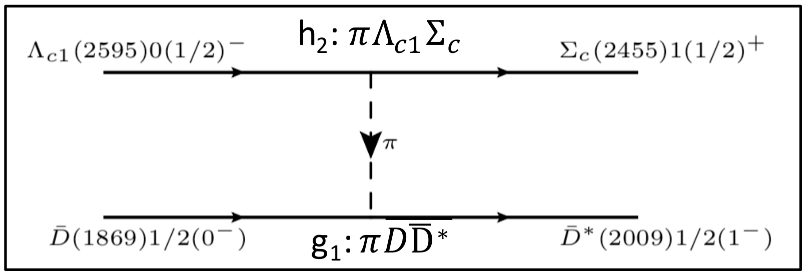
A two channel study of the $P_c(4450)$



A two channel study of the $P_c(4450)$

- There is standard one-pion exchange in the $\Sigma_c \bar{D}^*$ channel but no one-pion exchange in the $\Lambda_{c1} D$ channel—not our concern here

- In the off diagonal channel



$$\langle \Lambda_{c1}\bar{D}|V_{\rm OPE}(\vec{r})|\Sigma_c\bar{D}^*\rangle = \omega_\pi \tau \vec{\epsilon}\cdot\hat{r} W_E(r)$$

$\tau = \sqrt{3}$ for I =1/2; $\tau = 0$ for I =3/2

$$W_E(r) = \frac{g_1 h_2 \mu_\pi^2}{4\pi\sqrt{2} f_\pi^2}\frac{e^{-\mu_\pi r}}{\mu_\pi r}\left(1 + \frac{1}{\mu_\pi r}\right)$$

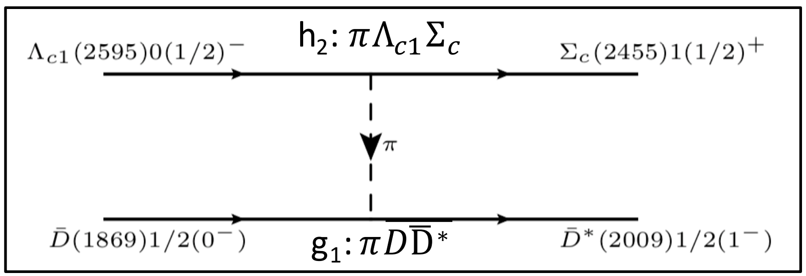
$$\omega_\pi = m(\Lambda_{c1}) - m(\Sigma_c)$$

$$\mu_\pi = \sqrt{m_\pi^2 - \omega_\pi^2}$$

A two channel study of the $P_c(4450)$

- There are standard one-pion exchange in the $\Sigma_c \bar{D}^*$ channel but no one-pion exchange in the $\Lambda_{c1} D$ channel—not our concern here

- In the off diagonal channel



$$|\mu_\pi| = \sqrt{\omega_\pi^2 - m_\pi^2} = 5\sim 35\ MeV \ll m_\pi$$

- In the $\mu_\pi = 0$ limit, for I=1/2, one has the following potential

$$\langle \Lambda_{c1}\bar{D}|V_{\rm OPE}(\vec{r})|\Sigma_c\bar{D}^*\rangle = \frac{g_1 h_2 \omega_\pi}{4\pi f_\pi^2}\sqrt{\frac{3}{2}}\frac{\vec{\epsilon}\cdot\hat{r}}{r^2} + \mathcal{O}(\mu_\pi^2 r^2)$$

A $1/r^2$ potential!!!

A $1/r^2$ potential and Efimov effect

- At zero energy the reduced Schroedinger equation for the s-wave

$$-u''(r) + \frac{g}{r^2} u(r) = 0$$

For a finite energy analysis, see M. Bawin and S. A. Coon, Phys. Rev. A 67, 042712 (2003).

  - **The above equation is scale invariant under the transformation $r \longrightarrow \lambda_0 r$,**
  - **The consequence is if E_n is the binding energy, so is $E_{n+1} = E_n/\lambda_0^2$**
  - **This resembles the three-body system if thinking of r as the hyper-radius ρ**

- For $g > -1/4$, the above equation has a power law solution

$$u(r) = c_+ r^{\frac{1}{2}+\nu} + c_- r^{\frac{1}{2}-\nu} \qquad \nu = \sqrt{1/4 + g},$$

- **For $g < -1/4$, the solution enjoys discrete scale invariance**

$$u(r) = c r^{1/2} \sin\left(\nu \log \Lambda_2 r\right) \quad \nu = \sqrt{-1/4 - g}$$

$$\lambda_0 = e^{\pi/\nu}$$

Λ_2 encodes short-distance physics and breaks the exact scale invariance

A surprising finding

The $\Sigma_c \bar{D}^* - \Lambda_{c1} D$ interaction behaves like a $\mathbf{1/r^2}$ potential, which may lead to **discrete scale invariance in a two-body hadronic system—an** Efimov like effect

$J^p = 3/2^-$

- Four coupled-channels

$$\Sigma_c\bar{D}^*(^2D_{3/2})\quad \Sigma_c\bar{D}^*(^4S_{3/2}),\ \Sigma_c\bar{D}^*(^4D_{3/2})\ \text{and}\ \Lambda_{c1}\bar{D}(^2P_{3/2})$$

- The Schroedinger equation reads

$$-\mathbf{u}'' + \left[2\mu_{P_c^*}\mathbf{V}_{\mathrm{OPE}} + \frac{\mathbf{L}^2}{r^2}\right]\mathbf{u} = 0$$

$$2\mu_{P_c^*}\mathbf{V}_{\mathrm{OPE}} + \frac{\mathbf{L}^2}{r^2} = \frac{\mathbf{g}(\frac{3}{2}-)}{r^2} = \frac{1}{r^2}\begin{pmatrix} 6 & 0 & 0 & g \\ 0 & 0 & 0 & g \\ 0 & 0 & 6 & -g \\ g & g & -g & 2 \end{pmatrix}$$

diagonalization

$$-u_i'' + \frac{g_i}{r^2}u_i = 0$$

$$g_i = \{6, 2, 3 + \sqrt{9+3g^2}, 3 - \sqrt{9+3g^2}\}$$

$J^p = 3/2^-$: no discrete scale invariance

- Four coupled-channels

  $\Sigma_c\bar{D}^*(^2D_{3/2})$ $\Sigma_c\bar{D}^*(^4S_{3/2})$, $\Sigma_c\bar{D}^*(^4D_{3/2})$ and $\Lambda_{c1}\bar{D}(^2P_{3/2})$

- The Schroedinger equation reads

$$-u_i'' + \frac{g_i}{r^2}u_i = 0$$

$$g_i = \{6, 2, 3 + \sqrt{9 + 3g^2}, 3 - \sqrt{9 + 3g^2}\}$$

- Negative $\boldsymbol{g_i}(< -\frac{1}{4})$ can trigger discrete scale invariance, which means $|g| > \frac{5}{4\sqrt{3}} \sim 0.7217$.

- With $g_1 = 0.59 \pm 0.01 \pm 0.07$ from $D^* \to D\pi/\gamma$, this requires $|h_2| > 1.21^{+0.25}_{-0.19}$, well above $h_2 = 0.60 \pm 0.07$ from **the CDF value** extracted from $\Lambda_{c1} \to \Sigma_c\pi$

- For $J^p = 1/2^-, 3/2^+, 5/2^+, 5/2^-$ the same conclusion

$J^p = 1/2^+$: discrete scale invariance likely

- Three coupled-channels

$$\Sigma_c \bar{D}^* (^2P_{1/2}) \;\; \Sigma_c \bar{D}^* (^4P_{1/2}) \text{ and } \Lambda_{c1} \bar{\tilde{D}} (^2S_{1/2})$$

- The Schroedinger equation reads

$$\mathbf{g}\left(\frac{1}{2}^+\right) = \begin{pmatrix} 2 & 0 & g \\ 0 & 2 & -\sqrt{2}g \\ g & -\sqrt{2}g & 0 \end{pmatrix}.$$

- The attractive eigenvalue is $1-\sqrt{1+3g^2}$, which requires $|g| > \frac{\sqrt{3}}{4} \sim 0.4330$ and $h_2 = 0.73^{+0.11}_{-0.06}$, marginally overlapping with that of CDF $h_2 = 0.60 \pm 0.07$

Long and short range consequences

- The approximate scale invariance of the Schroedinger equations has two consequences: **long range** and **short range**

- **The long range consequence** leads to the appearance of a geometric spectrum, depending on **how far the systems are from $\boldsymbol{\mu_\pi = 0}$**

- For $\mu_\pi \neq 0$, **scale invariance holds for**

$$R_s < r < \frac{1}{|\mu_\pi|}$$

The existence of a geometric excited state requires **the relative size of the scale invariant window to be bigger than the discrete scaling factor**

- For the Pc*, $|R_s \mu_\pi| \simeq 10\sim20$, requiring $|g_-| \geq 1$, **which is considerably larger than 1/4**

Long and short range consequences

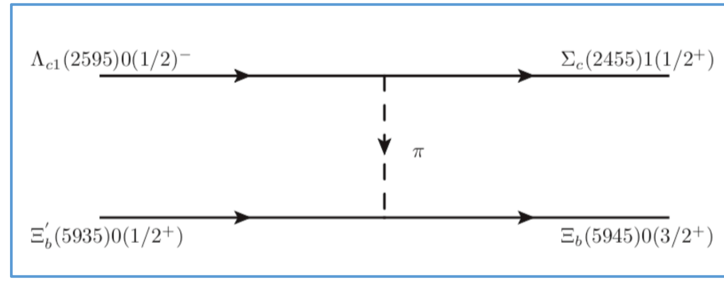
- The observation of geometric states in hadron and atomic physics shares a similar difficulty: **the fine-tuning of the pion mass (hadrons) or the scattering length (atoms).**

- In atomic physics, one can turn to a magnetic field

- In hadron physics, one can fine-tune the pion mass in the lattice or increase $|g_{-}|$, by **having a larger reduced mass** (two bottom hadrons) or **exchanging a kaon**.
  - For the first way out, we strongly encourage our lattice QCD colleagues to pursue such a study
  - We will explore the 2nd and 3rd options in the following.

（1） To have a larger reduced mass

$\Lambda_{c1}\overline{\Xi_b} - \Sigma_c\overline{\Xi_b}'$ baryonia system



- The potential

$$\langle \Sigma_c\bar{\Xi}_b{}'|V_{\rm OPE}(\vec{r})|\Lambda_{c1}\bar{\Xi}_b\rangle = \frac{g_3 h_2 \omega_\pi}{8\pi f_\pi^2}\frac{\sigma_2\cdot\hat{r}}{r^2} + \mathcal{O}(\mu_\pi^2 r^2)$$ g_3: $\overline{\Xi_b}\,\overline{\Xi_b}'\,\pi$

- Considering the following partial wave channels

$$0^+ = \Sigma_c\bar{\Xi}_b{}'({}^3P_0) - \Lambda_{c1}\bar{\Xi}_b({}^1S_0),$$
$$0^- = \Sigma_c\bar{\Xi}_b{}'({}^1S_0) - \Lambda_{c1}\bar{\Xi}_b({}^3P_0),$$
$$1^- = \Sigma_c\bar{\Xi}_b{}'({}^3S_1 - {}^3D_1) - \Lambda_{c1}\bar{\Xi}_b({}^1P_1 - {}^3P_1).$$

$$\mathbf{g}(0^+) = \begin{pmatrix} 2 & g \\ g & 0 \end{pmatrix},$$
$$\mathbf{g}(0^-) = \begin{pmatrix} 0 & g \\ g & 2 \end{pmatrix},$$
$$\mathbf{g}(1^-) = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{3}}g & -\sqrt{\frac{2}{3}}g \\ 0 & 6 & -\sqrt{\frac{2}{3}}g & -\frac{1}{\sqrt{3}}g \\ \frac{1}{\sqrt{3}}g & -\sqrt{\frac{2}{3}}g & 2 & 0 \\ -\sqrt{\frac{2}{3}}g & -\frac{1}{\sqrt{3}}g & 0 & 2 \end{pmatrix}$$

For $|g| > 3/4$, the attractive eigenvalues will trigger discrete scale invariance, which with $g_3 = 0.973^{+0.019}_{-0.042}$ **requires $|h_2| > 0.67^{+0.03}_{-0.02}$, overlapping with the CDF value**

Long and short range consequences

- Even if the vector force is not enough to trigger discrete scale invariance **it will still play a remarkable role in binding**

- Suppose the binding mechanism is s-wave short range physics, one has for $|r \leq R_s|$

$$V(r) = V_{\mathrm{OPE}}(r)\theta(r - R_s) + \frac{C_0(R_s)}{4\pi R_s^2}\delta(r - R_s)$$

  - In the one-channel problem of $g > -\frac{1}{4}$ and in the absence of tensor OPE, the relative strength of C_0 is $v + 1/2$ of that required to bind if $g = 0$ (for $|\mu_\pi \mathrm{R}_s| <$),
  - Thus if $v = 0$ ($g \to -1/4$) the short-range potential only has to be half the normal strength to be able to bind the system.

- If the binding mechanism is standard OPE or other intermediate physics, the number will change a bit but not the qualitative effect

  - For the $3/2^-$ Pc*, the number is 70%;
  - For the heavy baryonium, the number is 46% (0^-) or 53% (1^-)

Bottom line

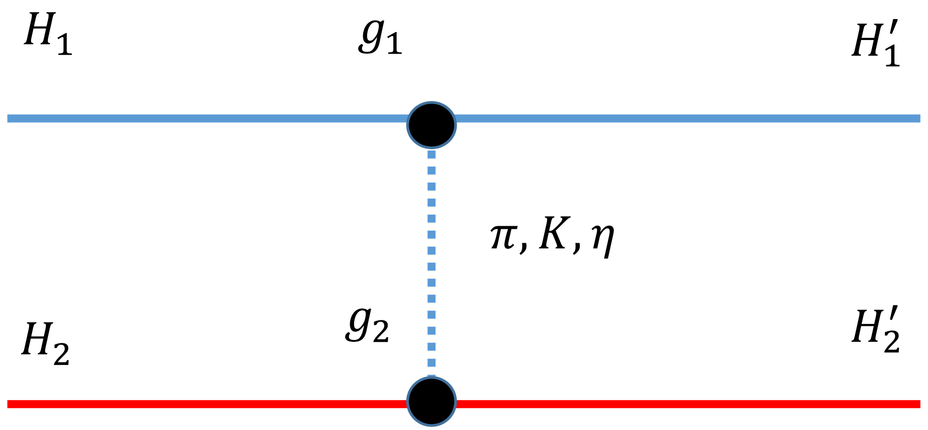
- If binding happens for distances in which the present picture is valid, **short-range physics is not necessary**
  - $3/2^-$, $1/2^+$, 0.94 fm, 0.92 fm
  - $0^-, 1^-, 1^+$, 0^+ baryoia 0.40 fm, 0.84 fm, 0.87fm,0.86 fm

two-pion exchange and hadron finite-size effects dominate for $r < \frac{m_\pi}{2} \sim 0.7$ fm,

- Two systems bind in p-wave, where the vector force effectively induces the existence of a channel behaving much like an s-wave

In short, the vector force induces a series of binding mechanisms which do not require the ratio m_π/μ_π to be particularly large (a factor of 2–3 is probably enough) and which in a few cases lead to predictions of new molecules.

For the 1/r^2 force to work



- H_1 and H_1' are of opposite parity but with the same spin
- H_2 and H_2' are of the same parity and their spin differs by one
- $m(H_{1/2}) - m(H_{1/2}') \approx m(\phi)$, **implying long range interaction**
- **The larger $g_1, g_2, m(\phi)$, the smaller $m(H_{1/2}) - m(H_{1/2}')$-m(ϕ), the stronger the attraction**

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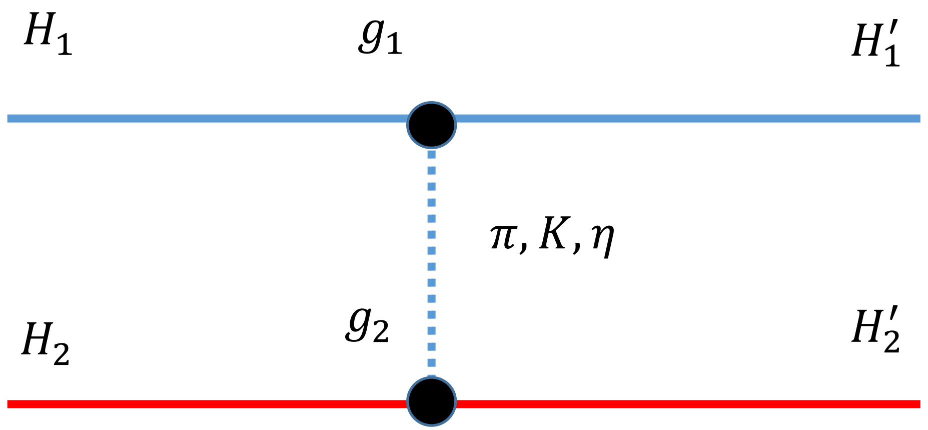
A coulomb like force: a modification of the $1/r^2$ case



- H_1 and H_1' are of opposite parity but with the same spin
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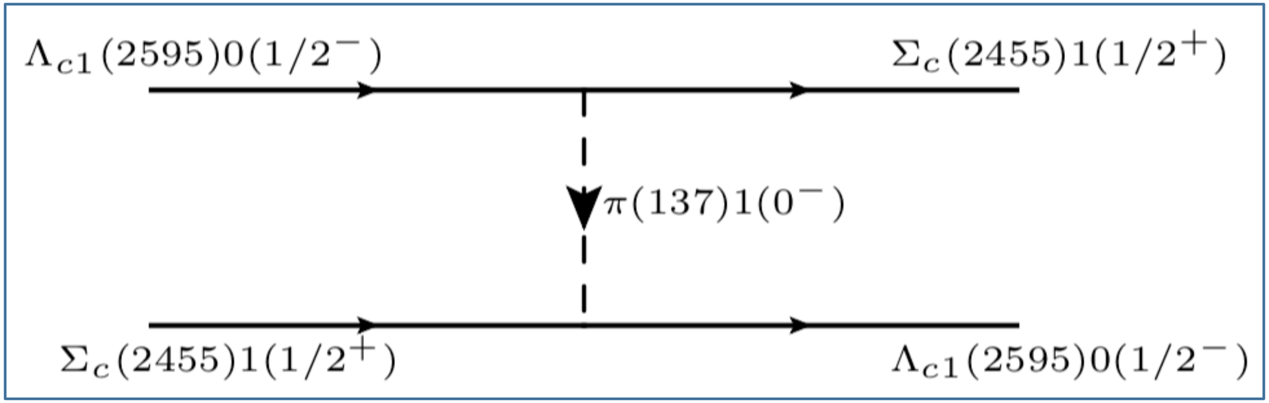
- $m(H_{1/2}) - m(H_{1/2}') \approx m(\phi)$, **implying long range interaction**
- **The larger g_1, g_2, $m(\phi)$, the smaller $m(H_{1/2}) - m(H_{1/2}')$- m(ϕ), the stronger the attraction**

A doubly charmed baryon $Y_{cc}(5050)$



$$V_{\rm OPE}(r) = -\frac{h_2^2\omega_\pi^2}{4\pi f_\pi^2}\frac{e^{-\mu_\pi r}}{r}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

where $\mu_\pi^2 = m_\pi^2 - \omega_\pi^2$ with $\omega_\pi = m_{\Lambda_{c1}} - m_{\Sigma_c}$

Attraction appears in spin 0 channel:

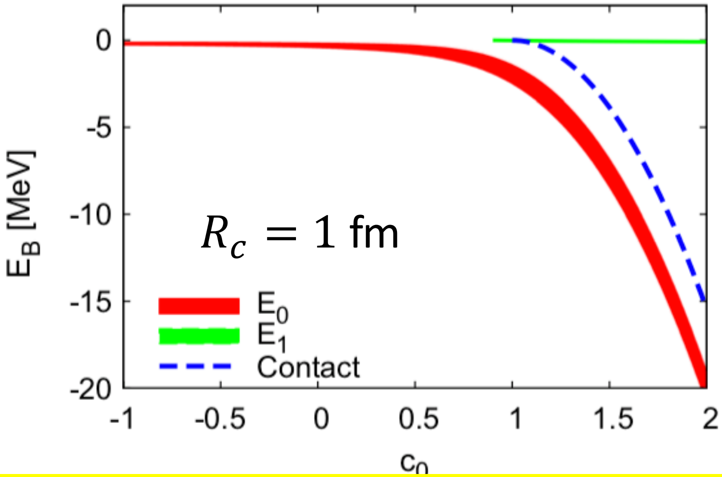
$$|Y_{cc}\rangle = \frac{1}{\sqrt{2}}\{|\Lambda_{c1}\Sigma_c\rangle + |\Sigma_c\Lambda_{c1}\rangle\}$$

A doubly charmed baryon $Y_{cc}(5050)$

Assuming again, that the one-baryon exchange is only valid above a certain cutoff radius

$$V(r) = V_{\rm OPE}(r)\theta(r - R_c) + \frac{C_0}{4\pi R_c^2}\delta(r - R_c).$$

Introducing a reduced coupling $c_0 = -\frac{2\mu_Y C_0}{4\pi R_c}$ and $c_0 \geq 1$ generates a bound state with binding momentum $\gamma = (c_0 - 1)/R_c$



- $c_0 \to -\infty$, $E_B = -0.09^{+0.06}_{-0.08}$ MeV
- $c_0 \to 1$, $E_B = -1.9^{+0.5}_{-0.6}$ MeV
- $c_0 > 0.9^{+0.2}_{-0.4}1$, a shallow excited state appears

$R_c = 1$ fm probably lies in an intermediate zone dominated by two-pion exchange and other contributions which might be attractive. Hence we expect the fundamental state of the doubly charmed Ycc to be deeper than the predictions from OPE alone

A hidden charmed baryon $Y_{c\bar{c}}$ (5050)

Replacing Σ_c with $\overline{\Sigma_c}$ will lead to a formation of **hidden charmed baryon $Y_{c\bar{c}}$(5050)** in both **spin 0 and 1**. The discussion is more involved because it involves annihilation.

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Rapid Communications

Are there near-threshold Coulomb-like Baryonia?

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The $\Lambda_c(2590)\Sigma_c$ system ... mass-shell. Owing to the opposite intrinsic parity of the $\Lambda_c(2590)$ and Σ_c, the pion is exchanged in S-wave. This gives rise to a Coulomb-like force that might be able to bind the system. If one takes into account that the pion is not exactly on the mass shell, there is a shallow S-wave state, which we generically call the $Y_{cc}(5045)$ and $Y_{c\bar{c}}(5045)$ for the $\Lambda_c(2590)\Sigma_c$ and $\Lambda_c(2590)\bar{\Sigma}_c$ systems respectively. For the baryon-antibaryon case this Coulomb-like force is independent of spin: the $Y_{c\bar{c}}(5045)$ baryonia will appear either in the spin $S = 0$ or $S = 1$ configurations with G-parities $G = (-1)^{L+S+1}$. For the baryon-baryon case the Coulomb-like force is attractive in the spin $S = 0$ configuration, for which a doubly charmed molecule is expected to form near the threshold. This type of spectrum might be very well realized in other molecular states composed of two opposite parity hadrons with the same spin and a mass difference close to that of a pseudo-Goldstone boson, of which a few examples include the $\Lambda(1405)N$, $\Lambda(1520)\Sigma^*$, $\Xi(1690)\Sigma$, $D^*_{s0}(2317)D$ and $D^*_{s1}(2460)D^*$ molecules.

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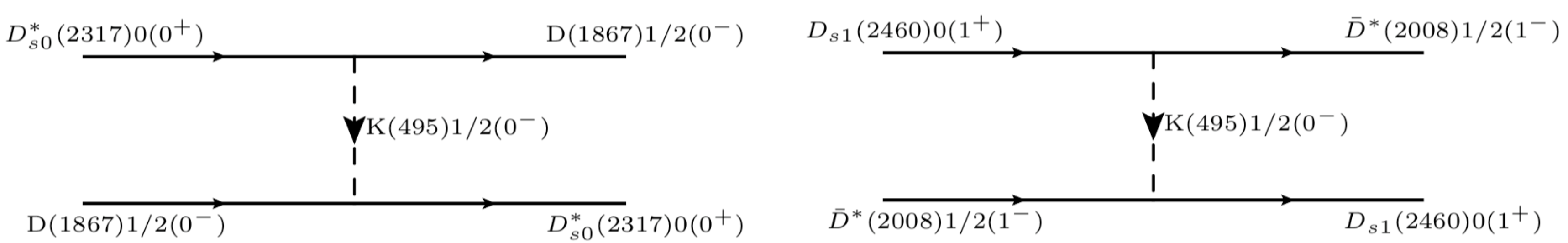
Exotic doubly charmed mesons

- **A straightforward extension of the above idea is investigating the exchange of the kaon**

- **In such a case, $D_{s0}^*(2317)D$ and $D_{s1}(2460)D^*$ are two interesting systems**



Using the $D_{s0}^*(2317)D$ system as one example

Exotic doubly charmed mesons

- In the following basis: $\{DD^*_{s0}, D^*_{s0}D\}\ \{D^*D^*_{s1}, D^*_{s1}D^*\}$

$$V(\vec{q}) = -h^2 \frac{\omega_K^2}{f_\pi^2} \frac{1}{m_K^2 - \omega_K^2 + \vec{q}^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\omega_K = m_{D^*_{s0}} - m_D \text{ or } m_{D^*_{s1}} - m_{D^*}$$

- The effective range is set by $\mu_K^2 = m_K^2 - \omega_K^2$, about 200 MeV

- For the following linear combinations, the interaction is attractive

$$\frac{1}{\sqrt{2}}\left[|DD^*_{s0}\rangle + |D^*_{s0}D\rangle\right],$$
$$\frac{1}{\sqrt{2}}\left[|D^*D^*_{s1}\rangle + |D^*_{s1}D^*\rangle\right]$$

$$V(r) = -h^2 \frac{\omega_K^2}{f_\pi^2} \frac{e^{-\mu_K r}}{4\pi r}$$

Exotic doubly charmed mesons

- The system will bind for $$\lambda_B = \frac{2\mu_H}{\mu_K}\frac{\omega_K^2}{4\pi f_\pi^2}h^2 \geq 1.68$$

which means for DD_{s0}^* and DD_{s1}, $|h| > 0.43$ and 0.40, respectively

- The requirement is satisfied, because $h \approx 0.5 \sim 0.9$ as deduced from the D meson decay

$$\begin{aligned}\Gamma(D_0 \to D\pi) &= \Gamma(D_0 \to D\pi^0) + \Gamma(D_0 \to D\pi^\pm)\\ &= \frac{3}{2}\Gamma(D_0 \to D\pi^\pm)\\ &= \frac{3}{2}\frac{m_D}{m_{D_0}}\frac{q_\pi}{2\pi}\frac{h^2}{f_\pi^2}(m_{D_0} - m_D)^2\,,\end{aligned}$$

$D_0^0 \to h = 0.61 \pm 0.07$
$D_0^0 \to h = 0.50 \pm 0.06$
$D_0^0 \to h = 0.8 \pm 0.2$

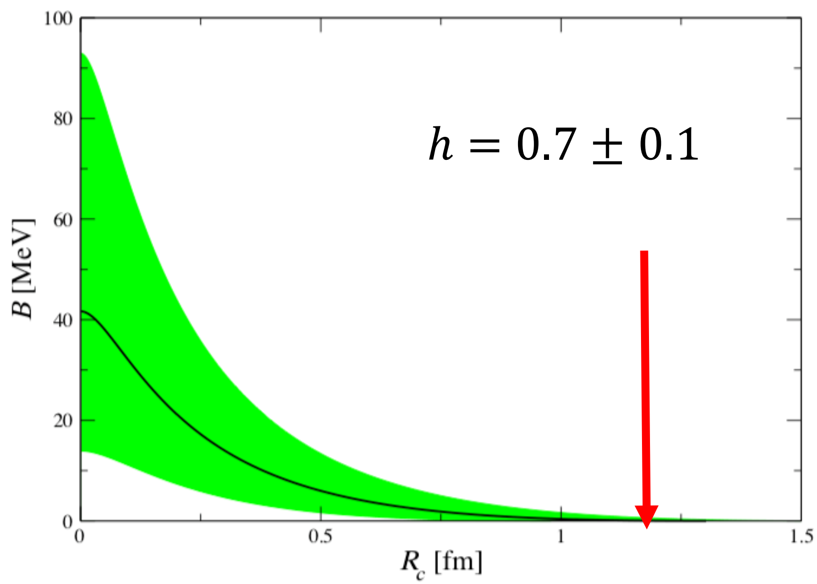
arXiv:1207.6940

- Concretely, one has $E_B = -40^{+30}_{-50}\ MeV \quad E_B = -50^{+30}_{-50}\ MeV$

Introducing a cutoff

- **Suppose the potential is only valid for large radius**

$$V(r; R_c) = V(r)\,\theta(r - R_c)$$



The system will bind for $R_c \leq 1.3 \pm 0.3\ fm$

For $R_c = 0.5\ fm$, $E_B = -6^{+4}_{-7}$ MeV

Understanding the results in EFT

- The kaon exchange can be rewritten

$$V_{\rm OKE}(\vec{q}) = -\frac{2\pi}{\mu_H\,\Lambda_{\rm OKE}}\,\frac{\mu_K^2}{\mu_K^2+\vec{q}^{\,2}} \qquad \Lambda_{\rm OKE} = \frac{2\pi}{\mu_H}\frac{f_\pi^2\mu_K^2}{h^2\omega_K^2} \simeq 50^{+40}_{-20}\,{\rm MeV}$$

- We count $V_{\rm OKE}(\vec{q}) \sim \frac{2\pi}{MQ}$, which is enhanced

- Heavy quark symmetry implies

$$V_C(\vec{q}, DD_{s0}^*) = C_{0a}\,,$$
$$V_C(\vec{q}, D^*D_{s1}^*) = C_{0a} + \vec{S}_1\cdot\vec{S}_2\,C_{0b}$$

- *M. P. Valderrama, Phys. Rev. D85, 114037 (2012), arXiv:1204.2400 [hep-ph].*
- *J.-X. Lu, L.-S. Geng, and M. Pavon Valderrama, (2017), arXiv:1706.02588 [hep-ph].*

Understanding the results in EFT

- In an natural scaling, the contact terms count as of subleading

$$C_{0a} \sim \frac{2\pi}{M^2} \quad , \quad C_{0b} \sim \frac{2\pi}{M^2}$$

- In an unnatural scaling, they also appear at LO (fine tuning)
  - Even in a worst case scenario, there exists a repulsive core at the cutoff radius

$$V_{\mathrm{EFT}} = V_{\mathrm{OKE}}(r)\,\theta(r - R_c) + C_0(R_c)\,\frac{\delta(r - R_c)}{4\pi R_c^2}$$

The system will still bind for $R_c \leq 1.3^{+0.3}_{-0.3}\ fm$

Summary and outlook

- We have studied the $\Sigma_c\bar{D}^* - \Lambda_{c1}D$ off diagonal interaction in an attempt to better understand the $P_c(4450)$ and accidentally identified a new binding mechanism that leads to discrete scale invariance
- We have identified two other similar mechanisms that are of long-range nature and can lead to relatively robust predictions of molecular states, namely $\Lambda_c(2590)\Sigma_c(\overline{\Sigma_c})$--$Y_{cc/\bar{c}}(5045)$, $D_{s0}^*(2317)D$ and $D_{s0}^*(2317)D^*$
- Many similar but more sophisticated studies are underway to explore the proposed mechanisms

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Thanks for your attention！

June 19th, 2018

Summary and outlook

$\Xi'_b(5935)^-$

$J^P = \frac{1}{2}^+$ Status: ∗∗∗

$\Xi'_b(5935)^-$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
5935.02±0.02±0.05	[1] AAIJ	15H	LHCB	pp at 7, 8 TeV

[1] Not independent of the mass difference measurement below. Observed in $\Xi_b^0 \pi^-$ channel with $\Xi_b^0 \to \Xi_c^+ \pi^-$ and $\Xi_c^+ \to p K^- \pi^+$.

$\Xi_b(5945)^0$

$J^P = \frac{3}{2}^+$ Status: ∗∗∗

Quantum numbers are based on quark model expectations.

$\Xi_b(5945)^0$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
5949.8±1.4 OUR AVERAGE				
5949.8±0.1±1.4	[1] AAIJ	16AE	LHCB	pp at 7, 8 TeV
5948.9±0.8±1.4	[2] CHATRCHYAN	12S	CMS	pp at 7 TeV, 5.3 fb^{-1}