Study of $K_S^0$ pair and $\eta_c(1S)$, $\eta_c(2S)$ and non-resonant $\eta'\pi\pi$ production in two-photon collisions at Belle

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On behalf of the Belle collaboration
IHEP & UCAS, Beijing

PWA10/ATHOS5 2018
16-20 July 2018, IHEP
Motivation of single-tag two-photon process

Reaction:
\[ e^+ e^- \rightarrow e^\pm (\text{undetected } e^\mp) \text{ hadrons} \]

- Study strong interaction in low energy region, where pQCD can’t be applied;
- Measure \( Q^2 \) dependence of Transition Form Factor (TFF);
- Provide input for a data-driven estimate of the hadronic light-by-light contribution significant for the problem of muon g-2.
Motivation of no-tag two-photon process

- Lowest heavy-quakonium $\eta_c(1S)$, plus J/psi, $\eta_b(1S)$ and $\Upsilon(1S)$, as benchmarks for the fine tuning of input parameters in QCD calculation.
- Attempt to measure $\Gamma_{\gamma\gamma}$ for $\eta_c(2S)$ and to address the discrepancy between data and QCD predictions.

- Improved precision in both data and QCD predictions at higher W mass would provide more sensitive comparisons.

- Pseudo-scalar meson pairs were measured by Belle [1]
  - Charged-meson pairs: $\pi^+\pi^-$, $K^+K^-$. Neutral-meson pairs: $K^0_SK^0_S$, $\pi^0\pi^0$, $\eta\pi^0$, $\eta\eta$.
  - Pseudo-scalar tensor pair $\eta'f_2(1270)$ and three-body final state $\eta'\pi\pi$ would provide new information to validate QCD models.

KEKB and Belle Detector

Tsukuba, Japan

$3.5 \text{ GeV } e^+ \text{ on } 8 \text{ GeV } e^-$

$W_{\text{CM}} = M(\Upsilon(4S,5S))$

3km circumference

$\sim 11\text{mrad crossing angle}$

$\text{L}_{\text{peak}} = 2.1 \times 10^{34}/\text{cm}^2/\text{s}$

TRISTAN tunnel
\[ \gamma^* \gamma \rightarrow K_S^0 K_S^0 \]

Dataset: 759 fb$^{-1}$

PRD 97, 052003 (2018)
No-tag results for $K_S^0 K_S^0$ process

Maximum at the $f_2'(1525)$ peak

$f_2(1270)/a_2(1320)$ destructive interference

Two-photon coupling of $f_0(1710)$

No data near the $K_S^0 K_S^0$ mass threshold

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PTEP 2013, 123C01 (2013)

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<table>
<thead>
<tr>
<th>Interference</th>
<th>$N_{\chi_{c0}}$</th>
<th>$N_{\chi_{c2}}$</th>
<th>$-2\ln\mathcal{L}/ndf$</th>
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<td>not included</td>
<td>$248.3^{+17.9}_{-17.2}$</td>
<td>$53.0^{+8.1}_{-7.4}$</td>
<td>$57.34/73$</td>
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<td>included</td>
<td>$266 \pm 53$</td>
<td>$53^{+14}_{-12}$</td>
<td>$57.22/71$</td>
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<table>
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<tr>
<th>Interference</th>
<th>$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c0})$ (eV)</th>
<th>$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c2})$ (eV)</th>
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<tr>
<td>not included</td>
<td>$8.09 \pm 0.58 \pm 0.83$</td>
<td>$0.268^{+0.041}_{-0.037} \pm 0.028$</td>
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<tr>
<td>included</td>
<td>$8.7 \pm 1.7 \pm 0.9$</td>
<td>$0.27^{+0.07}_{-0.06} \pm 0.03$</td>
</tr>
</tbody>
</table>

Belle 2007

| 7.00 $\pm$ 0.65 $\pm$ 0.71 | 0.31 $\pm$ 0.05 $\pm$ 0.03 |

PDG 2012

| 7.3 $\pm$ 0.5 | 0.297 $\pm$ 0.026 |
Reconstructed mass, angles and Energy of the Signal candidates

- **Reconstructed $K_S^0$ mass**

  - $W < 1.3$ GeV
  - $W = 1.3 - 3.0$ GeV
  - $W = 3.0 - 4.0$ GeV

- **Tag-$e$ angle**

- **$K_S^0$ angle**

- **$K_S^0$ energy**

  (in lab. frame, for the signal-candidate events)
\( \gamma^*\gamma \rightarrow K_S^0 K_S^0 \)

**Background processes**

Rejection of non-exclusive background, \( K_S^0 K_S^0 X \) using \( |\Sigma p_t^*| \) vs. \( E_{ratio} \)

- **Control region**
  - \( W < 1.3 \text{ GeV} \)
  - \( \Sigma p_t^* \) (GeV/c)

- **Signal region**
  - \( W = 1.3 - 2.6 \text{ GeV} \)
  - \( E_{ratio} \)

14% background only for \( W < 1.3 \text{ GeV} \)
\( \gamma^* \gamma \rightarrow K_S^0 K_S^0 \)

**Partial decay width of \( \chi_{cJ} \) mesons**

Assume that in total 7 events (3 events) peaking near the \( \chi_{c0} (\chi_{c2}) \) mass are purely from the charmonium (backgrounds are estimated <1 event in total)

Q\(^2\) dependence \( \Gamma_{\gamma^* \gamma} / \Gamma_{\gamma \gamma} \)

The first measurement of \( \chi_{cJ} \) production in high-Q\(^2\) single-tag two-photon collisions.

Solid curve: SBG [1] with the charmonium-mass scale (much favored).
Dashed curve: With the \( \rho \)-mass scale (VDM like)

Threshold enhancement, may be associated with $f_0(980)/a_0(980)$.

No $f_2(1270)/a_2(1320)$ is seen.
\[ \gamma^* \gamma \rightarrow K_S^0 K_S^0 \]

**Partial Wave Analysis for TFF of \( f_2'(1525) \)**

Waves: \( S, D_0, D_1, \) and \( D_2 \), contribute (\( W < 1.8 \text{ GeV} \))

\[
\frac{d\sigma(\gamma^* \gamma \rightarrow K_S^0 K_S^0)}{d\Omega} = \sum_{n=0}^{2} t_n \cos(n \varphi^*)
\]

\[
t_0 = |S Y_0^0 + D_0 Y_2^0|^2 + |D_2 Y_2^2|^2 + 2 \varepsilon_0 |D_1 Y_1^1|^2
\]

\[
t_1 = 2 \varepsilon_1 \Re(D_1 \ast |Y_2^1| (S Y_0^0 + D_0 Y_2^0))
\]

\[
t_2 = -2 \varepsilon_1 \Re(D_1 \ast |Y_2^1| (S Y_0^0 + D_0 Y_2^0))
\]

\( \varepsilon_0, \varepsilon_1 \) are variables that depend on \( x = \frac{q_i \cdot q_2}{p_1 \cdot p_2} \)

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**Parameterization of amplitudes**

\( S \) and \( D_1 \) amplitudes:

\[
S = A_{BW} e^{i \phi_{BW}} + B_S e^{i \phi_{BS}},
\]

\[
D_i = \sqrt{r_{ifa}(Q^2)(A_{f_2(1270)} - A_{a_2(1320)})} e^{i \phi_{f_2 D_i}}
\]

\[
+ \sqrt{r_{ifp}(Q^2)} A_{f'_2(1525)} e^{i \phi_{f'_2 D_i}}
\]

\[
+ B_{D_i} e^{i \phi_{D_i}}.
\]

\( r_{ifa}(Q^2) \) is fraction of \( f'_2(1525) \) contribution in D wave.

\[ r_{0fp} + r_{1fp} + r_{2fp} = 1 \]

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**Transition Form Factors**

\[
A_R^J(W) = \frac{F_R(Q^2)}{\sqrt{1 + \frac{Q^2}{m_R^2} \sqrt{\frac{8\pi(2J + 1)m_R}{W}}}}
\]

\[
\times \sqrt{\Gamma_{tot}(W) \Gamma_{\gamma \gamma}(W) B(K_S^0 K_S^0)} \frac{m_R^2 - W^2 - im_R \Gamma_{tot}(W)}{m_R^2 - W^2 - im_R \Gamma_{tot}(W)}
\]

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\[ 2018/7/20 \]
$\gamma^* \gamma \rightarrow K_S^0 K_S^0$

PWA results in W dependence at $Q^2$ bins

- Non-zero $D_0$ and $D_1$ components in the $f_2'(1525)$.
- No $f_2(1270)/a_2(1320)$ is seen.
- An enhancement near the threshold (0.995 GeV).
\( \gamma^*\gamma \rightarrow K_S^0K_S^0 \)

\( f_2'(1525) \) TFF results

The obtained helicity-0, -1, and -2 TFF of the \( f_2'(1525) \) meson as a function of \( Q^2 \).

- Shorter error bars: statistical
- Longer error bars: statistical and systematic
- Shaded areas: overall systematic on \( \Gamma_{\gamma\gamma} \).

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- Schuler, Berends, van Glick (SBG)

helicity-0 and -2 agree well with SBG.
helicity-1 -- slightly smaller, but not inconsistent.
\( \gamma^* \gamma \rightarrow \pi^0 \pi^0 \)

Dataset: 759 fb\(^{-1}\)

PRD 93, 032003 (2016)
Reconstructed $Q^2$, Energy and Angle of the Signal candidates

$\gamma^* \gamma \rightarrow \pi^0 \pi^0$

Pion Energy

Data

Signal MC

Pion Angle

Data

Signal MC
\( \gamma^* \gamma \rightarrow \pi^0 \pi^0 \)

**Background processes**

The wrong-sign events, that the tagged e\(^+\)(e\(^-\)) have wrong charge sign.

**Graphs:**

- (a) Data
- (b) Signal MC
- (c) \( W = 0.5 - 1.0 \text{ GeV} \)
- (d) \( W = 0.5 - 1.0 \text{ GeV} \)
- (e) \( W = 1.0 - 1.5 \text{ GeV} \)
- (f) \( W = 1.5 - 2.0 \text{ GeV} \)
$\gamma^* \gamma \to \pi^0 \pi^0$

W dependence and $\gamma^* \gamma$ cross section at $Q^2$ bins

Peaks corresponding to $f_0(980)$ and $f_2(1270)$. 
$\gamma^* \gamma \rightarrow \pi^0 \pi^0$

**TFF results**

- hel.-2 TFF of $f_2(1270)$ agrees with the prediction by Ref.[4] and Ref. [5].
- hel.-0 and 1 TFF, a factor of 1.5 – 2 smaller than the prediction by Ref.[4].
- TFF of $f_0(980)$; agree well with the prediction by Ref.[4] for $Q^2 < 10 \text{GeV}^2$

less steeper $Q^2$ dependence for $Q^2 > 10 \text{GeV}^2$

- **$f_2(1270)$ (hel.-2)**
- **$f_2(1270)$ (hel.-1)**
- **$f_2(1270)$ (hel.-0)**
- **$f_0(980)$**

Based on application of heavy quark approximation to light quarks
Based on sum rules
$$\gamma \gamma \rightarrow \eta' \pi^+ \pi^-$$

Dataset: 941 fb$^{-1}$

arXiv: 1805.03044
Submitted to PRD
\( \gamma \gamma \rightarrow \eta' \pi^+ \pi^- \)

Simultaneous Fit for \( \eta_c(1S) \) and \( \eta_c(2S) \)

<table>
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<th></th>
<th>( \eta_c(1S) )</th>
<th>( \eta_c(2S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma \rho )</td>
<td>( 1728^{+69}_{-68} )</td>
<td>( 945^{+38}_{-37} )</td>
</tr>
<tr>
<td>( n_s )</td>
<td>( 65^{+14}_{-13} )</td>
<td>( 41^{+9}_{-8} )</td>
</tr>
<tr>
<td>( M ) (MeV/c(^2))</td>
<td>( 2984.6 \pm 0.7 \pm 2.2 )</td>
<td>( 3635.1 \pm 3.7 \pm 2.9 )</td>
</tr>
<tr>
<td>( \Gamma ) (MeV)</td>
<td>( 30.8^{+2.8}_{-2.2} \pm 2.5 )</td>
<td>( 11.3 ) [fixed]</td>
</tr>
<tr>
<td>( \Gamma_{\gamma\gamma} \mathcal{B} ) (eV)</td>
<td>( 65.4 \pm 2.6 \pm 6.9 )</td>
<td>( 5.6^{+1.2}_{-1.1} \pm 1.1 )</td>
</tr>
</tbody>
</table>
Discussion on $\Gamma_{\gamma\gamma}$ of $\eta_c(2S)$

- Defining the ratio $R = \frac{\Gamma_{\gamma\gamma}(\eta_c(2S))B(\eta_c(2S))}{\Gamma_{\gamma\gamma}(\eta_c(1S))B(\eta_c(1S))}$, which is directly measured,

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>BaBar($K\bar{K}\pi$)[1]</th>
<th>CLEO[2]</th>
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<tbody>
<tr>
<td>$R$</td>
<td>$(8.6 \pm 2.6) \cdot 10^{-2}$</td>
<td>$(10.6 \pm 2.0) \cdot 10^{-2}$</td>
<td>$(18 \pm 5 \pm 2) \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

so, we have $R_B = \frac{B(\eta_c(2S)\rightarrow\eta'\pi\pi)}{B(\eta_c(1S)\rightarrow\eta'\pi\pi)} \approx \frac{B(\eta_c(2S)\rightarrow K\bar{K}\pi)}{B(\eta_c(1S)\rightarrow K\bar{K}\pi)}$ within error.

- Assuming $R_B \approx 1$ [3] and using the world average value $\Gamma_{\gamma\gamma}(\eta_c(1S)) = 5.1 \pm 0.4$ keV, we obtain $\Gamma_{\gamma\gamma}(\eta_c(2S)) = 0.44 \pm 0.13$ keV for $\eta'\pi\pi$ (this) and $0.54 \pm 0.11$ keV for BaBar($K\bar{K}\pi$) [1].

Both $\Gamma_{\gamma\gamma}(\eta_c(2S))$ values by Belle and BaBar are lower than $0.92 \pm 0.28$ keV from CLEO [2].

- QCD predictions for two-photon decay width of $\eta_c(2S)$ are ranged from 1.4 to 5.7.

- It is essential to have precise measurement of either $B(\eta_c(2S)\rightarrow K_sK\pi)$ or $B(B\rightarrow K\eta_c(2S))$.

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\[ \gamma \gamma \to \eta' \pi^+ \pi^- \]

**Study of \( \eta_c(1S) \to \eta' f_0(2080) \) decay with \( f_0(2080) \to \pi^+ \pi^- \)**

Black dots and red circles for events selected in \( \eta_c(1S) \) signal and sideband regions.

No enhanced structure is seen in the Dalitz distributions for the \( \eta_c(1S) \to a_2^\pm \pi^\mp \) with \( a_2^\pm \to \eta' \pi^\pm \).
Study of $\eta_c(1S) \rightarrow \eta' f_0(2080)$ decay with $f_0(2080) \rightarrow \pi^+ \pi^-$

$\gamma \gamma \rightarrow \eta' \pi^+ \pi^-$

$M = 2083^{+63}_{-66} \pm 32$ MeV, $\Gamma = 178^{+60}_{-178} \pm 55$ MeV
\( \gamma\gamma \rightarrow \eta' \pi^+ \pi^- \)

**Result of \( \sigma(\gamma\gamma \rightarrow \eta' f_2(1270)) \)**

- **Green dashed** is the leading term QCD predictions for neutral meson pairs \( \sim 1/W^{10} \) [1]
- No prediction for for \( \gamma\gamma \rightarrow \eta' f_2(1270) \).
- Assuming \( \sigma \sim 1/w^n \).
- The **red solid line** is the fitted value of \( n = 5.1 \pm 1.0 \) for \( |\cos\theta^*|<1 \) and \( n = 7.5 \pm 2.0 \) for \( |\cos\theta^*|<0.6 \).

\( \gamma \gamma \rightarrow \eta' \pi^+ \pi^- \)

**Result of \( \sigma(\gamma \gamma \rightarrow \eta' \pi \pi) \)**

(a). Structure near 1.8 GeV/c^2 is contributed from X(1835) or \( \eta(1760) \) [1].

(b) Enhancement at 2.1 GeV/c^2 is possible contribution from \( \gamma \gamma \rightarrow I(2100) \rightarrow \eta' f_0(980) \).

$\gamma\gamma \rightarrow \eta'\pi^+\pi^-$

Cross Section in $|\cos\theta^*|$

- Black dots with error bar are the $|\cos\theta^*|$ dependent cross sections in data
  $$\gamma\gamma \rightarrow \eta'f_2(1270)$$

Red lines, normalized to the data, follow a $1/\sin^4\theta$ behavior.

Measured cross section after subtracting the $\gamma\gamma \rightarrow \eta'f_2(1270)$ contribution in W region above 2.26 GeV. The distributions in data comparable with a uniform distribution (red lines).
Summary

Single-tag two-photon results

- Cross section for $\gamma^* \gamma \rightarrow K^0_s K^0_s$ has been measured for $2M(K^0_s) < W < 2.6$ GeV, $3 \text{ GeV}^2 < Q^2 < 30 \text{ GeV}^2$
- $Q^2$ dependence of $\Gamma_{\gamma^* \gamma} \chi c_0$ and $\chi c_2$ has been measured.
- $Q^2$ dependence of $f_2'(1525)$ TFF has been measured.
- First measurement for $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ with $Q^2$ up to 30 GeV$^2$.
- $Q^2$ dependence of $f_2(1270)$ TFF has been measured.

No-tag two-photon results

- First observation of $\eta_c(2S) \rightarrow \eta' \pi \pi$ with a significance $5.5\sigma$ including systematic error.
- First observation of $\eta_c(1S) \rightarrow \eta' f_0(2080)$ decay with $f_0(2080) \rightarrow \pi^+ \pi^-$ with a significance $20\sigma$.
- Measurements of pseudo-scalar tensor pair $\eta' f_2(1270)$ production, as well as that of $\eta' \pi \pi$, are made for the first time.

Thanks for your attention!
Backup
Experimental analysis of Single-tag $K^0_S K^0_S$ 

$e^+e^- \rightarrow e (e) K^0_S K^0_S$, $K^0_S \rightarrow \pi^+ \pi^-$ \hspace{1cm} 759 fb$^{-1}$

**Topology:** 1 electron(or positron) and 4 charged pions

**Event Selection Criteria:**
- **for tracks** 5 tracks satisfy $p_t > 0.1$ GeV/c, $\geq 2$ of them satisfy $p_t > 0.4$ GeV/c, 1 of them satisfies **e-identification** and $p > 1.0$ GeV/c
- **for $K^0_S$s** Charged $\pi/K$ separation
  - Reconstructed $K^0_S K^0_S$ masses (two-dimensional cut): $492.6 < \text{ave}[M(K^0_S)s] < 502.6$ MeV/c$^2$ and $\text{diff}[M(K^0_S)s] < 10$ MeV/c$^2$
  - $K^0_S$ decay vertex: $0.3 < v_r < 8$ cm
    - (a finite decay flight length in the $r\phi$ plane)

**Kinematical cuts** (Energy/momentum conservation and transverse-momentum balance)

$$E_{\text{ratio}} = \frac{E_{K^0_s K^0_s}^{\text{measured}}}{E_{K^0_s K^0_s}^{\text{expected}}}$$ and $$|\Sigma p_t^*|$$ satisfy

$$\sqrt{\left(\frac{E_{\text{ratio}} - 1}{0.04}\right)^2 + \left(\frac{|\Sigma p_t^*|}{0.1 \text{ GeV/c}}\right)^2} \leq 1$$

Masuda et al. (Belle), PRD 97, 052003 (2018)
Partial Wave Analysis for TFF of $f'_2(1525)$

Applied for $W < 1.8$ GeV. We take into account partial waves up to $J=2$. $J=1$ does not couple with $K^0_S K^0_S$ ($\rightarrow J^p = 0^+ \text{ and } 2^+$)

$$\frac{d\sigma(\gamma^*\gamma \to K^0_S K^0_S)}{d\Omega} = \sum_{n=0}^{2} t_n \cos(n\varphi^*),$$

$\gamma^*\gamma$ c.m. frame
$z^*$ axis // $\gamma^*$
$x^*z^*$plane includes tag-e

$$t_0 = |SY_0^0 + D_0 Y_2^0|^2 + |D_2 Y_2^2|^2 + 2\epsilon_0 |D_1 Y_2^1|^2,$$
$$t_1 = 2\epsilon_1 \Re \left[ (D_2^* Y_2^2) - S^* Y_0^0 - D_0^* Y_2^0 \right] D_1 \Re Y_2^1,$$
$$t_2 = -2\epsilon_0 \Re \left[ D_2^* Y_2^2 \right] (SY_0^0 + D_0 Y_2^0).$$

$S, D_0$, etc. --- Partial-wave amplitudes
$\epsilon_0, \epsilon_1$ --- Spin-dependent flux factor ratios for the virtual photon
$Y_j^m$ --- Spherical harmonics

Resonance amplitude for $f'_2$, etc.

$$A_R^J(W) = F_R(Q^2) \sqrt{1 + \frac{Q^2}{m_R^2}} \sqrt{\frac{8\pi(2J+1)m_R}{W}}$$
$$\times \frac{\sqrt{\Gamma_{tot}(W)\Gamma_{\gamma\gamma}(W)B(K^0_S K^0_S)}}{m_R^2 - W^2 - im_R \Gamma_{tot}(W)}$$

TFF of $f'_2$ for helicity $i = \lambda$

$$\sqrt{r_{ifp} F_{f2p}} (i = 0, 1, 2)$$
$$r_{0fp} + r_{1fp} + r_{2fp} = 1$$
From Uehara-san’s DIS 2018 report

Formalism of PWA and parametrizations

**Problems:** Low statistics

- Only 3 out of $S$, $D_0$, $D_1$ and $D_2$ are independent
- Non-unique solution (multiple solutions for resonances)

→ Parametrization of the amplitudes with modelled $W$ and $Q^2$ dependences

\[
S = A_{BW} e^{i\phi_{BW}} + B_{SE} e^{i\phi_{BS}},
\]
\[
D_i = \sqrt{r_{ifq}(Q^2)(A_{f_2(1270)} - A_{a_2(1320)})} e^{i\phi_{f_2D_i}}
+ \sqrt{r_{ifp}(Q^2)A_{f_2'(1525)}} e^{i\phi_{f_2D_i}}
+ B_{Di} e^{i\phi_{BD_i}},
\]

\[
A_{BW}(W) = \sqrt{\frac{8\pi m_S}{W}} \frac{f_S}{m_S - W^2 - im_{gs}g_S} \times \frac{1}{(Q^2/m_0^2 + 1)^{p_S}},
\]

Nominal fit $B_s = 0$

\[
B_s = \frac{\beta_{a_S} (W_0/W)^{b_S}}{(Q^2/m_0^2 + 1)^{c_{DS}}},
\]
\[
B_{D0} = \frac{\beta_{a_{D0}} (W_0/W)^{b_{D0}}}{(Q^2/m_0^2 + 1)^{c_{D0}}},
\]
\[
B_{D1} = \frac{\beta_{a_{D1}} (W_0/W)^{b_{D1}}}{(Q^2/m_0^2 + 1)^{c_{D1}}},
\]
\[
B_{D2} = \frac{\beta_{a_{D2}} (W_0/W)^{b_{D2}}}{(Q^2/m_0^2 + 1)^{c_{D2}}},
\]

\[
\beta = \sqrt{1 - 4m_{K^0_S}/W^2} \text{ is the } K^0 \text{ velocity}
\]

\[
r_{0f_p} : r_{1f_p} : r_{2f_p} = k_0 Q^2 : k_1 \sqrt{Q^2} : 1
\]

-Destructive interference between $f_2(1270)$ and $a_2(1320)$
- $r_i(Q^2)$ and TFF for $f_2(1270)$ and $a_2(1320)$ are the same;
use the values obtained in single-tag $\pi^0\pi^0$

Determine each component and the relative phase by a fit
Angular dependence and the PWA fit

Due to a lack of statistics, we use $Q^2$-integrated angular differential cross section derived with the following convention (MC generated isotropically):

$$\frac{d^2\sigma}{d|\cos \theta^*|d|\phi^*|} \propto N_{\text{EXP}}(|\cos \theta^*|,|\phi^*|)/N_{\text{MC}}(|\cos \theta^*|,|\phi^*|)$$

$Q^2$: integrated over the full range between 3 and 30 GeV$^2$

$W$: 4 bins

| $|\cos \theta^*|$ dependence ($|\phi^*|$ integrated) |
|-----------------------------------------------|
| ![Graph 1](1.1 GeV) ![Graph 2](1.3 GeV) ![Graph 3](1.5 GeV) ![Graph 4](1.7 GeV) |

| $|\phi^*|$ dependence ($|\cos \theta^*|$ integrated) |
|-----------------------------------------------|
| ![Graph 5](1.1 GeV) ![Graph 6](1.3 GeV) ![Graph 7](1.5 GeV) ![Graph 8](1.7 GeV) |

We regard this as the angular dependence at $<Q^2> = 6.5$ GeV$^2$

Fit:
Black: total
Red: $t_0$
Blue: $t_1 \cos \phi^*$
Magenta: $t_2 \cos 2\phi^*$

The fit is applied to the two-dimensional angular-dependence data.
Forward enhancement is from the helicity-0 component.
From PDG 2017

<table>
<thead>
<tr>
<th>Decay</th>
<th>Branching fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c(1S) \rightarrow K\bar{K}\pi$</td>
<td>$(7.3 \pm 0.5)%$</td>
</tr>
<tr>
<td>$\eta_c(2S) \rightarrow K\bar{K}\pi$</td>
<td>$(1.9 \pm 1.2)%$</td>
</tr>
<tr>
<td>$B \rightarrow K(\eta_c(1S) \rightarrow K_SK\pi)$</td>
<td>$(2.7 \pm 0.6) \times 10^{-5}$</td>
</tr>
<tr>
<td>$B \rightarrow K(\eta_c(2S) \rightarrow K_SK\pi)$</td>
<td>$(3.4^{+2.3}_{-16}) \times 10^{-6}$</td>
</tr>
</tbody>
</table>
Possible intermediate from $\gamma\gamma \rightarrow I(2100) \rightarrow \eta'f_0(980)$

$2.0 < W < 2.2\, GeV/c^2$

- In $f_0(980)$ signal region $0.86 < M(\pi\pi) < 1.10\, GeV/c^2$.
- $I(2100)$ with statistic significance $3.5\sigma$. 