Polarization observables in $e^+e^- \to$ baryon antibaryon

UPPSALA UNIVERSITET

Andrzej Kupsc
with E. Perotti, G. Fäldt, S. Leupold and J.J. Song

Motivation:

$e^+e^- \to B_1\bar{B}_2$
and J/ψ decays to $B_1\bar{B}_2$

BESIII

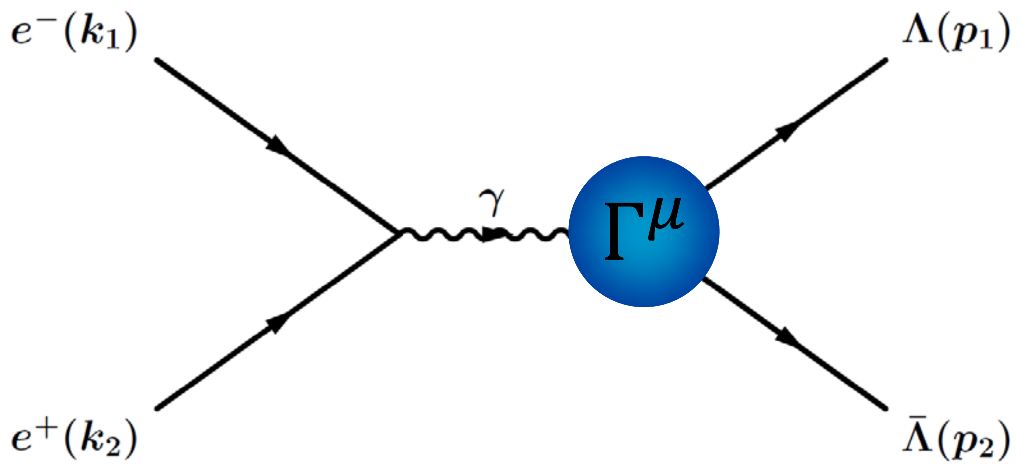
Helicity formalism for

$$e^+e^- \to \gamma^* \to B_{1/2}\bar{B}_{1/2}$$
$$\to B_{3/2}\,\bar{B}_{1/2}$$
$$\to B_{3/2}\bar{B}_{3/2}$$

and the sequential decays

PWA10/ATHOS5, IHEP, 2018-07-20

$e^+e^- \to \gamma^* \to B\overline{B}$ (spin 1/2)



$$s = (p_1 + p_2)^2$$
$$q = p_1 - p_2$$

$$\Gamma^\mu(p_1, p_2) = -ie\left[\gamma^\mu F_1(s) + i\frac{\sigma^{\mu\nu}}{2M_B}q_\nu F_2(s)\right]$$

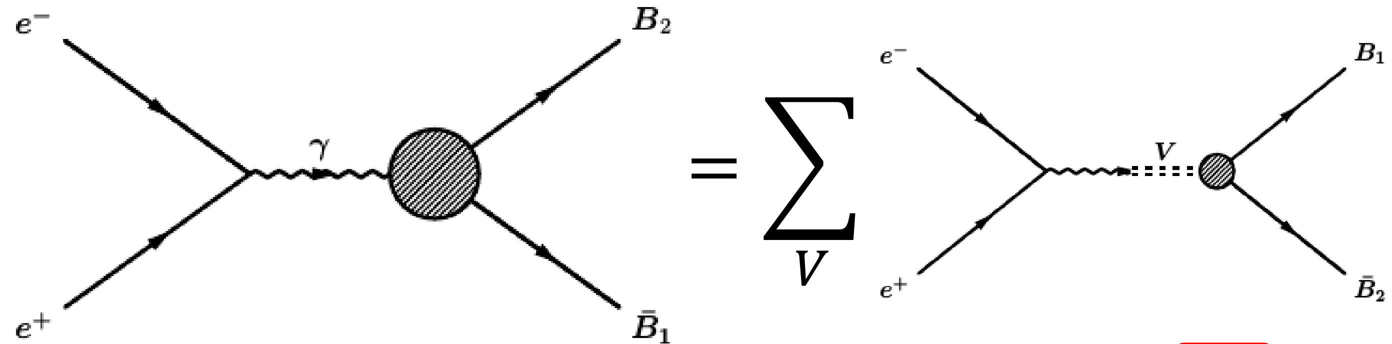
F_1 (Dirac) and F_2 (Pauli) Form Factors

Sachs Form Factors (FFs) ⇔ helicity amplitudes:

$$G_M(s) = F_1(s) + F_2(s), \quad G_E(s) = F_1(s) + \tau F_2(s)$$

$$\tau = \frac{s}{4M_B^2}$$

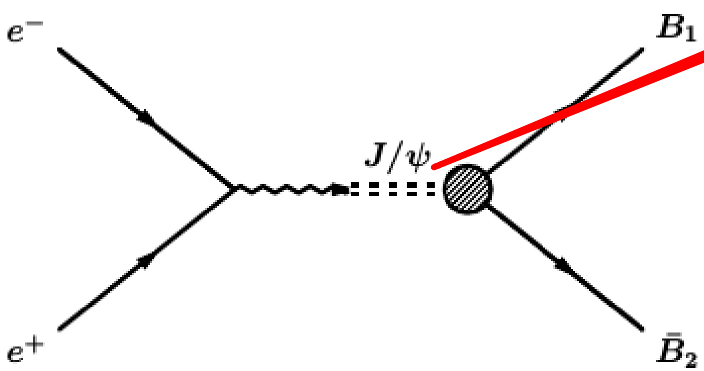
Baryon FFs (continuum):



Baryons B_1 and $\bar{B}_2$
(spin 1/2, 3/2, ...)

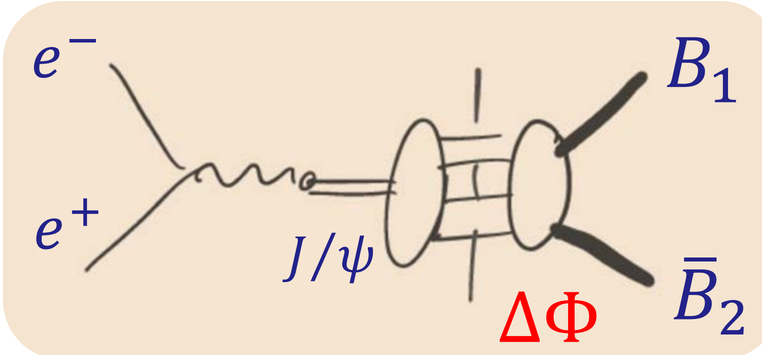
$V = \gamma, \rho, \omega, \varphi, \ldots, J/\psi, \psi(2S), \ldots$

vs J/ψ decay:



Both processes described by two complex FFs: relative phase $\Delta\Phi$

Cabibbo, Gatto PR124 (1961)1577



Time like spin ½ baryon FFs:
Dubnickova, Dubnicka, Rekalo
Nuovo Cim. A109 (1996) 241
Gakh, Tomasi-Gustafsson Nucl.Phys. A771 (2006) 169
Czyz, Grzelinska, Kuhn PRD75 (2007) 074026
Fäldt EPJ A51 (2015) 74; EPJ A52 (2016)141

Charmonia decays:
Fäldt, Kupsc PLB772 (2017) 16

$e^+e^- \rightarrow \gamma^* \rightarrow B\bar{B}$

The process at Born level is described by two complex FFs:

$G_M(s)$, $G_E(s)$

⇨ at given energy $\sqrt{s}$ three real parameters (neglecting overall phase):

- ❑ cross section (σ)
- ❑ angular distribution parameter (α_ψ) or R
- ❑ and relative phase ($\Delta\Phi$)

$$\alpha_\psi = \frac{\tau - R^2}{\tau + R^2} \qquad R = \left| \frac{G_E}{G_M} \right| \qquad G_E = R G_M e^{i\Delta\Phi}$$

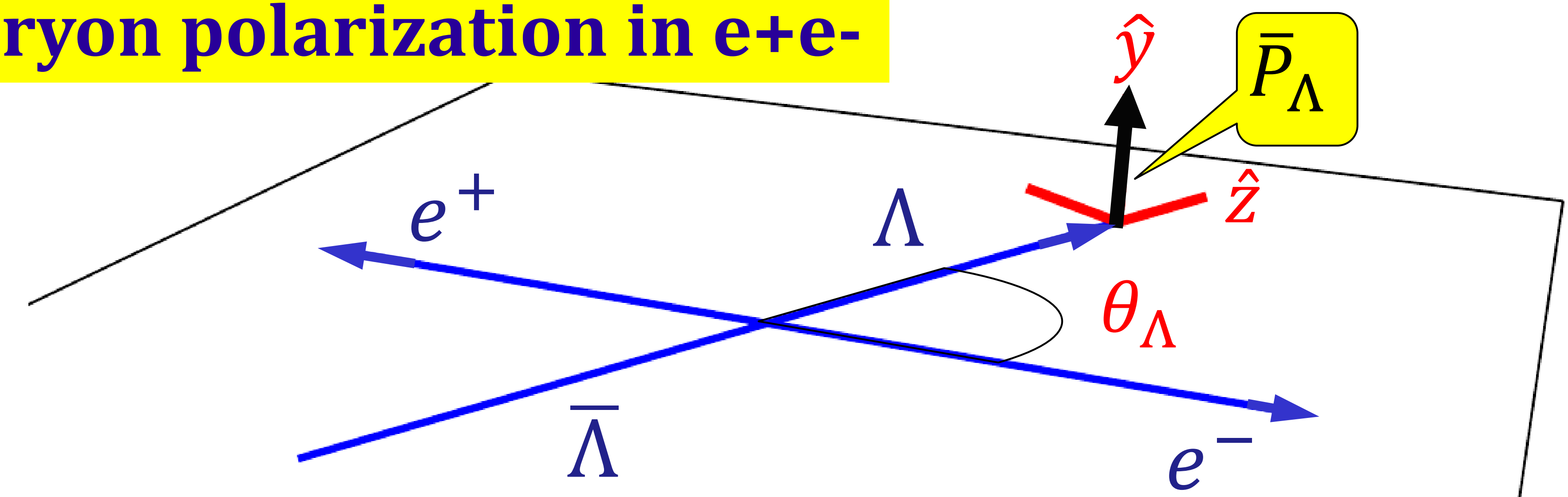
Baryon angular distribution:

(well known)

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha_\psi \cos^2\theta \qquad -1 \leq \alpha_\psi \leq 1$$

Phase predicted/expected for continuum
but neglected/not expected for the decays

Baryon polarization in e+e-



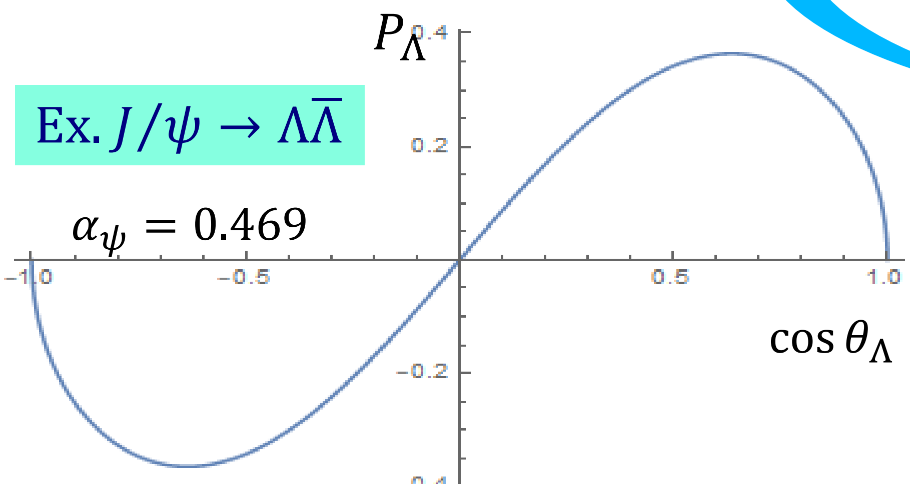
For unpolarized e+e- beams ⇨ transverse polarization:

$$\bar{P}_Y(\cos\theta_\Lambda) = \frac{\sqrt{1-\boldsymbol{\alpha_\psi}^2}\cos\theta_\Lambda\sin\theta_\Lambda}{1+\boldsymbol{\alpha_\psi}\cos^2\theta_\Lambda}\sin(\boldsymbol{\Delta\Phi})$$

$\Delta\Phi \neq 0$

Ex. $J/\psi \to \Lambda\bar{\Lambda}$

$\alpha_\psi = 0.469$

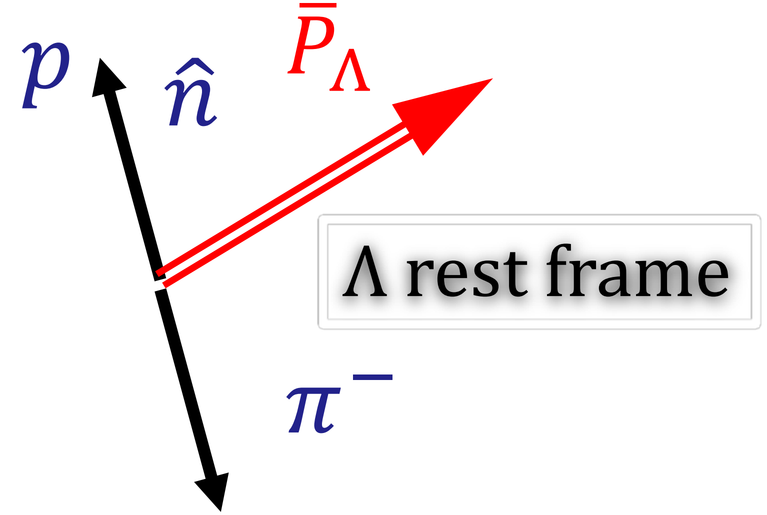


Max P_Λ= 36.4%

if $\Delta\Phi = \frac{\pi}{2}$

Weak decay $\Lambda \to p\pi^-$

$$\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1 + \alpha_{-}\hat{n}\,\bar{P}_{\Lambda})$$



p=100 MeV/c

Hyperon polarization determined from its own decay

Polarization of Λ is determined using this decay in all experiments.
Relies on:

α_{-}=0.642±0.013

World average based on 1963-75 experiments

Hyperon properties

α_-

hyperon	Mass [GeV/c²]	$c\tau$ [cm]	decay (BF)	α	ϕ
$\Lambda(uds)$	1.116	7.9	$p\pi^-$ (63.9%)	**0.642 ± 0.013**	$-6.5° \pm 3.5°$
	α_0		$n\pi^0$ (35.8%)		
$\bar{\Lambda}(\bar{u}\bar{d}\bar{s})$			$\bar{p}\pi^+$ (63.9%)	**−0.71 ± 0.08**	–
$\Sigma^-(dds)$	1.197	4.4	$n\pi^-$ (99.8%)	-0.068 ± 0.008	$10° \pm 15°$
$\Sigma^+(uus)$	1.189	2.4	$p\pi^0$ (51.6%)	-0.980 ± 0.017	$36° \pm 34°$
			$n\pi^+$ (48.3%)	-0.068 ± 0.013	$167 \pm 20°$
$\Xi^0(uss)$	1.315	8.7	$\Lambda\pi^0$ (99.5%)	-0.406 ± 0.085	$21° \pm 12°$
$\Xi^-(dss)$	1.321	5.1	$\Lambda\pi^-$ (99.8%)	-0.458 ± 0.012	$-2.1° \pm 0.8°$

α_+

CP violating asymmetries

$$A_{CP} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}$$

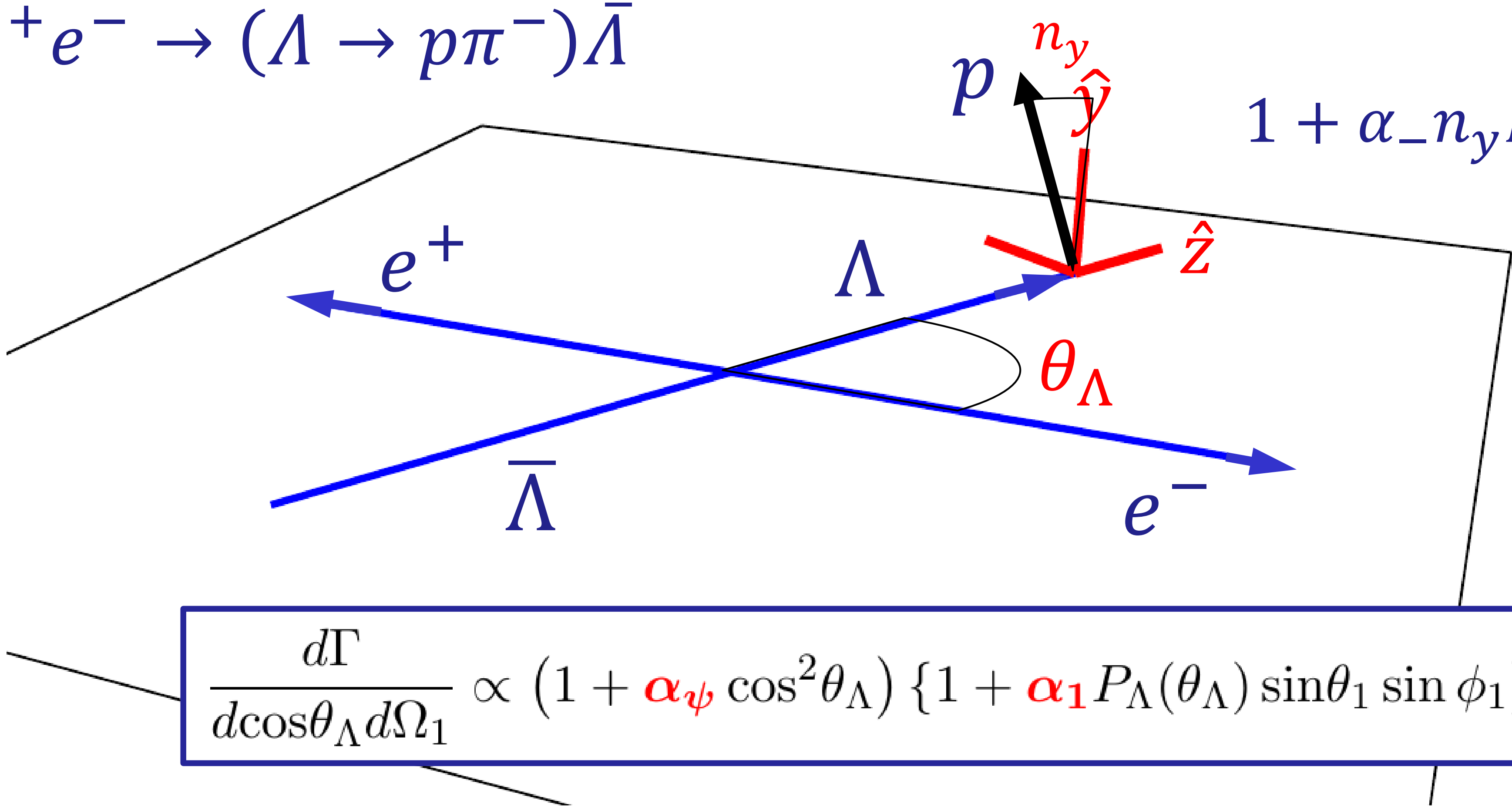
$$\beta = \sqrt{1 - \alpha^2}\,\sin\phi$$

$$B_{CP} = \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}}$$

In sequential decays also ϕ (β) is accessible

Inclusive angular distributions

$$e^+e^- \rightarrow (\Lambda \rightarrow p\pi^-)\bar{\Lambda}$$



$$1 + \alpha_- n_y P_y$$

$$\frac{d\Gamma}{d\cos\theta_\Lambda d\Omega_1} \propto \left(1 + \boldsymbol{\alpha_\psi} \cos^2\theta_\Lambda\right) \{1 + \boldsymbol{\alpha_1} P_\Lambda(\theta_\Lambda) \sin\theta_1 \sin\phi_1\}$$

$\Lambda \rightarrow p\pi^-$: $\Omega_1 = (\cos\theta_1, \phi_1)$: $\alpha_1 \rightarrow \alpha_-$

Hyperon polarization determined from
angular distribution of the nucleon from the weak decay

Exclusive angular distributions

Two decay modes for $\bar{\Lambda}$:

$e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$: $\alpha_2 \to \alpha_+$

$e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{n}\pi^0)$: $\alpha_2 \to \bar{\alpha}_0$

$\bar{\Lambda} \to \bar{p}\pi^+ (or\ \bar{n}\pi^0)$: $\Omega_2 = (\cos\theta_2, \phi_2)$

$\Lambda \to p\pi^-$: $\Omega_1 = (\cos\theta_1, \phi_1)$: $\alpha_1 \to \alpha_-$

$d\Gamma \propto \mathcal{W}(\boldsymbol{\xi})d\boldsymbol{\xi} = \mathcal{W}(\boldsymbol{\xi})d\cos\theta_\Lambda d\Omega_1 d\Omega_2$ $\quad \boldsymbol{\xi} : (\cos\theta_\Lambda, \Omega_1, \Omega_2)$ 5D PhSp

$$\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) = & 1 + \alpha_\psi \cos^2\theta_\Lambda \\
& + \alpha_1\alpha_2 \left(\sin^2\theta_\Lambda \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta_\Lambda \cos\theta_1 \cos\theta_2\right) \\
& + \alpha_1\alpha_2 \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)\left\{\sin\theta_\Lambda \cos\theta_\Lambda \left(\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2\right)\right\} \\
& + \alpha_1\alpha_2\, \alpha_\psi \left(\cos\theta_1 \cos\theta_2 - \sin^2\theta_\Lambda \sin\theta_1 \sin\theta_2 \sin\phi_1 \sin\phi_2\right) \\
& + \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)\sin\theta_\Lambda \cos\theta_\Lambda \left(\alpha_1 \sin\theta_1 \sin\phi_1 + \alpha_2 \sin\theta_2 \sin\phi_2\right)
\end{aligned}$$

Cross section

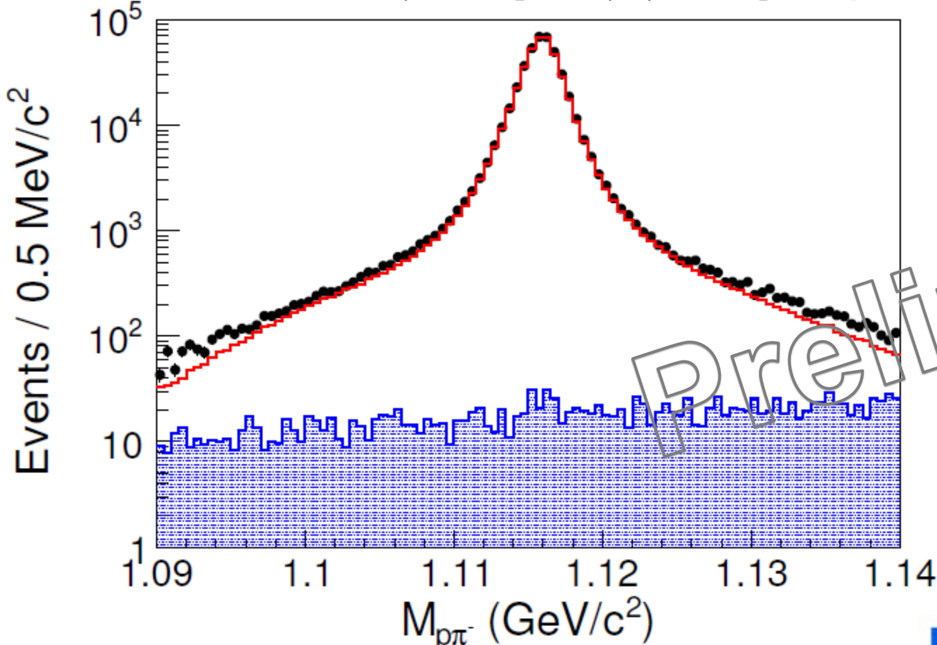
Spin correlations

Polarizations

$\Delta\Phi \neq 0 \Rightarrow$ independent determination of α_1 and α_2!

Fäldt, Kupsc PLB772 (2017) 16

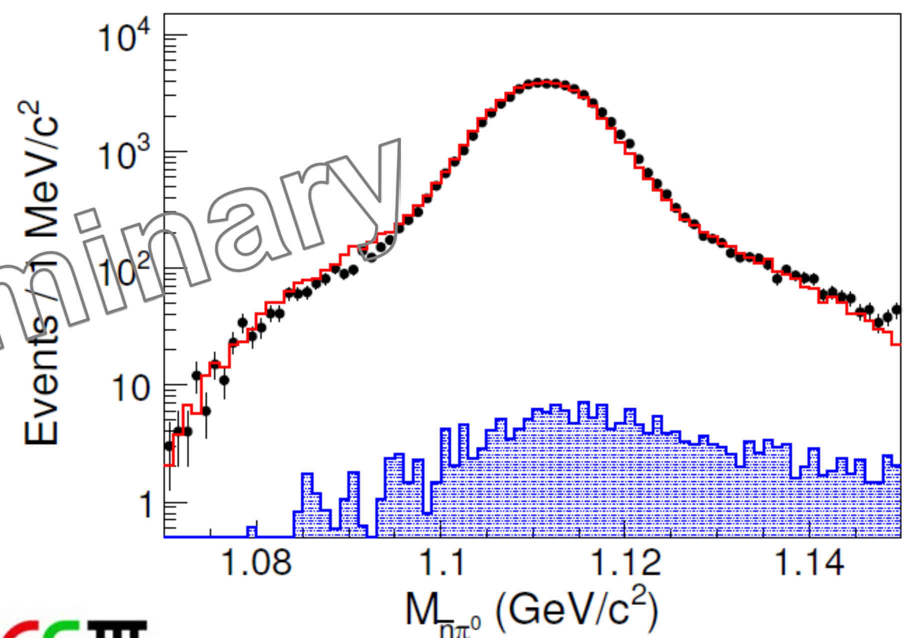
$e^+e^- \rightarrow (\Lambda \rightarrow p\pi^-)(\bar{\Lambda} \rightarrow \bar{p}\pi^+)$



420593 events

399 background

$e^+e^- \rightarrow (\Lambda \rightarrow p\pi^-)(\bar{\Lambda} \rightarrow \bar{n}\pi^0)$



47009 events

66 background

Global unbinned maximum log likelihood fit in **5D space** to the two data sets with the likelihood function constructed from probability function:

$$\mathcal{C}(\alpha_\psi, \Delta\Phi, \alpha_-, \alpha_2)\mathcal{W}(\boldsymbol{\xi}_i; \alpha_\psi, \Delta\Phi, \alpha_-, \alpha_2)$$

Where $\mathcal{C}(\alpha_\psi, \Delta\Phi, \alpha_-, \alpha_2)$ is the normalization factor obtained from $\mathcal{W}(\boldsymbol{\xi}_i; \alpha_\psi, \Delta\Phi, \alpha_-, \alpha_2)$ weighted sum for flat phase space model MC events after detector reconstruction.

Fit validation method

16 parameters for each θ:
I(θ), polarizations (6)
Spin correlations (9)

$$\rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\nu} C_{\mu\bar{\nu}}\sigma_\mu \otimes \sigma_{\bar{\nu}}$$

$$\mathcal{W}(\boldsymbol{\xi}) = \mathcal{I}(\theta)\Big\{1 + \alpha_\Lambda \sum_k \boxed{P_k(\theta)}\mathbf{n}_k + \alpha_{\bar\Lambda}\sum_{\bar k}\boxed{P_{\bar k}(\theta)}\mathbf{n}_{\bar k} + \alpha_\Lambda\alpha_{\bar\Lambda}\sum_{\bar k k}\boxed{C_{\bar k k}(\theta)}\mathbf{n}_{\bar k}\mathbf{n}_k\Big\}$$

polarizations (6)

Spin correlations (9)

$$P_y(\theta) = \sqrt{1-\alpha_\psi^2}\,\frac{\cos\theta\sin\theta}{1+\alpha_\psi\cos^2\theta}\sin(\Delta\Phi)$$

$$P_{\bar y}(\theta) = P_y(\theta).$$

$$\mathcal{I}(\theta) = 1 + \alpha_\psi\cos^2\theta.$$

$$C_{\bar z z}(\theta)\mathcal{I}(\theta) = -\alpha_\psi + \cos^2\theta$$

$$C_{\bar x x}(\theta)\mathcal{I}(\theta) = -\sin^2\theta$$

$$C_{\bar y y}(\theta)\mathcal{I}(\theta) = -\alpha_\psi\sin^2\theta$$

$$C_{\bar x z}(\theta)\mathcal{I}(\theta) = -\sqrt{1-\alpha_\psi^2}\cos\theta\sin\theta\cos(\Delta\Phi)$$

$$C_{\bar z x}(\theta) = C_{\bar x z}(\theta)$$

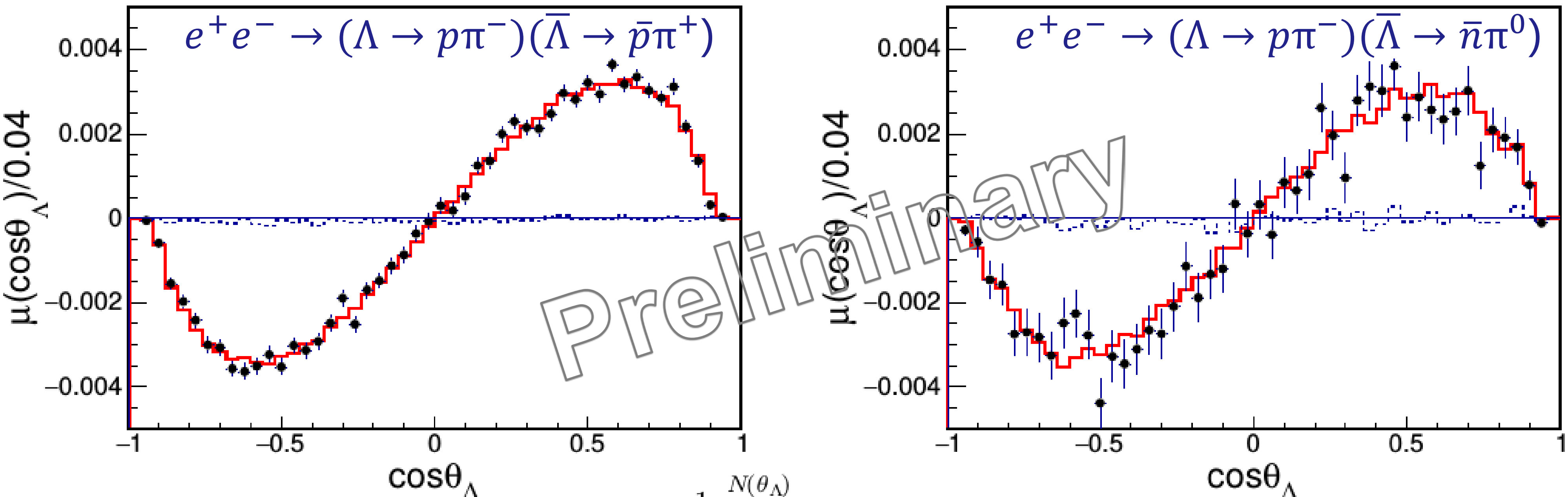
moments:

$$M(\theta) = \sum_i^{N(\theta)} \mathbf{n}_\mu^i\mathbf{n}_\nu^i$$

(uncorrected for acceptance)

Fit results

$\Delta\Phi = 42.3^{o} \pm 0.6^{o} \pm 0.5^{o}$



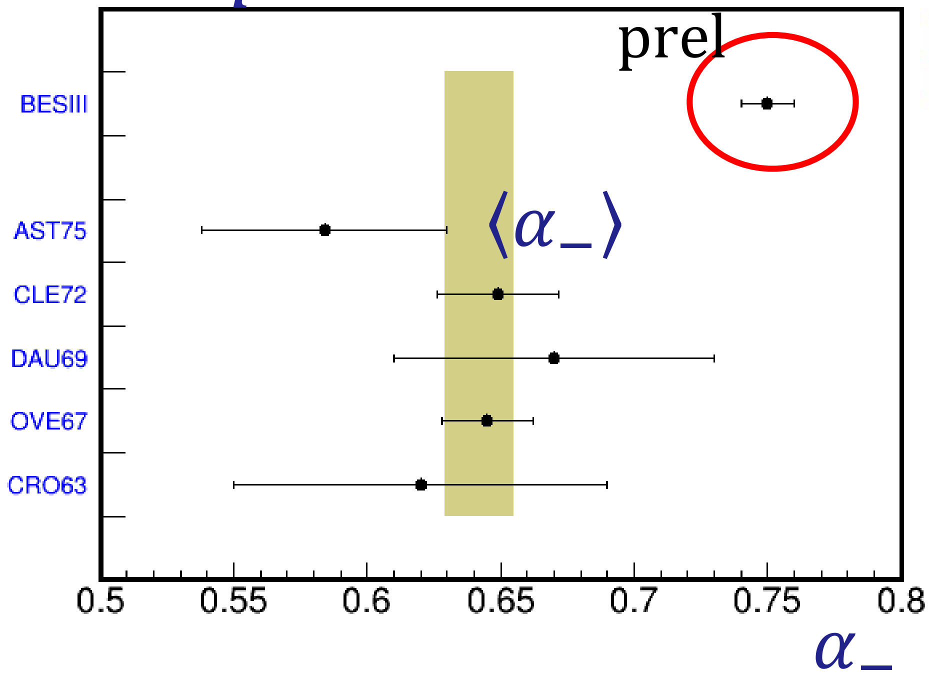
Moment: $\mu(\cos\theta_\Lambda) = \frac{1}{N}\sum_{i}^{N(\theta_\Lambda)}(\sin\theta_1^i \sin\phi_1^i - \sin\theta_2^i \sin\phi_2^i)$

Parameters	This work	Previous results
α_ψ	$\mathbf{0.461 \pm 0.006 \pm 0.007}$	0.469 ± 0.027 BESIII
$\Delta\Phi$ (rad)	$\mathbf{0.740 \pm 0.010 \pm 0.008}$	–
α_-	$\mathbf{0.750 \pm 0.009 \pm 0.004}$	0.642 ± 0.013 PDG
α_+	$\mathbf{-0.758 \pm 0.010 \pm 0.007}$	-0.71 ± 0.08 PDG
$\bar{\alpha}_0$	$\mathbf{-0.692 \pm 0.016 \pm 0.006}$	–
A_{CP}	$\mathbf{-0.006 \pm 0.012 \pm 0.007}$	0.006 ± 0.021 PDG
$\bar{\alpha}_0/\alpha_+$	$\mathbf{0.913 \pm 0.028 \pm 0.012}$	–

BESIII

Summary of the $J/\psi \to \Lambda\bar{\Lambda}$ analysis

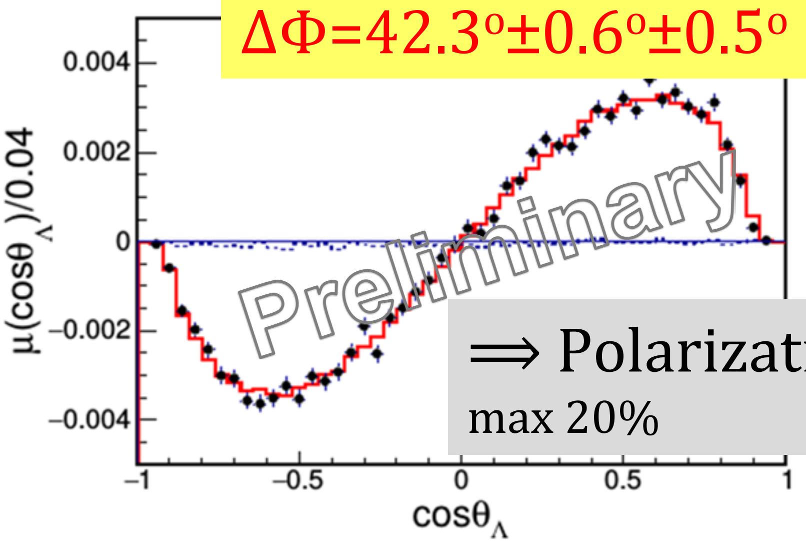
$\Lambda \to p\pi^-$: α_-=0.750±0.009±0.004



BESIII

17(3)% larger than PDG average
> 5 σ difference

ΔΦ=42.3°±0.6°±0.5°



⟹ Polarization:
max 20%

CP test: $A_{CP} = \dfrac{\alpha_- + \alpha_+}{\alpha_- - \alpha_+}$

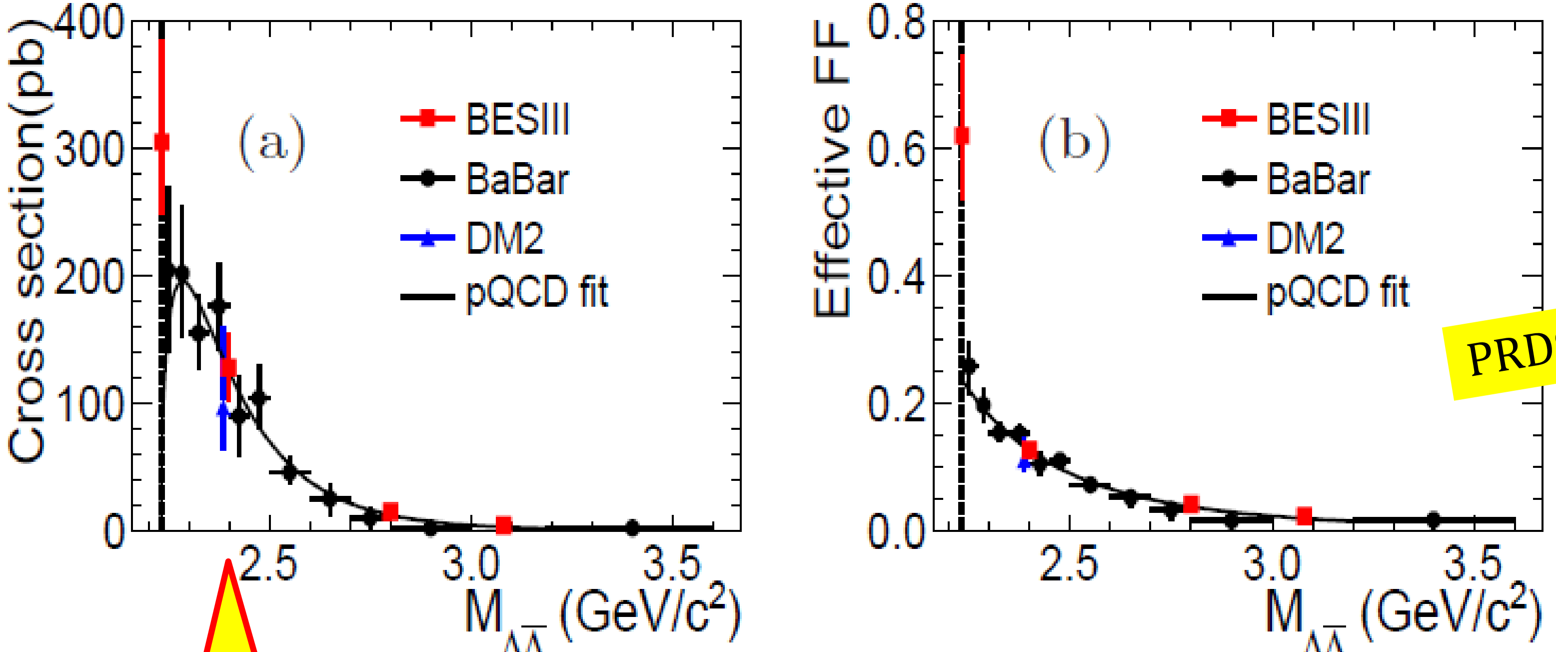
A_{CP} = -0.006 ± 0.012± 0.007 prel

Previous result (using αP product):

A_{CP}= 0.013 ± 0.021
PS185 PRC54(96)1877

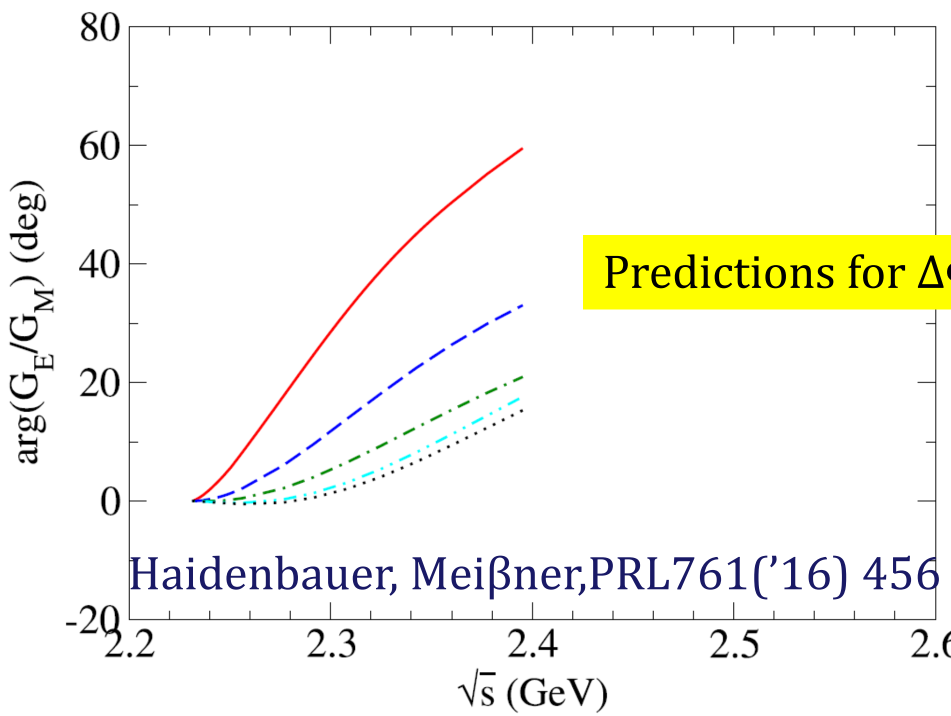
CKM $A_{CP} \sim 10^{-4}$

$e^+e^- \to \gamma^* \to \Lambda\bar{\Lambda}$ (continuum)



PRD97('18) 032013

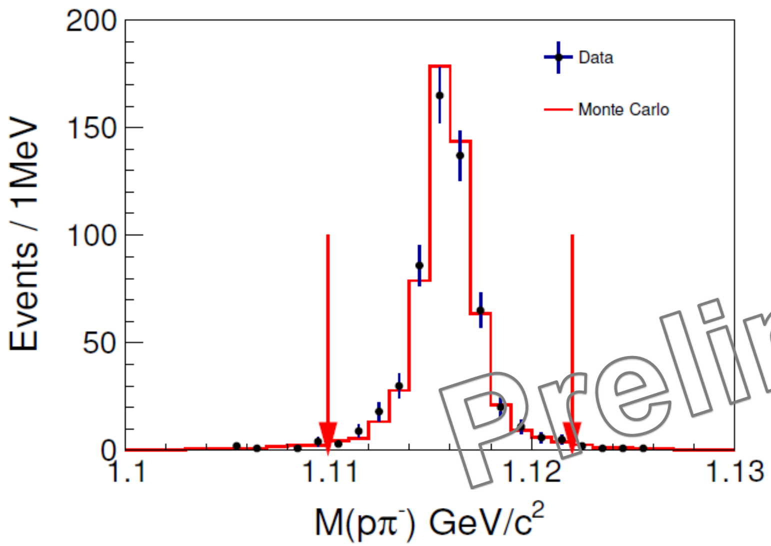
$\sqrt{s} = 2.396$ GeV
66.9 pb^{-1}

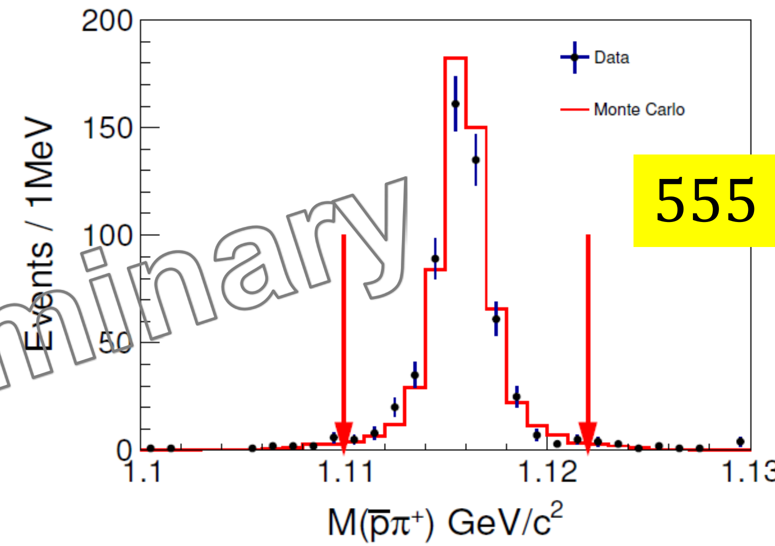


Predictions for ΔΦ

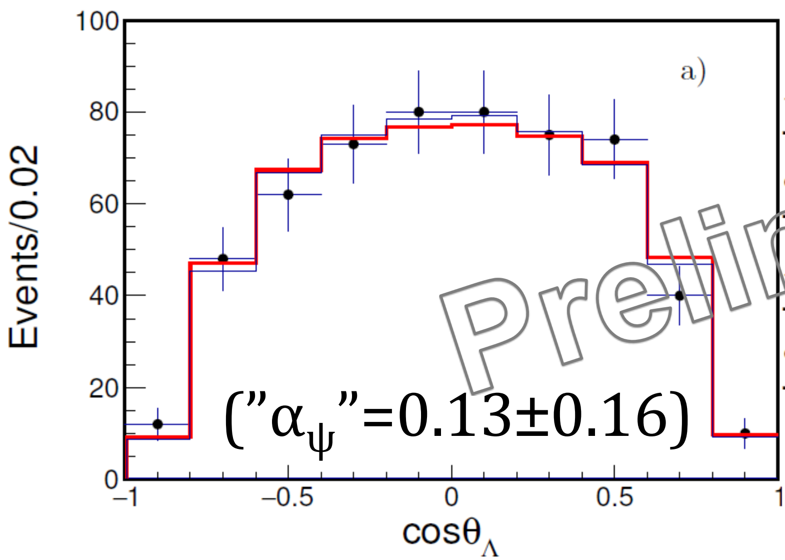
Haidenbauer, Meiβner,PRL761('16) 456

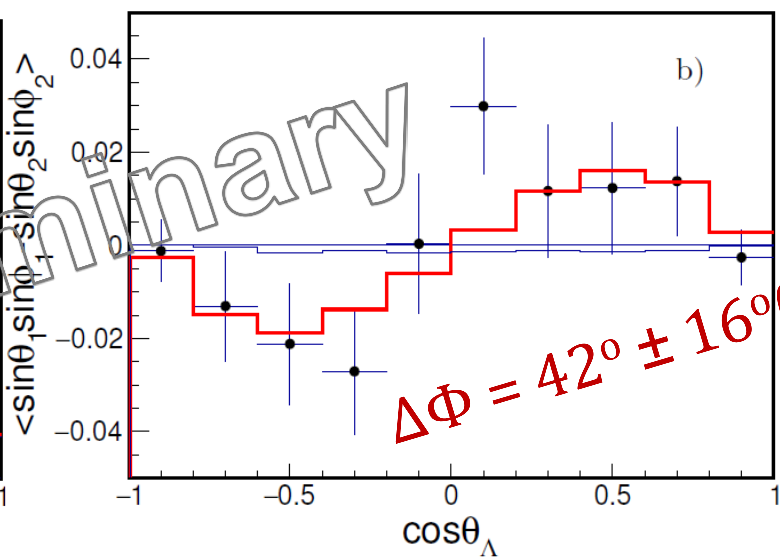
$e^+e^- \to \gamma^* \to \Lambda\bar{\Lambda}$ (continuum: 2.396 GeV)





555 events selected





$\Delta\Phi = 42^o \pm 16^o(\text{stat.}) \pm 8^o(\text{sys.})$ [PDG α_]

$37^o \pm 12^o \pm 6^o$ [BESIII α_]

$R = 0.94 \pm 0.16(\text{stat.}) \pm 0.03(\text{sys.}) \pm 0.02(\alpha\_)$

The same fit as for $J/\psi \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$ but $\alpha_- = \alpha_+$ and fixed

$\overline{\text{Hyperon-hyperon}}$ pair production at BESIII

$2.0 \text{ GeV} \leq \sqrt{s} \leq 4.6 \text{ GeV}$

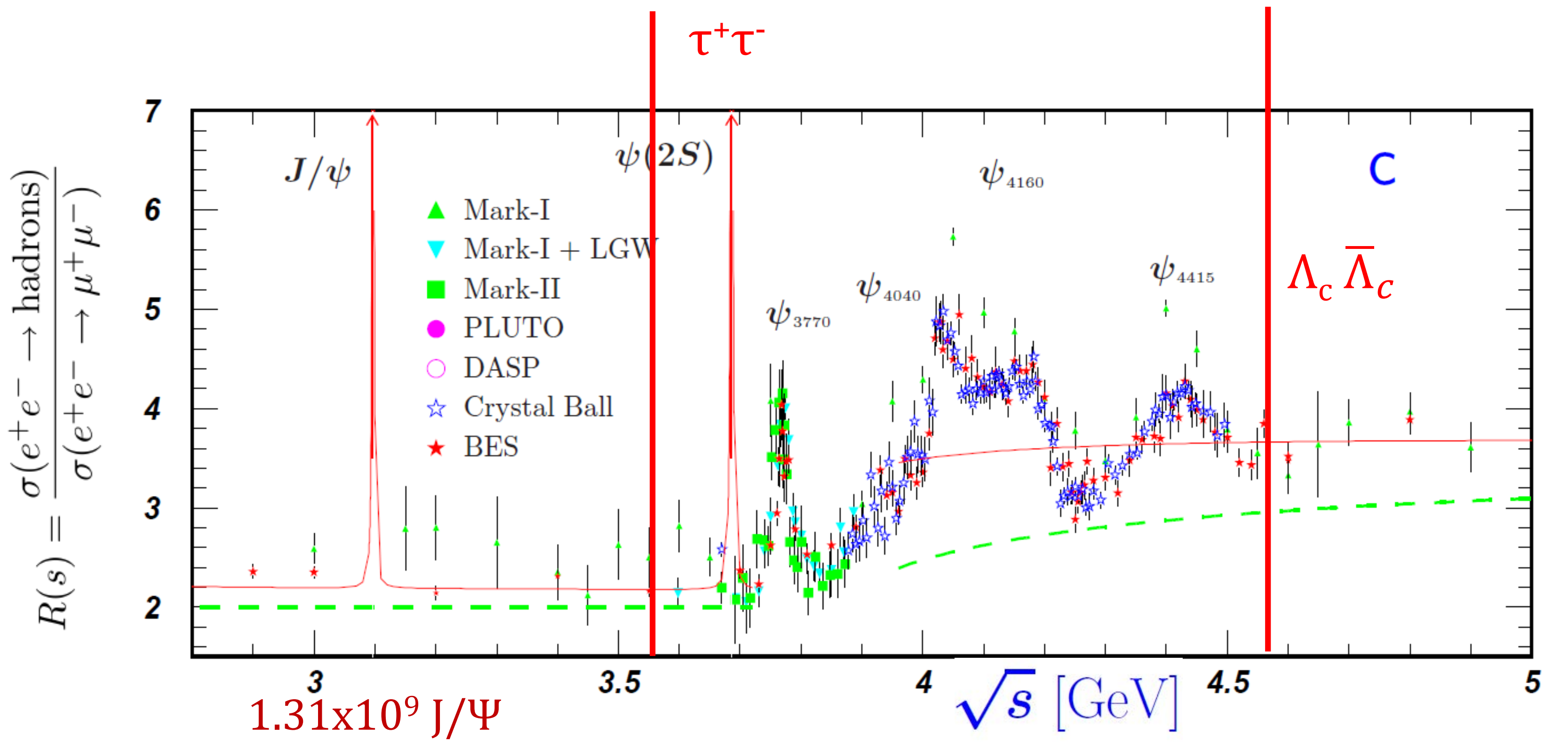
Thresholds:

$\Lambda\bar{\Lambda}$: 2.231 GeV $\Sigma^+\bar{\Sigma}^-$ 2.379 GeV

$\Sigma^0\bar{\Sigma}^0$ 2.385 GeV $\Sigma^-\bar{\Sigma}^+$ 2.395 GeV

$\Xi^0\bar{\Xi}^0$ 2.630 GeV $\Xi^-\bar{\Xi}^+$ 2.643 GeV

$\Lambda\bar{\Sigma}^0$ 2.308 GeV ($\Omega\bar{\Omega}$ 3.345 GeV)



$1.31\text{x}10^9$ J/Ψ

$J/\psi, \psi(2S) \to B\bar{B}$

Decay mode	Events	$\mathcal{B}(\times 10^{-4})$
$J/\psi \to \Lambda\bar{\Lambda}$	440675 ± 670	$19.43 \pm 0.03 \pm 0.33$
$\psi(2S) \to \Lambda\bar{\Lambda}$	31119 ± 187	$3.97 \pm 0.02 \pm 0.12$
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	111026 ± 335	$11.64 \pm 0.04 \pm 0.23$
$\psi(2S) \to \Sigma^0\bar{\Sigma}^0$	6612 ± 82	$2.44 \pm 0.03 \pm 0.11$
$J/\psi \to \Sigma(1385)^0\bar{\Sigma}(1385)^0$	102762 ± 852	10.71 ± 0.09
$J/\psi \to \Xi^0\bar{\Xi}^0$	134846 ± 437	11.65 ± 0.04
$\psi(2S) \to \Sigma(1385)^0\bar{\Sigma}(1385)^0$	2214 ± 148	0.69 ± 0.05
$\psi(2S) \to \Xi^0\bar{\Xi}^0$	10839 ± 123	2.73 ± 0.03
$J/\psi \to \Xi^-\bar{\Xi}^+$	42811 ± 231	10.40 ± 0.06
$J/\psi \to \Sigma(1385)^-\bar{\Sigma}(1385)^+$	42595 ± 467	10.96 ± 0.12
$J/\psi \to \Sigma(1385)^+\bar{\Sigma}(1385)^-$	52523 ± 596	12.58 ± 0.14
$\psi(2S) \to \Xi^-\bar{\Xi}^+$	5337 ± 83	2.78 ± 0.05
$\psi(2S) \to \Sigma(1385)^-\bar{\Sigma}(1385)^+$	1375 ± 98	0.85 ± 0.06
$\psi(2S) \to \Sigma(1385)^+\bar{\Sigma}(1385)^-$	1470 ± 95	0.84 ± 0.05

Only α_ψ extracted

BESIII

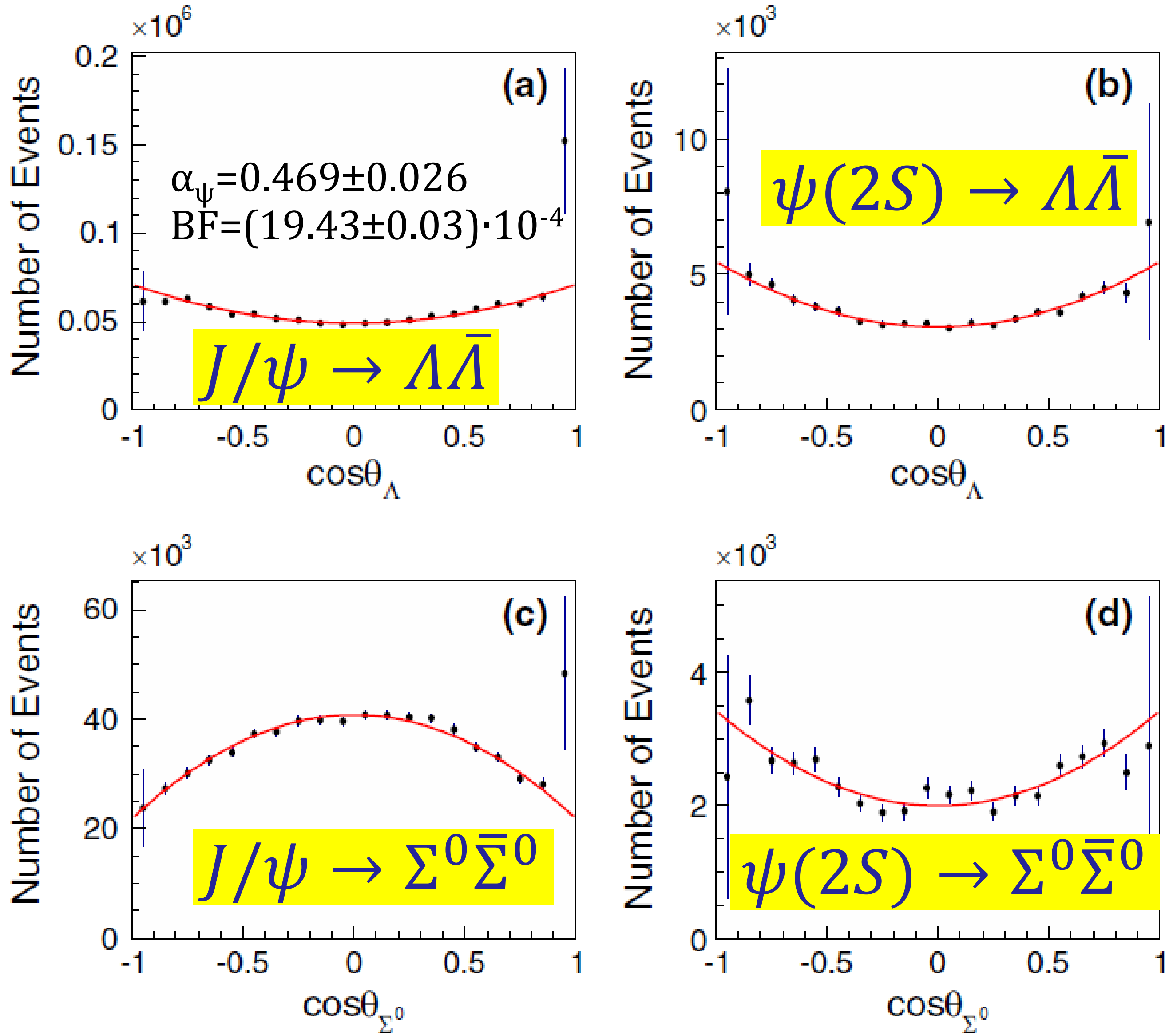
Phys. Rev. D 93, 072003 (2016)
PLB770(2017)217

Phys. Rev. D 95, 052003 (2017)

J/ψ and (4.48)x10^8 ψ(2S)

	$\mathcal{B}(\times 10^{-4})$
$J/\psi \to \Xi(1530)^-\bar{\Xi}^+$	5.9 ± 1.5
$J/\psi \to \Xi(1530)^0\bar{\Xi}^0$	3.3 ± 1.4
$J/\psi \to \Sigma(1385)^-\bar{\Sigma}^+$	3.1 ± 0.5
$\psi(2S) \to \Omega^-\bar{\Omega}^+$	0.47 ± 0.10

$J/\psi, \psi(2S) \to B\bar{B}$



α_ψ measurements at BESIII

BESIII,PRD 95, 052003 (2017)

Generalized formalism: motivation

What if the phase is non-zero also for other hyperon antihyperon in J/ψ or ψ(2S) decays?

⇨ e.g. cascades: $\mathbf{\Xi^-\Xi^+ \rightarrow \Lambda\pi^- \Lambda\pi^+ \rightarrow p\pi^-\pi^- p\pi^+\pi^+}$

could also measure ϕ_{Ξ^-}, ϕ_{Ξ^+} $(\beta = \sqrt{1-\alpha^2}\sin\phi)$

and do more sensitive (10x) CP test:

$$B_{CP} = (\beta_{\Xi^-} + \beta_{\Xi^+})/(\beta_{\Xi^-} - \beta_{\Xi^+})$$

⇨ Formalism for J=3/2 baryons:

$$\begin{aligned} e^+e^- \rightarrow \gamma^* &\rightarrow B_{1/2}\,\bar{B}_{1/2} \\ &\rightarrow B_{3/2}\,\bar{B}_{1/2} \\ &\rightarrow B_{3/2}\,\bar{B}_{3/2} \end{aligned}$$

⇨ Application to spectroscopy: polarization information in quasi-two body production:

$$e^+e^- \rightarrow J/\psi \rightarrow B_A^*\,\bar{B}_B^*$$

Jacobs-Wicks helicity formalism:

Production density matrix $e^+e^- \rightarrow \mathrm{B}_1\overline{\mathrm{B}}_2$

$$\rho_{B_1\bar{B}_2}^{\lambda_1,\lambda_2;\lambda_1',\lambda_2'} \propto A_{\lambda_1,\lambda_2}\, A^*_{\lambda_1',\lambda_2'}\, \rho_1^{\lambda_1-\lambda_2,\lambda_1'-\lambda_2'}(\theta_1)$$

Initial state (e+e-) spin density matrix for single γ^* processes:

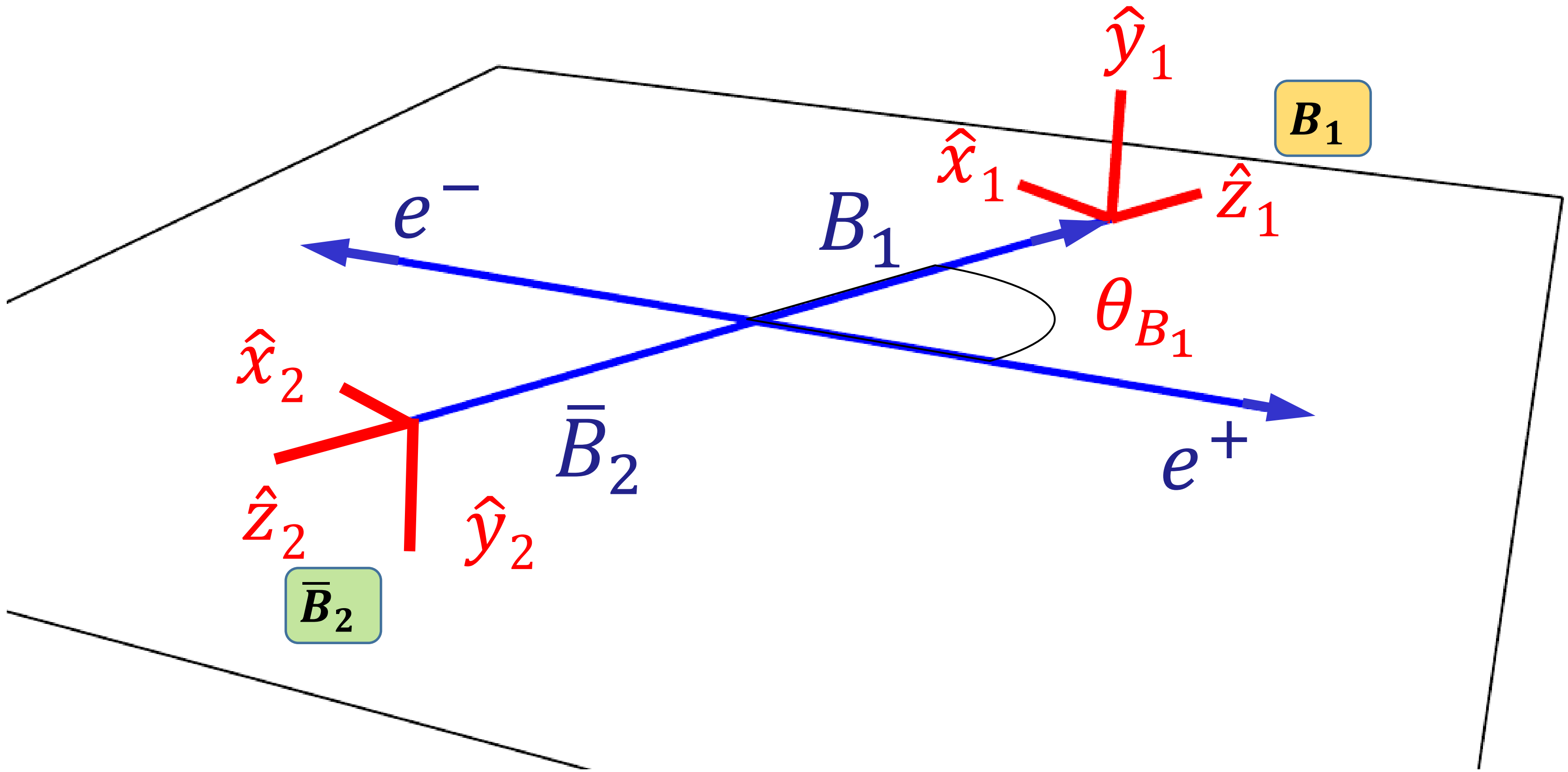
$$\rho_1^{i,j}(\theta) := \sum_{k=\pm 1} D^{1*}_{k,i}(0,\theta,0)\, D^1_{k,j}(0,\theta,0)$$

$$\rho_1 = \begin{pmatrix} \frac{1+\cos^2\theta}{2} & \frac{\cos\theta\sin\theta}{\sqrt{2}} & \frac{\sin^2\theta}{2} \\ \frac{\cos\theta\sin\theta}{\sqrt{2}} & \sin^2\theta & -\frac{\cos\theta\sin\theta}{\sqrt{2}} \\ \frac{\sin^2\theta}{2} & -\frac{\cos\theta\sin\theta}{\sqrt{2}} & \frac{1+\cos^2\theta}{2} \end{pmatrix}$$

valid for continuum
and for J/ψ, ψ(2S),...

$$D^{1*}_{k,i}(0,\theta,0)\, D^1_{k,j}(0,\theta,0) = D^{1*}_{k,i}(\phi,\theta,0)\, D^1_{k,j}(\phi,\theta,0)$$

Helicity reference frames



$e^+e^- \to \mathrm{B}_1\overline{\mathrm{B}}_2$ production

TWO SPIN $\frac{1}{2}$ BARYONS: $B_{1/2}\overline{B}_{1/2}$

$$A_{1/2,1/2} = A_{-1/2,-1/2} = h_1$$

$$A_{1/2,-1/2} = A_{-1/2,1/2} = h_2$$

$$\begin{pmatrix} h_1 & h_2 \\ h_2 & h_1 \end{pmatrix}$$

$$h_1 = \sqrt{1-\alpha_\psi}/\sqrt{2}$$

$$h_2 = \sqrt{1+\alpha_\psi}\exp(-i\Delta\Phi)$$

$$\rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}}\,\sigma_\mu \otimes \sigma_{\bar{\nu}}$$

$$C_{00} = 2(1+\alpha_\psi\cos^2\theta_1)\,,$$

$$C_{0\bar{y}} = 2\sqrt{1-\alpha_\psi^2}\sin\theta_1\cos\theta_1\sin(\Delta\Phi)\,,$$

$$C_{x\bar{x}} = 2\sin^2\theta_1\,,$$

$$C_{x\bar{z}} = 2\sqrt{1-\alpha_\psi^2}\sin\theta_1\cos\theta_1\cos(\Delta\Phi)\,,$$

$$C_{y0} = -C_{0\bar{y}}\,,$$

$$C_{y\bar{y}} = \alpha_\psi C_{x\bar{x}}\,,$$

$$C_{z\bar{x}} = -C_{x\bar{z}}\,,$$

$$C_{z\bar{z}} = -2(\alpha_\psi + \cos^2\theta_1)\,.$$

$$h_1 = G_M\,,\; h_2 = \sqrt{2\tau}G_E$$

cross check

SPIN $\frac{1}{2}$ AND $\frac{3}{2}$ BARYON: $B_{1/2}\overline{B}_{3/2}$

$$A_{\lambda_1,\lambda_2} = -A_{-\lambda_1,-\lambda_2}$$

$$\begin{pmatrix} h_3 & h_1 & h_2 & 0 \\ 0 & -h_2 & -h_1 & -h_3 \end{pmatrix}$$

TWO SPIN $\frac{3}{2}$ BARYONS: $B_{3/2}\overline{B}_{3/2}$

$$A_{\lambda_1,\lambda_2} = A_{-\lambda_1,-\lambda_2}$$

$$\begin{pmatrix} h_4 & h_3 & 0 & 0 \\ h_3 & h_1 & h_2 & 0 \\ 0 & h_2 & h_1 & h_3 \\ 0 & 0 & h_3 & h_4 \end{pmatrix}$$

inclusive (single particle)

$$\rho_{\overline{3/2}}(\theta) = \begin{pmatrix} m_{11} & c_{12} & c_{13} & 0 \\ c_{12}^* & m_{22} & im_{23} & c_{13}^* \\ c_{13}^* & -im_{23} & m_{22} & -c_{12}^* \\ 0 & c_{13} & -c_{12} & m_{11} \end{pmatrix}$$

$$m_{11} = \frac{1+\cos^2\theta}{2}|h_3|^2$$

$$m_{22} = |h_1|^2\sin^2\theta + \frac{1+\cos^2\theta}{2}|h_2|^2$$

$$m_{23} = \sqrt{2}\,\Im(h_1 h_2^*)\cos\theta\sin\theta$$

$$c_{12} = -\frac{h_3 h_1^*\cos\theta\sin\theta}{\sqrt{2}}$$

$$c_{13} = \frac{1}{2}h_3 h_2^*\sin^2\theta$$

4 complex FFs ⇨ 6 global parameters

Polarization of a spin 3/2 particle:

$$r^1_{-1} \to P_y \quad r^1_0 \to P_x \quad r^1_1 \to P_z$$

$$\rho_{3/2} = r_0 \left(Q_0 + \frac{3}{4} \sum_{M=-1}^{1} r^1_M Q^1_M + \frac{3}{4} \sum_{M=-2}^{2} r^2_M Q^2_M + \frac{3}{4} \sum_{M=-3}^{3} r^3_M Q^3_M \right)$$

M.G.Doncel, L.Michel, P.Minnaert Nucl. Phys. B38, 477(1972)

real coefficients,
scalable J=1/2,3/2,...

$$Q^L_M \to Q_\mu \, , \mu = 0, \dots, 15$$

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu$$

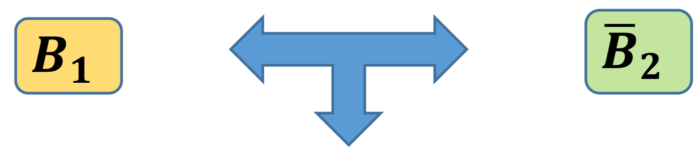
Degree of polarization

$$d(\rho_{3/2}) = \sqrt{\sum_{L=1}^{3} \sum_{M=-L}^{L} (r^L_M)^2}$$

Two particle density matrices:

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu \qquad \rho_{1/2} = \frac{1}{2}\sum_\mu I_\mu \sigma_\mu$$

$$\rho_{3/2,\overline{3/2}} = \sum_{\mu=0}^{15}\sum_{\bar{\nu}=0}^{15} C_{\mu\bar{\nu}} Q_\mu \otimes Q_{\bar{\nu}} \qquad \rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}} \sigma_\mu \otimes \sigma_{\bar{\nu}}$$



Respective helicity ref. frames

$$\rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}} \sigma_\mu \otimes \sigma_{\bar{\nu}}$$

4x4 matrix 6 non zero elements

$$\rho_{1/2,\overline{3/2}} = \sum_{\mu=0}^{3}\sum_{\bar{\nu}=0}^{15} C_{\mu\bar{\nu}} \sigma_\mu \otimes Q_{\bar{\nu}}$$

4x16 matrix 30 non zero elements

$$\rho_{3/2,\overline{3/2}} = \sum_{\mu=0}^{15}\sum_{\bar{\nu}=0}^{15} C_{\mu,\bar{\nu}} Q_\mu \otimes Q_{\bar{\nu}}$$

16x16 matrix 66 non zero elements

Decay chains:

$$\rho_{1/2,\overline{3/2}} = \frac{1}{2}\sum_{\mu=0}^{3}\sum_{\bar{\nu}=0}^{15} C_{\mu\bar{\nu}}\,\sigma_\mu \otimes Q_{\bar{\nu}}$$

$$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + 0^- \quad e.g.\ \Lambda \rightarrow p + \pi^-$$

$$\rho^{(f)}_{1/2,\overline{3/2}} = \frac{1}{2}\sum_{\mu=0}^{3}\sum_{\bar{\nu}=0}^{15} C_{\mu\bar{\nu}} \left(\sum_{\kappa=0}^{3} a_{\mu\kappa}\sigma^d_\kappa\right) \otimes Q_{\bar{\nu}}$$

$$\sigma_\mu \rightarrow \sum_{\nu=0}^{3} a_{\mu,\nu}\sigma^d_\nu$$

$$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+ + 0^- \quad e.g.\ \Omega^- \rightarrow \Lambda + K^-$$

$$\rho^{(f)}_{1/2,\overline{1/2}} = \frac{1}{2}\sum_{\mu=0}^{3}\sum_{\bar{\nu}=0}^{15} C_{\mu\bar{\nu}}\,\sigma_\mu \otimes \left(\sum_{\kappa=0}^{3} b_{\bar{\nu}\kappa}\sigma^d_\kappa\right)$$

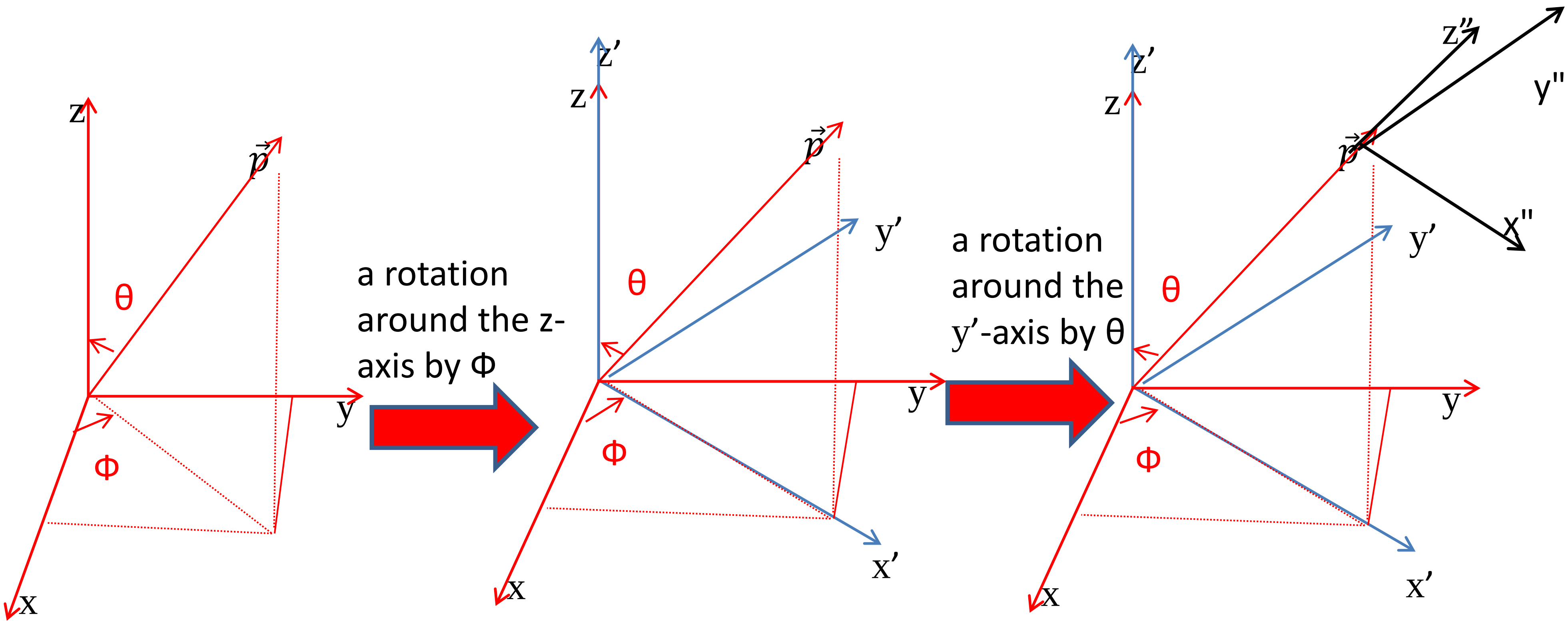
$$Q_\mu \rightarrow \sum_{\nu=0}^{3} b_{\mu,\nu}\sigma^d_\nu$$

PhD students' helicity rotations:

$$\begin{pmatrix} \cos\theta_m \cos\phi_m & \cos\theta_m \sin\phi_m & -\sin\theta_m \\ -\sin\phi_m & \cos\phi_m & 0 \\ \cos\phi_m \sin\theta_m & \sin\theta_m \sin\phi_m & \cos\theta_m \end{pmatrix}$$

$$|p, \theta, \phi, \lambda_1, \lambda_2\rangle := R(\phi, \theta, 0)|p, \lambda_1, \lambda_2\rangle$$

$$\mathcal{D}^{J*}_{\kappa,\lambda}(\Omega) = \mathcal{D}^{J*}_{\kappa,\lambda}(\phi, \theta, 0)$$



Decay matrices:

$$a_{\mu\nu} = \frac{1}{4\pi} \sum_{\lambda,\lambda'=-1/2}^{1/2} B_\lambda B^*_{\lambda'} \times$$

$$\boxed{\sigma_\mu \to \sum_{\nu=0}^{3} a_{\mu,\nu} \sigma^d_\nu}$$

4×4 decay matrix: $a_{\mu,\nu}$

$$\sum_{\kappa,\kappa'=-1/2}^{1/2} (\sigma_\mu)^{\kappa,\kappa'} (\sigma_\nu)^{\lambda',\lambda} \mathcal{D}^{1/2*}_{\kappa,\lambda}(\Omega) \mathcal{D}^{1/2}_{\kappa',\lambda'}(\Omega).$$

$$\alpha_D = -2\Re(A^*_S A_P) = |B_{1/2}|^2 - |B_{-1/2}|^2$$
$$\beta_D = -2\Im(A^*_S A_P) = 2\Im(B_{1/2} B^*_{-1/2})$$
$$\gamma_D = |A_S|^2 - |A_P|^2 = 2\Re(B_{1/2} B^*_{-1/2}),$$

$$\mathcal{D}^{J*}_{\kappa,\lambda}(\Omega) = \mathcal{D}^{J*}_{\kappa,\lambda}(\phi, \theta, 0)$$

$$\boxed{Q_\mu \to \sum_{\nu=0}^{3} b_{\mu,\nu} \sigma^d_\nu}$$

16×4 decay matrix: $b_{\mu,\nu}$

$$b_{\mu\nu} = \frac{1}{2} \sum_{\lambda,\lambda'=-1/2}^{1/2} B_\lambda B^*_{\lambda'} \times$$

$$\sum_{\kappa,\kappa'=-3/2}^{3/2} (Q_\mu)^{\kappa,\kappa'} (\sigma_\nu)^{\lambda',\lambda} \mathcal{D}^{3/2*}_{\kappa,\lambda}(\Omega) \mathcal{D}^{3/2}_{\kappa',\lambda'}(\Omega).$$

$$\alpha_D = -2\Re(A^*_P A_D) = |B_{1/2}|^2 - |B_{-1/2}|^2$$
$$\beta_D = -2\Im(A^*_P A_D) = 2\Im(B_{1/2} B^*_{-1/2})$$
$$\gamma_D = |A_P|^2 - |A_D|^2 = 2\Re(B_{1/2} B^*_{-1/2}),$$

Decay matrix $a_{\mu,\nu}$

$$
\begin{aligned}
a_{00} &= 1 \\
a_{03} &= \alpha_D \\
a_{10} &= \alpha_D \cos\phi \sin\theta \\
a_{11} &= \gamma_D \cos\theta \cos\phi - \beta_D \sin\phi \\
a_{12} &= -\beta_D \cos\theta \cos\phi - \gamma_D \sin\phi \\
a_{13} &= \sin\theta \cos\phi \\
a_{20} &= \alpha_D \sin\theta \sin\phi \\
a_{21} &= \beta_D \cos\phi + \gamma_D \cos\theta \sin\phi \\
a_{22} &= \gamma_D \cos\phi - \beta_D \cos\theta \sin\phi \\
a_{23} &= \sin\theta \sin\phi \\
a_{30} &= \alpha_D \cos\theta \\
a_{31} &= -\gamma_D \sin\theta \\
a_{32} &= \beta_D \sin\theta \\
a_{33} &= \cos\theta\,.
\end{aligned}
$$

$$
\boldsymbol{P}_\Lambda = \frac{(\alpha_Y + \boldsymbol{P}_Y \cdot \hat{\boldsymbol{p}})\hat{\boldsymbol{p}} + \beta_Y \boldsymbol{P}_Y \times \hat{\boldsymbol{p}} + \gamma_Y \hat{\boldsymbol{p}} \times \boldsymbol{P}_Y \times \hat{\boldsymbol{p}}}{1 + \alpha_Y \boldsymbol{P}_Y \cdot \hat{\boldsymbol{p}}}
$$

Example: $e^+e^- \to \Omega^-\overline{\Omega}^+$

$e^+e^- \to \Omega^-\overline{\Omega}^+$

with $\Omega^- \to \Lambda K^-$
and $\Lambda \to p\pi^-$

Single tag

$$A = \begin{pmatrix} \mathbf{h}_4 & \mathbf{h}_3 & 0 & 0 \\ \mathbf{h}_3 & \mathbf{h}_1 & \mathbf{h}_2 & 0 \\ 0 & \mathbf{h}_2 & \mathbf{h}_1 & \mathbf{h}_3 \\ 0 & 0 & \mathbf{h}_3 & \mathbf{h}_4 \end{pmatrix}$$

(Complex) Form Factors

$$\mathbf{h}_k \to h_k \exp(i\phi_k)$$

$$\rho_{3/2,\overline{3/2}}^{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}} = \sum_{\kappa=\pm 1} D^{1*}_{\kappa,\lambda_1-\lambda_2}(0,\theta_\Omega,0) D^{1}_{\kappa,\lambda_{1'}-\lambda_{2'}}(0,\theta_\Omega,0) A_{\lambda_1\lambda_2} A^*_{\lambda_{1'}\lambda_{2'}}$$

Single tag angular distributions

Single 3/2-spin baryon density matrix is

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu = \sum_{\mu=0}^{15} C_{\mu,0} Q_\mu$$

Angular distribution (using decay matrices in helicity frames):

$$W = \sum_{\mu=0}^{15}\sum_{\kappa=0}^{3} C_{\mu,0}\, b^{\Omega}_{\mu,\kappa}\, a^{\Lambda}_{\kappa,0}$$

decay 1/2->1/2+0 (Λ->pπ)

decay 3/2->1/2+0 (Ω->ΛK)

$$\alpha_\psi = \frac{h_2^2 - 2(h_1^2 - h_3^2 + h_4^2)}{h_2^2 + 2(h_1^2 + h_3^2 + h_4^2)}$$

$$r_0 = (1+\cos^2\theta_\Omega)(h_2^2 + 2h_3^2) + 2\sin^2\theta_\Omega(h_1^2 + h_4^2)$$

$$r_1 = 2\sin 2\theta_\Omega \frac{2\Im(\mathbf{h}_1\mathbf{h}_2^*) + \sqrt{3}\Im(\mathbf{h}_3^*(\mathbf{h}_1 + \mathbf{h}_4))}{\sqrt{30}}$$

$$r_6 = -\frac{2\sin^2\theta_\Omega(h_1^2 - h_4^2) + h_2^2(\cos^2\theta + 1)}{\sqrt{3}}$$

$$r_7 = \sqrt{2}\sin 2\theta_\Omega \frac{\Re(\mathbf{h}_3^*(\mathbf{h}_4 - \mathbf{h}_1))}{\sqrt{3}}$$

$$r_8 = 2\sin^2\theta_\Omega \frac{\Re(\mathbf{h}_3\mathbf{h}_2^*)}{\sqrt{3}}$$

$$r_{10} = 2\sin^2\theta_\Omega \frac{\Im(\mathbf{h}_3\mathbf{h}_2^*)}{\sqrt{3}}$$

$$r_{11} = 2\sin 2\theta_\Omega \frac{\Im(\sqrt{3}\mathbf{h}_2\mathbf{h}_1^* + \mathbf{h}_3^*(\mathbf{h}_1 + \mathbf{h}_4))}{\sqrt{15}}$$

$$\frac{d\Gamma}{d\cos\theta_\Omega} = 1 + \alpha_\psi \cos^2\theta_\Omega$$

Example: sequential decays

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu \qquad \rho_\Omega = \sum_{\mu=0}^{15} r_\mu(\theta_\Omega; h_1, h_2, h_3, h_4) Q_\mu$$

$$\frac{3}{2}^+ \to \frac{1}{2}^+ + 0^- \qquad \Omega^- \to \Lambda + K^-$$

$$Q_\mu \to \sum_{\nu=0}^{3} b_{\mu,\nu} \sigma_\nu^d$$

$$\rho_\Lambda = \sum_{\mu=0}^{15} \sum_{\kappa=0}^{3} r_\mu \cdot b_{\mu,\kappa}^{\Omega}(\theta_\Lambda, \phi_\Lambda; \alpha_\Omega, \beta_\Omega, \gamma_\Omega) \sigma_\kappa^\Lambda$$

$$\Upsilon_\Omega = \cos(\phi_\Omega)\sqrt{(1-\alpha_\Omega)^2}$$

$$\beta_\Omega = \sin(\phi_\Omega)\sqrt{(1-\alpha_\Omega)^2}$$

$$\alpha_\Omega^2 + \beta_\Omega^2 + \gamma_\Omega^2 = 1$$

$$\sigma_\mu \to \sum_{\nu=0}^{3} a_{\mu,\nu} \sigma_\nu^d$$

$$\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^- \qquad \Lambda \to p + \pi^-$$

$$\rho_p = \sum_{\mu=0}^{15} \sum_{\kappa,\nu=0}^{3} r_\mu \cdot b_{\mu,\kappa}^{\Omega} \cdot a_{\kappa,\nu}^{\Lambda}(\theta_p, \phi_p; \alpha_\Lambda, \beta_\Lambda, \gamma_\Lambda) \sigma_\nu^p$$

$$Tr\rho_p \rightarrow \frac{d\Gamma}{d\xi} = \sum_{\mu=0}^{15} \sum_{\kappa=0}^{3} r_\mu b_{\mu,\kappa}^{\Omega} a_{\kappa,0}^{\Lambda}$$

FFs vs helicity amplitudes for $e^+e^- \to B_{3/2}\,\overline{B}_{3/2}$

$$\Gamma_{\alpha\beta\mu} := g_{\alpha\beta}\left(F_1(q^2)\,\gamma_\mu + F_2(q^2)\,\frac{i\sigma_{\mu\nu}q^\nu}{2m}\right) + \frac{q_\alpha\,q_\beta}{m^2}\left(F_3(q^2)\,\gamma_\mu + F_4(q^2)\,\frac{i\sigma_{\mu\nu}q^\nu}{2m}\right)$$

J. G. Körner and M. Kuroda
Phys. Rev. D 16, 2165 (1977)

$$h_4 = 2m\,(F_1 + \tau\,F_2)\ ,$$
$$h_1 = 2m\left(1 - \frac{4}{3}\tau\right)(F_1 + \tau\,F_2) + 2m\,\frac{4}{3}\tau\,(1-\tau)\,(F_3 + \tau\,F_4)\ ,$$
$$h_3 = \sqrt{\frac{2}{3}}\,\sqrt{q^2}\,(F_1 + F_2)\ ,$$
$$h_2 = -\frac{2}{3}\sqrt{2q^2}\,[-(1-2\,\tau)\,(F_1+F_2) - 2\,\tau\,(1-\tau)\,(F_3+F_4)]$$

At threshold:

$$h_4 \approx -3\,h_1 \approx \sqrt{\frac{3}{2}}\,h_3 \approx -\frac{3}{\sqrt{8}}\,h_2$$

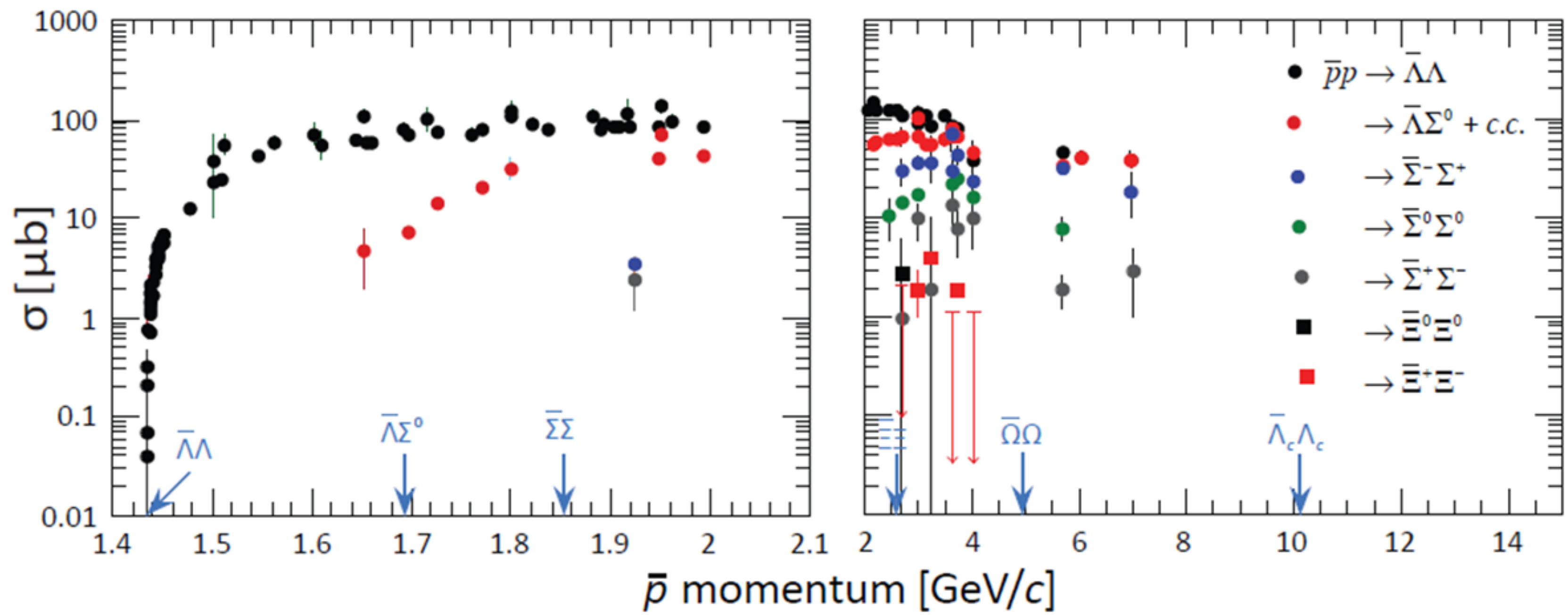
$$r_6 = \frac{1}{5\sqrt{3}}(1 - 3\cos^2\theta_1)$$
$$r_7 = \frac{1}{5}\sin 2\theta_1$$
$$r_8 = -\frac{1}{5}\sin^2\theta_1 .$$

Only $^3S_1 \neq 0$ but $d(\rho_{3/2}) = \sqrt{\sum_1^{15} r_\mu^2} = \frac{2}{5\sqrt{3}} \approx 23\%$ ⇨ spin filtering

Outlook: hyperon-$\overline{\text{hyperon}}$ pair from $\bar{p}p$

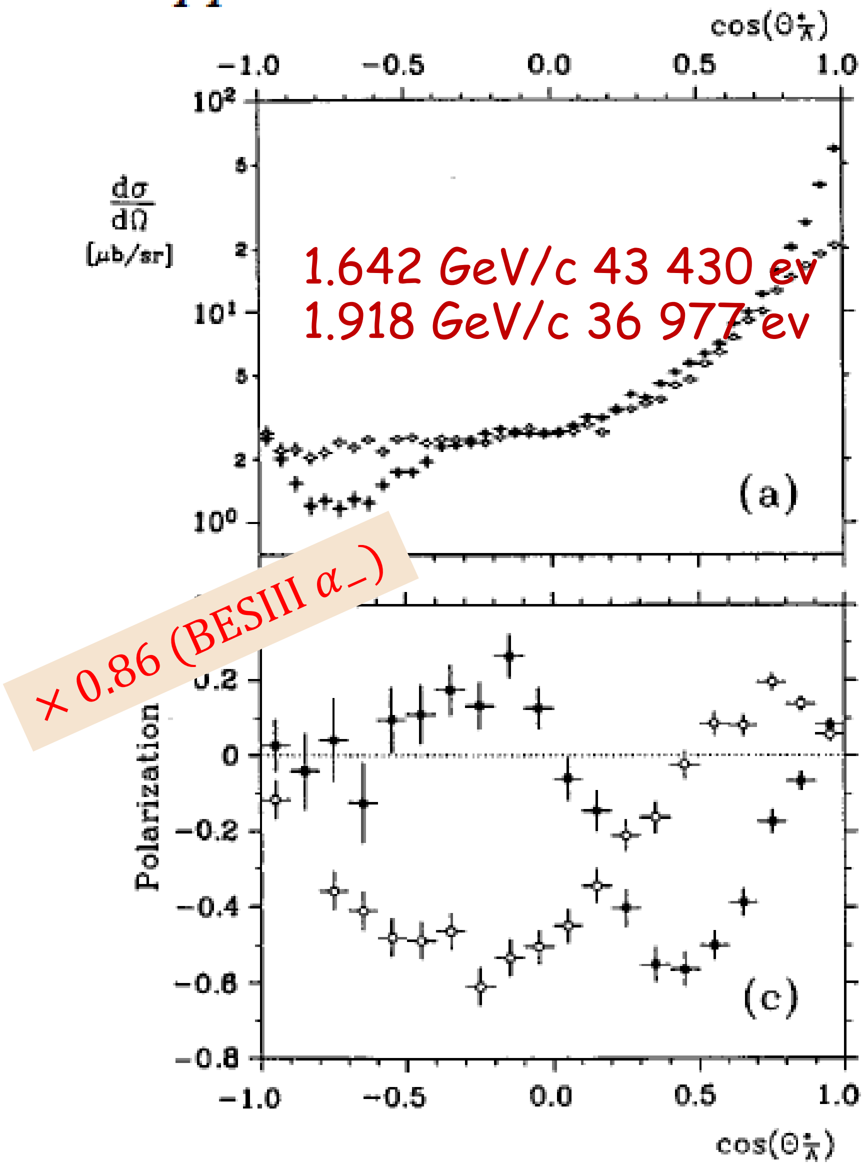
Reaction	σ (μb)	Efficiency (%)	Rate (with 10^{31} $cm^{-1}s^{-1}$)
$\bar{p}p \rightarrow \bar{\Lambda}\Lambda$	64	10	30 s^{-1}
$\bar{p}p \rightarrow \bar{\Lambda}\Sigma^0$	~40	30	30 s^{-1}
$\bar{p}p \rightarrow \bar{\Xi}^+\Xi^-$	~2	20	2 s^{-1}
$\bar{p}p \rightarrow \bar{\Omega}\Omega$	~0.002	30	~4 h^{-1}
$\bar{p}p \rightarrow \bar{\Lambda}_c\Lambda_c$	~0.1	35	~2 day^{-1}

$\Rightarrow$ **$\bar{\text{P}}$anda**



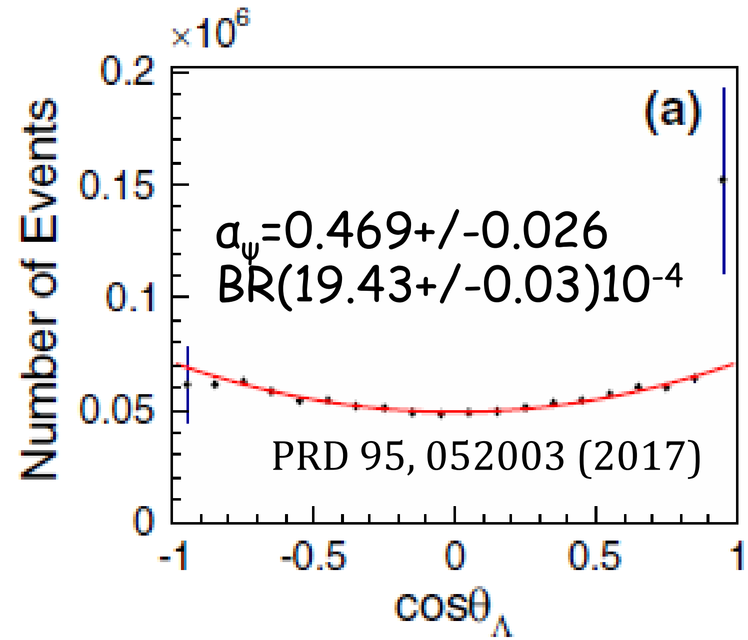
Nuclear Physics News, Vol. 27, No. 3, 2017

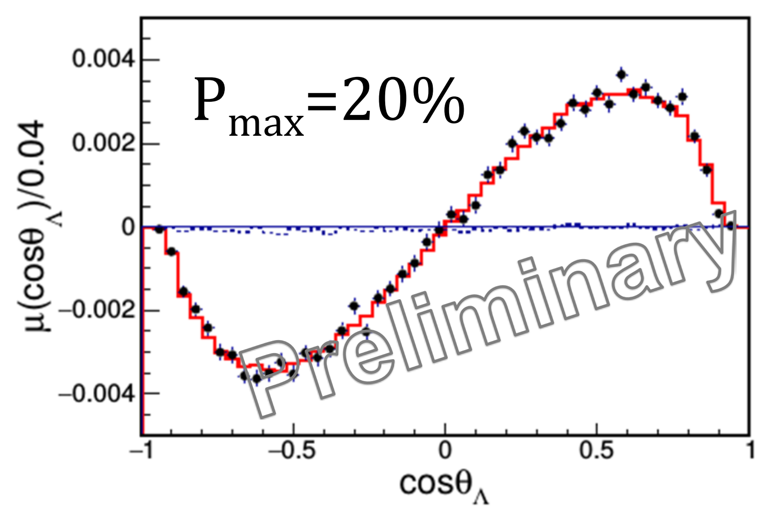
$\bar{p}p \rightarrow \bar{\Lambda}\Lambda$



PS185, PRC54 (1996) 1877

$e^+e^- \rightarrow \gamma^* \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$





5 parameters at each θ_Λ
Can't determine Λ decay param.

2 global parameters
extract Λ decay par. α

Conclusions: BESIII results

Polarization in $e^+e^- \to \Lambda\bar{\Lambda}$ observed both at J/ψ (**unexpected**) and continuum: 2.396 GeV (predicted) [phase close to 40°]

J/ψ and ψ' decays into hyperon-antihyperon:
unique spin entangled system for CP tests and for determination of (anti-)hyperon decay parameters (**polarization is essential!**)

Presented results use $1.31\cdot10^9$ J/ψ but 10^{10} J/ψ are being collected

17(3)% larger value for the $\Lambda \longrightarrow p\pi^-$ decay asymmetry (α_-)
⇒ calls for reinterpretation of **all** Λ polarization measurements!

α_-: 0.642±0.012 (PDG) ⇨ 0.750±0.009±0.004

BESIII Prel.

Conclusions: general framework

Helicity frames density matrices for $\boldsymbol{e^+e^-} \to \mathbf{B_1\overline{B}_2}$ with scalable (for higher spins) bases

Decay matrices allowing a modular approach for sequential decays:
$\mathbf{B_1\overline{B}_2}$ spin correlations are preserved

Uses two angle helicity rotation convention
⇨ simpler rotations and simpler expressions

Multi-dimensional unbinned MLL fits with systematic method of fit verification

[Baryons with spin J $\geq \frac{3}{2}$ are produced polarized ⇨ spin filtering]

Application: spectroscopy 2,3,… body final states with hyperons: $e^+e^- \to J/\psi \to \Sigma(1350)\bar{\Sigma}(1350), pK^-\Lambda, \Lambda\bar{\Sigma}^-\pi^+$ …

Thank you!