POLE STRUCTURE OF LOW ENERGY πN SCATTERING AMPLITUDES

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Regular Article - Theoretical Physics

New insights on low energy πN scattering amplitudes

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Abstract The S- and P- wave phase shifts of low-energy pion-nucleon scatterings are analysed using Peking University representation, in which they are decomposed into various terms contributing either from poles or branch cuts. We estimate the left-hand cut contributions with the help of tree-level perturbative amplitudes derived in relativistic baryon chiral perturbation theory up to $\mathcal{O}(p^2)$. It is found that in S_{11} and P_{11} channels, contributions from known resonances and cuts are far from enough to saturate experimental phase shift data – strongly indicating contributions from low lying poles undiscovered before, and we fully explore possible physics behind. On the other side, no serious disagreements are observed in the other channels.

1 Introduction

on axiomatic S-matrix arguments. It has been successfully applied to investigate $\pi\pi$ and πK scatterings and, in particular, corroborate the existences of σ and κ resonances [19,21]. The use of PKU representation to study πN scatterings may help us not only to enrich our knowledge of the amplitude structure but also to gain a fresh look at relevant physics in a much more rigorous manner.

The PKU representation factorizes the partial wave two-body elastic scattering S matrix in the form [21]

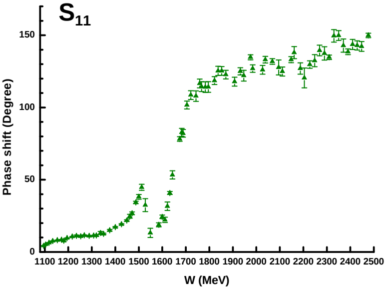
$$S(s)=\prod_b \frac{1-i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_b-s_L}{s_R-s_b}}}{1+i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_b-s_L}{s_R-s_b}}}\prod_v \frac{1+i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_v^l-s_L}{s_R-s_v^l}}}{1-i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_v^l-s_L}{s_R-s_v^l}}} \times\prod_r \frac{M_r^2-s+i\rho(s)sG_r}{M_r^2-s-i\rho(s)sG_r}e^{2i\rho(s)f(s)}, \quad (1)$$

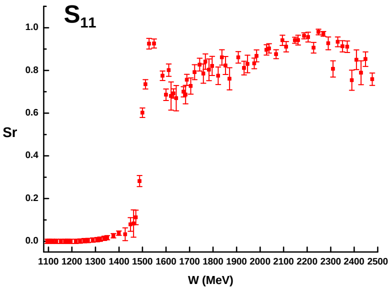
CONTENTS

1. Introduction

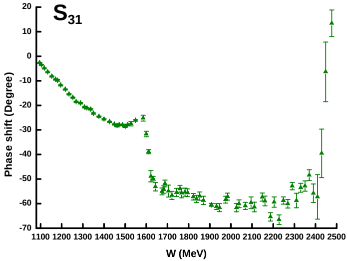
The pion-nucleon scattering

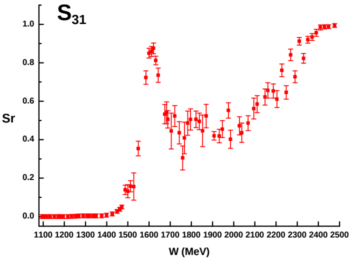
- The πN scattering $\rightarrow$ one of the most fundamental and important processes in nuclear or hadron physics
- Decades of researching
- Various experiments and phenomena ($L_{2I\ 2J}$ convention, $W = \sqrt{s}$, $S_r = 1 - \eta^2$)[SAID: WI 08]

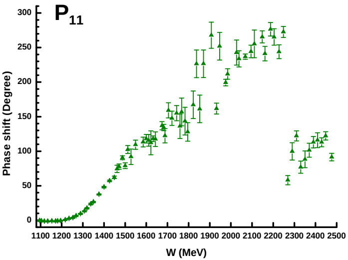


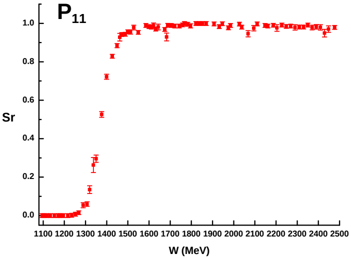


THE PION-NUCLEON SCATTERING









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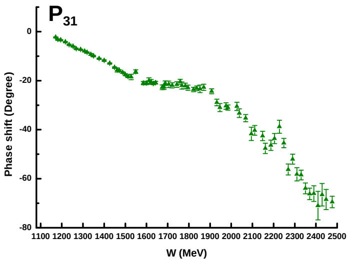

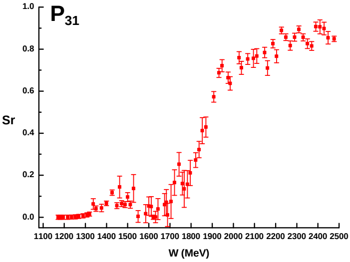

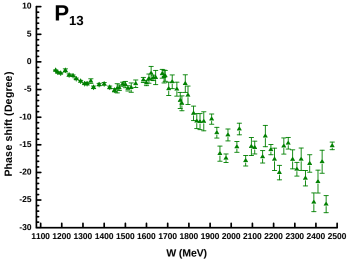

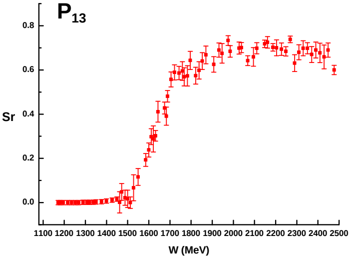

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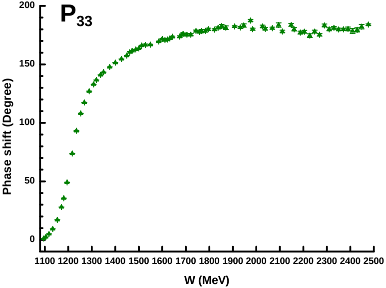
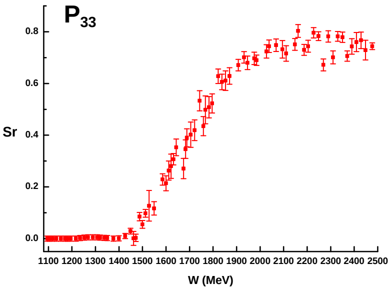




THEORETICAL DISCUSSIONS

- Problems to study
  - Low energy properties:
    πN σ-term, subthreshold expansions

    [C. Ditsche et. al. 2012 JHEP][Y. H. Chen et. al. 2013 PRD][Hoferichter et. al. 2016 Phys.Rept.]
  - Intermediate resonances: $\Delta(1232)$, $N^*(1440)$, $N^*(1535)\cdots$
- Methods
  - Perturbative calculation

    [J.M. Alarcón et. al. 2012 RPD][Y. H. Chen et. al. 2013 PRD]
  - Couple channel Lippmann-Schwinger Equation

    [O. Krehl et. al. 2000 PRC]
  - Dispersion technique [A. Gasparyan and M.F.M. Lutz 2010 NPA]
  - Roy-Steiner equation

    [C. Ditsche et. al. 2012 JHEP][Hoferichter et. al. 2016 Phys.Rept.]

S_{11} AND P_{11} CHANNELS

- S_{11} channel ($L_{2I\ 2J}$ convention): $N^*(1535)$

  [N. Kaiser et. al. 1995 PLB][J. Nieves et. al. 2000 PRD]

  - lies above the P- wave first resonance $N^*(1440)$
  - large couple channel effects with πN and ηN
- P_{11} channel: $N^*(1440)$ (Ropper resonance), various puzzles
  - low mass, large decay width, coupling to σN channel...

    [O. Krehl et. al. 2000 PRC]
  - two-pole structure? [R. A. Arndt et. al. 1985 PRD]
  - second sheet complex branch cut in P_{11} channel?

    [S. Ceci et. al. 2011 PRC]
- A method is needed to examine the relevant channels carefully and to exhume more physics behind
  - low energy
  - model independent

PKU REPRESENTATION

- Peking University (PKU) representation: elastic two-body scatterings

$$S = \prod_i S_i \times S_{cut}$$

- S_i: pole terms, $S_{cut} = e^{2i\rho(s)f(s)}$: left-hand cuts and right hand inelastic cut – background.

$$f(s) = \frac{s}{2\pi \mathrm{i}} \int_{\mathrm{L}} ds' \frac{\mathrm{disc} f(s')}{(s'-s)s'} + \frac{s}{2\pi \mathrm{i}} \int_{\mathrm{R}'} ds' \frac{\mathrm{disc} f(s')}{(s'-s)s'}$$

- $f(0) \equiv 0$ [Z. Y. Zhou and H. Q. Zheng 2006 NPA]

PKU REPRESENTATION

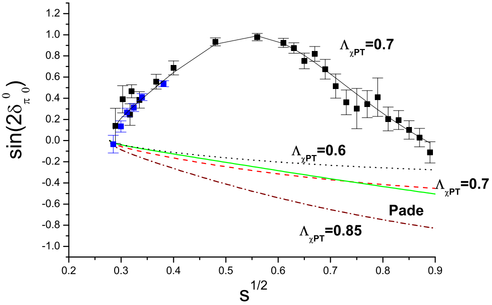
- $f(s)$ perturbatively calculated, poles as parameters (input or fit)
- Corresponding to the Ning Hu representation in QM

  [N. Hu 1948 PR]
- Advantages
  - rigorous and universal
  - separated $S \to$ additive phase shift
  - sensitive to (not too) distant poles
  - definite sign of the phase shifts $\to$ figuring out hidden contributions
- Applications
  - the $\pi\pi$ elastic scattering $\to$ existence of the σ particle $(f_0(500))$[Z. G. Xiao and H. Q. Zheng 2001 NPA]
  - the πK elastic scattering $\to$ κ resonance $(K^*(800))$

    [H. Q. Zheng et. al. 2004 NPA]

THE EXISTENCE OF σ

The left-hand cut contribution (negative definite)
$\rightarrow$ the existence of σ particle [Z. G. Xiao and H. Q. Zheng 2001 NPA]



$M_\sigma = 457 \pm 15$ MeV, $\Gamma_\sigma = 551 \pm 28$ MeV [Z. Y. Zhou et. al 2005 JHEP]
$M_\sigma = 441^{+16}_{-8}$ MeV, $\Gamma_\sigma = 544^{+18}_{-25}$ MeV [I. Caprini et. al. 2006 PRL]

THE EXISTENCE OF κ

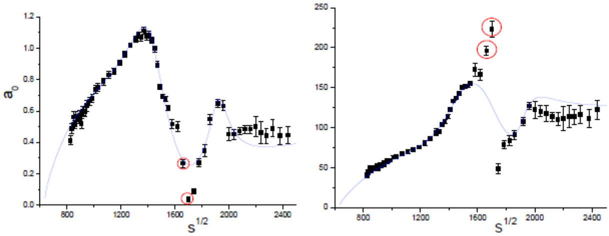
κ exists if the scattering length is not far from the value obtained from χPT.

Conclusions (almost) model independent.

(H.Q. Zheng, et. al., Nucl.Phys.A733:235-261,2004)

Taking $f(0) = 0$ into account:

Z. Y. Zhou and H. Q. Zheng, Nucl. Phys. **A755** (2006) 212.



2. Theoretical framework

LAGRANGIAN

- Covariant baryon chiral perturbation theory, $SU(2)$ case.
- Lagrangians [N. Fettes et. al. 2000 Ann. Phys.]
- $\mathcal{O}(p^1)$:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\big(\mathrm{i}\not{D} - M + \frac{1}{2} g \not{u} \gamma^5\big) N$$

- $\mathcal{O}(p^2)$ ("$\langle\rangle$" stands for trace in isospin space):

$$\mathcal{L}_{\pi N}^{(2)} = c_1 \langle \chi_+ \rangle \bar{N} N - \frac{c_2}{4M_N^2} \langle u^\mu u^\nu \rangle (\bar{N} D_\mu D_\nu N + \text{h.c.}) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle \bar{N} N - \frac{c_4}{4} \bar{N} \gamma^\mu \gamma^\nu \left[u^\mu, u^\nu\right] N$$

CONVENTIONS

- Conventions

$$D_\mu = \partial_\mu + \Gamma_\mu$$
$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u + u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger\right]$$
$$u_\mu = \mathrm{i}\left[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u - u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger\right]$$
$$\chi_\pm \equiv u^\dagger \chi u^\dagger \pm u\chi^\dagger u$$
$$\chi = 2B_0(s + ip)$$
$$h_\nu^{\ \mu} = \left[D_\nu, u^\mu\right] + \left[D^\mu, u_\nu\right]$$

In calculation $2B_0 s \to 2B_0 m_q = m_\pi^2$, other sources are switched off

ISOSPIN DECOMPOSITION

- Symmetric part vs. anti-symmetric part

$$T(\pi^a + N_i \to \pi^{a'} + N_f) = \chi_f^\dagger \left(\delta^{aa'} T^S + \frac{1}{2}\left[\tau^{a'}, \tau^a\right] T^A \right) \chi_i$$

- Isospin channels

$$T^{I=1/2} = T^S + 2T^A$$

$$T^{I=3/2} = T^S - T^A$$

HELICITY STRUCTURE

- Lorentz structure

$$T^{S,A} = \bar{u}(p', s')\Big[A^{S,A}(s,t) + \frac{1}{2}(\not{q} + \not{q}')B^{S,A}(s,t)\Big]u(p,s)$$
$$= \bar{u}(p', s')\Big[D^{S,A}(s,t) + \frac{\mathrm{i}\sigma^{\mu\nu}q_\nu q'_\mu}{2M}B^{S,A}(s,t)\Big]u(p,s)$$

where $D = A + (s-u)B/(4M_N)$

- Helicity amplitudes ($z_s = \cos\theta$)

$$T_{++} = (\frac{1+z_s}{2})^{\frac{1}{2}}[2M_N A(s,t) + (s - m_\pi^2 - M_N^2)B(s,t)]$$
$$T_{+-} = -(\frac{1-z_s}{2})^{\frac{1}{2}} s^{-\frac{1}{2}}[(s - m_\pi^2 + M_N^2)A(s,t) + M_N(s + m_\pi^2 - M_N^2)B(s,t)]$$

- Partial wave projection

$$T^J_{++} = \frac{1}{32\pi}\int_{-1}^{1} dz_s T_{++}(s, t(s,z_s))d^J_{-1/2,-1/2}(z_s)$$
$$T^J_{+-} = \frac{1}{32\pi}\int_{-1}^{1} dz_s T_{+-}(s, t(s,z_s))d^J_{1/2,-1/2}(z_s)$$

Channels to be analyzed

$L_{2I\ 2J}$ convention

$$T(S_{11}) = T_{++}(I = 1/2, J = 1/2) + T_{+-}(I = 1/2, J = 1/2)$$
$$T(S_{31}) = T_{++}(I = 3/2, J = 1/2) + T_{+-}(I = 3/2, J = 1/2)$$
$$T(P_{11}) = T_{++}(I = 1/2, J = 1/2) - T_{+-}(I = 1/2, J = 1/2)$$
$$T(P_{31}) = T_{++}(I = 3/2, J = 1/2) - T_{+-}(I = 3/2, J = 1/2)$$
$$T(P_{13}) = T_{++}(I = 1/2, J = 3/2) + T_{+-}(I = 1/2, J = 3/2)$$
$$T(P_{33}) = T_{++}(I = 3/2, J = 3/2) + T_{+-}(I = 3/2, J = 3/2)$$

Each channel satisfies unitarity condition.

PKU REPRESENTATION

- PKU representation

$$S(s) = \prod_b \frac{1 - i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_b - s_L}{s_R - s_b}}}{1 + i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_b - s_L}{s_R - s_b}}} \prod_v \frac{1 + i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s'_v - s_L}{s_R - s'_v}}}{1 - i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s'_v - s_L}{s_R - s'_v}}}$$
$$\prod_r \frac{M_r^2 - s + i\rho(s)sG_r}{M_r^2 - s - i\rho(s)sG_r} e^{2i\rho(s)f(s)}$$

- s_b: bound states. s'_v: virtual states (sheet I). z_r: resonances (sheet II).
- $s_L = (m_1 - m_2)^2, \;\; s_R = (m_1 + m_2)^2, \;\; \rho(s) = \sqrt{s - s_L}\sqrt{s - s_R}/s.$

$$M_r^2 = \mathrm{Re}[z_r] + \mathrm{Im}[z_r]\frac{\mathrm{Im}[\sqrt{(z_r - s_R)(z_r - s_L)}]}{\mathrm{Re}[\sqrt{(z_r - s_R)(z_r - s_L)}]}$$

$$G_r = \frac{\mathrm{Im}[z_r]}{\mathrm{Re}[\sqrt{(z_r - s_R)(z_r - s_L)}]}$$

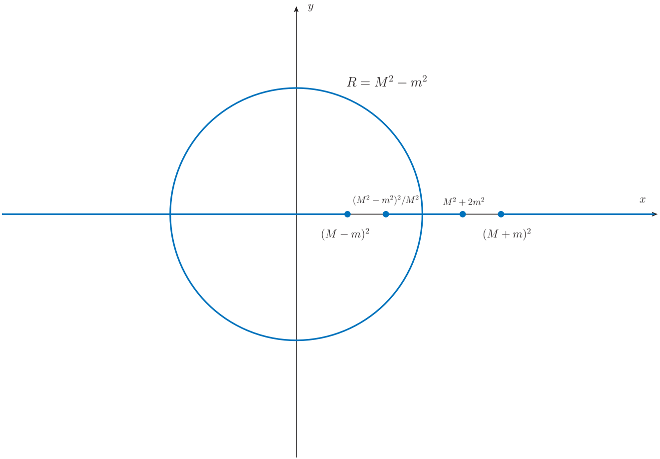
Phase shift components

- PKU representation $\rightarrow$ conventionally additive phase shift
- Phase shift contributions
  - bound states $\rightarrow$ negative phase shift
  - virtual states (usually hidden !) $\rightarrow$ positive phase shift
  - resonances $\rightarrow$ positive phase shift
  - left hand cut $\rightarrow$ (empirically) negative phase shift (proved in quantum mechanical potential scatterings)

    [T. Regge 1958 Nuovo Cimento]

BRANCH CUT STRUCTURE OF PARTIAL WAVE πN ELASTIC SCATTERING AMPLITUDE

[S. W. MacDowell 1959 PR][J. Kennedy and T. D. Spearman 1961 PR]



Tree level left-hand cut

- Tree level left-hand cut of S
  - $(-\infty, (M_N - m_\pi)^2] \rightarrow$ From logs and relativistic kinematics!
  - $[(M_N^2 - m_\pi^2)^2/M_N^2, M_N^2 + 2m_\pi^2] \rightarrow u$ channel nucleon exchange $\rightarrow$ very small
- The main contribution of $f(s)$ (with a cut-off s_c)

$$f(s) = \frac{s}{\pi} \int_{s_c}^{(M_N - m_\pi)^2} \frac{\sigma(w) dw}{w(w-s)}$$

- The dispersion spectral function

$$\sigma(w) = \mathrm{Im}\left\{ \frac{\ln |S_{\text{tree}}|}{2i\rho(w)} \right\} = -\frac{\ln |1 + 2i\rho(w) T_{\text{tree}}|}{2\rho(w)}$$

negative definite

- Right-hand inelastic cuts are omitted for the moment

3. Numerical results

TREE-LEVEL QUALITATIVE ANALYSIS

- Values of the constants (s_c determined by $N^*(1440)$ shadow pole position)

$$F = 0.0924 \text{ GeV},\ g = 1.267,\ s_c = -0.08 \text{ GeV}^2$$

$$M_N = 0.9383 \text{ GeV},\ m_\pi = 0.1396 \text{ GeV}$$

- $\mathcal{O}(p^2)$ K-Matrix phase shift:

$$T = T_{\text{tree}}/(1 - i\rho T_{\text{tree}}),\ \delta = \arctan(\rho T_{\text{tree}})$$

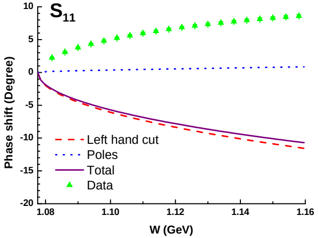
- Data from computer code SAID (WI 08) http://gwdac.phys.gwu.edu/
- K-Matrix fit to S_{11}, S_{31}, P_{11}, P_{31}, P_{13} channels, $W = \sqrt{s} \in [1.08, 1.16]$ GeV, χ^2/d.o.f. $= 1.850$.

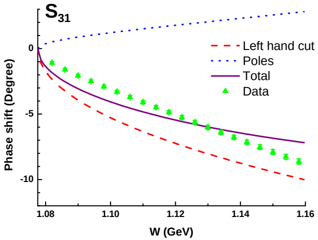
$$c_1 = -0.841,\ c_2 = 1.170,\ c_3 = -2.618,\ c_4 = 1.677$$

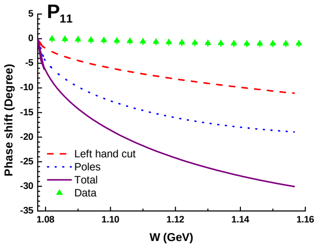
- Known poles[A.V. Anisovich et. al. 2012 Eur. Phys. J. A] $\sqrt{s_p}^{\text{II}} = M_p - i|\Gamma_{\pi N} - \Gamma_{\text{inel.}}|/2$

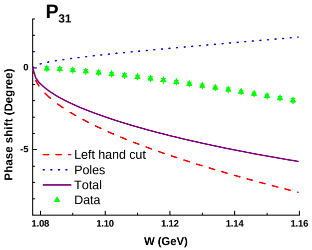
TREE LEVEL PHASE SHIFT RESULTS

$L_{2I\ 2J}$ convention, $W = \sqrt{s}$, data: green triangles [SAID: WI 08]









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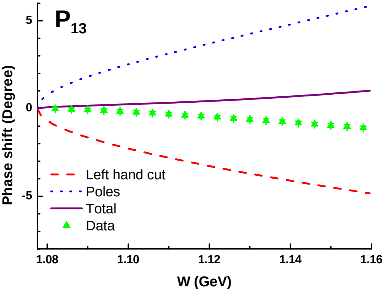
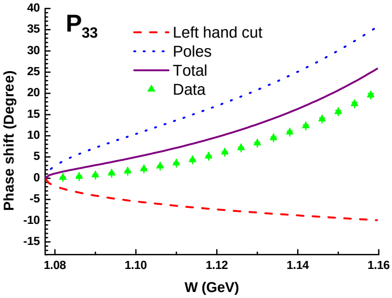




DISCREPANCIES IN S_{11} AND P_{11} CHANNELS

- Large missing positive contributions
- Possible interpretations
  - one loop contributions? numerical uncertainties?
  - contributions from other branch cuts?
  - hidden poles - virtual states, crazy resonances below threshold, or some extremely broad states?
- Once subtraction, logarithmic form $\rightarrow$ not sensitive to chiral orders and numerical details

Right-hand inelastic cut

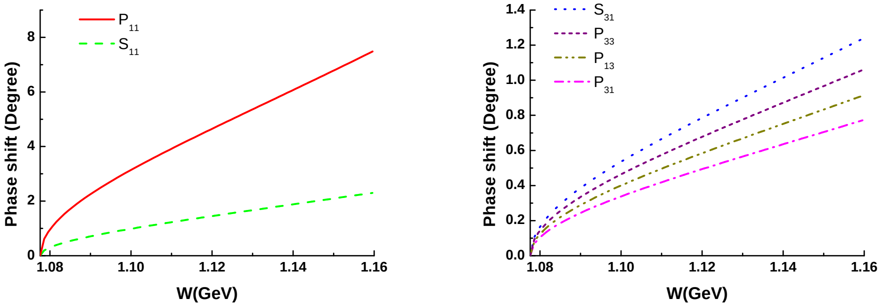
- Right-hand inelastic cut contribution $\rightarrow$ positive definite

$$f_{\mathsf{R}'}(s) = \frac{s}{\pi}\int_{(2m+M)^2}^{\Lambda_{\mathsf{R}}^2}\frac{\sigma(w)dw}{w(w-s)}$$

$$\sigma(w) = -\left\{\frac{\ln[\eta(w)]}{2\rho(w)}\right\}$$

- η: inelasticity, from SAID WI 08 data and extrapolation
- Cut-off: $\Lambda_{\mathsf{R}} = 4.00$GeV

RIGHT-HAND INELASTIC CUT



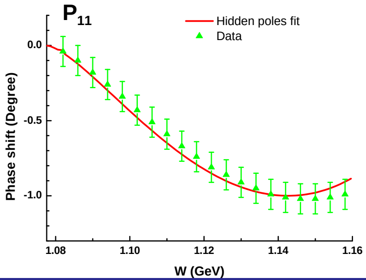
Far from enough!!

4. Hidden contributions

FINDING P_{11} HIDDEN POLE

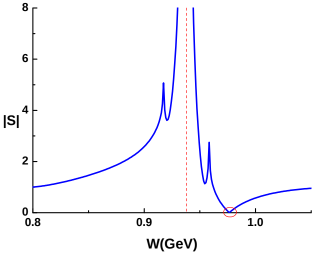
- P- wave: $\delta(k) \sim \mathcal{O}(k^3)$
- Initially one resonance $\rightarrow$ two virtual states $\rightarrow$ one survives, the other is nearly absorbed by the point $(M_N - m_\pi)^2$
- $s_c = -9$ GeV^2, virtual pole: 980 MeV, $\chi^2_{P_{11}}$/d.o.f $= 0.201$.
- An extra CDD pole is needed in P_{11} channel

[A. Gasparyan and M.F.M. Lutz 2010 NPA]



P_{11} CHANNEL: SHADOW POLE OF THE NUCLEON

- Analytical continuation: $S^{\text{II}} = 1/S^{\text{I}}$.
  Second sheet poles $\to$ first sheet zeros.
- Expansion: $S^{\text{I}} \sim a/(s - M_N^2) + b + \cdots$
- Arbitrary non-zero $b \to$ the virtual state
- Perturbative calculation $\to$ virtual state at 976 MeV; fit $\to 980$ MeV



FINDING S_{11} HIDDEN POLE

- $s_c = -0.08$ GeV^2, $\Lambda_\mathrm{R} = 4.00$ GeV.
- Hidden pole $\rightarrow$ a "crazy resonance" below threshold $(0.861 \pm 0.053) - (0.130 \pm 0.075)i$ GeV

s_c (GeV^2)	Pole position (GeV)	Fit quality χ^2/d.o.f
-0.08	$0.808 - 0.055i$	0.109
-1	$0.822 - 0.139i$	0.076
-9	$0.883 - 0.195i$	0.034
∞	$0.914 - 0.205i$	0.018

S_{11} CHANNEL: LOWEST POTENTIAL-NATURE RESONANCE?

- S_{11} channel $\to$ no s-channel intermediate states $\to$ potential nature interaction
- Square-well potential (μ: reduced mass)

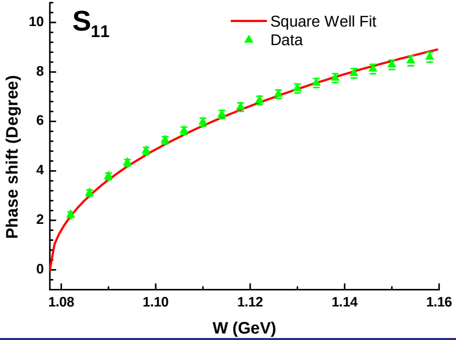
$$U(r) = 2\mu V(r) = \begin{cases} -2\mu V_0 & (r \leq L), \\ 0 & (r > L), \end{cases}$$

- Phase shift ($k' = (k^2 + 2\mu V_0)^{1/2}$)

$$\delta_{\text{sw}}(k) = \arctan\left[\frac{k\tan k'L - k'\tan kL}{k' + k\tan(kL)\tan(k'L)}\right]$$

S_{11} CHANNEL: LOWEST POTENTIAL-NATURE RESONANCE?

- Fit result (20 data): $L = 0.829$ fm and $V_0 = 144$ MeV, $\chi^2_{\text{sw}}/\text{d.o.f} = 0.740$
- Pole position: $k = -346i$ MeV $\to 0.872 - 0.316i$ GeV. Hidden pole fit $(0.861 \pm 0.053) - (0.130 \pm 0.075)i$ GeV



4. Summary

Summary

- PKU representation which separates phase shift contributions is employed to analyze πN elastic scatterings in s and p wave channels.
- The calculation of the left-hand cuts is under covariant baryon chiral perturbation theory at tree level.
- The S_{11} and P_{11} channels contain significant disagreements between "known poles $+$ cut" and the experiment, missing large positive contributions. (reliable, independent of numerical details)
- S_{11} channel contains a hidden resonance below threshold, while in P_{11} channel the nucleon pole induces a companionate virtual state.

Thank you !!

Back up

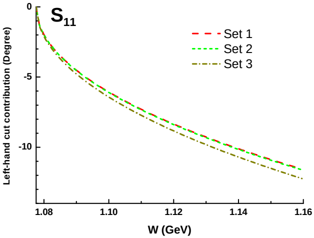
Poles

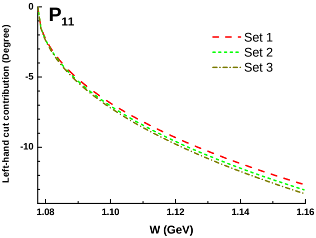
- Intermediate particles

Channel	$I(J^P)$	Intermediate particles
S_{11}	$\frac{1}{2}(\frac{1}{2}^-)$	$N^*(1535), N^*(1650), N^*(1895)$
S_{31}	$\frac{3}{2}(\frac{1}{2}^-)$	$\Delta(1620), \Delta(1900)$
P_{11}	$\frac{1}{2}(\frac{1}{2}^+)$	$N, N^*(1440), N^*(1710), N^*(1880)$
P_{31}	$\frac{3}{2}(\frac{1}{2}^+)$	$\Delta(1910)$
P_{13}	$\frac{1}{2}(\frac{3}{2}^+)$	$N^*(1720), N^*(1900)$
P_{33}	$\frac{3}{2}(\frac{3}{2}^+)$	$\Delta(1232), \Delta(1600), \Delta(1920)$

DETERMINATION OF COEFFICIENTS c_i?

- Set 1: this work
  Set 2: $\mathcal{O}(p^3)$ fit in [Y. H. Chen et. al. 2013 PRD]
  Set 3: [D. Siemens et. al. 2017 PLB]
- Different choices have little impact on the left-hand cut contributions!





$\mathcal{O}(p^3)$ PRELIMINARY RESULTS

- The same cut-off condition
- Chiral order does not impact on the existence of the S_{11} and P_{11} states
- $\mathcal{O}(p^3)$ greatly improves the fit quality in other channels that are impossible to fit the data at $\mathcal{O}(p^2)$, and there may be some indications of new hidden structures.