Model Independent Partial Wave Analysis for π^0 Photoproduction

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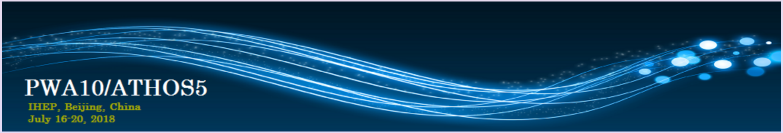

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Institut Ruđer Bošković

UNIVERSITY OF TUZLA

Mainz-Tuzla-Zagreb collaboration

Mainz

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Zagreb

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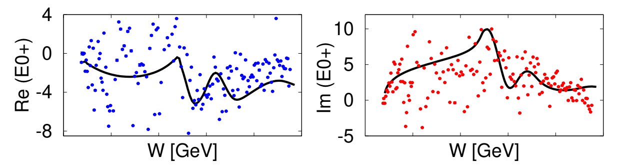
First PWA workshop, Abilene, 2004

Third PWA workshop, Tuzla, 2006

Outline

- Problems in the unconstrained single energy partial wave analysis
- Constrained PWA-Imposing the fixed-t analyticity
- Preliminary results
    - π^0 photoproduction
- Conclusions

Problems in the unconstrained single energy partial wave analysis



- One of the main problems of single energy PWA (SE PWA) are ambiguities of partial waves.
- First attempt: Requiring smoothness of partial waves as a function of energy. It was shown that it was not enough.
- One must impose more stringent constraints taking into account analyticity of scattering amplitudes.
- Dispersion relations? Not easy to apply!
- Powerful and elegant method has been proposed by Pietarinen. The method is called Pietarinen's expansion method.

Problems in the unconstrained single energy partial wave analysis

In a series of papers E. Pietarinen proposed a substitute for dispersion relations. In his method invariant amplitudes are expanded in terms of analytic functions having the same analytic structure.

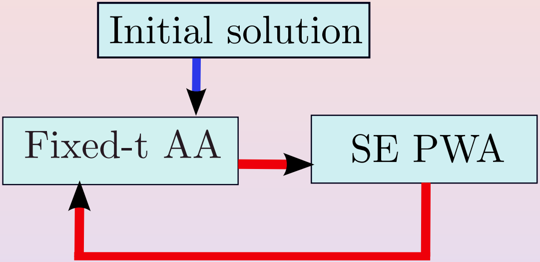
- E. Pietarinen: Amplitude analysis using fixed-t analyticity of invariant amplitudes
- E. Pietarinen, Nuovo Cim. 12 (1972) 522
- E. Pietarinen, Nucl. Phys. B49 (1972) 315 Discussion of uniqueness problem
- S. Bowcock, H. Burkhardt, Rep. Prog Phys 38 (1975) 1099
- E. Pietarinen, Nucl. Phys. 8107 (1976) 21 Discussion of uniqueness problem
- J. Hamilton, J. L. Peterson, New developments in dispersion theory, Vol.1, Nordita, 1975.

Problems in the unconstrained single energy partial wave analysis

We use the same approach as it was done in famous KH80 analysis of πN scattering data.
We already used the same approach for η photoproduction, and the results are published in paper
H. Osmanović, *et al.*, Phys. Rev. C 97, (2018) 015207.

- The method consists of two separated analysis:-
  - Fixed-t amplitude analysis (Fixed-t AA) - determination of the invariant scattering amplitudes from exp. data at a given fixed-t value
  - Constrained single energy partial wave analysis - SE PWA.
- Fixed-t AA and SE PWA are coupled. Results from one analysis are used as constraint in another in an iterative procedure.

Coupled fixed-t amplitude analysis and single energy PWA

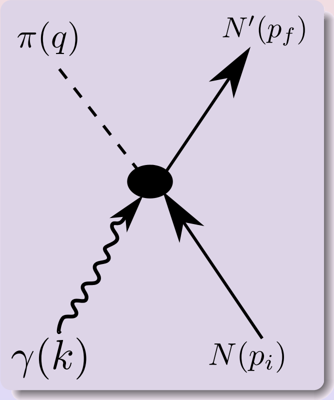


Connection between SE PWA and fixed-t AA

- Multipoles obtained from SE PWA at a given set of energies are used to calculate helicity amplitudes which are used as constraint in the fixed-t amplitude analysis.
- The whole procedure has to be iterated until reaching reasonable agreement in two subsequent iterations

Pion photoproduction-kinematics

p_i -four momentum of incoming nucleon
p_f - four momentum of outgoing nucleon
k - four momentum of incident photon
q - four momentum of π meson
Mandelstam variables:
$s = w^2 = (p_i + k)^2$
$t = (q - k)^2$
$u = (p_i - q)^2$
$\nu = \frac{s-u}{4m}$
$s + t + u = 2m^2 + m_\pi^2$;
m - mass of nucleon,
m_π - mass of π meson



Pion photoproduction-kinematics

- In pion photoproduction we consider four reactions:

$$\gamma + p \rightarrow \pi^0 + p; \qquad \gamma + p \rightarrow \pi^+ + n$$

$$\gamma + n \rightarrow \pi^0 + n; \qquad \gamma + n \rightarrow \pi^- + p$$

- For each reaction we have four independent Invariant amplitudes B_1, B_2, B_6 and B_8 defined as in I. G. Aznauryan, Phys. Rev C67, (2013) 015209.
- The invariant amplitudes can be decomposed into three isospin combinations: $B^{(\pm)}$, $B^{(0)}$.
- $B^{(\pm)}$ describe absorption of isovector photon ($I_3 = 1$); $B^{(0)}$ decribes absorption of isoscalar photon ($I_3 = 0$)

Pion photoproduction-kinematics

Invariant amplitudes describing reactions are linear combinations of above defined IA:

- $B_i(\gamma + p \to \pi^0 + p) = B_i^{(+)} + B_i^{(0)}$
- $B_i(\gamma + n \to \pi^0 + n) = B_i^{(+)} - B_i^{(0)}$
- $B_i(\gamma + n \to \pi^+ + n) = \sqrt{2}(B_i^{(-)} + B_i^{(0)})$
- $B_i(\gamma + n \to \pi^- + p) = -\sqrt{2}(B_i^{(-)} - B_i^{(0)})$

In order to get a full insight into πN system by studing the pion photoproduction, one needs data on three physical reactions to determine **4 x 3** amplitudes.

Studying **particular** reaction gives a linear combination of multipoles with different values of isospin - no isospin separation.

I'll present results of PWA of $\gamma + p \to \pi^0 + p$ data.

Observables, amplitudes and multipoles in π^0 photoproduction

16 observables

Spin Observable	Type
σ_0 $\hat{\Sigma}$ $\hat{T}$ $\hat{P}$	$\mathcal{S}$ (single spin)
$\hat{G}$ $\hat{H}$ $\hat{E}$ $\hat{F}$	$\mathcal{BT}$ (beam–target)
$\hat{O}_{x'}$ $\hat{O}_{z'}$ $\hat{C}_{x'}$ $\hat{C}_{z'}$	$\mathcal{BR}$ (beam–recoil)
$\hat{T}_{x'}$ $\hat{T}_{z'}$ $\hat{L}_{x'}$ $\hat{L}_{z'}$	$\mathcal{TR}$ (target–recoil)

Observables are represented by one set of four complex amplitudes:

- CGLN amplitudes ($F_k(W, \cos\theta)$, $k = 1, 2, 3, 4$)
- helicity amplitudes ($H_k(W, \cos\theta)$, $k = 1, 2, 3, 4$)
- invariant amplitudes ($B_k(s, t)$, $k = 1, 2, 6, 8$)

Amplitudes are given by expansion in terms of electric ($E_{\ell\pm}$) and magnetic ($M_{\ell\pm}$) multipoles.

Observables, amplitudes and multipoles in π^0 photoproduction

Example, differential cross section in terms of helicity amplitudes

$$\frac{d\sigma}{d\Omega} = \frac{q}{2k}(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2)$$

Expansion of CGLN in terms of multipoles (truncated - up to $\ell = L_{max}$)

$$F_1 = \sum_{\ell \geq 0}^{L_{max}} \{(\ell M_{\ell+} + E_{\ell+})P'_{\ell+1} + [(\ell+1)M_{\ell-} + E_{\ell-}]P'_{\ell-1}\},$$

$$F_2 = \sum_{\ell \geq 1}^{L_{max}} [(\ell+1)M_{\ell+} + \ell M_{\ell-}]P'_{\ell},$$

$$F_3 = \sum_{\ell \geq 1}^{L_{max}} [(E_{\ell+} - M_{\ell+})P''_{\ell+1} + (E_{\ell-} - M_{\ell-})P''_{\ell-1}]$$

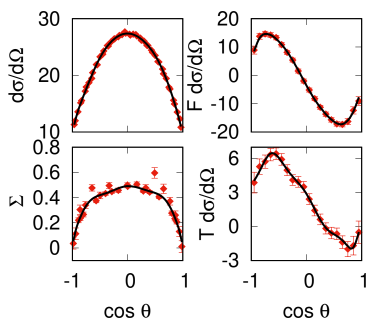
$$F_4 = \sum_{\ell \geq 2}^{L_{max}} (M_{\ell+} - E_{\ell+} - M_{\ell-} - E_{\ell-})P''_{\ell}$$

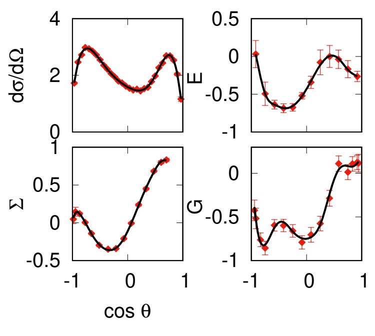
Some concepts in analysis of experimental data

- Energy dependent (ED)
  partial waves (multipoles) are parameterized as a function of energy (model dependent). In this talk we will use four ED solutions.
- Single energy (SE)
  multipoles are determined at a single energy.
- Amplitude analysis (AA)
  amplitudes are parameterized as a function of energy in energy range where data are available.

Unconstrained single energy partial wave analysis

We fitted experimental data for $\gamma p \to \pi^0 p$ reaction.
$L_{max} = 5$, $40-$ real parameters. Multipoles with $L > 5$ are set to zero.

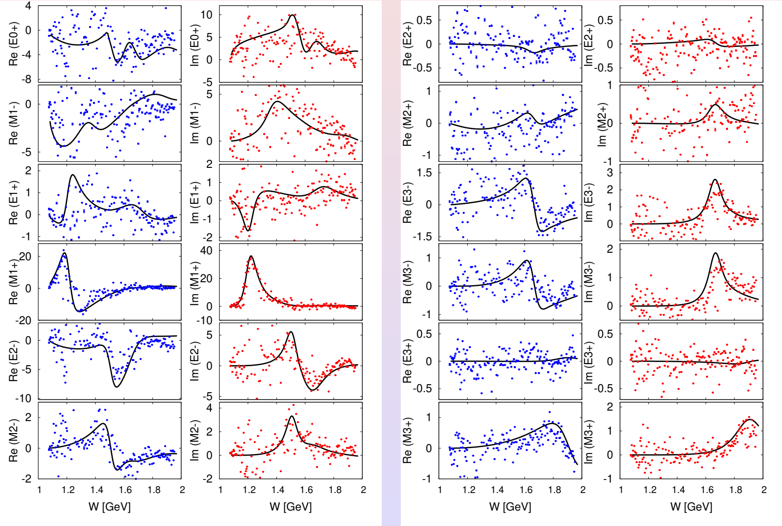




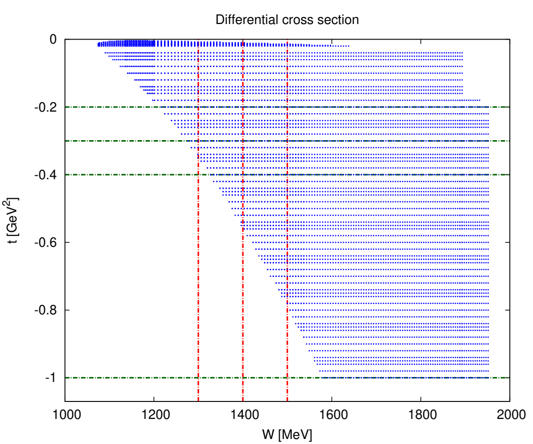
$E = 0.3 GeV; W = 1.201 GeV$ $E = 1.0 GeV; W = 1.66 GeV$

Unconstrained SE PWA

(MAID-ED -black line).



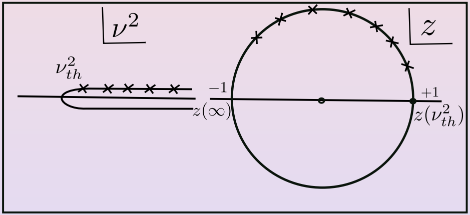
Problem is more serious - UNIQUENESS. How to resolve it?



SE PWA is performed along red lines. Fixed-t AA is performed along green lines.

Blue dots-experimental data in the physical region of π^0 photoproduction.

Analytic structure of invariant amplitudes in pion photoproduction. Apart from nucleon poles, crossing symmetric invariant amplitudes are analytic function in a complex ν^2 plane $\nu_{th}^2 \leq \nu^2 < \infty$, ($\nu_{th} = m_\pi + \frac{t}{4m}$).



Conformal mapping:

$$z = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}}$$

Points on the cut in complex ν^2 plane is mapped on the circle.

Fixed-t amplitude analysis of π^0 photoproduction data

For a given t invariant amplitudes are represented by Pietarinen series (I. G. Aznauryan, Phys. Rev. C 67, 015209 (2003)):

$$B_1 = B_{1N} + \sum_{i=0}^{N} b_{1i} z^i, \quad B_2 = B_{2N} + \sum_{i=0}^{N} b_{2i} z^i$$

$$B_6 = B_{6N} + \sum_{i=0}^{N} b_{6i} z^i, \quad B_8 = \frac{B_{8N}}{\nu} + \sum_{i=0}^{N} b_{8i} z^i$$

B_{iN} are known nucleon pole contributions. B_i are crossing symmetric invariant amplitudes.

Fixed-t amplitude analysis

Coefficients $\{b_{1i}\}$, $\{b_{2i}\}$, $\{b_{6i}\}$ and $\{b_{8i}\}$ are obtained by minimizing a quadratic form

$$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi_{conv}$$

χ^2_{data} contains data at a given t-value.
χ^2_{PW} contains as the "data" the helicity amplitudes from the SE PWA analysis.
Φ_{conv} is Pietarinen's convergence test function
(H. Osmanović, *et al.*, Phys. Rev. C 97, (2018) 015207,
E. Pietarinen, Nucl. Phys. B 107, 21 (1976)).

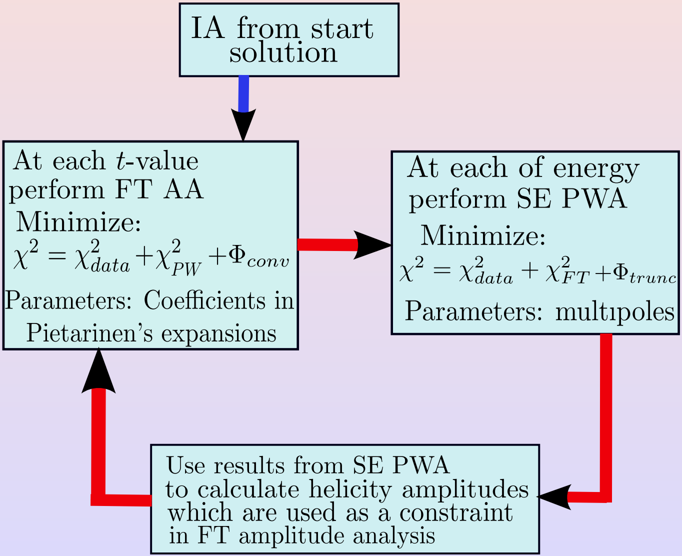
$$\Phi = \Phi_1 + \Phi_2 + \Phi_6 + \Phi_8$$

$$\Phi_k = \lambda_k \sum_{i=0}^{N} (b_{ki})^2 (n+1)^3, \quad k = 1, 2, 6, 8$$

λ_k are adjustable weight parameters.

In a first iteration helicity amplitudes are calculated from initial, already existing PW solution.
In subsequent iterations helicity amplitudes are calculated from multipoles obtained in SE PWA of the same set of experimental data.

Fixed-t amplitude analysis



π^0 photoproduction data base -up to 8 measured observables

1. Differential cross section σ_0

A2MAMI:	D. Hornidge, PRL 111 (2013) 062004	$E = 146 \ldots 795 MeV$
A2MAMI:	P. Adlarson et al., PRC 92 (2015) 024617	$E = 218 \ldots 1445 MeV$

2. Beam asymmetry Σ

A2MAMI:	D. Hornidge, PRL 111 (2013) 062004	$E = 146 \ldots 318 MeV$
GRAAL:	O. Bartalini, EPJ A 26 (2005) 399	$E = 551 \ldots 1487 MeV$
A2MAMI:	R. Leukel, PhD thesis (2001) Mainz University	$E = 235 \ldots 445 MeV$

3. Target asymmetry T

A2MAMI:	J. R. M. Annand, PRC 93, (2016) 055209	$E = 425 \ldots 1445 MeV$
CBELSA/TAPS:	J. Hartmann, PRL 113 (2014) 062001	$E = 670 \ldots 930 MeV$
A2MAMI:	P. Otte, PhD thesis (2015), Mainz University	$E = 145 \ldots 419 MeV$
A2MAMI:	S. Schumann et al., PLB 750 (2015) 252	$E = 145 \ldots 170 MeV$

4. Recoil asymmetry P

CBELSA/TAPS:	J. Hartmann, PRL 113 (2014) 062001	$E = 670 \ldots 930 MeV$

5. Double-polarisation asymmetry E

DAPHNE/MAMI:	I. Preobrajenski, PhD thesis (2001)	$E = 300 \ldots 790 MeV$
CBELSA/TAPS:	M. Gottschall, PRL 112 (2014), 012003	$E = 700 \ldots 2000 MeV$
A2MAMI:	J. Linturi, PhD thesis (2015), Mainz University	$E = 225 \ldots 1400 MeV$

6. Double-polarisation asymmetry F

A2MAMI:	J. R. M. Annand, PRC 93, (2016) 055209	$E = 425 \ldots 1445 MeV$
A2MAMI:	P. Otte, PhD thesis (2015), Mainz University	$E = 145 \ldots 419 MeV$

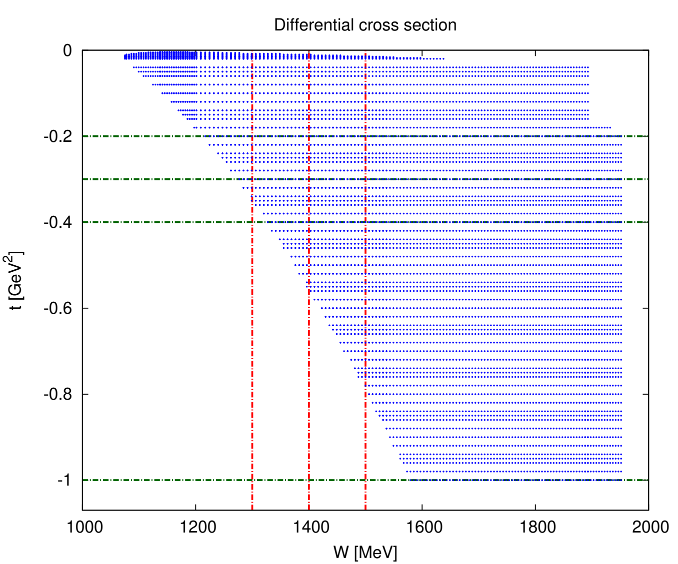
7. Double-polarisation asymmetry G

DAPHNE/MAMI:	J. Ahrens, EPJA 26 (2005) 135	$E = 340$MeV
CBELSA/TAPS:	A. Thiel, PRL 109 (2012), 102001	$E = 620 \ldots 1120 MeV$

8. Double-polarisation asymmetry H

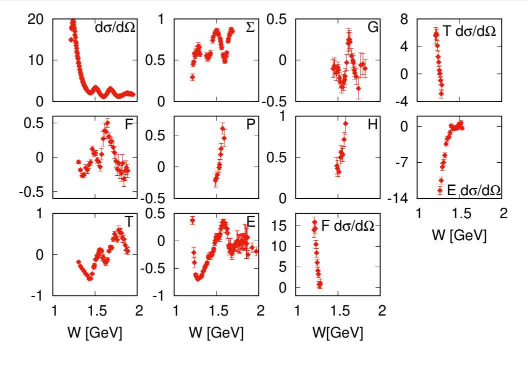
CBELSA/TAPS:	J. Hartmann, PRL 113 (2014) 062001	$E = 670 \ldots 930 MeV$

Preparing input data

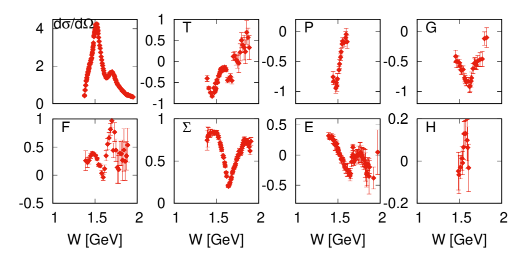


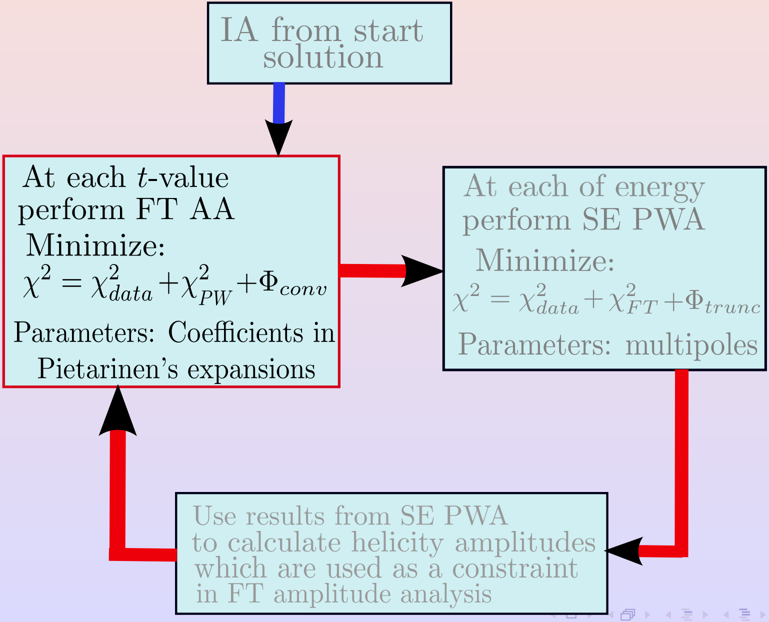
π^0 photoproduction data base in FT

Observables at $t = -0.2\,GeV^2$



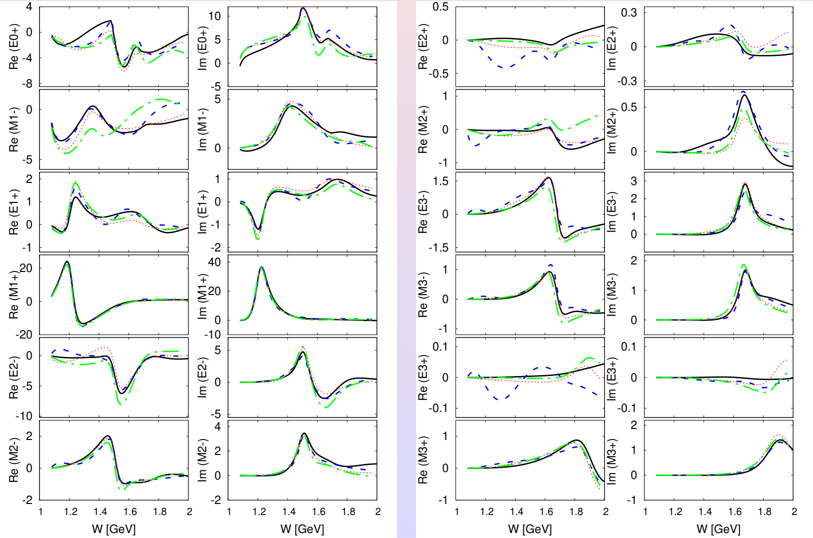
Observables at $t = -0.5\,GeV^2$



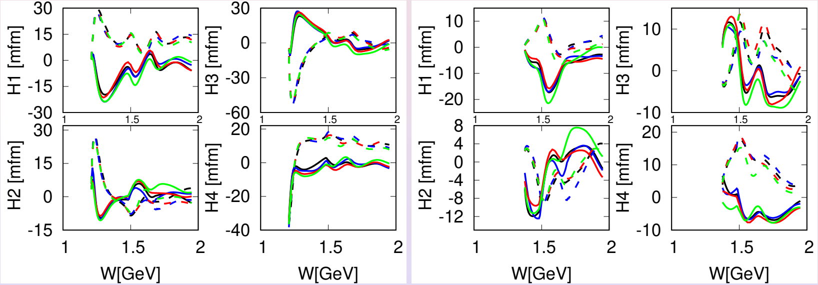

IA from start solution
At each t-value perform FT AA
Minimize:
$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi_{conv}$
Parameters: Coefficients in Pietarinen's expansions
At each of energy perform SE PWA
Minimize:
$\chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc}$
Parameters: multipoles
Use results from SE PWA to calculate helicity amplitudes which are used as a constraint in FT amplitude analysis

Fixed-t amplitude analysis-initial solutions

In order to explore model dependence of our solution we started with four ED solutions (BnGa-black, JüBo-blue, MAID-green, SAID-red) solutions.



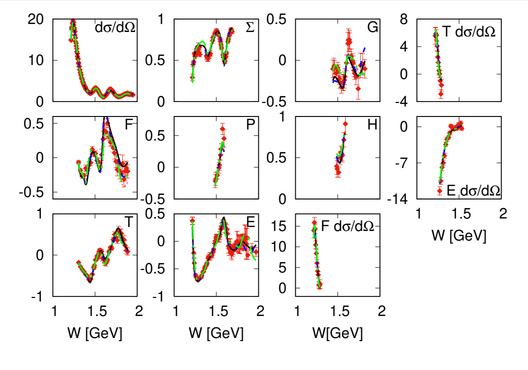
Helicity amplitudes from initial solutions (BnGa-black, JüBo-blue, MAID-green, SAID-red)



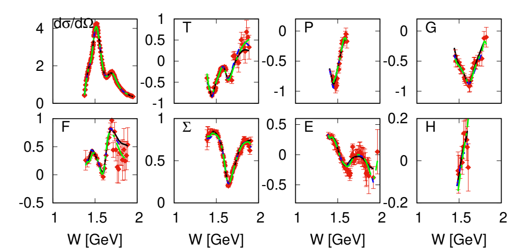
$t = -0.2 GeV^2$; $t = -0.5 GeV^2$

Fixed-t amplitude analysis

Fit to the data (BnGa-black, JüBo-blue, MAID-green, SAID-red)

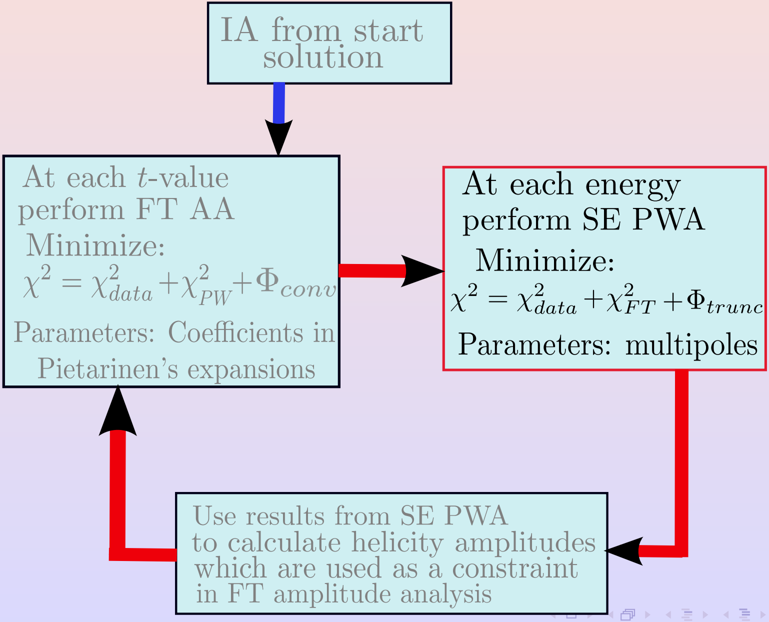


Fit to the data $t = -0.2 GeV^2$

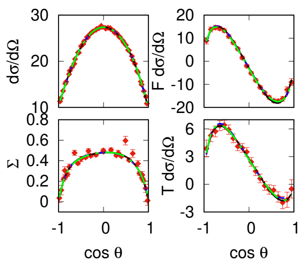


Fit to the data $t = -0.5 GeV^2$

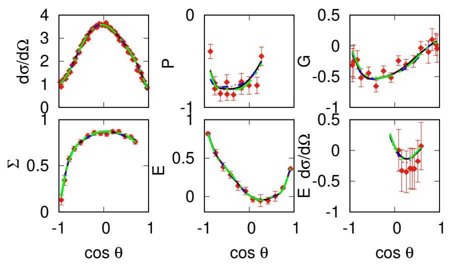
Constrained SE PWA-real data



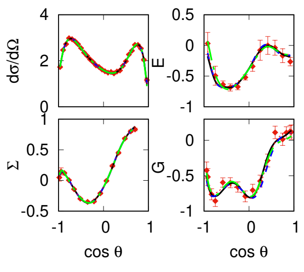
Constrained SE PWA-fit to the data



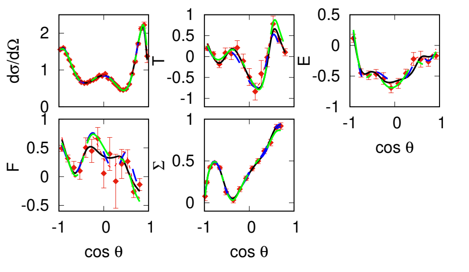
(a) E=0.300GeV; W=1.201GeV



(b) E=0.700GeV;W=1.481GeV

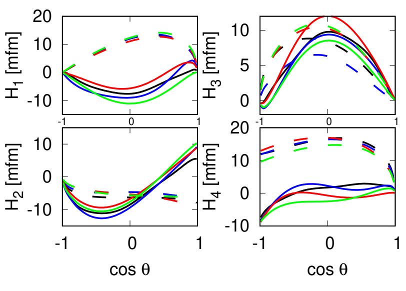


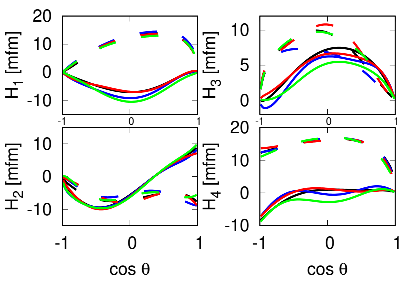
(c) E=1.000GeV;W=1.660GeV



(d)E=1.400GeV;W=1.872GeV

Helicity amplitudes at $W = 1481.01 MeV$.



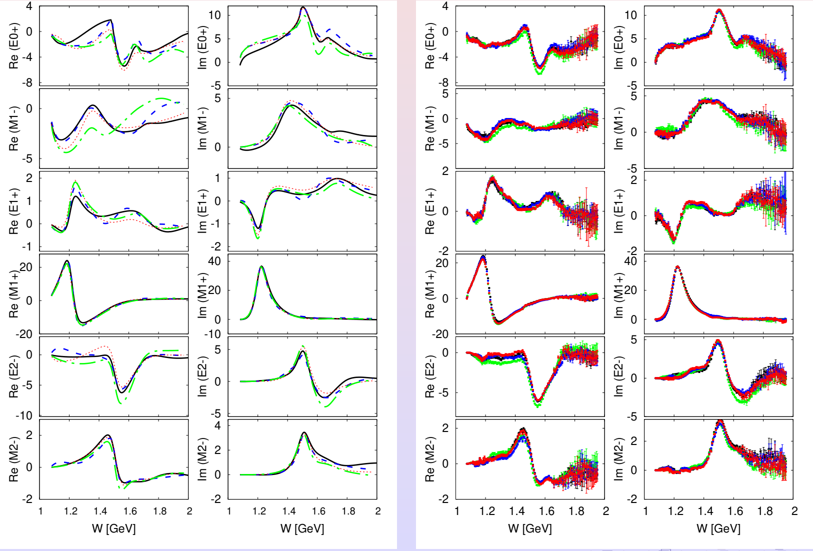


(a) Helicity amplitudes-initial solutions

(b) Helicity amplitudes after three iterations obtained using ED solutions as a constraint.

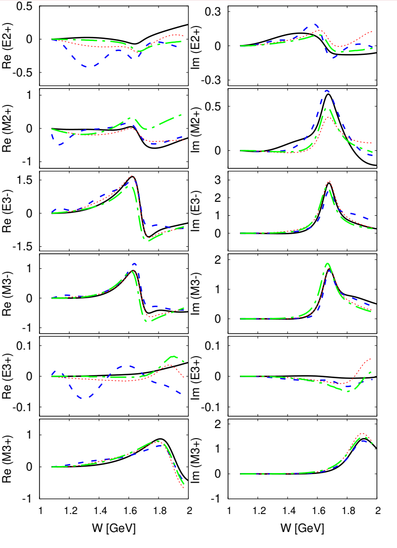
Constrained SE PWA-multipoles

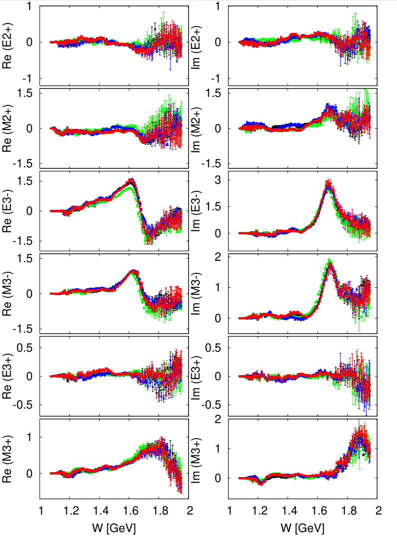
SE solutions obtained using ED solutions BnGa-black, JüBo-blue, MAID-green, SAID-red as a constraint.



Constrained SE PWA-multipoles

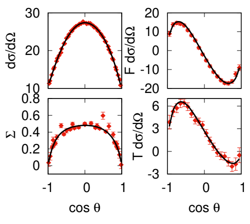
SE solutions obtained using ED solutions BnGa-black, JüBo-blue, MAID-green, SAID-red as a constraint.





Constrained SE PWA-fit to the data

We calculated avarage value of SE solutions, and used them as constraint in our procedure.



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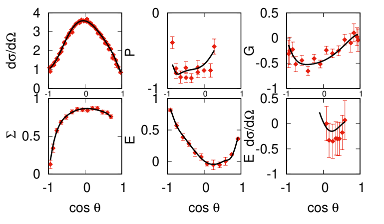
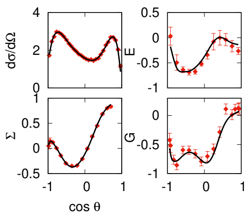


(b) E=0.700GeV;W=1.481GeV



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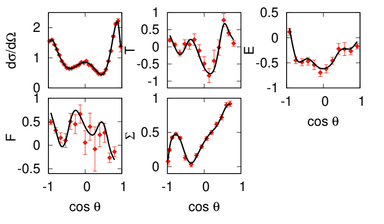
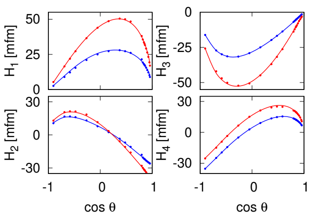


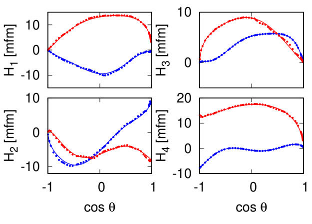
(d) E=1.400GeV;W=1.872GeV

Constrained SE PWA-helicity amplitudes

To be pointed out: Real and imaginary parts of helicity amplitudes (blue and red dots- Re FT and Im FT) are obtained from independent fixed-t AA at different t-values.



(a) E=0.300GeV; W=1.201GeV



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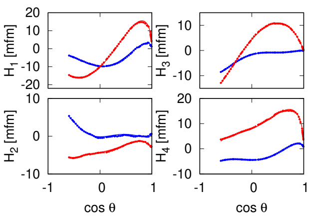
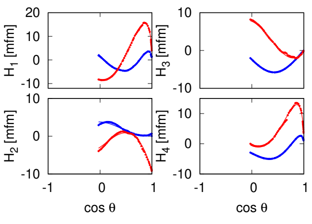


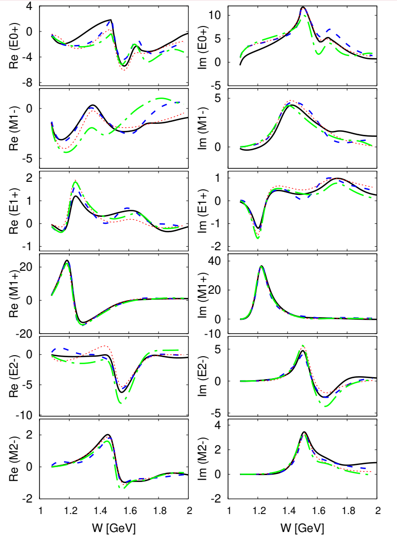
(c) E=1.000GeV;W=1.660GeV

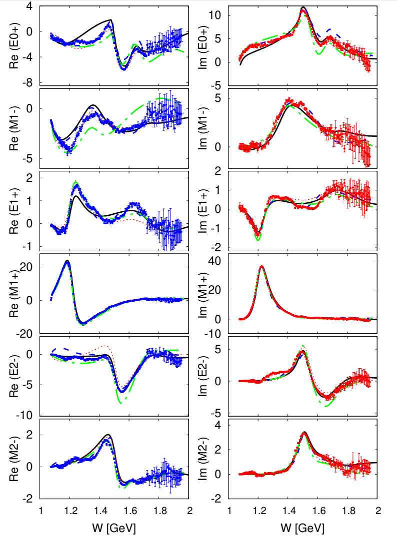


(d) E=1.400GeV;W=1.872GeV

Constrained SE PWA-multipoles

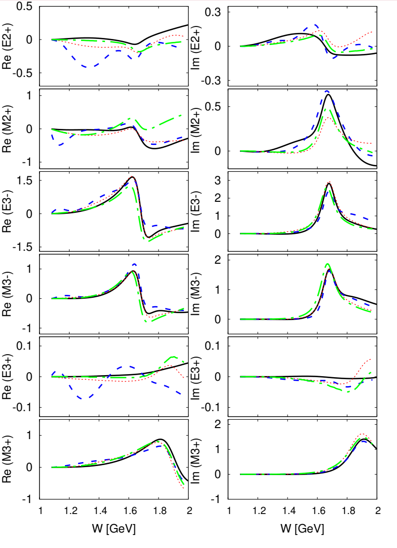
We calculated avarage value of SE solutions, and used them as constraint in our procedure.

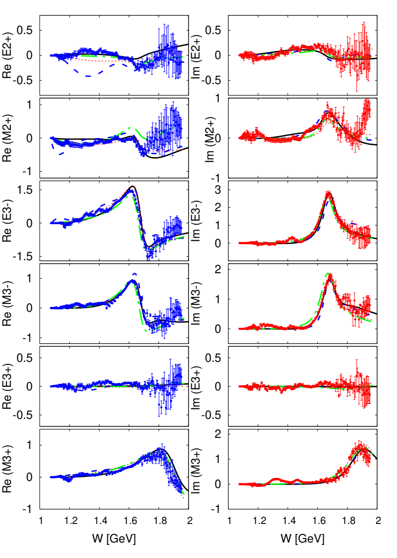




Constrained SE PWA-multipoles

We calculated avarage value of SE solutions, and used them as constraint in our procedure.

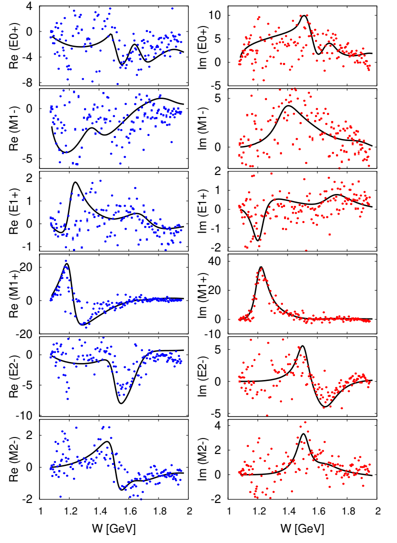


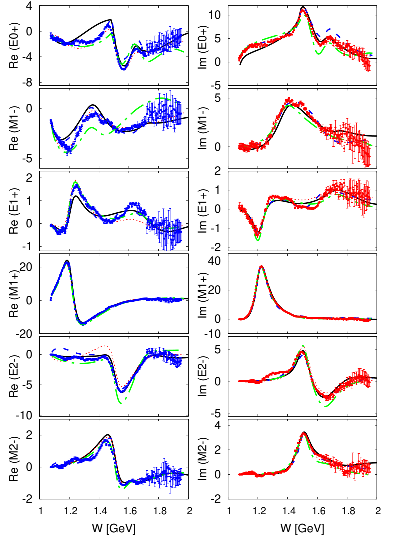


Conclusions

Unconstrained -SE PWA $\Downarrow$

SE PWA with **Fixed – t** constraint $\Downarrow$

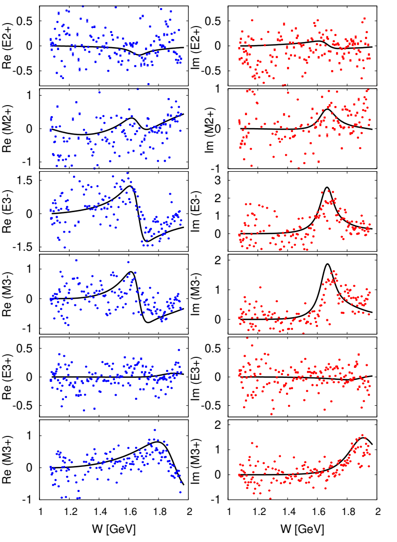




Conclusions

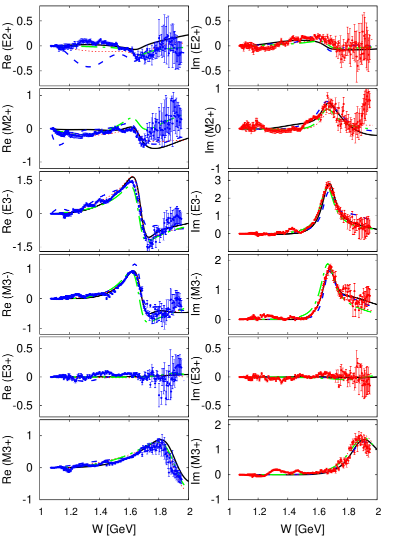
Unconstrained -SE PWA

⇓



SE PWA with **Fixed − t** constraint

⇓



Next step and a final goal:

PWA analysis of all pion photoproduction data simultaneously.
It will make it possible to

- Determine isospin 3/2 and 1/2 multipoles
- Extract parameters of resonances