Model independent PWA in light and heavy meson decays - Traps and remedies

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PWA with fixed shapes of isobars vs. freed isobars

PWA with established isobars:

Partial waves are labelled as $i, \varepsilon = J^{PC} M^{\varepsilon} \xi \pi L$

The mass-independent PWA events density:

$\mathcal{I}(\tau) = \sum_{\varepsilon} \sum_{r} |\sum_{i} T^{\varepsilon}_{ir} \psi^{\varepsilon}_{i}(\tau)|^2$

The decay amplitude $\psi^{\varepsilon}_{i}(\tau)$ contains angular part and $\pi^-\pi^+$ isobar Breit-Wigner function and is bose-symmetrized for (1) $\leftrightarrow$ (3) of $\pi^-_{(1)}\pi^+_{(2)}\pi^-_{(3)}$ system:

$\psi^{\varepsilon}_{i}(\tau) = A^{\varepsilon}_{q}(\Omega_{12}, \Omega^*_1) BW^{\varepsilon}_{q,k}(m_{12}) + A^{\varepsilon}_{q}(\Omega_{32}, \Omega^*_3) BW^{\varepsilon}_{q,k}(m_{32})$

where $q = J^{PC} M \nu L$ and ν is spin of $\pi\pi$ system

PWA with freed isobars:

The fixed amplitude of $\pi^-\pi^+$ isobar is replaced by sum of step-like functions with complex coefficients: $BW_{q,k}(m) \rightarrow \sum_{\beta} \omega_{q,\beta} \Pi_{\beta}(m)$ gives:

$\hat{\psi}^{\varepsilon}_{q,\beta} = A^{\varepsilon}_{q}(\Omega_{12}, \Omega^*_1) \Pi_{\beta}(m_{12}) + A^{\varepsilon}_{q}(\Omega_{32}, \Omega^*_3) \Pi_{\beta}(m_{32})$

Integral matrix: $INT_{q\beta, q'\beta'} = \int \hat{\psi}_{q\beta} \hat{\psi}^*_{q'\beta'} d\Phi_3(\tau)$

Analysis with freed isobars - Zero Modes

The whole free-isobarred amplitude for limited $J^{PC}M^{\epsilon}$ sector:

$\Psi^{\varepsilon}_{J^{PC}M} = \sum_a \sum_\beta \tilde{\omega}^{\varepsilon}_{J^{PC}M,a,\beta}(A^{\varepsilon}_{J^{PC}M,a}(\Omega_{12},\Omega^*_1)\Pi_\beta(m_{12})+A^{\varepsilon}_{J^{PC}M,a}(\Omega_{32},\Omega^*_3)\Pi_\beta(m_{32}))$

where $a=\nu, L$. Notation again: $J^{PC}M^{\varepsilon}(\pi\pi)_\nu\ \pi\ L$

Continuous ambiguities - Zero Modes:

We have found ambiguities for freed amplitudes $\tilde{\omega}^{\varepsilon}_{J^{PC}M,a,\beta}$ inside same $J^{PC}M^{\epsilon}$, they are always real-valued functions z^a_β so that

$\tilde{\omega}^a_\beta \rightarrow \tilde{\omega}^a_\beta + \sum_z C_z z^a_\beta$ will not change the whole amplitude $\Psi^{\varepsilon}_{J^{PC}M}(\tau)$.

They arise if:

a) freed amplitudes have bose-symmetrization (+usually more then one $a=\nu, L$ with $(\pi\pi)_\nu$ freed independently)

b) simultaneous different freed isobars in different combinations of final mesons

Strategy for resolving the ambiguity:

a) find some solution for 2π amplitudes $\tilde{\omega}_{a\beta}$ (ambiguous $\rightarrow$ infinite uncertainties)

b) modify PWA fit covariance matrix - add finite eigenvalues for eigenvectors in ZM direction $\rightarrow CoV^{-1}$ provides determination of ZM for correction

c) perform model-dependent fit to 2π amplitude:

$C_a\ BW_{a\beta}(M_0,\Gamma_0)\ I_{a\beta,a\beta} + \sum_z C_z z_{a\beta}$ to $\tilde{\omega}^a_\beta$

d) Substract zero mode(s) $\sum_z C_z z_{a\beta}$ from $\tilde{\omega}_{a\beta}$ to get "corrected" values $\omega_{a\beta}$

Two "simple" analythical examples

1) $1^{++} \to (\eta\pi^-)_S\ \pi^+\ P + (\eta\pi^+)_S\ \pi^-\ P + (\pi^+\pi^-)_S\ \eta\ P$

$\Psi_{1^{++}} = (\vec{p}_\eta + \vec{p}_{\pi^-}) a_0(m_{\eta\pi^-}) + (\vec{p}_\eta + \vec{p}_{\pi^+}) a_0(m_{\eta\pi^+}) + (\vec{p}_{\pi^-} + \vec{p}_{\pi^+}) \sigma(m_{\pi^-\pi^+})$

$a_0(m) + C$ and $\sigma(m) + C$ $\to$ no change for $\Psi_{1^{++}}$ (3-vectors cancel) $z(m) = 1$.

2) $1^{-+} \to (\pi^{-(1)}\pi^{+(2)})_P\ \pi^{-(3)}\ P + (\pi^{-(3)}\pi^{+(2)})_P\ \pi^{-(1)}\ P$

$\Psi_{1^{-+}} = [\vec{p_1} x \vec{p_2}]\rho(m_{12}) + [\vec{p_3} x \vec{p_2}]\rho(m_{32})$

$\rho(m) + C$ $\to$ no change for $\Psi_{1^{-+}}$ as $[\vec{p_3} x \vec{p_2}] = -[\vec{p_1} x \vec{p_2}]$ so $z(m) = 1$

Non-relativistic Zemach formalism gives simpliest formulas for those cases

$$0^{-+} \to (\pi^{-(1)}\pi^{+(2)})_S\ \pi^{-(3)}\ S + (\pi^{-(3)}\pi^{+(2)})_S\ \pi^{-(1)}\ S + (\pi^{-(1)}\pi^{+(2)})_P\ \pi^{-(3)}\ P + (\pi^{-(3)}\pi^{+(2)})_P\ \pi^{-(1)}\ P$$

Using non-relativistic Zemach formalism for Dalitz-plot analysis:

$$\psi_{0^{-+}\sigma\pi S} = \sigma(m_{12}) + \sigma(m_{32})$$

$$\psi_{0^{-+}\rho\pi P} = \tfrac{1}{4}(m_{12}^2 + 2m_{32}^2 - m_{3\pi}^2 - 3m_\pi^2)\rho(m_{12}) + \tfrac{1}{4}(m_{32}^2 + 2m_{12}^2 - m_{3\pi}^2 - 3m_\pi^2)\rho(m_{32})$$

$$\Psi_{0^{-+}} = \psi_{0^{-+}\sigma\pi S} + \psi_{0^{-+}\rho\pi P}$$

$\rho(m) + 4C$ and $\sigma(m) + C(m_{3\pi}^2 + 3m_\pi^2 - 3m^2)$ no change for $\Psi_{0^{-+}}$

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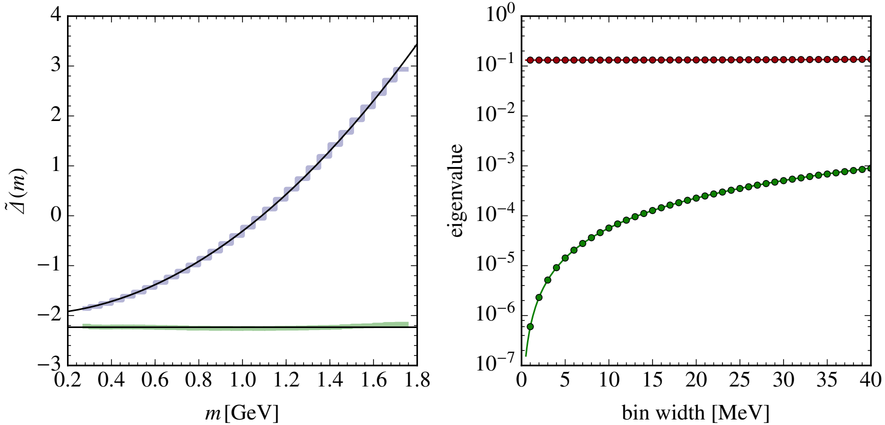
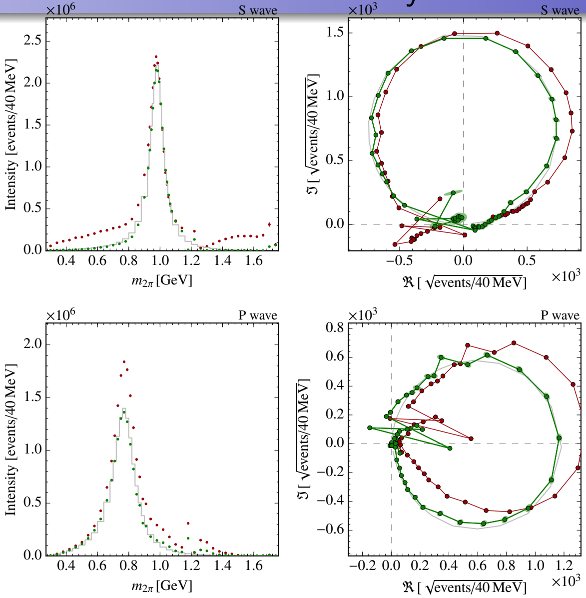
$D \to \pi^-\pi^+\pi^-$ - numerical Zero Modes



Zero modes in $0^{-+}(\pi\pi)_S\pi S$ and $0^{-+}(\pi\pi)_P\pi P$

dependence of the 2 smallest eigenvalues on the $m(2\pi)$ bin width

$D \to \pi^-\pi^+\pi^-$ - MC study of freed PWA



Grey: input MC shapes, Red: uncorrected $\tilde{\omega}_{a\beta}$, Green: corrected for ZM $\omega_{a\beta}$

Models of free isobarred COMPASS fits:

The basic fit includes 11 different $J^{PC}M^{\varepsilon}(\pi\pi)_{\nu}\pi L$ free-isobarred amplitudes:

$J^{PC}M^{\varepsilon}$	$(\pi\pi)_{\nu}$ L				ZM
$0^{-+}0^{+}$	$(\pi^+\pi^-)_S\pi S$;	$(\pi^+\pi^-)_P\pi P$			+
$1^{++}0^{+}$	$(\pi^+\pi^-)_S\pi P$;	$(\pi^+\pi^-)_P\pi S$			+
$1^{++}1^{+}$		$(\pi^+\pi^-)_P\pi S$			
$2^{-+}0^{+}$	$(\pi^+\pi^-)_S\pi D$;	$(\pi^+\pi^-)_P\pi P$;	$(\pi^+\pi^-)_P\pi F$;	$(\pi^+\pi^-)_D\pi S$	+
$2^{-+}1^{+}$		$(\pi^+\pi^-)_P\pi P$			
$2^{++}1^{+}$		$(\pi^+\pi^-)_P\pi D$			

+72 waves with fixed isobares left

Adding one more free isobarred wave:

$1^{-+}1^{+}$	$(\pi^+\pi^-)_P\pi P$	+

71 waves with fixed isobares left

Adding 2 additional free isobarred waves:

$4^{++}1^{+}$	$(\pi^+\pi^-)_P\pi G$;	$(\pi^+\pi^-)_D\pi F$;	

70 waves with fixed isobares left

Adding 3 additional free isobarred waves:

$3^{++}0^{+}$	$(\pi^+\pi^-)_P\pi D$;	$(\pi^+\pi^-)_D\pi P$;	$(\pi^+\pi^-)_F\pi S$	+

69 waves with fixed isobares left

3 TYPES OF AMPLITUDES in FREE-ISOBARRED PWA

- $(\pi\pi)_P\pi$, $(\pi\pi)_D\pi$, $(\pi\pi)_F\pi$ corresponding to ONE fixed-isobarred wave $\rho\pi$, $f_2\pi$, $\rho_3\pi$
  3 types of values:
  - total intensity of freed wave, summing contribution of all step-like amplitudes and their mutual overlaps (blue)
  - modelled total intensity - using smooth fitting function projected to the center of each step, production phase (magenta)
  - 88 wave isobarred PWA intensity, phase (black)
- $(\pi\pi)_S\pi L$ with $L = 0, 1, 2$ corresponding to several fixed-isobarred waves like $f_0(600)\pi$, $f_0(980)\pi$, $f_0(1500)\pi$
  3 types of values:
  - intensity, phase (magenta)
  - intensity, phase in 88 isobarred-PWA (if exists!) (black)
- Amplitudes with remaining fixed isobars 3 types of values:
  - Remaining fixed-isobarred 69 waves intensity, phase (red)
  - 88 wave fully isobarred PWA intensity, phase (black)

TWO stages of modelling

- Fitting only amplitudes with $(\pi\pi)_P\pi$, $(\pi\pi)_D\pi$, $(\pi\pi)_F\pi$ by Breit-Wigner shapes of $\rho(770)$, $f_2(1260)$ and $\rho_3(1690)$ and resolving Zero modes, if any.
- Fitting $(\pi\pi)_S\pi$ by $f_0(600)\pi$, $f_0(980)\pi$, $f_0(1500)\pi$ - each resonance - separately in relatively narrow $m_{2\pi}$ region and adding complex linear background as $C_0 + C_1 m_{2\pi}$

Each fit is done independently in ($m_{3\pi}$, t') - bin

1) We have obtained free isobarred amplitudes $\omega_{q\beta}$. In case of ZM, they are corrected in model-dependent way.
To construct total intensity $J^{PC}M^{\varepsilon}\xi\pi L$ - Spin-total approach is used:

$I_q^{\varepsilon} = \sum \omega_{q\beta}\omega_{q\beta'} I\bar{N}T_{q\beta,q\beta'}$
This intensity corresponds to one-wave intensity in fixed PWA - only if there is one isobar ξ for a given $J^{PC}M^{\varepsilon}(\pi\pi)_{\nu}\pi L$

2) At each stage of modeling $\tilde{\omega}_{q\beta}$ there is corresponding smooth function:

$\hat{\omega}_{q\beta} = C_q \; BW_{q\beta}(M_0, \Gamma_0) \; I_{q\beta,q\beta}$

Analogous, the following spin-total intensity could be obtained:

$\hat{I}_q^{\varepsilon} = \sum \hat{\omega}_{q\beta}\hat{\omega}_{q\beta'} I\bar{N}T_{q\beta,q\beta'}$

Modelled phase:
$\hat{\phi} = arg(C_q)$

3) There are corresponding intensities and phases in fixed-isobar PWA (here 88 wave-set)

CONCLUSIONS

- The method of PWA with freed isobars is worked out and X-checked btw. rootPwa and compassPwa
  - The continuous ambiguities for freed 2 body amplitudes were discovered
  - The method of their resolving is implemented
  - Various methods of modeling of 2 body amplitudes
- We applied the PWA with freed isobars to
  - Toy Monte Carlo for $D \to \pi^- \pi^+ \pi^-$
  - COMPASS $\pi^- p \to \pi^- \pi^+ \pi^- p$ (Highly preliminary results are demonstrated)
- Work in progress ...