Controlling uncertainties in electroweak precision studies

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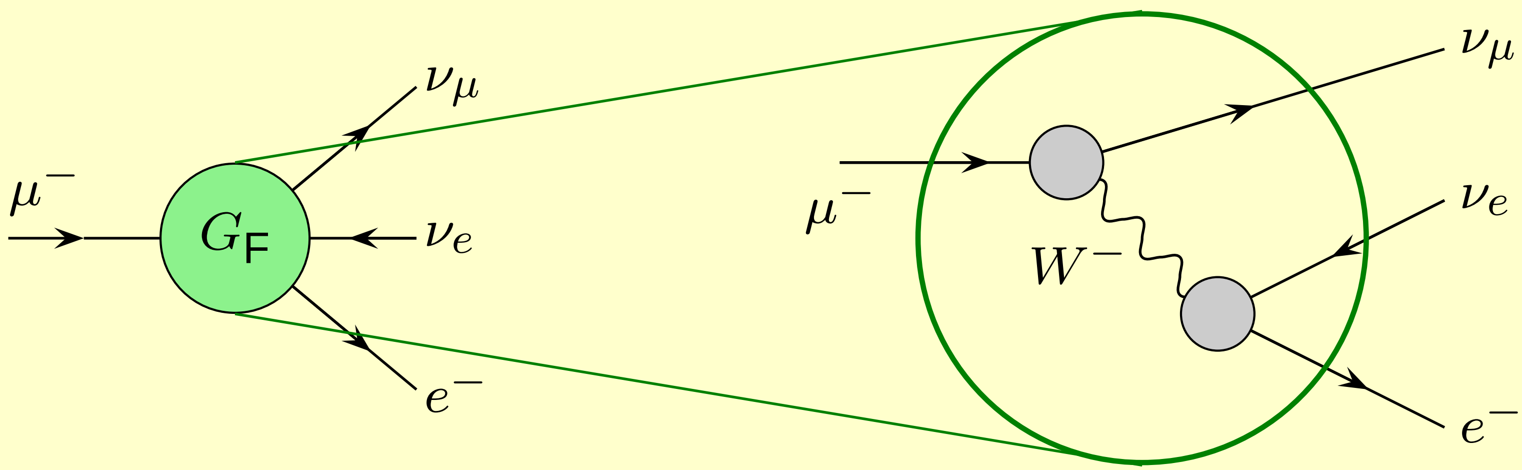
- "Other" electroweak parameters needed for global fit
- Theory error estimates
- Controlling uncertainties in EW precision tests
- Role of beam polarization

W mass prediction

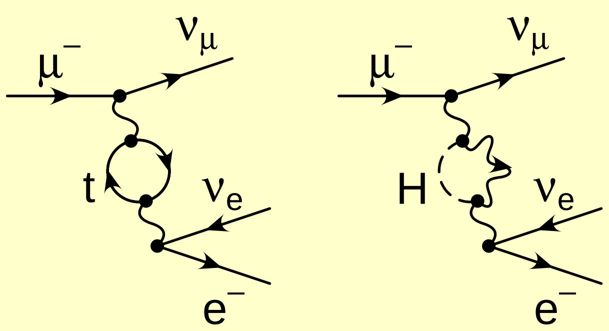
W-boson mass can be calculated from muon decay rate:



μ decay in Standard Model

$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections



⟶ Dependence on M_Z, m_t, $\Delta\alpha$, ...

⟶ Comparison to direct measurement of M_W

Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\mathrm{CM}} \sim M_{\mathrm{Z}}$:

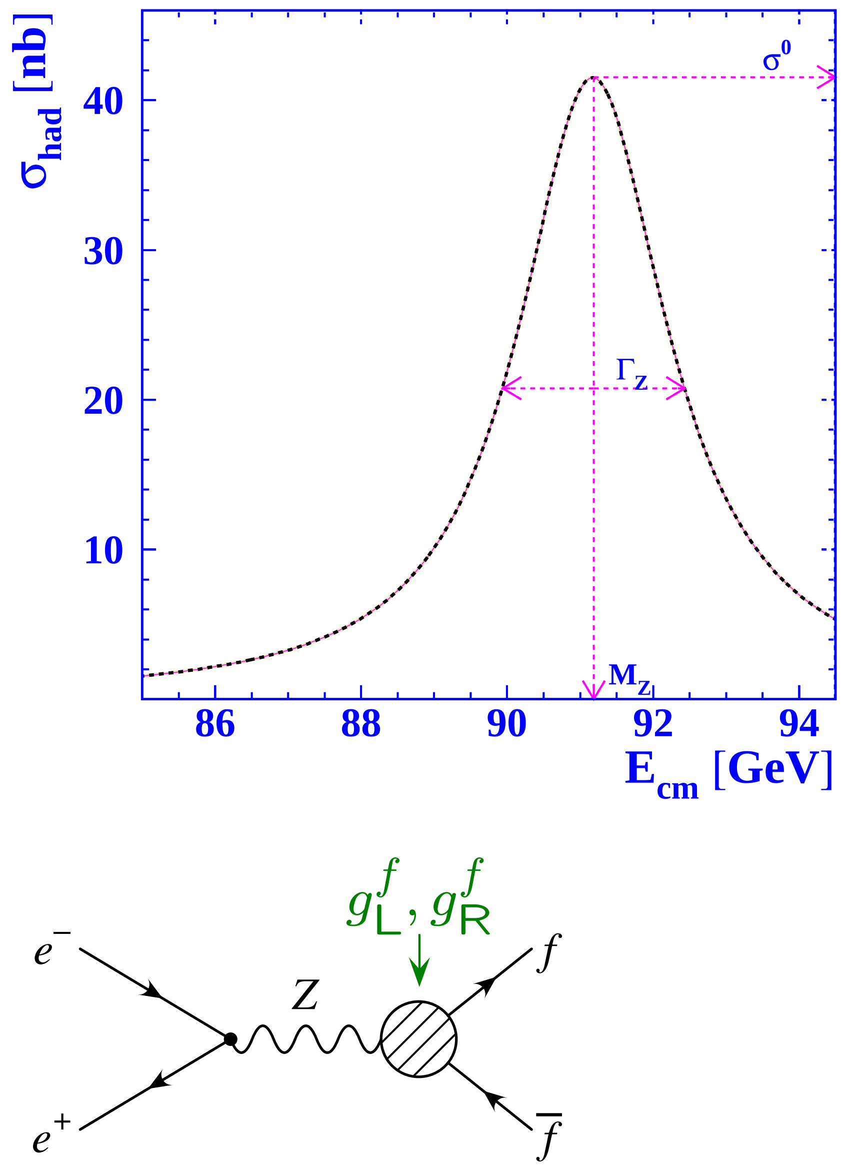
- Mass M_{Z}
- Width $\Gamma_{\mathrm{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\mathrm{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\mathrm{Z}}^2)^2+M_{\mathrm{Z}}^2\Gamma_{\mathrm{Z}}^2} = \frac{12\pi}{M_{\mathrm{Z}}^2}R_eR_f$

$$\Gamma_{ff} = C\left[(g_{\mathrm{L}}^f)^2 + (g_{\mathrm{R}}^f)^2\right]$$

$\longrightarrow$ Dependence on α_{s}, m_{t}, M_{Z}, ...

Main exp. systematic uncertainties:

- Beam energy calibration
- Particle ID and acceptance

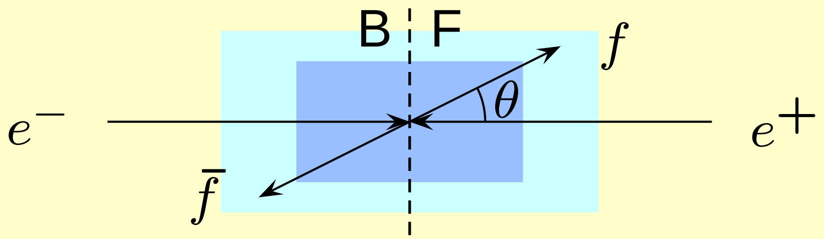


Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1-4\sin^2\theta^f_{\mathrm{eff}})}{1+(1-4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$



Main systematic uncertainties:

- For $f = b$: charge tagging, jet clustering
- For $f = \mu$: calibration of $\sqrt{s}$, muon angle

Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$, $\qquad \langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$

Z-pole asymmetries

Left-right asymmetry: (using polarization e^- beams)

$$A_{\mathrm{LR}} \equiv \frac{1}{P_{e^-}} \frac{\sigma_\mathrm{L} - \sigma_\mathrm{R}}{\sigma_\mathrm{L} + \sigma_\mathrm{R}} = \mathcal{A}_e + \Delta A_{\gamma Z} + \Delta A_\gamma$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$

Limited by systematic uncertainty of P_{e^-}

0.5% at SLD, 0.1% possible in future

Karl, List '17

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\text{LR}} = \sqrt{\frac{(\sigma_{++}+\sigma_{-+}-\sigma_{+-}-\sigma_{--})(-\sigma_{++}+\sigma_{-+}-\sigma_{+-}+\sigma_{--})}{(\sigma_{++}+\sigma_{-+}+\sigma_{+-}+\sigma_{--})(-\sigma_{++}+\sigma_{-+}+\sigma_{+-}-\sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

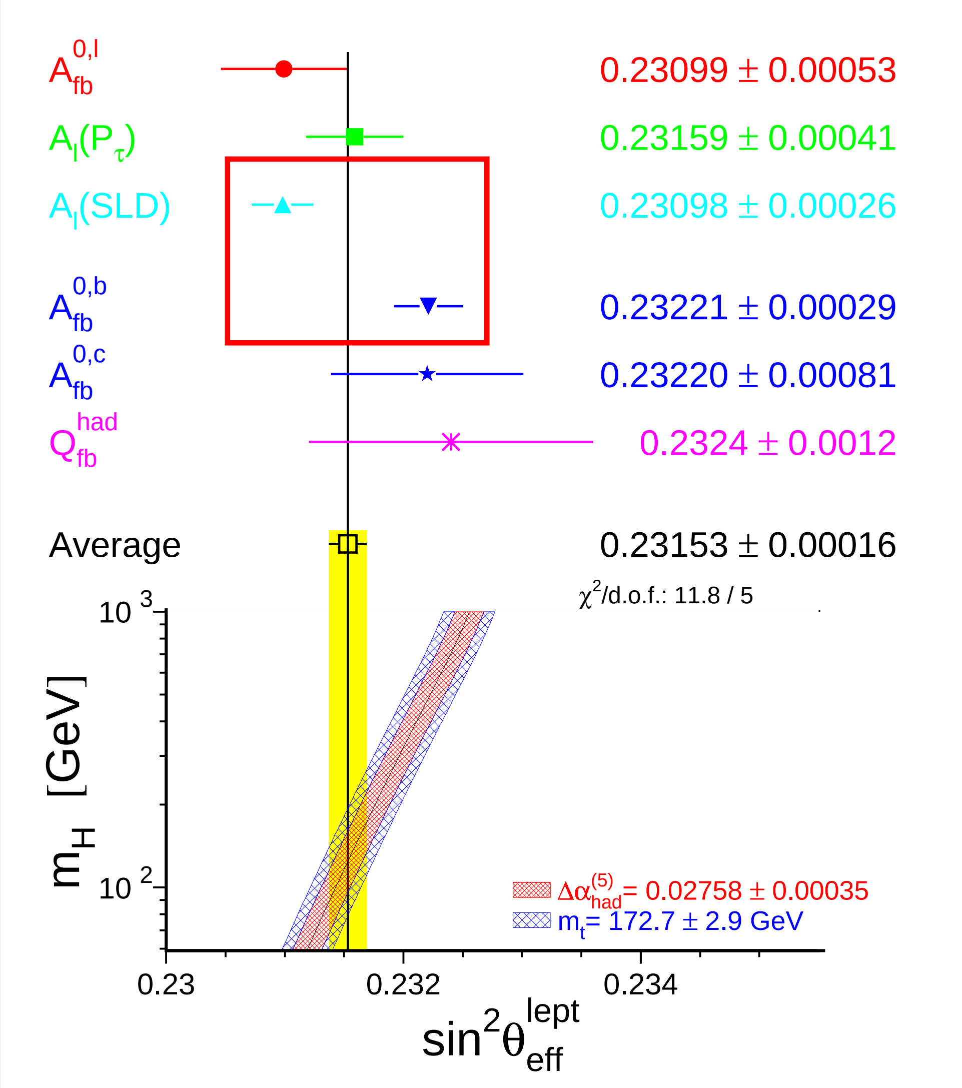
Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\text{LR}} \approx 10^{-4}$ $\Rightarrow$ $\delta \sin^2\theta^{\ell}_{\text{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

Present status of $\sin^2\theta^\ell_{\rm eff}$

- Most precise determination from $A_{\rm LR}$ at SLD and $A^b_{\rm FB}$ at LEP
- Disagreement by $\sim 4\sigma$ $\longrightarrow$ Underestimated systematics?
- Default at CEPC, FCC-ee: $\sin^2\theta^\ell_{\rm eff}$ from $A^{\mu\mu}_{\rm FB}$



LEP EWWG ’05

Future precision estimates

	Current exp.	ILC/GigaZ	CEPC	FCC-ee
$M_{\rm W}$ [MeV]	15	3–4	1	1
$M_{\rm Z}$ [MeV]	2.3	–	0.5	0.1
Γ_Z [MeV]	2.3	0.8	0.5	0.1
$R_\ell = \Gamma_{\rm Z}^{\rm had}/\Gamma_{\rm Z}^{\ell}$ [10^{-3}]	25	10	2	1
$R_b = \Gamma_{\rm Z}^{b}/\Gamma_{\rm Z}^{\rm had}$ [10^{-5}]	66	14	4.3	6
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	16	1.3	$<$1	0.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC/CEPC/FCC-ee!

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_W, Z-pole observables

  Freitas, Hollik, Walter, Weiglein ’00
  Awramik, Czakon ’02
  Onishchenko, Veretin ’02
  Awramik, Czakon, Freitas, Weiglein ’04
  Awramik, Czakon, Freitas ’06
  Hollik, Meier, Uccirati ’05,07
  Awramik, Czakon, Freitas, Kniehl ’08
  Freitas ’13,14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch ’16,18

- Approximate 3- and 4-loop results (to ρ parameter)

  Chetyrkin, Kühn, Steinhauser ’95
  Faisst, Kühn, Seidensticker, Veretin ’03
  Boughezal, Tausk, v. d. Bij ’05
  Schröder, Steinhauser ’05
  Chetyrkin et al. ’06
  Boughezal, Czakon ’06

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_{W}	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$, $\alpha\alpha_{\mathrm{s}}^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_{\mathrm{s}}$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta^\ell_{\mathrm{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Theory and parametric uncertainties

	CEPC	perturb. error with 3-loop†	Param. error CEPC**	main source
M_W [MeV]	1	1	2.1	m_t, $\Delta\alpha$
Γ_Z [MeV]	0.5	0.15	0.15	m_t, α_s
R_b [10^{-5}]	4.3	5	< 1	
$\sin^2\theta^\ell_\mathrm{eff}$ [10^{-5}]	$<$1	1.5	2	m_t, $\Delta\alpha$

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\mathrm{s})$

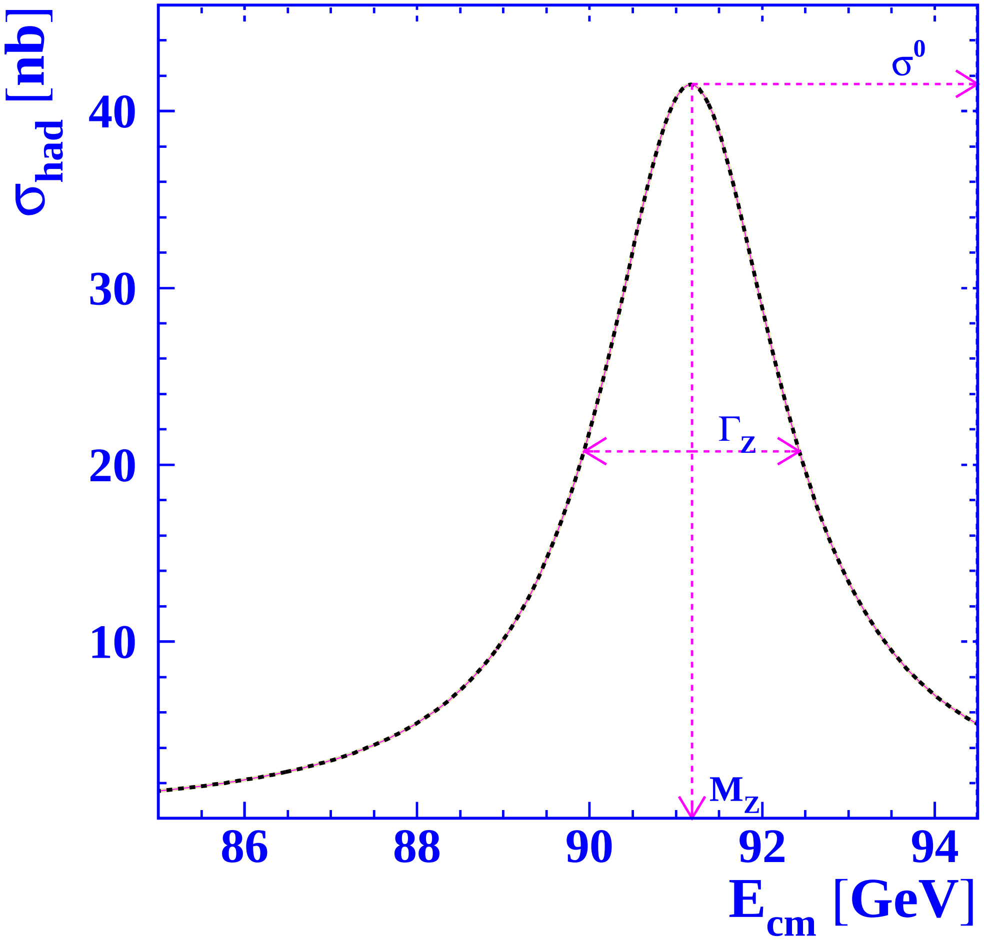
(N_f^n = at least n closed fermion loops)

Parametric inputs:

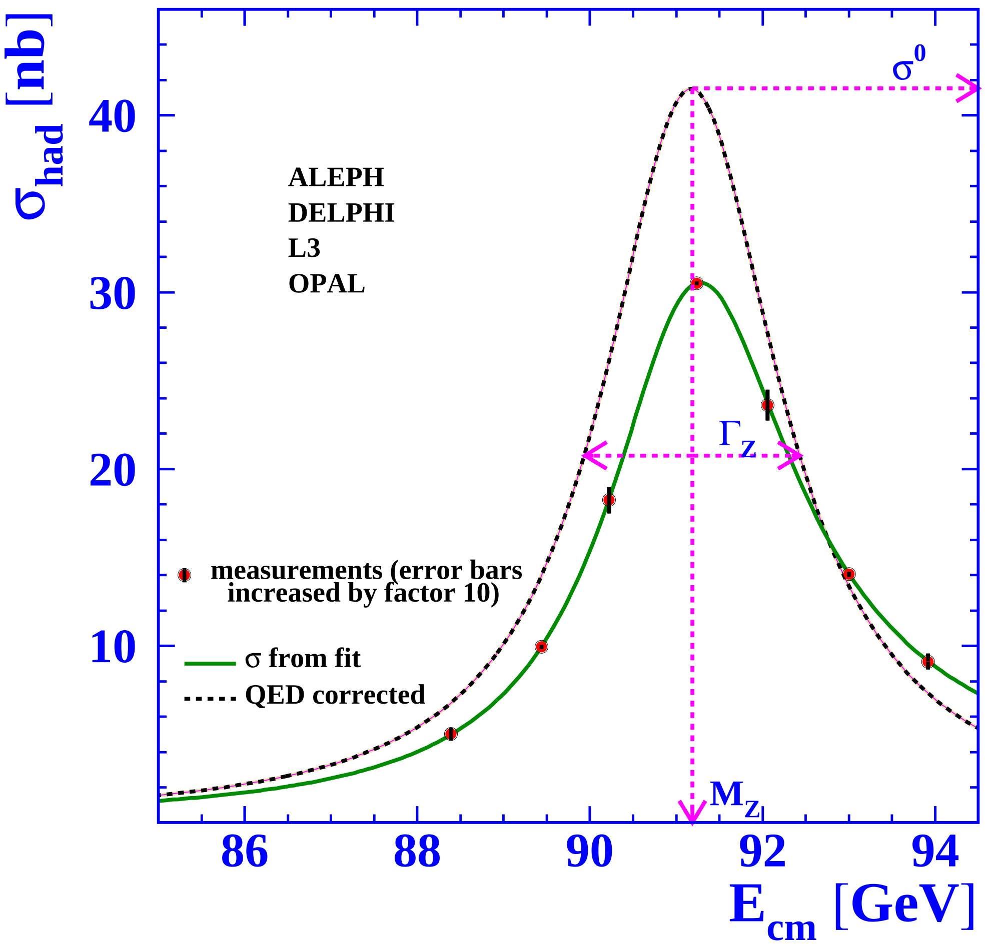
****CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_\mathrm{s} = 0.0002$, $\delta M_Z = 0.5$ MeV

also: $\delta(\Delta\alpha) = 5\times 10^{-5}$

Real vs. pseudo-observables



Real vs. pseudo-observables



LEP EWWG '05

→ Large effects from initial-state QED radiation

→ Theory input necessary to extract relevant EWPOs ("pseudo-observables")

→ talk by F. Piccinini

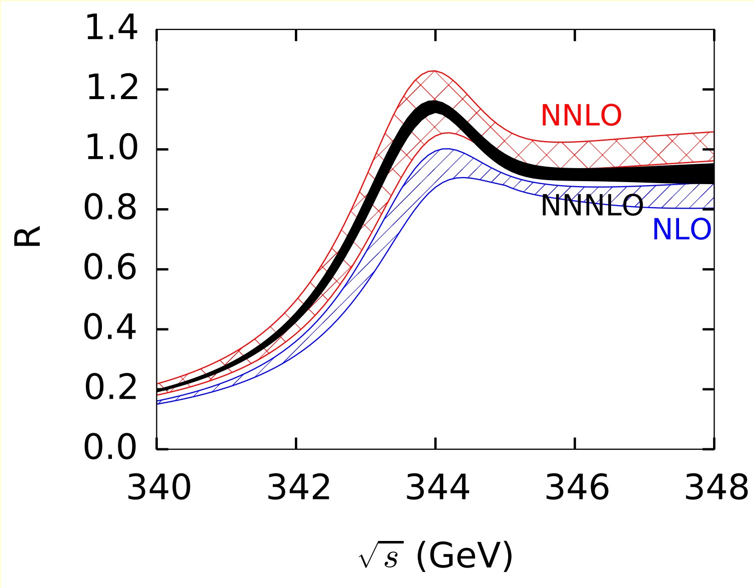
Other electroweak precision parameters

- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape
  $\longrightarrow$ Main uncertainties: B-field calibration, QED
  $\longrightarrow$ δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV could be achievable

- m_t: Current status $\delta m_t \sim 0.4$ GeV at LHC — PDG '18
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16; Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV — $\longrightarrow$ talk by Blondel

$$\begin{aligned} \delta m_t = & \; [10...50 \text{ MeV}]_{\text{stat}} \\ & \oplus [< 40 \text{ MeV}]_{\text{exp. sys.}} \\ & \oplus [\ll 50 \text{ MeV}]_{\text{QCD}} \\ & \oplus [10 \text{ MeV}]_{\text{mass def.}} \\ & \oplus [10...20 \text{ MeV}]_{\alpha_s} \\ \sim & \; 50...100 \text{ MeV} \end{aligned}$$



Beneke et al. '15

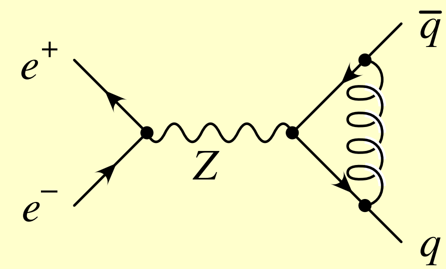
Other electroweak precision parameters

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD from Υ spectroscopy:
    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10
    $\longrightarrow$ Difficulty in evaluating systematics
  - e^+e^- event shapes and DIS: $\alpha_s \sim 0.114$
    Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13
    $\longrightarrow$ Subject to sizeable non-pertubative power corrections
    $\longrightarrow$ Systematic uncertainties in power corrections?
  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18
    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation
    Pich '14; Boito et al. '15,18

Other electroweak precision parameters

- α_{s}: d'Enterria, Skands, et al. '15
  - Electroweak precision ($R_\ell = \Gamma_Z^{\mathrm{had}}/\Gamma_Z^\ell$):

    $\alpha_{\mathrm{s}} = 0.120 \pm 0.003$ PDG '18

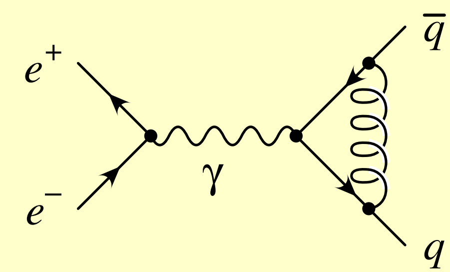
    $\longrightarrow$ No (negligible) non-perturbative QCD effects

    FCC: $\delta R_\ell \sim 0.001$

    $\Rightarrow \quad \delta\alpha_{\mathrm{s}} < 0.0002$ (subj. to theory error)

    Caviat: R_ℓ could be affected by new physics

  - $R = \frac{\sigma[ee \to \mathrm{had.}]}{\sigma[ee \to \mu\mu]}$ at lower $\sqrt{s}$

    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_{\mathrm{s}} = 0.110 \pm 0.015$

    Kühn, Steinhauser, Teubner '07

    $\longrightarrow$ dominated by s-channel photon, less room for new physics

    $\longrightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_{\mathrm{s}} \sim 0.0001$

Impact of new physics

Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3})$ $(\Lambda \gg M_Z)$

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi\, \Phi^\dagger (D^\mu \Phi) \qquad \alpha \Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

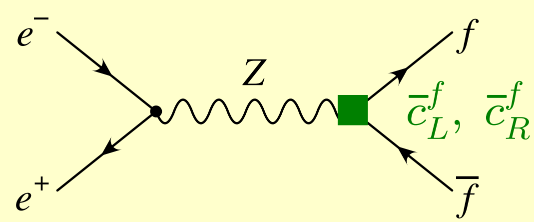
$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi \qquad \alpha \Delta S = -e^2 v^2 \frac{c_{BW}}{\Lambda^2}$$

$$O_{LL}^{(3)e} = (\bar{L}_L^e \sigma^a \gamma_\mu L_L^e)(\bar{L}_L^e \sigma^a \gamma^\mu L_L^e) \qquad \Delta G_F = -\sqrt{2}\frac{c_{LL}^{(3)e}}{\Lambda^2}$$

$$O_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_R \gamma^\mu f_R) \qquad f = e, \mu\, \tau, b, lq$$

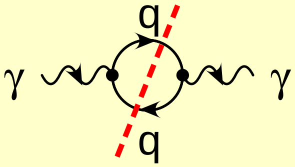
$$O_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_L \gamma^\mu F_L) \qquad F = \begin{pmatrix}\nu_e \\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}, \begin{pmatrix}u, c \\ d, s\end{pmatrix}, \begin{pmatrix}t \\ b\end{pmatrix}$$

$$O_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_L \sigma_a \gamma^\mu F_L)$$

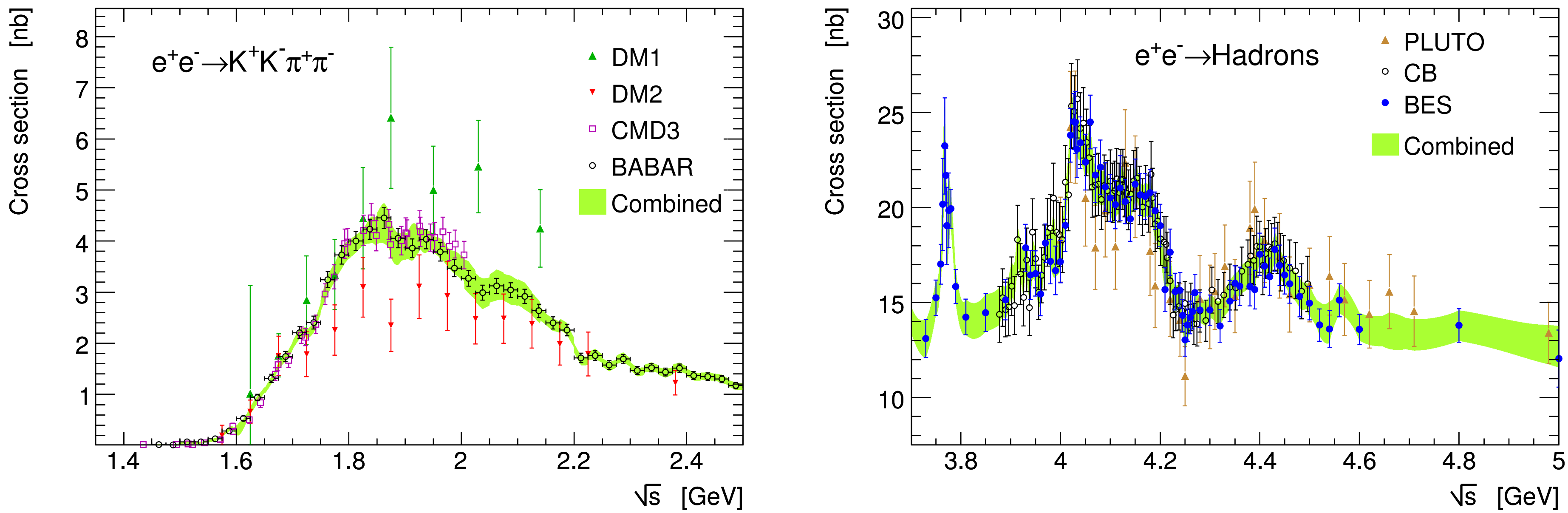


Shift of finestructure constant

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$
- Hadronic effects from $e^+e^- \to$ had. data
- Last 5 years: new data from BaBar, VEPP, BES
- No significant improvement from including τ data
- Robust precision $\sim 10^{-4}$



Davier et al. '17; Jegerlehner '17
Keshavarzi, Nomura, Teubner '18



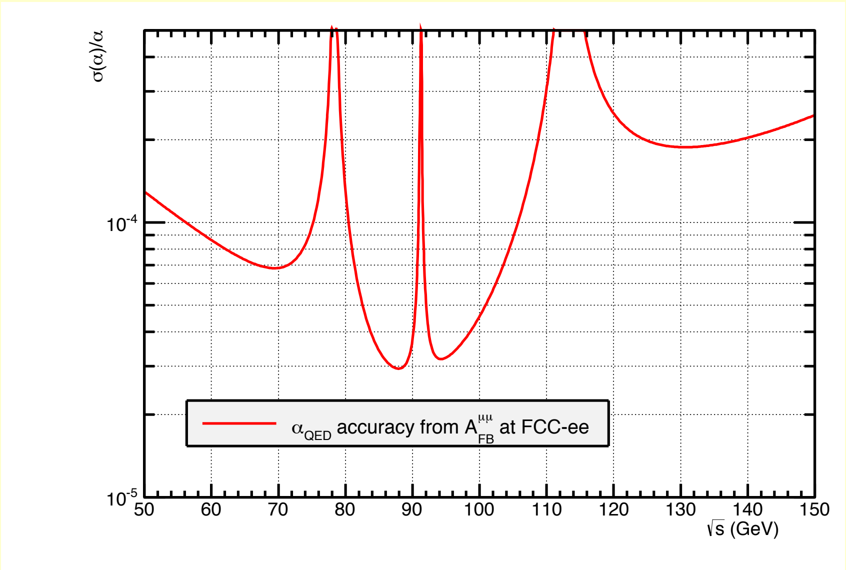
Davier et al. '17

Shift of finestructure constant

- $\Delta\alpha_{\mathrm{had}}$: Could be limiting factor
  a) From $e^+e^- \to$ had. using dispersion relation
  Current: $\delta(\Delta\alpha_{\mathrm{had}}) \sim 10^{-4}$
  Improvement to $\delta(\Delta\alpha_{\mathrm{had}}) \sim 5 \times 10^{-5}$ likely
  b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
  (i.e. $A_{\mathrm{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)
  $\to \delta_{\mathrm{th}}(\Delta\alpha_{\mathrm{had}}) \sim 3 \times 10^{-5}$

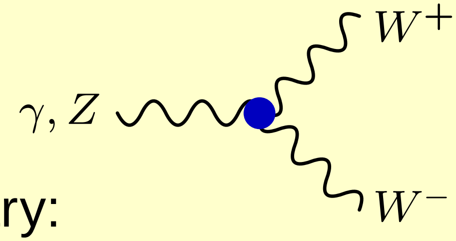
  Janot '15

  Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including 2/3-loop corrections for γ-exchange and box contributions



WW threshold and new physics

γWW and ZWW couplings:



General coupling structure consistent with Lorentz symmetry:

$$\mathcal{L} = -ig_V \Big[g_1^V (W^+_{\mu\nu} W^{-\mu} - W^-_{\mu\nu} W^{+\mu}) V^\nu + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W^{+\nu}_\mu W^{-\rho}_\nu V_\rho{}^\mu$$
$$+ ig_4^V W^+_\mu W^-_\nu V^{\mu\nu} + ig_5^V \epsilon^{\mu\nu\rho\sigma} W^+_\mu \overleftrightarrow{\partial}_\rho W^-_\nu V_\sigma$$
$$+ \tilde{\kappa}_V W^+_\mu W^-_\nu \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}_V}{M_W^2} W^{+\nu}_\mu W^{-\rho}_\nu \tilde{V}_\rho{}^\mu \Big] \qquad (V = \gamma, Z)$$

Hagiwara, Peccei, Zeppenfeld, Hikasa ’87

EM gauge invariance: $g_1^\gamma = 1$

SU(2) invariance imposes more constraints

WW threshold and new physics

Extension of SM by **higher-dimensional operators**:

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_i c_i \, \mathcal{O}_i^{(d)}$$

Operators must satisfy SM gauge invariance

Leading CP-even gauge boson operators ($d = 6$):

$$\mathcal{O}_{\mathsf{WWW}} = \mathrm{Tr}[W_{\mu\nu} W^{\nu\rho} W_\rho{}^\mu]$$

$$\mathcal{O}_{\mathsf{W}} = (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{\mathsf{B}} = (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

Hagiwara, Ishihara, Szalapski, Zeppenfeld '93

$\longrightarrow$ Only 3 indep. parameters

Relations between anom. couplings (aGC) and eff. theory (EFT):

$$g_1^{\mathsf{Z}} = 1 + c_{\mathsf{W}} \frac{M_{\mathsf{Z}}^2}{2\Lambda^2} \qquad \kappa_\gamma = 1 + (c_{\mathsf{W}} + c_{\mathsf{B}}) \frac{M_{\mathsf{W}}^2}{2\Lambda^2}$$

$$\lambda_\gamma = \lambda_{\mathsf{Z}} = c_{\mathsf{WWW}} \frac{3g^2 M_{\mathsf{W}}^2}{2\Lambda^2} \qquad \kappa_{\mathsf{Z}} = 1 + \left(c_{\mathsf{W}} - c_{\mathsf{B}} \frac{s^2}{c^2} \right) \frac{M_{\mathsf{W}}^2}{2\Lambda^2}$$

WW threshold and polarization

- CP-even dim-6 operators can be constrained from final-state distributions
- No significant improvement from beam polarization

$\mathcal{P}_{-80,-20} : \mathcal{P}_{-80,+20} : \mathcal{P}_{+80,-20} : \mathcal{P}_{+80,+20}$	Δg_1^Z	$\Delta\kappa_\gamma$	$\Delta\lambda_\gamma$	$\Delta\mathcal{P}_{e^-}$	$\Delta\mathcal{P}_{e^+}$
1:1:1:1	1.88	1.73	2.66	0.0009	0.0014
1:4:4:1	1.92	1.68	2.79	0.0010	0.0017
0:1:1:0	1.89	1.69	2.85	0.0015	0.0023
1:4:4:1 and $\lvert\mathcal{P}_e\rvert \neq -\lvert\mathcal{P}_e\rvert$	1.92	1.68	2.79	0.0016	0.0018

Rosca ’16

WW threshold and polarization

- Strongly coupled models / mixing of Higgs with heavy states:
  Other aGC possible

- Independent constraint of all parameters requires
  $e^{\pm}$ **longitudinal and transverse** polarization

Table 6. Same as Table 5, but for symmetry (b) and with the L–R-parameterisation. We write again $\tilde{h}_{\pm} = \mathrm{Im}(g_1^{\mathrm{R}} \pm \kappa_{\mathrm{R}})/\sqrt{2}$. Using this parameterisation, a maximum number of couplings can be measured without transverse beam polarisation. In the γ–Z-parameterisation, the four couplings $\mathrm{Im}\, g_1^{\gamma}$, $\mathrm{Im}\, g_1^{Z}$, $\mathrm{Im}\, \kappa_{\gamma}$ and $\mathrm{Im}\, \kappa_Z$ are not measurable without transverse polarisation

	$\mathrm{Im}\, g_1^{\mathrm{L}}$	$\mathrm{Im}\, \kappa_{\mathrm{L}}$	$\mathrm{Im}\, \lambda_{\mathrm{L}}$	$\mathrm{Im}\, g_5^{\mathrm{L}}$	$\tilde{h}_-$	$\tilde{h}_+$	$\mathrm{Im}\, \lambda_{\mathrm{R}}$	$\mathrm{Im}\, g_5^{\mathrm{R}}$
No polarisation	2.7	1.7	0.48	2.5	11	–	3.1	17
$(P_l^-, P_l^+) = (\mp 80\%, 0)$	2.6	1.2	0.45	2.0	4.5	–	1.4	4.3
$(P_l^-, P_l^+) = (\mp 80\%, \pm 60\%)$	2.1	0.95	0.37	1.6	2.5	–	0.75	2.3
$(P_t^-, P_t^+) = (80\%, 60\%)$	2.6	1.2	0.46	2.0	3.7	3.2	0.98	4.4

$\sqrt{s} = 500$ GeV

Diehl, Nachtmann, Nagel '03

Summary

- **Electroweak precision tests** at future e^+e^- colliders probe physics at beyond the Standard Model at **multi-TeV scale**
- Uncertainties from experimental **systematics**, **perturbative** and **non-perturbative** theory require much work, but can also be mitigated through choice of measurements and analysis
- **Direct determination** of α_s, m_t, $\Delta\alpha$ at e^+e^- colliders is important
- Other lower-energy experiments can provide additional input:
  BELLE II, BES, **...**
- **Beam polarization** can access more information, but non essential for SM program

Backup slides

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors:

$$\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

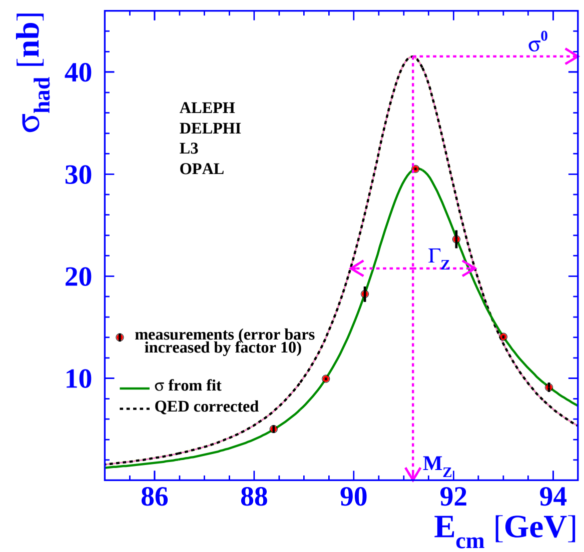
Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Skrzypek '92
Montagna, Nicrosini, Piccinini '97

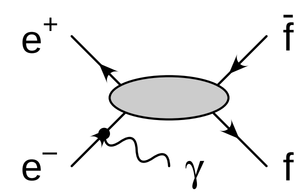
Soft photons (resummed) + collinear photons

$$\mathcal{R}_{\text{ini}} = \frac{\zeta(1 - s'/s)^{\zeta-1}}{\Gamma(1-\zeta)} e^{-\gamma_E \zeta + 3\alpha L/2\pi}$$
$$- \frac{\alpha}{\pi} L \left(1 + \frac{s'}{s}\right) + \alpha^2 L^2 \ldots + \alpha^3 L^3 \ldots$$

$$\zeta = \frac{2\alpha}{\pi}(L - 1)$$
$$L = \log \frac{s}{m_e^2}$$

LEP EWWG '05





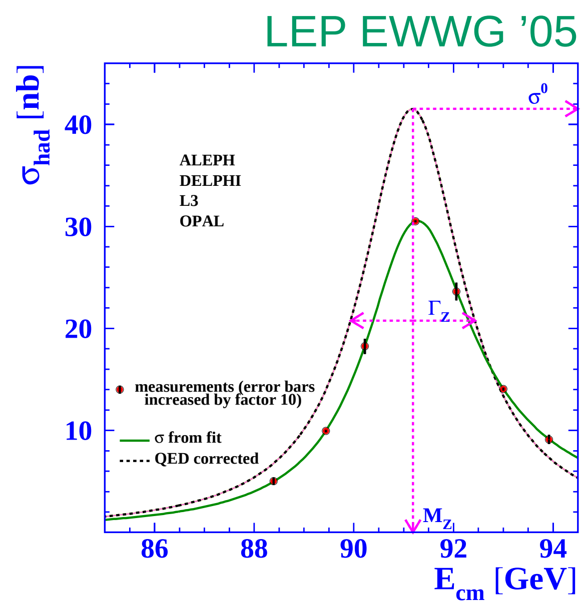
Z lineshape

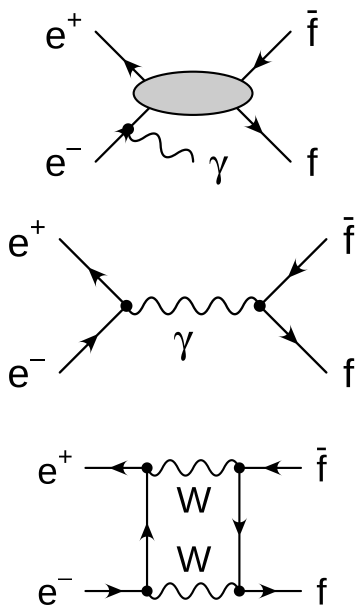
- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_\gamma + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$





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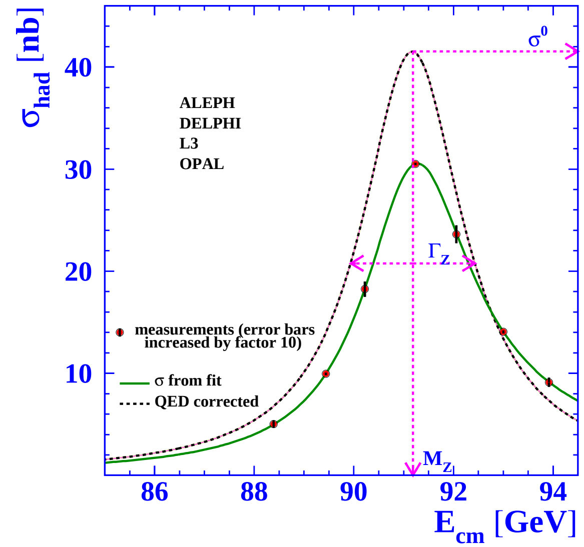
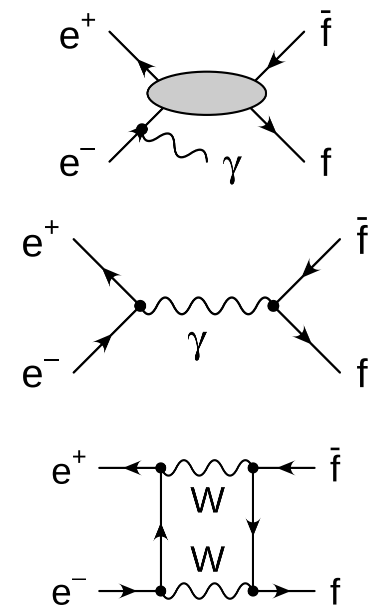
$$\sigma_{\text{Z}} = \frac{R}{(s - \overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2\overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

LEP EWWG '05





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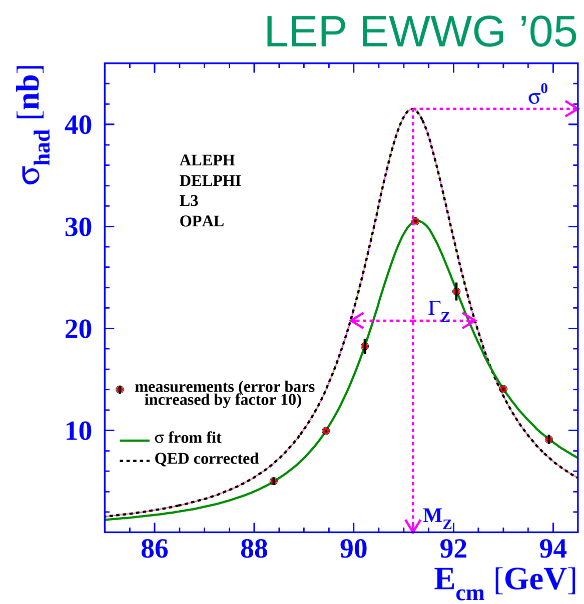
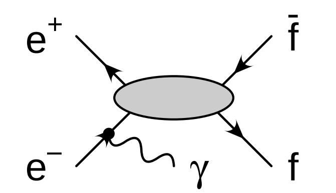
- In experimental analyses:

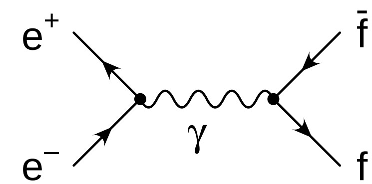
$$\sigma \sim \frac{1}{(s-M_{\mathrm{Z}}^2)^2 + s^2\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2}$$

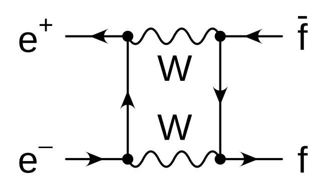
$$\overline{M}_{\mathrm{Z}} = M_{\mathrm{Z}}\Big/\sqrt{1+\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2} \approx M_{\mathrm{Z}} - 34\ \mathrm{MeV}$$

$$\overline{\Gamma}_{\mathrm{Z}} = \Gamma_{\mathrm{Z}}\Big/\sqrt{1+\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2} \approx \Gamma_{\mathrm{Z}} - 0.9\ \mathrm{MeV}$$







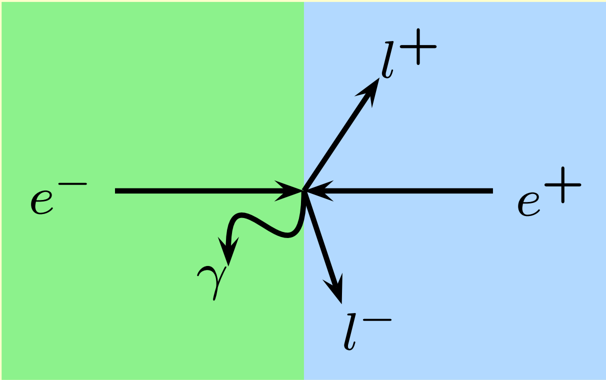


QED radiation in Z asymmetries

QED radiation in principle cancels in asymmetries, e.g. $A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$

Some effects from detector acceptance and cuts

Typical indfluence $< 10^{-3}$



Implementation of QED effects:

a) Analytical formulae, e.g. ZFITTER

Arbuzov, Bardin, Christova, Kalinovskaya, Riemann, Riemann, ...

b) Monte Carlo event generator, e.g. KORALZ

Jadach, Ward, ...

Other electroweak precision parameters

- α_s:

  Most precise determination using Lattice QCD from Υ spectroscopy:

  $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

  But e^+e^- event shapes and DIS prefer $\alpha_s \sim 0.114$

  Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

- Impact on EWPOs:

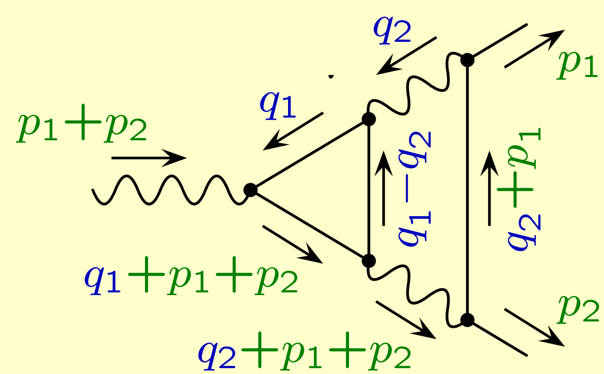
$$\delta\alpha_s = 0.005 \quad \Rightarrow \quad \delta M_W \approx 3.5 \text{ MeV}$$

$$\delta \sin^2\theta^\ell_{\text{eff}} \approx 2 \times 10^{-5}$$

Currently not dominant, but similar magnitude as intrinsic theory error

Theory calculations

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, p_2, \ldots, m_1, m_2, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods must be automizable, stable, fastly converging
- Need procedure for isolating divergent pieces

- Subtraction of QED radiation contributions

  $\longrightarrow$ Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**, $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A$_{\text{FB}}$**

  $(L = \log \frac{s}{m_e^2})$

  Berends, Burgers, v.Neerven '88
  Kniehl, Krawczyk, Kühn, Stuart '88
  Beenakker, Berends, v.Neerven '89
  Skrzypek '92; Montagna, Nicrosini, Piccinini '97

  $\longrightarrow$ $\mathcal{O}(0.1\%)$ uncertainty on σ_Z, A_{FB}

  $\longrightarrow$ Improvement needed for ILC/FCC-ee

- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

  see, e.g., Bardin, Grünewald, Passarino '99

  $\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

  $\longrightarrow$ Sensitivity to some NP beyond EWPO