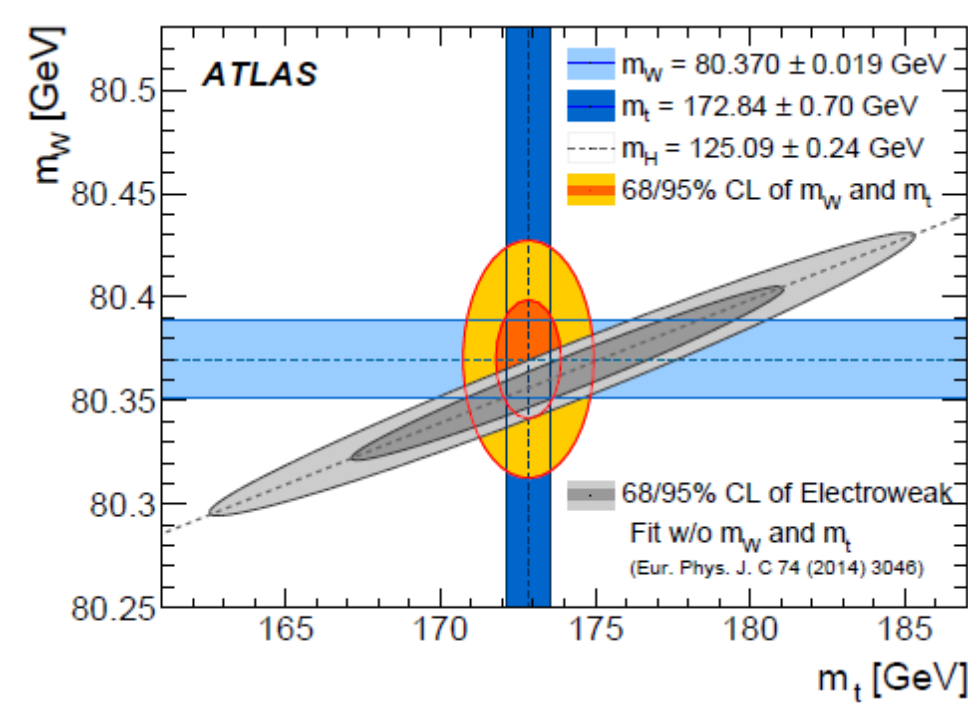
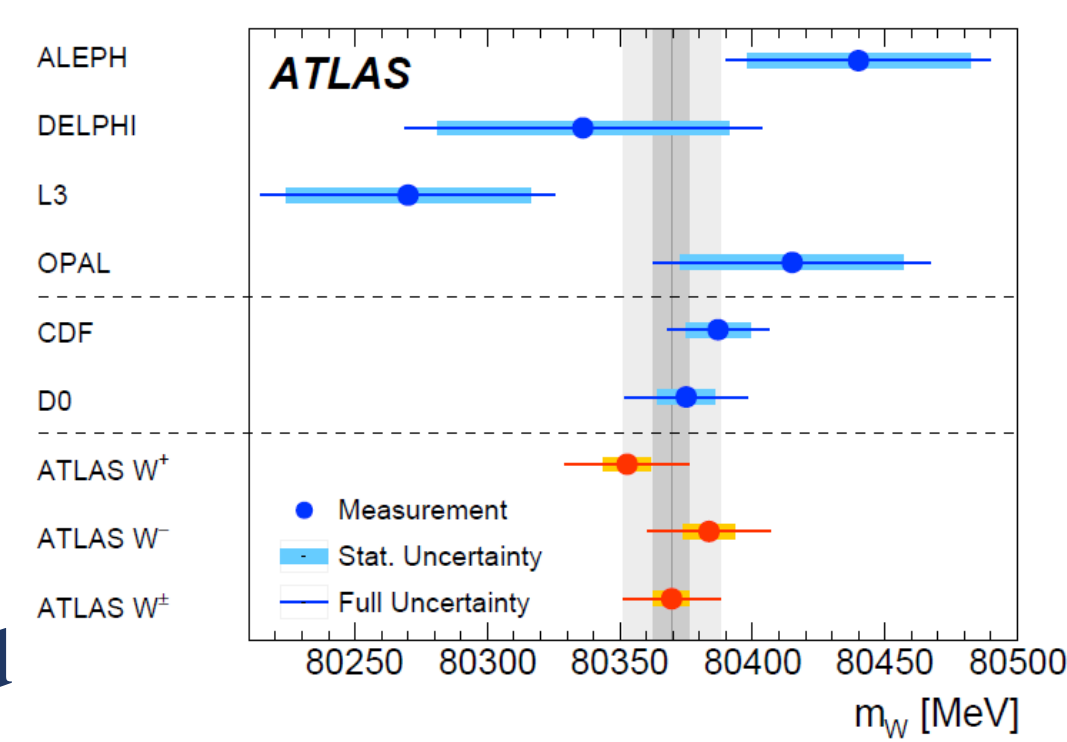


Introduction

- The W boson mass plays a central role in precision EW measurements and in constraint on the SM model through global fit.
- The direct measurement suffers the large uncertainties, such as QED and EW corrections, modeling of hadronization and so on.
- Threshold scan method is sensitive to the number of events, which leads a high precision measurement if there is large data sample around W -pair threshold.



Threshold scan method

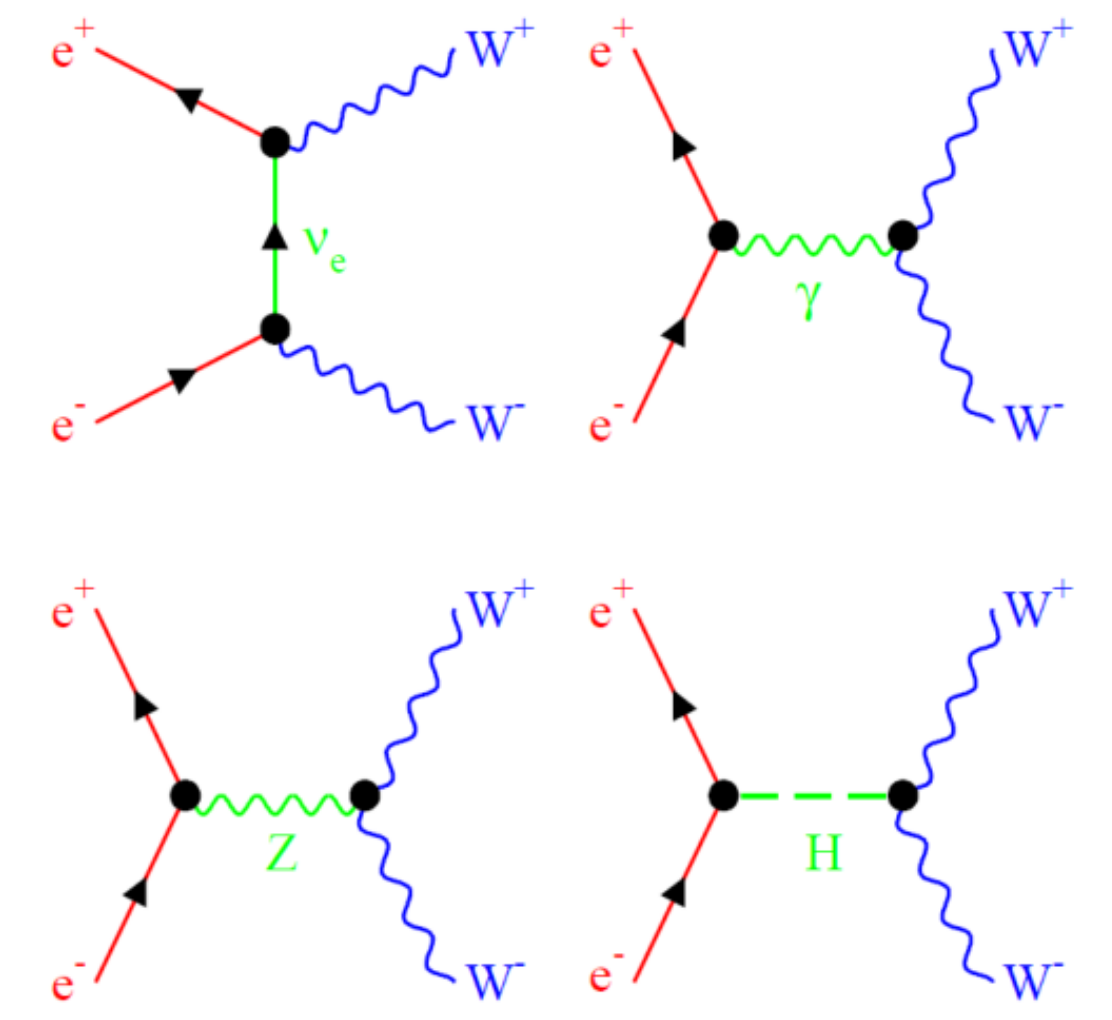
The matrix element for on-shell W -pair production at Born level:

$$M = \sqrt{2}e^2[M^\gamma + M^Z + M^\nu(\Theta)]\Delta\sigma(-1)d_{\Delta\sigma,\Delta\lambda}^J(\Theta)$$

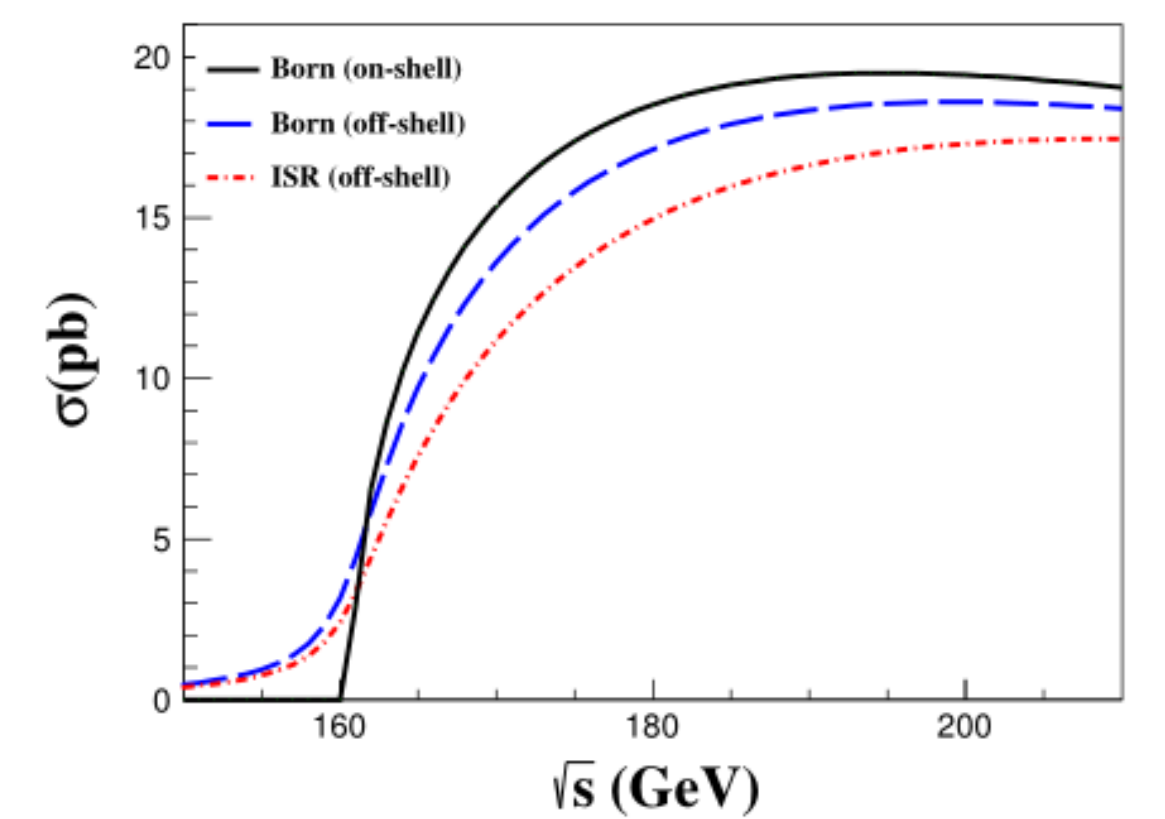
$$M^\nu = \frac{1}{2\sin^2\theta_W\beta}\delta_{|\Delta\sigma|,1}[B_{\lambda\lambda}^\nu - \frac{1}{1+\beta^2-2\beta\cos\Theta}C_{\lambda\lambda}^\nu]$$

$$M^Z = \beta[\delta_{|\Delta\sigma|,1} - \frac{1}{2\sin^2\theta_W}\delta_{|\Delta\sigma|,1}]\frac{s}{s-M_Z^2}A_{\lambda\lambda}^Z$$

$$M^\gamma = -\beta\delta_{|\Delta\sigma|,1}A_{\lambda,\lambda}^\gamma$$



- $\sigma_{WW}(m_W, \Gamma_W, \sqrt{s}) = \frac{N_{obs} - N_B}{L\epsilon}$
- $m_W(\Gamma_W)$ can be obtained by fitting the N_{obs} , with theoretical calculation of σ_{WW}
- The precision of $m_W(\Gamma_W)$ is dependent on the L, ϵ, \sqrt{s} , which are associated with the data-taking scheme.



Statistical uncertainty

$$\Delta\sigma_{WW} = \sigma_{WW} \times \frac{\Delta N_{WW}}{N_{WW}} = \sigma_{WW} \times \frac{\sqrt{N_{WW} + N_{bkg}}}{N_{WW}}$$

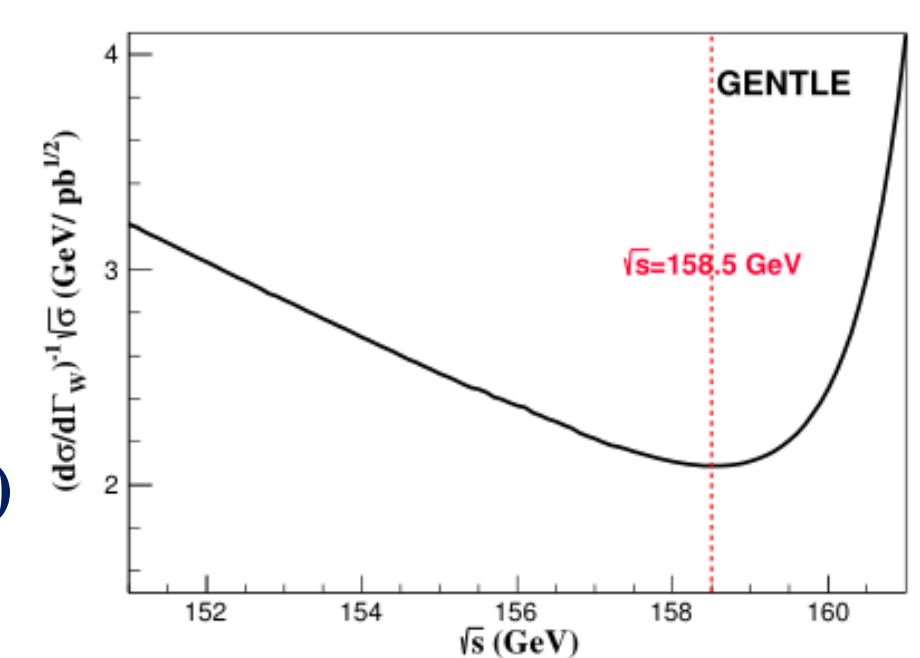
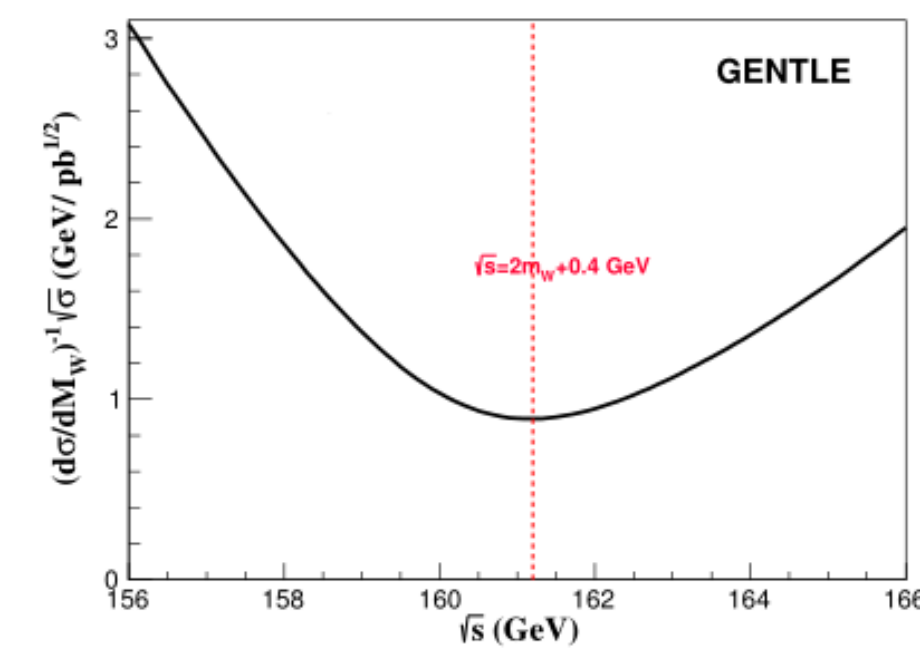
$$= \sqrt{\frac{\sigma_{WW}}{L\epsilon P}} \quad (P = \frac{N_{WW}}{N_{WW} + N_{bkg}})$$

$$\Delta m_W = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

$$\Delta\Gamma_W = \left(\frac{\partial\sigma_{WW}}{\partial\Gamma_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial\sigma_{WW}}{\partial\Gamma_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

With $L=3.2ab^{-1}$, $\epsilon=0.8$, $P=0.9$:

$\Delta m_W=0.6$ MeV, $\Delta\Gamma_W=1.4$ MeV (individually)

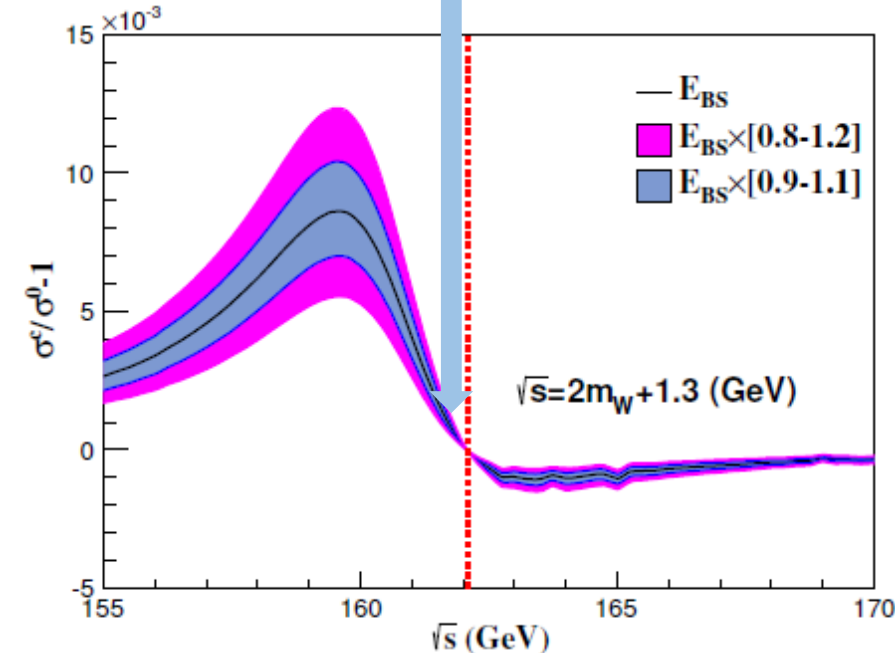
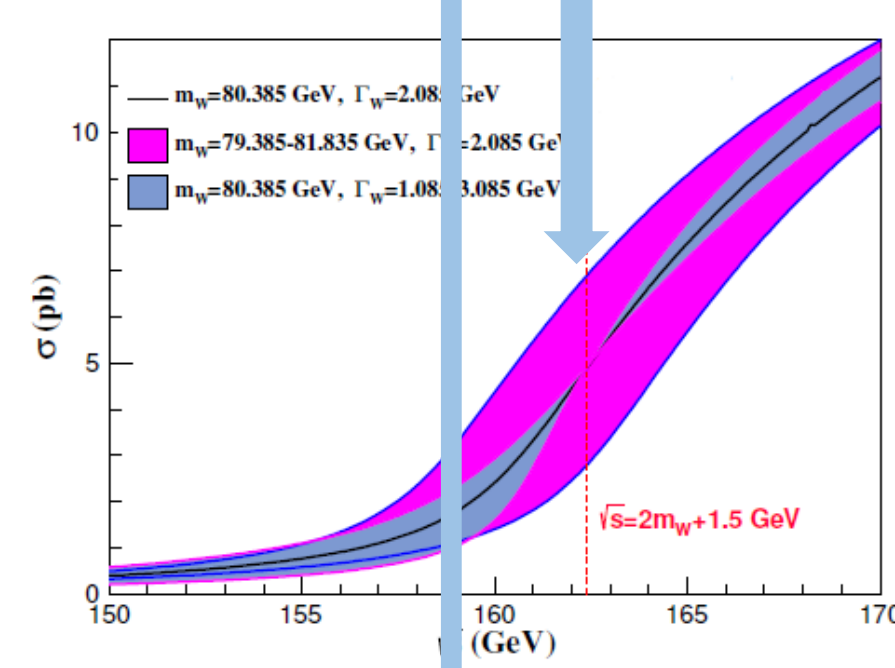
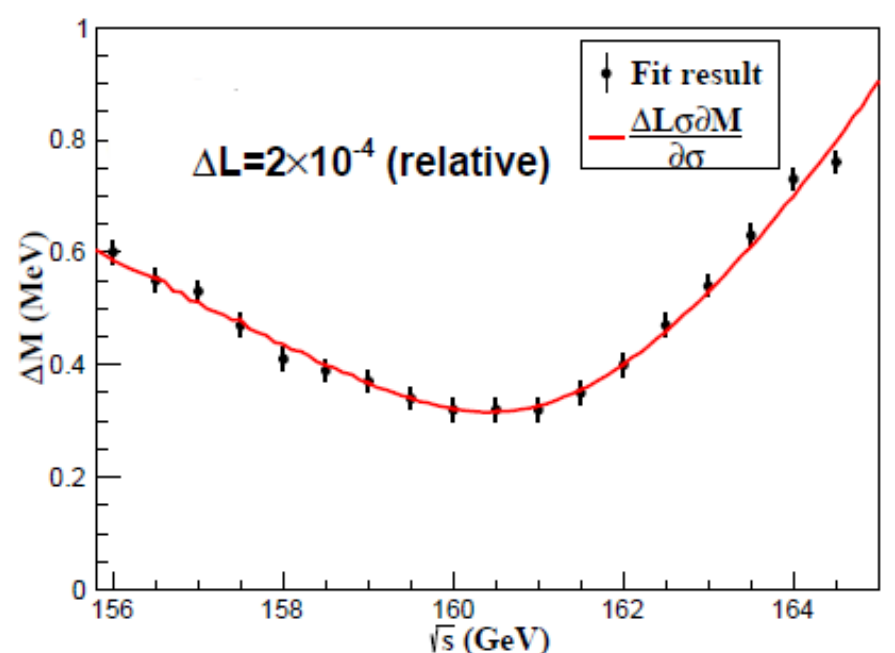
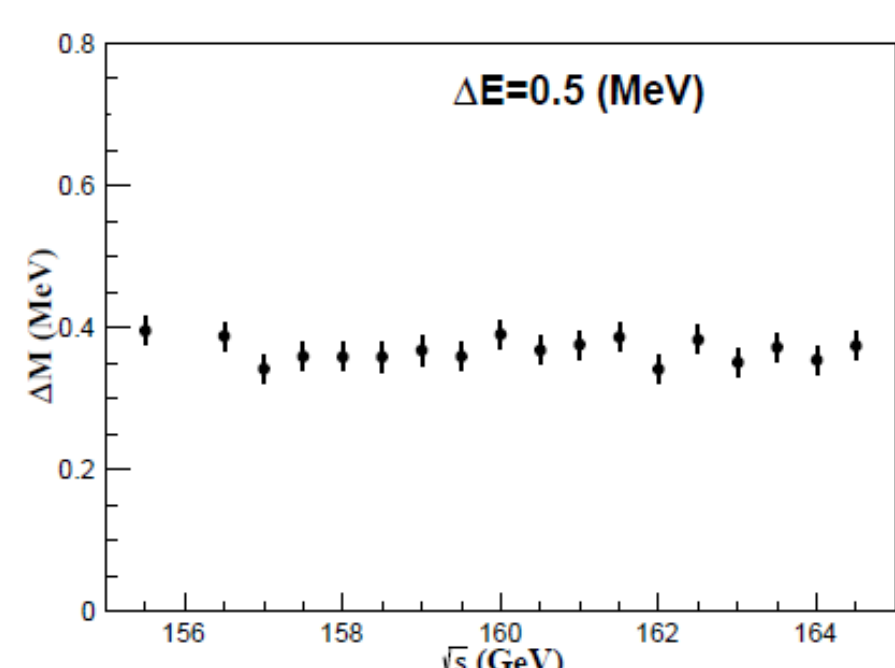


Systematic uncertainty

Systematic uncertainties:

- Uncorrelated: $\Delta E, \Delta E_{BS}$
- Correlated: $\Delta L, \Delta\epsilon, \Delta\sigma_{WW}, \Delta\sigma_B$

The m_W almost insensitive to $\Delta\Gamma_W$ and ΔE_{BS} around 161.2 GeV



Optimized data-taking scheme

The observe events is fitted with minimum chisq method. With the consideration of the correlated systematic uncertainties, the χ^2 is constructed as:

$$\chi^2 = \sum_i^n \frac{(y_i - h \cdot x_i)^2}{\delta_i^2} + \frac{(h - 1)^2}{\delta_c^2}$$

When the correlations are not include, the h returns to 1.

Define $T \equiv m_W + A \cdot \Gamma_W$ to optimize data-taking scheme ($A = 0.1$, importance factor)

Data taking scheme	One point	Two points	Three points
• Smallest $\Delta m_W, \Delta\Gamma_W$ (stat.)			
• Large sys. Uncertainties			
• Only for m_W or Γ_W , without correlation			
• Measure m_W and Γ_W simultaneously			
• Without the correlation			
• Measure m_W and Γ_W simultaneously, with the correlation			
• Maybe increase the $\Delta m_W, \Delta\Gamma_W$ (stat.)			

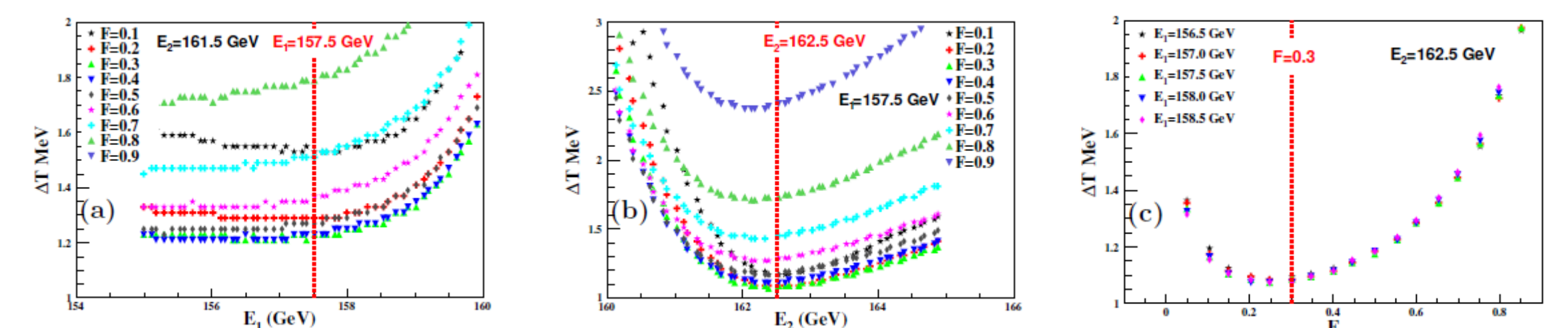
Configurations	This study	FCC work
m_W (GeV)	80.385 ± 0.015	
Γ_W (GeV)	2.085 ± 0.042	
L (ab^{-1})	3.2	15
E_{BS} (%)	0.1	-
ϵ	0.8	0.75
P	0.9	-
ΔE_{BS} (%)	0.1	-
ΔE (MeV)	0.5	0.24
ΔL (%)		
$\Delta\epsilon$ (%)	10^{-4}	10^{-4}
$\Delta\sigma_{WW}$ (%)		
$\Delta\sigma_B$ (%)	10^{-4}	10^{-3}

With $L = 3.2 ab^{-1}, \epsilon P = 0.72$

The optimized number of data-taking point is 3:

Data taking scheme	mass or width	δ_{stat} (stat.)	ΔE	ΔE_{BS}	δ_{sys}^{corr}	Total
One point	Δm_W (MeV)	0.68	0.37	-	0.44	0.90
Two points	Δm_W (MeV)	0.81	0.38	-	0.48	1.02
	$\Delta\Gamma_W$ (MeV)	2.72	0.54	0.50	0.22	2.83
Three points	Δm_W (MeV)	0.81	0.34	-	0.40	0.97
	$\Delta\Gamma_W$ (MeV)	2.73	0.58	0.36	0.20	2.82

E_1	157.5 GeV
E_2	162.5 GeV
F_1	0.3
E_3	161.5 GeV
F_2	0.9



Conclusions

The future Circular Electron Positron Colliders, such as the CEPC and FCC-ee, are proposed to make precise measurement of the Higgs boson, test the Standard Model, explore physic beyond the Standard Model, and son on. One of the important goal of these colliders is operating at a center-of-mass energy around the W -pair threshold to measure the W boson mass with high precision. In this paper, the optimization of the data taking scheme is performed. The study shows that in case of taking data at three energy points and with $L=3.2 ab^{-1}$, the precision of W mass ~ 1 MeV, as well as the precision of 2.8 MeV of its width, can be achieved.