

Theory status of the W -pair threshold scan

Christian Schwinn

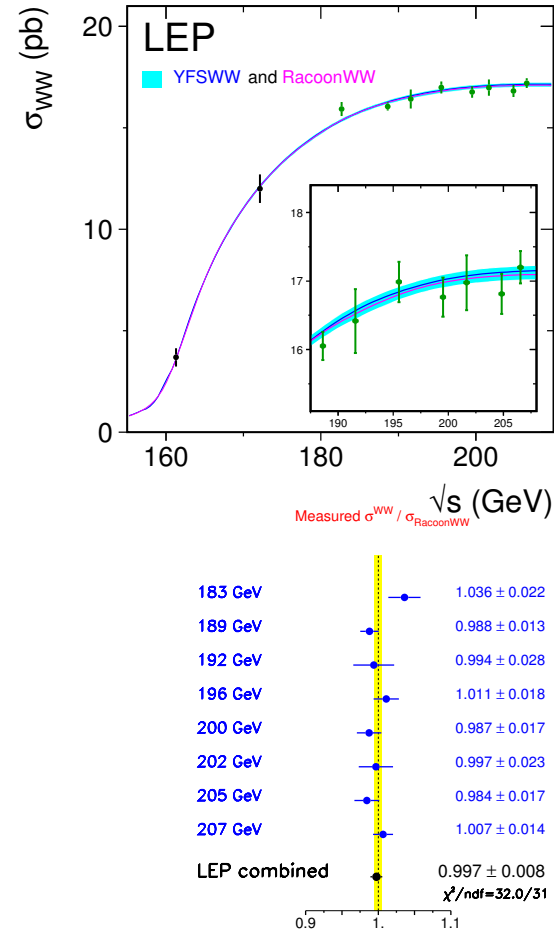
— RWTH Aachen —

13 November, 2018

W-pair production

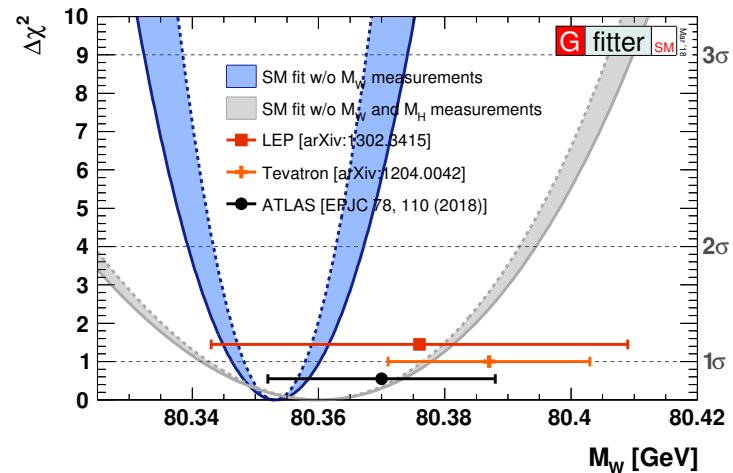
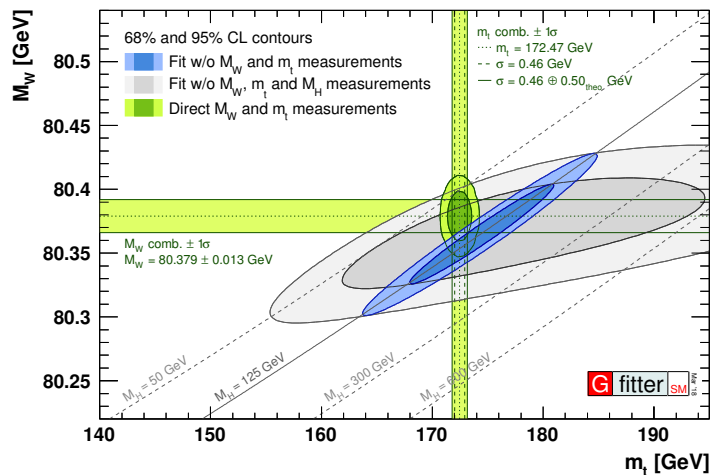
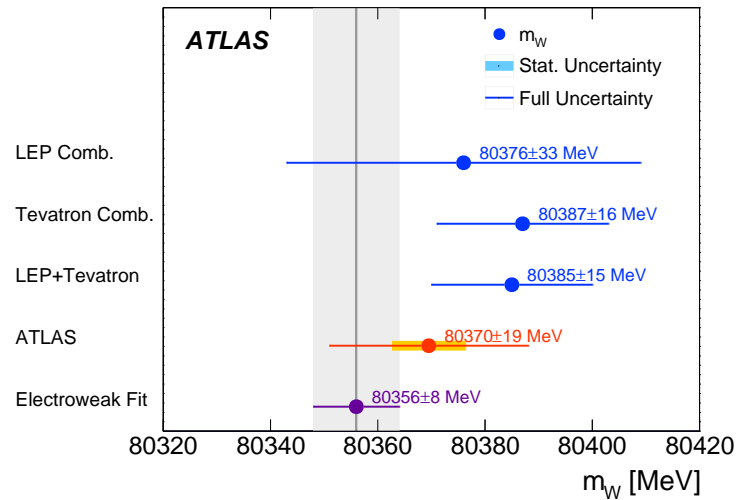
Success story at LEP2:

- σ_{WW} : 1%-level agreement with NLO theory
- ⇒ test of EW-sector of SM at quantum level
- measurement of branching ratios (lepton universality)
- bounds on anomalous triple vector-boson couplings
- ⇒ test of non-abelian structure
- W-mass measurement from kinematic reconstruction (+ σ_{WW} at threshold)



M_W key observable of SM

Current status LEP+Tevatron+LHC $\Delta M_W \sim 15$ MeV



M_W key observable of SM

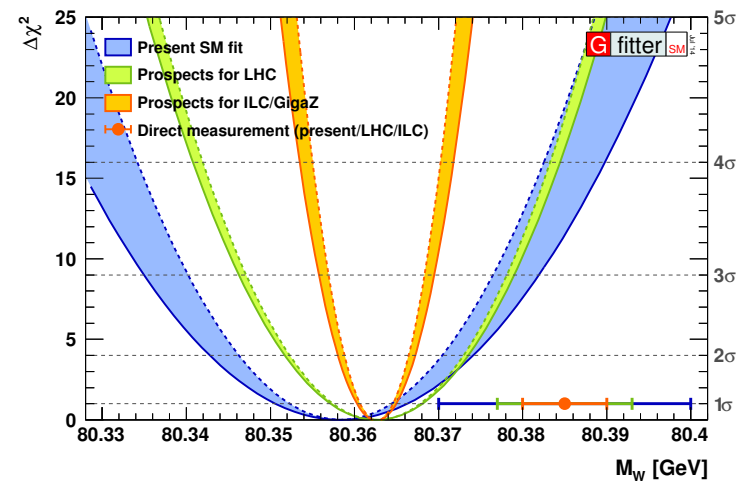
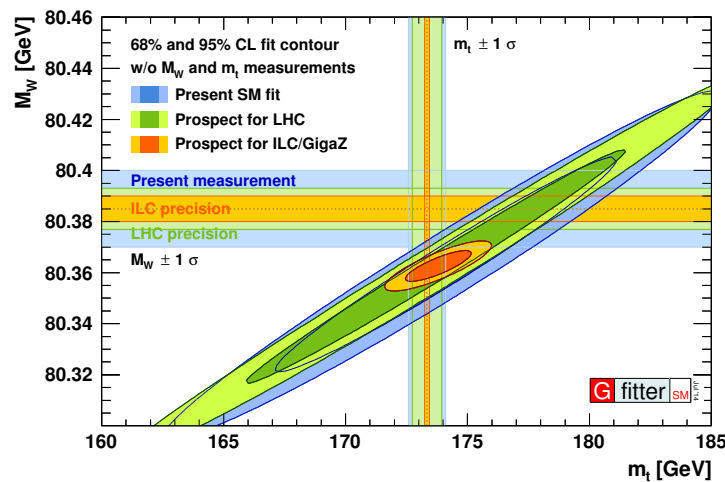
Current status LEP+Tevatron+LHC $\Delta M_W \sim 15$ MeV

Prospects at future e^-e^+ colliders

$$\Delta M_W \sim 3\text{-}4 \text{ MeV (ILC)}, 1 \text{ MeV (FCCee/CEPC)}$$

	LHC	LHC	ILC/GigaZ	ILC	ILC	ILC	TLEP	SM prediction
\sqrt{s} [TeV]	14	14	0.091	0.161	0.161	0.250	0.161	-
\mathcal{L} [fb $^{-1}$]	300	3000		100	480	500	3000 \times 4	-
ΔM_W [MeV]	8	5	-	4.1-4.5	2.3-2.9	3.6	1.2	4.2(3.0)
$\Delta \sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	36	21	1.3	-	-	-	0.3	3.0(2.6)

(Snowmass EW report 13)



FCCee study: (1703.01626 [hep-ph])

- single point $\sqrt{s} = 161.4 \text{ GeV}$
 $\Delta M_W \simeq 0.25 \text{ MeV}$

with 15 ab^{-1} if

$$\delta\sigma_{WW}^{\text{th.}} < 0.6 \text{ fb} (\approx 0.01\%)$$

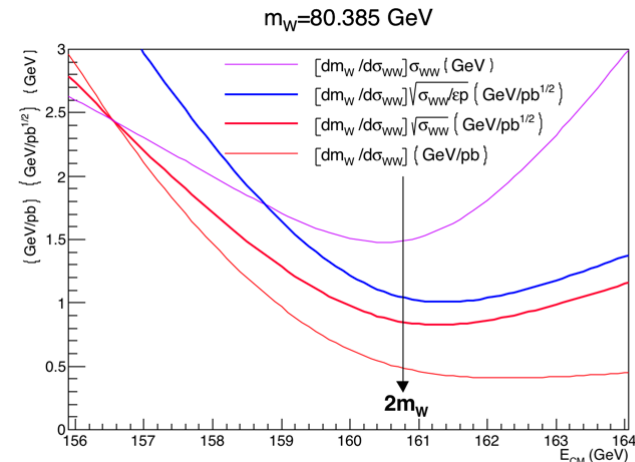
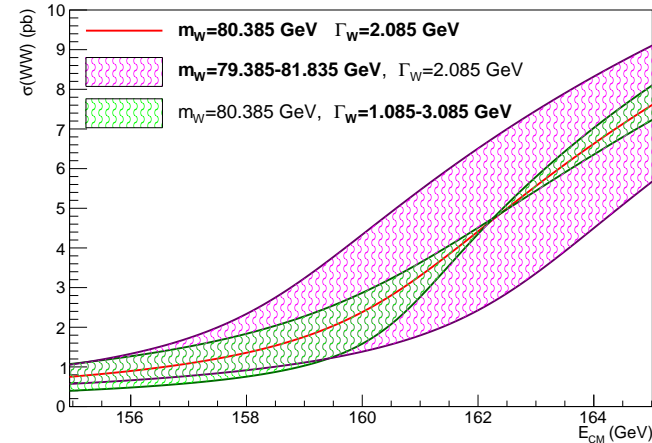
- simultaneous fit
 $\Delta M_W \simeq 0.41 \text{ MeV}, \Delta\Gamma_W \simeq 1.10 \text{ MeV}$

from two points

$$\sqrt{s} = 157.5, 162.3 \text{ GeV}$$

CEPC CDR scenario:

- 4 points
 $\sqrt{s} = 157.5, 161.5, 162.3, 172.0 \text{ GeV}$
- with 2.6 ab^{-1}
 $\Delta M_W \simeq 1 \text{ MeV}, \Delta\Gamma_W \simeq 2.8 \text{ MeV}$



W-pair production near threshold

- For $\Gamma_W \rightarrow 0$ expect threshold behaviour

$$\sigma_{WW} \propto \beta = \sqrt{1 - \frac{4M_W^2}{s}}$$

- Finite decay width ($E = \sqrt{s} - 2M_W$)

$$\beta \rightarrow \frac{1}{\sqrt{2}} \sqrt{\frac{E}{M_W} + \sqrt{\frac{E^2}{M_W^2} + \frac{\Gamma^2}{M_W^2}}}$$

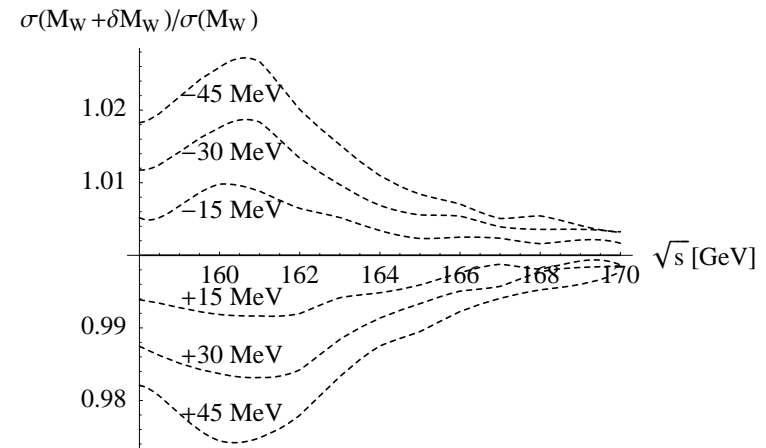
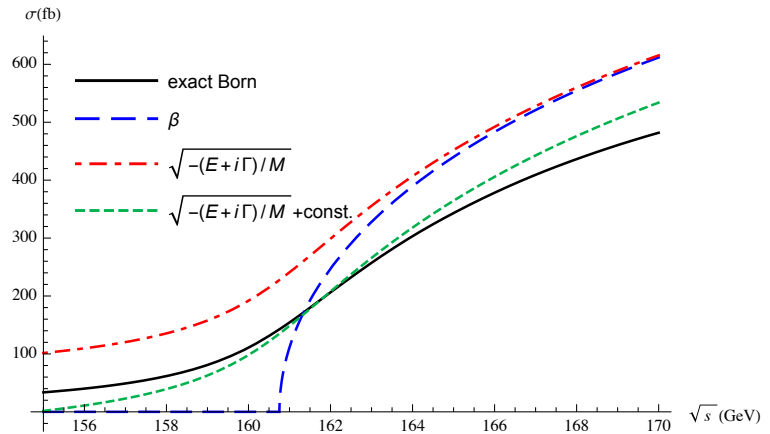
- constant shift

(phase-space constraints;
non-resonant contributions)

- Sensitivity to M_W

$$\Delta\sigma \sim 1\% \Leftrightarrow \Delta M_W \sim 15 \text{ MeV}$$

maximum near $\sqrt{s} = 161 \text{ GeV}$

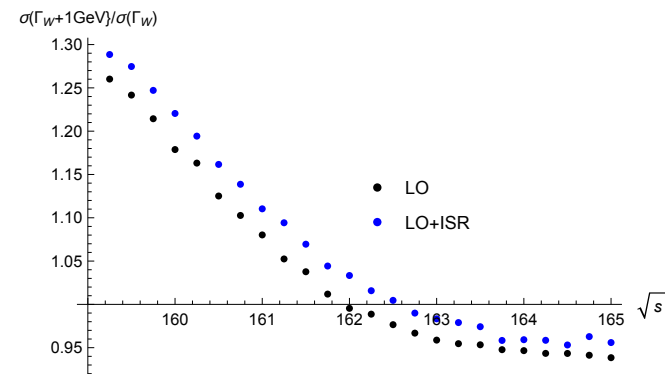
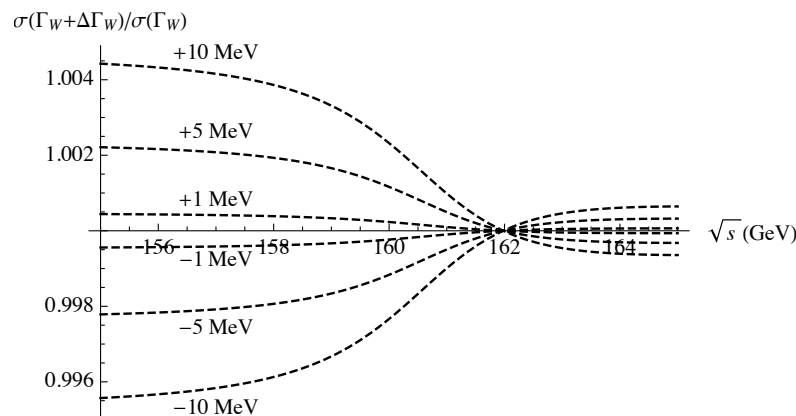


Sensitivity to W -width

- $\Gamma_W^{\text{exp.}} = 2.085 \pm 0.042 \text{ GeV}$; EW fit: $\Gamma_W = 2.091 \pm 0.001 \text{ GeV}$ (Gfitter, 18)

Consistent variation of width in theory calculations?

- implicitly $\sigma_{e^-e^+ \rightarrow 4f} \propto \Gamma_{W \rightarrow f_1 \bar{f}_2}^{\text{SM}} \Gamma_{W \rightarrow f_3 \bar{f}_4}^{\text{SM}} / \Gamma_W^2$
- ⇒ rescale by $(\Gamma_W / \Gamma_W^{\text{SM}})^2$ to keep $\text{BR}_{W \rightarrow f \bar{f}}$ constant (physics motivation?)
- sensitivity to Γ_W below threshold: $\Delta \Gamma_W = 10 \text{ MeV} \Leftrightarrow \Delta \sigma \sim 4\%$
- point with $\frac{d\sigma}{d\Gamma_W} = 0$ from interplay of $\beta \Leftrightarrow \text{const. terms in } \sigma$
- stability under radiative corrections? ISR effect: 500 MeV.



NLO corrections near threshold

$$s - 4M_W^2 \sim M_W \Gamma_W \Rightarrow \beta \sim \sqrt{\Gamma_W / M_W}$$

Schematic structure of total cross section:

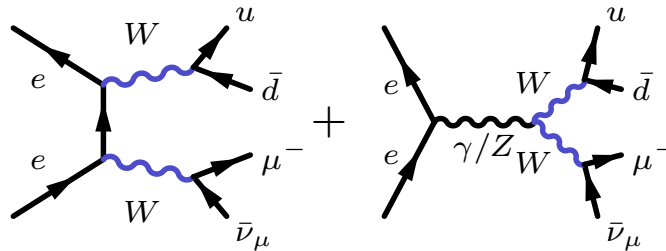
$$\Delta^{(1)} \sigma_{WW \rightarrow 4f} |_{s \approx 4M_W^2} \propto \beta \alpha \left[\frac{1}{\beta} + \ln \beta \ln \left(\frac{m_e}{M_W} \right) + \ln \left(\frac{m_e}{M_W} \right) + C^{(1)} \right] + \text{const} + \mathcal{O}(\beta)$$

Enhanced corrections in **threshold limit**

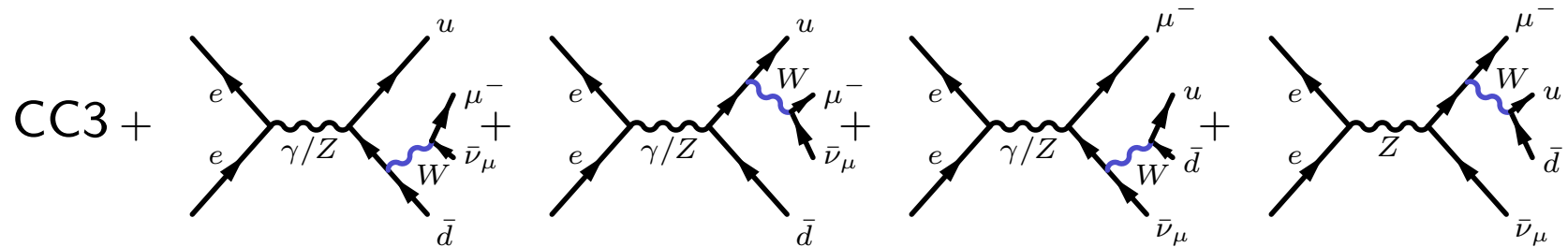
- **mass logarithms** $\ln \left(\frac{m_e}{M_W} \right)$: resum in ISR structure function
- **Coulomb corrections** $\sim \alpha/\beta \sim$ screened by finite W -width
 \Rightarrow Coulomb corrections $\sim \alpha^n (M_W / \Gamma_W)^{n/2} \sim \alpha^{n/2}$
enhanced but resummation not necessary
 (Method for all-order resummation known $\Rightarrow t\bar{t}$)
- **soft $\ln \beta$ corrections** $\sim \alpha \ln \alpha \sim 0.04$
 \Rightarrow resummation not necessary

4-fermion production at tree level, e.g. $e^-e^+ \rightarrow \mu^-\bar{\nu}_\mu u\bar{d}$

Double resonant ('signal') diagrams (CC3):



But 10 diagrams in total:



Only sum gauge invariant

Need consistent scheme for finite width effects: (Beenakker et al. 96)

\sqrt{s}	200 GeV	500 GeV	1 TeV	5 TeV
Running width	672.96	225.45	62.17	123.76
Constant width	673.08	224.05	56.90	2.212

NLO calculations in double pole approximation

- **Factorizable** EW corrections to production, decay of **on-shell W s**

- **Nonfactorizable** soft photon corrections

(Berends et al. 98; Denner et al. 99)

Vanish for σ_{tot} : Fadin/Khoze/Martin; Melnikov/Yakovlev 94)

- Implemented in Monte-Carlo programs used at LEP2:

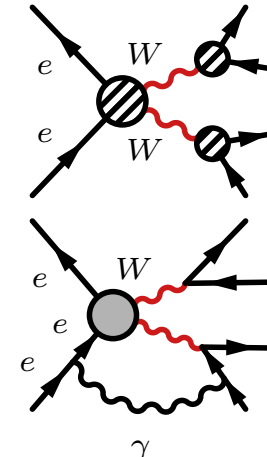
RacoonWW (Denner et al. 99), YFSWW (Jadach et al. 99)

- Estimate of DPA accuracy at NLO

$$\Delta\sigma_{\text{DPA}} \sim \frac{\Gamma_W}{M_W} \times \frac{\alpha}{\pi} \sim \mathcal{O}(0.1\%)$$

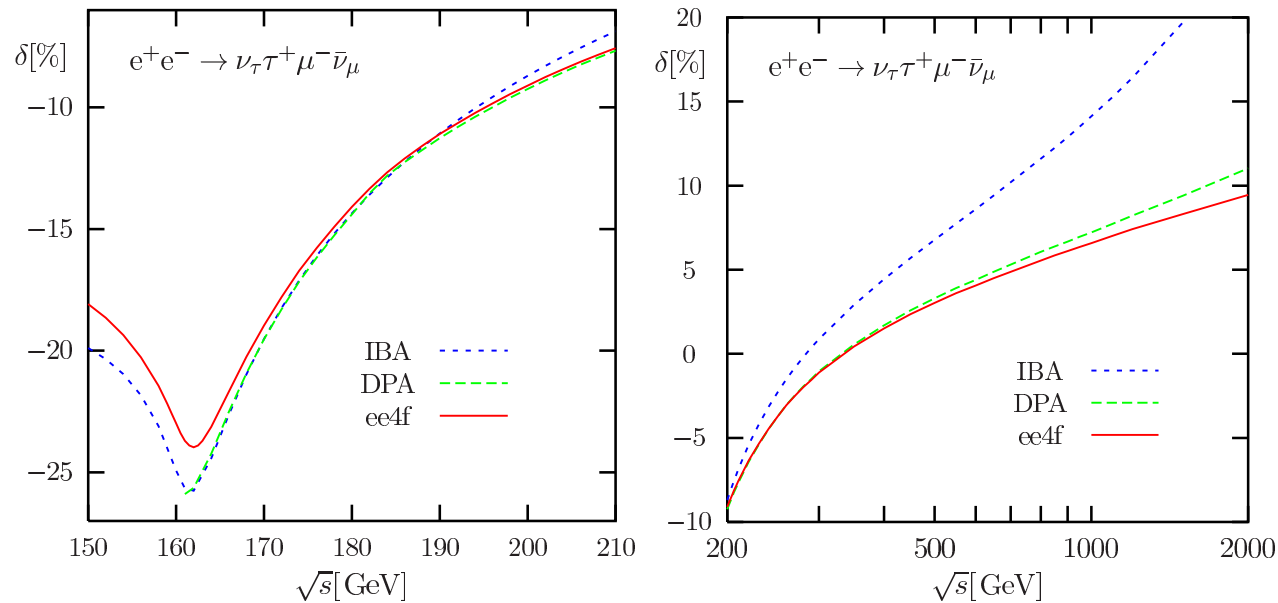
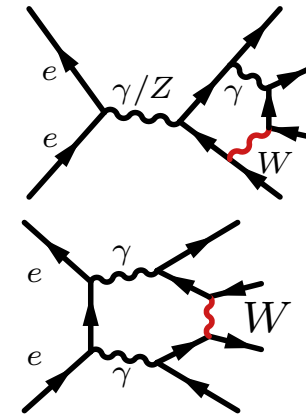
- Loss of accuracy at production threshold

$$\Delta\sigma_{\text{DPA}} \sim \frac{\Gamma_W}{\sqrt{s} - 2M_W} \times \frac{\alpha}{\pi} \sim \mathcal{O}(1\%)$$



Full NLO calculation for $e^+e^- \rightarrow 4f$ (Denner, Dittmaier, Roth, Wieders 05)

- More than 1000 1-loop diagrams, 5, 6-point loop integrals
- ⇒ pioneering methods for six-point diagrams
now automated for LHC: RECOLA, OpenLoops, MadLoops
- complex mass scheme for W decay width
- fully differential calculation
- not easy to incorporate higher-order effects
- DPA not sufficient at threshold and for $\sqrt{s} > 500$ GeV

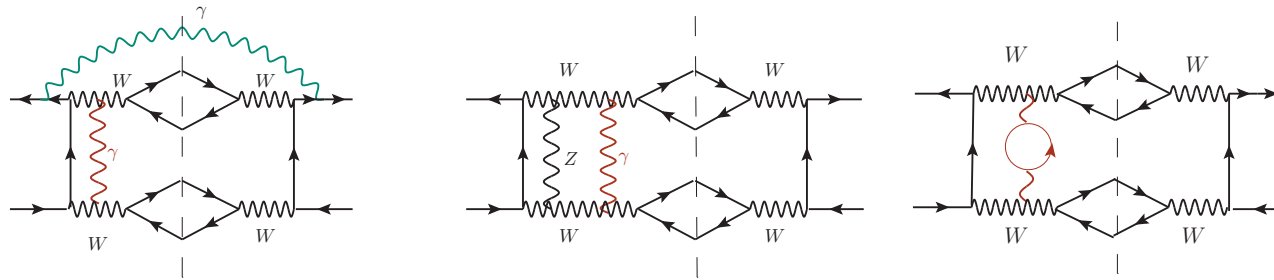


EFT expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$ (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

Leading NNLO corrections

- 2nd Coulomb correction $\sim \alpha^2/\beta^2 \sim \alpha$ (Fadin et al. 95)
- Coulomb-enhanced corrections $\sim \alpha^2/\beta \sim \alpha^{3/2}$ (Actis et al. 08)



- Numerical effect: $\Delta\sigma_{WW} \sim 5\text{‰}$; $[\delta M_W] \lesssim 3 \text{ MeV}$

\sqrt{s} [GeV]	$\sigma(e^-e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d})$ (fb)			
	NLO _{EFT}	NLO _{ee4f} [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7‰)	0.15 (1.3‰)
170	397.8	404.5	0.25 (0.6‰)	1.6 (3.9‰)

ISR: resum leading logs

$$\beta_e = \frac{2\alpha}{\pi} \left(2 \log \left(\frac{2M_W}{m_e} \right) - 1 \right)$$

in electron structure functions:

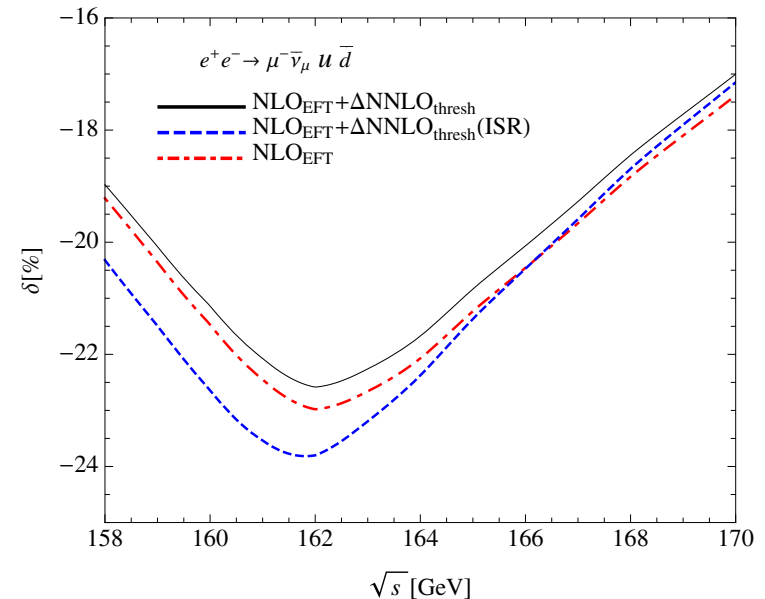
(Skrzypek 92)

$$\sigma_{\text{NLO}}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}^{\text{LL}}(x_1) \Gamma_{ee}^{\text{LL}}(x_2) (\sigma_{\text{tree}} + \Delta \hat{\sigma}_{\text{NLO}})$$

ISR for tree only \Leftrightarrow also for NLO:

$\sim 2\%$ NLL $\mathcal{O}(\alpha\beta_e)$ effect

\Rightarrow NLL resummation important



Implementation of state-of-the art calculations in public tools?

- **NLO-EW** $e^-e^+ \rightarrow 4f$ now possible with standard tools
(RECOLA, OpenLoops, MadLoops)
but not (yet) optimized for e^-e^+ (ISR, Beamstrahlung; \Rightarrow talk by Shao)
- **Two-loop Coulomb-enhanced** corrections for differential observables doable; (related: $t\bar{t}$ with Coulomb resummation in WHIZARD)
(no guarantee of formal accuracy for general distributions)

Implementation of state-of-the art calculations in public tools?

- **NLO-EW** $e^-e^+ \rightarrow 4f$ now possible with standard tools
(RECOLA, OpenLoops, MadLoops)
but not (yet) optimized for e^-e^+ (ISR, Beamstrahlung; \Rightarrow talk by Shao)
- **Two-loop Coulomb-enhanced** corrections for differential observables doable; (related: $t\bar{t}$ with Coulomb resummation in WHIZARD)
(no guarantee of formal accuracy for general distributions)

Full NNLO in EFT for total cross section

- Soft $\log \beta$ terms can be adapted from QCD results
- NNLO $\log(m_e/M_W)$ terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections
(from amplitudes for $e^+e^- \rightarrow W^+W^-$ at threshold: border of current capabilities)

resulting uncertainty from cross-section calculation

$$\Delta\sigma_{\text{hard}}^{(2)} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)\% \text{ for estimate } c^{(2)} = (c^{(1)})^2$$

Full NNLO for $e^+e^- \rightarrow 4f$: completely new methods needed

W mass measurement from threshold scan:

- error $\lesssim 1$ MeV needs $\sigma(e^+e^- \rightarrow 4f)$ with accuracy $< 1\text{‰}$

Full NLO corrections to $e^+e^- \rightarrow 4f$

(Denner et al. 05)

Coulomb-enhanced NNLO corrections

for σ_{tot} near threshold in EFT approach:

(Actis et al. 08)

5‰ correction; remaining uncertainty $\Delta M_W \lesssim 3$ MeV

Next possible steps

- Implementation of Coulomb-enhanced NNLO corrections near threshold for distributions
- Logarithmic NNLO corrections in EFT

Full NNLO in EFT required for ultimate CEPC/FCCee precision

Estimate impact of uncertainties on mass measurement:

- Assume measurements O_i at $\sqrt{s} = 160, 161, \dots, 164, 170$ GeV
- Minimize

$$\chi^2(\delta M_W) = \sum_{i=1}^6 \frac{(O_i - E_i(\delta M_W))^2}{2\sigma_i^2}.$$

where $E_i(\delta M_W)$: theoretical calculation for $M_W = M_W^{\text{ref}} + \delta M_W$

- take O_i : NLO EFT , E_i : Estimate of error

Example:

2nd Coulomb correction

$\Rightarrow \mathcal{O}(\text{‰})$ (Fadin et al. 95)

$\Rightarrow [\delta M_W]_{C2} < 4 \text{ MeV}$

