Energy-Energy Correlation for Higgs Decay

analytically at next-to-leading order

Vladyslav Shtabovenko

Zhejiang University

in collaboration with
M.-x. Luo, T.-Z. Yang and H. X. Zhu

The 2018 International Workshop on the
High Energy Circular Electron-Positron Collider
12th of November 2018

浙江大学 1897 ZHEJIANG UNIVERSITY

浙江大学
ZHEJIANG UNIVERSITY

Outline

1. **Motivation**
   - Why a new e^+e^- collider
   - Event shape variables
   - Energy-Energy correlation
2. **Higgs Energy-Energy Correlation**
   - Definition
   - Results
   - Nonperturbative corrections
3. **Summary and Outlook**

- Maturing plans for new e^+e^- collider(s):
- Circular Electron Positron Collider (CEPC) in China [CEPC-SPPC Study Group, 2018] ?
- International Linear Collider (ILC) in Japan [Behnke et al., 2013]?
- Compact Linear Collider (CLIC) [Aicheler et al., 2012] or Future Circular Collider (FCC-ee) in Europe [Bicer et al., 2014]?
- A hadron collider is primarily a discovery machine.
- Mission accomplished: Discovery of an elementary scalar boson by the LHC in 2012.
- Does this particle behave exactly as the SM Higgs?
- Need a precision machine to study the Higgs sector in more details.

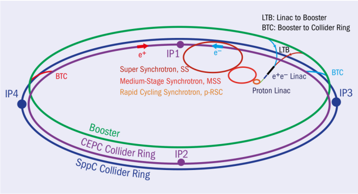


Fig. 1. The overall CEPC-SppC schematic layout.

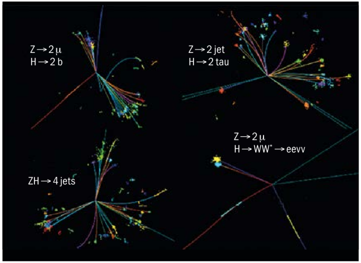


Fig. 2. Simulated Higgs-boson signals with different decay final states for 240 GeV electron–positron collisions envisaged at CEPC, using a PFA-oriented detector design.

Source: [CERN Courier June 2018].

- CEPC: expect 10^6 Higgs boson events over a period of 7 years! [An et al., 2018]
- Prospects to determine the Higgs couplings with an accuracy below 1%!
- Yet, the new machine would not be *just* a Higgs factory.
- Many possible measurements constitute a rich physics program.
- Lower $\sqrt{s}$ to the Z-boson peak: 10^9 Z-bosons.
- Operate at WW-threshold: 10^8 W-bosons.
- CEPC could operate as a super Z- and W-boson factory.

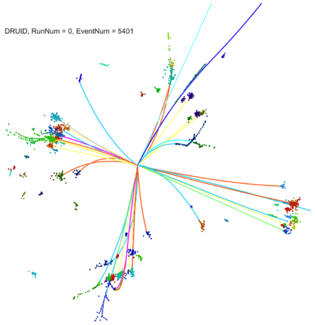


Fig. 3. A simulated $e^+e^- \to ZH \to q\bar{q}\ b\bar{b}$ event reconstructed with the ARBOR algorithm. Different types of reconstructed final state particles are represented in different colors.

Source: [An et al., 2018].

- Precision electroweak measurements, rare decays, flavor physics.
- Clean environment to search for physics beyond the Standard Model.
- Strong sector: Precision QCD studies (jet physics, *event shape observables*)

- Original motivation for the event shape observables: Verify QCD by inventing observables that
  - can be reliably calculated in pQCD (IR- and collinear safety, nonperturbative corrections suppressed),
  - are easy to extract from the experimental data (kinematics of the final states).
- Noteworthy properties
  - Characterization of the event topologies
  - Sensitivity to QCD radiation (soft gluon emissions)
  - Can be employed for the determination of α_s (especially in 3-jet events)
  - Probe our understanding of QCD (resummations, subtractions, mathematical structure, development of event generators ...)
- Examples:
  - Thrust T [Brandt et al., 1964; Farhi, 1977].
  - C-parameter [Parisi, 1978; Donoghue et al., 1979; Ellis et al., 1981].
  - Wide B_W and total B_T jet broadenings [Rakow & Webber, 1981; Ellis & Webber, 1986; Catani et al., 1992].
  - Normalized heavy jet mass M_H^2/s [Clavelli, 1979].
  - Transition from 3-jet to 2-jet final states in the Durham jet algorithm y_{23} [Catani et al., 1991; Brown & Stirling, 1990, 1992; Stirling, 1991]
  - Energy-energy correlations [Basham et al., 1978].
- The six "classical" event shape observables C, M_H^2/s, B_W, B_T, T and y_{23} were measured by the LEP experiments with high precision: ALEPH [Heister et al., 2004], DELPHI [Abdallah et al., 2004], L3 [Achard et al., 2004], OPAL [Abbiendi et al., 2005].

- Connection between QCD event shape observables and the Higgs sector?
- Consider event shape variables in hadronic Higgs decays! [see also today's talk of Yin-Qiang Gong]
- Hard process: $H \to \mathrm{partons}$ instead of $\gamma^*/Z_0 \to \mathrm{partons}$.
- Dominant production channel at an e^+e^- collider: Higgs-Strahlung.

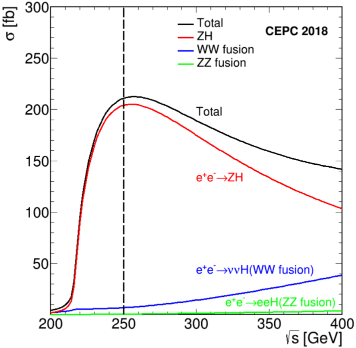


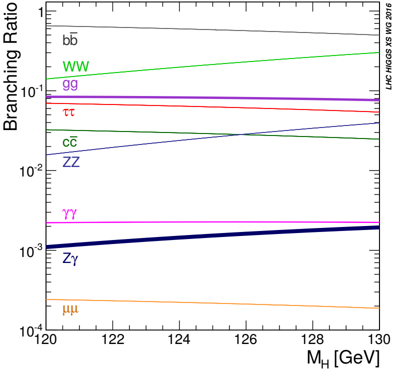
Fig. 8. Production cross sections of $e^+e^- \to ZH$ and $e^+e^- \to (e^+e^-/\nu\bar{\nu})H$ as functions of $\sqrt{s}$ for a 125 GeV SM Higgs boson. The vertical indicates $\sqrt{s} = 250$ GeV, the energy assumed for most of the studies summarized in this paper.

Source: [An et al., 2018].

Process	Cross section	Events in 5.6 ab^{-1}
Higgs boson production, cross section in fb		
$e^+e^- \to ZH$	204.7	1.15×10^6
$e^+e^- \to \nu_e\bar{\nu}_e H$	6.85	3.84×10^4
$e^+e^- \to e^+e^- H$	0.63	3.53×10^3
Total	212.1	1.19×10^6
Background processes, cross section in pb		
$e^+e^- \to e^+e^-(\gamma)$ (Bhabha)	850	4.5×10^9
$e^+e^- \to q\bar{q}(\gamma)$	50.2	2.8×10^8
$e^+e^- \to \mu^+\mu^-(\gamma)$ [or $\tau^+\tau^-(\gamma)$]	4.40	2.5×10^7
$e^+e^- \to WW$	15.4	8.6×10^7
$e^+e^- \to ZZ$	1.03	5.8×10^6
$e^+e^- \to e^+e^- Z$	4.73	2.7×10^7
$e^+e^- \to e^+\nu W^- / e^-\bar{\nu}W^+$	5.14	2.9×10^7

Source: [An et al., 2018].

- Partonic decay channels: $H \to b\bar{b}$, $H \to c\bar{c}$, $H \to gg$.
- Event shape observables from gluon-inititated events are particularly interesting!
- But $H \to gg$ is also much more suppressed as compared to $H \to b\bar{b}$!



Source: [de Florian et al., 2016].

Decay mode	Branching ratio	Relative uncertainty
$H \to b\bar{b}$	57.7%	+3.2%, −3.3%
$H \to c\bar{c}$	2.91%	+12%, −12%
$H \to \tau^+\tau^-$	6.32%	+5.7%, −5.7%
$H \to \mu^+\mu^-$	2.19×10^{-4}	+6.0%, −5.9%
$H \to WW^*$	21.5%	+4.3%, −4.2%
$H \to ZZ^*$	2.64%	+4.3%, −4.2%
$H \to \gamma\gamma$	2.28×10^{-3}	+5.0%, −4.9%
$H \to Z\gamma$	1.53×10^{-3}	+9.0%, −8.8%
$H \to gg$	8.57%	+10%, −10%
Γ_H	4.07 MeV	+4.0%, −4.0%

Source: [An et al., 2018].

- LHC: too large background to measure $H \to gg$ decays.
- CEPC: difficult, but feasible!
- Use the recoil mass method.
- Crucial ingredient: Jet tagging performance of the CEPC detector.
- Expect some $\mathcal{O}(10^3)$ events.

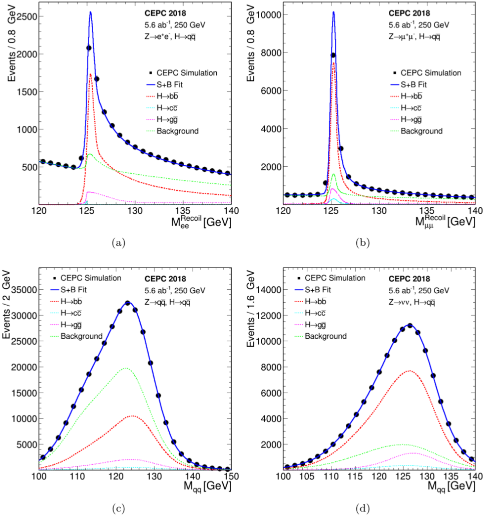
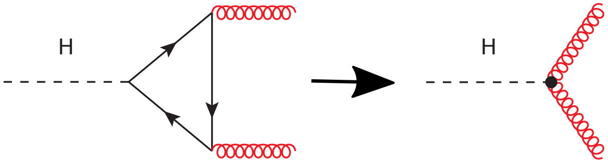


Fig. 11. ZH production with $H \to b\bar{b}/c\bar{c}/gg$: the recoil mass distributions of (a) $Z \to e^+e^-$ and (b) $Z \to \mu^+\mu^-$; the dijet mass distributions of Higgs boson candidates for (c) $Z \to q\bar{q}$ and (d) $Z \to \nu\bar{\nu}$. The markers and their uncertainties represent expectations from a CEPC dataset of $5.6\,\mathrm{ab}^{-1}$ whereas the solid blue curves are the fit results. The dashed curves are the signal and background components. Contributions from other decays of the Higgs boson are included in the background.

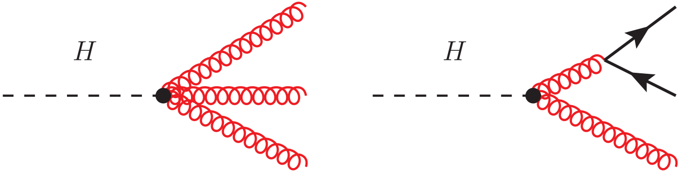
Source: [An et al., 2018].

- Study the gluonic Higgs decay using the Effective Field Theory (EFT) approach.
- Since $m_{b,c,s,u,d} \ll m_t$, top quark loops give the largest contribution to $H \to gg$.
- By integrating out m_t we obtain Higgs Effective Field Theory (HEFT)



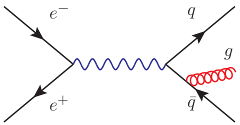
$$\mathcal{L}_{\text{HEFT,int}} = -\frac{1}{4}\lambda H G_{\mu\nu,a} G^{\mu\nu,a}$$

- HEFT: tree-level couplings between Higgs and 2, 3 or 4 gluons.
- Consider hard processes with at least 3 partons: $H \to ggg, \quad H \to gq\bar{q}$



- Interesting event shape observable: Energy-Energy correlation function (EEC).

- EEC [Basham et al., 1978] is a classical hadronic observable in e^+e^- annihilation: $e^+e^- \to \gamma^*/Z^0 \to a + b + X$.



- Formal definition

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma(\chi)}{d\cos\chi} = \sum_{a,b} \int \frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)\, d\sigma_{a+b+X}, \quad \cos\theta_{ab} = \hat{\boldsymbol{p}}_a \cdot \hat{\boldsymbol{p}}_b.$$

- Two calorimeters at relative angle χ measure the energies of the hadrons a and b.
- EEC: differential angular distribution of the energy flow through the calorimeters.
- Measures the energies between all the pairs of hadrons produced in each event
- Can be computed in pQCD by the virtue of the momentum sum rule

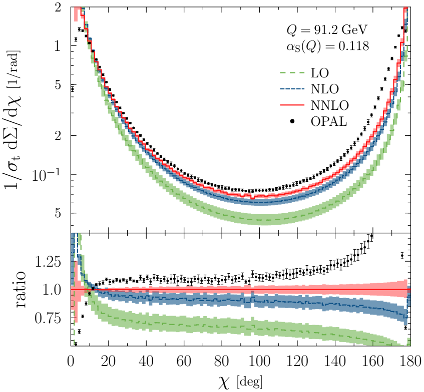
$$\sum_h \int_0^1 dx\, x\, D_{h/q}(x, \mu_F^2) = 1.$$

- A good probe of QCD, as already the LO contribution starts with α_s.
- Also interesting for the determination of α_s, c. f. [Kardos et al., 2018].

- The analytic form of the EEC at LO is known since 40 years [Basham et al., 1978]

$$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\Sigma(\chi)}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi} C_F \frac{3-2z}{4(1-z)z^5}\left[3z(2-3z)+2(2z^2-6z+3)\log(1-z)\right] + \mathcal{O}(\alpha_s^2), \quad \text{with } z=(1-\cos\chi)/2.$$

- Phenomenological purposes: reliable numerical results at NNLO [Del Duca et al., 2016; Tulipánt et al., 2017].



Source: [Tulipánt et al., 2017].

- Until recently, the only analytic result known for EEC was the LO calculation.
- The fully analytic NLO result became available this year [Dixon et al., 2018].

PHYSICAL REVIEW LETTERS **120,** 102001 (2018)

Editors' Suggestion

Analytical Computation of Energy-Energy Correlation at Next-to-Leading Order in QCD

Lance J. Dixon,[1,*] Ming-xing Luo,[2,†] Vladyslav Shtabovenko,[2,‡] Tong-Zhi Yang,[2,§] and Hua Xing Zhu[2,∥]

[1]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

[2]*Zhejiang Institute of Modern Physics, Department of Physics, Zhejiang University, Hangzhou 310027, China*

(Received 17 January 2018; published 9 March 2018)

The energy-energy correlation (EEC) between two detectors in e^+e^- annihilation was computed analytically at leading order in QCD almost 40 years ago, and numerically at next-to-leading order (NLO) starting in the 1980s. We present the first analytical result for the EEC at NLO, which is remarkably simple, and facilitates analytical study of the perturbative structure of the EEC. We provide the expansion of the EEC in the collinear and back-to-back regions through next-to-leading power, information which should aid resummation in these regions.

DOI: 10.1103/PhysRevLett.120.102001

- In our calculation we used the techniques of the IBP-reduction [Chetyrkin & Tkachov, 1981] and differential equations [Kotikov, 1991a, 1991b, 1991c; Bern et al., 1994; Remiddi, 1997; Gehrmann & Remiddi, 2000].
- Analytic EEC at NLO does not change much for the phenomenology.
- But: The framework we developed can be also applied to other processes!
- Our current interest: EEC in the gluonic Higgs decay.

- Formal definition

$$\frac{1}{\Gamma_{\text{tot}}}\frac{d\Sigma_H(\chi)}{d\cos\chi} = \sum_{a,b}\int \frac{E_a E_b}{m_H^2}\,\delta(\cos\theta_{ab} - \cos\chi)\, d\Gamma_{a+b+X}, \quad \cos\theta_{ab} = \hat{\boldsymbol{p}}_a \cdot \hat{\boldsymbol{p}}_b.$$

- LO hard processes: $H \to ggg$, $H \to q\bar{q}g$.
- NLO hard processes: $H \to gggg$, $H \to q\bar{q}gg$, $H \to q\bar{q}q\bar{q}$, $H \to q\bar{q}q'\bar{q}'$.
- Normalize w.r.t the partial decay width Γ_{tot} for $H \to gg$ in the limit $2m_t \gg m_H$.
- The $\mathcal{O}(\alpha_s)$ result is sufficient [Inami et al., 1983; Djouadi et al., 1996; Spira et al., 1995]

$$\Gamma_{\text{tot}} = \Gamma(H \to gg)^{\text{LO}} \underbrace{\left(1 + \frac{\alpha_s(\mu)}{2\pi}\left(\frac{95}{2} - \frac{7}{3}N_f + \frac{33 - 2N_f}{3}\ln\frac{\mu^2}{m_H^2}\right)\right)}_{\equiv b_H}$$

$$\approx \Gamma(H \to gg)^{\text{LO}}(1 + 0.78) \quad \text{for} \quad N_f = 5,\ \alpha_s(m_H) = 0.113$$

- Express the full NLO result as ($\beta_0 = 11C_A/3 - 4N_f T_f/3$)

$$\frac{1}{\Gamma_{\text{tot}}}\frac{d\Sigma_H(\chi)}{d\cos\chi} = \frac{1}{b_H} \times \left[\frac{\alpha_s(\mu)}{2\pi}A_H(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left(\beta_0 \ln\frac{\mu}{m_H}A_H(z) + B_H(z)\right) + \mathcal{O}(\alpha_s^3)\right].$$

$$\frac{1}{\Gamma_{\text{tot}}}\frac{d\Sigma_H(\chi)}{d\cos\chi} = \frac{1}{b_H} \times \left[\frac{\alpha_s(\mu)}{2\pi}A_H(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left(\beta_0 \ln\frac{\mu}{m_H}A_H(z) + B_H(z)\right) + \mathcal{O}(\alpha_s^3)\right].$$

- LO result (NEW!)

$$\frac{1}{\Gamma_{\text{tot}}}\frac{d\Sigma_H(\chi)}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi}\frac{1}{b_H}\Bigg\{ \times C_A\left[\frac{25z^3-156z^2+336z-216}{12(1-z)z^5} - \frac{(2z^4-14z^3+51z^2-74z+36)}{2(1-z)z^6}\ln(1-z)\right] - N_f T_f\left[\frac{25z^3-201z^2+390z-216}{6(1-z)z^5} - \frac{(z^4-17z^3+63z^2-83z+36)}{(1-z)z^6}\ln(1-z)\right]\Bigg\} + \mathcal{O}(\alpha_s^2).$$

- Color decomposition of the NLO coefficient $B_H(z)$

$$B_H(z) = C_A^2 B_{H,\text{lc}}(z) + C_A T_f N_f B_{H,\text{nlc}}(z) + (C_A - 2C_F)T_f N_f B_{H,\text{nnlc}}(z) + N_f^2 T_f^2 B_{H,N_f^2}(z).$$

- The full result is still unpublished.
- We show the explicit analytic expression only for $B_{H,\text{lc}}(z)$

- Building blocks: Pure functions $g_i^{(n)}$ of uniform transcendental weight $n \leq 3$

$$g_1^{(1)} = \log(1-z)\,, \qquad g_2^{(1)} = \log(z)\,, \qquad g_1^{(2)} = 2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\,,$$
$$g_2^{(2)} = \mathrm{Li}_2(1-z) - \mathrm{Li}_2(z)\,,$$
$$g_3^{(2)} = -2\,\mathrm{Li}_2\left(-\sqrt{z}\right) + 2\,\mathrm{Li}_2\left(\sqrt{z}\right) + \log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\log(z)\,, \qquad g_4^{(2)} = \zeta_2\,,$$
$$g_1^{(3)} = -6\left[\mathrm{Li}_3\left(-\frac{z}{1-z}\right) - \zeta_3\right] - \log\left(\frac{z}{1-z}\right)\left(2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\right)\,,$$
$$g_2^{(3)} = -12\left[\mathrm{Li}_3(z) + \mathrm{Li}_3\left(-\frac{z}{1-z}\right)\right] + 6\,\mathrm{Li}_2(z)\log(1-z) + \log^3(1-z)\,,$$
$$g_3^{(3)} = 6\log(1-z)\left(\mathrm{Li}_2(z) - \zeta_2\right) - 12\,\mathrm{Li}_3(z) + \log^3(1-z)\,,$$
$$g_4^{(3)} = \mathrm{Li}_3\left(-\frac{z}{1-z}\right) - 3\,\zeta_2\log(z) + 8\,\zeta_3\,,$$
$$g_5^{(3)} = -8\left[\mathrm{Li}_3\left(-\frac{\sqrt{z}}{1-\sqrt{z}}\right) + \mathrm{Li}_3\left(\frac{\sqrt{z}}{1+\sqrt{z}}\right)\right] + 2\mathrm{Li}_3\left(-\frac{z}{1-z}\right)$$
$$+ 4\zeta_2\log(1-z) + \log\left(\frac{1-z}{z}\right)\log^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\,.$$

- The same basis as in our NLO result for the standard EEC!

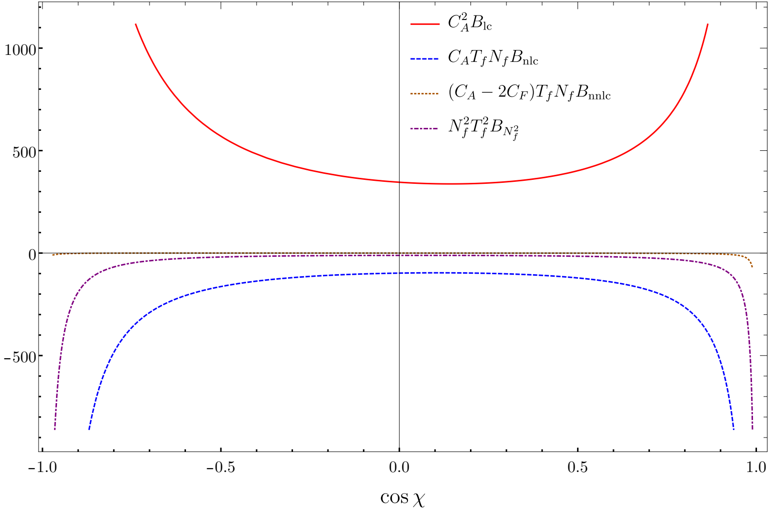
$$B_H(z) = C_A^2 B_{H,\mathrm{lc}}(z) + C_A T_f N_f B_{H,\mathrm{nlc}}(z) + (C_A - 2C_F) T_f N_f B_{H,\mathrm{nnlc}}(z) + N_f^2 T_f^2 B_{H,N_f^2}(z).$$

- Leading color coefficient $B_{H,\mathrm{lc}}(z)$

$$
\begin{aligned}
&-\frac{3240z^6 - 3240z^5 + 981z^4 - 207539z^3 + 1131821z^2 - 2416929z + 1546086}{8640(1-z)z^5} \\
&+\frac{2160z^7 - 2700z^6 + 4560z^5 - 975z^4 - 13190z^3 + 70367z^2 - 151398z + 92556}{1440(1-z)z^5} g_1^{(1)} \\
&-\frac{2160z^8 - 3780z^7 + 5640z^6 - 3909z^5 + 2317z^4 + 12434z^3 - 2958z^2 - 36449z + 22565}{1440(1-z)z^6} g_2^{(1)} \\
&+\frac{-168z^6 + 353z^5 - 605z^4 + 3080z^3 - 3860z^2 - 1967z + 4047}{240(1-z)z^6} g_1^{(2)} \\
&-\frac{-180z^7 + 90z^6 - 330z^5 + 75z^4 - 460z^3 + 3000z^2 - 8860z + 7833}{120z^6} g_2^{(2)} \\
&-\frac{3z^4 - 6z^3 + 9z^2 - 10z + 3}{4(1-z)z} g_1^{(3)} - \frac{2z^6 - z^5 + 7z^4 - 44z^3 + 156z^2 - 224z + 109}{12(1-z)z^6} g_2^{(3)} \\
&+\frac{1}{6(1-z)} g_3^{(3)} + \frac{1-2z}{2(1-z)z} g_4^{(3)} + \frac{2z^5 + z^4 + 2z^2 - z + 1}{4z^6} g_5^{(3)} + \text{one more line}
\end{aligned}
$$

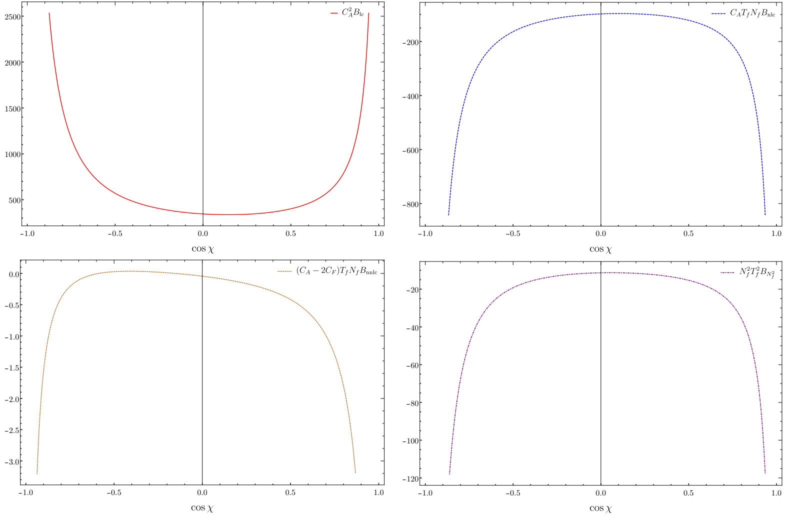
$$B_H(z) = C_A^2 B_{H,\mathrm{lc}}(z) + C_A T_f N_f B_{H,\mathrm{nlc}}(z) + (C_A - 2C_F) T_f N_f B_{H,\mathrm{nnlc}}(z) + N_f^2 T_f^2 B_{H,N_f^2}(z).$$

- In the central region the contributions from $B_{H,\mathrm{lc}}(z)$ and $B_{H,\mathrm{nlc}}(z)$ dominate

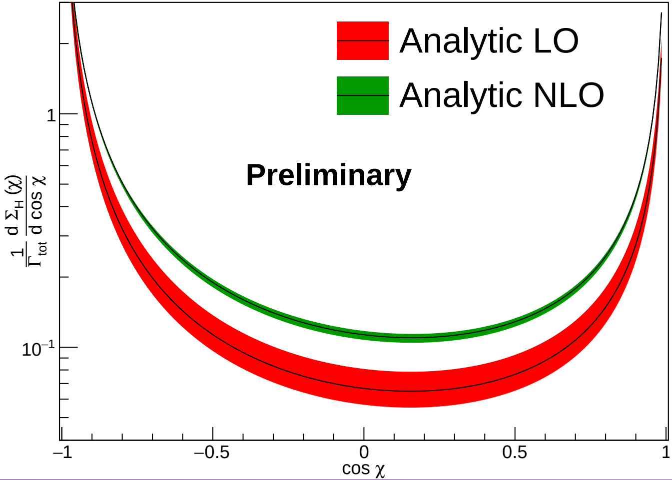


$$B_H(z) = C_A^2 B_{H,\mathrm{lc}}(z) + C_A T_f N_f B_{H,\mathrm{nlc}}(z) + (C_A - 2C_F) T_f N_f B_{H,\mathrm{nnlc}}(z) + N_f^2 T_f^2 B_{H,N_f^2}(z).$$

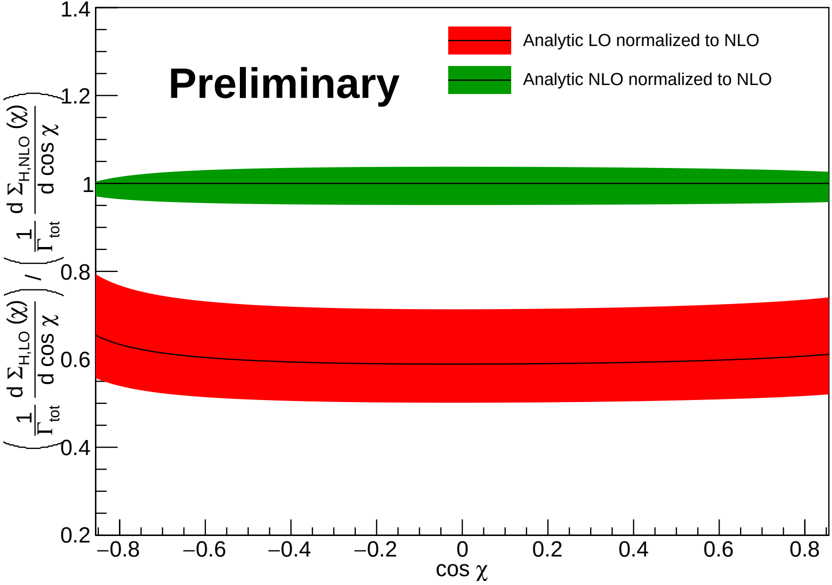
- Only $B_{H,\mathrm{lc}}(z)$ yields a positive contribution.



- Full analytic result: the NLO corrections are sizable.
- Sidebands: variation of μ in $\alpha_s(\mu)$ and $\log(\mu^2/m_H^2)$.
- Central value: $\mu = m_H$, uncertainties: $\mu = 1/2 m_H$ and $\mu = 2m_H$.
- Fixed-order calculation diverges when the measured particles are collinear ($\cos\chi \to 1$) or back-to-back ($\cos\chi \to -1$).
- Emission of soft/collinear particles requires resummation of logarithms.

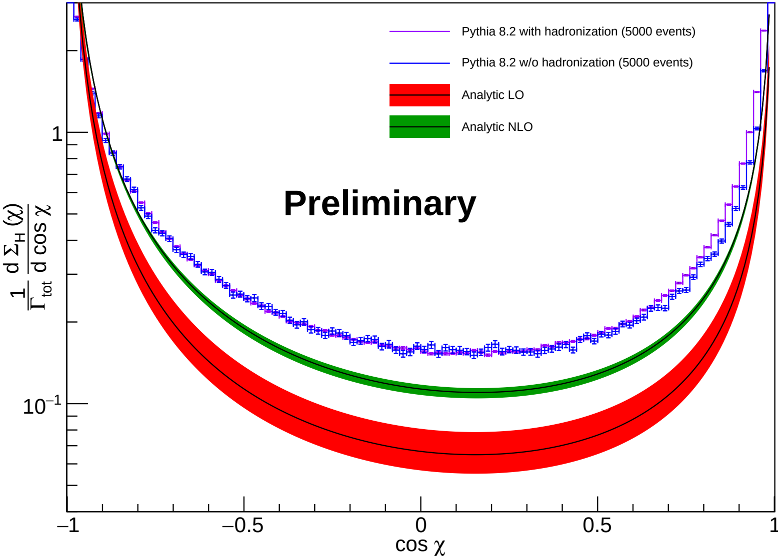


- Size of the NLO corrections: Normalize the curves to the NLO result at $\mu = m_H$.



- How large are the hadronization effects?
- The original estimate for standard EEC [Basham et al., 1978] $\sim 1/Q$.
- More careful treatment e. g. in the framework of the DMW model [Dokshitzer et al., 1999].
- Comparisons to real data (e. g. for α_s determination) require proper modelling.
- In view of the lack of experimental data, we use Pythia [Sjöstrand et al., 2015] for simulations.
- Hard process: $e^+e^- \to H \to gg$ at $\sqrt{s} = m_H$.
- Generate $N = 5000$ events with Pythia 8.2.15.
- Parton showering and hadronization are included.
- We include only the statistical uncertainties.
- For each event calculate Higgs EEC as $\sum_{i<j} \frac{2E_iE_j}{E_{\text{vis}}^2}\,\delta(\cos\theta_{ij} - \cos\chi)$
- Overall normalization of the histogram: $1/(\Delta\chi N)$, $\Delta\chi$ is the bin width.
- The normalization ensures that the area under the curve is unity.

- The simulation suggests that the hadronization effects are comparably small.
- However, Pythia was never tuned to gluon-initiated event shape observables!
- Realistic simulation: The LO hard matrix element in Pythia may not be sufficient.



- It is tempting to try to determine α_s by fitting the NLO prediction to Pythia.
- For simplicity, assume a very naive description of the simulated data

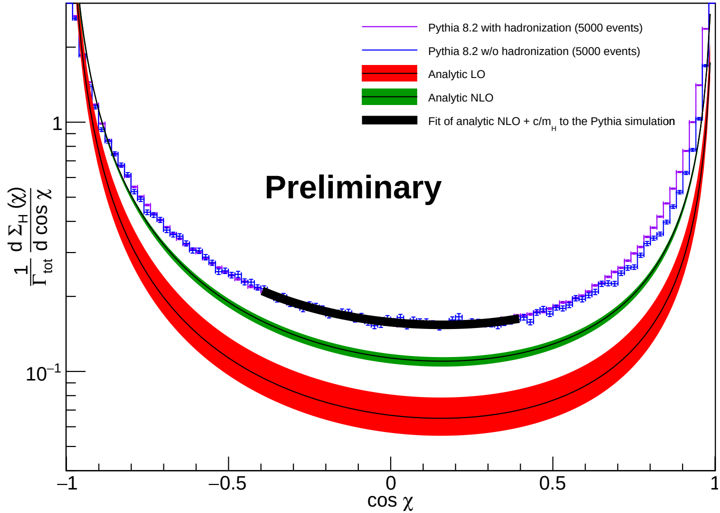
$$\left(\frac{1}{\Gamma_{\text{tot}}} \frac{d\Sigma_H(\chi)}{d\cos\chi} \right)_{\text{sim}} = \left(\frac{1}{\Gamma_{\text{tot}}} \frac{d\Sigma_H(\chi)}{d\cos\chi} \right)_{\text{pert}} + \frac{c}{m_H}.$$

- c is a nonperturbative parameter responsible for hadronization corrections.
- Perform a two parameter fit to determine $\alpha_s(m_H)$ and c, e. g. as in [Abdallah et al., 2003]
- Clearly a *toy model*, not a real α_s determination!
- Need to be sufficiently far away from $\cos\chi \to \pm 1$.
- Reasonable range $\cos\chi \in (-0.4; 0.4)$.

- Our binned maximum likelihood fit yields

$$c = (3.34 \pm 1.98)\ \text{GeV}, \quad \alpha_s(m_H) = 0.130 \pm 0.015, \quad \chi^2/\text{NDF} = 51/38$$

- This is just a fit to the Pythia simulation, not to the real data!
- It is not even clear how well Pythia can model this process.



Summary

- We introduced a new event shape observable, that connects the strong and the Higgs sectors.
- Higgs EEC could be potentially measured at CEPC or another future e^+e^- collider.
- We have already obtained the fully analytic NLO result in the Higgs EFT.
- The NLO corrections are sizable.
- A naive PYTHIA simulation suggests that hadronization effects are not too large.

Outlook

- How large are the NNLO corrections (at least numerically)?
- Better Monte Carlo + hadronization simulation for Higgs EEC?
- Explore prospects for α_s determination (expected number of reconstructed events, detector simulation, uncertainties, ...).