Studies of τ decays into two mesons

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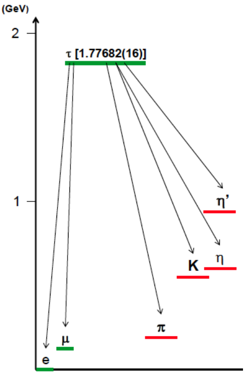
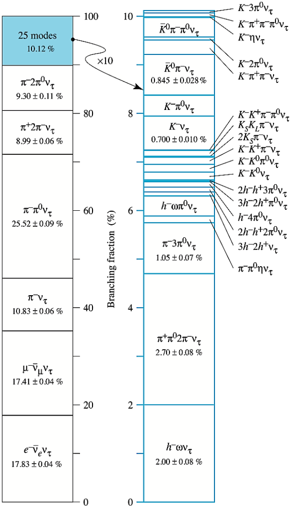
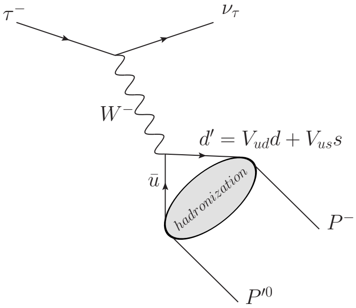
Outline

1. **Introduction**
2. **Analyses of τ decays into a pair of mesons**
   - $\tau \to K\pi\nu_\tau$
   - Combined analysis of $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$
   - $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$
   - $\tau \to \pi^-\pi^0\nu_\tau$ and $\tau \to K^-K_S\nu_\tau$
3. **Outlook**

Hadronic Tau decays







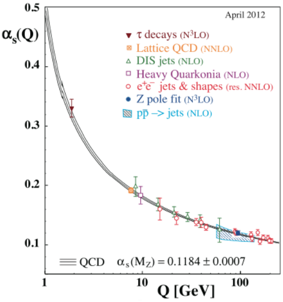
Test of QCD and ElectroWeak Interactions

Test of QCD and ElectroWeak Interactions

- Inclusive decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$

  Full hadron spectra (precision physics)

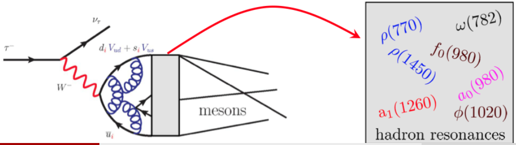
  Fundamental SM parameters: $\alpha_s(m_\tau), m_s, |V_{us}|$



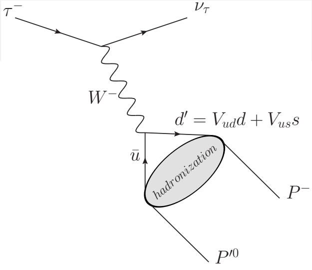
- Exclusive decays: $\tau^- \to (PP, PPP, ...)\nu_\tau$

  specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)



τ decays into two mesons



- $\tau^- \to P^-\nu_\tau : F_{\pi,K}$
- $\tau^- \to (2P)^-\nu_\tau$: reasonable good control
- $\tau^- \to (3P)^-\nu_\tau$: reasonable good control
- $\tau^- \to (>3P)^-\nu_\tau$: poor knowledge

$$\mathcal{M}\left(\tau^- \to P^-P'^0\nu_\tau\right) = \frac{G_F}{\sqrt{2}} V_{\rm CKM}\bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle P^-P'^0|d'\gamma^\mu u|0\rangle\,,$$

$$\langle P^-P'^0|d'\gamma^\mu u|0\rangle = \mathcal{C}_{P^-P'^0}\left\{\left(p_- - p_0 - \frac{\Delta_{P^-P'^0}}{s}q\right)^\mu F_V^{P^-P'^0}(s) + \frac{\Delta_{P^-P'^0}}{s}q^\mu F_S^{P^-P'^0}(s)\right\},$$

$$\frac{d\Gamma\left(\tau^-\to P^-P^0\nu_\tau\right)}{ds} = \frac{G_F^2|V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\rm had} C_{PP'}^2 \left(1-\frac{s}{M_\tau^2}\right)^2$$

$$\times\left\{\left(1+\frac{2s}{m_\tau^2}\right)\lambda_{P^-P^0}^{3/2}(s)|F_V^{P^-P^0}(s)|^2 + 3\frac{\Delta_{P^-P^0}^2}{s^2}\lambda_{P^-P^0}^{1/2}(s)|F_S^{P^-P^0}(s)|^2\right\}$$

$$\Delta_{P^-P^0} = m_{P^-}^2 - m_{P^0}^2\,,\quad C_{P'P^0} : \text{Clebsch} - \text{Gordon}$$

$\tau^- \to \nu_\tau$+strange

- Tau partial width to strange $\sim 3\%$

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

- $\tau \to (K\pi)^- \nu_\tau$ and $\tau \to K^- \eta^{(\prime)} \nu_\tau \longrightarrow$ this talk

$\tau \to K\pi\nu_\tau$

- Vector Form Factor:
  - Jamin, Pich and Portolés, PLB 640 (2006) 176-181
  - Jamin, Pich and Portolés, PLB 664 (2008) 78-83
  - Moussallam, EPJC 53 (2008)
  - Boito, Escribano and Jamin, EPJC 59 (2009) → this talk
  - Boito, Escribano and Jamin, JHEP 1009 (2010) 031 → this talk
  - Antonelli, Cirigliano, Lusiani and Passemar, JHEP 1310 (2013) 070
- Scalar Form Factor:
  - Jamin, Oller and Pich, Nucl. Phys. B587 (2002); PRD 74 (2006)

$\tau \to K\pi\nu_\tau$

- RχT with two resonances: $K^*(892)$ and $K^*(1410)$

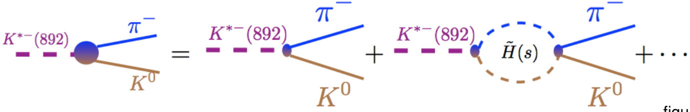


figure courtesy of D. Boito

$$\widetilde{F}_V^{K\pi}(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star}, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}}, \gamma_{K^{\star\prime}})},$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \mathrm{Re}\left[H_{K\pi}(s)\right] - i m_n \Gamma_n(s),$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)}\frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_n^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$

- We then have a phase with two resonances

$$\delta^{K\pi}(s) = \tan^{-1}\left[\frac{\mathrm{Im}F_V^{K\pi}(s)}{\mathrm{Re}F_V^{K\pi}(s)}\right]$$

Vector Form Factor: Dispersive representation

- Three subtractions: helps the convergence of the form factor and suppresses the the high-energy region of the integral

$$F_V^{K\pi}(s) = P(s)\exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta^{K\pi}(s')}{(s')^3(s'-s-i0)}\right]$$

- $\alpha_1 = \lambda_+^{'}$ and $\alpha_1^2 + \alpha_2 = \lambda_+^{''}$ low energies parameters

$$F_V^{K\pi}(t) = 1 + \frac{\lambda_+^{'}}{M_{\pi^-}^2} t + \frac{1}{2}\frac{\lambda_+^{''}}{M_{\pi^-}^4} t^2$$

- $s_{\rm cut}$: cut-off to check stability
- Parameters to Fit: λ_+' , λ_+'', $m_{K^*}, \gamma_{K^*}, m_{K^{*\prime}}, \gamma_{K^{*\prime}}$

Fits to the $\tau^- \to K_S\pi^-\nu_\tau$ Belle data

Boito, Escribano and Jamin, EPJC 59 (2009)

	$s_{cut} = 3.24\ GeV^2$	$s_{cut} = 4\ GeV^2$	$s_{cut} = 9\ GeV^2$	$s_{cut} \to \infty$
m_{K^*} [MeV]	943.32 ± 0.59	943.41 ± 0.58	943.48 ± 0.57	943.49 ± 0.57
γ_{K^*} [MeV]	66.61 ± 0.88	66.72 ± 0.86	66.82 ± 0.85	66.82 ± 0.85
$m_{K'^*}$ [MeV]	1407 ± 44	1374 ± 30	1362 ± 26	1362 ± 26
$\gamma_{K'^*}$ [MeV]	325 ± 149	240 ± 100	216 ± 86	215 ± 86
$\gamma \times 10^2$	-5.2 ± 2.0	-3.9 ± 1.5	-3.5 ± 1.3	-3.5 ± 1.3
$\lambda'_+ \times 10^3$	24.31 ± 0.74	24.66 ± 0.69	24.94 ± 0.68	24.96 ± 0.67
$\lambda''_+ \times 10^4$	12.04 ± 0.20	11.99 ± 0.19	11.96 ± 0.19	11.96 ± 0.19
χ^2/n.d.f.	74.2/79	75.7/79	77.2/79	77.3/79

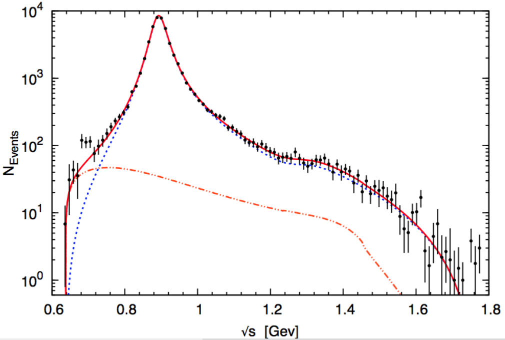
Data from Belle:

Epifanov et. at., PL B654 (2007)

$$\frac{dN_{\rm events}}{d\sqrt{s}} = \frac{1}{2}\frac{2}{3}\frac{N_{\rm events}}{\Gamma_\tau \bar{B}}\frac{d\Gamma_{K\pi}}{d\sqrt{s}}\Delta_{\rm bin}$$

$$N_{\rm events} = 53110\,,$$

$$\Delta_{\rm bin} = 11.5\ \text{MeV}$$



Determination of physical resonance parameters

- Important: the pole! $\sqrt{s_{\rm pole}} = M_R - \frac{i}{2}\Gamma_R$

$$D(m_n,\gamma_n) \quad \equiv \quad m_n^2 - s_{\rm pole} - \kappa_n {\rm Re}\left[H_{K\pi}(s_{\rm pole})\right] - i m_n \Gamma_n(s_{\rm pole}) = 0$$

Model Parameters	Pole Positions
(m_{K^*},γ_{K^*}) [MeV]	(M_{K^*},Γ_{K^*}) [MeV]
$(943.41 \pm 0.59\,,66.72 \pm 0.87)$	$(892.0 \pm 0.9\,,46.2 \pm 0.4)$
$(m_{K^{*\prime}},\gamma_{K^{*\prime}})$ [MeV]	$(M_{K^{*\prime}},\Gamma_{K^{*\prime}})$ [MeV]
$(1374 \pm 30\,,240 \pm 100)$	$(1276^{72}_{-77}\,,198^{+61}_{-87})$

$K^*(892)$ $\quad I(J^P) = \frac{1}{2}(1^-)$

$K^*(892)$ MASS

CHARGED ONLY, PRODUCED IN τ LEPTON DECAYS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
895.47±0.20±0.74	53k	[6] EPIFANOV	07	BELL	$\tau^- \to K^0_S\pi^-\nu_\tau$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
892.0 ±0.5		[7] BOITO	10	RVUE	$\tau^- \to K^0_S\pi^-\nu_\tau$
892.0 ±0.9		[8,9] BOITO	09	RVUE	$\tau^- \to K^0_S\pi^-\nu_\tau$
895.3 ±0.2		[8,10] JAMIN	08	RVUE	$\tau^- \to K^0_S\pi^-\nu_\tau$
896.4 ±0.9	11970	[11] BONVICINI	02	CLEO	$\tau^- \to K^-\pi^0\nu_\tau$
895 ±2		[12] BARATE	99R	ALEP	$\tau^- \to K^-\pi^0\nu_\tau$

$K^*(892)$ WIDTH

CHARGED ONLY, PRODUCED IN τ LEPTON DECAYS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMEN
46.2±0.6±1.2	53k	[26] EPIFANOV	07	BELL	$\tau^- \to$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
46.5±1.1		[27] BOITO	10	RVUE	$\tau^- \to$
46.2±0.4		[28,29] BOITO	09	RVUE	$\tau^- \to$
47.5±0.4		[28,30] JAMIN	08	RVUE	$\tau^- \to$
55 ±8		[31] BARATE	99R	ALEP	$\tau^- \to$

Determination of physical resonance parameters

- Important: the pole! $\sqrt{s_{\rm pole}} = M_R - \frac{i}{2}\Gamma_R$

$$D(m_n,\gamma_n) \equiv m_n^2 - s_{\rm pole} - \kappa_n {\rm Re}\left[H_{K\pi}(s_{\rm pole})\right] - i m_n \Gamma_n(s_{\rm pole}) = 0$$

Model Parameters	Pole Positions
(m_{K^*}, γ_{K^*}) [MeV]	(M_{K^*}, Γ_{K^*}) [MeV]
$(943.41 \pm 0.59\,, 66.72 \pm 0.87)$	$(892.0 \pm 0.9\,, 46.2 \pm 0.4)$
$(m_{K^{*\prime}}, \gamma_{K^{*\prime}})$ [MeV]	$(M_{K^{*\prime}}, \Gamma_{K^{*\prime}})$ [MeV]
$(1374 \pm 30\,, 240 \pm 100)$	$(1276^{72}_{-77}\,, 198^{+61}_{-87})$

$K^*(1410)$ $\quad I(J^P) = \frac{1}{2}(1^-)$

$K^*(1410)$ MASS

VALUE (MeV)	DOCUMENT ID	TECN	CHG	COMMENT
1414±15 OUR AVERAGE	Error includes scale factor of 1.3.			
1380±21±19	ASTON 88	LASS	0	11 $K^- p \to K^- \pi^+ n$
1420± 7±10	ASTON 87	LASS	0	11 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1276^{+72}_{-77}	[1,2] BOITO 09	RVUE		$\tau^- \to K^0_S \pi^- \nu_\tau$
1367±54	BIRD 89	LASS	−	11 $K^- p \to \overline{K}^0 \pi^- p$
1474±25	BAUBILLIER 82B	HBC	0	8.25 $K^- p \to \overline{K}^0 2\pi n$
1500±30	ETKIN 80	MPS	0	6 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$

[1] From the pole position of the $K\pi$ vector form factor in the complex s-plane and using EPIFANOV 07 data.
[2] Systematic uncertainties not estimated.

$K^*(1410)$ WIDTH

VALUE (MeV)	DOCUMENT ID	TECN	CHG	COMMENT
232± 21 OUR AVERAGE	Error includes scale factor of 1.1.			
176± 52±22	ASTON 88	LASS	0	11 $K^- p \to K^- \pi^+ n$
240± 18±12	ASTON 87	LASS	0	11 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
198^{+61}_{-87}	[3,4] BOITO 09	RVUE		$\tau^- \to K^0_S \pi^- \nu_\tau$
114±101	BIRD 89	LASS	−	11 $K^- p \to \overline{K}^0 \pi^- p$
275± 65	BAUBILLIER 82B	HBC	0	8.25 $K^- p \to \overline{K}^0 2\pi n$
500±100	ETKIN 80	MPS	0	6 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$

[3] From the pole position of the $K\pi$ vector form factor in the complex s-plane and using EPIFANOV 07 data.
[4] Systematic uncertainties not estimated.

Fits to $\tau^- \to K_S\pi^-\nu_\tau$ data+restrictions from $K_{\ell 3}$

Boito, Escribano and Jamin
JHEP 1009 (2010) 031

$$\chi^2 = \chi^2_\tau + \left(\frac{\bar{B}_{K\pi} - B^{\rm exp}_{K\pi}}{\sigma^{\rm exp}_{B_{K\pi}}}\right)^2 + \left(\lambda^{\rm th}_+ - \lambda^{\rm exp}_+\right) V^{-1} \left(\lambda^{\rm th}_+ - \lambda^{\rm exp}_+\right), \quad \lambda^{\rm th,exp}_+ = \begin{pmatrix} \lambda'^{\rm th,exp}_+ \\ \lambda''^{\rm th,exp}_+ \end{pmatrix}$$

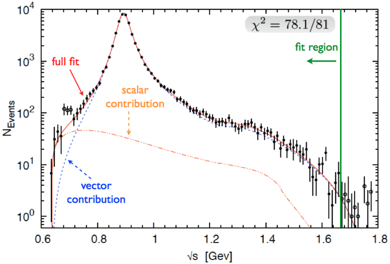
	$s_{\rm cut} = 3.24$ GeV^2	$s_{\rm cut} = 4$ GeV^2	$s_{\rm cut} = 9$ GeV^2	$s_{\rm cut} \to \infty$
$B_{K\pi}$	0.429 ± 0.009	$0.427 \pm 0.008\%$	$0.426 \pm 0.008\%$	$0.426 \pm 0.008\%$
$(B^{\rm th}_{K\pi})$	(0.426%)	(0.425%)	(0.423%)	(0.423%)
M_{K^*} [MeV]	892.04 ± 0.20	892.02 ± 0.20	892.03 ± 0.19	892.03 ± 0.19
Γ_{K^*} [MeV]	46.58 ± 0.38	46.52 ± 0.38	46.48 ± 0.38	46.48 ± 0.38
$M_{K^{*\prime}}$ [MeV]	1257^{+30}_{-45}	1268^{+25}_{-32}	1270^{+24}_{-29}	1271^{+24}_{-29}
$\Gamma_{K^{*\prime}}$ [MeV]	321^{+95}_{-76}	238^{+75}_{-57}	206^{+67}_{-50}	205^{+67}_{-50}
$\gamma \times 10^2$	$-8.2^{+2.2}_{-3.5}$	$-5.4^{+1.4}_{-2.0}$	$-4.4^{+1.2}_{-1.6}$	$-4.4^{+1.2}_{-1.6}$
$\lambda'_+ \times 10^3$	25.43 ± 0.30	25.49 ± 0.30	25.55 ± 0.30	25.55 ± 0.30
$\lambda''_+ \times 10^4$	12.31 ± 0.10	12.20 ± 0.10	12.12 ± 0.10	12.12 ± 0.10
χ^2/n.d.f.	77.9/81	78.1 /81	79.0 /81	79.1/81

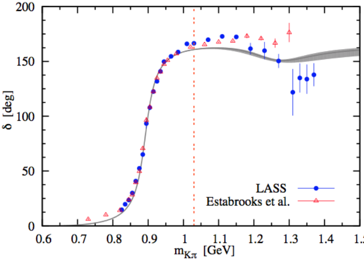
$\lambda'^{\rm ,exp}_+ = 24.9(1.1) \times 10^{-3}$

$\lambda''^{\rm ,exp}_+ = 16(5) \times 10^{-4}$

$\rho_{\lambda'_+,\lambda''_+} = -0.95$

exp. average: NA48, KLOE, ISTRA+KTeV (Antonelli et.al. 1005.2323)

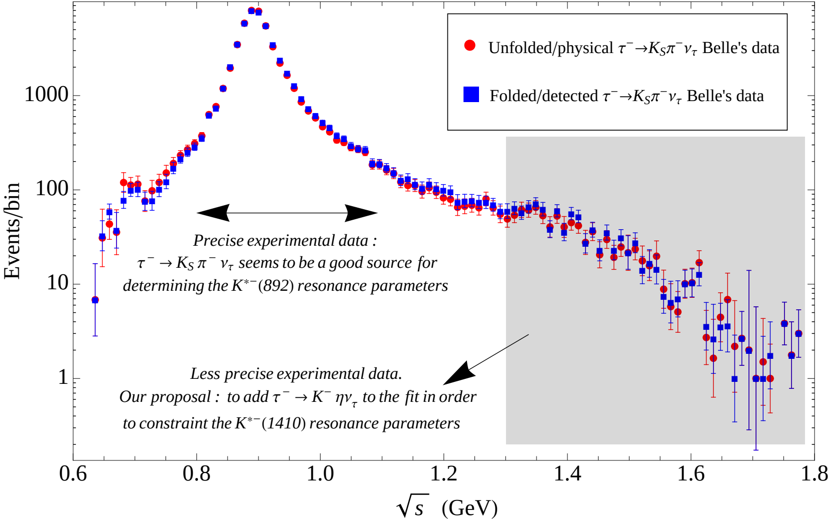




Combined analysis of $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$

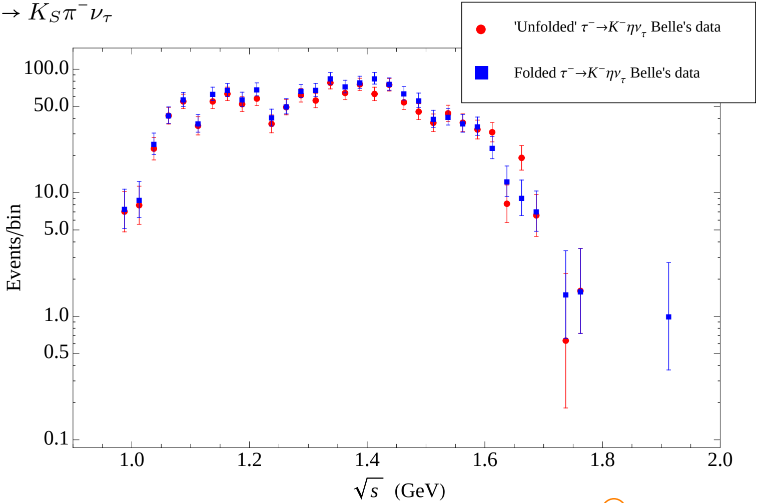
- Reason for a simultaneous fit to $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ Belle data

(Epifanov et. al. Phys. Lett. **B 654 (2007) 65**)



Combined analysis of $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$

- Unfolding $\tau^- \to K^-\eta\nu_\tau$ Belle's data through an "unfolding" function from $\tau^- \to K_S\pi^-\nu_\tau$



∘ Experimentalist: To provide unfolded data would be really useful ☺

∘ Theorists: To provide theoretical models to be fitted by experimentalists

- We relate the experimental data with the differential decay distribution from theory through

$$\frac{dN_{events}}{d\sqrt{s}} = N_{events}\Delta_{bin}\frac{1}{\Gamma_\tau BR(\tau\to P^-P^0\nu_\tau)}\frac{d\Gamma\left(\tau^-\to P^-P^0\nu_\tau\right)}{d\sqrt{s}}$$

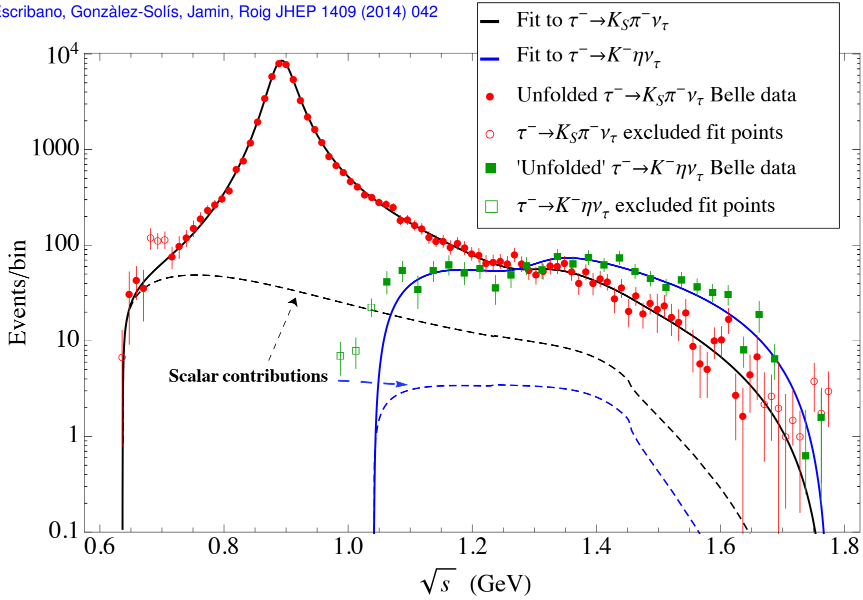
$$\frac{d\Gamma\left(\tau^-\to P^-P^0\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{32\pi^3 s}S_{EW}|V_{us}F_+^{P^-P^0}(0)|^2\left(1-\frac{s}{M_\tau^2}\right)^2$$
$$\times\left\{\left(1+\frac{2s}{M_\tau^2}\right)q^3_{P^-P^0}(s)|\widetilde{F}_+^{P^-P^0}(s)|^2+\frac{3\Delta^2_{P^-P^0}}{4s}q_{P^-P^0}(s)|\widetilde{F}_0^{P^-P^0}(s)|^2\right\}$$

- $P^-P^0 = K_S\pi^- \to BR^{Belle}_{exp} = 0.404\%$ $\quad N_{events} = 53113$ $\quad \Delta_{bin} = 0.0115$ GeV/bin
- $P^-P^0 = K^-\eta \to BR^{Belle}_{exp} = 1.58\cdot 10^{-4}$ $\quad N_{events} = 1271$ $\quad \Delta_{bin} = 0.025$ GeV/bin
- $\Gamma_\tau = 2.265\cdot 10^{-12}$
- Function minimised in our fit

$$\chi^2 = \sum_{bin}\left(\frac{\mathcal{N}^{th}-\mathcal{N}^{exp}}{\sigma_{\mathcal{N}^{exp}}}\right)^2 + \sum_{K_S\pi^-,K^-\eta}\left(\frac{\bar{B}^{th}-\bar{B}^{exp}}{\sigma_{\bar{B}^{exp}}}\right)^2$$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

Escribano, Gonzàlez-Solís, Jamin, Roig JHEP 1409 (2014) 042



Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Different choices regarding linear slopes and resonance mixing parameters ($s_{cut} = 4$ GeV^2)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{th}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
χ^2/n.d.f.	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Reference fit results obtained for different values of s_{cut}

Parameter \ $s_{\rm cut}$ (GeV^2)	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
χ^2/n.d.f.	$105.8/105$	$108.1/105$	$111.0/105$	$111.1/105$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Central results including the largest variation of s_{cut}

$M_{K^{*-}(892)} = 892.03 \pm 0.19$ MeV
$\Gamma_{K^{*-}(892)} = 46.18 \pm 0.44$ MeV
} no gain

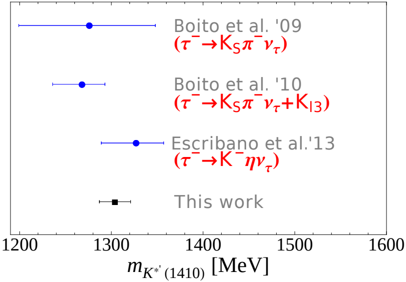
$M_{K^{*-}(1410)} = 1305^{+16}_{-18}$ MeV
$\Gamma_{K^{*-}(1410)} = 168^{+65}_{-59}$ MeV
} improvement

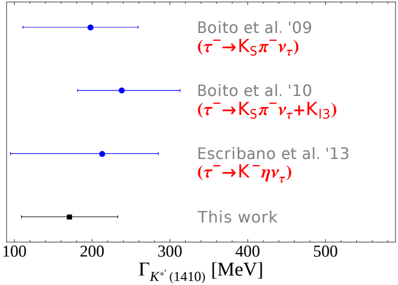
$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$

$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$

$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$

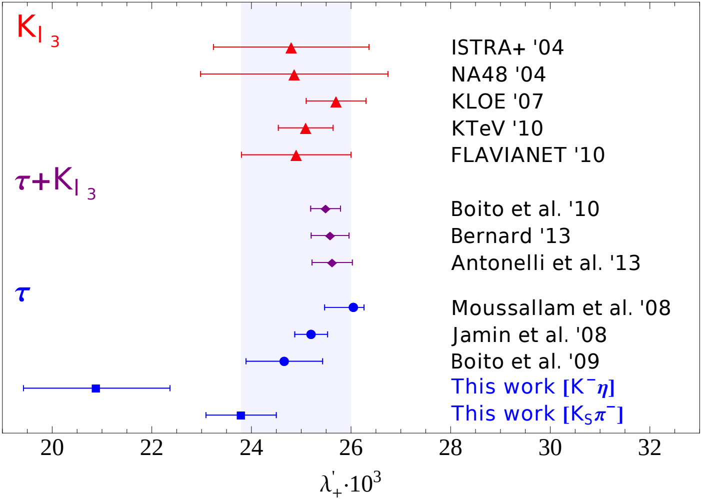
$\chi^2/d.o.f = 108.1/105 = 1.03$





Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

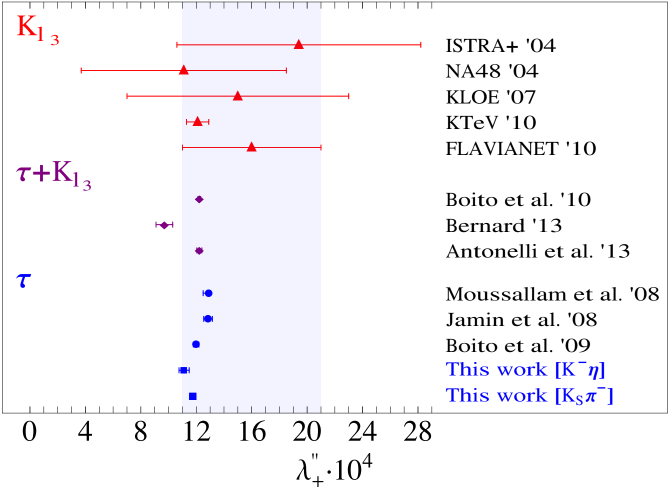
$$\left.\begin{aligned} \lambda'_{K\pi} &= (23.9 \pm 0.9) \cdot 10^{-3} \\ \lambda'_{K\eta} &= (20.9 \pm 2.7) \cdot 10^{-3} \end{aligned}\right\} \text{isospin violation?}$$



Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

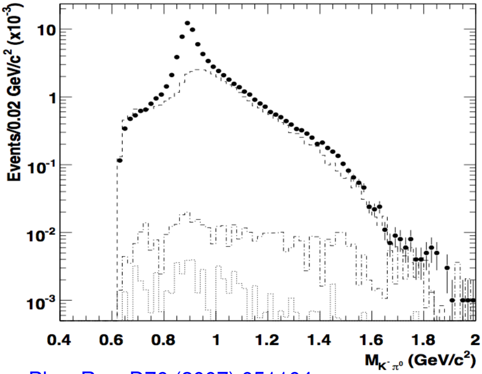
$\lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4}$

$\lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4}$



Prospects of improvement

- Call 1: to release $\tau^- \to K^-\eta\nu_\tau$ acceptance corrected
- Call 2: to provide $\tau^- \to K^-\pi^0\nu_\tau$ data (acceptance corrected)



Phys.Rev. D76 (2007) 051104

- Call 3: $K^-\eta \to K^-\eta$ scattering $\to K^-\eta$ phase shift

Applications of the $K\pi$ Form Factors

- Dispersive representation of the $K\pi$ form factor suited to describe both $\tau \to K\pi\nu_\tau$ and $K_{\ell 3}$ decays

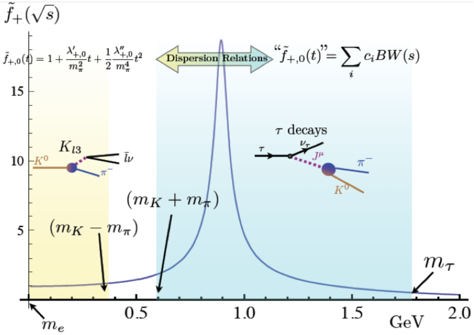


figure from D. Boito

- $K_{\ell 3}$ decays are the main route towards the determination of $|V_{us}|^2$

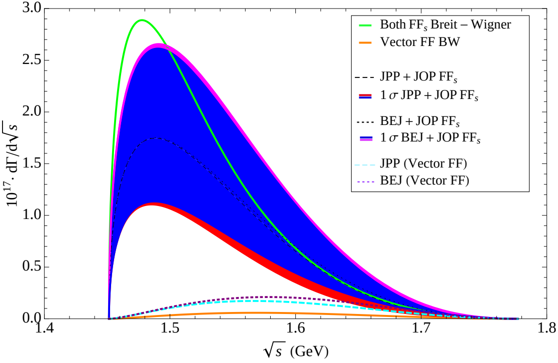
$$\Gamma_{K_{\ell 3}} \propto |V_{us}|^2 |F_+(0)|^2 I_{K_{\ell 3}} \,, \quad I_{K_{\ell 3}} = \frac{1}{m_K^8}\int dt(p.s.)\left[\widetilde{F}_+(t)^2 + \eta(t,m_\ell)\widetilde{F}_0(t)^2\right]$$

$|F_+(0)|$ ↓ ChPT, lattice

$\widetilde{F}_+(t)$, $\widetilde{F}_0(t)$ ↓ RChT+ dispersion relations

Predictions for the $\tau^- \to K^-\eta'\nu_\tau$ decay

- Decay dominated by the scalar Form Factor ($\sim 90\%$ of the BR)



Source	Branchig ratio
Breit-Wigner	$(1.45^{+3.80}_{-0.87}) \cdot 10^{-6}$
Exponential representation	$(1.00^{+0.37}_{-0.29}) \cdot 10^{-6}$
Dispersion relation	$(1.03^{+0.37}_{-0.29}) \cdot 10^{-6}$
Experimental bound	$< 2.4 \cdot 10^{-6}$ at 90% C.L.

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays

Motivations

- $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ belong to the second-class current processes unobserved in Nature so far (Weinberg '58)

$$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y} C|X\rangle = (-1)^I C|X\rangle$$
$$G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \quad \neq \quad G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$$

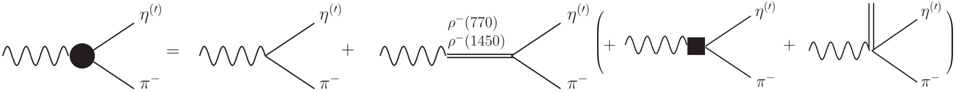
- It is an isospin violating process $(\underline{m_u \neq m_d}, e \neq 0)$
- Sensitive to the intermediate vector and scalar resonances $(\rho, \rho', a_0, a_0' ...)$ coupled to the $\bar{u}d$ operator

Purposes

- To describe the participating hadronic form factors
  - Resonance Chiral Theory (this talk)
  - Dispersive parametrization (Moussallam'14)
- To predict the decay spectra and to estimate the branching ratios
- To stimulate people from B-factories (Belle-II) to measure these decays

$\pi\eta^{(\prime)}$ form factors in resonance chiral theory

- The Vector contribution current occurs via π^0-η-η' mixing

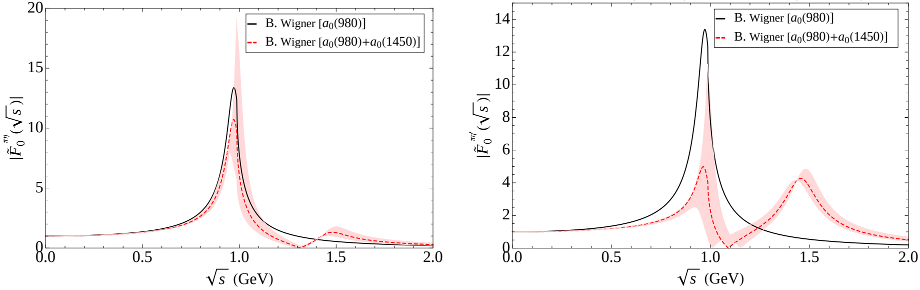


$$\begin{pmatrix} F_V^{\pi^-\eta}(s) \\ F_V^{\pi^-\eta'}(s) \end{pmatrix} = \underbrace{\begin{pmatrix} \varepsilon_{\pi\eta} \\ \varepsilon_{\pi\eta'} \end{pmatrix}}_{\text{suppresion}} \times \left[1 + \sum_{V=\rho,\rho',\rho''} \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} \right] = \begin{pmatrix} \varepsilon_{\pi\eta} \\ \varepsilon_{\pi\eta'} \end{pmatrix} \times F_V^{\pi}(s)$$

cancel each other

$F_V^{\pi}(s)$ ↓ Pion Vector Form Factor
Interpolating function from Belle data

- The Scalar contribution Breit-Wigner with $a_0(980)$ and $a_0(1450)$



Scalar Form Factor: Omnès integral

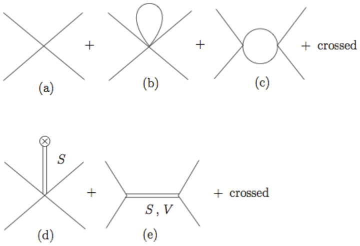
- Analyticity and elastic unitarity through the Omnès solution

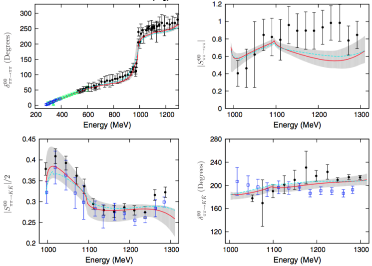
$$F_0^{\pi^-\eta^{(\prime)}}(s) = P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right] = P(s)\Omega(s)$$

- Elastic unitarity: Form factor phase= $\delta_{\pi^-\eta^{(\prime)}}$ $2\to 2$ elastic scattering

$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan\frac{\mathrm{Im}\,t_{1,0}(s)}{\mathrm{Re}\,t_{1,0}(s)}, \quad t_{1,0}(s) = \frac{N_{1,0}(s)}{1+g(s)N_{1,0}(s)} = \frac{N(s)}{D(s)}$$

- $N_{1,0}$: $U(3)\times U(3)$ amplitudes in $R\chi$T (Guo-Oller: Phys.Rev. D84 (2011) 034005)





$$\tilde{c}_d = c_d/\sqrt{3}$$
$$\tilde{c}_m = c_m/\sqrt{3}$$
$$c_d = 19.8^{+2.0}_{-5.2}\ \text{MeV}$$
$$c_m = 41.9^{+3.9}_{-9.2}\ \text{MeV}$$
$$M_{a_0,S_8} = 1397^{+73}_{-61}\ \text{MeV}$$
$$M_{S_1} = 1100^{+30}_{-63}\ \text{MeV}$$

Scalar Form Factor: Omnès integral

- Analyticity and elastic unitarity through the Omnès solution

$$F_0^{\pi^-\eta^{(\prime)}}(s) = P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right] = P(s)\Omega(s)$$

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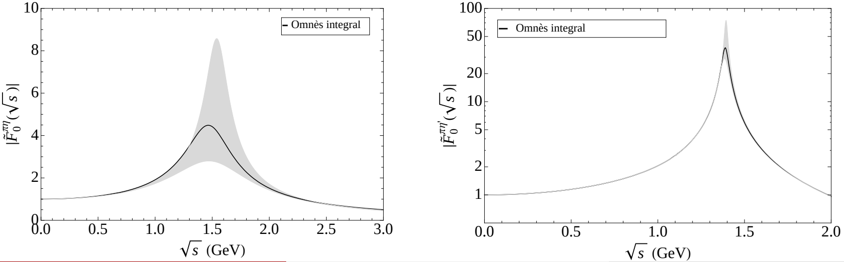
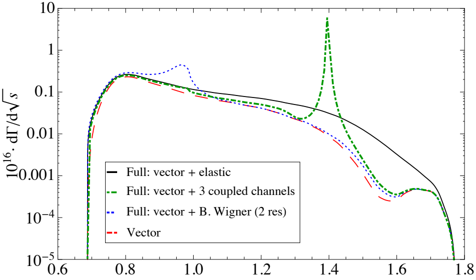
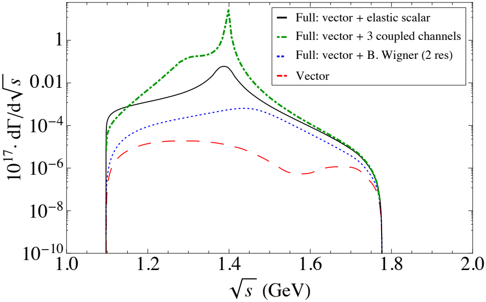
$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan\frac{\mathrm{Im}t_{1,0}(s)}{\mathrm{Re}t_{1,0}(s)}\,, \quad t_{1,0}(s) = \frac{N_{1,0}(s)}{1+g(s)N_{1,0}(s)} = \frac{N(s)}{D(s)}$$

- $N_{1,0}$: $U(3)\times U(3)$ amplitudes in $R\chi$T (Guo-Oller: Phys.Rev. D84 (2011) 034005)



$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$: Invariant mass distribution and Branching Ratio





- $\tau^- \to \pi^- \eta \nu_\tau$
  - Theory predictions: $BR \sim 1 \times 10^{-5}$ (Escribano'16, Moussallam'14)
  - BaBar: $BR < 9.9 \cdot 10^{-5}$ 95% CL , Belle: $BR < 7.3 \cdot 10^{-5}$ 90% CL
- $\tau^- \to \pi^- \eta' \nu_\tau$
  - Theory predictions: $BR \sim [10^{-7}, 10^{-6}]$ (Escribano'16)
  - BaBar: $BR < 4 \cdot 10^{-6}$ 90% CL

Challenging for Belle II

$\tau \to \pi^-\pi^0\nu_\tau$

- Governed by the pion vector form factor $F_V^\pi(s)$
- Enters the description on many physical observables
- Interest: $\sim 65\%$ of $(g-2)_\mu$, LFV hadronic tau decays etc.
- Extraction of the $\rho(770)$ meson parameters
- Sensitive to the $\rho(1450)$ and $\rho(1700)$ resonances
- Extensively studied object
  - Gounaris-Sakurari (1968)
  - Guerrero and Pich, PL B412 (1997) 382
  - Pich and Portolés, PRD 63, 093005 (2001) 382
  - Hanhart, PL B715, 170 (2012) (2012)
  - Dumm and Roig, EPJC 73 2528 (2013) (2013)
  - Celis, Cirigliano and Passemar, PRD 89, 013008 (2014)

Dispersive representation

Celis, Cirigliano and Passemar, PRD 89, 013008 (2014)

- Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right],$$

- Form Factor phase: $\tan\delta(s) = \mathrm{Im}\widetilde{F}_V(s)/\mathrm{Re}\widetilde{F}_V(s)$

$$\tilde{F}_V(s) = \frac{\tilde{M}_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})s}{\tilde{M}_\rho^2 - s + \kappa_\rho(s)\mathrm{Re}[A_\pi(s) + \frac{1}{2}A_K(s)] - i\tilde{M}_\rho\tilde{\Gamma}_\rho(s)} - \frac{\alpha' e^{i\phi'} s}{D(\tilde{M}_{\rho'}, \tilde{\Gamma}_{\rho'})} - \frac{\alpha'' e^{i\phi''} s}{D(\tilde{M}_{\rho''}, \tilde{\Gamma}_{\rho''})}, \quad (6)$$

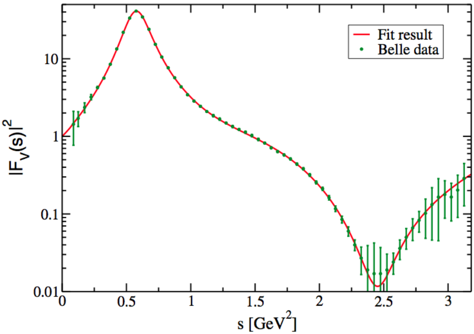
with

$$D(\tilde{M}_R, \tilde{\Gamma}_R) = \tilde{M}_R^2 - s + \kappa_R(s)\mathrm{Re}A_\pi(s) - i\tilde{M}_R\tilde{\Gamma}_R(s). \quad (7)$$

In this equation $\tilde{M}_R$ and $\tilde{\Gamma}_R$ are model parameters. $\tilde{\Gamma}_R$ and κ_R are given by

$$\tilde{\Gamma}_R(s) = \tilde{\Gamma}_R \frac{s}{\tilde{M}_R^2} \frac{(\sigma_\pi^3(s) + 1/2\sigma_K^3(s))}{(\sigma_\pi^3(\tilde{M}_R^2) + 1/2\sigma_K^3(\tilde{M}_R^2))},$$
$$\kappa_R(s) = \frac{\tilde{\Gamma}_R}{\tilde{M}_R} \frac{s}{\pi(\sigma_\pi^3(\tilde{M}_R^2) + 1/2\sigma_K^3(\tilde{M}_R^2))}, \quad (8)$$

Fujikawa et. al. PRD78 072006 (Belle)



Determination of physical resonance parameters

- Important: the pole! $\sqrt{s_{\text{pole}}} = M_R - \frac{i}{2}\Gamma_R$

Source	Model Parameters	Pole Positions
	(m_ρ, γ_ρ) [MeV]	(M_ρ, Γ_ρ) [MeV]
Dumm'13	$(843.0 \pm 0.50\,, 206.0 \pm 0.1)$	$(759 \pm 2\,, 146 \pm 6)$
	$(m_{\rho'}, \gamma_{\rho'})$ [MeV]	$(M_{\rho'}, \Gamma_{\rho'})$ [MeV]
Celis'14/Dumm'13	$(1497 \pm 7\,, 785 \pm 51)$	$(1440 \pm 80\,, 320 \pm 80)$
	$(m_{\rho''}, \gamma_{\rho''})$ [MeV]	$(M_{\rho''}, \Gamma_{\rho''})$ [MeV]
Celis'14/Dumm'13	$(1685 \pm 30\,, 800 \pm 31)$	$(1720 \pm 90\,, 180 \pm 90)$

$\rho(1450)$ $\qquad I^G(J^{PC}) = 1^+(1^{--})$

See our mini-review under the $\rho(1700)$.

$\pi\pi$ MODE

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$1350 \pm 20 {}^{+20}_{-30}$	63.5k	[1] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^- p$
1493 ± 15		[2] LEES	12G	BABR	$e^+e^- \to \pi^+\pi^-\gamma$
$1446 \pm 7 \pm 28$	5.4M	[3,4] FUJIKAWA	08	BELL	$\tau^- \to \pi^-\pi^0\nu_\tau$
1328 ± 15		[5] SCHAEL	05C	ALEP	$\tau^- \to \pi^-\pi^0\nu_\tau$
1406 ± 15	87k	[3,6] ANDERSON	00A	CLE2	$\tau^- \to \pi^-\pi^0\nu_\tau$
~ 1368		[7] ABELE	99C	CBAR	$0.0\ \bar{p}d \to \pi^+\pi^-\pi^- p$
1348 ± 33		BERTIN	98	OBLX	$0.05–0.405\ \bar{n}p \to 2\pi^+\pi^-$
1411 ± 14		[8] ABELE	97	CBAR	$\bar{p}n \to \pi^-\pi^0\pi^0$
$1370 {}^{+90}_{-70}$		ACHASOV	97	RVUE	$e^+e^- \to \pi^+\pi^-$
1359 ± 40		[6] BERTIN	97C	OBLX	$0.0\ \bar{p}p \to \pi^+\pi^-\pi^0$
1282 ± 37		BERTIN	97D	OBLX	$0.05\ \bar{p}p \to 2\pi^+2\pi^-$
1424 ± 25		BISELLO	89	DM2	$e^+e^- \to \pi^+\pi^-$
1265.5 ± 75.3		DUBNICKA	89	RVUE	$e^+e^- \to \pi^+\pi^-$
1292 ± 17		[9] KURDADZE	83	OLYA	$0.64–1.4\ e^+e^- \to \pi^+\pi^-$

$\rho(1450)$ WIDTH

VALUE (MeV)	DOCUMENT ID
400±60 OUR ESTIMATE This is only an educated guess; the error given is the error on the average of the published values.	

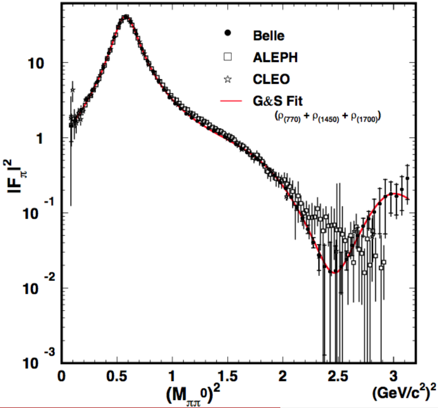
$\pi\pi$ MODE

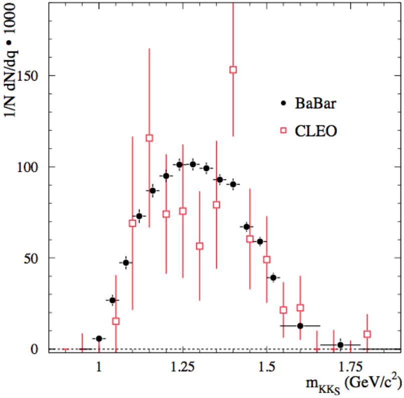
• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$460 \pm 30 {}^{+40}_{-45}$	63.5k	[1] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^- p$
427 ± 31		[2] LEES	12G	BABR	$e^+e^- \to \pi^+\pi^-\gamma$
$434 \pm 16 \pm 60$	5.4M	[3,4] FUJIKAWA	08	BELL	$\tau^- \to \pi^-\pi^0\nu_\tau$
468 ± 41		[5] SCHAEL	05C	ALEP	$\tau^- \to \pi^-\pi^0\nu_\tau$
455 ± 41	87k	[3,6] ANDERSON	00A	CLE2	$\tau^- \to \pi^-\pi^0\nu_\tau$
~ 374		[7] ABELE	99C	CBAR	$0.0\ \bar{p}d \to \pi^+\pi^-\pi^- p$
275 ± 10		BERTIN	98	OBLX	$0.05–0.405\ \bar{n}p \to \pi^+\pi^+\pi^-$
343 ± 20		[8] ABELE	97	CBAR	$\bar{p}n \to \pi^-\pi^0\pi^0$
310 ± 40		[6] BERTIN	97C	OBLX	$0.0\ \bar{p}p \to \pi^+\pi^-\pi^0$
236 ± 36		BERTIN	97D	OBLX	$0.05\ \bar{p}p \to 2\pi^+2\pi^-$
269 ± 31		BISELLO	89	DM2	$e^+e^- \to \pi^+\pi^-$
391 ± 70		DUBNICKA	89	RVUE	$e^+e^- \to \pi^+\pi^-$
218 ± 46		[9] KURDADZE	83	OLYA	$0.64–1.4\ e^+e^- \to \pi^+\pi^-$

Combined analysis of the $\tau \to \pi^-\pi^0\nu_\tau$ and $\tau \to K^-K_S\nu_\tau$

- To determine the $\rho(1450)$ and $\rho(1700)$ mass and width with improved precision (SG-S and Roig, in preparation)
- $\tau^- \to \pi^-\pi^0\nu_\tau$ measurement by Belle (2008) (0805.3773)
- Measurement of the $\tau^- \to K^-K_S\nu_\tau$ decay by BaBar (1806.10280)





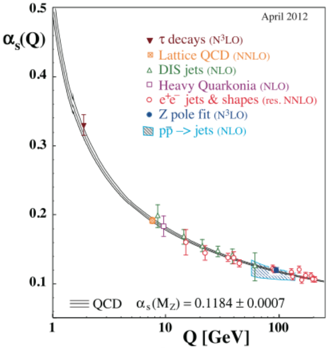
Outlook

- Tau physics is a very rich field to test QCD and EW
- Important experimental activities: Belle (II), BaBar, LHCb, BESIII
- τ decays into two mesons are a privileged laboratory to access the non-perturbative regime of QCD
- Form Factors from dispersion relations with subtractions
  - Extraction of the $K^*(892)$ parameters from a fit to $\tau \to K_S \pi^- \nu_\tau$
  - Extraction of the $K^*(1410)$ from $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$
  - Predictions $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ are challenging for Belle-II
  - $F_V^\pi(s)$: important for testing QCD dynamics and the SM and NP
  - $\tau^- \to K_S K^- \nu_\tau$: extraction of the $\rho(1450)$ and $\rho(1700)$ parameters
- A lot of interesting physics to be done in the tau sector

Back-up

Quantum Chromodynamics

- Hadrons interact strongly: could perturbation theory be applied to describe strong interactions?
- **Quantum Chromodynamics** is a renormalizable QFT but
  1. with **asymptotic freedom**: it looks like QED, but only at very high energies
  2. with **confinement**: at low energies the gluons bind the quarks together



Chiral Perturbation Theory

- **Effective Field Theory** of QCD at low energies
- **Mesons** as explicit **degrees of freedom**

$$U(\Phi) = \exp\left(i\sqrt{2}\Phi/f\right), \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

- Expansion organized in terms of the momentum and quark masses

$$\mathcal{L}_{ChPT} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots, \quad \mathcal{L}_2 = \frac{f^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle + \frac{f^2}{4}\langle U^\dagger \chi + \chi^\dagger U \rangle$$

- Valid up to the first resonance: $\sim \rho$ mass (0.7 GeV)
- Large-N$_C$: include the η_1 singlet
- **Resonance Chiral Theory**: To test low-and intermediate-energies

Resonance Chiral Theory

- Mesons and resonances as explicit degrees of freedom
- To explore low-and intermediate-energies

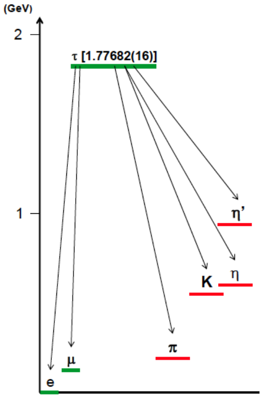
$$\mathcal{L}_{R\chi T} = \mathcal{L}_2 + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_S + \mathcal{L}_P$$

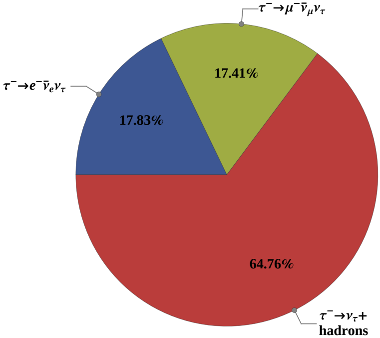
$$\mathcal{L}_V = i\frac{G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + \dots$$

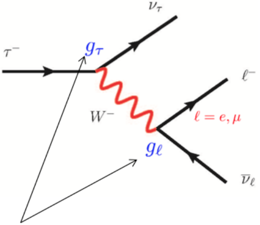
$$u^\mu = iu^\dagger D^\mu U u^\dagger$$

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & \rho^+ & K^{\star +} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \end{pmatrix}$$

Decay Spectrum of the τ lepton







Leptonic decays

Charged current universality $g_\tau = g_\mu = g_e$

Hadronic Matrix Element

- Taking the divergence we obtain on the L.H.S

$$\langle 0|\partial_\mu(\bar{s}\gamma^\mu u)|K^+\eta^{(\prime)}\rangle = i(m_s - m_u)\langle 0|\bar{s}u|K^+\eta^{(\prime)}\rangle = i\Delta_{K\pi}C^S_{K^-\eta^{(\prime)}}F_0^{K^-\eta^{(\prime)}}(s) \quad (1)$$

where $\Delta_{PQ} = M_P^2 - M_Q^2$, $C^S_{K^-\eta} = 1/\sqrt{6}$, $C^S_{K^-\eta'} = 2/\sqrt{3}$

- on the R.H.S (vector current not conserved)

$$iq_\mu\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle = iC^V_{K\eta^{(\prime)}}\left[(m^2_{\eta^{(\prime)}} - m^2_{K^-})F_+^{K^-\eta^{(\prime)}}(s) - sF_-^{K^-\eta^{(\prime)}}(s)\right] \quad (2)$$

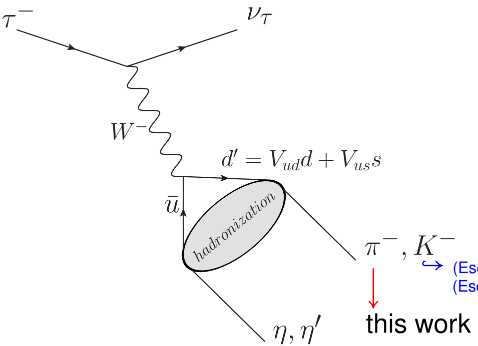
- Equating eqs. (1,2) allows us to relate $F_-^{K^-\eta^{(\prime)}}(s)$ with $F_0^{K^-\eta^{(\prime)}}(s)$ as

$$F_-^{K^-\eta^{(\prime)}}(s) = -\frac{\Delta_{K^-\eta^{(\prime)}}}{s}\left[\frac{C^S_{K\eta^{(\prime)}}}{C^V_{K\eta^{(\prime)}}}\frac{\Delta_{K\pi}}{\Delta_{K^-\eta^{(\prime)}}}F_0^{K^-\eta^{(\prime)}}(s) + F_+^{K^-\eta^{(\prime)}}(s)\right] \quad (3)$$

- The hadronic matrix element finally reads ($q^\mu = (p_{\eta^{(\prime)}} + p_{K^-})^\mu$+ and $q^2 = s$)

$$\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle =$$
$$\left[(p_{\eta^{(\prime)}} - p_K)^\mu + \frac{\Delta_{K^-\eta^{(\prime)}}}{s}q^\mu\right]C^V_{K\eta^{(\prime)}}F_+^{K^-\eta^{(\prime)}}(s) + \frac{\Delta_{K\pi}}{s}q^\mu C^S_{K\eta^{(\prime)}}F_0^{K^-\eta^{(\prime)}}(s) \quad (4)$$

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$: Decay amplitude



↪ (Escribano, González-Solís and Roig JHEP 1310 (2013) 039)
(Escribano, González-Solís, Jamin and Roig JHEP 1409 (2014) 042)

The decays proceed through the not yet evidenced second class current:

$$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y} C|X\rangle = (-1)^I C|X\rangle$$

$$G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \quad \neq \quad G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$$

G-Parity violation

$$\mathcal{M} \overset{q^2 << M_W^2}{=} \frac{G_F}{\sqrt{2}} V_{ud} \bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle \pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu(1 \underbrace{-\gamma^5}_{0^-,1^+ \nrightarrow 0^+,1^-})u|0\rangle$$

The hadronic matrix element is generally parametrized as

$$\langle \pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = C^V_{\pi^-\eta^{(\prime)}}\left[(p_{\eta^{(\prime)}} - p_{\pi^-})^\mu F_+^{\pi^-\eta^{(\prime)}}(s) - (p_{\eta^{(\prime)}} + p_{\pi^-})^\mu F_-^{\pi^-\eta^{(\prime)}}(s)\right]$$

Predictions for the $\tau^- \to K^-\eta^{(\prime)}\nu_\tau$ decays

- $\tau^- \to K_S\pi^-\nu_\tau$ Fit results (Boito-Escribano-Jamin Eur.Phys.J. C59 (2009))

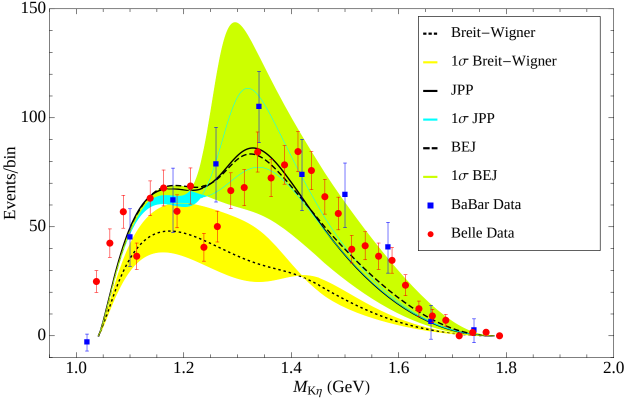
	$s_{\rm cut} = 3.24$ GeV^2	$s_{\rm cut} = 4$ GeV^2	$s_{\rm cut} = 9$ GeV^2	$s_{\rm cut} \to \infty$
m_{K^*} [MeV]	943.32 ± 0.59	943.41 ± 0.58	943.48 ± 0.57	943.49 ± 0.57
γ_{K^*} [MeV]	66.61 ± 0.88	66.72 ± 0.86	66.82 ± 0.85	66.82 ± 0.85
$m_{K'^*}$ [MeV]	1407 ± 44	1374 ± 30	1362 ± 26	1362 ± 26
$\gamma_{K'^*}$ [MeV]	325 ± 149	240 ± 100	216 ± 86	215 ± 86
$\gamma \times 10^2$	-5.2 ± 2.0	-3.9 ± 1.5	-3.5 ± 1.3	-3.5 ± 1.3
$\lambda'_+ \times 10^3$	24.31 ± 0.74	24.66 ± 0.69	24.94 ± 0.68	24.96 ± 0.67
$\lambda''_+ \times 10^4$	12.04 ± 0.20	11.99 ± 0.19	11.96 ± 0.19	11.96 ± 0.19
χ^2/n.d.f.	74.2/79	75.7/79	77.2/79	77.3/79

- Our $K\pi$ system is $K^-\pi^0$ instead of $K_S\pi^-$
- Mass difference (~ 10 MeV) strongly correlated with λ'_+ and λ''_+
- No $\tau^- \to K^-\pi^0\nu_\tau$ data available. We fit $\tau^- \to K_S\pi^-\nu_\tau$ data using $K^-\pi^0$ masses

Parameter	Best fit with $K^-\pi^0$ masses	Best fit
$\lambda'_+ \times 10^3$	22.2 ± 0.9	24.7 ± 0.8
$\lambda''_+ \times 10^4$	10.3 ± 0.2	12.0 ± 0.2
$M_{K^\star}$ (MeV)	892.1 ± 0.6	892.0 ± 0.9
$\Gamma_{K^\star}$ (MeV)	46.2 ± 0.5	46.2 ± 0.4
$M_{K^{\star\prime}}$ (GeV)	1.28 ± 0.07	1.28 ± 0.07
$\Gamma_{K^{\star\prime}}$ (GeV)	$0.16^{+0.10}_{-0.07}$	$0.20^{+0.06}_{-0.09}$
γ	-0.03 ± 0.02	-0.04 ± 0.02

Predictions for the $\tau^- \to K^-\eta\nu_\tau$ decay

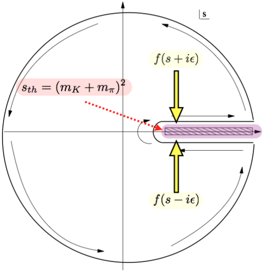
- Decay dominated by the vector Form Factor ($\sim 96\%$ of the BR)



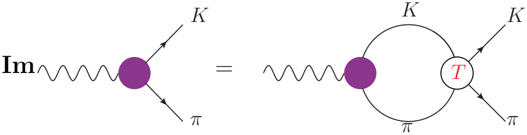
Source	Branchig ratio	χ^2/dof
Breit-Wigner	$\left(0.78^{+0.17}_{-0.10}\right)\cdot 10^{-4}$	8.3
Exponential representation	$\left(1.47^{+0.14}_{-0.08}\right)\cdot 10^{-4}$	1.9
Dispersion relation	$(1.49 \pm 0.05)\cdot 10^{-4}$	1.5
Experimental value	$(1.52 \pm 0.08)\cdot 10^{-4}$	-

Vector Form Factor: Dispersive representation

- Analyticity and elastic unitarity through a dispersion relation



$$F_+^{K\pi}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}F_+^{K\pi}(s')}{s'-s-i\varepsilon}$$



$$\mathrm{Im}F_+^{K\pi}(s) = \sigma_{K\pi}(s)F_+^{K\pi}T^*(s) = F_+^{K\pi}\sin\delta^{K\pi}(s)e^{-i\delta^{K\pi}(s)}$$

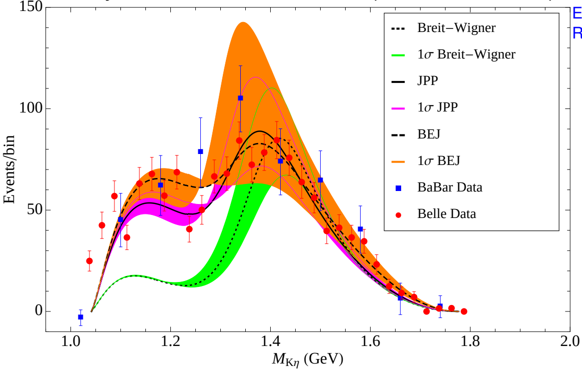
- Watson theorem: phase of $F_+^{K\pi}(s)$ is $\delta^{K\pi}(s)$ in the elastic approx.

- Omnès solution (Omnès '58)

$$F_+^{K\pi}(s) = P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta^{K\pi}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right]$$

Fits to the $\tau^- \to K^-\eta\nu_\tau$ BaBar and Belle data

- Decay dominated by the vector Form Factor ($\sim 96\%$ of the BR)



Escribano, Gonzàlez-Solís, Roig, JHEP 1310 (2013) 039

Source	Branchig ratio$\cdot 10^4$	χ^2/dof	$K^\star(1410)$ Mass	$K^\star(1410)$ Width	γ
Breit-Wigner	$\left(0.96^{+0.21}_{-0.15}\right)$	5.0			-0.174 ± 0.007
Exponential representation	(1.42 ± 0.04)	1.4	1332^{+16}_{-18} MeV	220^{+26}_{-24} MeV	$-0.078^{+0.012}_{-0.014}$
Dispersion relation	(1.55 ± 0.08)	0.8	1327^{+30}_{-38} MeV	213^{+72}_{-118} MeV	$-0.051^{+0.012}_{-0.036}$
Experimental value	(1.52 ± 0.08)	-			

Channel	$K^\star(1410)$ Mass	$K^\star(1410)$ Width	γ
$K\pi$ mode	1277^{+35}_{-41} MeV	218^{+95}_{-66} MeV	$-0.049^{+0.019}_{-0.016}$
$K\eta$ mode (this work)	1330^{+27}_{-41} MeV	217^{+68}_{-122} MeV	$-0.065^{+0.025}_{-0.050}$

Scalar Form Factor

- Central unitarity relation

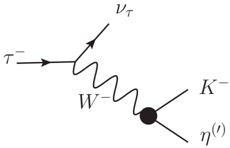
$$\mathrm{Im}F_i(s) = \sigma_j(s)F^j(s)T^{i\to j}(s)^*$$

- Coupled channels dispersion relations (Jamin, Oller, Pich Nucl.Phys.B622 (2002))

$$F_0^{K\pi}(s) = \frac{1}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\sigma_{K\pi}(s')F_0^{K\pi}(s')T^*_{K\pi\to K\pi}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{K\eta}}^{\infty} ds' \frac{\sigma_{K\eta}(s')F_0^{K\eta}(s')T^*_{K\eta\to K\pi}(s')}{s'-s-i\varepsilon}$$

$$F_0^{K\eta}(s) = \frac{1}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\sigma_{K\pi}(s')F_0^{K\pi}(s')T^*_{K\pi\to K\eta}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{K\eta}}^{\infty} ds' \frac{\sigma_{K\eta}(s')F_0^{K\eta}(s')T^*_{K\eta\to K\eta}(s')}{s'-s-i\varepsilon}$$

$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$: Amplitude and decay width



$$\stackrel{q^2\ll M_W^2}{=} \frac{G_F}{\sqrt{2}} V_{us}\bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu(1-\underbrace{\not{\gamma^5}}_{0^-,1^+\nrightarrow 0^+,1^-})u|0\rangle$$

$$\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle = \left[(p_{\eta^{(\prime)}}-p_K)^\mu + \frac{\Delta_{K^-\eta^{(\prime)}}}{s}q^\mu\right]C^V_{K\eta^{(\prime)}}F_+^{K^-\eta^{(\prime)}}(s) + \frac{\Delta_{K\pi}}{s}q^\mu C^S_{K\eta^{(\prime)}}F_0^{K^-\eta^{(\prime)}}(s$$

$$\frac{d\Gamma\left(\tau^-\to K^-\eta^{(\prime)}\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2M_\tau^3}{32\pi^3 s}S_{EW}\underbrace{|V_{us}}_{suppression}F_+^{K^-\eta^{(\prime)}}(0)|^2\left(1-\frac{s}{M_\tau^2}\right)^2$$

$$\left\{\left(1+\frac{2s}{M_\tau^2}\right)q^3_{K\eta^{(\prime)}}(s)|\widetilde{F}_+^{K^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{K\eta^{(\prime)}}}{4s}q_{K\eta^{(\prime)}}(s)|\widetilde{F}_0^{K^-\eta^{(\prime)}}(s)|^2\right\}$$

$F_+^{K^-\eta}(0) = F_+^{K^-\pi}(0)\cos\theta_P\,,\ F_+^{K^-\eta'}(0) = F_+^{K^-\pi}(0)\sin\theta_P\,,$

$\theta_P = (-13.3\pm 0.5)^\circ$
$V_{us}\cdot F_+^{K^-\pi}(0) = 0.2163(5)\,,\ K_{\ell 3}$

Scalar Form Factor: Closed expression

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i,j} \frac{(1 - s/s_{p_i})}{(1 - s/s_{z_j})} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of $D(s)^{-1} = (1 + g(s)N(s))^{-1}$

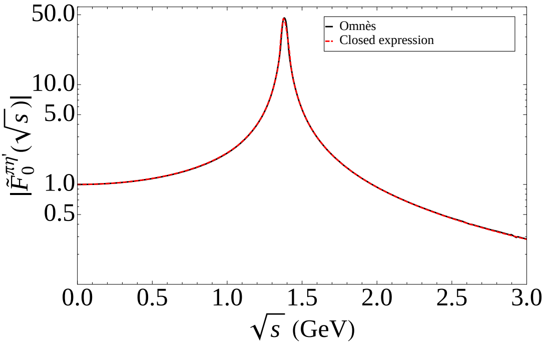
Iwamura, Kurihara, Takahashi '77
Kamal '79, Kamal, Cooper '80
Jamin, Oller, Pich '01

$s_0 = 0$

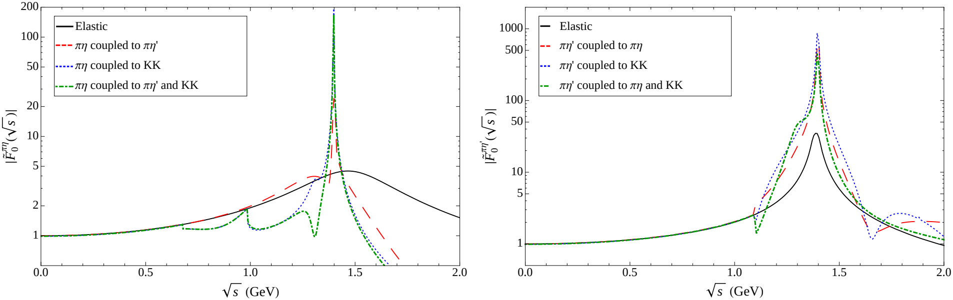
$F_0(s_0) = F_0^{\pi^-\eta',BW}(0) = 0.05$

$s_{z_1} = 1.397$ GeV

$N(s) = N_{\pi\eta'\to\pi\eta'}$



$\pi^-\eta^{(\prime)}$ Form Factors: recapitulate



- Vector Form Factor:
  - Driven by the $\pi^-\pi^0$ vector form factor
- Scalar Form Factor
  1. Breit-Wigner: with $a_0(980)$ and $a_0(1450)$ resonances
  2. Omnès solution: analyticity+elastic final state interactions
  3. Closed Form: coupled-channels

Scalar Form Factor: Coupled channels case (closed expression)

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i,j} \frac{(1 - s/s_{p_i})}{(1 - s/s_{z_j})} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of det$D(s)^{-1}$

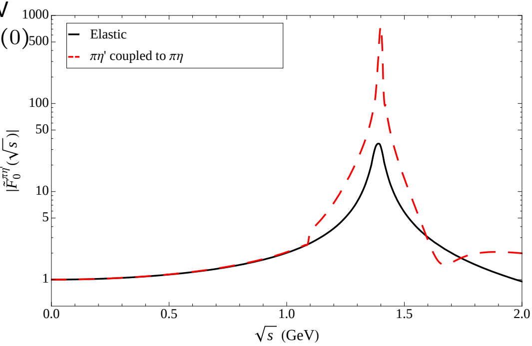
Iwamura, Kurihara, Takahashi PTF 58 (1977)
Kamal '79, Kamal, Cooper '80

$$F_0(s) = \begin{pmatrix} F_0^{\pi\eta}(s) \\ F_0^{\pi\eta'}(s) \end{pmatrix}, \quad \begin{array}{l} s_{z_1} = 1.397 \text{ GeV} \\ F_0(s_0) = F_0^{BW}(0) \end{array}$$

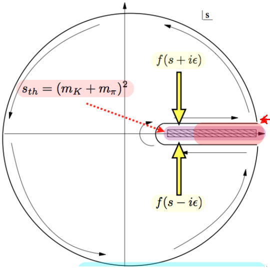
$$D(s) = \mathbb{1} + g(s) N(s),$$

$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 \\ 0 & g_{\pi\eta'} \end{pmatrix},$$

$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix},$$

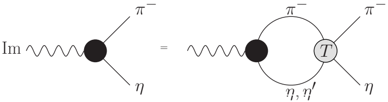


Scalar Form Factor: Coupled channels case

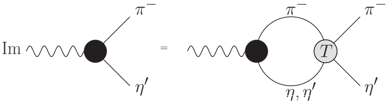


$$F_0^i(s) = \frac{1}{\pi}\sum_{j=1}^{2}\int_{s_i}^{\infty} ds' \frac{\Sigma_j(s')F_0^j(s')T_0^{i\to j}(s')^\star}{(s'-s-i\varepsilon)}$$

Other cuts $(K\bar{K}, \pi\eta'...)$



$$F_0^{\pi\eta}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta}(s')}{s'-s-i\varepsilon}$$

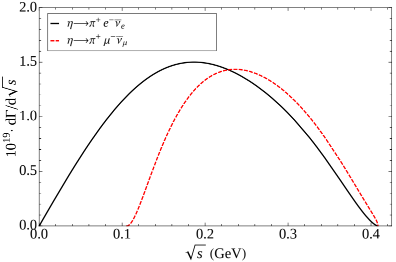


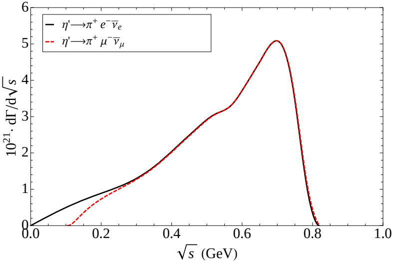
$$F_0^{\pi\eta'}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta'}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta'}(s')}{s'-s-i\varepsilon}$$

Branching Ratio estimates: $\eta^{(\prime)} \to \pi^+ \ell^- \bar{\nu}_\ell \quad (\ell = e, \mu)$

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{G_F^2 |V_{ud} F_+(0)|^2 (C_V^{\pi\eta})^2 (s - m_l^2)^2}{24\pi^3 M_\eta^3 s}$$

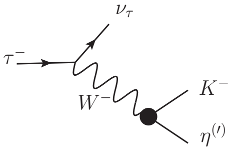
$$\left\{ (2s + m_\ell^2) q_{\pi\eta}^3 |\widetilde{F}_+(s)|^2 + \frac{3}{4s} \Delta_{\pi\eta}^2 {m_\ell}^2 q_{\pi\eta} |\widetilde{F}_0(s)|^2 \right\}$$





Decay	Descotes-Genon, Moussallam '14	Our estimate
$\eta \to \pi^+ e^- \bar{\nu}_e + c.c.$	$\sim 1.40 \cdot 10^{-13}$	$0.6 \cdot 10^{-13}$
$\eta \to \pi^+ \mu^- \bar{\nu}_\mu + c.c.$	$1.02 \cdot 10^{-13}$	$0.4 \cdot 10^{-13}$
$\eta' \to \pi^+ e^- \bar{\nu}_\mu + c.c.$		$1.7 \cdot 10^{-17}$
$\eta' \to \pi^+ \mu^- \bar{\nu}_\mu + c.c.$		$1.7 \cdot 10^{-17}$

$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$: Amplitude and decay width



$$\overset{q^2 \ll M_W^2}{=} \frac{G_F}{\sqrt{2}} V_{ud}\bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu(1-\underbrace{\cancel{\gamma^5}}_{0^-,1^+ \nrightarrow 0^+,1^-})u|0\rangle$$

$$\leftrightarrow \Delta_{PQ} = M_P^2 - M_Q^2$$

$$\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = \left[(p_{\eta^{(\prime)}} - p_\pi)^\mu + \frac{\Delta_{\pi^-\eta^{(\prime)}}}{s}q^\mu\right] C^V_{\pi\eta^{(\prime)}} F_+^{\pi\eta^{(\prime)}}(s) + \frac{\Delta^{QCD}_{K^0K^+}}{s} q^\mu C^S_{\pi^-\eta^{(\prime)}} F_0^{\pi^-\eta^{(\prime)}}(s)$$

$$\frac{d\Gamma\left(\tau^-\to\pi^-\eta^{(\prime)}\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{24\pi^3 s} S_{EW}|V_{ud}|^2 |F_+^{\pi^-\eta^{(\prime)}}(0)|^2 \left(1-\frac{s}{M_\tau^2}\right)^2$$

$$\left\{\left(1+\frac{2s}{M_\tau^2}\right) q^3_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_+^{\pi^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{\pi^-\eta^{(\prime)}}}{4s} q_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_0^{\pi^-\eta^{(\prime)}}(s)|^2\right\}$$

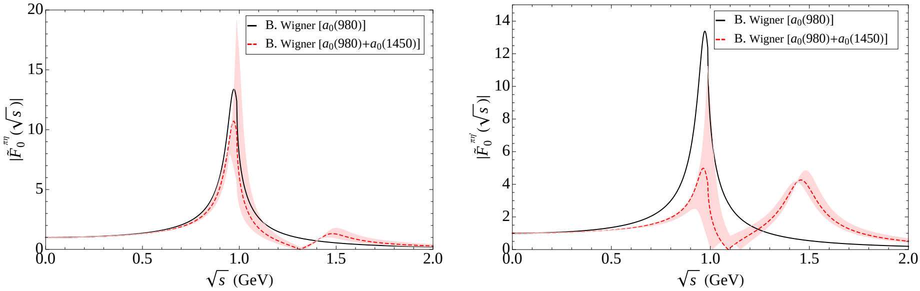
$$\widetilde{F}_{+,0}^{\pi^-\eta^{(\prime)}}(s) = \frac{F_{+,0}^{\pi^-\eta^{(\prime)}}(s)}{F_{+,0}^{\pi^-\eta^{(\prime)}}(0)}, \quad F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{C^S_{\pi^-\eta^{(\prime)}}}{C^V_{\pi^-\eta^{(\prime)}}}\frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}} F_0^{\pi^-\eta^{(\prime)}}(0)$$

Scalar Form Factor: Breit-Wigner

- Resonance Chiral Theory imposing $1/s$ fall-off for $s \to \infty$

$$F_S^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}} \frac{M_S^2+\Delta_{\pi^-\eta^{(\prime)}}}{M_S^2-s-iM_S\Gamma_S(s)}$$

- Breit-Wigner with 2 resonances: $a_0(980)$ and $a_0(1450)$



Scalar Form Factor: Coupled channels case (closed expression)

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i,j} \frac{(1 - s/s_{p_i})}{(1 - s/s_{z_j})} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of det$D(s)^{-1}$

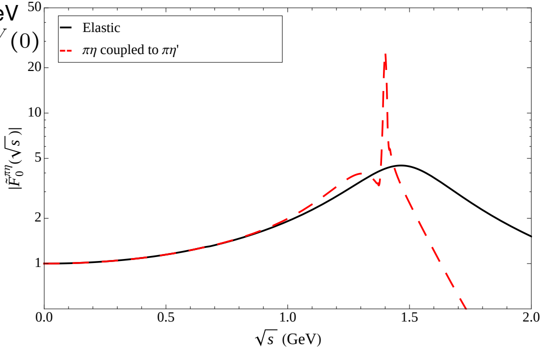
Iwamura, Kurihara, Takahashi PTF 58 (1977)
Kamal '79, Kamal, Cooper '80

$$F_0(s) = \begin{pmatrix} F_0^{\pi\eta}(s) \\ F_0^{\pi\eta'}(s) \end{pmatrix}, \quad s_{z_1} = 1.397 \text{ GeV}, \quad F_0(s_0) = F_0^{BW}(0)$$

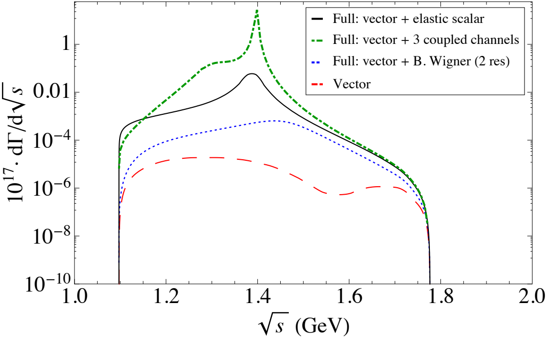
$$D(s) = \mathbb{1} + g(s) N(s),$$

$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 \\ 0 & g_{\pi\eta'} \end{pmatrix},$$

$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix},$$



$\tau^- \to \pi^- \eta' \nu_\tau$: Invariant mass distribution and Branching Ratio



	BR_V	BR_S	BR	Reference
	$< 10^{-7}$	$[0.2, 1.3] \cdot 10^{-6}$	$[0.2, 1.4] \cdot 10^{-6}$	Nussinov, Soffer '99
	$[0.14, 3.4] \cdot 10^{-8}$	$[0.6, 1.8] \cdot 10^{-7}$	$[0.61, 2.1] \cdot 10^{-7}$	Paver, Riazuddin '11
	$1.11 \cdot 10^{-8}$	$2.63 \cdot 10^{-8}$	$3.74 \cdot 10^{-8}$	Volkov, Kostunin '12
THIS WORK	$[0.3, 5.7] \cdot 10^{-10}$	$[2 \cdot 10^{-11}, 7 \cdot 10^{-10}]$	$[0.5 \cdot 10^{-10}, 1.3 \cdot 10^{-9}]$	Breit-Wigner (1 res)
	$[0.3, 5.7] \cdot 10^{-10}$	$[5 \cdot 10^{-11}, 2 \cdot 10^{-9}]$	$[0.8 \cdot 10^{-10}, 2.6 \cdot 10^{-9}]$	Breit-Wigner (2 res)
	$[0.3, 5.7] \cdot 10^{-10}$	$[2 \cdot 10^{-9}, 4 \cdot 10^{-8}]$	$[2.6 \cdot 10^{-9}, 4 \cdot 10^{-8}]$	Elastic Omnès solution
	$[0.3, 5.7] \cdot 10^{-10}$	$[2 \cdot 10^{-7}, 2 \cdot 10^{-6}]$	$[2 \cdot 10^{-7}, 2 \cdot 10^{-6}]$	2 cc ($\pi^-\eta'$ to $\pi^-\eta$)
	$[0.3, 5.7] \cdot 10^{-10}$	$[3 \cdot 10^{-7}, 3 \cdot 10^{-6}]$	$[3 \cdot 10^{-7}, 3 \cdot 10^{-6}]$	2 cc ($\pi^-\eta'$ to K^-K^0)
	$[0.3, 5.7] \cdot 10^{-10}$	$[1 \cdot 10^{-7}, 1 \cdot 10^{-6}]$	$[1 \cdot 10^{-7}, 1 \cdot 10^{-6}]$	3 coupled channels

$$BR_{exp}^{BaBar} < 4 \cdot 10^{-6}\,(90\%CL)\,, \quad BR_{exp}^{CLEO} < 7.4 \cdot 10^{-5}\,(90\%CL)$$