Optimizing Higgs factories by modifying the recoil mass

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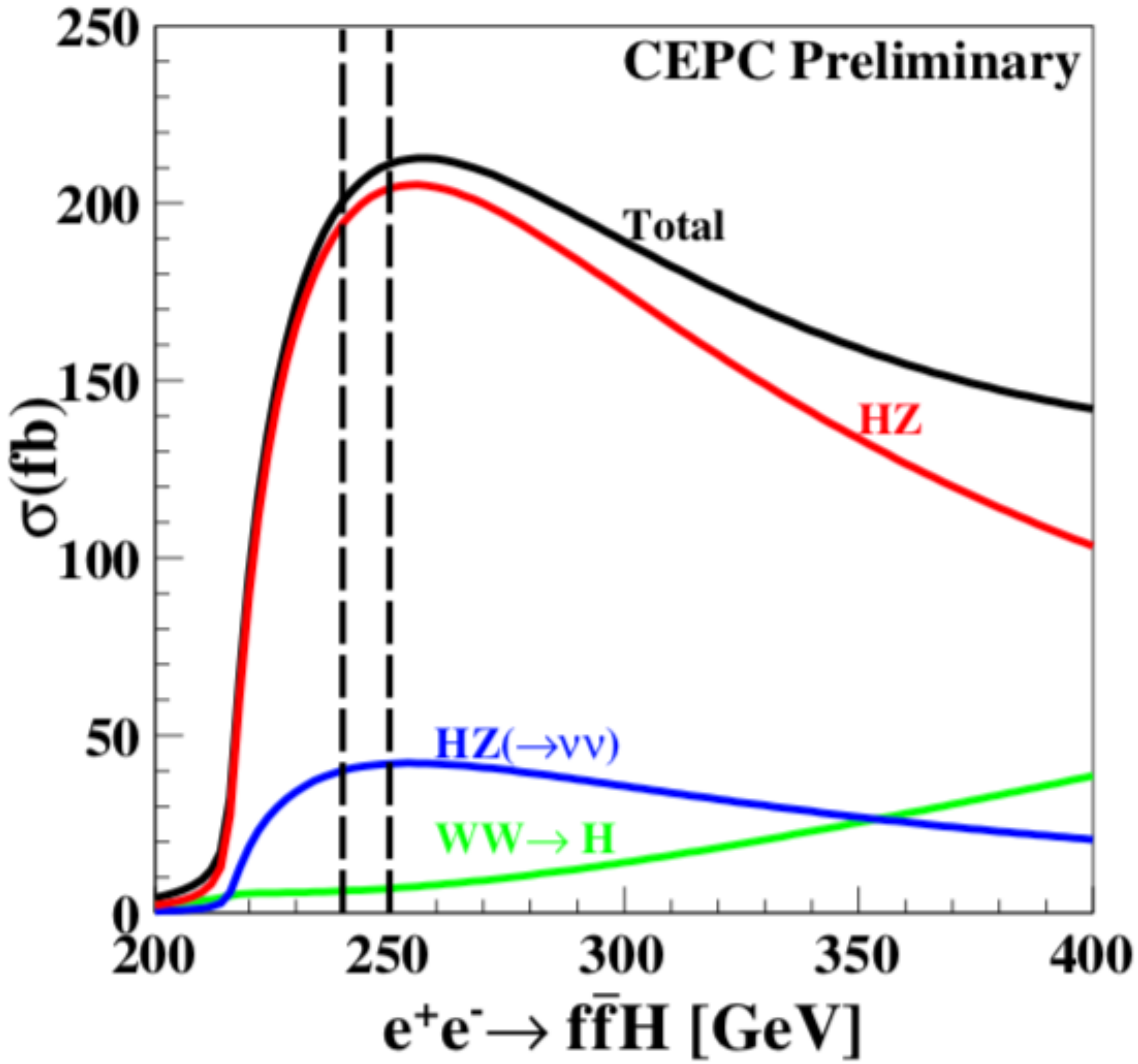
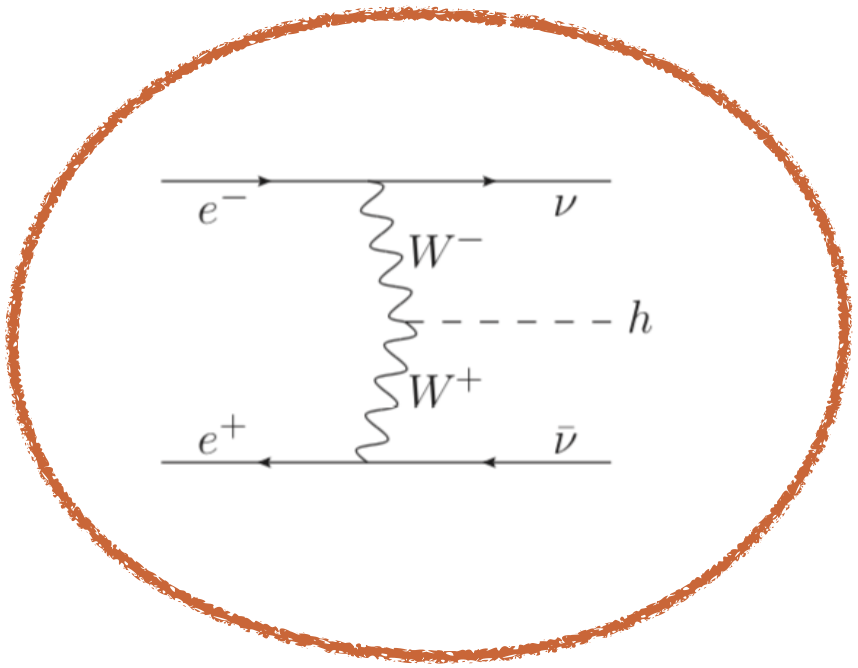
arXiv: 1709.08654

in collaboration with Jiayin Gu

2018 CEPC Workshop

measuring WW-fusion @ 240-250GeV

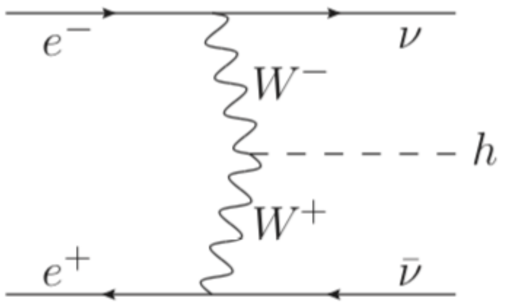




Estimated accuracy:
10.8%(250/fb) for WW->H->bb at ILC;
arXiv:1403.7734
0.28% (5/ab) for H->bb
CEPC CDR

measuring WW-fusion @ 240-250GeV

Complementary to Higgs-strahlung process



- **kappa frame:** contraint hWW coupling;
  input for the determination of higgs width;

$$\Gamma_h \propto \frac{\Gamma(h \to b\bar{b})}{\mathrm{BR}(h \to b\bar{b})} \propto \frac{\sigma(\nu\bar{\nu}h,\ h \to b\bar{b})}{\mathrm{BR}(h \to b\bar{b}) \cdot \mathrm{BR}(h \to WW^*)},$$

- **EFT frame**: WW-fusion and the Higgs-strahlung process probe different combinations of EFT parameters.

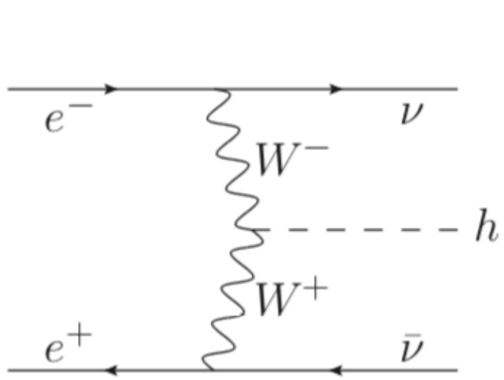
~focus on H->bb channel~

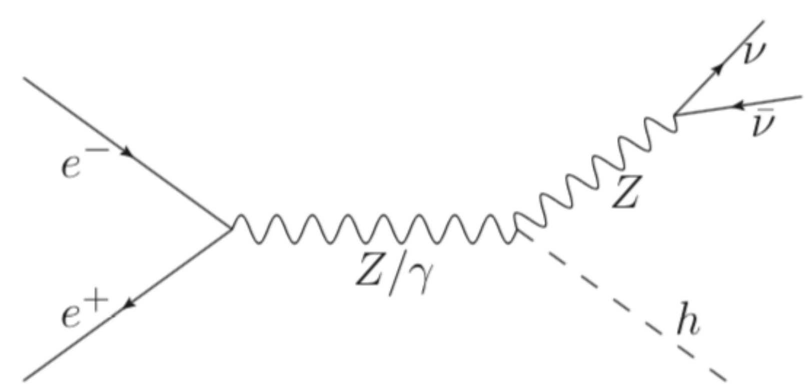
Challenge of measuring@ 240-250GeV

- Small rate, 6.72 fb @250GeV with unpolarised beam;
- The rate of its dominant background Higgs-strahlung process with $Z \to \nu\bar{\nu}$, 27 fb @250GeV with unpolarised beam;

Process	expected	pre-selection	Cut1	Cut2	Cut3	Cut4	Cut5	Cut6	Cut7	Cut8
$\nu\bar{\nu}H$(fusion)	3426	2663	2070	2023	1577	1053	965	547	519	507
$\nu\bar{\nu}H(ZH)$	1.4×10^4	10918	8356	8356	7448	4860	4594	2574	2546	2546

arXiv:1403.7734

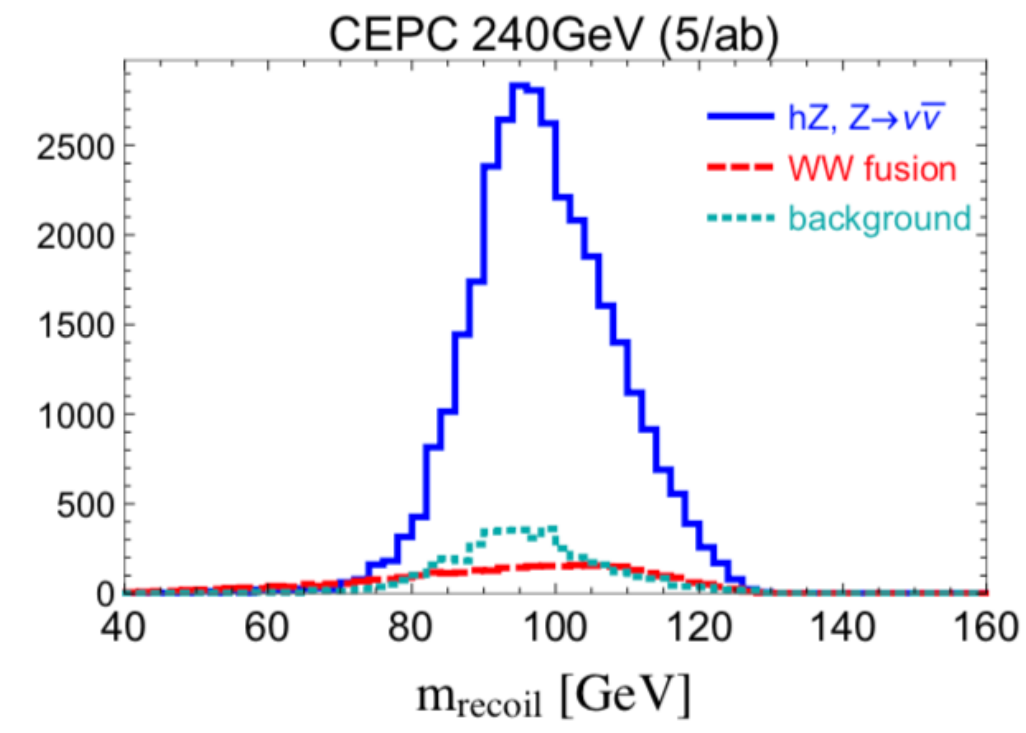




Challenge of measuring @ 240-250GeV

- observable to suppress bg: recoil mass, limited by the detector resolutions (especially for hadronic Higgs decay) and other effects, e.g. beam energy spread.

$$m_{\rm recoil}^2 = s - 2\sqrt{s}\, E_h^{\rm rec} + (m_h^{\rm rec})^2\,,$$



Optimisation and Fit

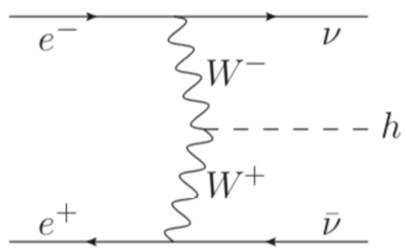
Collider scenarios:

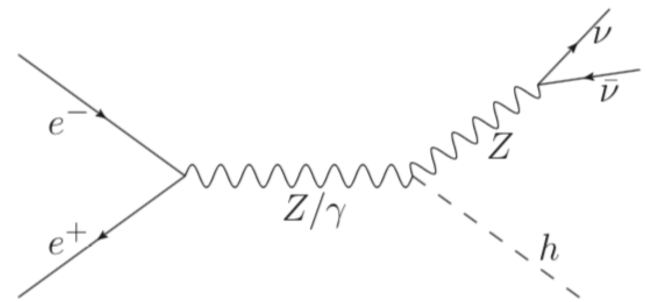
- **CEPC:** @240GeV, 5/ab data, unpolarised beam;
- **ILC:** @250 GeV, 2/ab data,
  polarised beam $P(e^-, e^+) = (\pm 0.8, \pm 0.3)$

Detector simulation:

- follow the ILD cards studied in arXiv:1306.6329

Optimisation





$$m^2_{\text{recoil}} = s - 2\sqrt{s}\,E_h^{\text{rec}} + (m_h^{\text{rec}})^2\,,$$

As Higgs mass uncertainty could be measured to around MeV, apply Higgs mass constraint?

$$(m^E_{\text{recoil}})^2 = s - 2\sqrt{s}\,E_h^{\text{rec}} + m_h^2\,,$$

$$(m^p_{\text{recoil}})^2 = s - 2\sqrt{s}\,\sqrt{m_h^2 + |\vec{p}_h^{\text{rec}}|^2} + m_h^2\,,$$

Optimisation

Define:

$$m_{\text{recoil}} = m_{\text{recoil}}^{\text{true}}(1+\delta_m)$$

$$m_{\text{recoil}}^{E} = m_{\text{recoil}}^{\text{true}}(1+\delta_m^{E})$$

$$m_{\text{recoil}}^{p} = m_{\text{recoil}}^{\text{true}}(1+\delta_m^{p})$$

$$|\vec{p}_h^{\,\text{rec}}| = |\vec{p}_h|(1+\delta_p)$$

$$E_h^{\text{rec}} = E_h(1+\delta_E)$$

$$\delta_m \approx -\frac{1}{m_Z^2}\left[(\sqrt{s}-E_h)\,E_h\,\delta_E + |\vec{p}_h|^2\,\delta_p\right]$$

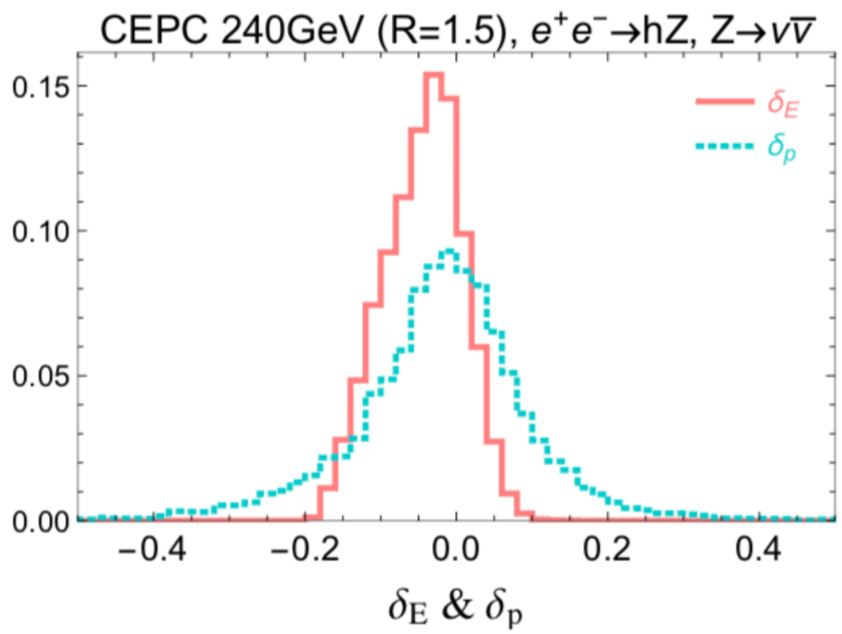
$$\delta_m^E \approx -\frac{\sqrt{s}}{m_Z^2}\,E_h\,\delta_E\,,$$

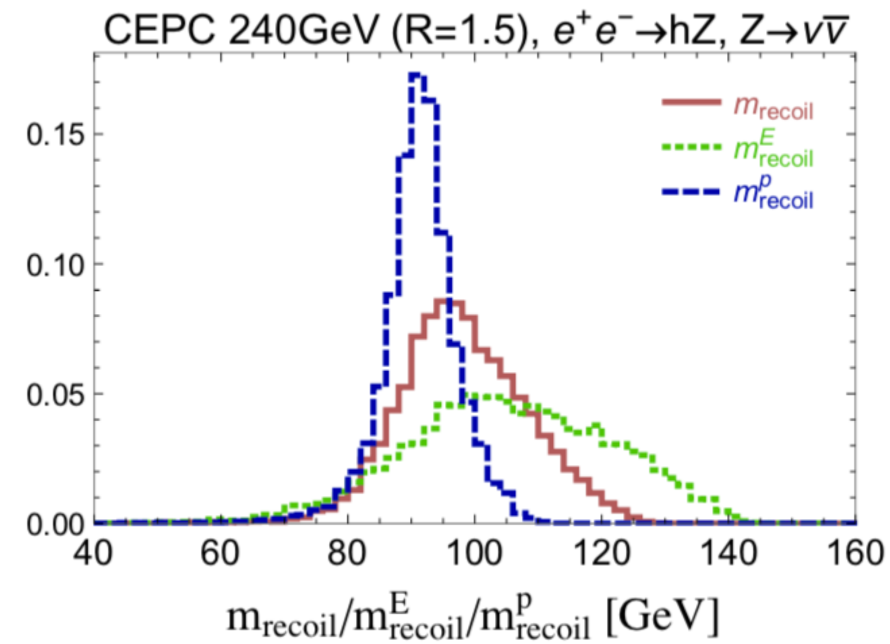
$$\delta_m^p \approx -\frac{\sqrt{s}}{m_Z^2}\,\frac{|\vec{p}_h|^2}{E_h}\,\delta_p\,.$$

With $|\vec{p}_h| \approx 51\,\text{GeV}$ and $E_h \approx 135\,\text{GeV}$ at $\sqrt{s} = 240\,\text{GeV}$, and $|\vec{p}_h| \approx 62\,\text{GeV}$ and $E_h \approx 140\,\text{GeV}$ at $\sqrt{s} = 250\,\text{GeV}$

$$\delta_m \;/\; \delta_m^E \;/\; \delta_m^p \approx \begin{cases} -1.7\,\delta_E - 0.32\,\delta_p \;/\; -3.9\,\delta_E \;/\; -0.57\,\delta_p & \text{at } 240\,\text{GeV} \\ -1.9\,\delta_E - 0.46\,\delta_p \;/\; -4.2\,\delta_E \;/\; -0.83\,\delta_p & \text{at } 250\,\text{GeV} \end{cases}$$

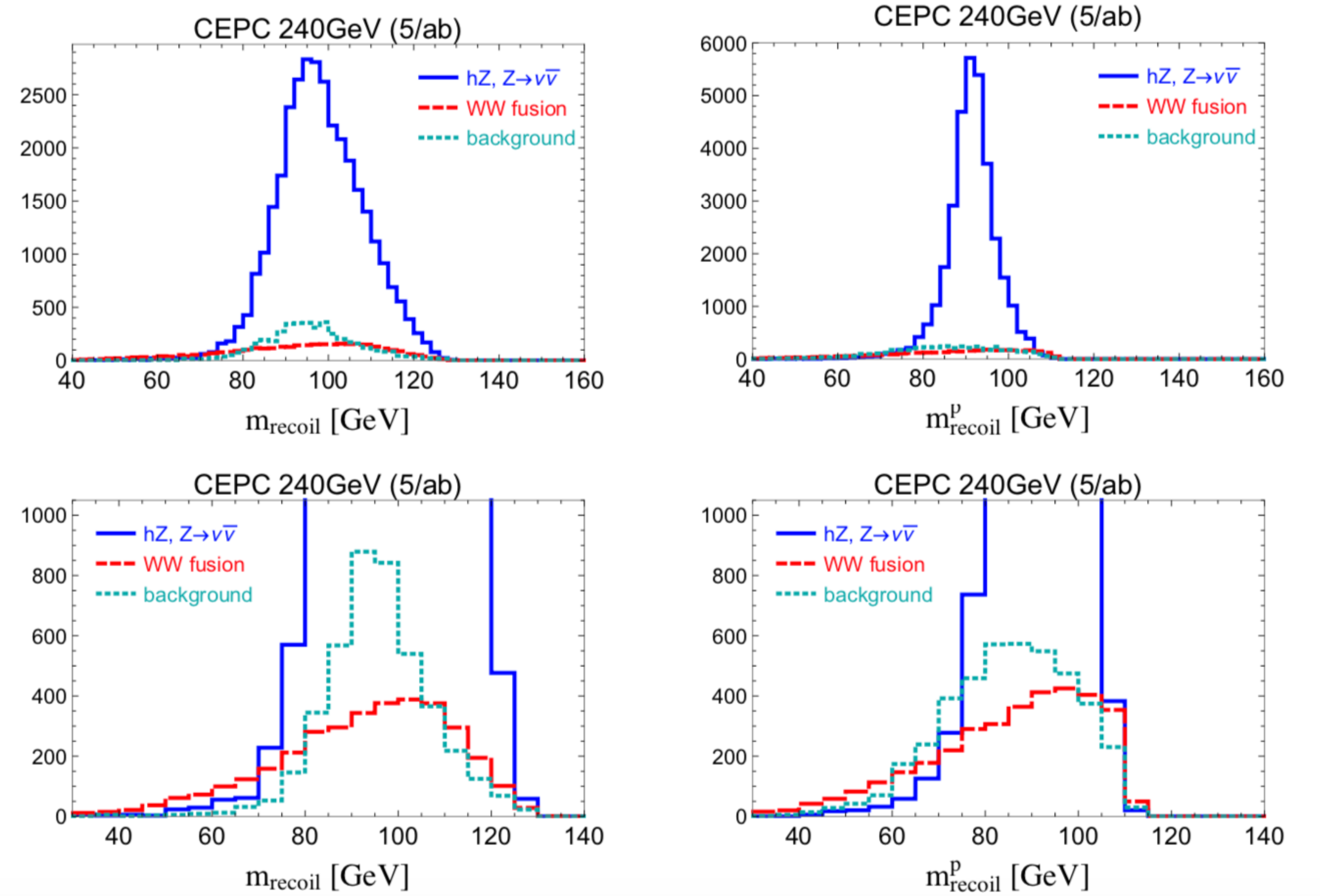
Optimisation





- A Higgs-mass-window of 105GeV-135GeV is applied;
- Radius of R=1.5 is used in the jet clustering algorithm;
- Didn't apply energy correction due to detector geometry defects;
- m^p_{recoil} and m_{recoil} performance are better in constructing the recoil mass and will be our focus in our study.

Optimisation

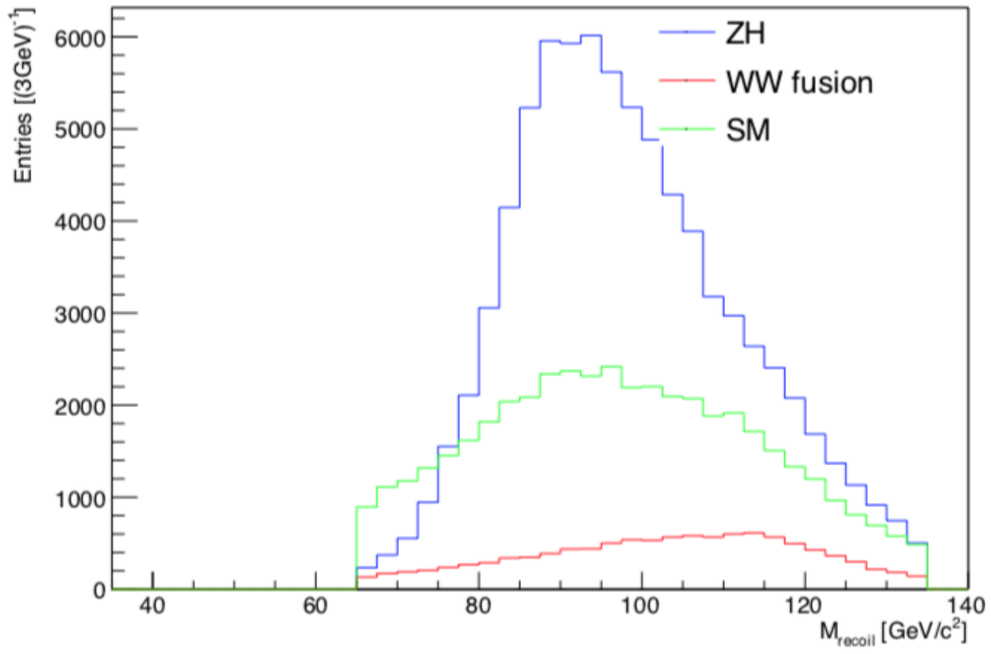


- Didn't consider the interference between signal and background;
- Didn't include the effects of ISR photon;
- Rescaled $\nu\bar{\nu}b\bar{b}$ background to the total yield;
- m^{p}_{recoil} looks better in distinguishing hZ and WW-fusion distributions.

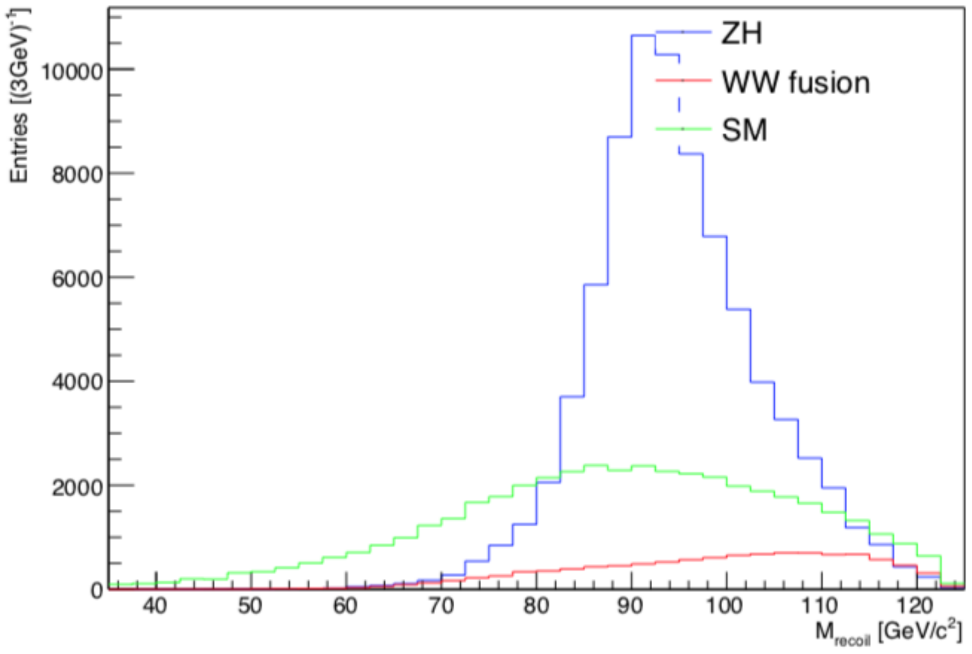
Optimisation

Happy to have the more dedicated simulations today from our experimental colleagues.

From Hao Liang's talk



Raw: M_{recoil}



Refined: M_{recoil}

Chi-square Fit

Fit to binned m^{p}_{recoil} and m_{recoil} distribution to extract the precisions of hZ and WW cross section.

CEPC 240 GeV, 5 ab^{-1}, uncertainties normalized to SM predictions

m_{recoil}	3-parameter fit				fixing σ_{bg}		
	uncertainty	correlation matrix			uncertainty	correlation matrix	
		σ_{hZ}	$\sigma_{WW\to h}$	σ_{bg}		σ_{hZ}	$\sigma_{WW\to h}$
σ_{hZ}	0.024	1	0.28	-0.95	0.0077	1	-0.61
$\sigma_{WW\to h}$	0.058		1	-0.47	0.051		1
σ_{bg}	0.20			1			

m^{p}_{recoil}	3-parameter fit				fixing σ_{bg}		
	uncertainty	correlation matrix			uncertainty	correlation matrix	
		σ_{hZ}	$\sigma_{WW\to h}$	σ_{bg}		σ_{hZ}	$\sigma_{WW\to h}$
σ_{hZ}	0.0071	1	0.098	-0.35	0.0066	1	-0.45
$\sigma_{WW\to h}$	0.083		1	-0.87	0.041		1
σ_{bg}	0.082			1			

- A little bit worse than the literature for fitting to m_{recoil};
- Focus on the comparison between m^{p}_{recoil} and m_{recoil}.

see Hao Liang's talk for a more dedicated simulation.

Kappa-framework

In the kappa framework

- As an independent parameter, improve the constraint on the hWW coupling by 20-30%;
- Higgs width improved by 20-30%;

$$\Gamma_h \propto \frac{\Gamma(h \to b\bar{b})}{\mathrm{BR}(h \to b\bar{b})} \propto \frac{\sigma(\nu\bar{\nu}h,\ h \to b\bar{b})}{\mathrm{BR}(h \to b\bar{b}) \cdot \mathrm{BR}(h \to WW^*)},$$

EFT-framework

see Jiayin's talk

- Could preserve gauge invariance;
- A good parameterization of the the effects of new physics, easy to map to models that satisfies the assumptions of the framework;
- Take account of connections among different measurements: operators that contribute to both Higgs process and di-boson process could be constrained by the triple gauge coupling measurements, e.g. c_{ZZ} .

EFT-framework

Focus on Higgs and di-boson measurements at 240GeV with the following assumptions:

see Jiayin's talk

- gauge invariance;
- CP even operators;
- Z pole observable and W mass to be SM-like;
- custodial symmetry conservation;
- no flavor violating Yukawa couplings;
- hgg and $ht\bar{t}$ coupling cannot be independently constrained yet.

EFT-framework

The relevant part in the Lagrangian are:

see Jiayin's talk

$$\mathcal{L}_{hVV} = \frac{h}{v}\Big[(1+\delta c_W)\frac{g^2v^2}{2}W_\mu^+W^{-\mu} + (1+\delta c_Z)\frac{(g^2+g'^2)v^2}{4}Z_\mu Z^\mu$$
$$+ c_{WW}\frac{g^2}{2}W_{\mu\nu}^+W^{-\mu\nu} + c_{W\Box}\, g^2(W_\mu^-\partial_\nu W^{+\mu\nu} + \text{h.c.})$$
$$+ c_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a G^{a\,\mu\nu} + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A^{\mu\nu} + c_{Z\gamma}\frac{e\sqrt{g^2+g'^2}}{2}Z_{\mu\nu}A^{\mu\nu}$$
$$+ c_{ZZ}\frac{g^2+g'^2}{4}Z_{\mu\nu}Z^{\mu\nu} + c_{Z\Box}\, g^2 Z_\mu\partial_\nu Z^{\mu\nu} + c_{\gamma\Box}\, gg' Z_\mu\partial_\nu A^{\mu\nu}\Big]$$

$$\mathcal{L}_{hff} = -\frac{h}{v}\sum_{f=t,c,b,\tau,\mu} m_f(1+\delta y_f)\bar{f}_R f_L + \text{h.c.}$$

$$\begin{aligned}\mathcal{L}_{\text{tgc}} = \; & igs_{\theta_W}A^\mu(W^{-\nu}W_{\mu\nu}^+ - W^{+\nu}W_{\mu\nu}^-) \\ & + ig(1+\delta g_1^Z)c_{\theta_W}Z^\mu(W^{-\nu}W_{\mu\nu}^+ - W^{+\nu}W_{\mu\nu}^-) \\ & + ig\left[(1+\delta\kappa_Z)c_{\theta_W}Z^{\mu\nu} + (1+\delta\kappa_\gamma)s_{\theta_W}A^{\mu\nu}\right]W_\mu^-W_\nu^+ \\ & + \frac{ig}{m_W^2}(\lambda_Z c_{\theta_W}Z^{\mu\nu} + \lambda_\gamma s_{\theta_W}A^{\mu\nu})W_v^{-\rho}W_{\rho\mu}^+ \,,\end{aligned}$$

A.Falkowski, el.

EFT-framework

$$\mathcal{L}_{hVV} = \frac{h}{v}\Big[(1+\delta \cancel{c_W})\frac{g^2v^2}{2}W^+_\mu W^{-\mu} + (1+\delta c_Z)\frac{(g^2+g'^2)v^2}{4}Z_\mu Z^\mu$$
$$+ \cancel{c_{WW}}\frac{g^2}{2}W^+_{\mu\nu}W^{-\mu\nu} + \cancel{c_{W\Box}}\, g^2(W^-_\mu \partial_\nu W^{+\mu\nu} + \text{h.c.})$$
$$+ c_{gg}\frac{g_s^2}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A^{\mu\nu} + c_{Z\gamma}\frac{e\sqrt{g^2+g'^2}}{2}Z_{\mu\nu}A^{\mu\nu}$$
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see Jiayin's talk

gauge invariance

$$\mathcal{L}_{hff} = -\frac{h}{v}\sum_{f=t,c,b,\tau,\mu} m_f(1+\delta y_f)\bar{f}_R f_L + \text{h.c.}$$

$$\begin{aligned}\mathcal{L}_{\text{tgc}} = \; & i g s_{\theta_W} A^\mu (W^{-\nu}W^+_{\mu\nu} - W^{+\nu}W^-_{\mu\nu}) \\ & + ig(1+\delta \cancel{g_1^Z})c_{\theta_W} Z^\mu (W^{-\nu}W^+_{\mu\nu} - W^{+\nu}W^-_{\mu\nu}) \\ & + ig\left[(1+\delta\cancel{\kappa_Z})c_{\theta_W}Z^{\mu\nu} + (1+\delta\cancel{\kappa_\gamma})s_{\theta_W}A^{\mu\nu}\right]W^-_\mu W^+_\nu \\ & + \frac{ig}{m_W^2}(\lambda_Z c_{\theta_W} Z^{\mu\nu} + \cancel{\lambda_\gamma} s_{\theta_W} A^{\mu\nu})W^{-\rho}_v W^+_{\rho\mu}\,,\end{aligned}$$

A.Falkowski, el.

$$\delta c_Z\,,\quad c_{ZZ}\,,\quad c_{Z\Box}\,,\quad \bar{c}_{\gamma\gamma}\,,\quad \bar{c}_{Z\gamma}\,,\quad \bar{c}_{gg}^{\text{eff}}\,,\quad \delta y_c\,,\quad \delta y_b\,,\quad \delta y_\tau\,,\quad \delta y_\mu\,,\quad \lambda_Z\,,$$

EFT-framework

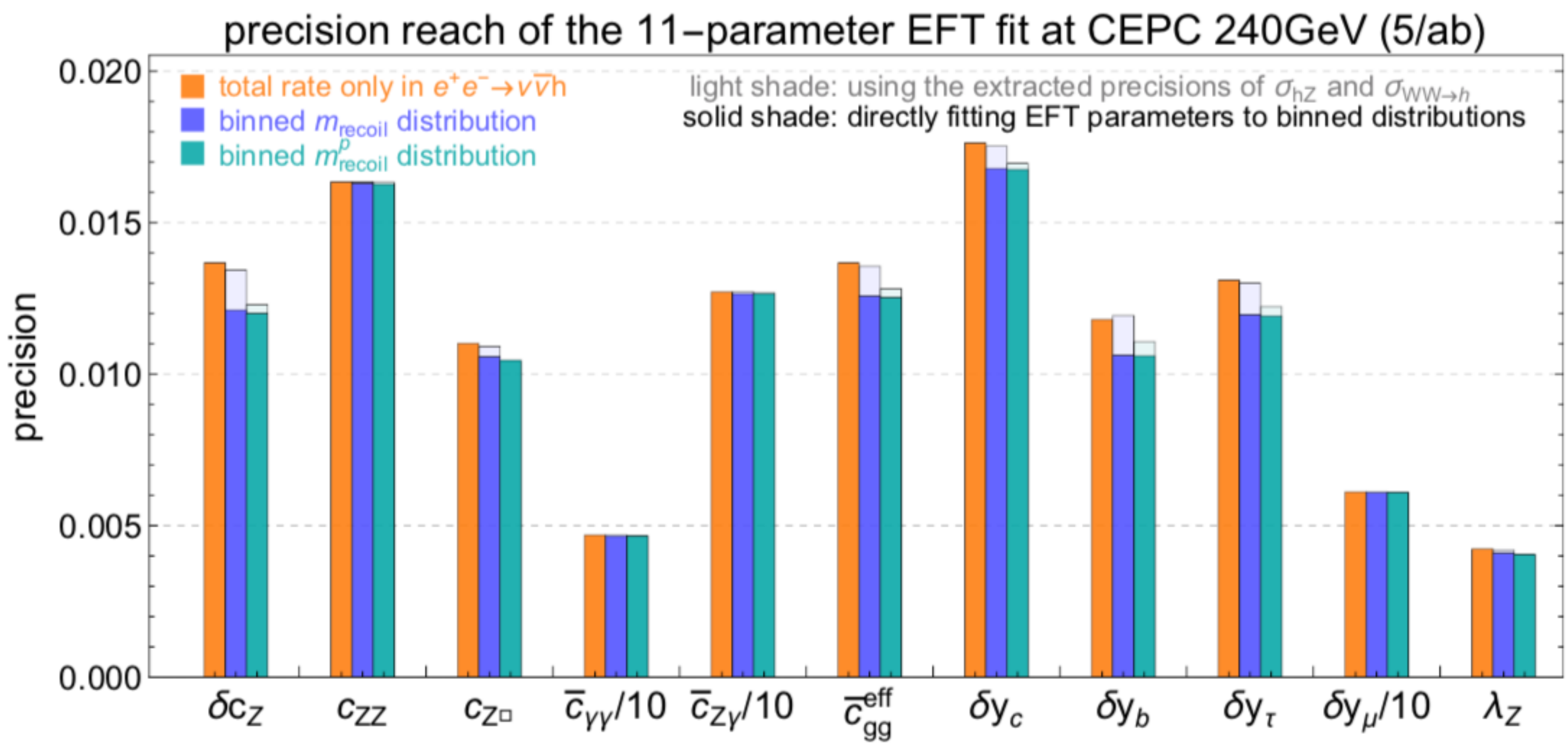
Table of measurement inputs

CEPC 240 GeV, 5 ab^{-1}, unpolarized beams						
$e^+e^- \to hZ$		$e^+e^- \to WW$				
$\sigma(e^+e^- \to hZ)$	0.50%		uncertainty	correlation matrix		
	$\sigma(hZ) \times \mathrm{BR}$			$\delta g_{1,Z}$	$\delta\kappa_\gamma$	λ_Z
$h \to b\bar{b}$	0.24%★	$\delta g_{1,Z}$	6.4×10^{-3}	1	0.068	-0.93
$h \to c\bar{c}$	2.5%	$\delta\kappa_\gamma$	3.5×10^{-3}		1	-0.40
$h \to gg$	1.2%	λ_Z	6.3×10^{-3}			1
$h \to \tau\tau$	1.0%	Angular observables in $e^+e^- \to hZ,\ h \to b\bar{b},\ Z \to \ell^+\ell^-$ are also included.				
$h \to WW^*$	1.0%					
$h \to ZZ^*$	4.3%					
$h \to \gamma\gamma$	9.0%					
$h \to \mu\mu$	12%					
$h \to Z\gamma$	25%					

see Kaili's talk for updated number

- For the ★, Z invisible decay mode contribution is removed;
- Expected precisions from CEPC pre-CDR, need to be updated.

EFT-framework



- Total rate fit;
- Fit to the extracted precisions of cross sections without taking into account of their correlations.
- Fit to binned distribution: could be better by a factor as large as10%

Applying to HZ measurement?

Z decay mode	ΔM_H (MeV)	$\Delta\sigma(ZH)/\sigma(ZH)$	$\Delta g(HZZ)/g(HZZ)$
ee	14	2.1%	
$\mu\mu$	6.5	0.9%	
$ee+\mu\mu$	5.9	0.8%	0.4%
$q\bar{q}$		0.65%	0.32%
$ee+\mu\mu+q\bar{q}$		0.51%	0.25%

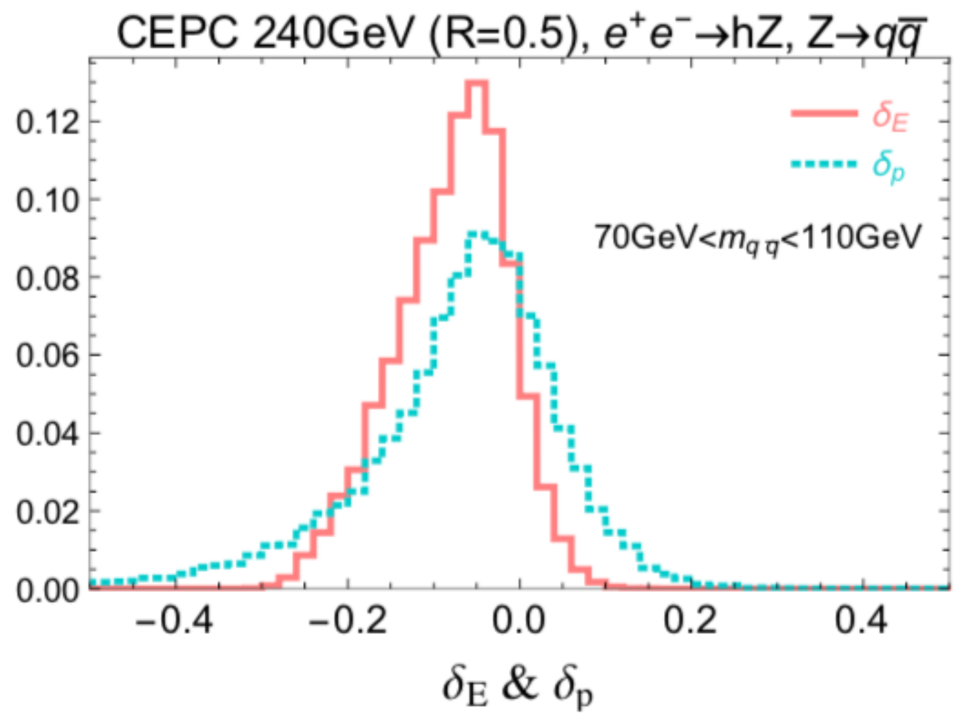
CEPC CDR

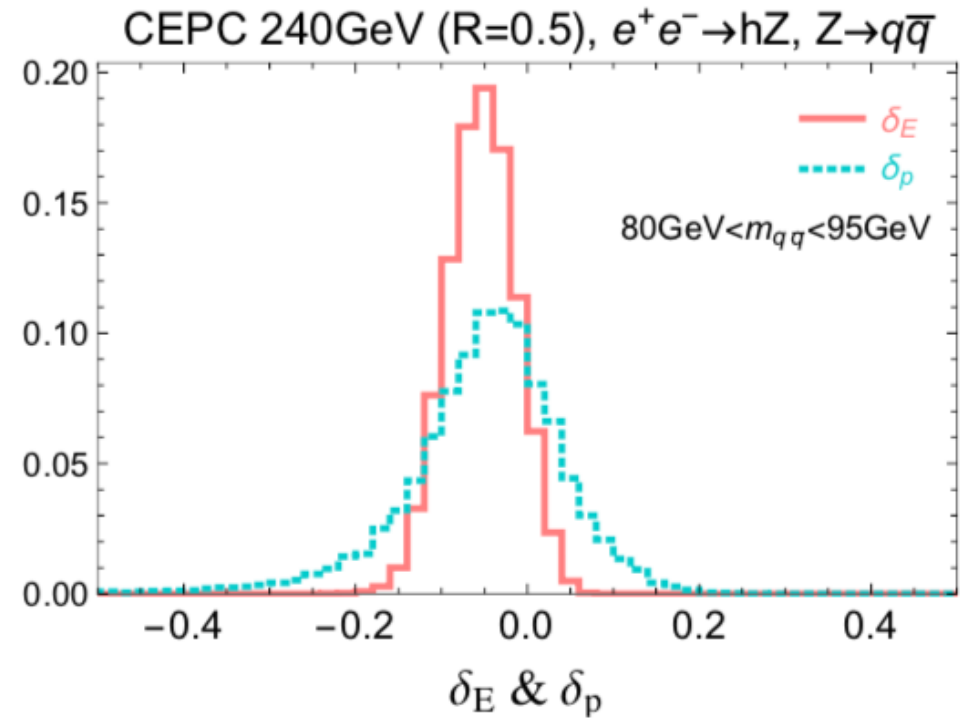
- Hadronic channel gains by larger decay branching ratio, suffers from jet resolution, etc.

Applying to HZ measurement?

$E_Z \sim 104.7$ GeV, $|p_Z| \sim 51.8$ GeV

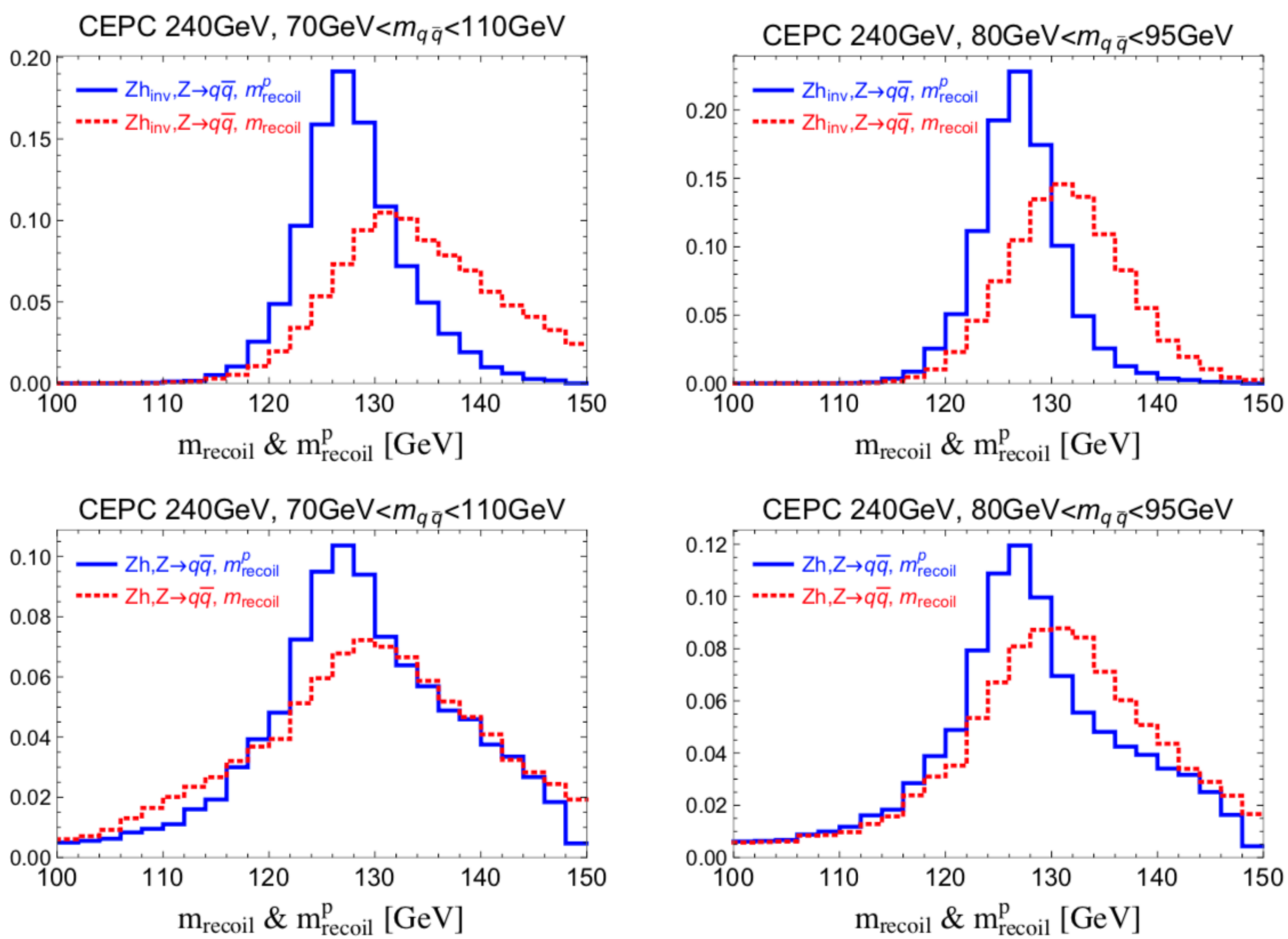
$$\delta_m \;/\; \delta_m^E \;/\; \delta_m^p \approx \begin{cases} -0.91\,\delta_E - 0.17\,\delta_p \;/\; -1.6\,\delta_E \;/\; -0.39\,\delta_p & \text{at } 240\,\text{GeV} \\ -0.99\,\delta_E - 0.25\,\delta_p \;/\; -1.8\,\delta_E \;/\; -0.56\,\delta_p & \text{at } 250\,\text{GeV} \end{cases}$$





- larger deviations of measured energy and momentum from the true ones, the invariant mass further away from its true value;

Applying to HZ measurement?



- Comparison shows the effect of combinatory problem for hadronically Higgs decay;

Applying to HZ measurement?

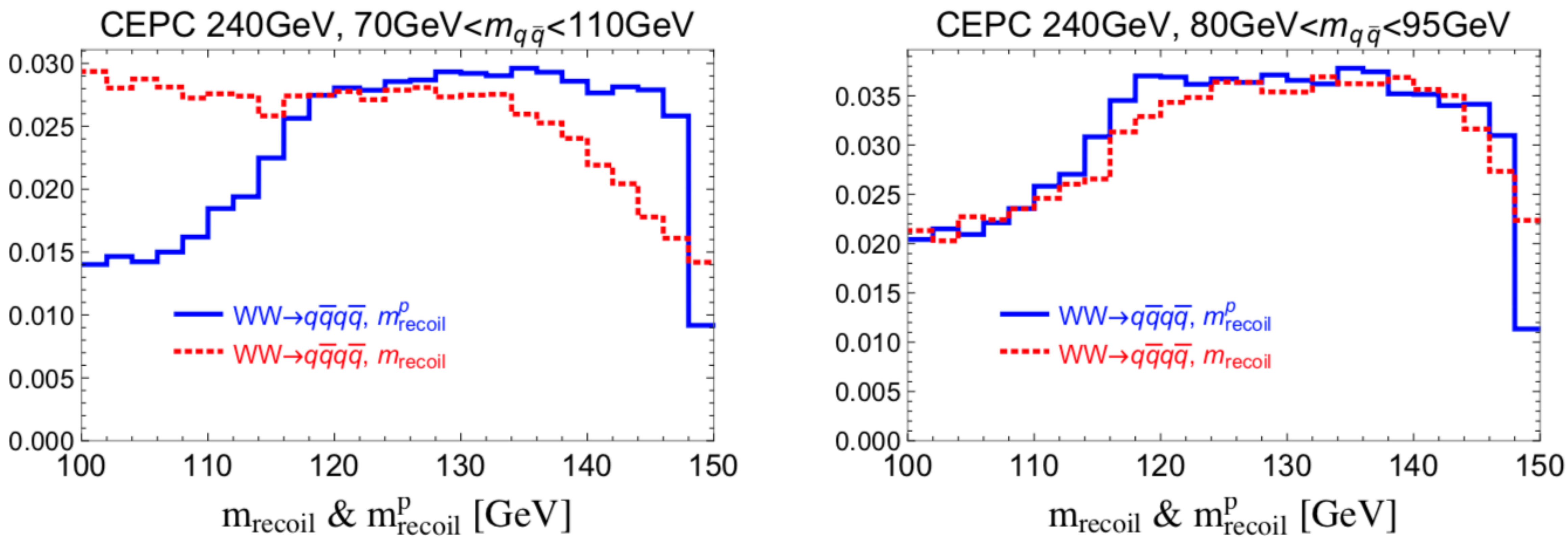


Figure 8: The distributions of $m_{\rm recoil}$ and $m^p_{\rm recoil}$ for $W^+W^- \to q\bar{q}q\bar{q}$ at CEPC 240 GeV with a Z-mass-window cut of 70 GeV $< m_{q\bar{q}} <$ 110 GeV (left panel) or 80 GeV $< m_{q\bar{q}} <$ 95 GeV (right panel).

Summary and discussions

- We studied how to improve measurements where recoil mass of hadronic events are crucial, e.g.WW-fusion and ZH process;
- We also did kappa-frame and EFT fit including the WW-fusion information, especially including the kinematics distribution information.
- Machine learning, thorough kinematic fit?
  ——Easy physical understanding of these more complicated method, indication of detector improvement to balance energy resolution/momentum resolution.

Thank you

Backup

measuring WW-fusion process

Complementary to Higgs-strahlung process

- **kappa frame:** contraint hWW coupling;
  input for the determination of higgs width;

$$\Gamma_h \propto \frac{\Gamma(h \to b\bar{b})}{\mathrm{BR}(h \to b\bar{b})} \propto \frac{\sigma(\nu\bar{\nu}h,\ h \to b\bar{b})}{\mathrm{BR}(h \to b\bar{b}) \cdot \mathrm{BR}(h \to WW^*)}\ ,$$

- **EFT frame**: WW-fusion and the Higgs-strahlung process probe different combinations of EFT parameters: e.g. cww= czz+ \alpha cz\gamma

  could measure czz via ZH, and cz\gamma via WW

Optimisation and Fit

Collider scenarios:

- **CEPC:** @240GeV, 5/ab data, unpolarised beam;
- **ILC:** @250 GeV, 2/ab data,
  polarised beam P(e-, e+) = (±0.8,±0.3)

Detector simulation:

- follow the ILD cards studied in arXiv:1306.6329 ,
  tagging efficiency:

Challenge of measuring @ 240-250GeV

—Small rate, 6.72 fb @250GeV with unpolarised beam;

—The rate of its dominant background Higgs-strahlung process with Z decay to vv, 27 fb @250GeV with unpolarised beam;

Process	expected	pre-selection	Cut1	Cut2	Cut3	Cut4	Cut5	Cut6	Cut7	Cut8
$\nu\bar{\nu}H(\text{fusion})$	3426	2663	2070	2023	1577	1053	965	547	519	507
$\nu\bar{\nu}H(ZH)$	1.4×10^4	10918	8356	8356	7448	4860	4594	2574	2546	2546
$\nu_l\bar{\nu}_l b\bar{b}$	3.05×10^4	23012	1040	1040	878	421	390	224	193	187
$\nu_l\bar{\nu}_l q\bar{q}$	1.19×10^5	88998	5548	5545	4714	2408	2271	15	9	9
$q\bar{q}l^+l^-$	2.99×10^5	153540	6196	5922	1760	588	508	65	38	36
$q\bar{q}l\nu$	1.73×10^6	1.15×10^6	181973	177193	134047	22654	20533	111	73	65
$q\bar{q}q\bar{q}$	3.91×10^6	1.15×10^6	782	728	3	1	0	0	0	0
$q\bar{q}$	26.02×10^6	17.27×10^6	852321	794892	1507	1199	683	289	152	152
BG	32.104×10^6	19.846×10^6	1.047×10^6	985320	142909	27271	24385	1404	465	449

arXiv:1403.7734

ILC 250 GeV, 2 ab^{-1}, uncertainties normalized to SM predictions

m_{recoil}	3-parameter fit				fixing σ_{bg}		
	uncertainty	correlation matrix			uncertainty	correlation matrix	
		σ_{hZ}	$\sigma_{WW\to h}$	σ_{bg}		σ_{hZ}	$\sigma_{WW\to h}$
σ_{hZ}	0.049	1	0.47	-0.97	0.011	1	-0.69
$\sigma_{WW\to h}$	0.063		1	-0.63	0.045		1
σ_{bg}	0.31			1			

m^{p}_{recoil}	3-parameter fit				fixing σ_{bg}		
	uncertainty	correlation matrix			uncertainty	correlation matrix	
		σ_{hZ}	$\sigma_{WW\to h}$	σ_{bg}		σ_{hZ}	$\sigma_{WW\to h}$
σ_{hZ}	0.010	1	0.21	-0.51	0.0088	1	-0.46
$\sigma_{WW\to h}$	0.059		1	-0.83	0.033		1
σ_{bg}	0.088			1			

$$\left.\frac{\sigma_{\nu\bar{\nu}h}}{\sigma_{\nu\bar{\nu}h}^{\mathrm{SM}}}\right|_{240\,\mathrm{GeV}}^{\mathrm{unpolarized}} = 1 + 1.7\,\delta c_Z + 1.3\,c_{ZZ} + 2.9\,c_{Z\Box} + 0.051\,c_{Z\gamma} + 0.14\,c_{\gamma\Box} + 0.23\,\delta c_W - 0.0026\,c_{WW} - 0.065\,c_{W\Box}\,.$$

CEPC 240 GeV (with unpolarized beams) m_{recoil}										
	bin index [GeV]									
	75	80	85	90	95	100	105	110	115	130
σ_{SM} [fb]	0.15	0.18	0.38	0.78	1.2	1.3	1.1	0.74	0.47	0.34
δc_Z	0.97	1.4	1.6	1.7	1.8	1.9	1.9	1.9	1.8	1.9
c_{ZZ}	0.50	0.95	1.1	1.3	1.3	1.4	1.4	1.4	1.4	1.4
$c_{Z\Box}$	1.5	2.2	2.6	2.9	3.0	3.0	3.2	3.1	3.1	3.0
$c_{Z\gamma}$	0.021	0.035	0.044	0.051	0.052	0.054	0.056	0.056	0.055	0.055
$c_{\gamma\Box}$	0.075	0.11	0.13	0.14	0.15	0.15	0.15	0.15	0.15	0.15
δc_W	0.93	0.62	0.37	0.22	0.15	0.13	0.14	0.17	0.20	0.29
c_{WW}	-0.011	-0.0066	-0.0038	-0.0023	-0.0016	-0.0013	-0.0013	-0.0016	-0.0019	-0.0023
$c_{W\Box}$	-0.30	-0.18	-0.11	-0.060	-0.038	-0.032	-0.033	-0.036	-0.037	-0.049

CEPC 240 GeV (with unpolarized beams) m^{p}_{recoil}								
	bin index [GeV]							
	75	80	85	90	95	100	105	115
σ_{SM}[fb]	0.22	0.24	0.59	1.7	2.4	0.99	0.32	0.11
δc_Z	0.95	1.4	1.7	1.8	1.9	2.0	1.8	1.4
c_{ZZ}	0.54	0.99	1.2	1.3	1.4	1.4	1.4	1.0
$c_{Z\Box}$	1.6	2.3	2.6	3.0	3.2	3.3	3.1	2.0
$c_{Z\gamma}$	0.021	0.035	0.045	0.053	0.056	0.059	0.054	0.044
$c_{\gamma\Box}$	0.075	0.11	0.13	0.15	0.16	0.16	0.15	0.11
δc_W	0.92	0.61	0.33	0.14	0.075	0.11	0.33	0.99
c_{WW}	-0.0095	-0.0062	-0.0034	-0.0014	-0.00082	-0.0012	-0.0025	-0.0075
$c_{W\Box}$	-0.28	-0.17	-0.092	-0.037	-0.019	-0.025	-0.056	-0.13

$$
\begin{aligned}
\delta c_W &= \delta c_Z + 4\delta m\,, \\
c_{WW} &= c_{ZZ} + 2 s^2_{\theta_W} c_{Z\gamma} + s^4_{\theta_W} c_{\gamma\gamma}\,, \\
c_{W\Box} &= \frac{1}{g^2 - g'^2}\left[g^2 c_{Z\Box} + g'^2 c_{ZZ} - e^2 s^2_{\theta_W} c_{\gamma\gamma} - (g^2 - g'^2) s^2_{\theta_W} c_{Z\gamma}\right]\,, \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2}\left[2 g^2 c_{Z\Box} + (g^2 + g'^2) c_{ZZ} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{Z\gamma}\right]\,,
\end{aligned}
$$

$$
\begin{aligned}
\delta g_{1,Z} &= \frac{1}{2(g^2 - g'^2)}\left[-g^2 (g^2 + g'^2) c_{Z\Box} - g'^2 (g^2 + g'^2) c_{ZZ} + e^2 g'^2 c_{\gamma\gamma} + g'^2 (g^2 - g'^2) c_{Z\gamma}\right]\,, \\
\delta\kappa_\gamma &= -\frac{g^2}{2}\left(c_{\gamma\gamma}\frac{e^2}{g^2 + g'^2} + c_{Z\gamma}\frac{g^2 - g'^2}{g^2 + g'^2} - c_{ZZ}\right)\,. \qquad \text{(A.6)}
\end{aligned}
$$

$$\bar{c}_{\gamma\gamma} \simeq \frac{c_{\gamma\gamma}}{8.3 \times 10^{-2}}\,, \qquad \bar{c}_{Z\gamma} \simeq \frac{c_{Z\gamma}}{5.9 \times 10^{-2}}\,, \qquad \bar{c}_{gg} \simeq \frac{c_{gg}}{8.3 \times 10^{-3}}\,.$$

$$\frac{\Gamma_{gg}}{\Gamma_{gg}^{\mathrm{SM}}} \simeq 1 + 2\bar{c}_{gg}^{\mathrm{eff}} \simeq 1 + 2\,\bar{c}_{gg} + 2.10\,\delta y_t - 0.10\,\delta y_b\,,$$

Applying to HZ measurement?

Z decay mode	ΔM_H (MeV)	$\Delta\sigma(ZH)/\sigma(ZH)$	$\Delta g(HZZ)/g(HZZ)$
ee	14	2.1%	
$\mu\mu$	6.5	0.9%	
$ee+\mu\mu$	5.9	0.8%	0.4%
$q\bar{q}$		0.65%	0.32%
$ee+\mu\mu+q\bar{q}$		0.51%	0.25%