Radiative corrections @Z pole

Fulvio Piccinini

INFN, Sezione di Pavia

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- present and future conceivable theoretical knowledge of Z peak in view of TeraZ @ future e^+e^- colliders
- Mini Workshop on Precision EW and QCD Calculations for the FCC Studies: Methods and Techniques
  12-13 January 2018 CERN

Standard Model Theory for the FCC-ee: The Tera-Z

Report on the 1st Mini workshop: Precision EW and QCD calculations for the FCC studies: methods and tools, 12-13 January 2018, CERN, Geneva

`https://indico.cern.ch/event/669224/`

A. Blondel[1], J. Gluza[*,2], S. Jadach[3], P. Janot[4], T. Riemann[2,5] (editors),
A. Akhundov[6], A. Arbuzov[7], R. Boels[8],
S. Bondarenko[7], S. Borowka[4], C.M. Carloni Calame[9], I. Dubovyk[8,5],
Ya. Dydyshka[10], W. Flieger[2], A. Freitas[11], K. Grzanka[2], T. Hahn[12], T. Huber[13],
L. Kalinovskaya[10], R. Lee[14], P. Marquard[5], G. Montagna[15], O. Nicrosini[9],
C. G. Papadopoulos[16], F. Piccinini[9], R. Pittau[17], W. Płaczek[18], M. Prausa[19],
S. Riemann[5], G. Rodrigo[20], R. Sadykov[10], M. Skrzypek[3], D. Stöckinger[21], J. Usovitsch[22],
B.F.L. Ward[23,12], S. Weinzierl[24], G. Yang[25], S.A. Yost[26]

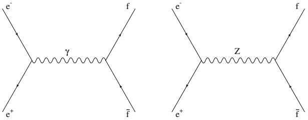
with an exhaustive list of references

Observable	Present value	±	error	FCC-ee Stat.	FCC-ee Syst.	Source and dominant exp. error
m_Z (keV/c^2)	91186700	±	2200	5	100	Z line shape scan Beam energy calibration
Γ_Z (keV)	2495200	±	2300	8	100	Z line shape scan Beam energy calibration
R_ℓ^Z ($\times 10^3$)	20767	±	25	0.06	1	Ratio of hadrons to leptons Acceptance for leptons
$\alpha_s(m_Z)$ ($\times 10^4$)	1196	±	30	0.1	1.6	R_ℓ^Z above
R_b ($\times 10^6$)	216290	±	660	0.3	<60	Ratio of $b\bar{b}$ to hadrons Stat. extrapol. from SLD [7]
σ^0_{had} ($\times 10^3$) (nb)	41541	±	37	0.1	4	Peak hadronic cross-section Luminosity measurement
$N_\nu (\times 10^3)$	2991	±	7	0.005	1	Z peak cross sections Luminosity measurement
$\sin^2\theta_W^{eff} (\times 10^6)$	231480	±	160	3	2 - 5	$A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z)(\times 10^3)$	128952	±	14	4	small	$A_{FB}^{\mu\mu}$ off peak
$A_{FB}^{b,0}$ ($\times 10^4$)	992	±	16	0.02	<1	b-quark asymmetry at Z pole Jet charge
$A_{FB}^{pol,\tau}$ ($\times 10^4$)	1498	±	49	0.15	<2	τ polar. and charge asymm. τ decay physics
m_W (keV/c^2)	803500	±	15000	600	300	WW threshold scan Beam energy calibration
Γ_W (keV)	208500	±	42000	1500	300	WW threshold scan Beam energy calibration
$\alpha_s(m_W)(\times 10^4)$	NA		NA	3	small	R_ℓ^W
$N_\nu(\times 10^3)$	2920	±	50	0.8	small	Ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV/c^2)	172740	±	500	20	small	$t\bar{t}$ threshold scan QCD errors dominate
Γ_{top} (MeV/c^2)	1410	±	190	40	small	$t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	m = 1.2	±	0.3	0.08	small	$t\bar{t}$ threshold scan QCD errors dominate
$t\bar{t}Z$ couplings		±	30%	<2%	small	$E_{CM} = 365$GeV run

similar requirements to theory@Z pole from CEPC and FCC-ee

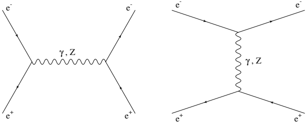
Radiative corrections required for two processes

* s-channel $f\bar{f}$ production



  - differential cross sections
  - Z decay widths

* t-channel small angle Bhabha scattering (large angle included above)



* rad. corr. means
  - Feynman diagrams calculations with bosonic γ, W, Z and fermionic insertions in all possible ways (virtual and real) to the tree-level amplitudes
  - development of simulation tools (MC event generators/MC integrators including consistently higher order amplitudes)

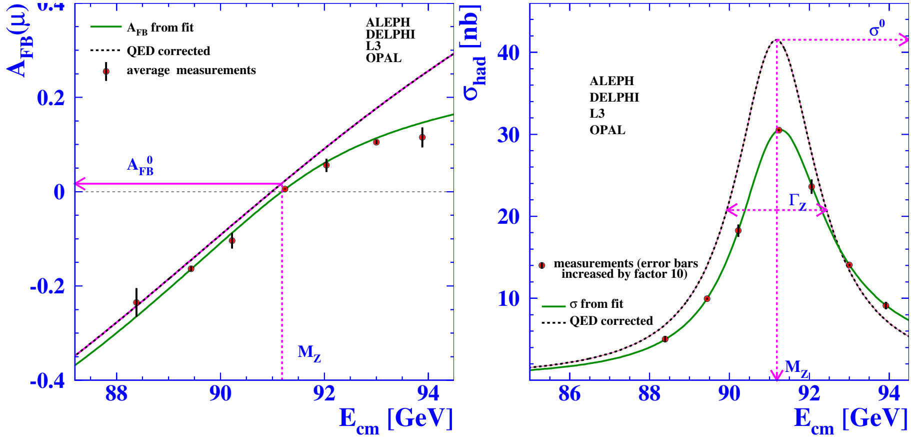
separation of QED and EW corrections

- among the complete SM higher order corrections, photonic (QED) corrections display characteristic features (similarly for QCD corrections, but they appear only for hadronic final states or in higher orders)
  - both virtual and real corrections are involved
  - they develop large IR/collinear logarithms $\sim \log\left(\frac{\Delta E}{Q^2}\right) \log\left(\frac{m^2}{Q^2}\right)$
  - where ΔE is some maximum real photon energy induced by the event selection for final state fermions
  - moreover, $\frac{\Gamma_Z}{M_Z} \sim 3\%$ inhibites hard photon radiation from the initial state

$\Longrightarrow$ large effects on observables, which depend also in a non-trivial way on the applied event selection

  - moreover QED corrections are a gauge-invariant subset of the whole SM corrections

Effect of QED ISR deconvolution



- LEP recipe: separate known large QED effects (and resum them) from $\mathcal{O}(\%)$ (or below) EW one-loop corrections not yet (in the ninetees) "measured"
  - note that m_t and m_H were not yet known, together with the non-abelian trilinear gauge coupling

$$d\sigma = \int dx_1 dx_2 D(x_1, Q^2) D(x_2, Q^2) d\hat{\sigma}(s') \delta(s' - x_1 x_2 s)$$

- by analytical integration over one x dimension, we get the convolution with the radiatior/flux function $H(z, Q^2)$
  - at the prize of being not fully exclusive on both leptons, being able to treat only simplified event selection (e.g. $s' > s_0$)
  - ~ 20 years ago one analytical integration allowed to avoid CPU time problems

$$\sigma_{\rm T}(s) = \int_{z_0}^{1} dz H(z; s) \hat{\sigma}_{\rm T}(zs) \qquad A_{FB}(s) = \frac{\pi \alpha^2 Q_e^2 Q_f^2}{\sigma_{\rm tot}} \int_{z_0}^{1} dz \frac{1}{(1+z)^2} H_{\rm FB}(z; s)\, \hat{\sigma}_{\rm FB}(zs)$$

- H functions known at $\mathcal{O}(\alpha^3)$ for cross sections and $\mathcal{O}(\alpha^2)$ for A_{FB}

ansatz for the kernel cross section

- model-independent parameterization of $\hat{\sigma}(e^+e^- \to f\bar{f})$

$$A_{\rm SM} = A_\gamma + A_Z + \text{non-factorizable}$$

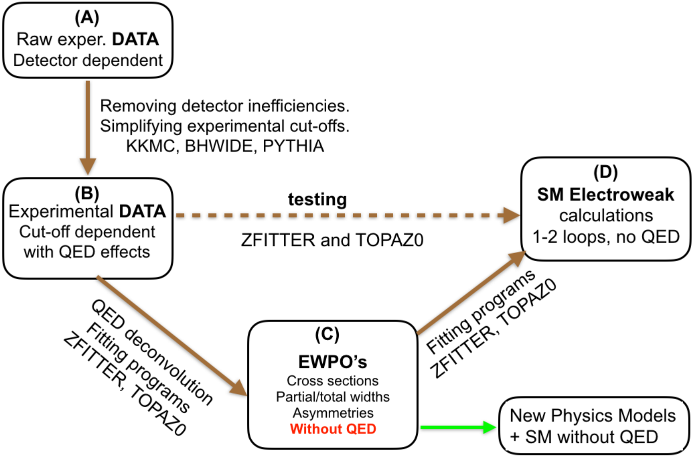
  - aim: write the Z-line shape in a model independent way

Borrelli, Consoli, Maiani, Sisto, NPB333 (1990) 357

$$\sigma^Z_{f\bar{f}} = \sigma^{\rm peak}_{f\bar{f}} \frac{s\Gamma_Z^2}{(s-M_Z)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\sigma^{\rm peak}_{f\bar{f}} = \frac{\sigma^0_{f\bar{f}}}{R_{\rm QED}}; \qquad \sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

- partial widths (or, even better, ratios) can be fitted from data and calculated within SM (and/or possible extension of it) to the desired accuracy
- calculating decay widths is easier w.r.t. a complete cross section
- also easier data combination between different experiments

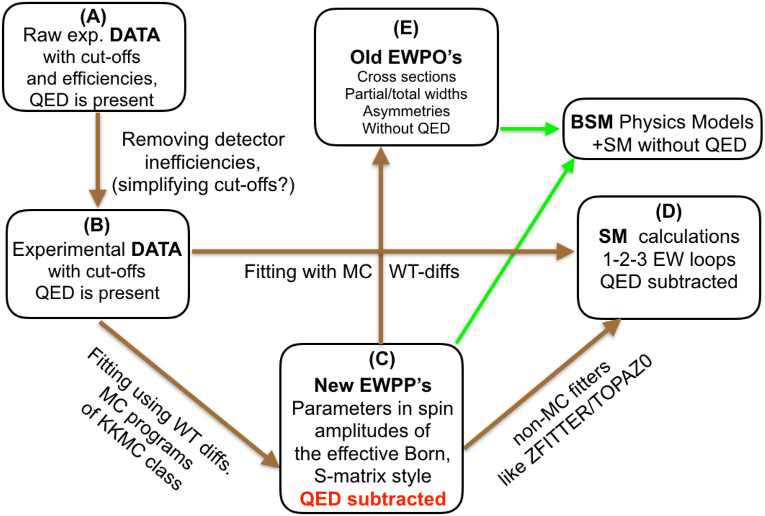


- for $\sim 0.1\%$ precision, it was thoroughly checked that any uncertainty in the procedure was below 0.01% level in the regions of interest

can we extend the method to higher pert. orders?

- The separation between QED and EW corrections becomes more complicated beyond one-loop
  - e.g. a QED virtual and real correction on top of a one-loop virtual EW contribution
  - numerically it is more convenient to adopt a method where the IR cancellations between real and virtual corrections is performed at the integrand level, as realized in the code KKMC
- role played by the initial-final state interference
  - it is naturally suppressed by the factor Γ_Z/M_Z, its typical size being $\sim 0.1\%$
  - however it is important for
    - total cross sections out of peak
    - asymmetry around the peak
  - resummation of IFI effects in presence of resonance available in the literature

M. Greco, G. Pancheri, Y. Srivastava, 1975



- subtraction of only QED or QED *and* EW (SM now well known effects)?

can we extend the method to higher pert. orders?

- kernel cross section should respect analyticity, unitarity and gauge-invariance
  - the general expression of any $2 \to 2$ massless fermion matrix element can be written in terms of 4 form-factors (4 independent helicities), which are computed perturbatively

$$\begin{aligned}\mathcal{M}_{\gamma}^{(0)}(e^-e^+ \to f^-f^+) &= \frac{4\pi i \alpha_{em}(s)}{s} Q_e Q_f \, \gamma_\alpha \otimes \gamma^\alpha, \\ \mathcal{M}_{Z}^{(0)}(e^-e^+ \to f^-f^+) &= 4ie^2 \frac{\chi_Z(s)}{s} \big[M_{vv}^{ef} \, \gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \\ &\quad - M_{va}^{ef} \, \gamma_\alpha \times \gamma^\alpha \gamma_5 + M_{aa}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \gamma_5 \big].\end{aligned}$$

  - $\Longrightarrow$ e.g. $A_{FB} = \frac{3}{4} A_e A_f +$terms that contain the γ exchange and box diagrams. The latter depend on two kinematic invariants and break factorization

- additionally, close to resonance $s \sim s_0$, the matrix element can be represented as a Laurent series

$$\mathcal{M} = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n B^{(n)}$$
$$s_0 = \bar{M}_Z^2 + i \bar{M}_Z \bar{\Gamma}_Z$$

- R and B are gauge invariant and the expansion should be done consistently considering that $s - s_0 \sim g^2$
- The procedure can be safely adopted for the complete two-loop calculation and then subtract consistently γ exchange and boxes
- however there are some challenges for complete two-loop calculations
  - ambiguities in the γ^5 definition in dimensional regularization for chiral fermions $\Longrightarrow$ no general solution beyond one-loop
    - this stimulated recent investigations of different regularization methods in four dimensions
  - singularities have to be extracted from diagrams: two main methods available, sector decomposition and Mellin-Barnes
  - numerical integration uncertainty, because of large cancellations among diagrams

- most recent achievement is the complete bosonic two-loop calculation to Z decay

Dubovik et al. 2018

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \overset{(A)}{\to} 7 \overset{(B)}{\to} 5$	$211 \overset{(A)}{\to} 84 \overset{(B)}{\to} 51$
Number of diagrams	15	$2383 \overset{(A,B)}{\to} 1074$	$490387 \overset{(A,B)}{\to} 120472$
Fermionic loops	0	150	17580
Bosonic loops	15	924	102892
Planar / Non-planar	15 / 0	981/133	84059/36413
QCD / EW	1 / 14	98 / 1016	10386/110086

$Z \to e^+e^-, \ldots$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \overset{(A)}{\to} 7 \overset{(B)}{\to} 5$	$211 \overset{(A)}{\to} 84 \overset{(B)}{\to} 51$
Number of diagrams	14	$2012 \overset{(A,B)}{\to} 880$	$397690 \overset{(A,B)}{\to} 91472$
Fermionic loops	0	114	13104
Bosonic loops	14	766	78368
Planar / Non-planar	14 / 0	782/98	65487/25985
QCD / EW	0 / 14	0 / 880	144/91328

Summary

- TeraZ data analysis will require SM EW predictions based on complete two loop calculations plus resummed higher orders QED effects and partial three/four loop contributions
- the data analysis strategy through the definition of pseudo-observables has to be carefully investigated
- at least in principle the method exists for a proper separation of QED effects from EW ones at higher orders
- the calculation of the radiative corrections to the hard scattering has to be defined in the pole scheme, in order to respect general properties like analyticity, unitarity and gauge invariance
- different seminumerical/analytical methods for dealing with multiloop diagrams already exist but progress and independent approaches are required
- construction of independent precision simulation tools is required, in order allow for detailed cross-checking and deliver robust th-predictions

Next Workshop on precision calculations: CERN, 7-11 January 2019

Thank you!