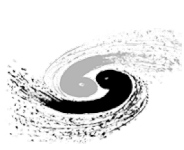


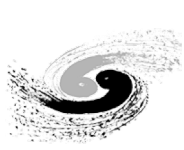
The 16 pairs of circular coils for EMF shielding of JUNO's detector

Teerapat Payupol,
Narumon Suwonjandee and
Burin Asavapibhop



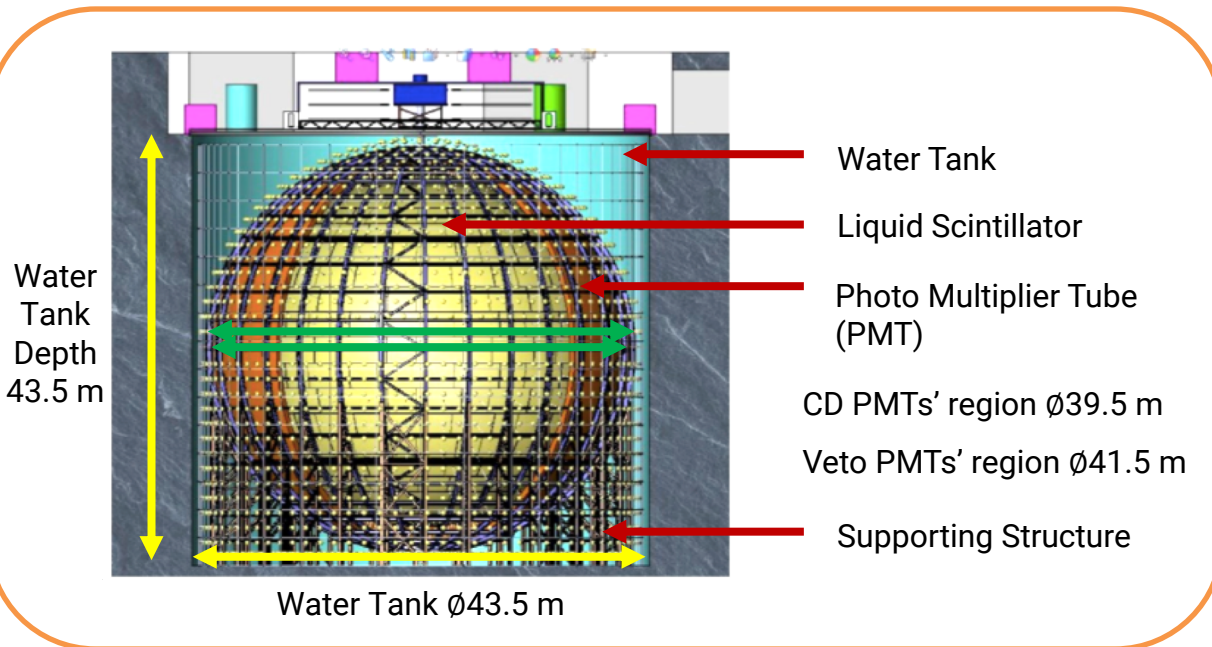
Outlines

- Introduction
- 16 pairs of circular coils with equal space
 - The coils with repeated current
- Mis-location study
- The coils with small curves
- Effect of EMF secular variation
- Conclusions



Introduction

- The Jiangmen Underground Neutrino Observatory (JUNO) consists of the Photo Multiplier Tubes (PMTs) that detect the light signal from neutrino's interactions.
- **The magnetic field can reduce the PMTs' efficiency**
- At JUNO's construction site, the Earth Magnetic Field (EMF) is approximately 0.448 G. Therefore, **the PMTs are necessary to be shielded from the EMF.**
- This study aims to design **current-carrying coils that generate magnetic field in the opposite direction of the EMF**, thus, the two field compensate each other.



EMF (Calculated)

X component = -0.37988 G
(Geographic North)

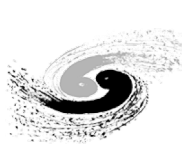
Y component = -0.01505 G
(Geographic West)

Z component = -0.23772 G (Down)

Norm of the EMF = 0.44839 G

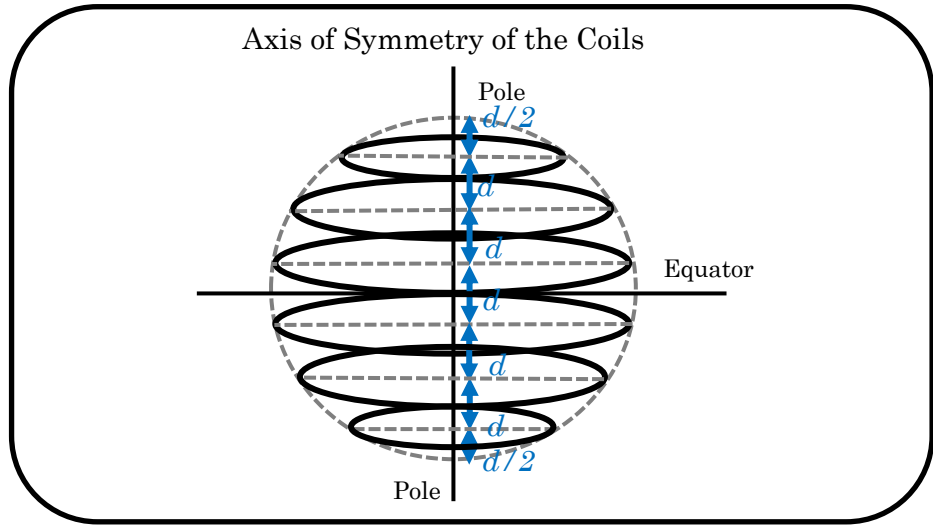
Declination = -2.269°

Inclination = 32.017°

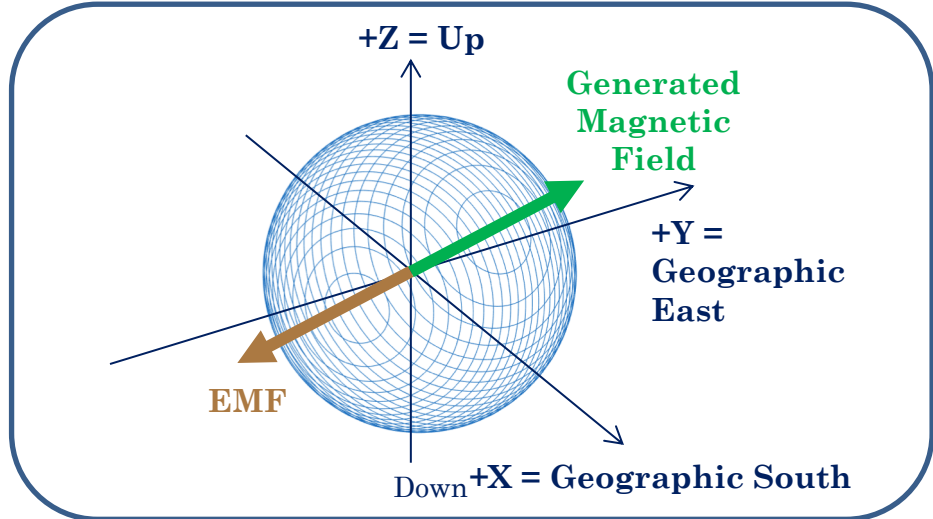


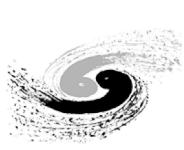
16 Pairs with Equal Space

- Spherical coils consist of **32** circular coils with the **same space of 1.36 m**.
- The axis of symmetry of the coils lays **exactly opposite to the EMF direction**.
- The currents are optimize at the spherical surface of diameter **39.5 m (CD PMTs)**



No.	Radii (m)	Locations (m)	Currents (A)
1	5.39	21.07	58.35
2	9.19	19.71	76.72
3	11.67	18.35	71.20
4	13.58	16.99	73.55
5	15.12	15.63	72.34
6	16.41	14.27	73.01
7	17.50	12.91	72.60
8	18.43	11.55	72.87
9	19.21	10.20	72.69
10	19.87	8.87	72.80
11	20.42	7.48	72.73
12	20.87	6.12	72.78
13	21.22	4.76	72.75
14	21.48	3.40	72.77
15	21.65	2.04	72.76
16	21.74	0.68	72.76



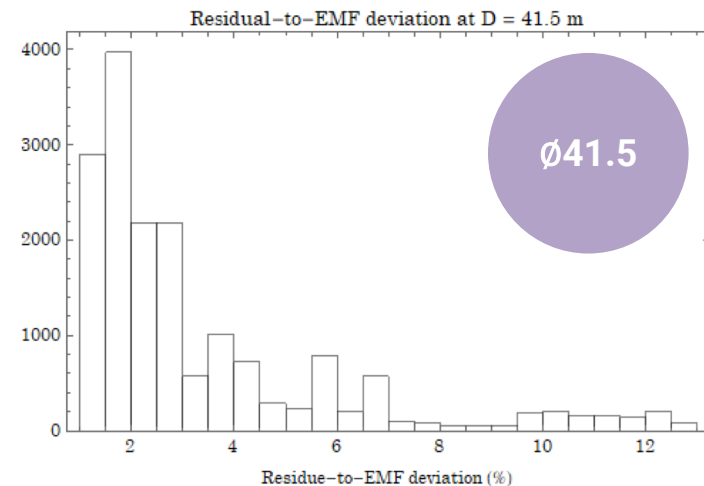
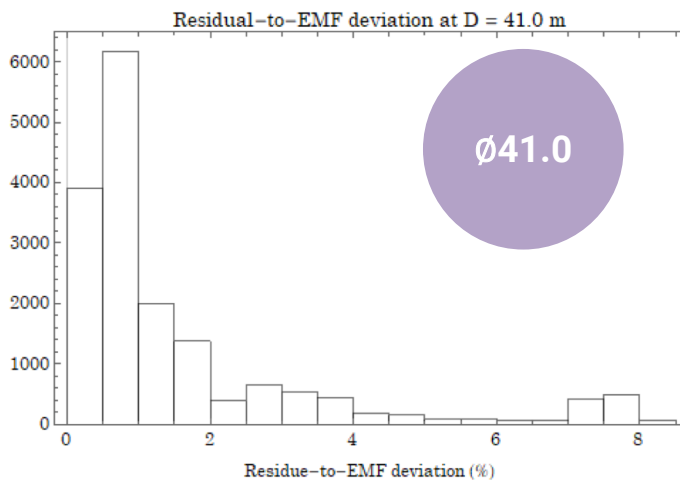
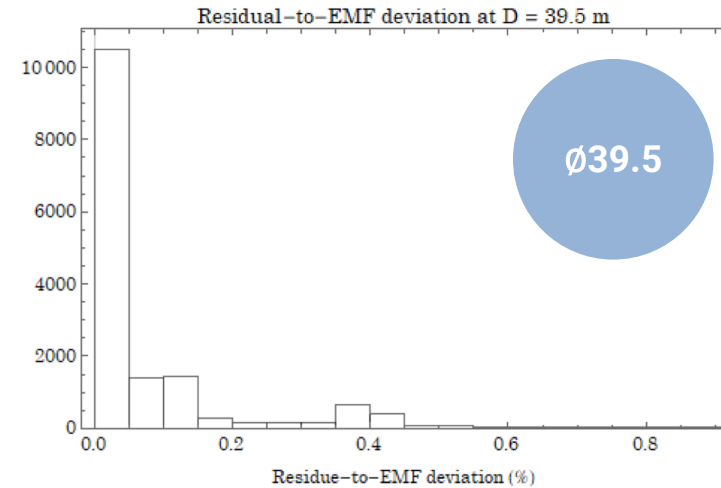
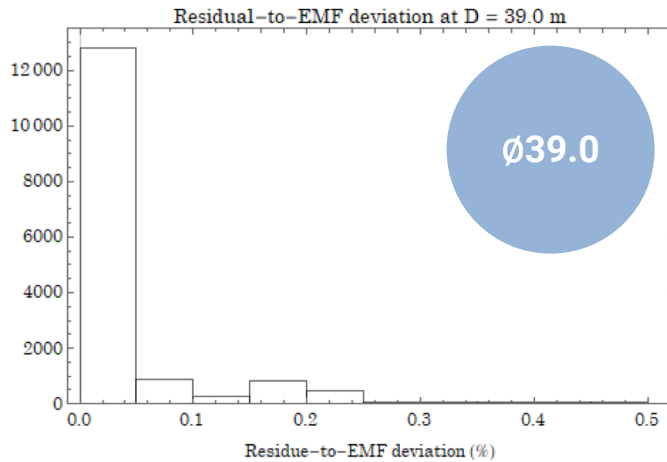


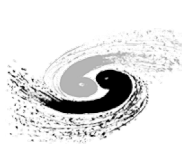
Results | 16 Pairs (1/2)

$$\text{Residue-to-EMF Deviation} = \frac{\sqrt{(B_x + EMF_x)^2 + (B_y + EMF_y)^2 + (B_z + EMF_z)^2}}{EMF} \times 100\%$$

Preliminary requirement

- The deviation < 10% at CD region ($\phi=39.5$ m)
- The deviation < 20% at veto region ($\phi=41.5$ m)





Results | 16 Pairs (2/2)

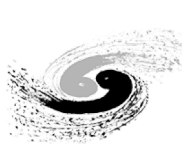
$$\text{Residue - to - EMF Deviation} = \frac{\sqrt{(B_x + EMF_x)^2 + (B_y + EMF_y)^2 + (B_z + EMF_z)^2}}{EMF} \times 100\%$$

Preliminary requirement

- The deviation < 10% at CD region ($\phi=39.5$ m)
- The deviation < 20% at veto region ($\phi=41.5$ m)

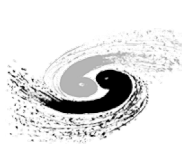
Diameter (m)	Max of residue-to-EMF deviation (%)	Mean of residue-to-EMF deviation (%)
41.8	17.45	5.50
41.5	12.65	3.40
41.0	8.01	1.62
40.0	4.08	0.45
39.5	2.95	0.28
39.0	2.32	0.17

- **At CD region, the maximum of residue-to-EMF deviation is less than 5% with mean value is less than 1%**
- **At veto region, the maximum of residue-to-EMF deviation is less than 15% with mean value is less than 5%**



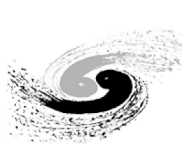
Current Optimization with Constraints

- Since currents in some coils are roughly the same (less than 3 A difference)
- The optimization was performed with 3 constraints
 - 1. The currents for coils no. 3-30 are the same**
 - 2. The currents for coils no. 2-31 are the same**
 - 3. The currents for all coils are the same**
- The optimization is done at the surface of $\emptyset = 39.5 \text{ m}$



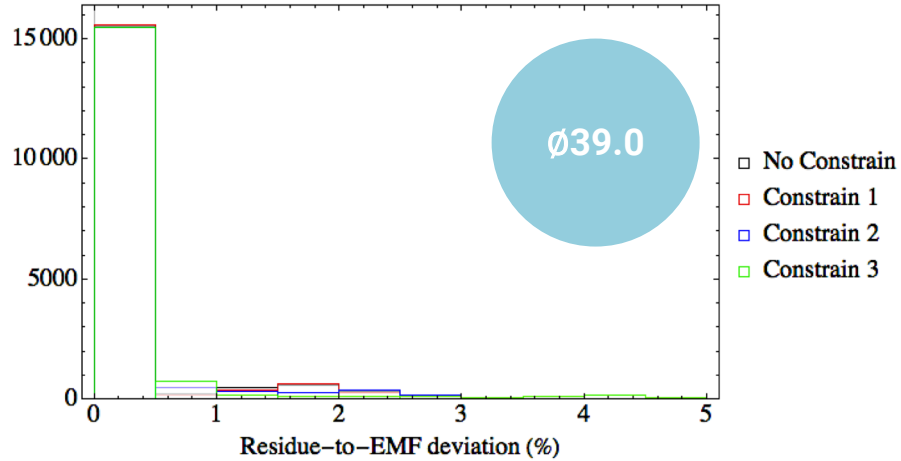
Result from Current Optimization with Constraints (1/3)

Coil	Radius (m)	Location (m)	Currents (A)			
			No Constraint	Constraint 1	Constraint 2	Constraint 3
1	5.39	21.07	58.35	59.24	61.60	72.73
2	9.19	19.71	76.72	75.76	72.78	72.73
3	11.67	18.35	71.20	72.75	72.78	72.73
4	13.58	16.99	73.55	72.75	72.78	72.73
5	15.12	15.63	72.34	72.75	72.78	72.73
6	16.41	14.27	73.01	72.75	72.78	72.73
7	17.50	12.91	72.60	72.75	72.78	72.73
8	18.43	11.55	72.87	72.75	72.78	72.73
9	19.21	10.20	72.69	72.75	72.78	72.73
10	19.87	8.87	72.80	72.75	72.78	72.73
11	20.42	7.48	72.73	72.75	72.78	72.73
12	20.87	6.12	72.78	72.75	72.78	72.73
13	21.22	4.76	72.75	72.75	72.78	72.73
14	21.48	3.40	72.77	72.75	72.78	72.73
15	21.65	2.04	72.76	72.75	72.78	72.73
16	21.74	0.68	72.76	72.75	72.78	72.73

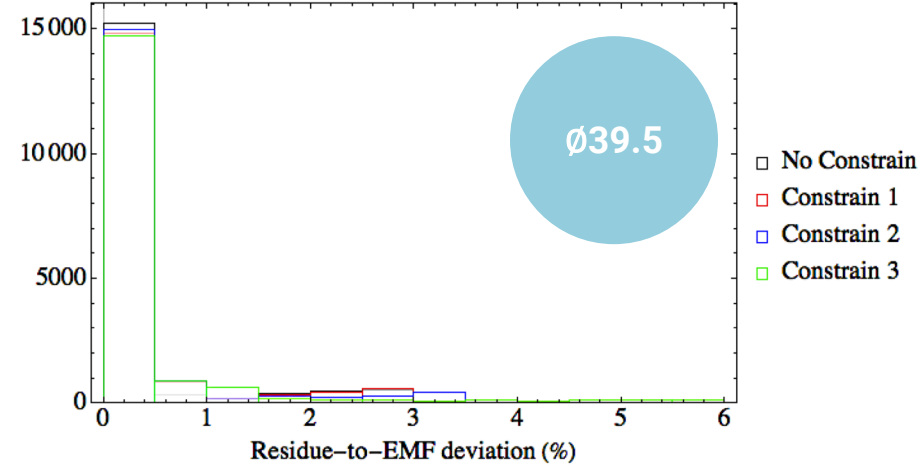


Result from Current Optimization with Constraints (2/3)

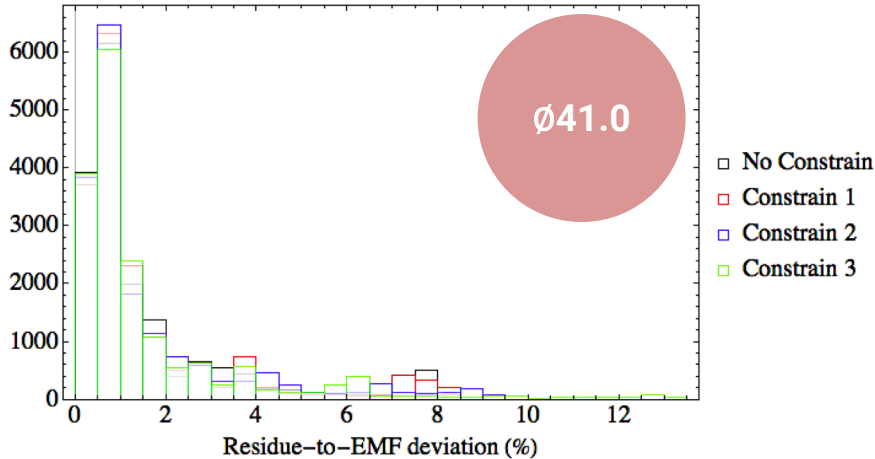
Residual-to-EMF deviation at D = 39.0 m



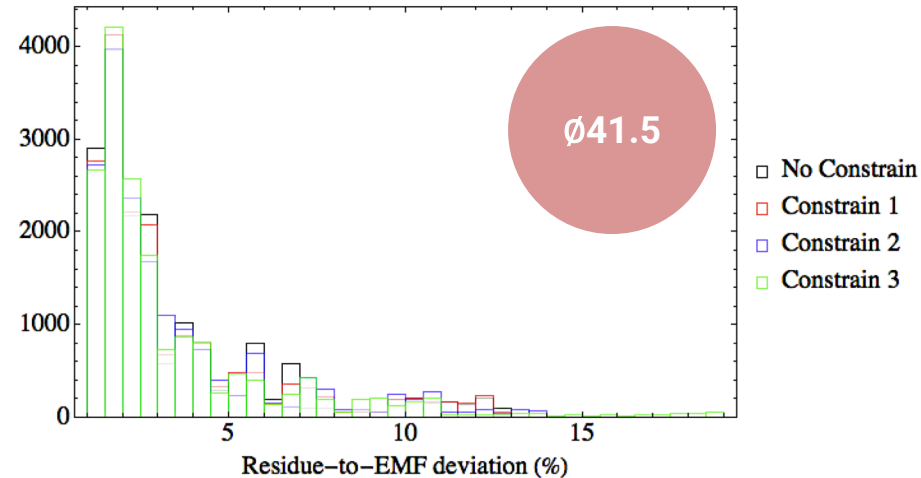
Residual-to-EMF deviation at D = 39.5 m

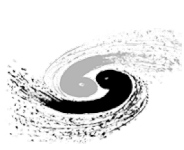


Residual-to-EMF deviation at D = 41.0 m



Residual-to-EMF deviation at D = 41.5 m

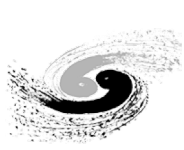




Result from Current Optimization with Constraints (3/3)

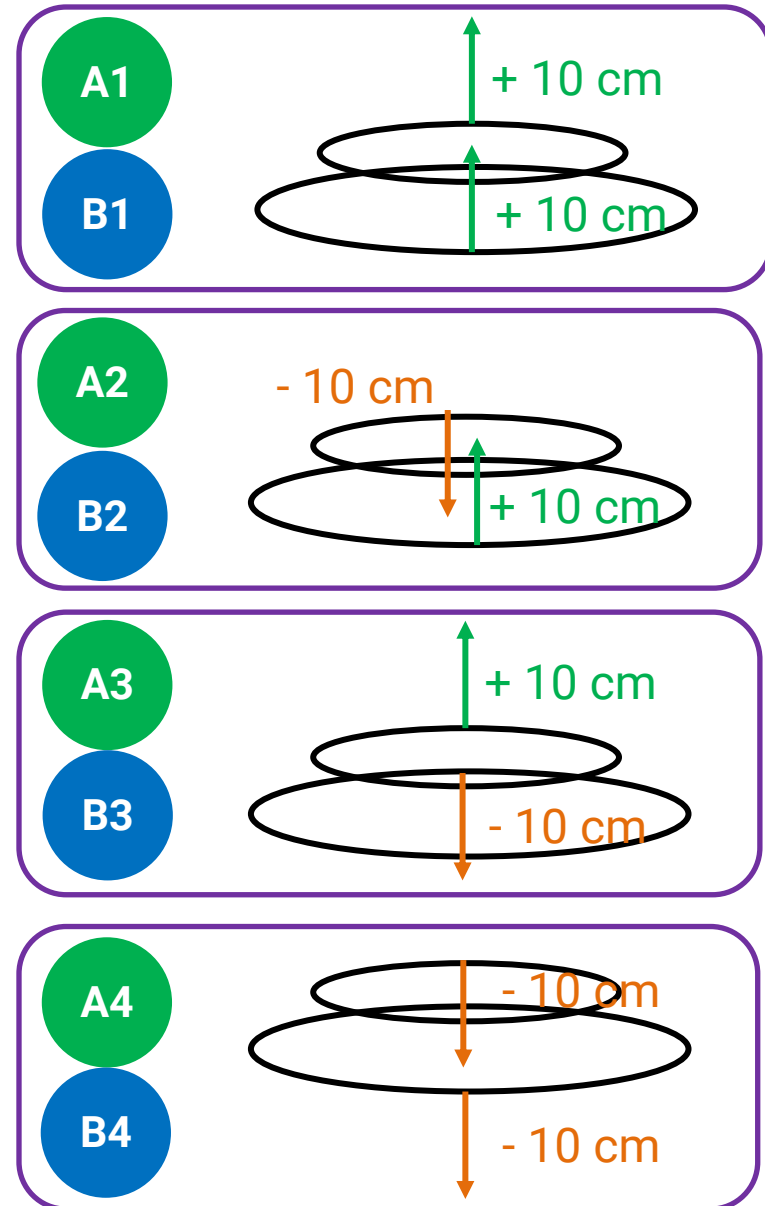
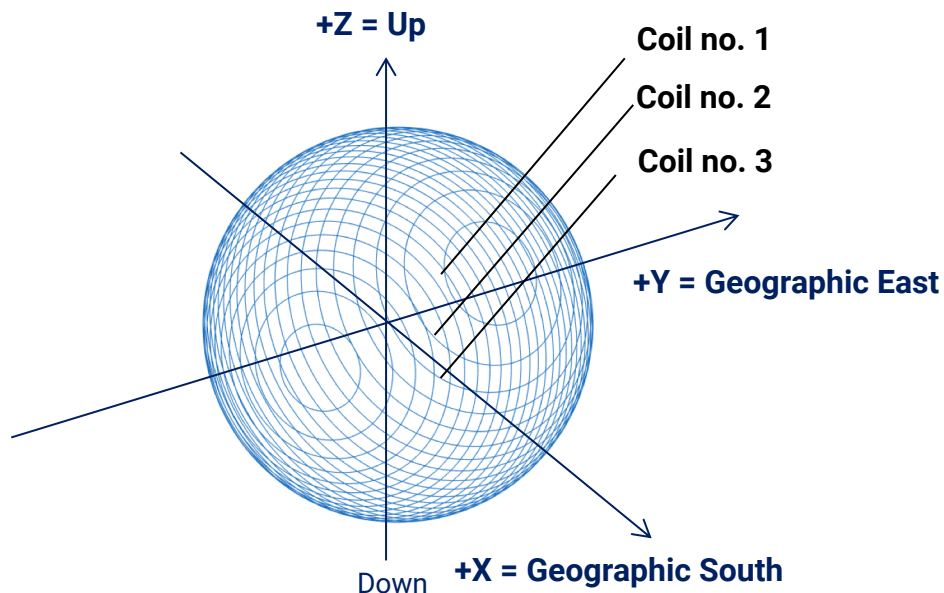
Diameter (m)	No Constrains		Constraint 1		Constraint 2		Constraint 3	
	Max of residue-to-EMF deviation (%)	Mean of residue-to-EMF deviation (%)	Max of residue-to-EMF deviation (%)	Mean of residue-to-EMF deviation (%)	Max of residue-to-EMF deviation (%)	Mean of residue-to-EMF deviation (%)	Max of residue-to-EMF deviation (%)	Mean of residue-to-EMF deviation (%)
41.8	17.45	5.50	17.05	5.50	18.03	2.23	24.00	5.58
41.5	12.65	3.40	12.64	3.40	13.69	3.40	18.73	3.48
41.0	8.01	1.62	8.22	1.62	9.06	1.62	13.04	1.69
40.0	4.08	0.45	4.09	0.48	4.64	0.48	7.23	0.52
39.5	2.95	0.28	2.95	0.28	3.43	0.29	5.65	0.34
39.0	2.32	0.17	2.14	0.18	2.57	0.19	4.53	0.24

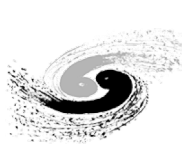
- **At CD region**, currents with **constraint 1 and 2** provide the maximum of residue-to-EMF deviation **less than 5%** and current with **constraint 3** give the maximum **less than 10%**
- **At veto region**, currents with **constraint 1 and 2** provide the maximum of residue-to-EMF deviation **less than 15%** and current with **constraint 3** give the maximum **less than 20%**
- The coils **composing of 30 coils with the same current + other 2 coils** can be utilized and provide acceptable efficiency as the case without constrain.



Mis-location Study

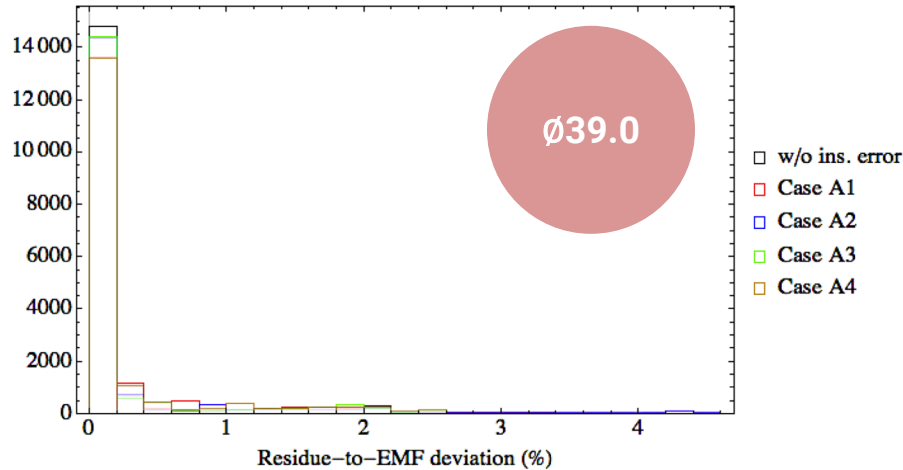
- Two coils are assumed to be mis-located for ± 10 cm.
 - **Case A_**: Coils no.1 and 2
 - **Case B_**: Coils no. 2 and 3
- Note:** The radii are assumed to change in order to make the circular coils remain in the same spherical surface and the currents without constraint are used



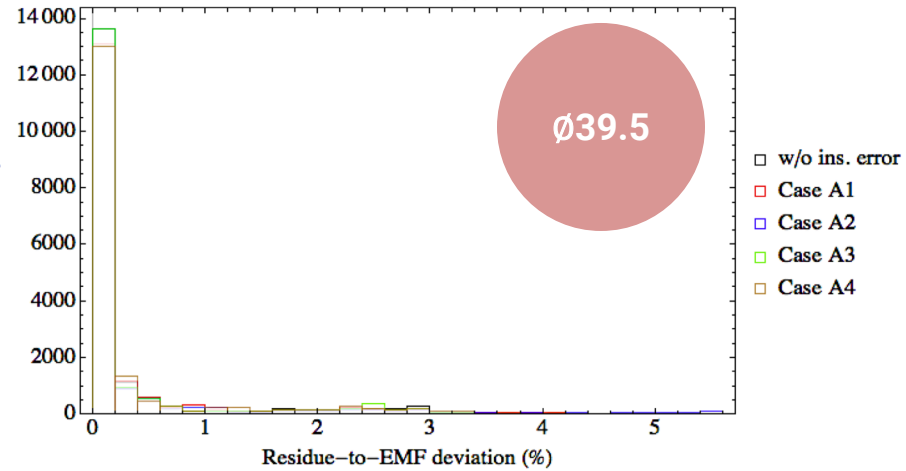


Result of Case A (1/2)

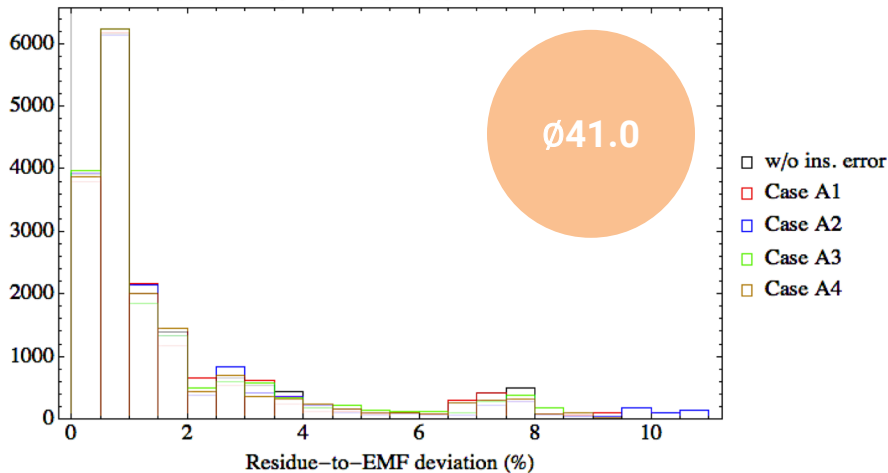
Residual-to-EMF deviation at D = 39.0 m



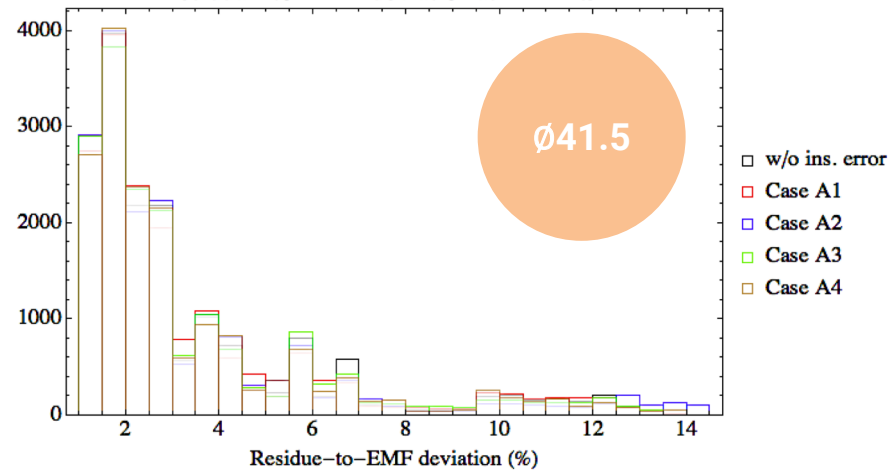
Residual-to-EMF deviation at D = 39.5 m

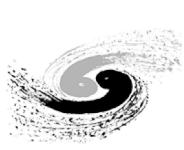


Residual-to-EMF deviation at D = 41.0 m



Residual-to-EMF deviation at D = 41.5 m

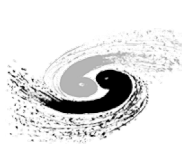




Result of Case A (2/2)

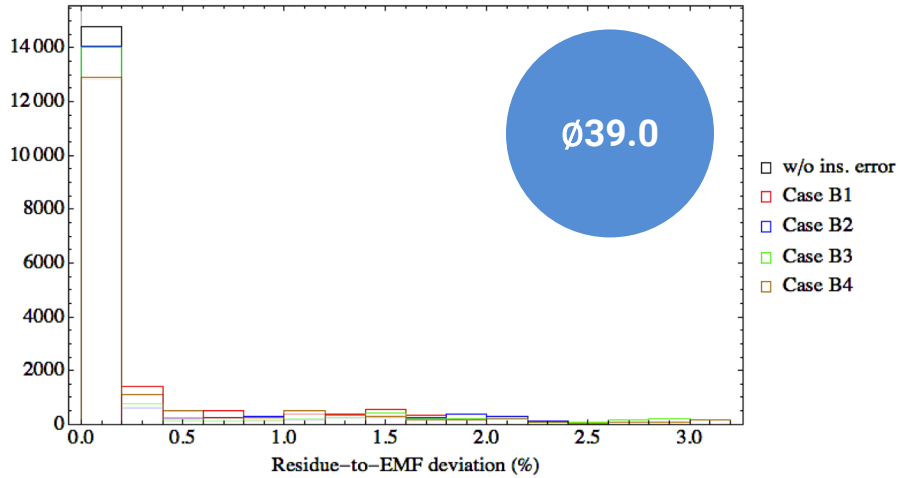
Diameter (m)	Residue-to-EMF Deviation (%)									
	without installation error		Case A1		Case A2		Case A3		Case A4	
	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean
41.8	17.45	5.50	17.56	5.54	18.47	5.53	18.04	5.49	18.31	5.47
41.5	12.65	3.40	13.53	3.44	14.34	3.44	13.18	3.39	13.71	3.38
41.0	8.01	1.62	9.41	1.66	10.91	1.67	8.22	1.61	9.01	1.61
40.0	4.08	0.45	5.32	0.50	6.87	0.53	4.29	0.47	4.51	0.49
39.5	2.95	0.28	4.11	0.34	5.50	0.35	3.26	0.29	3.74	0.34
39.0	2.32	0.17	3.24	0.25	4.43	0.25	2.57	0.20	3.10	0.26

- **At CD region**, the coils **cases A1, A3 and A4** provide the maximum of residue-to-EMF deviation **less than 5%** and current with **constrain A2** give the maximum **less than 10%**
 - Case A2 provide highest deviation possibly because the the smallest coil move away from the pole -> large unshielded area at the pole and large space btw. coil no. 2 and 3.
- **At veto region**, the coils **case A1 to A4** provide the maximum of residue-to-EMF deviation **less than 15%**.
- The construction company guarantees the installation precision of 2 cm -> mis-location coils are negligible.

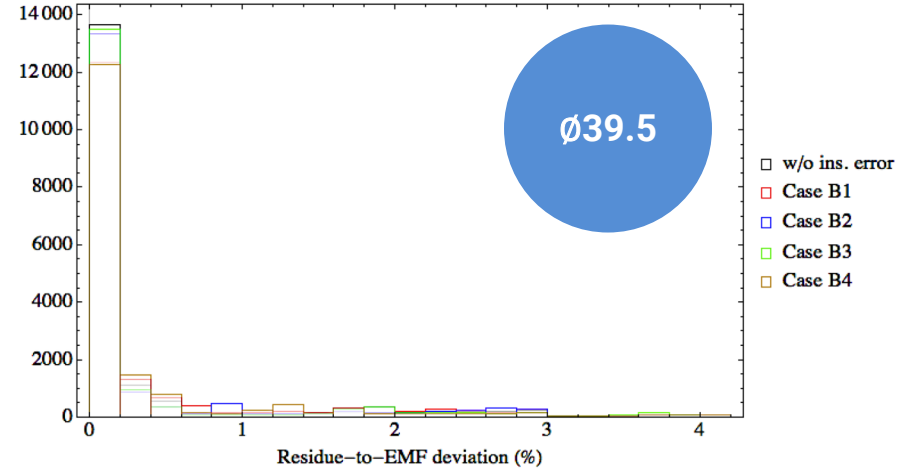


Result of Case B (1/2)

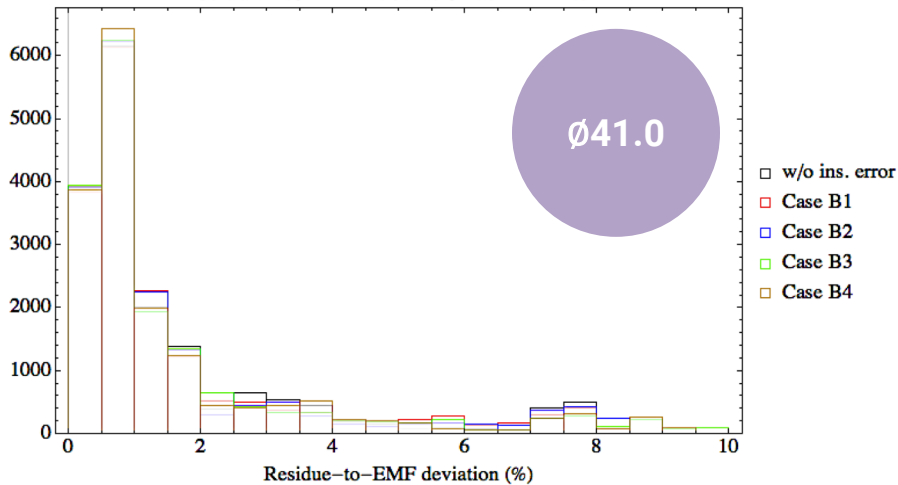
Residual-to-EMF deviation at D = 39.0 m



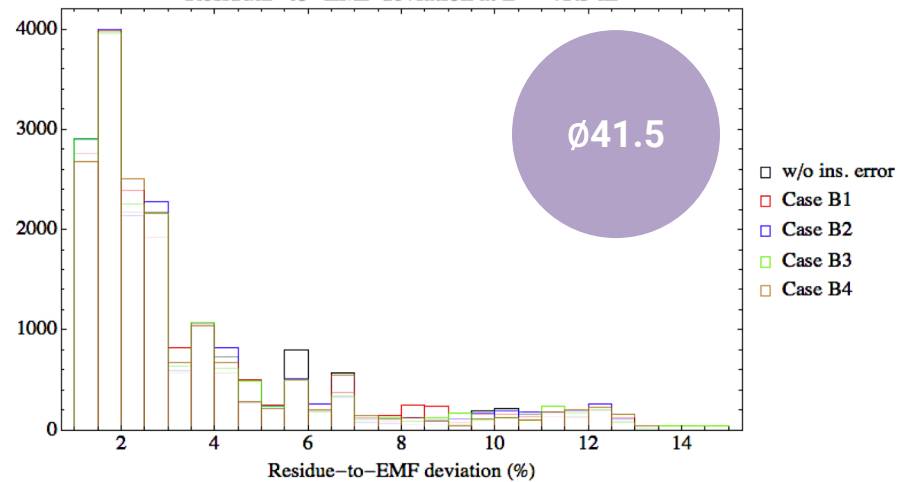
Residual-to-EMF deviation at D = 39.5 m

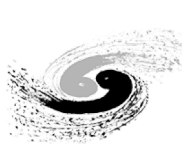


Residual-to-EMF deviation at D = 41.0 m



Residual-to-EMF deviation at D = 41.5 m

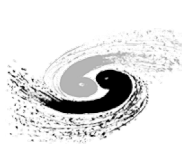




Result of Case B (2/2)

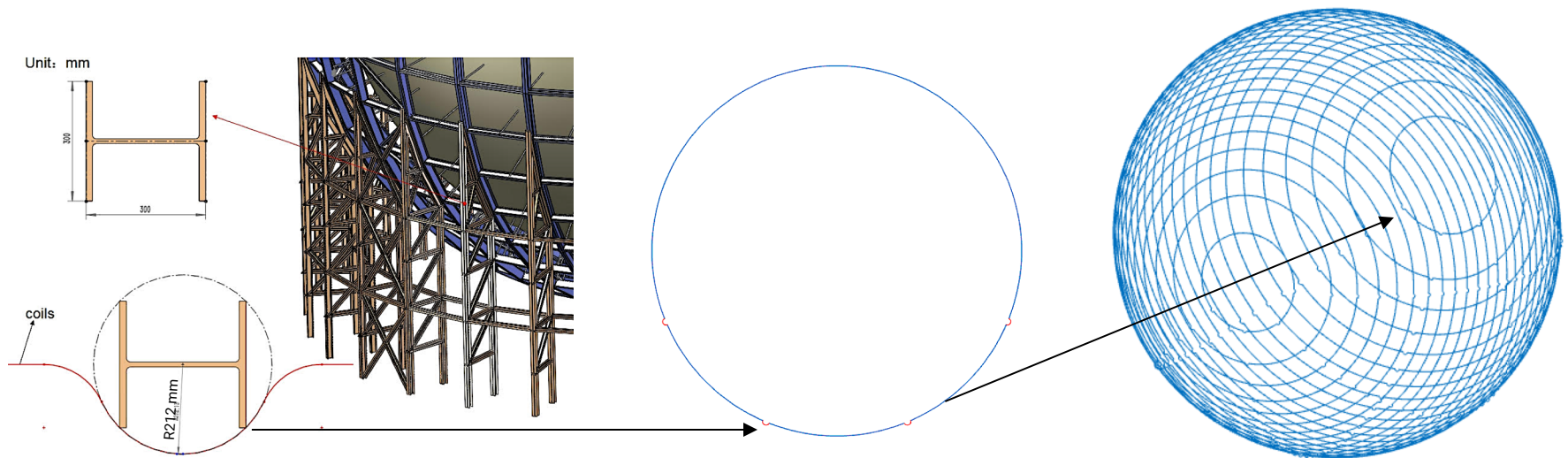
Diameter (m)	Residue-to-EMF Deviation(%)									
	without installation error		Case B1		Case B2		Case B3		Case B4	
	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean
41.8	17.45	5.50	17.85	5.52	17.45	5.53	19.19	5.53	17.63	5.50
41.5	12.65	3.40	12.92	3.42	12.65	3.43	14.62	3.44	13.13	3.41
41.0	8.01	1.62	8.14	1.64	8.31	1.66	9.88	1.68	9.12	1.65
40.0	4.08	0.45	4.09	0.52	4.13	0.52	5.14	0.55	5.29	0.55
39.5	2.95	0.28	2.96	0.36	3.00	0.33	3.84	0.36	4.07	0.39
39.0	2.32	0.17	2.33	0.28	2.32	0.23	2.92	0.26	3.15	0.31

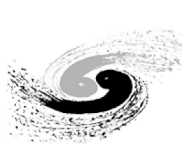
- **At CD region**, the coils **cases A1 to A4** provide the maximum of residue-to-EMF deviation **less than 5%**
- **At veto region**, the coils **cases A1 to A4** provide the maximum of residue-to-EMF deviation **less than 15%**.
- Again, the construction company guarantees the installation precision of 2 cm -> mis-location coils are negligible.



Coils with Small Curve

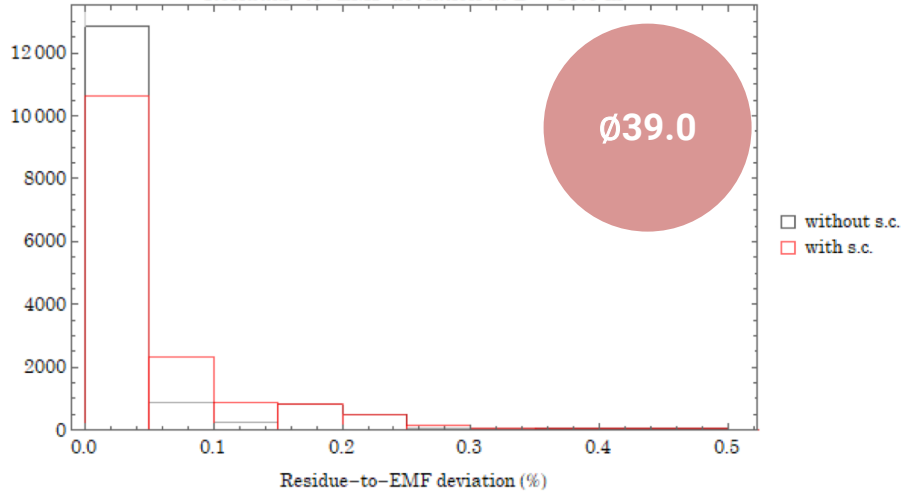
- It is possible that circular coils overlap with supporting truss.
- Some coils need to be installed with small curves.
- **Assumption:** the radius of small curves is 20 cm, there are 4 small curves on all circular coils -> total 128 small curves.



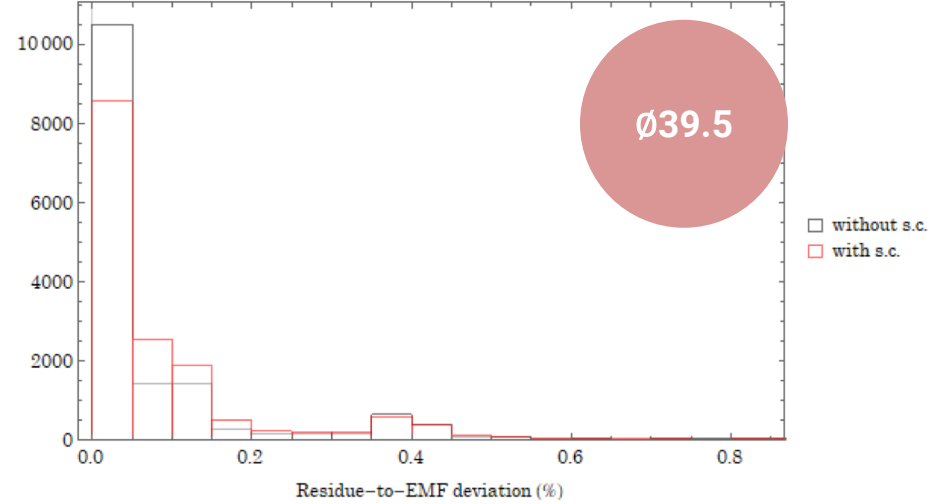


Result of Coils with Small Curve (1/2)

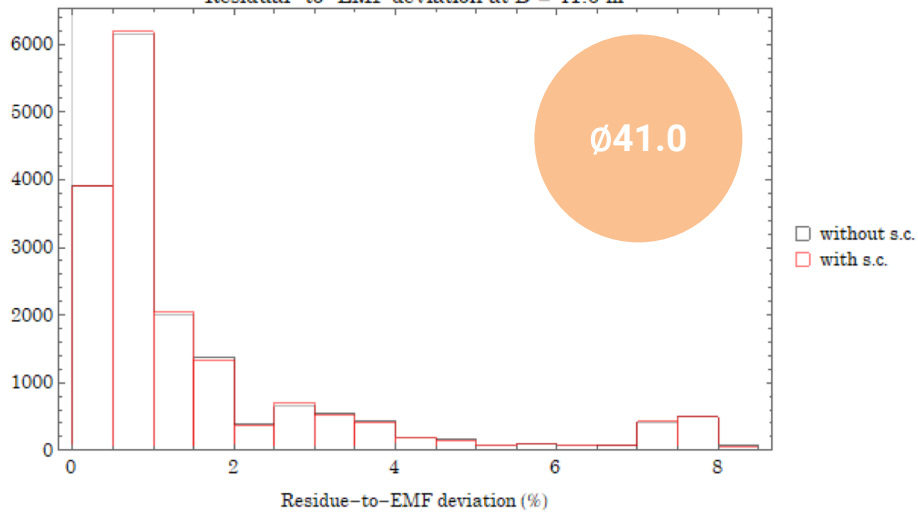
Residual-to-EMF deviation at D = 39.0 m



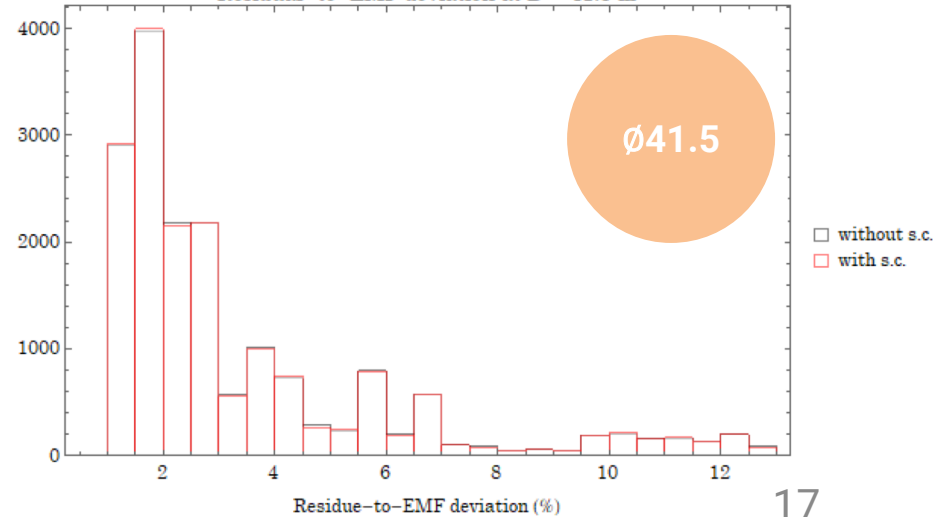
Residual-to-EMF deviation at D = 39.5 m

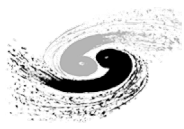


Residual-to-EMF deviation at D = 41.0 m



Residual-to-EMF deviation at D = 41.5 m

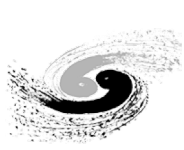




Result of Coils with Small Curve (2/2)

Diameter (m)	Residue-to-EMF Deviation(%)			
	Without Small Curve		With Small Curve	
	Max	Mean	Max	Mean
41.8	17.45	5.50	17.45	5.47
41.5	12.65	3.40	12.64	3.40
41.0	8.01	1.62	8.06	1.62
40.0	4.08	0.45	4.09	0.47
39.5	2.95	0.28	2.96	0.29
39.0	2.32	0.17	2.32	0.19

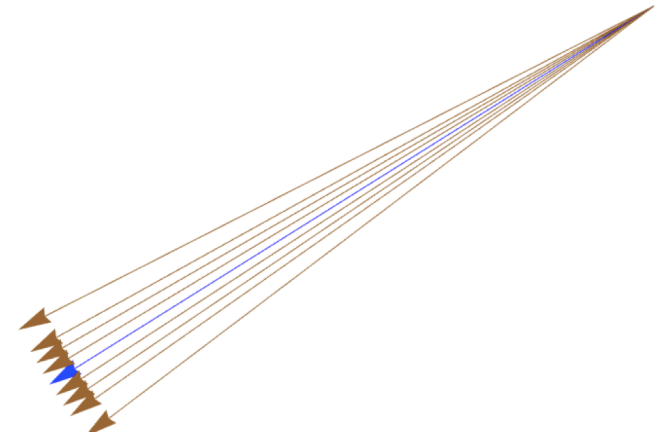
- **At CD region**, the coils with small curves provide higher max and mean of residue-to-EMF deviation as compared to the coils without small curves. However the deviation is **less than 5%**.
- **At veto region**, the coils with small curves provide roughly the same max and mean of residue-to-EMF deviation and the deviation is **less than 15%**.
- The small curves can be built to avoid the supporting truss and the residue-to-EMF deviations are remain acceptable.

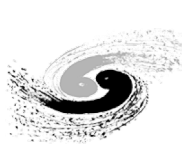


EMF Secular Variation

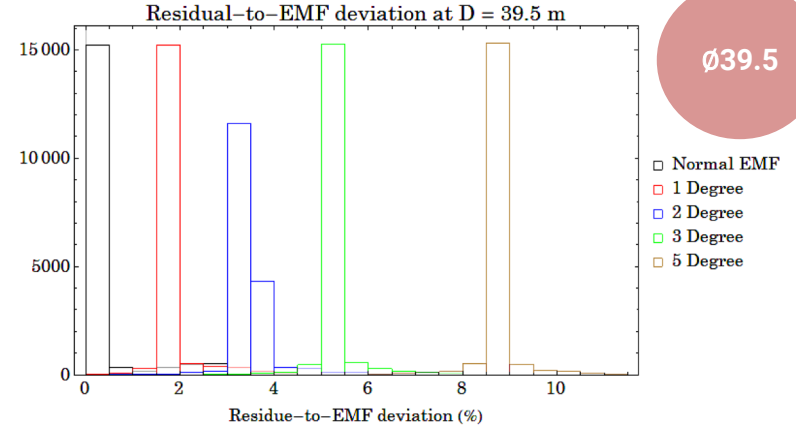
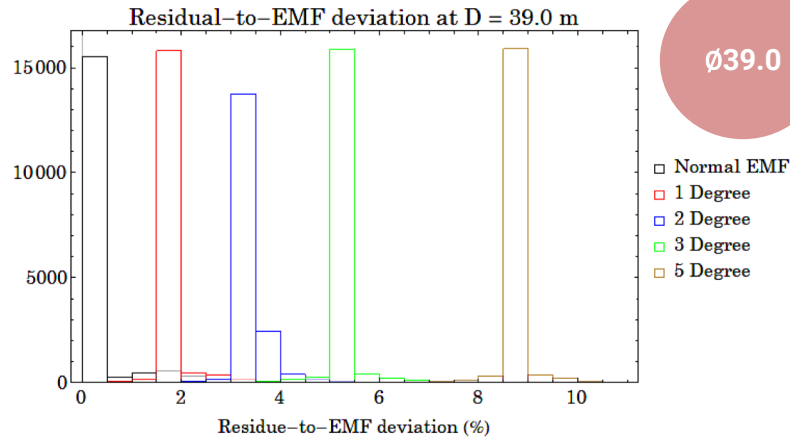
- EMF at Jiangmen changes $-0.0607^\circ/\text{year}$ for declination and $0.1437^\circ/\text{year}$ for inclination. (calculated by American National Centers for Environmental Information)
 - The dominant change is **inclination**, $\sim 2.87^\circ$ of inclination in 20 years.
- How will the residue be if the EMF fluctuates and we still using the same coil?
- Assumption: The EMF turns the direction for 1° , 2° and 5° of its inclination angle, both clockwise and anti-clockwise.
 - However, the total intensity remains the same.

Change in Angle ($^\circ$)	EMF _x (G)	EMF _y (G)	EMF _z (G)
5	0.3577	0.01505	0.2699
3	0.3669	0.01505	0.2573
2	0.3713	0.01505	0.2508
1	0.3757	0.01505	0.2443
-1	0.3839	0.01505	0.2310
-2	0.3879	0.01505	0.2243
-3	0.3918	0.01505	0.2174
-5	0.3991	0.01505	0.2037



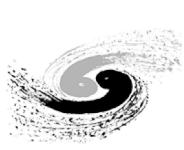


Result of EMF variation (1/2)

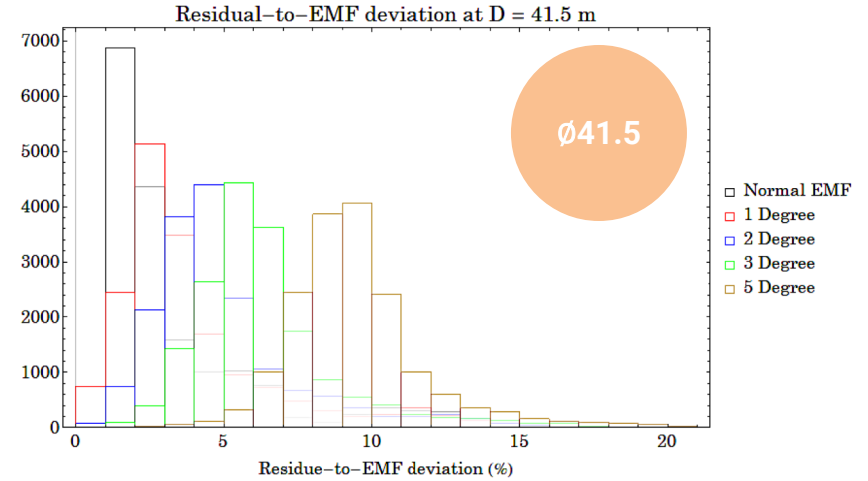
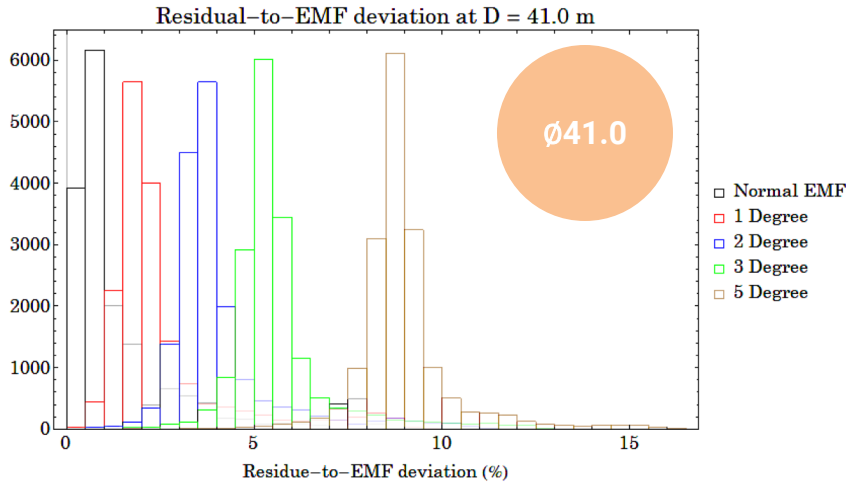


Change in Angle (°)	Residual-to-EMF deviation (%)	
	Max	Mean
5	10.62	8.73
3	7.13	5.25
2	5.39	3.51
1	3.68	1.79
0	2.32	0.17
-1	3.69	1.79
-2	5.43	3.51
-3	7.17	5.25
-5	10.66	8.73

Change in Angle (°)	Residual-to-EMF deviation (%)	
	Max	Mean
5	11.36	8.74
3	7.87	5.26
2	6.13	3.54
1	4.44	1.84
0	2.95	0.28
-1	4.44	1.84
-2	6.18	3.54
-3	7.92	5.26
-5	11.41	8.74

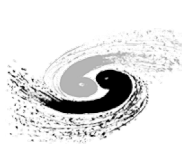


Result of EMF variation (2/2)



Change in Angle (°)	Residual-to-EMF deviation (%)	
	Max	Mean
5	16.38	8.96
3	12.99	5.62
2	11.30	4.02
1	9.63	2.54
0	8.01	1.62
-1	9.54	2.54
-2	11.17	4.03
-3	12.85	5.63
-5	16.24	8.97

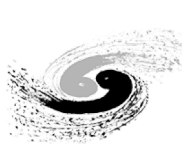
Change in Angle (°)	Residual-to-EMF deviation (%)	
	Max	Mean
5	20.68	9.44
3	17.30	6.33
2	15.62	4.93
1	13.97	3.83
0	12.65	3.40
-1	13.97	3.84
-2	15.51	4.95
-3	17.18	6.35
-5	20.55	9.46



Conclusions

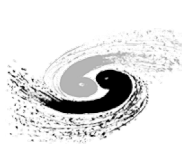
- The **16 pairs of circular coils** forming a sphere of **diameter 43.5 m** with equal space of **1.36 m** are simulated as a JUNO's compensation coils.
- The currents are optimized in the way that the **residual magnetic field is as low as possible on the CD PMT's region.**
 - **At CD region**, the **maximum of residue-to-EMF deviation less than 5%** with **mean value less than 1%**
 - **At veto region**, **the maximum of residue-to-EMF deviation less than 15%** with **mean value less than 5%**
- The coils **composing of 30 coils with the same current + other 2 coils** can be utilized and provide **maximum of residue-to-EMF deviation less than 5%** at CD region and **less than 15% at veto region.**
- **The 16 pairs of coils with two coils that mis-located for ± 10 cm** provide the **maximum of residue-to-EMF deviation less than 10% at CD region** and **less than 15% at veto region.**
 - Since, the construction company guarantees the installation precision of 2 cm -> mis-location coils are negligible.
- The small curves can be built to avoid the supporting truss and provide **maximum of residue-to-EMF deviation less than 5%** at CD region and **less than 15% at veto region.**
- When the EMF inclination angle changes less than 3° during JUNO's operating time, the maximum of residue-to-EMF deviation **less than 10% at CD region** and **less than 20% at veto region.**

Meet
Prelimi-
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Require-
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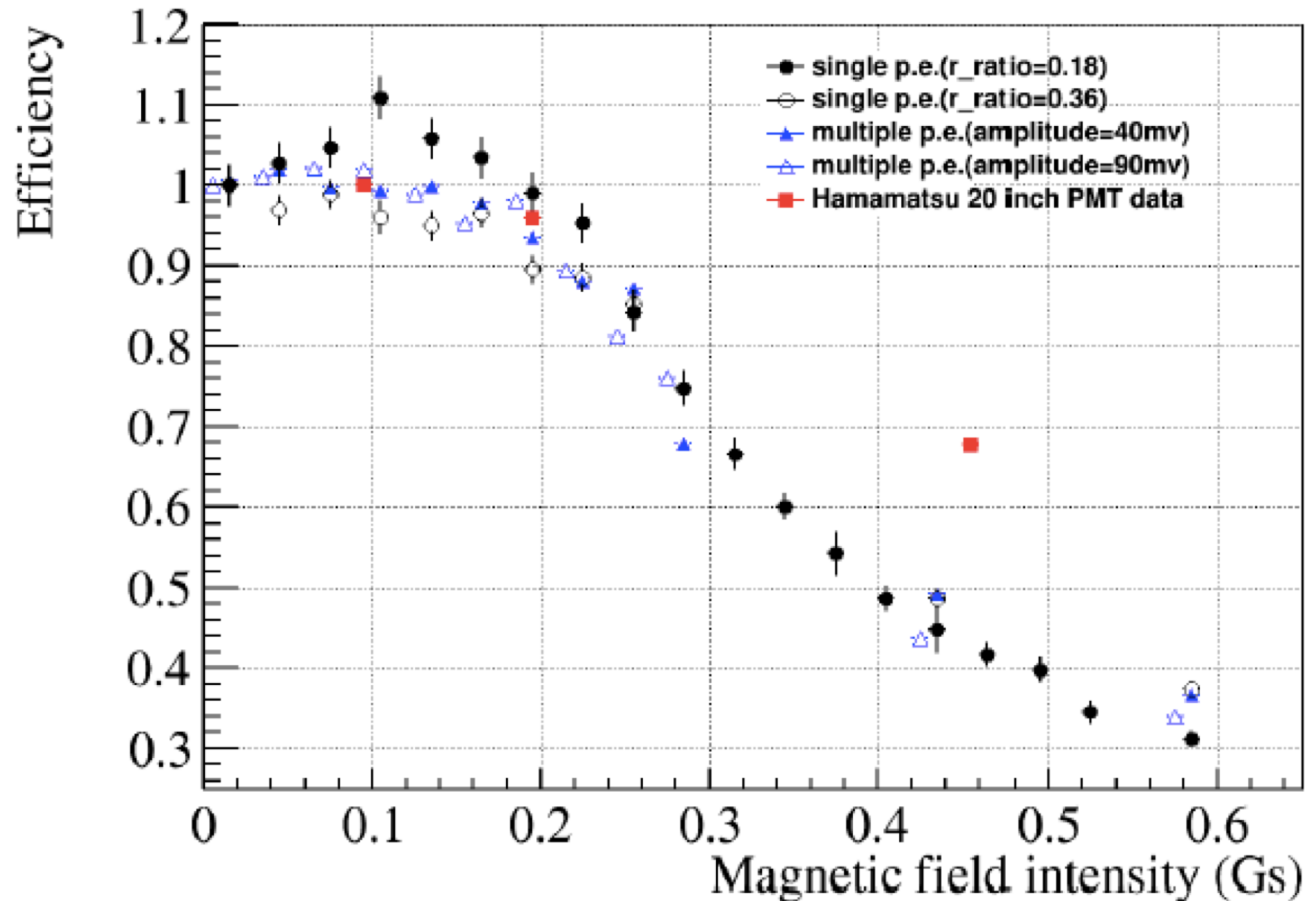


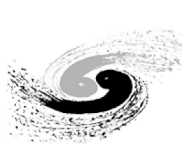
Thank you

Question?



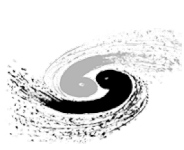
Back Up | PMT's efficiency VS Magnetic Field





Current Optimization (1/4)

- Points of Interest: point on sphere of diameter 39.0 m
- We want to find the currents flowing in each coil that generates B-field **close** to $(0.37988, 0.01505, 0.23772)$ G.
 - These values are referred as EMFx, EMFy and EMFz respectively.
 - The EMF was assumed to be uniform for this calculation.
- Use Least Square method in Mathematica for optimization of the currents
- We need 3 Matrices for optimization
 1. Matrix of coil-generated magnetic field $\rightarrow \mathbf{B}$
 2. Matrix of current's scaling factor or matrix of variable $\rightarrow \mathbf{x}$
 3. Matrix of goal of generated magnetic field (+EMF) $\rightarrow \mathbf{EMF}$



Current Optimization (2/4)

Step 1: Construct Matrix **B**

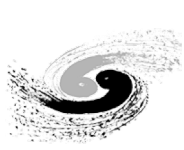
- Let m be the number of coils and n be the number of points of interest
- The magnetic field at **one point** is the magnetic field from all m coils.

- $$\begin{bmatrix} B_{x1}^{c1} & B_{x1}^{c2} & B_{x1}^{c3} & \dots & B_{x1}^{cm} \\ B_{y1}^{c1} & B_{y1}^{c2} & B_{y1}^{c3} & \dots & B_{y1}^{cm} \\ B_{z1}^{c1} & B_{z1}^{c2} & B_{z1}^{c3} & \dots & B_{z1}^{cm} \end{bmatrix}_{\{3 \times m\}}$$

- Now, I consider **n point**

- $$\begin{bmatrix} B_{x1}^{c1} & B_{x1}^{c2} & B_{x1}^{c3} & \dots & B_{x1}^{cm} \\ B_{y1}^{c1} & B_{y1}^{c2} & B_{y1}^{c3} & \dots & B_{y1}^{cm} \\ B_{z1}^{c1} & B_{z1}^{c2} & B_{z1}^{c3} & \dots & B_{z1}^{cm} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{xn}^{c1} & B_{xn}^{c2} & B_{xn}^{c3} & \dots & B_{xn}^{cm} \\ B_{yn}^{c1} & B_{yn}^{c2} & B_{yn}^{c3} & \dots & B_{yn}^{cm} \\ B_{zn}^{c1} & B_{zn}^{c2} & B_{zn}^{c3} & \dots & B_{zn}^{cm} \end{bmatrix}_{\{3n \times m\}}$$

- Each element is calculated **numerically with Radia** and initial currents $I_0 = 1$ A



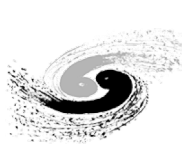
Current Optimization (3/4)

Step 2: Construct a matrix of currents scaling factors \mathbf{x}

- $\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}_{\{m \times 1\}}$
- $I = xI_0$

Step 3: Construct a matrix of constant EMF, **EMF**

- $\begin{bmatrix} EMFx \\ EMFy \\ EMFz \\ \vdots \\ EMFx \\ EMFy \\ EMFz \end{bmatrix}_{\{3n \times 1\}}$

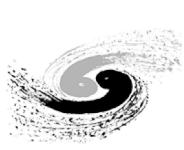


Current Optimization (4/4)

Step 4: Form the equation $\mathbf{mx}=\mathbf{b}$

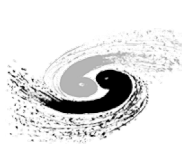
$$\begin{matrix} \bullet \\ \bullet \end{matrix}
 \begin{bmatrix}
 B_{x1}^{c1} & B_{x1}^{c2} & B_{x1}^{c3} & \dots & B_{x1}^{cm} \\
 B_{y1}^{c1} & B_{y1}^{c2} & B_{y1}^{c3} & \dots & B_{y1}^{cm} \\
 B_{z1}^{c1} & B_{z1}^{c2} & B_{z1}^{c3} & & B_{z1}^{cm} \\
 & & \vdots & & \\
 B_{xn}^{c1} & B_{xn}^{c2} & B_{xn}^{c3} & & B_{xn}^{cm} \\
 B_{yn}^{c1} & B_{yn}^{c2} & B_{yn}^{c3} & \dots & B_{yn}^{cm} \\
 B_{zn}^{c1} & B_{zn}^{c2} & B_{zn}^{c3} & & B_{zn}^{cm}
 \end{bmatrix}_{\{3n \times m\}}
 \times
 \begin{bmatrix}
 x_1 \\
 \vdots \\
 x_{16}
 \end{bmatrix}_{\{m \times 1\}}
 =
 \begin{bmatrix}
 EMF_x \\
 EMF_y \\
 EMF_z \\
 \vdots \\
 EMF_x \\
 EMF_y \\
 EMF_z
 \end{bmatrix}_{\{3n \times 1\}}$$

- Use Mathematica to find x that minimize norm of $\mathbf{mx}-\mathbf{b}$
- **The results are m values of the scaling factors, which are equal to optimized electric currents**
 - Then, these currents are used to calculate for magnetic field.



Current Optimization with Constrain (1/4)

- Constrain : The currents for coil pair 1-15 are the same
- Points of Interest: point on sphere of diameter 39.0 m
- We want to find the currents flowing in each coil that generates B-field close to (0.37988, 0.01505, 0.23772) G.
 - These values are referred as EMFx, EMFy and EMFz respectively.
 - The EMF was assumed to be uniform for this calculation.
- Use Least Square method in Mathematica for optimization of the currents
- We need 3 Matrices for optimization
 1. Matrix of coil-generated magnetic field -> **B**
 2. Matrix of current's scaling factor or matrix of variable -> **x**
 3. Matrix of goal of generated magnetic field (+EMF) -> **EMF**



Current Optimization (2/4)

Step 1: Construct Matrix **B**

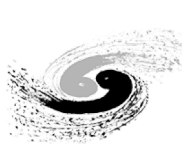
- Let n be the number of points of interest
- The magnetic field at **one point** is the superposition of magnetic field from all 32 coils.

- $$\begin{bmatrix} B_{x1}^{c1} & (B_{x1}^{c2} + \dots + B_{x1}^{c31}) & B_{x1}^{c32} \\ B_{y1}^{c1} & (B_{y1}^{c2} + \dots + B_{y1}^{c31}) & B_{y1}^{c32} \\ B_{z1}^{c1} & (B_{z1}^{c2} + \dots + B_{z1}^{c31}) & B_{z1}^{c32} \end{bmatrix}_{\{3 \times 3\}}$$

- Now, consider **n point**

- $$\begin{bmatrix} B_{x1}^{c1} & (B_{x1}^{c2} + \dots + B_{x1}^{c31}) & B_{x1}^{c32} \\ B_{y1}^{c1} & (B_{y1}^{c2} + \dots + B_{y1}^{c31}) & B_{y1}^{c32} \\ B_{z1}^{c1} & (B_{z1}^{c2} + \dots + B_{z1}^{c31}) & B_{z1}^{c32} \\ \vdots & \vdots & \vdots \\ B_{xn}^{c1} & (B_{xn}^{c2} + \dots + B_{xn}^{c31}) & B_{xn}^{c32} \\ B_{yn}^{c1} & (B_{yn}^{c2} + \dots + B_{yn}^{c31}) & B_{yn}^{c32} \\ B_{zn}^{c1} & (B_{zn}^{c2} + \dots + B_{zn}^{c31}) & B_{zn}^{c32} \end{bmatrix}_{\{3n \times 3\}}$$

- Each element is calculated **numerically with Radia** and initial currents $I_0=1$ A



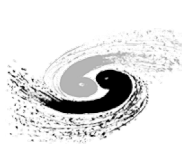
Current Optimization with Constrain (3/4)

Step 2: Construct a matrix of currents scaling factors \mathbf{x}

- $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{\{3 \times 1\}}$
- $I = xI_0$

Step 3: Construct a matrix of constant EMF, \mathbf{EMF}

- $\begin{bmatrix} EMFx \\ EMFy \\ EMFz \\ \vdots \\ EMFx \\ EMFy \\ EMFz \end{bmatrix}_{\{3n \times 1\}}$



Current Optimization with Constrain (4/4)

Step 4: Form the equation $\mathbf{mx}=\mathbf{b}$

$$\begin{bmatrix} B_{x1}^{c1} & (B_{x1}^{c2} + \dots + B_{x1}^{c31}) & B_{x1}^{c32} \\ B_{y1}^{c1} & (B_{y1}^{c2} + \dots + B_{y1}^{c31}) & B_{y1}^{c32} \\ B_{z1}^{c1} & (B_{z1}^{c2} + \dots + B_{z1}^{c31}) & B_{z1}^{c32} \\ \vdots & \vdots & \vdots \\ B_{xn}^{c1} & (B_{xn}^{c2} + \dots + B_{xn}^{c31}) & B_{xn}^{c32} \\ B_{yn}^{c1} & (B_{yn}^{c2} + \dots + B_{yn}^{c31}) & B_{yn}^{c32} \\ B_{zn}^{c1} & (B_{zn}^{c2} + \dots + B_{zn}^{c31}) & B_{zn}^{c32} \end{bmatrix}_{\{3n \times 3\}} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{\{3 \times 1\}} = \begin{bmatrix} EMFx \\ EMFy \\ EMFz \\ \vdots \\ EMFx \\ EMFy \\ EMFz \end{bmatrix}_{\{3n \times 1\}}$$

- Use Mathematica to find x that minimize norm of $\mathbf{mx}-\mathbf{b}$
- **The results are m values of the scaling factors, which are equal to optimized electric currents**
 - Then, these currents are used to calculate for magnetic field.