Freeze-out Hypersurface Calculation

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- Direct Method
- Intersection Method
- Projection Method

Introduction

Cooper-Frye formula

$$E\frac{dN_i}{d^3P}=\frac{dN_i}{dYp_T dp_T d\phi}=\frac{g_h}{(2\pi)^3}\int_\Sigma P^\mu d\Sigma_\mu f_h(x,p)$$

where $d\Sigma_\mu$ is the normal vector of a small piece of freeze-out hyper-surface, and Σ is a 3D hyper-surface in the 4D space and time.

Thermal distribution at freeze-out temperature T_f

$$f(p\cdot u)=\frac{1}{e^{((p\cdot u-u_i)/T_f)}\pm 1}$$

$\pm$： ferminons and bosons

u： flow velocity

All resonnances are assumed to freeze out from the same surface and decay into stable particles.

Introduction

The invariant energy of particle

$$E = p \cdot u = u^{\tau}[m_T \cosh(Y-\eta_s) - \vec{p}_\perp \cdot \vec{v}_\perp - \tau v_\eta m_T \sinh(Y-\eta_s)]$$

The normal vector for one piece of freeze-out hypersurface

$$d\Sigma_\mu = (\tau_f dxdyd\eta_s, -\tau_f d\tau dyd\eta_s, -\tau_f d\tau dxd\eta_s, -d\tau dxdy)$$

Four-momentum $P^\mu = (m_T \cosh(Y-\eta_s), \vec{P}_T, \frac{m_T \sinh(Y-\eta_s)}{\tau})$

$$P^\mu d\Sigma_\mu = (m_T \cosh(Y-\eta_s)\tau dxdyd\eta_s - P_T \cos\phi\, \tau dxdyd\eta_s$$
$$- P_T \sin\phi\, \tau dxdyd\eta_s - m_T \sinh(Y-\eta_s) d\tau dxdy)$$

So we can calculate all observables accoroding to $\frac{dN_i}{dYp_T dp_T d\phi}$

Direct Method

The normal vector

$$d\Sigma_{\mu} = (\tau_f dxdyd\eta_s, -\tau_f d\tau dyd\eta_s, -\tau_f d\tau dxd\eta_s, -d\tau dxdy)$$

Finite grid sizes $\Delta\tau$, Δx , Δy, and $\Delta\eta_s$ are used to calculate $d\Sigma_\mu$

In the τ-direction, a cuboidal volume $d\Sigma_\tau = \tau_f dxdyd\eta_s$ is recorded when the freeze-out temperature falls between $T(\tau_n, x, y, \eta_s)$ and $T(\tau_{n+1}, x, y, \eta_s)$

In every direction, the surface is independent

norm vector → low energy

freeze-out time and 4-velocity

→ Interpolation τ_n - τ_{n+1}

Direct Method

if $(E_{dec} - E_0(i,j,k))(E_1(i,j,k) - E_{dec}) < 0$, then $d\Sigma_0 = sign * \tau dx dy d\eta_s$.

if $(E_{dec} - E_0(i,j,k))(E_0(i+1,j,k) - E_{dec}) < 0$, then $d\Sigma_1 = sign * \tau d\tau dy d\eta_s$.

if $(E_{dec} - E_0(i,j,k))(E_0(i,j+1,k) - E_{dec}) < 0$, then $d\Sigma_2 = sign * \tau d\tau dx d\eta_s$.

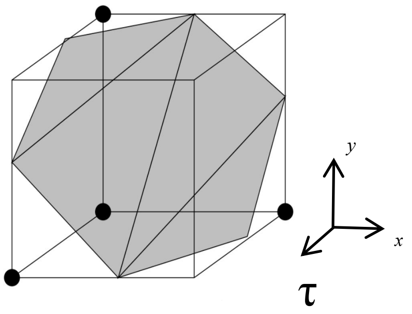
if $(E_{dec} - E_0(i,j,k))(E_0(i,j,k+1) - E_{dec}) < 0$, then $d\Sigma_3 = sign * d\tau dx dy$.

Note: the dreaction to be towards to the lowe energy density

if $(E_{dec} - E_0(i,j,k)) > 0$, then $sign = 1$; else $sign = -1$ for $d\Sigma_0$

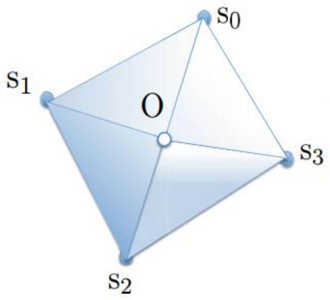
if $(E_{dec} - E_0(i,j,k)) > 0$, then $sign = -1$; else $sign = 1$ for $d\Sigma_1$,$d\Sigma_2$,$d\Sigma_3$

Intersection Method



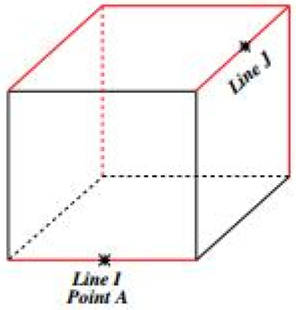
Hyper surface → smaller pieces

The area S of one piece of hyper surface inside an interpolation cube is approximated by the summation of the areas of a group of trangles.

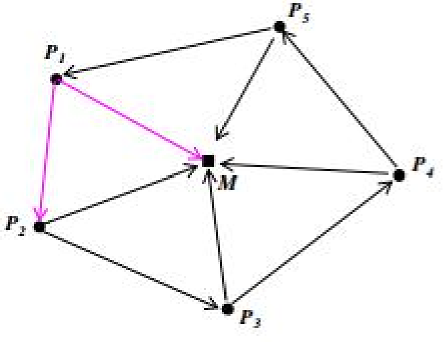


$$S = \sum_{i=1}^{N} \Delta O s_i s_{i+1}$$

Once know center point O and two neighboring intersections, the normal vector:



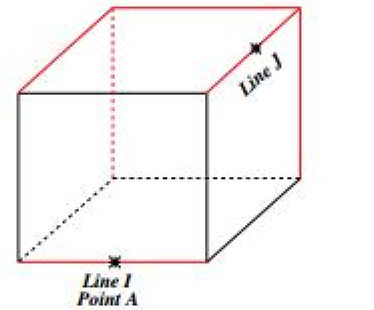
$$\vec{V}_{norm}^{2+1D} = \frac{1}{2}\begin{vmatrix} n & i & j \\ A_0 & A_1 & A_2 \\ B_0 & B_1 & B_2 \end{vmatrix} = nd\Sigma_0 + id\Sigma_1 + jd\Sigma_2$$



Intersection Method

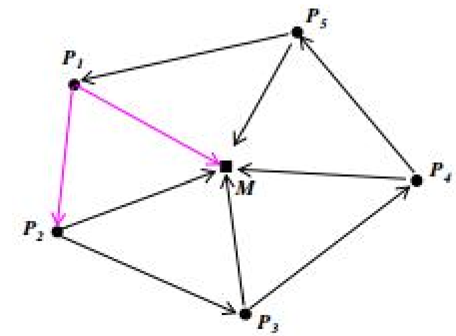
Step 1: Find all the intersection points on the cube

Note the distence between the middle points of two edges has 4 different values ($\sqrt{0.5}$, 1 , $\sqrt{1.5}$, $\sqrt{2}$), larger than or equal to $\sqrt{1.5}$,the weight is 0.



Step2: Intersections are ordered in to a circular sequence.

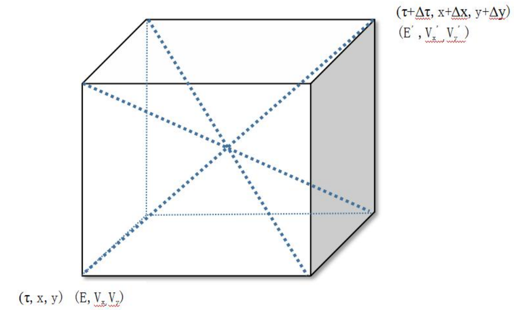
For intersection A, collect all adjacent edges that contain one intersection point. Find the edges that is closest to the edges which have been ordered.



Step3: Calculate hypersurface.

Acoording to the $\overrightarrow{P_1M}$ and $\overrightarrow{P_1P_2}$, calculate the area of the triangle ΔP_1P_2M

Projection Method



Aussmption: Only one intersection on a edge.

Step 1: Determine a cube, label every edge and corners.

Step 2: Identify intersection points whenever energy difference $\varepsilon - \varepsilon_{fr}$ changes sign between two gird points. (Only consider the diagonal corners.)

$$(E_{dec} - E_0(i,j,k))(E_1(i+di,j+dj,k+dk) - E_{dec}) < 0 \quad \text{and}$$
$$(E_{dec} - E_0(i+di,j,k))(E_1(i,j+dj,k+dk) - E_{dec}) < 0 \quad \text{and}$$

$$(E_{dec} - E_0(i,j+dj,k))(E_1(i+di,j,k+dk) - E_{dec}) < 0 \quad \text{and}$$

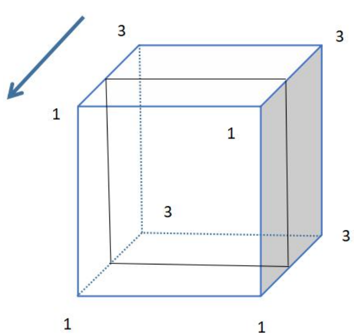
$$(E_{dec} - E_0(i,j,k+dk))(E_1(i+di,j+dj,k) - E_{dec}) < 0 \text{ then intersection=True}$$

Step 3: Identify the energy direction which towards the low energy.

$$VH_i = \frac{\sum_{i=i+\Delta i}(E_i - E_{fr})}{\sum(E_i - E_{fr})}, \; i = \tau, x, y, \; when \; E_i - E_{fr} > 0$$

$$VL_i = \frac{\sum_{i=i+\Delta i}(E_i - E_{fr})}{\sum(E_i - E_{fr})}, \; i = \tau, x, y, \; when \; E_i - E_{fr} < 0$$

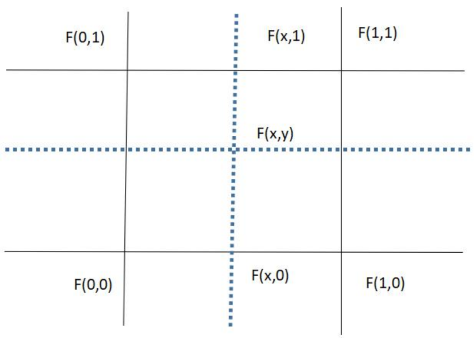
Energy flow vector $\quad \vec{V}_E = \overrightarrow{VL} - \overrightarrow{VH}$



Projection Method

Step 4: Calculate the position of intersection by linear Interpolation at every edge. Meanwhile, calculate the position of center of mass of all intersection.

Step 5: Calculate the velocity through intersection by 3D interpolation.



Firstly, do the interpolation along x direaction to calculate F(x,1) and F(y,1) acoording to F(0,0) , F(0,1),F(1,0),F(1,1)

Secondly, do the interpolation along x direaction to calculate F(x,y) acoording to F(x,1) ,F(y,1)

$$f(x,y) = f(0,0)(1-x)(1-y) + f(1,0)x(1-y) + f(0,1)(1-x)\mathrm{y} + \mathrm{f}(1,1)\mathrm{xy}$$

This interpolation easily extend to N-D interpolation.

$$f(x,y,z) = f(0,0,0)(1-x)(1-y)(1-z) + f(1,0,0)x(1-y)(1-z) + f(0,1,0)(1-x)\mathrm{y}(1-z) + \mathrm{f}(0,0,1)(1-x)(1-y)z$$
$$+ f(1,1,0)xy(1-z) + f(1,0,1)x(1-y)z + f(0,1,1)(1-x)\mathrm{y}z + \mathrm{f}(1,1,1)xyz$$

Projection Method

Step 6: Calculate the hypersurface.

Step 6.1 If N_{int} =3 intersections , then calculate the area of one piece of hypersurface.

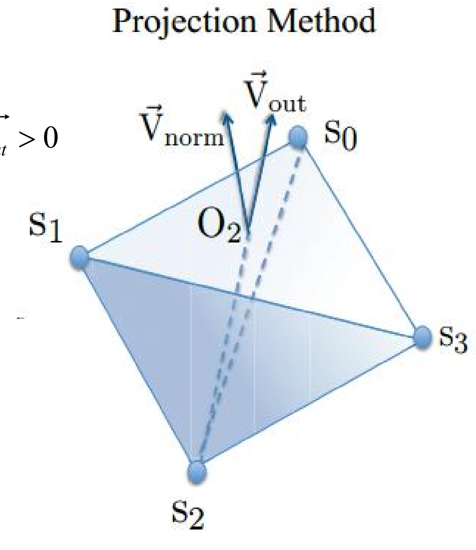
$$\vec{V}_{norm}^{2+1D} = \frac{1}{2}\begin{vmatrix} n & i & j \\ A_0 & A_1 & A_2 \\ B_0 & B_1 & B_2 \end{vmatrix} = nd\Sigma_0 + id\Sigma_1 + jd\Sigma_2$$

Identify sign acoording to $\overrightarrow{V_{norm}} \cdot \overrightarrow{V_E} > 0$

Step 6.2 If N_{int} =4 intersections , select any 4 intersections $s_{i=0.1,2,3}$ to construct a tetrahedron(四面体).

Step 6.3 Consider the norm vector on $\Delta S_0S_1S_3$ and out vector for S_2 , choose the outward normal normal vector that satisfies $\overrightarrow{V_{norm}} \cdot \overrightarrow{V_{out}} > 0$

Step 6.4 Then order the intersections on $\Delta S_0S_1S_3$



Projection Method

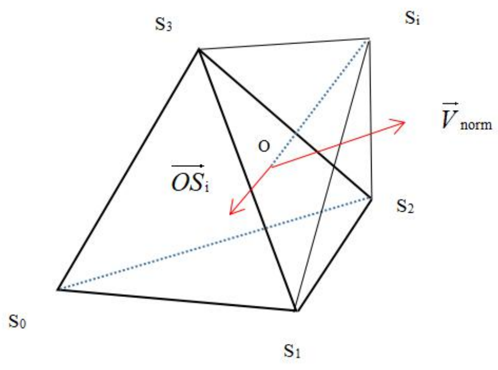
Step 6.5 If N_{int} >5 intersections , do step 6.1-6.5 firstly. Consider another intersection Si outside the first tetrahedron(四面体). Outside condition: $\overrightarrow{V_{\text{norm}}} \cdot \overrightarrow{OS_i} < 0$

Step 6.6 Find all possible outside intersections.

Step 6.7 Construct a new tetrahedron between S_i and each of these triangles. And remove the inner surface.

Step 6.8 Compare all new tetrahedron , remove the inner surface acoording to the order of intersection of one surface.

Step 6.9 Sum all the hypersurface, identify sign acoording to $\overrightarrow{V_{norm}} \cdot \overrightarrow{V_E} > 0$



Thank for your listening.