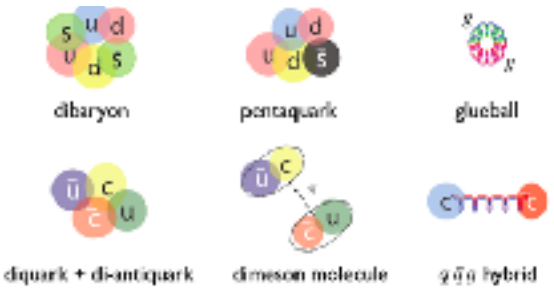
Lecture Plan

Lecture 1: Hadrons as laboratory for QCD:

- **Introduction to QCD**
- **Bare vs effective effective quarks and gluons : Quark Model is important**
- **Phenomenology of Hadrons**



Lecture 2: Complex analysis

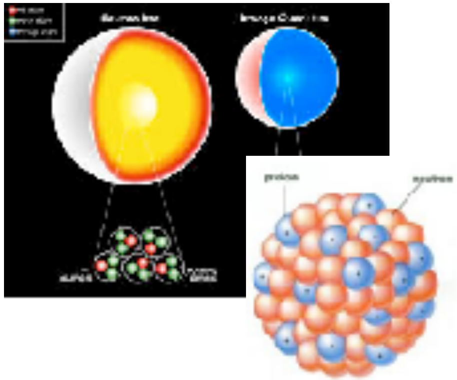
Lecture 3: Phenomenology of hadron reactions

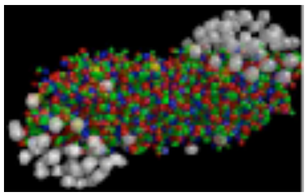
- **Kinematics and observables**
- **Space time picture of Parton interactions and Regge phenomena**
- **Properties of reaction amplitudes**

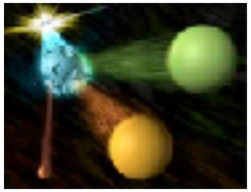
Lecture 4: How to extract resonance information from the data

- **Partial waves and resonance properties**
- **Amplitude analysis methods (spin complications)**

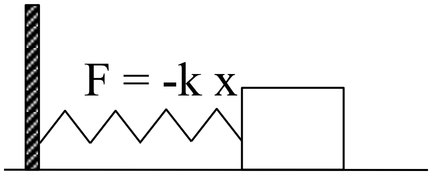
Why QCD and why Hadron Spectroscopy







- A single theory describing nuclear phenomena at distance scales O(10^{15}m) as well as O(10^{4}m).
- It builds from objects (quarks and gluons) that do not exist. Gluons are responsible for mass generation and color confinement.
- ~99% mass comes from interactions!
- Complex ground state (vacuum) and excited (hadrons) states (monopoles, vortices, …)
- Predicts existence of exotic matter, e.g. matter made from radiation (glueballs, hybrids) and novel plasmas.
- A possible template for physics beyond the Standard Model
- It is challenging !



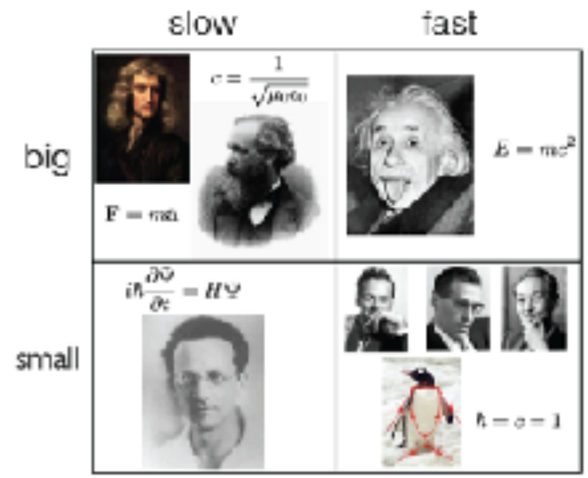
Stranger Things (of the Nuclear World)

What are the constituents of hadrons, (quarks and gluons) ?

small world (10^{-15}m)

of fast (v~c) particles

exerting ~1T forces !!!



$$\hbar = c = 1$$

[length] = [time] = [energy]$^{-1}$
= [momentum]$^{-1}$

Unit energy = 1GeV

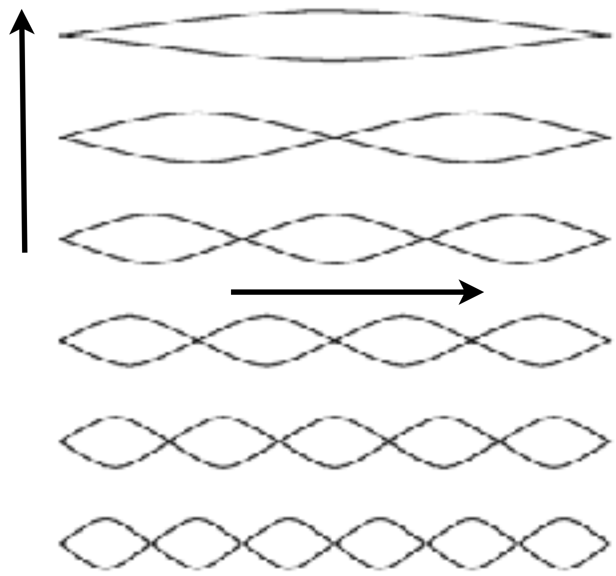
Unit lengt = 1GeV^{-1} = 0.197 fm

Particles vs Fields

In relativistic quantum mechanics (QFT) particles are emergent phenomena

(i.e. fields are not physically measurable but their "consequences" are, cf. potential vs electric field density)

"excitation of the aether" → field



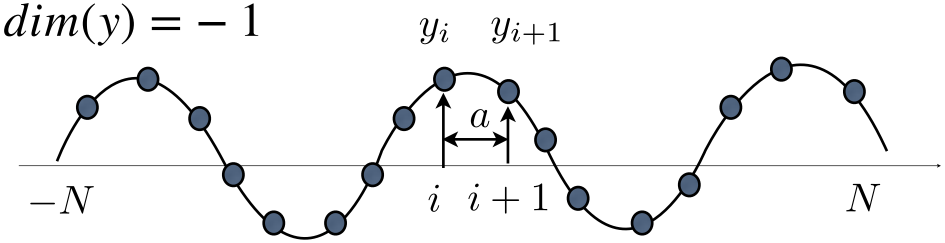
collective motion → particle

H = $H_{h.o}$ = harmonic oscillators

"bare" particles : eigenstates of $H_{h.o.}$

Relativity and QM makes things disappear !

$dim(T) = 2$
$dim(y) = -1$



$$L = \lim_{a \to 0} \left[\sum_i \frac{m\dot{y}_i^2}{2} - \sum_i \frac{T}{2a}(y_{i+1} - y_i)^2 \right]$$

$$L = \frac{1}{2}\int dx \left[\frac{1}{v^2}(\partial_t q)^2 - (\partial_x q)^2 \right] \qquad q = \sqrt{T}y \qquad v^2 = Ta/m$$

- The distance scale a IS the only mass scale left, e.g. E = O(a^{-1}) so there is NO continuum limit for energy. This a reflation of scale invariance of the continuum Hamiltonian.
- A physical theory, like QCD needs a continuum limit. This implies the scale invariance is broken (anomalous anomalous symmetry breaking). In QCD a natural scale emerges $\rightarrow \Lambda_{QCD}$.
- When this is possible (e.g. finite number of interactions) we deal with renormalizable theory, otherwise it is an (in)effective theory

$$H = \frac{1}{2a}\left[\sum_i v^2 p_i^2 + (q_{i+1} - q_i)^2 \right]$$

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 \qquad p_i = -i\partial/\partial q_i \qquad \omega^2 = v^2/a^2$$

Coupled harmonic oscillators, use normal modes to uncouple (Fourier transform)

$$H = \sum_n \omega_n a_n^\dagger a_n + \frac{1}{2}\sum_n \omega_n$$

$$\omega_n = vk_n \qquad k_n = \frac{2\pi n}{Na}$$

$$|n_1, n_2, n_3 \cdots\rangle = a_{n_1}^\dagger a_{n_2}^\dagger a_{n_3}^\dagger \cdots |0\rangle$$

associated with ladder operators, 1-particle with momentum k_1.1-particle with momentum k_2, etc.

$$\frac{\Delta k_n}{k_n} = \frac{1}{N}$$

k's quasi-continuous spectrum for large N

Lets see how this works in practice

In 0+1 dimension (Quantum Mechanics in 1 special dimension) find bound states of the Hamiltonian

$$H = p + \lambda\delta(x) = \sqrt{p^2} + \lambda\delta(x)$$

$$\left(\hat{p} + \lambda \delta(\hat{x})\right)|\psi\rangle = E|\psi\rangle$$

$$\langle k|\hat{p}|\psi\rangle + \lambda \langle k|\delta(\hat{x})|\psi\rangle = E\langle k|\psi\rangle$$

$$|k|\psi(k) + \lambda \int \frac{dk}{2\pi} \underbrace{\langle k|\delta(\hat{x})|k'\rangle}\psi(k') = E\psi(k)$$

$$= \int dx\, e^{i(k-k')x}\delta(x) = 1$$

$$|k|\psi(k) + \lambda \underbrace{\int_{-\infty}^{+\infty} \frac{dk}{2\pi}\psi(k)}_{N} = E\psi(k)$$

$$\psi(k) = \frac{-\lambda N}{|k| - E} \Rightarrow N = -\frac{\lambda N}{2\pi}\int_{-\infty}^{+\infty}\frac{dk}{|k|-E}$$

$\Rightarrow$ need $\lambda < 0$, $E < 0$ and cutoff $\pm\infty \to \pm\Lambda$

$$\Rightarrow 1 = \frac{(-\lambda)}{\pi}\int_0^{\Lambda}\frac{dk}{|k| + (-E)} = \frac{|\lambda|}{\pi}\ln\frac{\Lambda}{|E|}$$

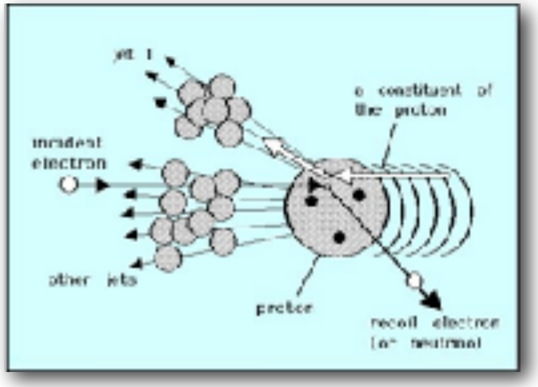
$$\Rightarrow |E| = \Lambda e^{-\frac{\pi}{|\lambda|}}$$

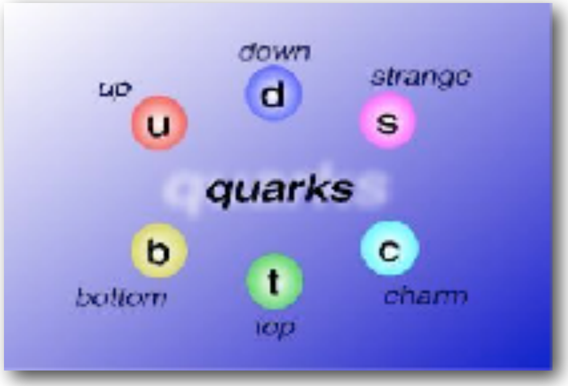
Hamiltonian is scale invariant!

Bare particles are eigenstates of free Hamiltonian

"Bare (free)" particles of QCD: quarks and gluons

e.g. because of asymptotic freedom measured in high energy collisions





- Gluon ~ 8 copies of a photon
- Photons do not cary electric charge : they only interact the matter (e.g.) electrons that do carry charge
- Gluons carry charge, i.e. interact with each other and with quarks.

Particles vs Fields: Hamiltonian vs Lagrangian

Legendre transformation

$$H(p_i, q_i) \longleftrightarrow L(q_i, \dot{q}_i)$$

Quantization

$$q_i = \vec{A}_i = \vec{A}(\vec{x}_i)$$

$$H\Psi[A_i] = E\Psi[A_i]$$

Ground state of H_0 (pure gauge) is Gaussian

Quantization

$$\langle q_i' | e^{-\beta H} | q_j' \rangle \to \int_{q'=q(\beta, q=q(0)} Dq e^{-S[q]}$$

Probabilistic interpretation

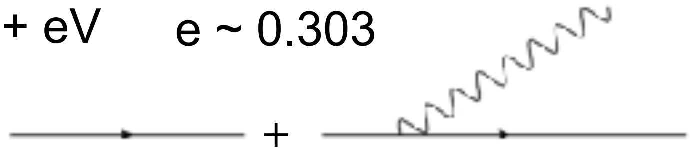
Quantum picture: particles, states, operators, etc. ⟷ Semiclassical Picture: path integral, classical solution (solitons), etc.

QED vs QCD

QED

- Bare particles are eigenstates of free Hamiltonian. If interactions are weak (e.g. QED) the "bare particle" ~ observed particle = (interacting particles)

$H_{QED} = H_{c.h.o.} + eV \qquad e \sim 0.303$

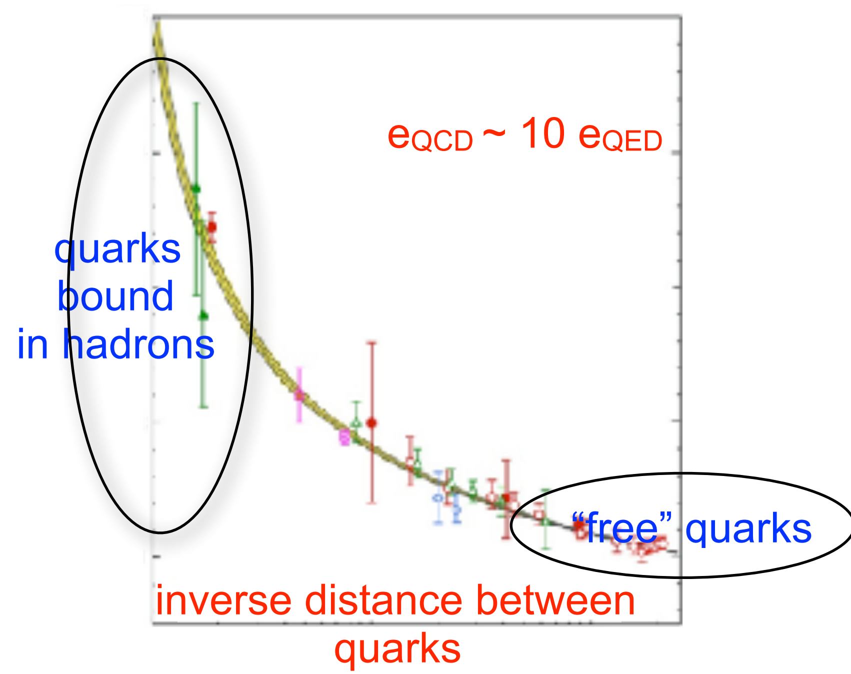
$|\text{electron}\rangle =$  $+ O(e^2)$

$|\text{bare electron}\rangle \qquad eV|\text{bare electron}\rangle$

- Bound states, aka positronium require all orders, but can nevertheless be understood in terms of "bare" particles

QCD

- Quarks in hadrons have the effective color charge e > 3-4. Therefore there is in principle no reason for them to retain their identify in presence of strong interactions …
…but it seems they do



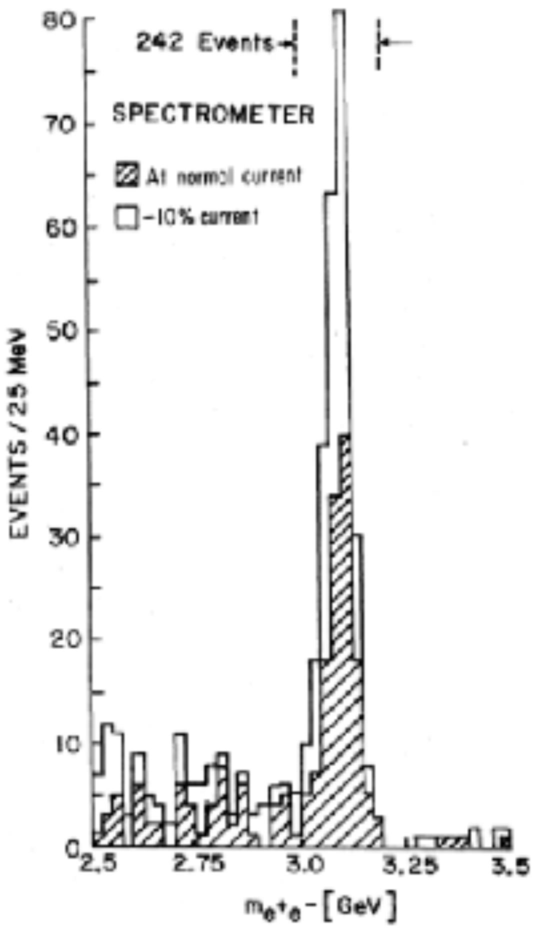
Discovery of quarks e.g. the J/ψ

Light quarks -> Deep Inelastic Scattering

A narrow resonance was discovered in the 1974 November revolution of particle physics" in two reactions:

Proton + Be => e^+ e^- + anything at the BNL J. J. Aubert et al., "Experimental observation of a heavy particle J," Phys. Rev. Lett. 33, 1404 (1974).

$$J/\psi = c\bar{c}$$



e^+e^- annihilation to hadrons in the SPEAR storage ring at Stanford
J. E. Augustin et al., "Discovery of a narrow resonance in e^+e^- annihilation," Phys. Rev. Lett. 33, 1406 (1974).

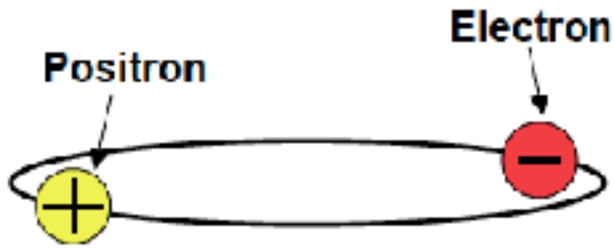
$$\text{mass} = 3096.87\ \text{MeV}$$

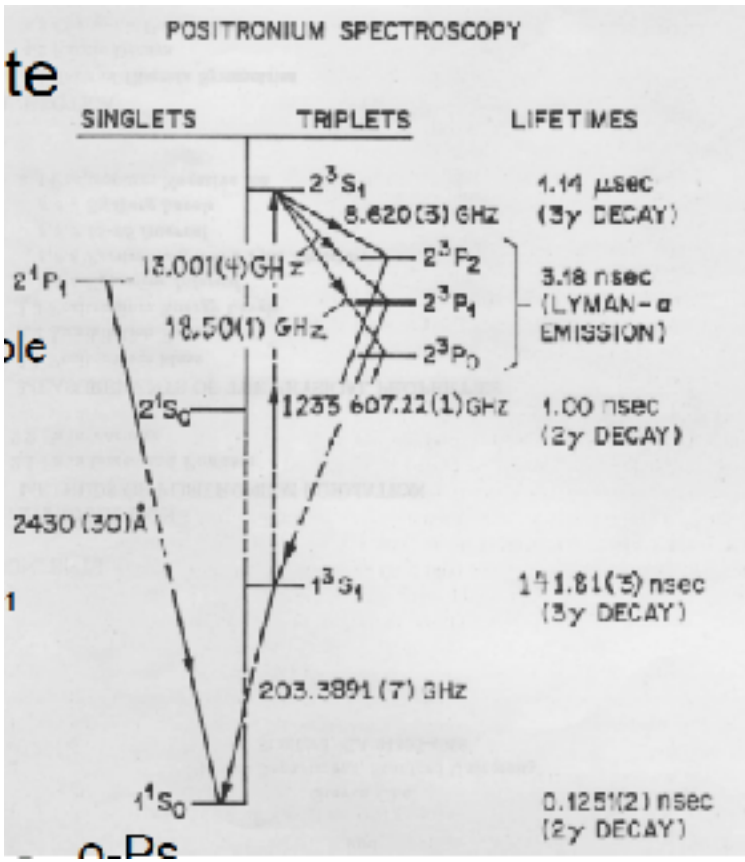
$$\Gamma = 87\ \text{keV}$$

10^3 longer lifetime !

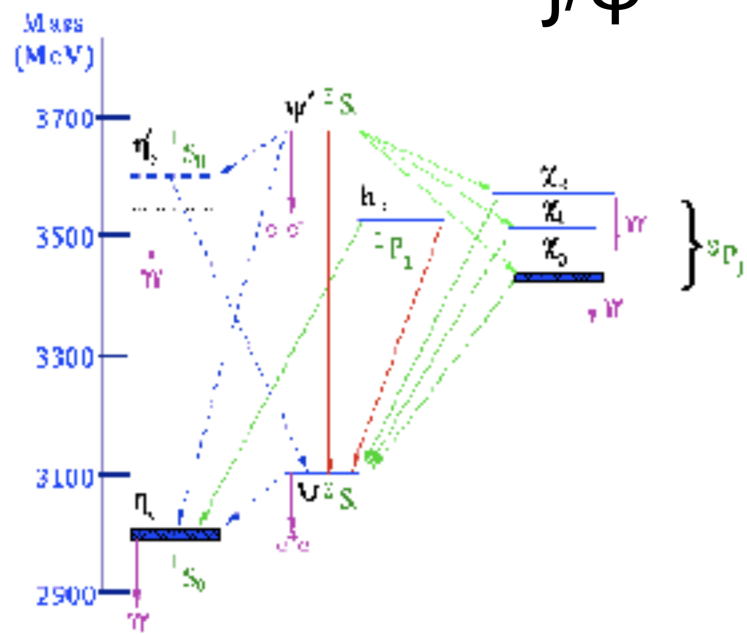
(weak interactions 10^{12}) typical hadronic width $= O(100\ \text{MeV})$

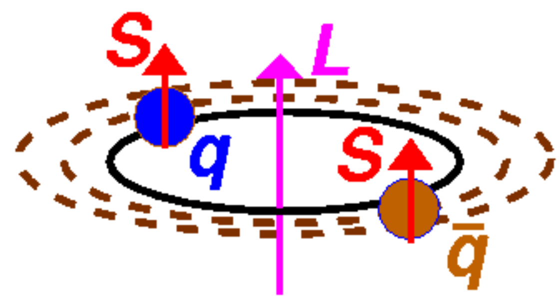
Charmonium spectrum





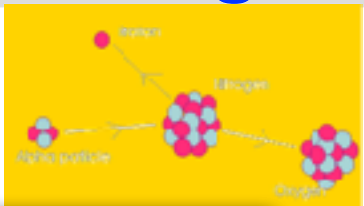
J/ψ





J/ψ is a bound state of $\bar{c}$ c

Hunting for Resonances

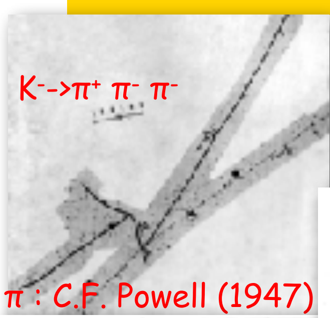


1909/1911 Rutherford/Geiger/Marsden discover the nucleus

1919 Rutherford discovers the proton

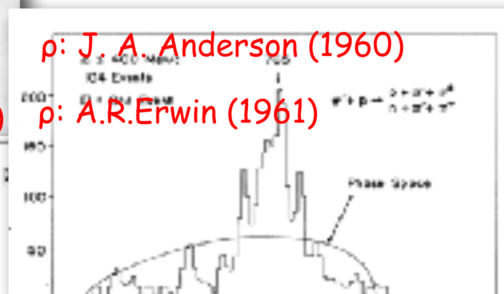
1932 Chadwick discovers the neutron

1940-now hundreds of resonances discovered



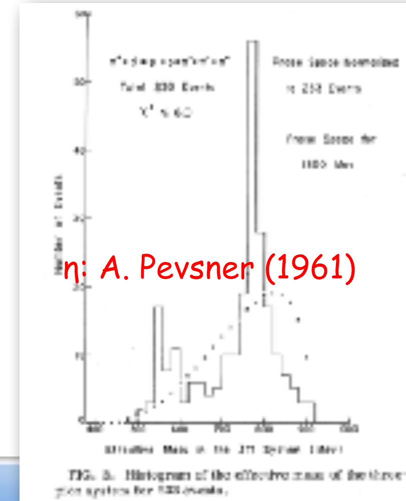
$K^- \to \pi^+ \pi^- \pi^-$

π : C.F. Powell (1947)



ρ: J. A. Anderson (1960)

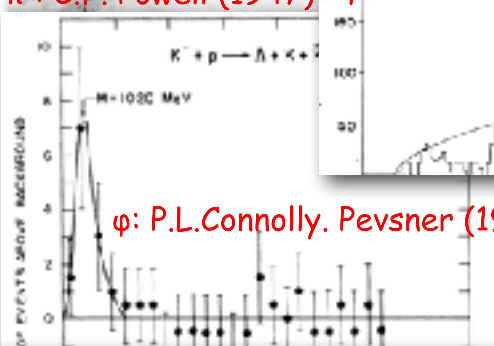
ρ: A.R.Erwin (1961)



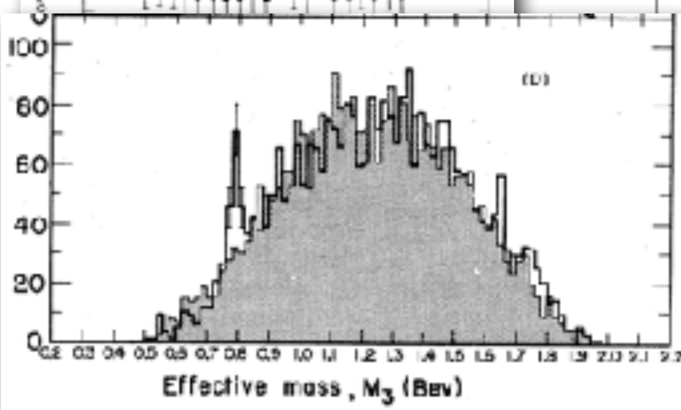
η: A. Pevsner (1961)

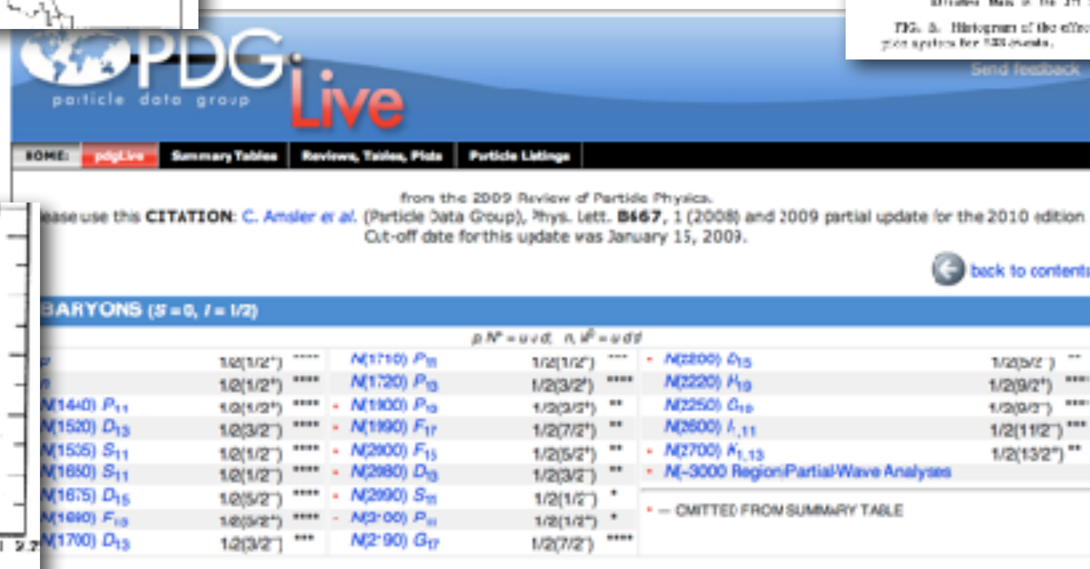
lifetime ~ 10^{-24} s

width ~ lifetime^{-1} = 150 MeV

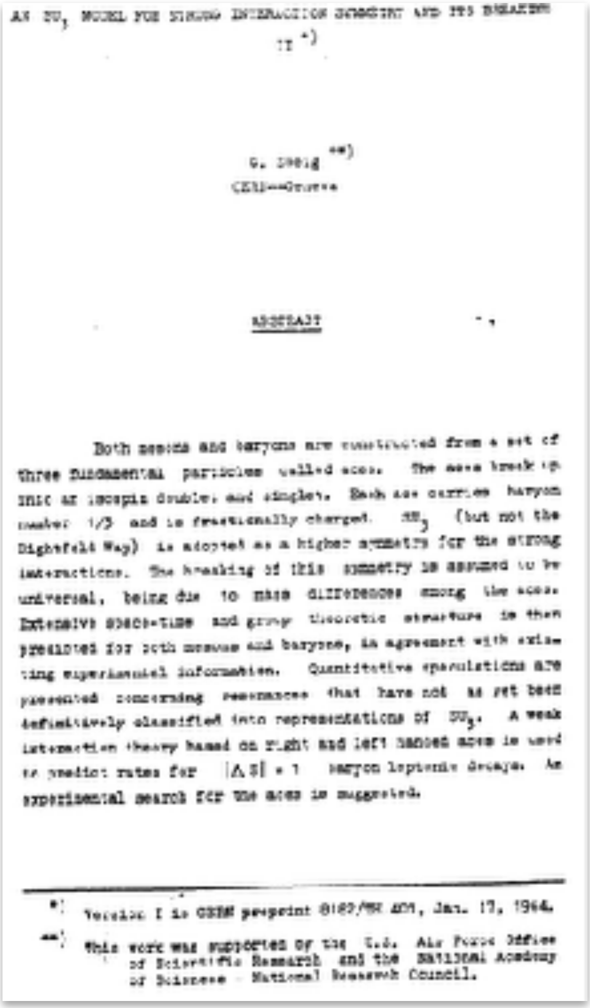


ϕ: P.L.Connolly. Pevsner (1962)





Quark Model : exploring flavor



AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig **)

CERN—Geneva

ABSTRACT

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number 1/3 and is fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. Quantitative speculations are presented concerning resonances that have not as yet been definitively classified into representations of SU_3. A weak interaction theory based on right and left handed aces is used to predict rates for $|\Delta S| = 1$ baryon leptonic decays. An experimental search for the aces is suggested.

*) Version I is CERN preprint 8182/TH 401, Jan. 17, 1964.

**) This work was supported by the U.S. Air Force Office of Scientific Research and the National Academy of Sciences – National Research Council.

PHYSICS LETTERS

A SCHEMATIC MODEL OF BARYONS AND MESONS *

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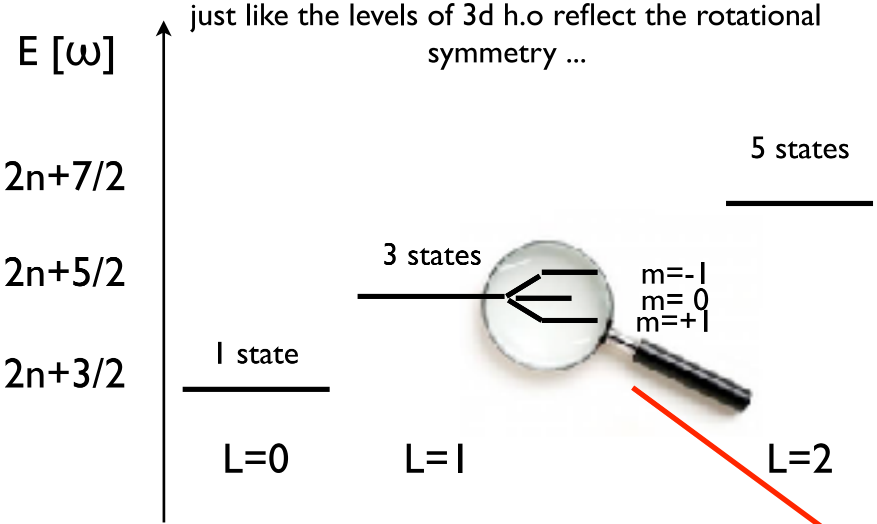
California Institute of Technology, Pasadena, California

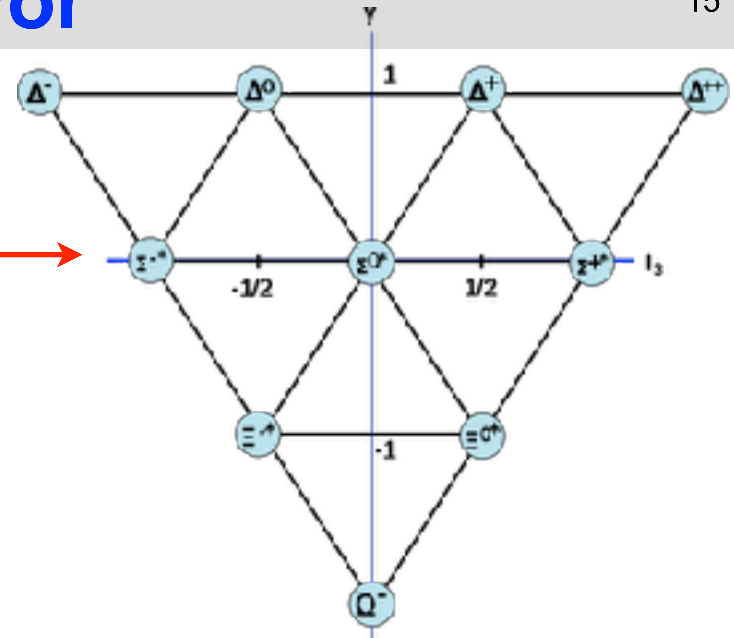
Received 4 January 1964

Quark Model : exploring flavor

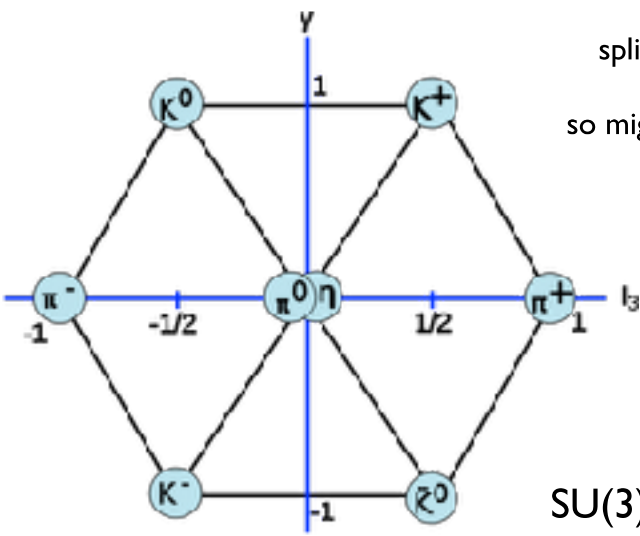
just like the levels of 3d h.o reflect the rotational symmetry ...



... mass patterns in hadron tables reflect the underlying dynamical symmetries

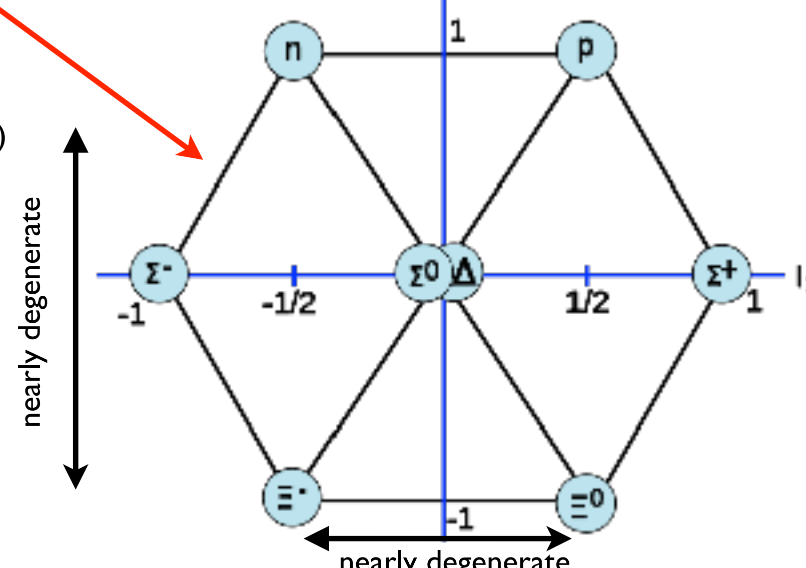


$$Y = S + B$$



splitting of L-levels is dynamical (centrifugal barrier) so might be splitting between SU(3) multiplets

SU(3) weight diagrams !



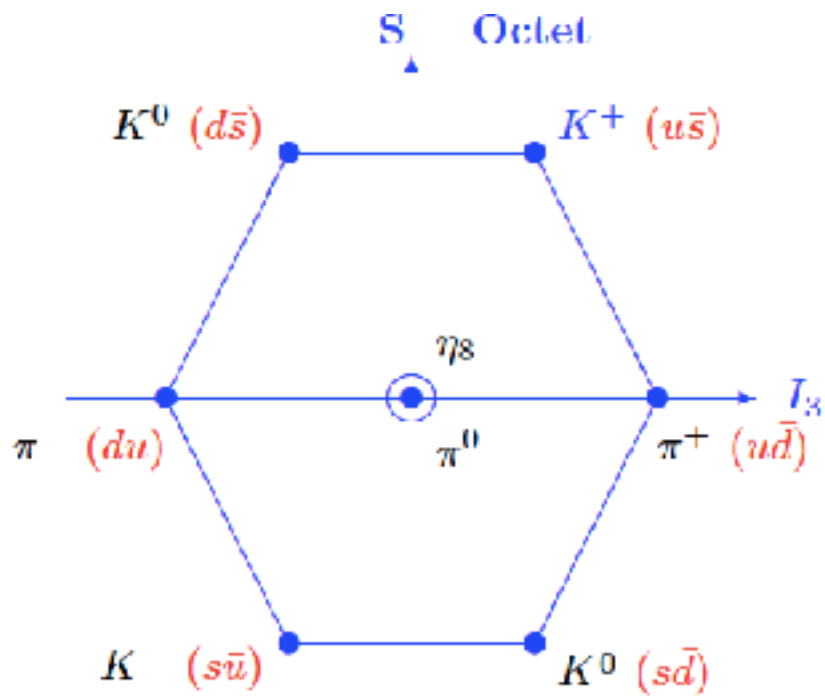
quarks and symmetries of hadrons

$$Q_u = +\frac{2}{3}e \quad Q_d = -\frac{2}{3}e \quad Q_s = -\frac{2}{3}e$$

baryons : 3 quarks, mesons : quark-antiquark

SU(3) fundamental vector

$$M_{ij} = (\bar{u}, \bar{d}, \bar{s})_i \begin{pmatrix} u \\ d \\ s \end{pmatrix}_j$$



$$\bar{3} \otimes 3 = 8 \oplus 1$$

$$M^a = (\bar{u}, \bar{d}, \bar{s})_i T^a_{ij} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_j$$

$$a = 1 \cdots 8$$

T^a = SU(3) generators

$$M^1 = (\bar{u}, \bar{d}, \bar{s})_i I_{ij} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_j$$

quarks and symmetries of hadrons

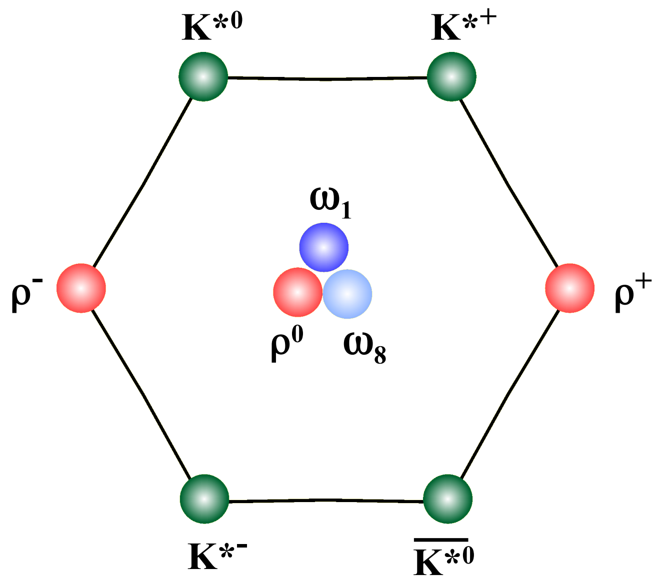
mixing of states

physical states are not quite SU(3) flavor eigenstates

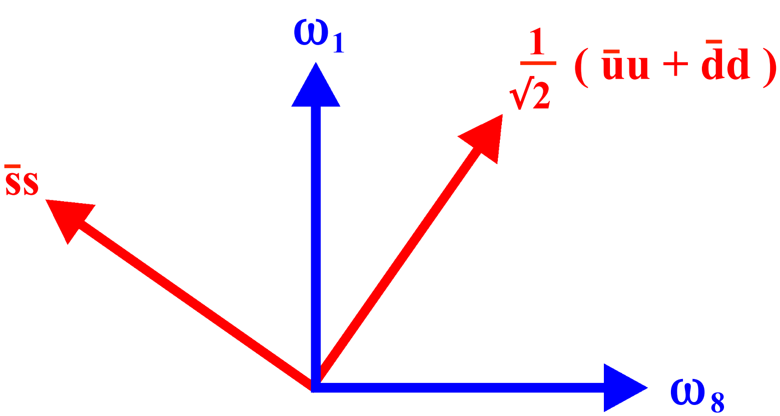
$$\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$$

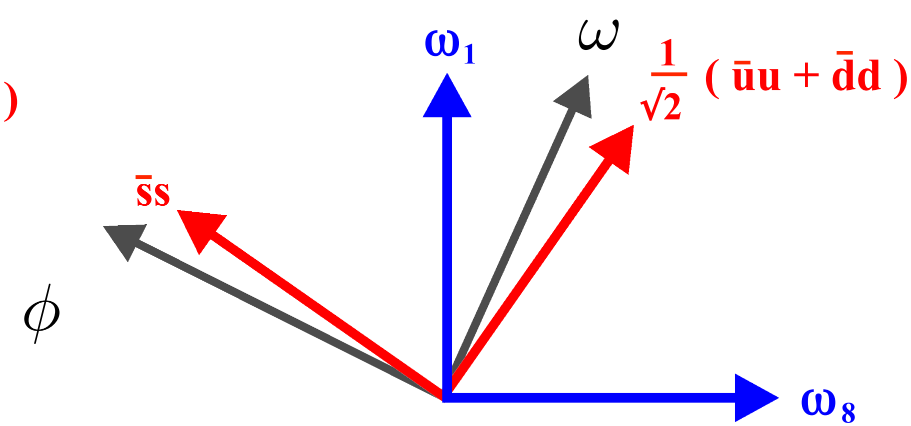
$$\omega_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s) \quad (M^1)$$

$$\omega_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \quad (M^8)$$

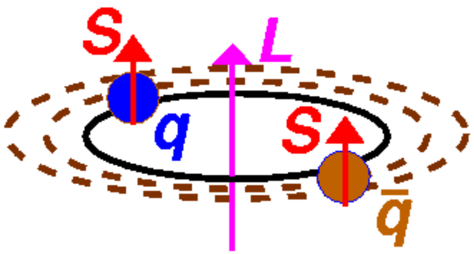


$$\omega_1, \omega_8 \to \omega(782), \phi(1020)$$





Light mesons



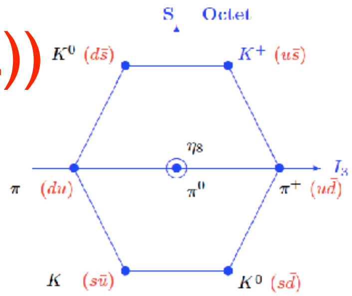
Light (u,d,s) Mesons :

Flavor = 8 + 1 ($SU_f(3)$)

Spin S = 1/2 x 1/2 = 0 + 1 (SU(2))

Orbital L = 0,1,2... (O(3))

Radial n = 0,1,2... (various)



$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

S,L => J,Parity (+,-), Charge conjugation)

$$P_Q = -P_{\bar{Q}}$$

$$C|Q\rangle = |\bar{Q}\rangle$$

8

J^{PC}	$I=1$	$I=0$
J^{-+}	π_J	η_J
J^{--}	ρ_J	ω_J
J^{+-}	b_J	h_J
J^{++}	a_J	f_J

$$\pi = 0^{-+} \rightarrow L=0, S=0, I=1$$

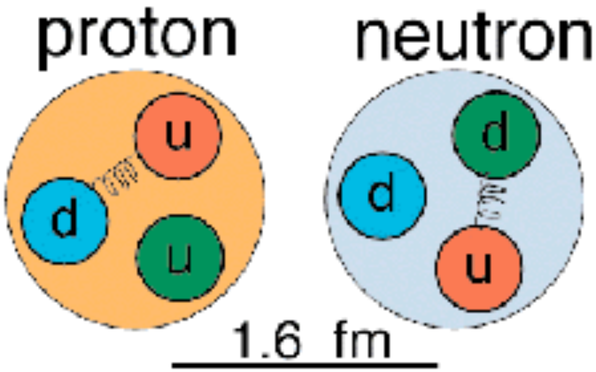
$$\rho_2 = 2^{--} \rightarrow L = even = 2, S=1, I=1$$

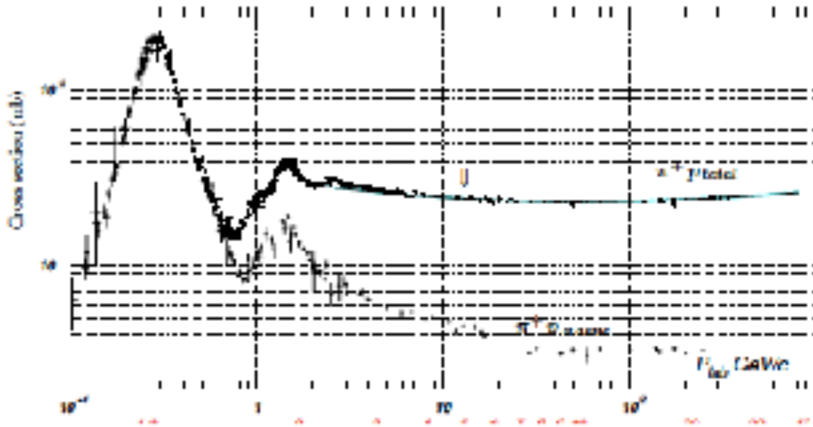
Need for color

(Greenberg, Fritzsch)

$$\Delta^{++}(1232\ \mathrm{MeV}) = uuu$$

Spin = 3/2 (quark spins aligned)





closest in mass to the nucleon
(symmetric spacial wave function)

need another q.number to make w.f. fully antisymmetric

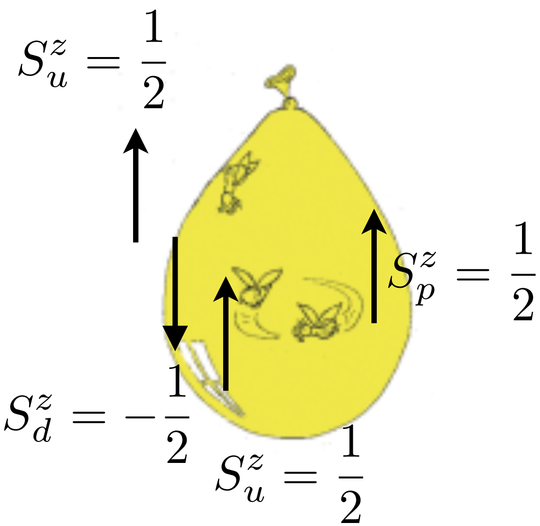
$$\Delta^{++}(1232\ \mathrm{MeV}) = \epsilon_{ijk} u_i u_j u_k \quad i, j, k = 1, 2, 3 \text{ or } R, G, B$$

“solves” the problem of the fractional electric charge: nature supports only color neutral entities >> color confinement <<

quark model

$$|B[8]\rangle = |Flavor\rangle_{8_{M_A}} \times |Spin\rangle_{8_{M_A}} + |Flavor\rangle_{8_{M_S}} \times |Spin\rangle_{8_{M_S}}$$

fully symmetric wave function (antisymmetric does not work!)
Color makes it into fully antisymmetric to respect Pauli principle



H. J. Lipkin

FERMILAB-Conf-84/125-T
November, 1984

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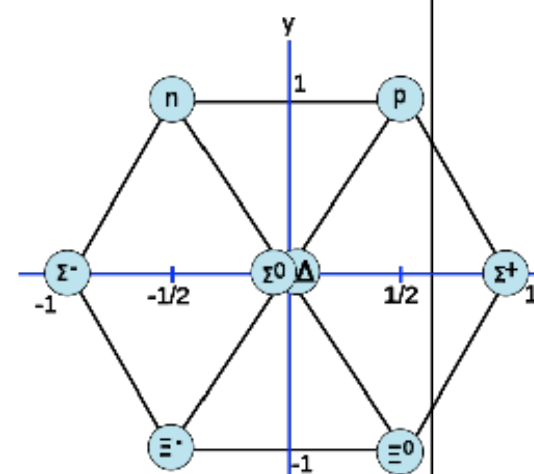
Baryon magnetic moments

Baryon Moment	1983 Data Ref[26]	From Naive Model[25]
μ(p)	2.793±0.000	2.79
μ(n)	-1.913±0.000	-1.86
μ(Λ)	-0.613±0.005	-0.58
μ(Σ⁺)	2.38±0.02	2.68
μ(Σ⁻)	-1.11±0.04[27]	-1.05
μ(Ξ°)	-1.25±0.014	-1.40
μ(Ξ⁻)	-0.60±0.04	-0.47

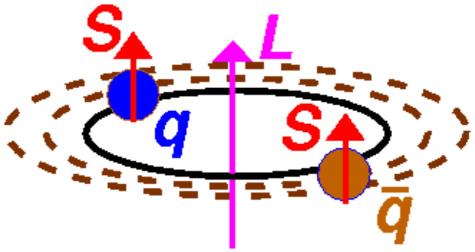
1995

$\mu_{\Omega^-} = (-2.019 \pm 0.054)\mu_N$ $\quad -1.84\mu_N$



better then 10% accuracy !!

quark model



orbital excitations

radial excitations

L=0	ρ,ω,φ,K*	1^{--}	
	π,η,η′,K	0^{-+}	
L=1	a_2,f_2,f'_2,K_2	2^{++}	
	a_1,f_1,f'_1,K_1	1^{++}	
	a_0,f_0,f'_0,K_0	0^{++}	
	b_1,h_1,h'_1,K_1	1^{+-}	

$J=L+S$ $\quad$ $^{(2S+1)}L_J$

$P=(-1)^{L+1}$

$C=(-1)^{L+S}$ $\quad$ $^1S_0 = 0^{-+}$

$G=C\,(-1)^I$ $\quad$ $^3S_1 = 1^{--}$

$$|J^{PC},n\rangle \to \Psi^{J^{PC},n}(\mathbf{r}_{q\bar{q}}, m_q m_{\bar{q}}, f_q f_{\bar{q}})\delta_{c_q c_{\bar{q}}}$$

$$S+L=J, \quad \Psi(\mathbf{r}_{q\bar{q}}) \to \Psi(|\mathbf{r}_{q\bar{q}}|)$$

$$H = \frac{\mathbf{p}_q^2}{2m_q} + \frac{\mathbf{p}_{\bar{q}}^2}{2m_{\bar{q}}} + V_C(r_{q\bar{q}}) + V_{SS} + V_{LS} + V_T + V_F$$

$m_u \sim m_d \sim 300$ GeV

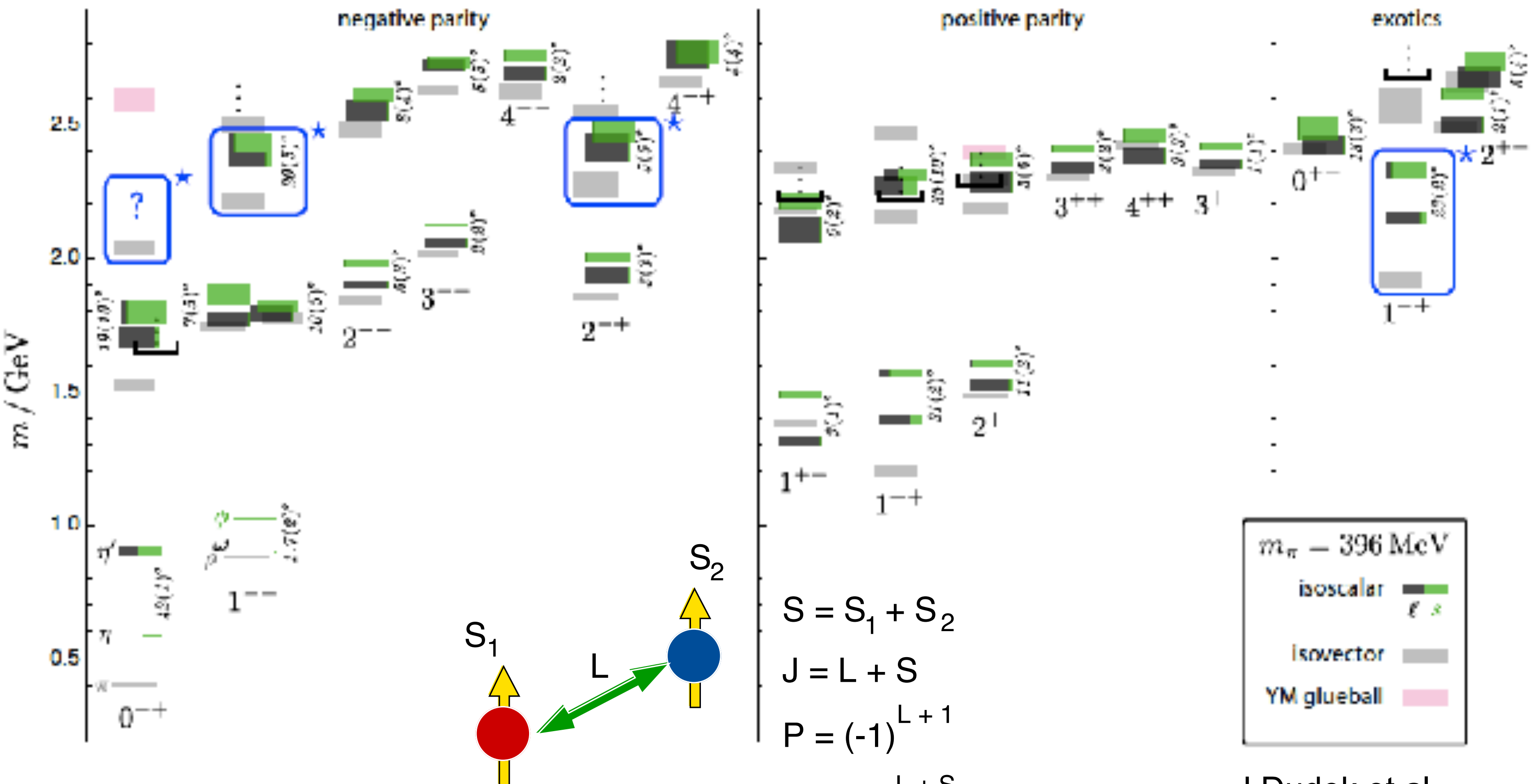
constituent quarks

$m_u \sim m_d \sim$ few GeV

bare quarks

"Evidence" for Constituent Quarks:Light Quark Hadrons

Spectrum of mesons containing u,d,s quarks from numerical QCD simulations (lattice) resembles spectrum of quark models.



$S = S_1 + S_2$

$J = L + S$

$P = (-1)^{L+1}$

$C = (-1)^{L+S}$

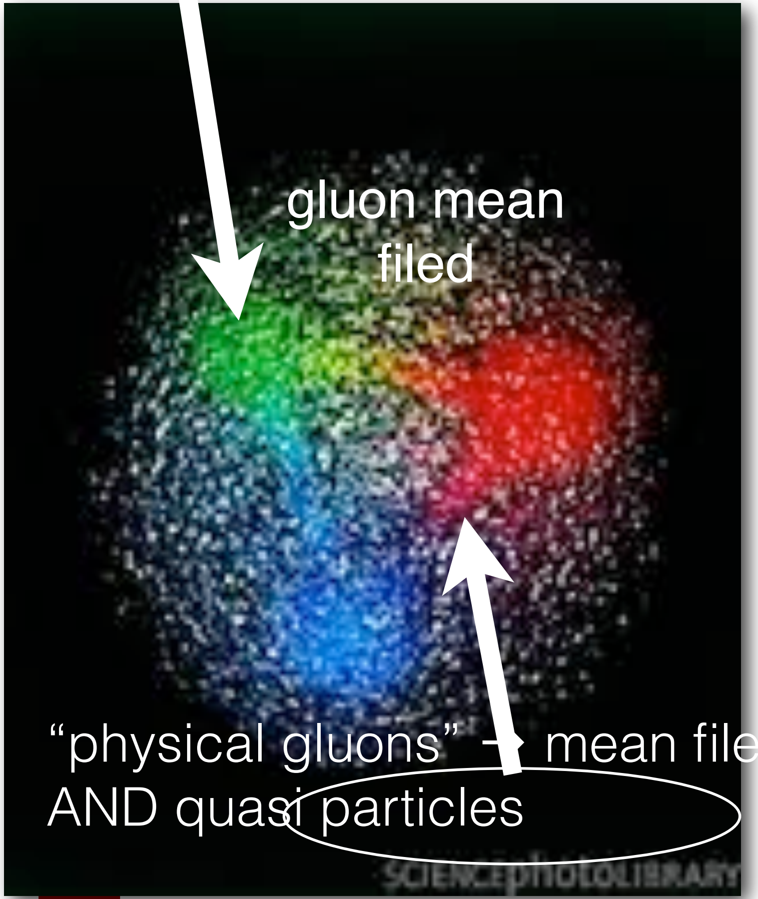
J.Dudek et al.

Plausible scenario

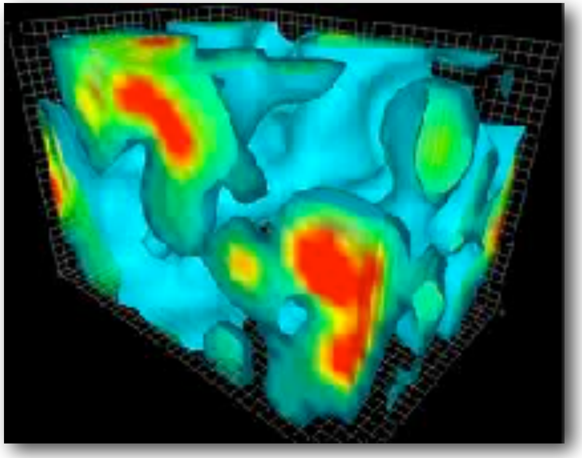
$H_{QCD} = H_{c.h.o.} +$ non-linear

finite energy, localized solutions:
solitons (monopoles, vortices , ...)

"physical quarks" → quasi particles in gluon mean filed



The QCD vacuum is not empty. Rather it contains quantum fluctuations in the gluon field at all scales. (Image: University of Adelaide)

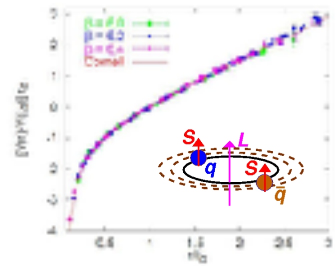


Monopoles have been long speculated to be candidate gluon filed configurations responsible for confinement

Emergence of constituent quarks

$$H = H_0 + V \qquad H_0 = \int d\mathbf{x} m_0 |\psi(\mathbf{x})|^2$$

Mean field approximation   Hartree + Fock (BCS theory)

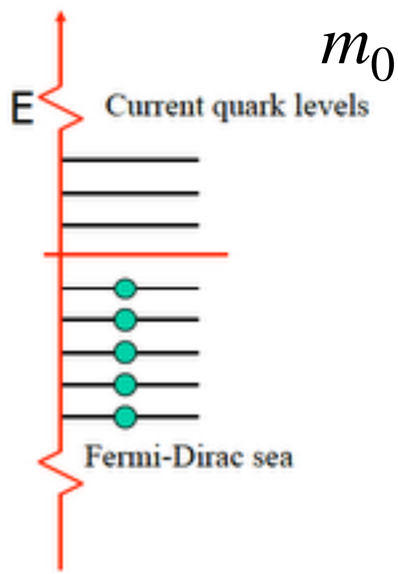


$$V = \int d\mathbf{x} d\mathbf{y} |\psi(\mathbf{x})|^2 V(\mathbf{x}-\mathbf{y}) |\psi(\mathbf{y})|^2$$

Instantaneous potential between (color) charges, e.g. Coulomb + Linear

$$|\psi(\mathbf{y})|^2 \rightarrow \langle |\psi(\mathbf{y})|^2 \rangle = \text{condensate}$$

$$m_0 \rightarrow m_0 + V \times \text{ condensate}$$

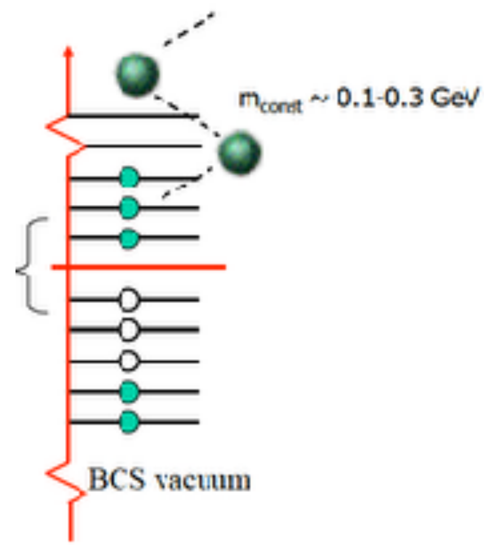


ground state contains a condensate of bare quarks

Interaction with the condensate increases energy of a quark added to the vacuum

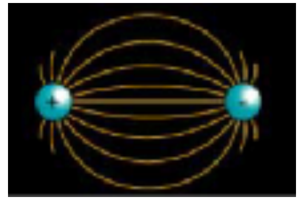
The condensate is “rigid” (Hartree-Fock) or

There is a back reaction on the condensate (BCS), Copper pars are formed

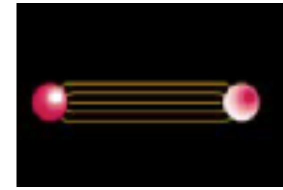


Monopole confining scenario

in “empty vacuum”



in “magnetic condensate”



Type-II supper conductor

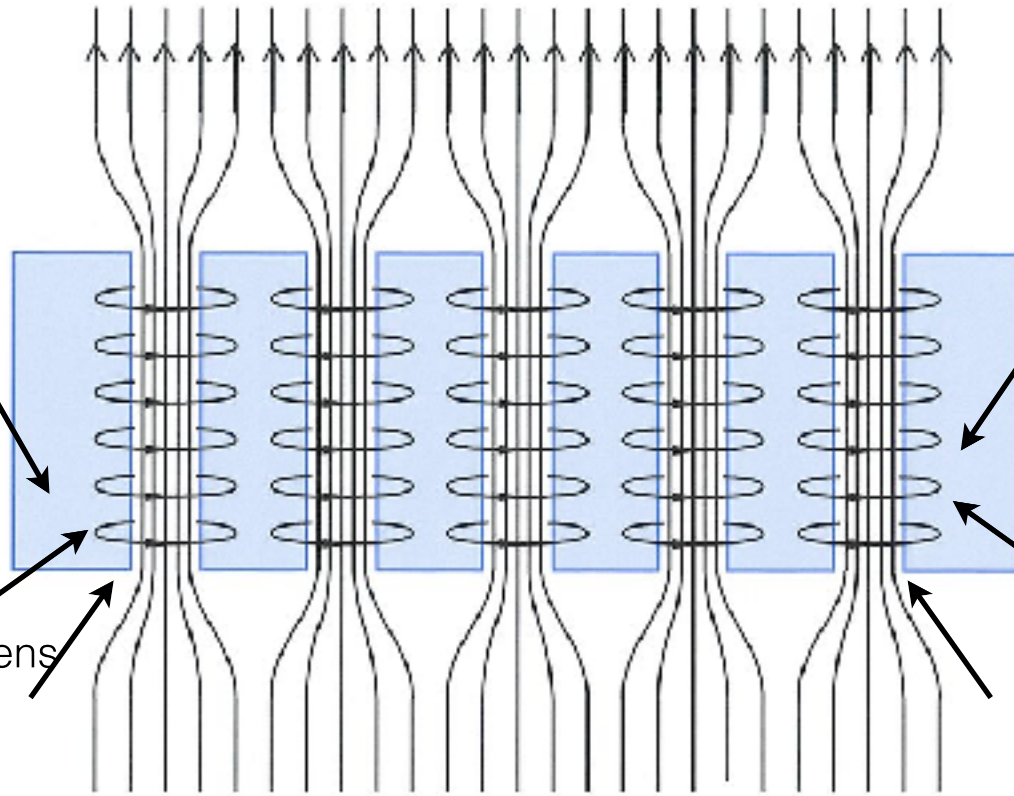
Dual Type-II supper conductor

QED

QCD

condensate of
electric charges

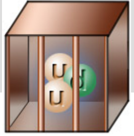
condensate of magneti
charges



electric current screens
magnetic lines

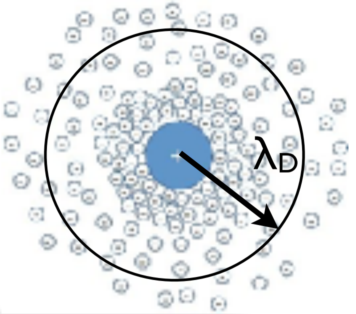
magnetic current
screens electric lines

Confinement in QCD



e.g. absence of isolated quarks applies to both screening and confinement

$$\int_{\partial V} \vec{E}\cdot d\vec{S} = Q \sim e^{-R/\lambda_D}$$



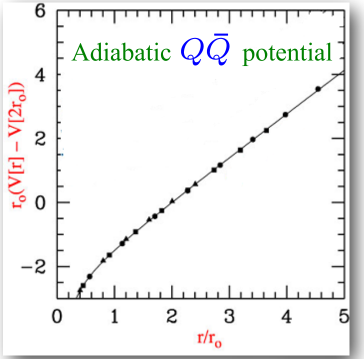
condensate (i.e. electrons in metal)

$$-\frac{e^2}{r} \to -\frac{e^2}{r} e^{-r/\lambda_D}$$

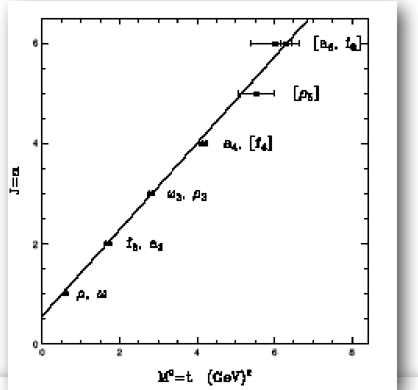
- absence of isolated quarks

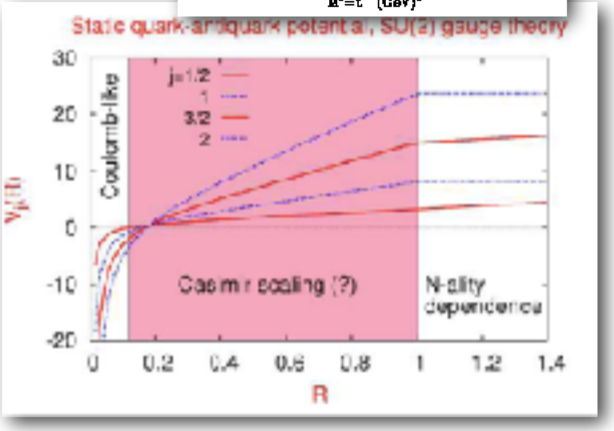
In absence of an order parameter we have to content with properties of confinement:

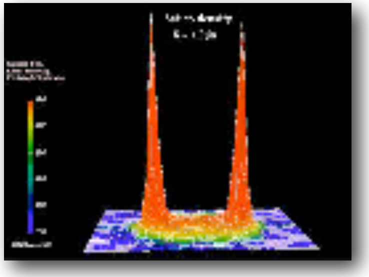
- linearly rising potential
- Regge trajectories
- Casimir and N-ality scaling
- string behavior

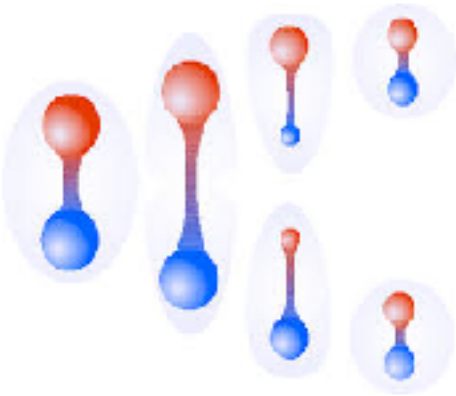


$r_0 = 0.5$ fm



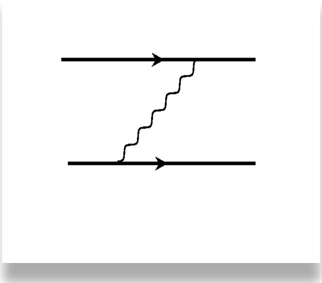






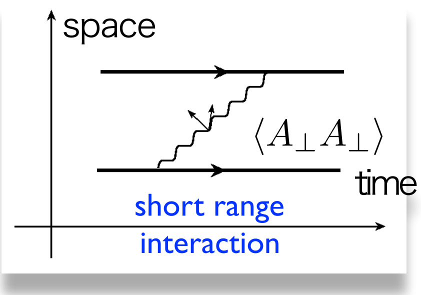
QCD vacuum and the role of gluons

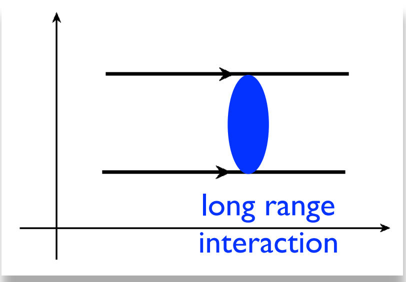
Gluons are responsible for confinement (aka effective potential between color charges) and are confined (aka contribute to the color charge)



Coulomb gauge

$$\nabla A^a(x) = 0$$





QCD as a many body theory

Remember this example

$$L = \frac{1}{2}\int dx \left[\frac{1}{v^2}(\partial_t q)^2 - (\partial_x q)^2\right]$$

QCD

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \bar{\psi}(\gamma_\mu D_\mu + m)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D_\mu = \partial_\mu + igA^a_\mu T^a$$

$$[T^a, T^b]_{ij} = if^{abc}T^c_{ij}$$

Variables:

$A^a_\mu(\mathbf{x};t)$ $\psi=\psi^i_\alpha(\mathbf{x};t)$

8 x 4 x 3N 3 x 4 x 3N

Parameters: g,m

Gauge freedom → redundant d.o.f
Gauge fixing → selects a physical d.o.f

- Weyl gauge, $A^a_{\mu=0}=0$

$$\vec{V}^a(\mathbf{x};t) \equiv A^a_{\mu=1\cdots 3}(\mathbf{x};t)$$

$$[V^a_i(\mathbf{x}'), \Pi^b_j(\mathbf{x})] = i\delta_{ab}\delta_{ij}\delta(\mathbf{x}'-\mathbf{x})$$

$$\left\{\psi(\mathbf{x}^i_\alpha, \psi^{\dagger,j}_\beta(\mathbf{x}')\right\} = \delta_{ij}\delta\alpha\beta\delta(\mathbf{x}'-\mathbf{x})$$

$$H = \int d\mathbf{x}\frac{1}{2}\left[\vec{\Pi}^a(\mathbf{x})\vec{\Pi}^a(\mathbf{x}) + \vec{B}^a(\mathbf{x})\vec{B}^a(\mathbf{x})\right] + \int d\mathbf{x}\psi^\dagger(\mathbf{x})\left(-i\vec{\alpha}\vec{D} + \beta m\right)\psi(\mathbf{x})$$

Constraint:
Gauss' law

$$G^a|\rangle = \left[\vec{\nabla}\vec{\Pi}^a + gf^{abc}\vec{V}^b\vec{\Pi}^c + g\psi^\dagger T^a\psi\right]|\rangle = 0$$

$-\vec{\nabla}\vec{E}^a$

Gluon charge Density $\rho_g(\mathbf{x})$

Quark charge Density $\rho_q(\mathbf{x})$

$$[G^a(\mathbf{x}'), G^b(\mathbf{x})] = if^{abc}\delta(\mathbf{x}'-\mathbf{x})G^c(\mathbf{x})$$

Generators of residual gauge symmetry, e.g.

$$[G^a(\mathbf{x}'), \rho^a(\mathbf{x})] = if^{abc}\delta(\mathbf{x}'-\mathbf{x})\rho^c(\mathbf{x})$$

Gribov region

Weyl: $3 \times (N_C^2 - 1) \times \mathcal{V}$ **d.o.f** $V_i^a(x)$ $H = H(V, -i\delta/\delta V)$

Gauss' law => $\mathcal{G}[V, -i\delta/\delta V]|\mathrm{Physcial}\rangle = 0$

Coulomb: => coordinate transformation $V_i^a \to A_i^a, \phi^a$

$$V_i^a = uA_i^a u^\dagger + \frac{i}{g} u \nabla_i u^\dagger \qquad u = u(\phi^a) = e^{iT^a \phi^a} \qquad \nabla_i A_i^a = 0$$

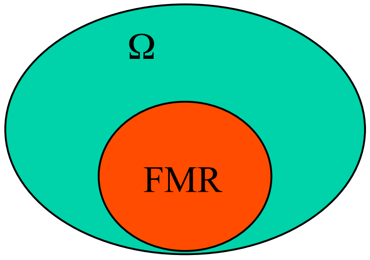
$$\mathcal{G}|\mathrm{Physical}\rangle = 0 \to \langle A, \phi|\mathrm{Physical}\rangle = \langle A|\mathrm{Physical}\rangle$$

$$H = H[A, -i\delta/\delta A]$$

$$\left.\begin{array}{l}(x,y,z) \to (r,\theta,\phi) \\ (x,y,z) \to (-r,\theta,\phi)\end{array}\right\}$$
$$r > 0$$

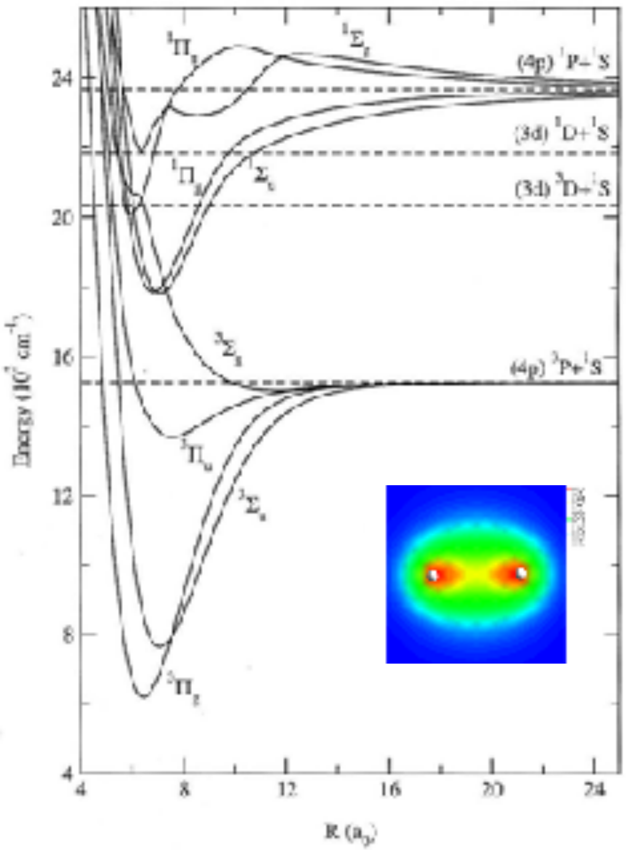
Gribov ambiguity
but single a patch
is complete

$$\mathrm{Jacobian} = -\nabla_i D[A]_i > 0$$



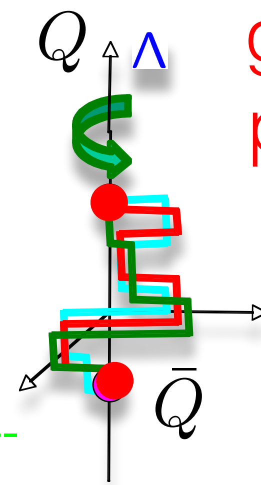
$$\int DA\mathcal{J}|\Psi[A]|^2 = \int_{FMR} DA\mathcal{J}|\Psi[A]|^2$$

Adiabatic potentials



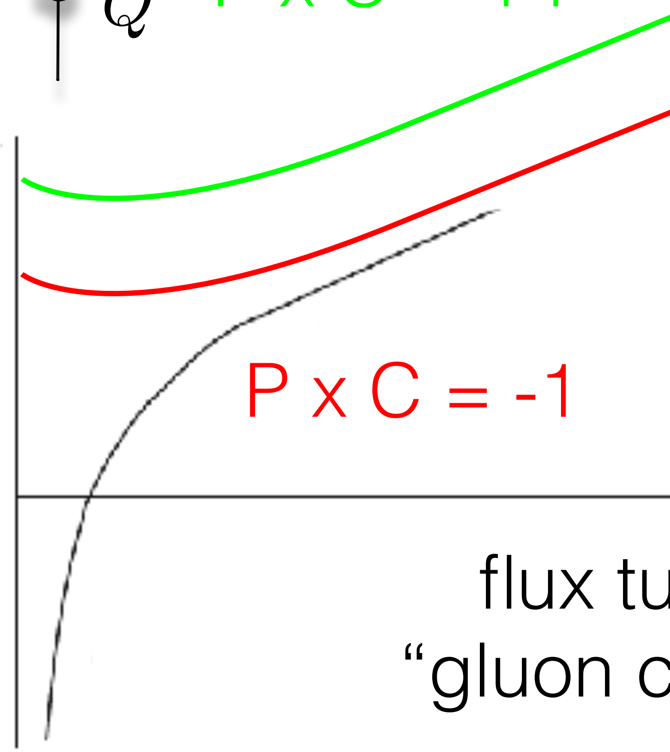
Potential energy curves for the excited valence states of Ca_2

gluons behave as physical particles with $J^{PC} = 1^{+-}$



P x C = +1

$J^{PC}=1^{--}$

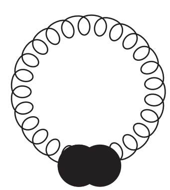


Energy of the gluon field

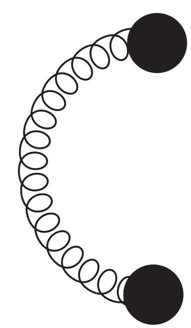
$J^{PC}=1^{+-}$

P x C = -1

R→0



glue-lump



flux tube
“gluon chain”

Coulomb gauge

Coulomb gauge Hamiltonian

$$B_i^a = \nabla_j A_k^a - \nabla_k A_j^a + g f^{abc} A_j^b A_k^c$$

$$H = \frac{1}{2}\int d\mathbf{x} \left[\mathcal{J}^{-1}\vec{\Pi}^a \mathcal{J}\vec{\Pi}^a + \vec{B}^a\vec{B}^a\right]$$

Jacobian (e.g. $r^{-1}\frac{d}{dr}r\frac{d}{dr}$)

$$\mathcal{J}(A) = Det\vec{\nabla}\mathcal{D}(\mathcal{A})$$

$$+\int d\mathbf{x}\psi^\dagger\left[-i\vec{\alpha}\left(\vec{\nabla} - igA^aT^a\right) + \beta m\right]\psi$$

$$+\frac{g^2}{2}\int d\mathbf{x}d\mathbf{y}\mathcal{J}^{-1}\rho^a(\mathbf{x})K_{ab}[A](\mathbf{x},\mathbf{y})\mathcal{J}\rho^b(\mathbf{y})$$

$$K = \frac{1}{\vec{\nabla}\mathcal{D}(A)}(-\vec{\nabla}^2)\frac{1}{\vec{\nabla}\mathcal{D}(A)} \qquad \rho^a = f^{abc}\vec{A}^b\vec{\Pi}^c + \psi^\dagger T^a\psi$$

$$H\left(\frac{\delta}{\delta A}, A\right)\Psi[A] = E\Psi[A], \quad \int \mathcal{D}A\mathcal{J}|\Psi[A]|^2 = \langle|\rangle$$

$$\bar{H} = \mathcal{J}^{1/2}H\mathcal{J}^{-1/2}, \; \bar{\Psi} = \mathcal{J}^{1/2}\Psi \qquad \int\mathcal{D}A|\bar{\Psi}[A]|^2 = \langle|\rangle$$

Example of calculation

H_0 is a h.o.

$$|0\rangle \sim \exp(-\int dx dy A(x) \omega_0(x-y) A(y))$$

$$H = H_0 + gV$$

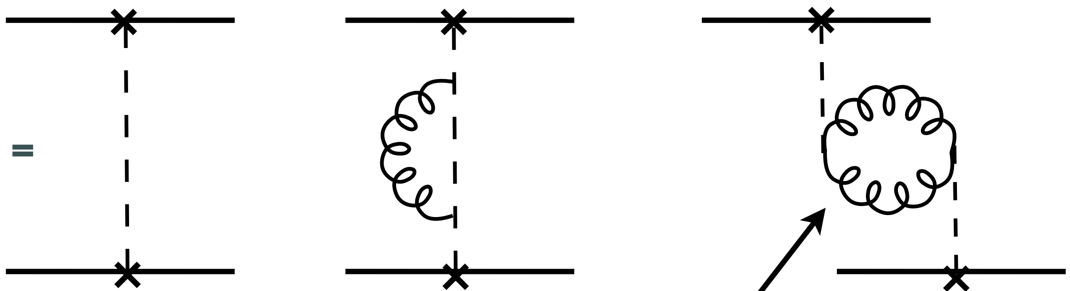
$$E = E_0 + gE_1 + g^2 E_2 + \cdots$$

calculate E for QQ in the perturbative QCD ground state

<H> enhanced in the IR from modes near horizon

real (quasi) particles propagating expected to be suppresses

E(R) =



$$E(R) = \frac{\alpha}{R}\left[1 + \frac{\alpha}{4\pi}[12 - 1]\log\left(\frac{1}{\Lambda R}\right)\right]$$

12 comes from the Coulomb potential

QCD

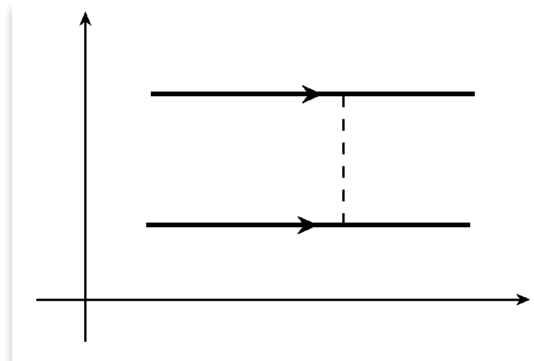
Debye screening

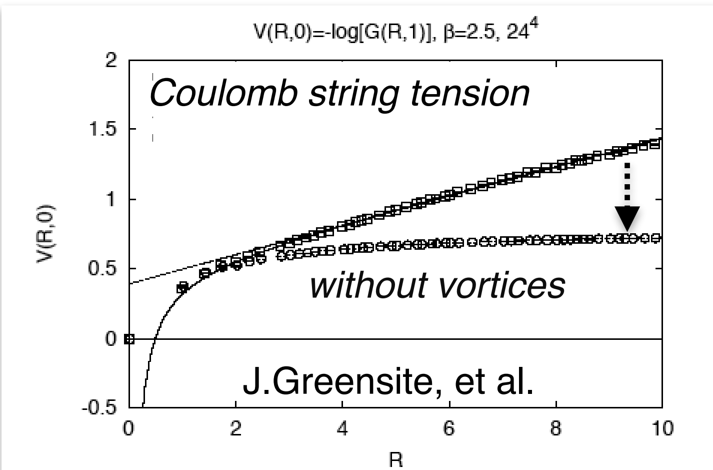
Confining Potential and the gluon condensate

$$H = H_{kin} + V \qquad H = H_{kin} + V$$

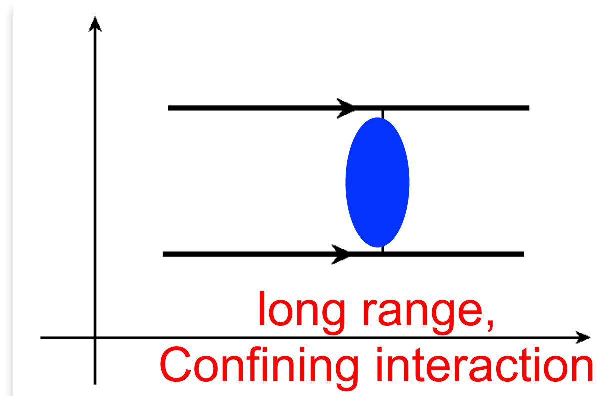
$$V = \int d\mathbf{x}d\mathbf{y}\,\rho(\mathbf{x})K[\mathbf{A},\mathbf{x},\mathbf{y}]\rho(\mathbf{y})$$

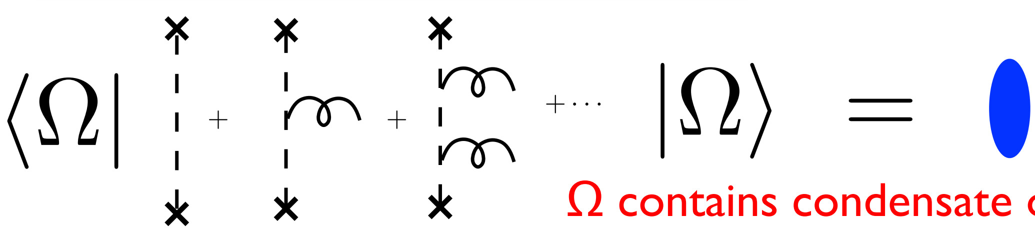
$$K \to -\frac{g^2}{\nabla^2} = \frac{\alpha}{|\mathbf{x}-\mathbf{y}|} =$$  $$V + \int d\mathbf{x}d\mathbf{y}\rho(\mathbf{x}V(\mathbf{x}-\mathbf{y})\rho(\mathbf{y})$$





- Coulomb "Potential" between external (i.e. quark charges) depends on the distribution of gluons.
- In presence of a gluon condensate it produces a Confining force been external color charge



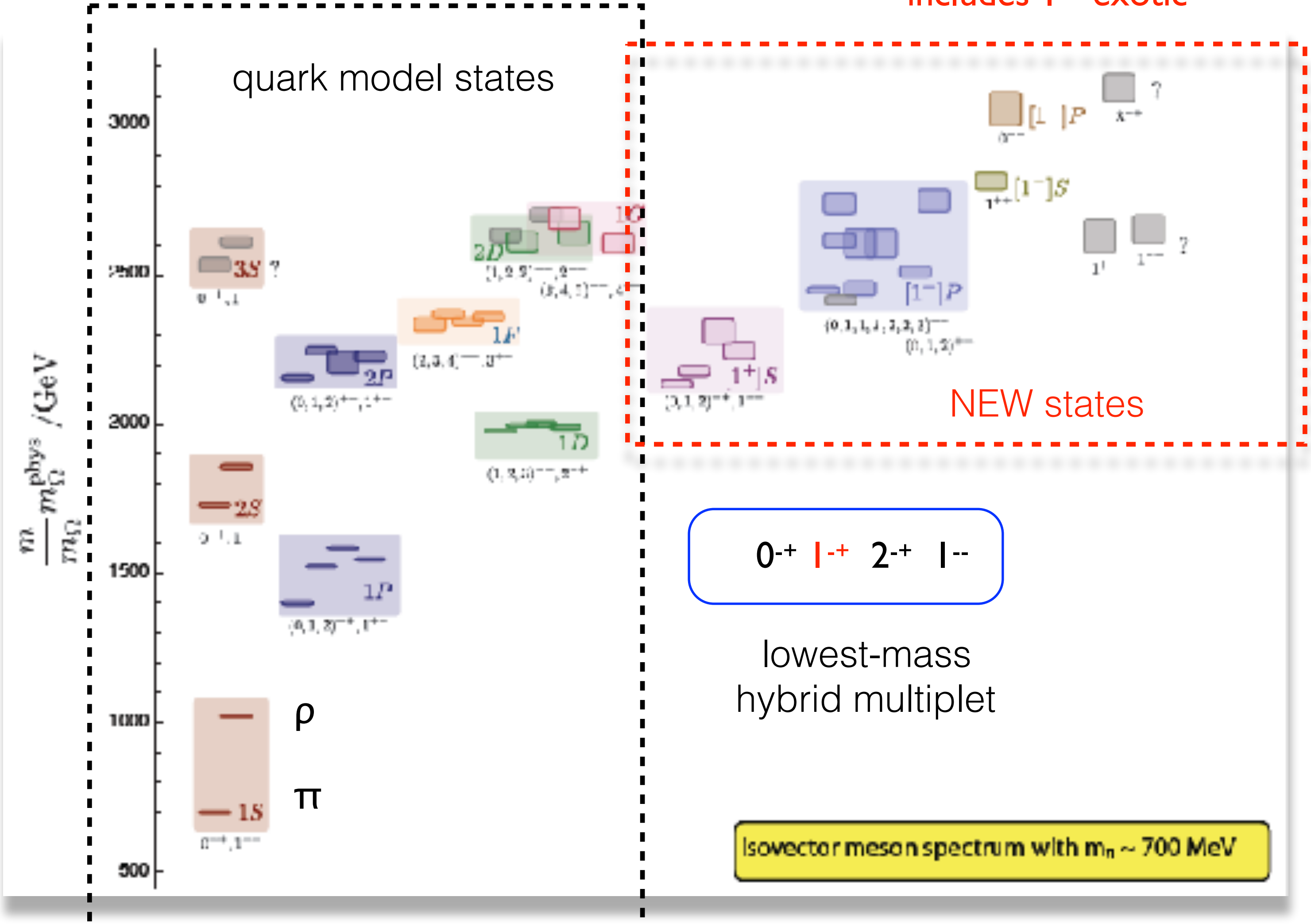


Ω contains condensate of monopoles, vortices, …

Meson Spectrum on the Lattice

new multiplets from lattice

large overlap with gluonic operators includes 1^{-+} exotic



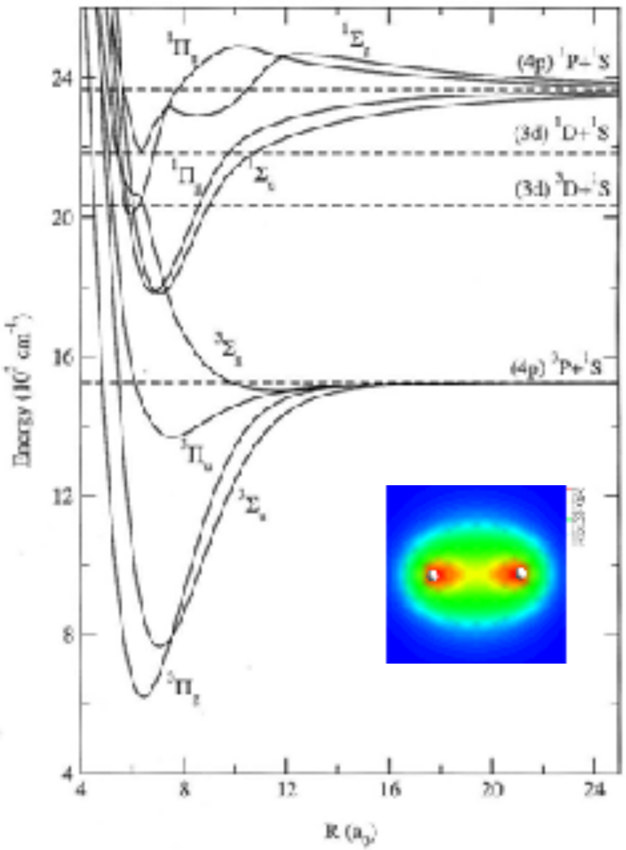
0^{-+} 1^{-+} 2^{-+} 1^{--}

lowest-mass hybrid multiplet

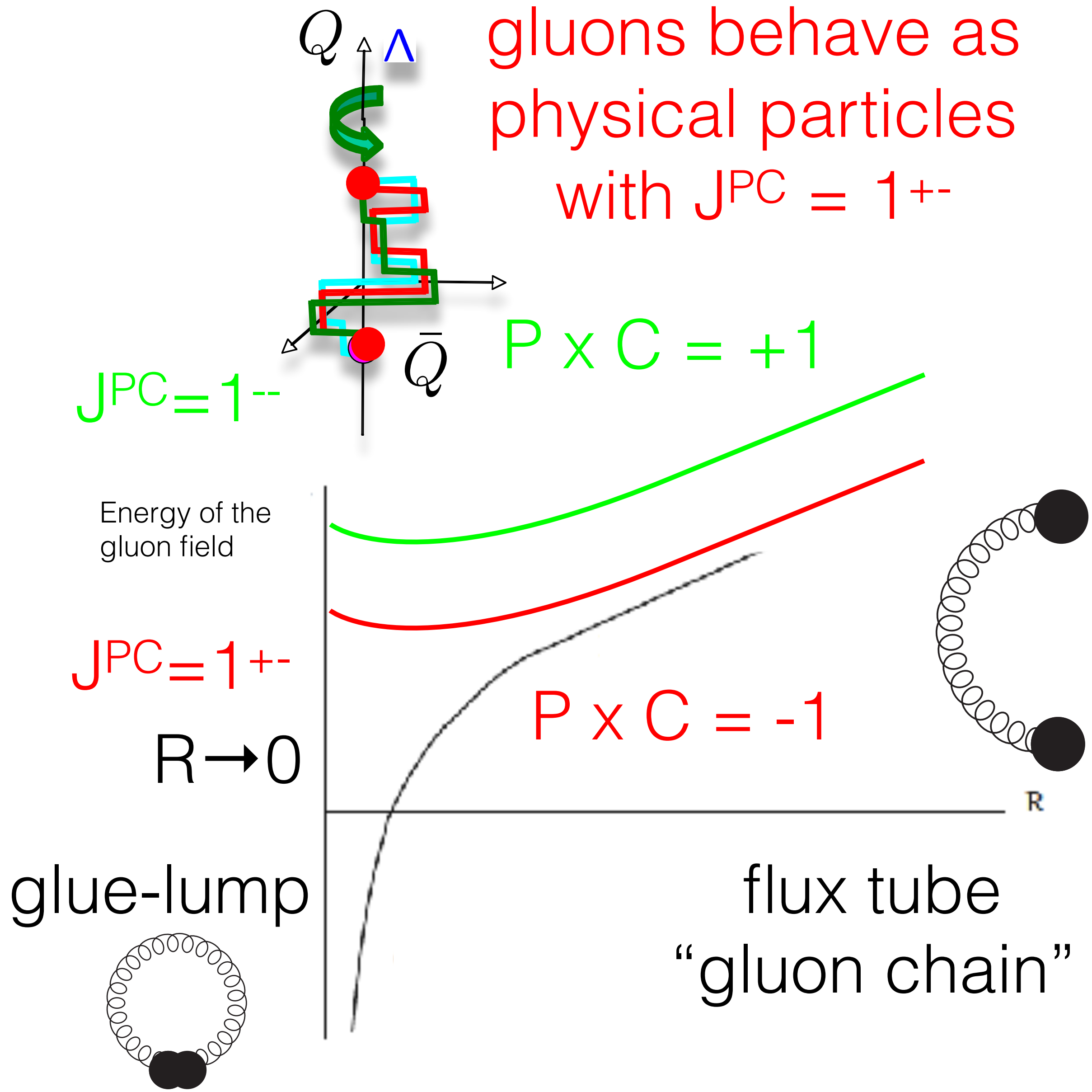
J.Dudek et al. JLab

Adiabatic potentials

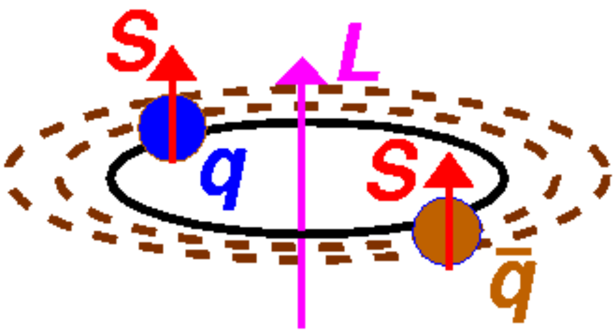
gluons behave as physical particles with $J^{PC} = 1^{+-}$



Potential energy curves for the excited valence states of Ca_2



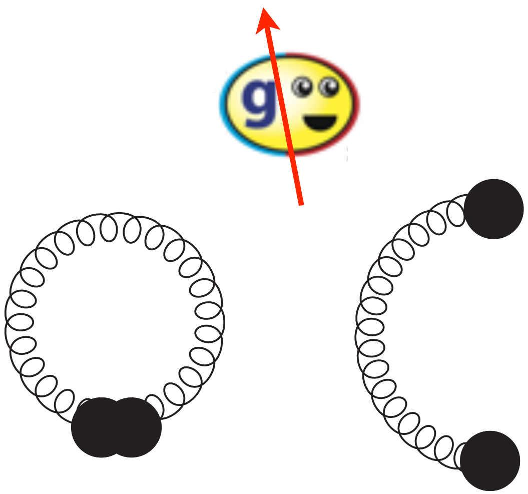
Quark Model with Gluons : Hybrid States



$$P_{q\bar{q}} = (-1)^{L+1}$$

$$C_{q\bar{q}} = (-1)^{L+S}$$

$$J_g^{PC} = 1^{+-}$$



J^{PC} glue

J^{PC} $Q\bar{Q}$

$$1^{+-} \times 0^{-+}_{S_{Q\bar{Q}}} = \boxed{1^{--}}$$

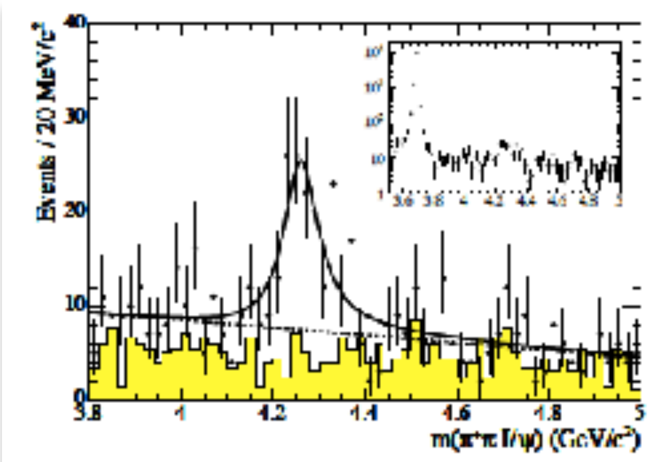
$$1^{+-} \times 1^{--}_{S_{Q\bar{Q}}=1} = \boxed{0^{-+}, 1^{-+}, 2^{-+}}$$

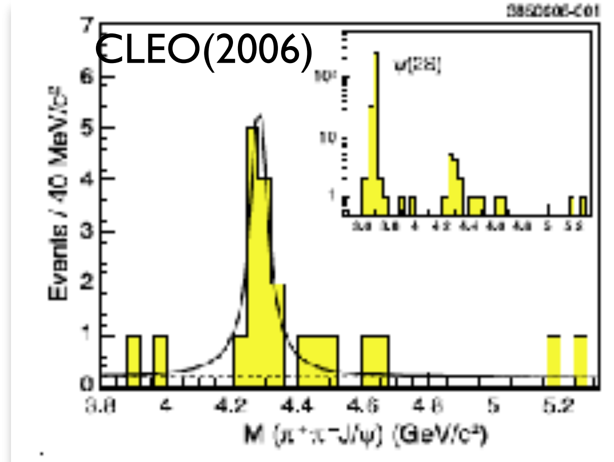
$J^{PC} = 1^{-+}$ is not a $q\bar{q}$ state

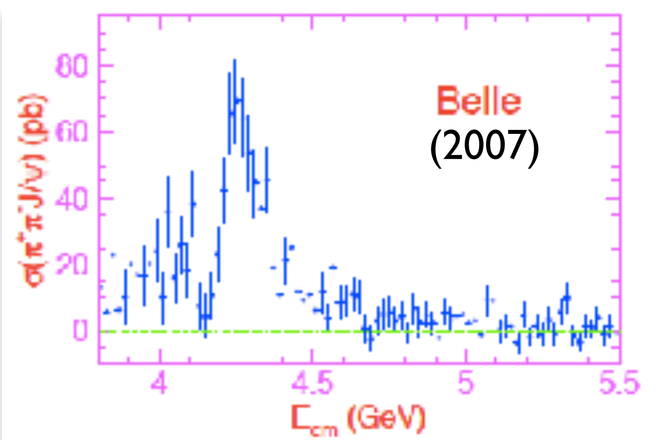
exotic quantum numbers

Y(4260) as Hybrid Candidate

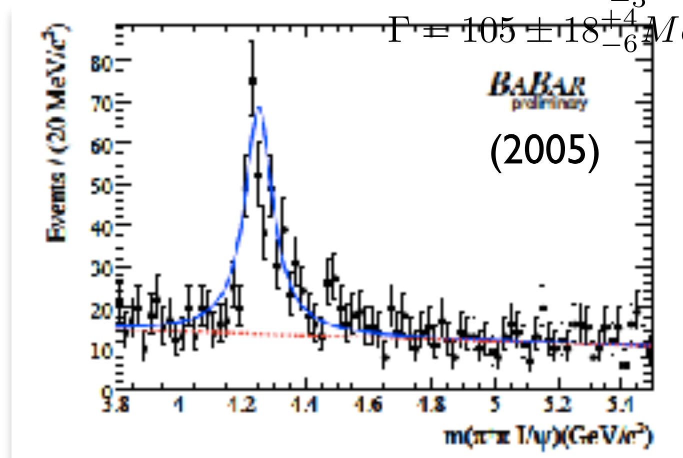
discovered by BaBar in J/ψ $\pi^+\pi^-$ (2005) confirmed by CLEO,Belle other modes from BaBar







$$M = 4252 \pm 6^{+2}_{-3} MeV$$
$$\Gamma = 105 \pm 18^{+4}_{-6} MeV$$



Theory: Hybrid candidate

Light quark exotic candidate

$\pi^- p \to \eta\pi^- p$

$M = 1370 \pm 16^{+50}_{-30}$ MeV / c^2

$\Gamma = 385 \pm 40^{+65}_{-105}$ MeV / c^2

$\pi^- p \to \eta\pi^0 n$

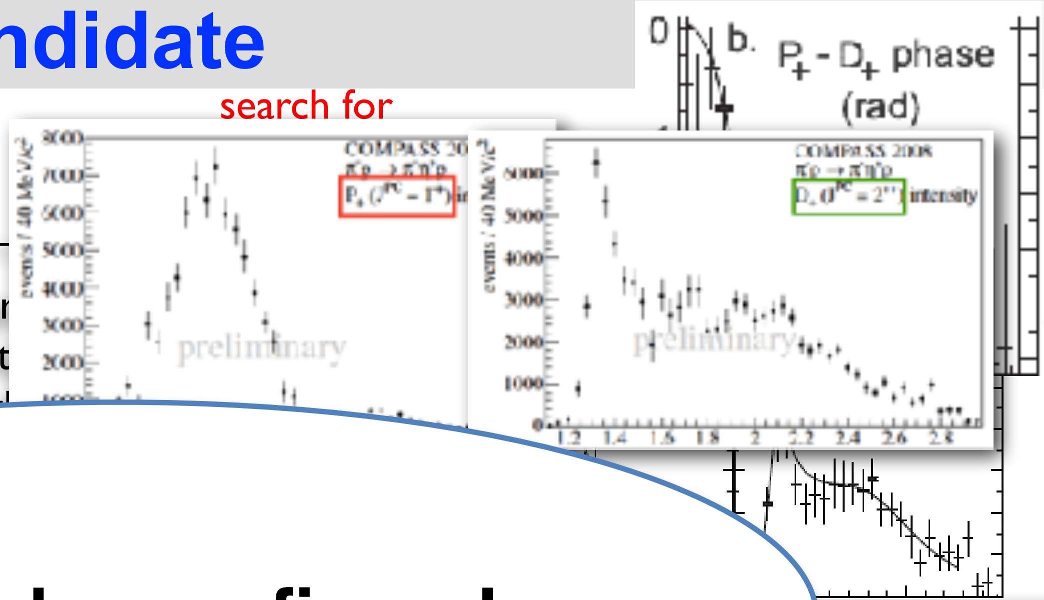
No consistent B-W interpr... possible but a weak ηπ int... exists and can reproduc...

$\pi^- p \to \eta'\pi$...

$\pi^- p \to \rho^0\pi^- p$

Γ = ...

search for



Need to be confirmed

E852 result

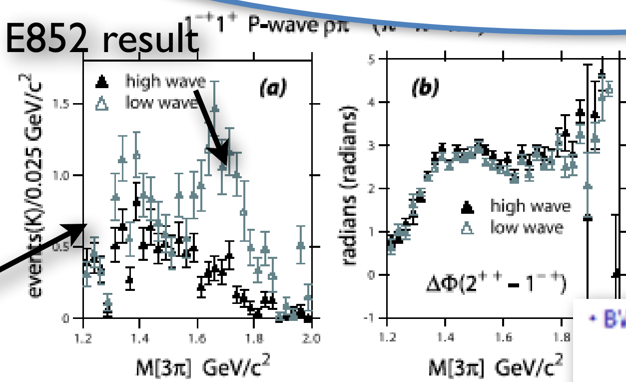
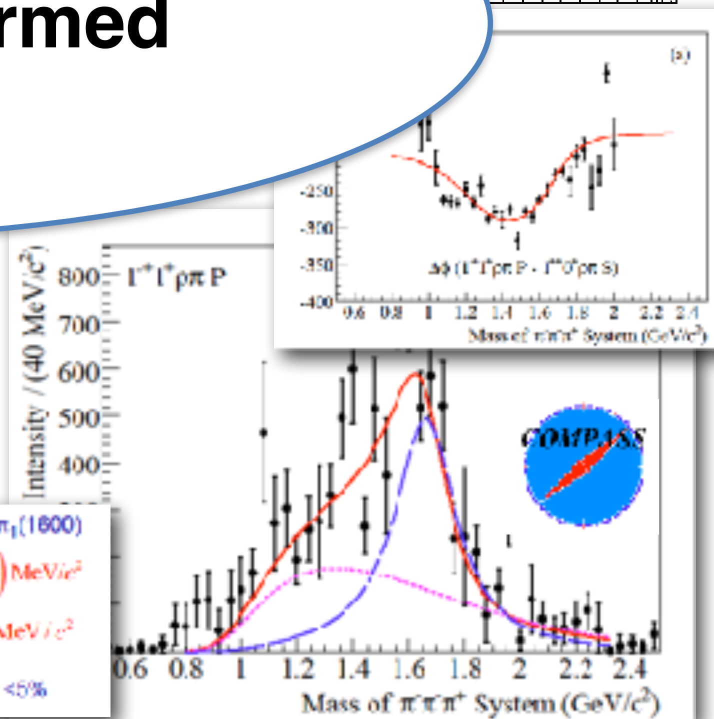


FIG. 25: (a) The $1^{-+}1^+$ P-wave $\rho\pi$ partial wave ... charged mode ($\pi^-\pi^-\pi^+$) for the high-wave set PWA a... low-wave set PWA and (b) the phase difference $\Delta\Phi$ b... the 2^{++} and 1^{-+} for the two wave sets.

$\pi^- p \to \pi_2^-(1600)p$

$\pi_2^- \to \rho^0\pi^-$

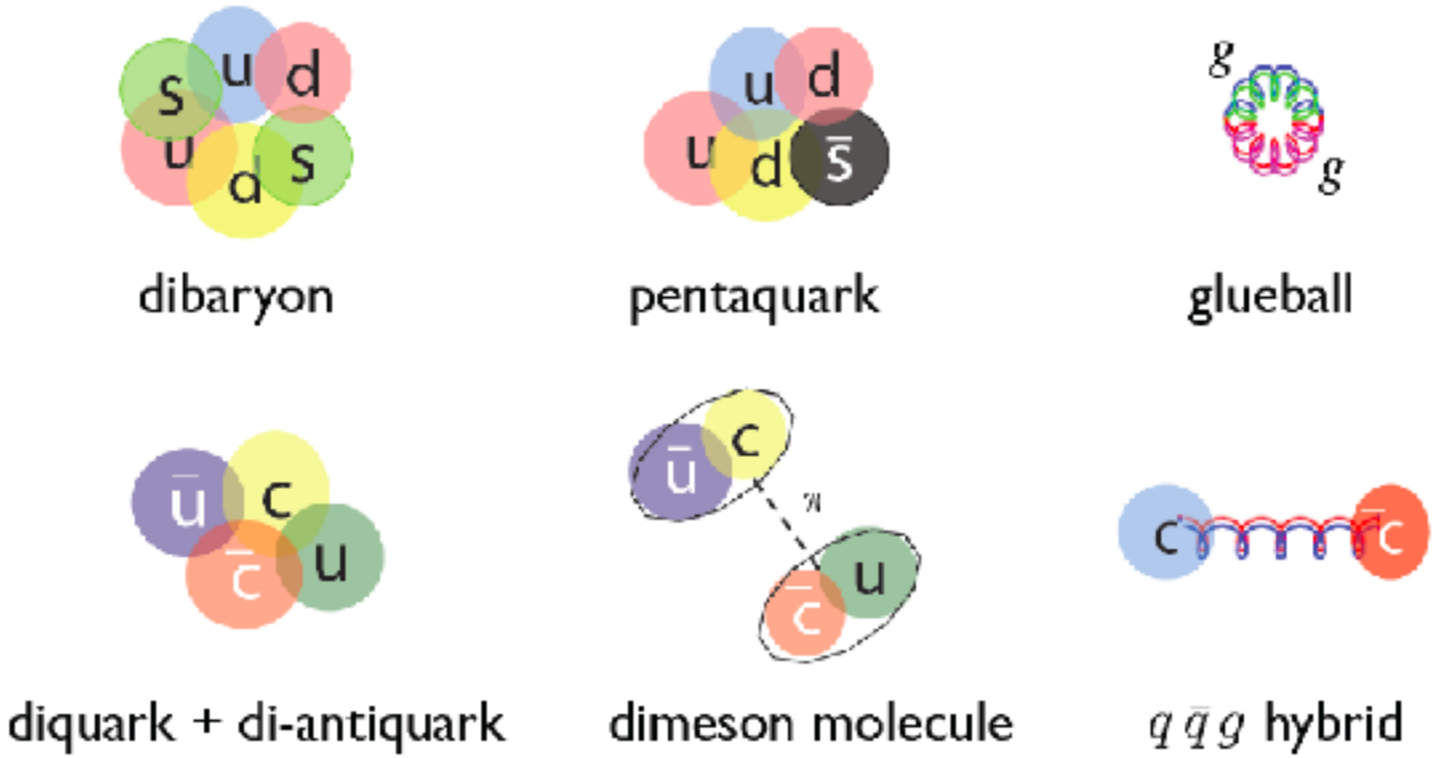
$\rho^0 \to \pi^+\pi^-$



- BW parameters for $\pi_1(1600)$
- $M = \left(1660 \pm 10^{+0}_{-64}\right)$ MeV/c^2
- $\Gamma = \left(269 \pm 21^{+42}_{-64}\right)$ MeV/c^2
- Leakage negligible: <5%

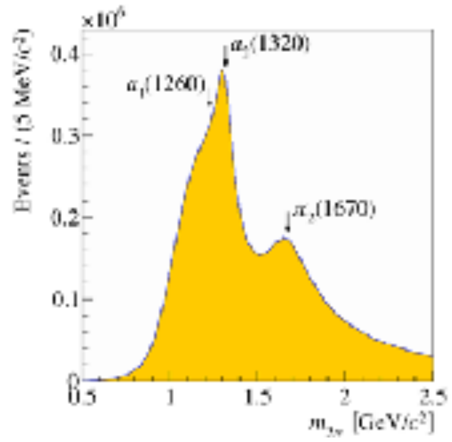
Beyond the quark model

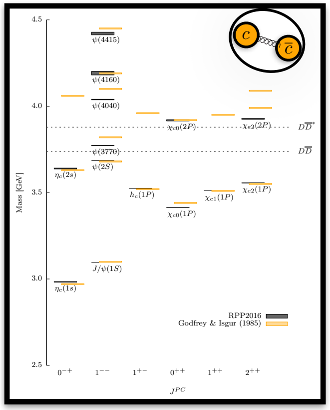
QCD: There are many other possible color singlets.



Identifying resonances

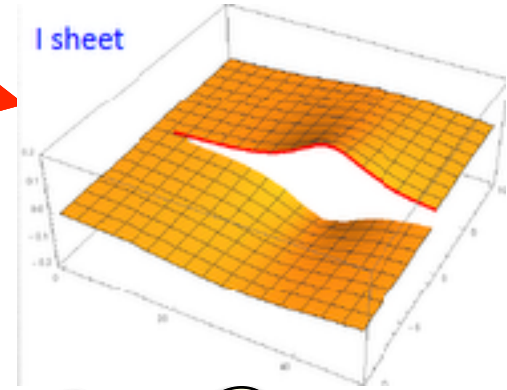
- Experimental or lattice signatures (rea axis data: cross section bumps and dips, energy levels)





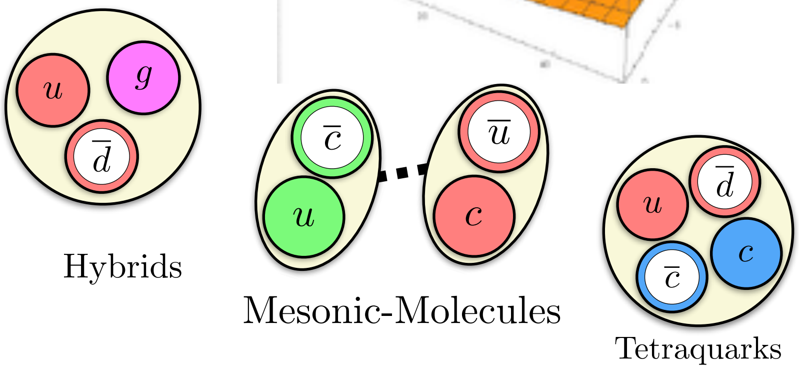
Reaction amplitudes

- Theoretical signatures (complex plane singularities: poles, cusps)



Microscopic Models

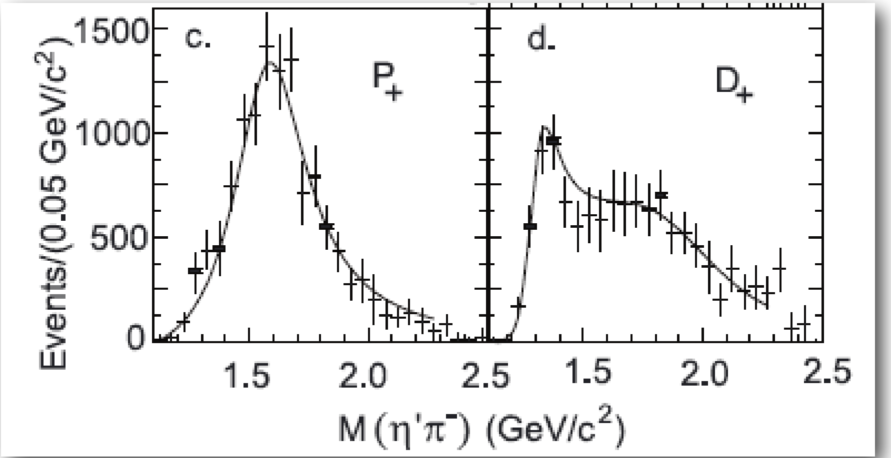
- What is the interpretation (constituent quarks, molecules, …) ?



A possible scenario (Lecture 1 summary)

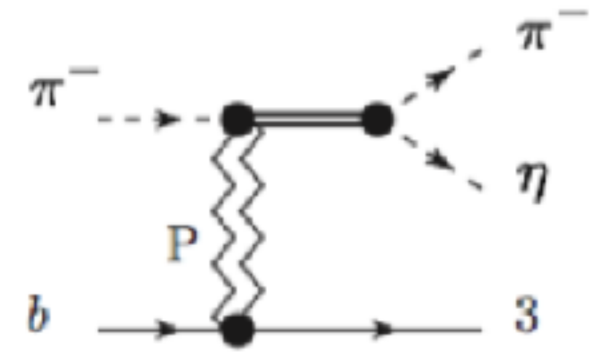
- QCD vacuum has gluon condensate in the form color monopolies, vortices,…
- The condensate leads to an effective, confining potential between color charges
- Light quarks propagating through this medium acquire effective mass
- Static color charges (i.e. “very heavy” quarks) inserted into the vacuum polarize the condensate and change the background gluon distribution
- For large separation between the charges this leads to formation of a chromo electric flux tube (aka dual superconductor)
- For small separation between charges, the effect of vacuum polarization can be described as quasi-particles.
- Once the have quarks are allowed to move the polarized gluon filed (the quasi-particle of the flux tube) can result in a new type of hadrons -> hybrid mesons or baryons.

The Golden Channel: ηπ

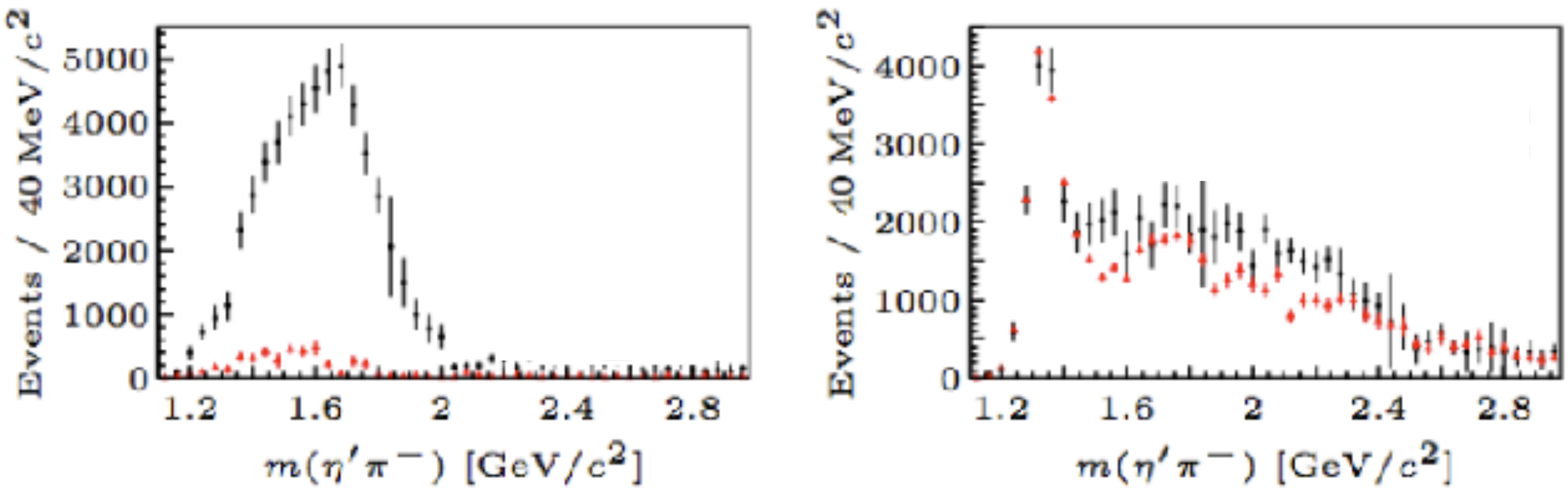


E852

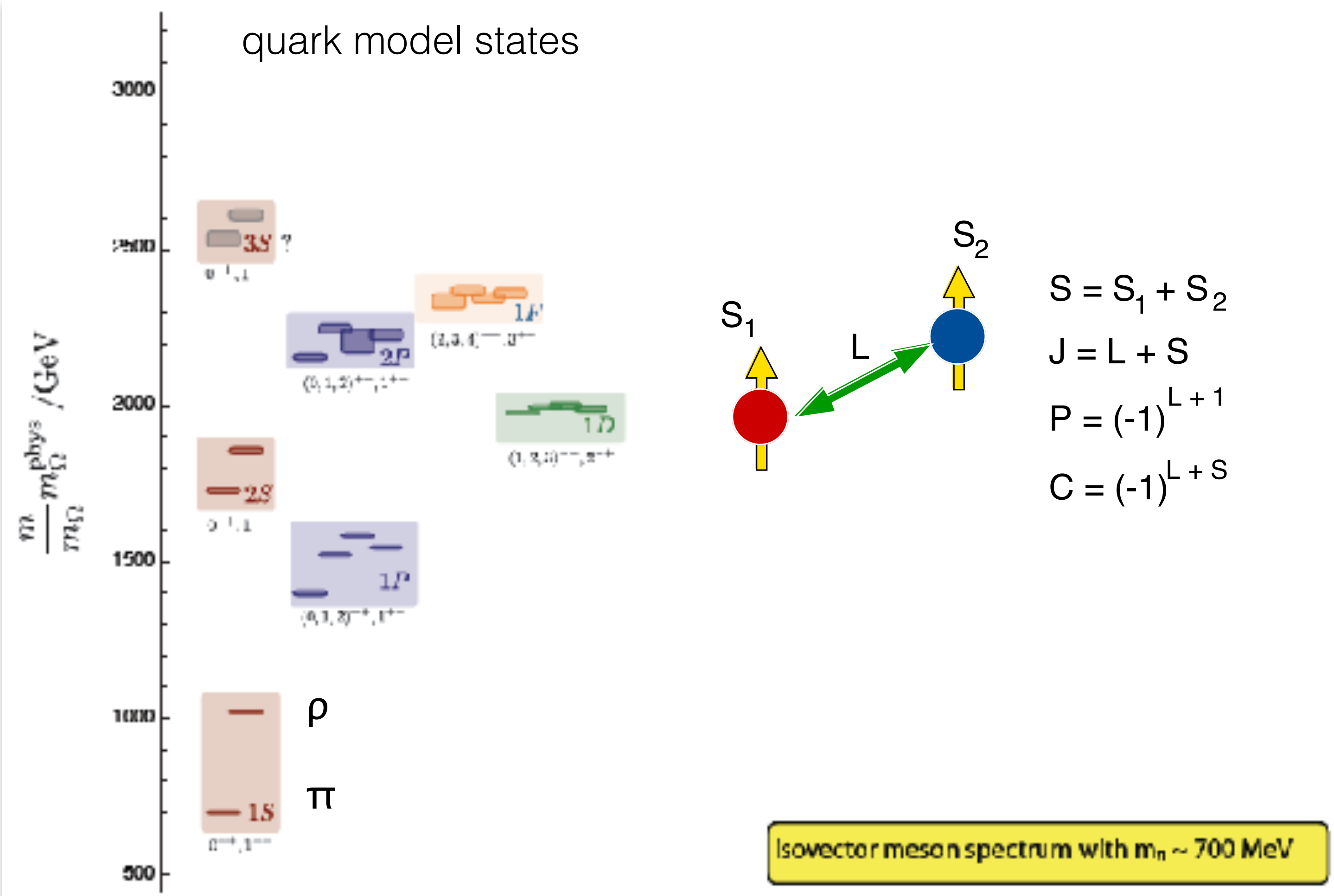
$\pi^- p \rightarrow \eta' \pi^- p$



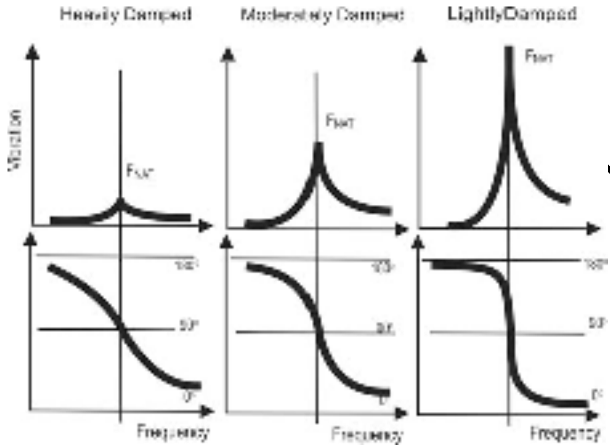
COMPASS



Quark Model (without quasi-gluons)



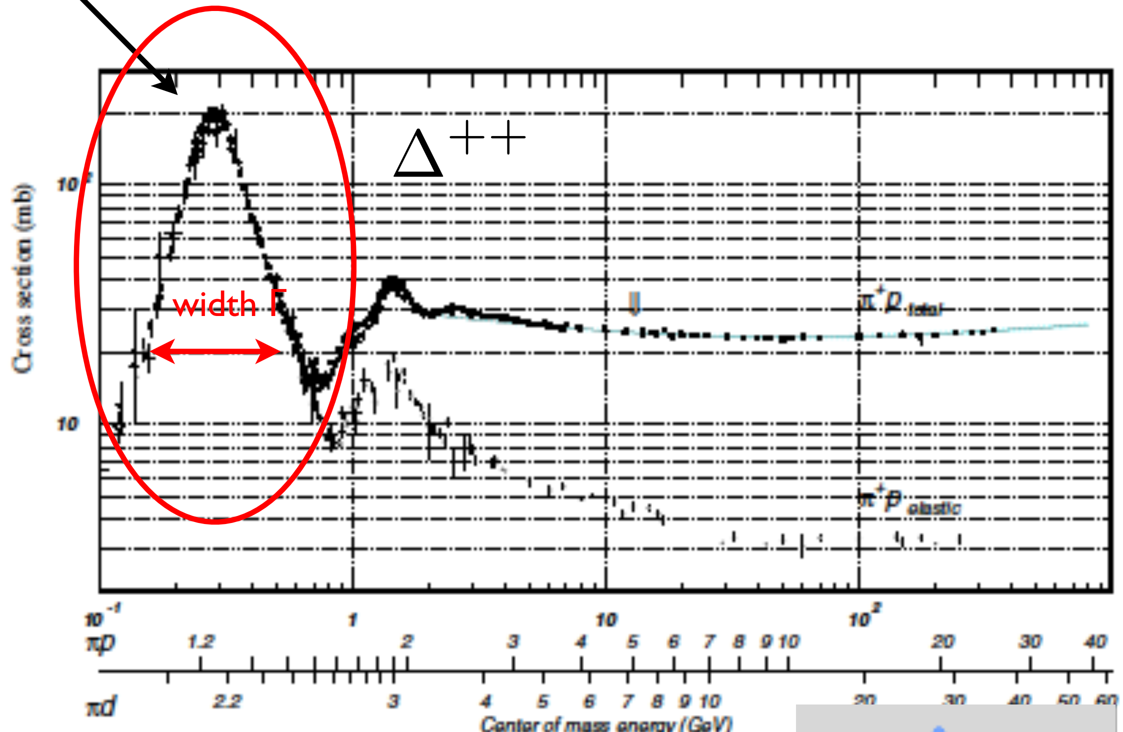
Hunting for Resonances : Amplitude Analysis



peak in intensity
(cross section)

In 1952, E. Fermi and collaborators measured the cross section for $\pi^+ p \to \pi^+ p$ and found it steeply raising.

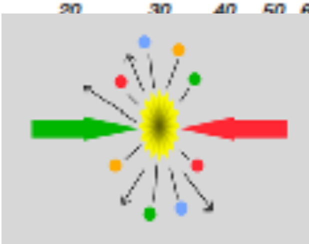
180^0 phase change
in the amplitude



mass ~ 30% above proton

lifetime ~ 4.5 x 10^{-24} s

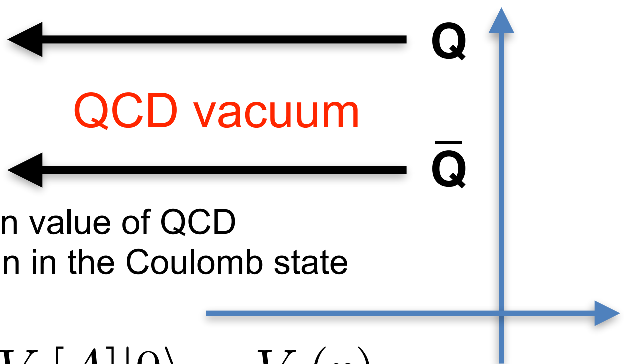
width ~ lifetime^{-1} = 150 MeV



How to Probe Gluons

1. Gluons in the vacuum:

   - Insert a quark pair and measure energy the instantaneous energy.



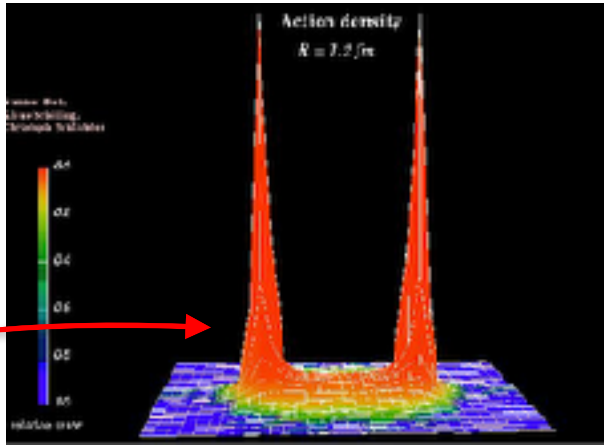
Expectation value of QCD Hamiltonian in the Coulomb state

$$\frac{1}{r} \rightarrow \langle 0|V_c[A]|0\rangle = V_c(r)$$

Coulomb state ≠ QCD eigenstate

2. Gluons in a physical e.g. quark-antiquark state:

   - Insert a quark pair, wait until it polarizes the vacuum and measure energy the state.



Wilson state = QCD eigenstate

$$|Q\bar{Q}\rangle \sim Q^\dagger \bar{Q}^\dagger |0\rangle$$

Coulomb state

$$|Q\bar{Q}\rangle = Q^\dagger \bar{Q}^\dagger |0\rangle + Q^\dagger \bar{Q}^\dagger g^\dagger |0\rangle + \cdots$$

Coulomb state + extra gluons

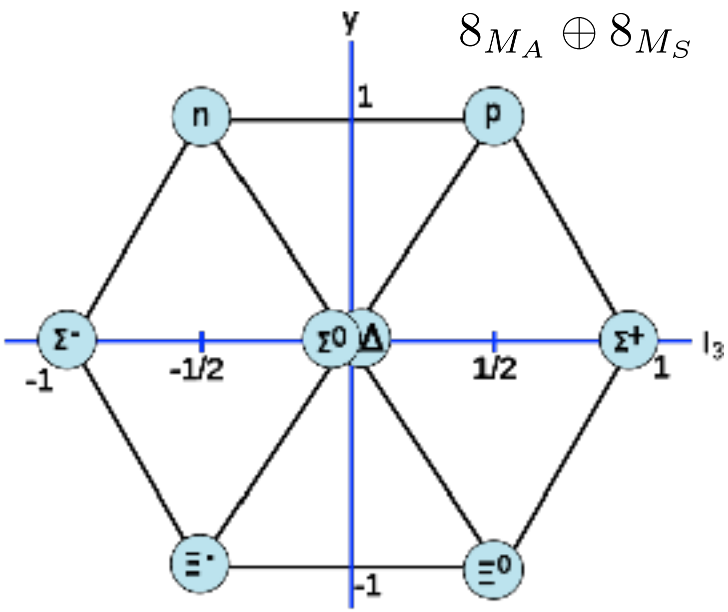
$$B_{ijk} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}_i \otimes \begin{pmatrix} u \\ d \\ s \end{pmatrix}_j \otimes \begin{pmatrix} u \\ d \\ s \end{pmatrix}_k$$

$$1_A = \epsilon_{ijk} B_{ijk}$$

not realized in nature (Pauli blocking)

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_{M_S} \oplus 8_{M_A} \oplus 1_A$$

$8_{M_A} \oplus 8_{M_S}$



10_S