Introduction to Quantum Chromodynamics (QCD)

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Four Lectures

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Physical Observables

Cross sections with identified hadrons
are
non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is not a perturbative scale

Purely infrared safe quantities

Observables with a lot of hadrons,
but,
without specifically identified hadron(s)

Identified hadrons – QCD factorization

Fully infrared safe observables

$$\sigma^{\rm total}_{e^+e^-\to{\rm hadrons}}$$

Fully inclusive with a lot of hadrons,

but,

without any specifically identified hadron(s)

in terms of particle type and momentum!

QCD: $$\sigma^{\rm total}_{e^+e^-\to{\rm hadrons}} = \sigma^{\rm total}_{e^+e^-\to{\rm partons}}$$

The simplest observable in QCD!

BESIII can measure it with precision

Fully infrared safe observables

Another example: $\sigma^{\text{Jets}}_{e^+e^-\to\text{hadrons}}$

with phase space constraints

Sufficiently inclusive with a lot of hadrons,
but,
without any specifically identified hadron(s)
in terms of particle type or nambers!

QCD: $\sigma^{\text{Jets}}_{e^+e^-\to\text{hadrons}} \approx \sigma^{\text{Jets}}_{e^+e^-\to\text{partons}}$

Jets – "trace" of energetic partons

Other examples: Thrust distribution in e^+e^- collisions, etc.

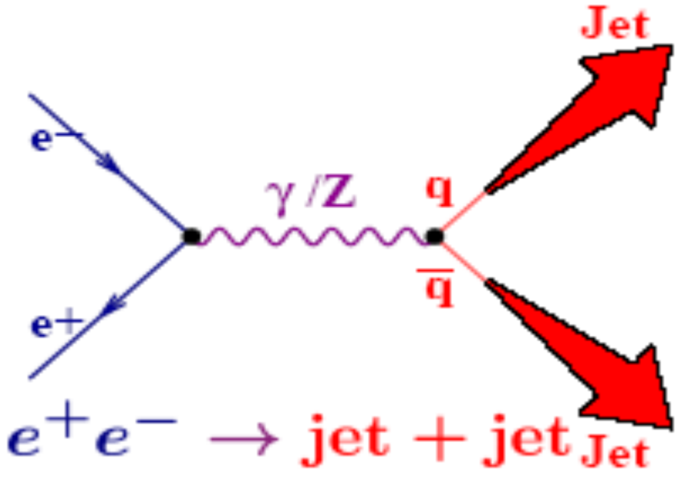
Jets – trace of partons

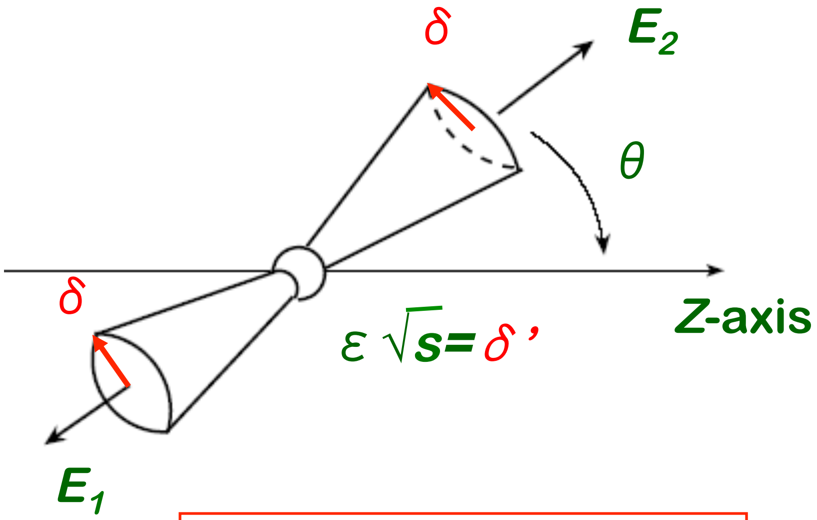
- **Jets – “total” cross-section with a limited phase-space**

  Not any specific hadron!

- Q: will IR cancellation be completed?
  - ✧ Leading partons are moving away from each other
  - ✧ Soft gluon interactions should not change the direction of an energetic parton → a “jet” – “trace” of a parton

- Many Jet algorithms

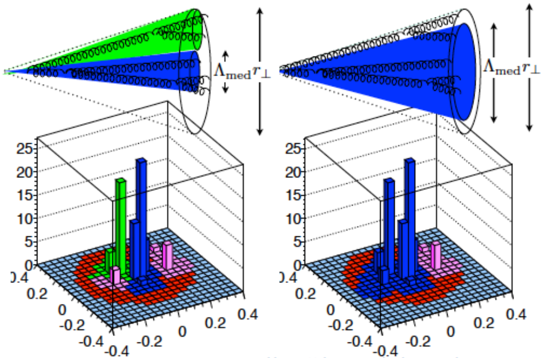




Sterman-Weinberg Jet

Jet finding algorithms – Not unique!

❑ Jet definition – how to combine particles into a jet



Casalderrey, Mehtar-Tani, Salgado, Tywoniuk 1210.7765

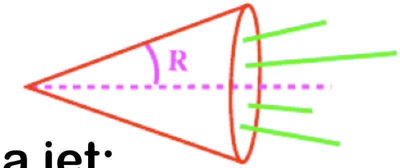
✧ Recombination algorithms (almost all e+e- cases):

- **Recombination metric:** $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$

  $M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

- **Combine particle pair (i,j) with the smallest y_{ij}:**

  $(i,j) \rightarrow k \qquad e.g.\ \mathrm{E\ scheme}:\ p_k = p_i + p_j$



✧ Cone jet algorithms (CDF, …, colliders):

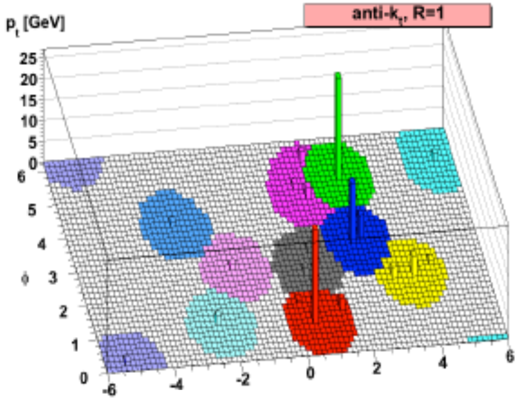
- **Cluster all particles into a cone of half angle R to form a jet:**
- **Recombination metric:** $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right)\frac{\Delta_{ij}^2}{R^2}$

  $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

- **Classical choices:**

  “*k*$_T$ algorithm” ($p=1$), “anti-*k*$_T$” ($p=-1$), …

- **Require a minimum visible jet energy:** $E_{jet} > \epsilon$
- **Particle could be outside the “cone”**



Cacciari, Salam, Soyez 0802.1189

Infrared safety for restricted cross sections

- For any observable with a phase space constraint, Γ,

$$\begin{aligned} d\sigma\left(\Gamma\right) \equiv & \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) \\ & + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) \\ & + \ldots \\ & + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots \end{aligned}$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$
are constraint functions
and invariant under
Interchange of n-particles

- Conditions for IRS of dσ (Γ):

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \;\; 0 \le \lambda \le 1$$

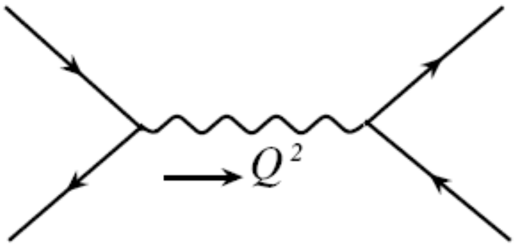
Physical meaning:

Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without the parton

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1$ for all n $\Rightarrow$ $\sigma^{(\mathrm{tot})}$

Two-jet cross section in e+e- collisions

- **Parton-Model = Born term in QCD:**



$$\sigma_{2\mathrm{Jet}}^{(\mathrm{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

- **Two-jet in pQCD:**

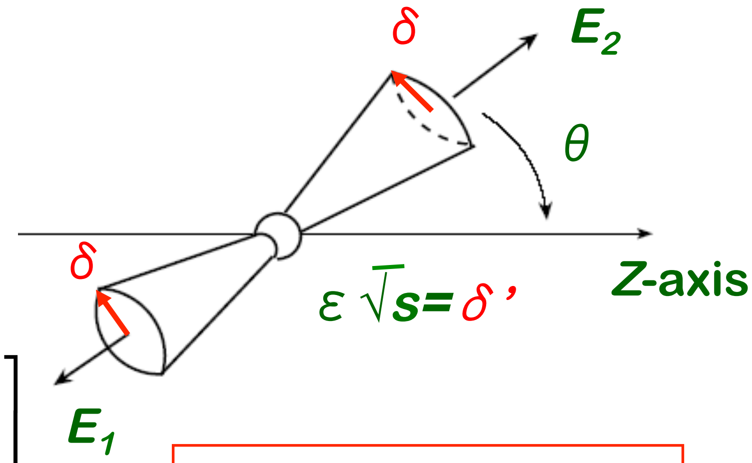
$$\sigma_{2\mathrm{Jet}}^{(\mathrm{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $C_n = C_n(\delta)$

- **Sterman-Weinberg jet:**

$$\sigma_{2\mathrm{Jet}}^{(\mathrm{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\times\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$

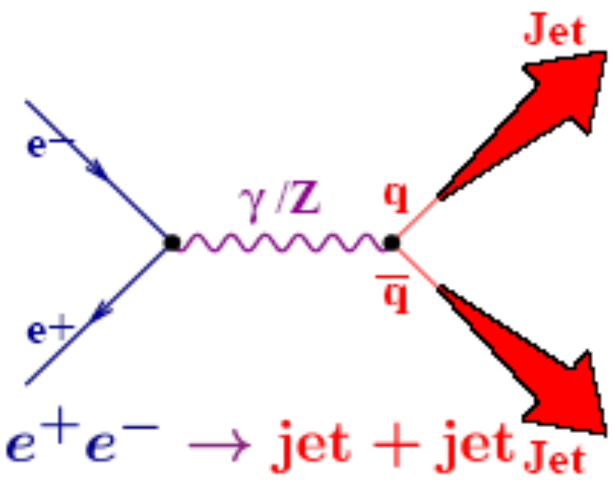


Sterman-Weinberg Jet

$$\sigma_{\mathrm{total}} = \sigma_{2\mathrm{Jet}} \qquad \text{as } Q\to\infty$$

An early clean two-jet event

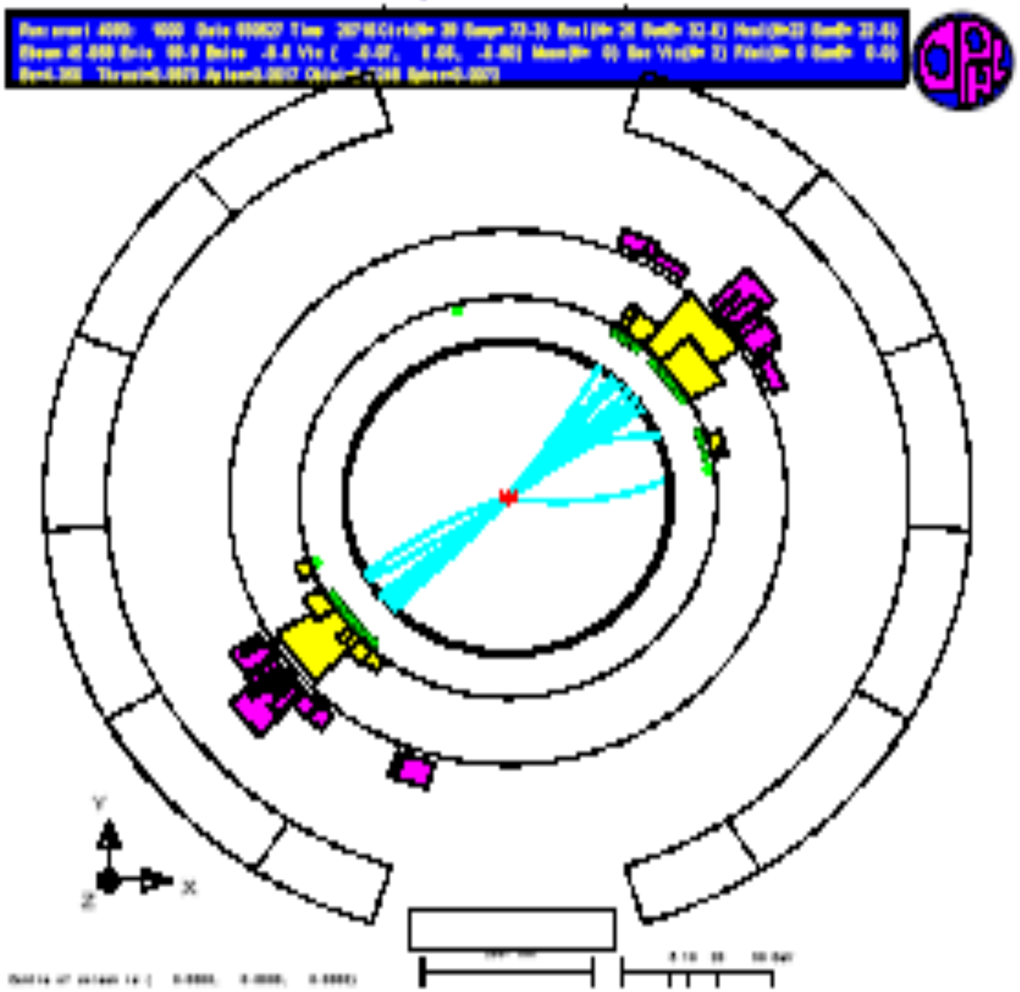
Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \rightarrow \text{jet} + \text{jet}$

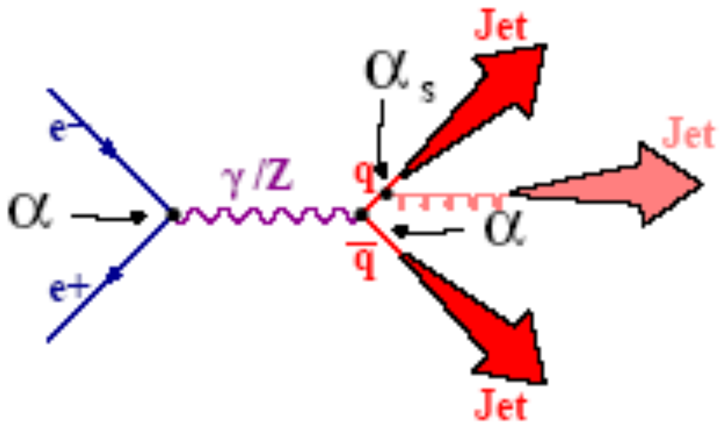
A clean trace of two partons – a pair of quark and antiquark

LEP ($\sqrt{s} = 90 - 205$ GeV)



Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

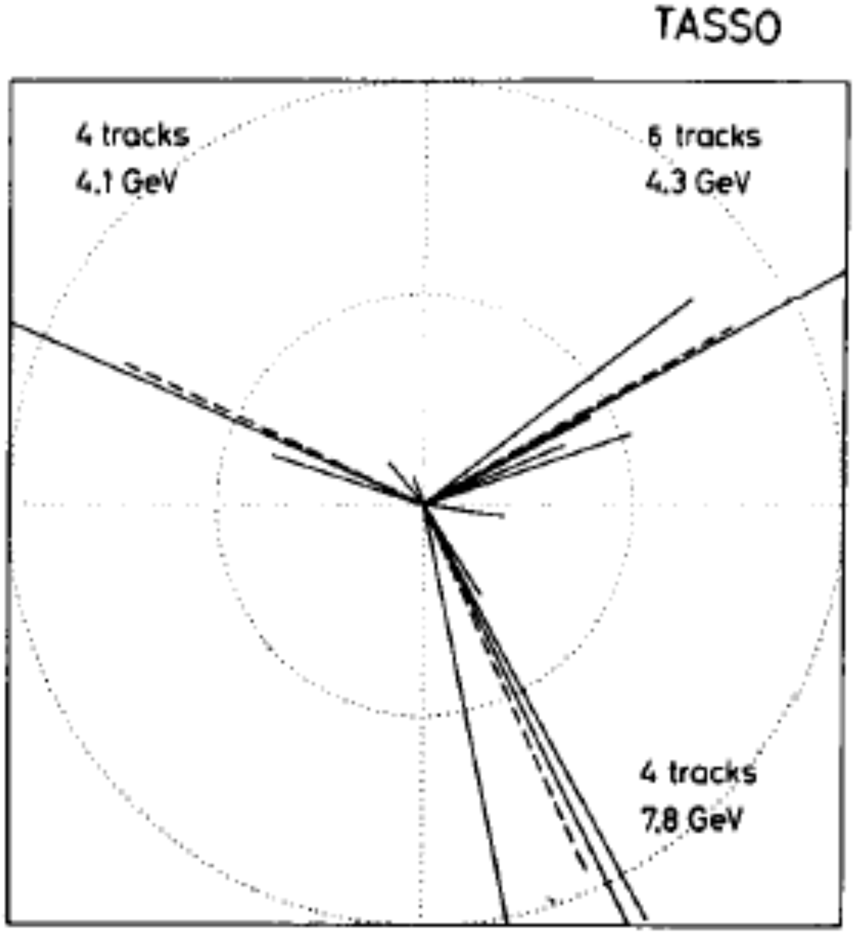
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

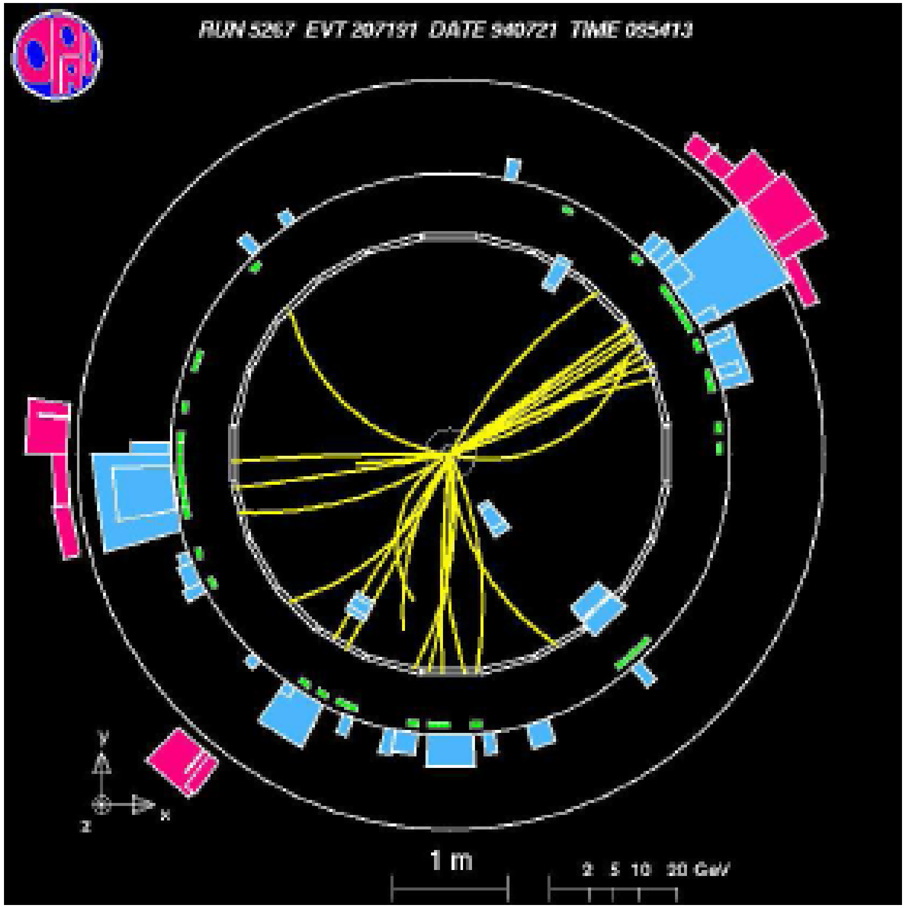
JADE Collab., Phys. Lett. B91 (1980) 142

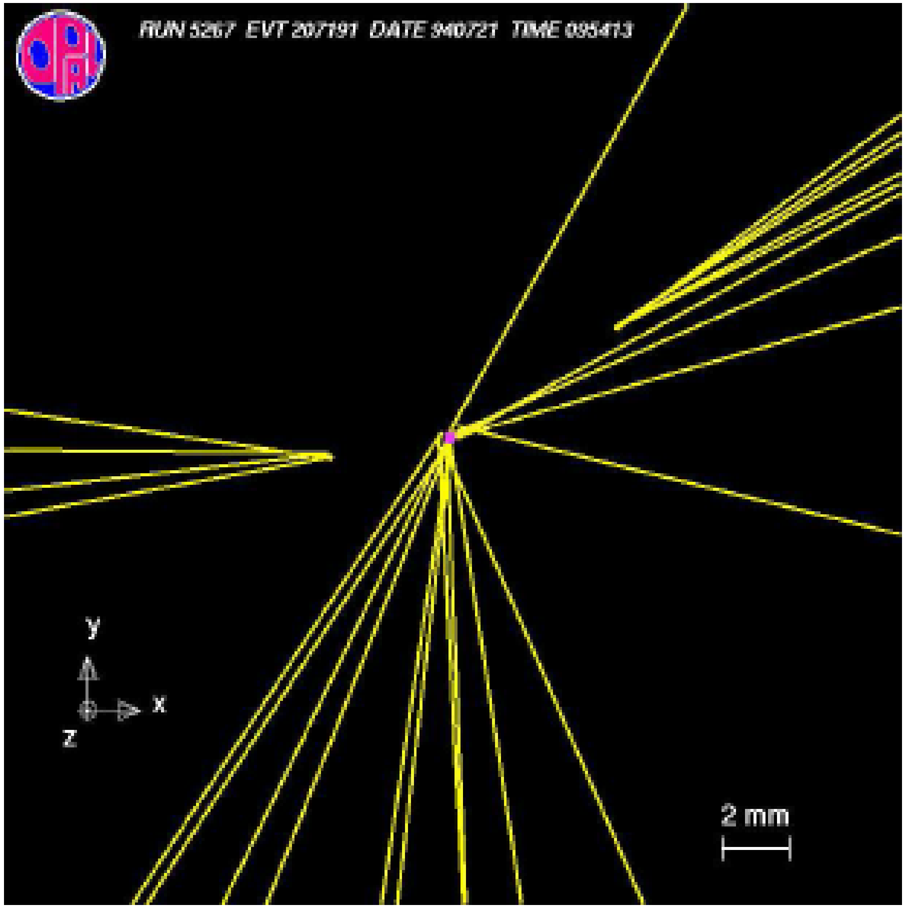
PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



Tagged three-jet event from LEP



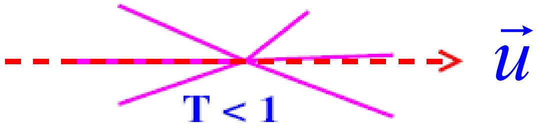


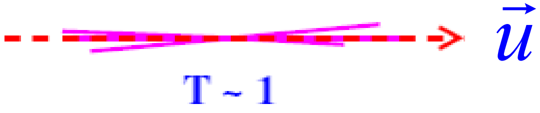
Gluon Jet

Thrust distribution

❑ **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}}\left(\frac{\sum_{i=1}^n \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^n \left|\vec{p}_i\right|}\right)$$





❑ **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to\text{hadrons}}}{dT} \quad \textbf{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

✧ Contribution from p=0 particles drops out the sum

✧ Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

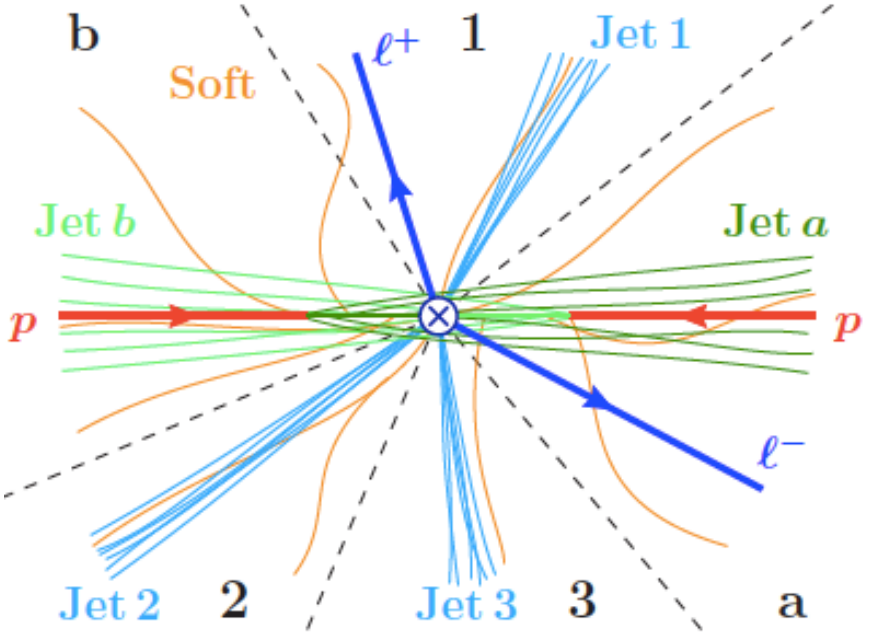
and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

N-Jettiness

- **Event structure:**

  $pp \to \text{leptons plus jets}$



- **N-Jettiness:**

  (Stewart, Tackmann, Waalewijin, 2010)

$$\tau_N = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

The sum include all final-state hadrons *excluding* more than N jets

Allows for an event-shape based analysis of multi-jets events (a generalization of Thrust)

- **N-infinitely narrow jets (jet veto):**

  As a limit of N-Jettiness: $\tau_N \to 0$

Generalization of the thrust distribution in e^+e^- initial-state identified hadron!

The harder question

❑ **Question:**

How to test QCD in a reaction with identified hadron(s)?
– to probe the quark-gluon structure of the hadron

❑ **Facts:**

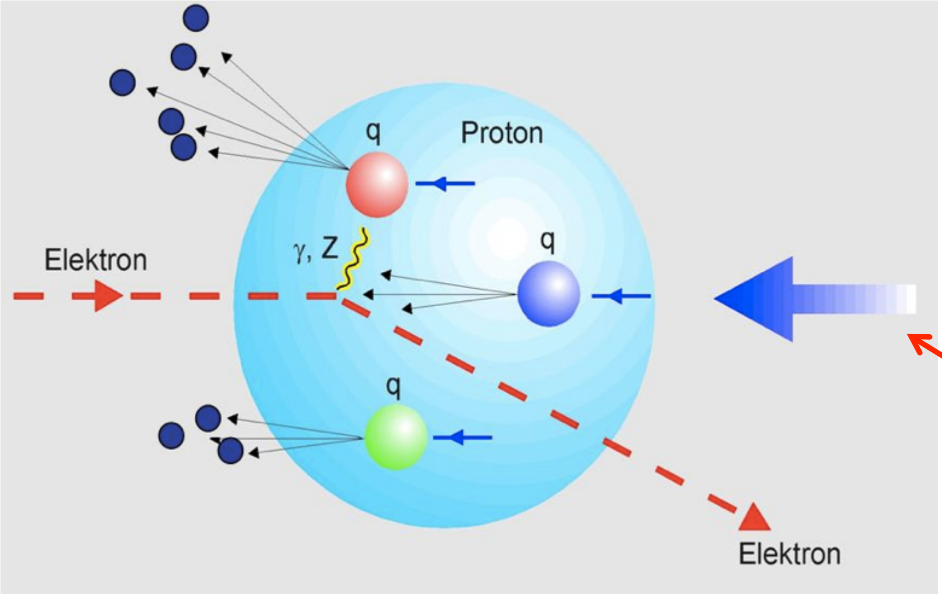
Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

❑ **Solution – Factorization:**

- ✧ Isolate the calculable dynamics of quarks and gluons
- ✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
  - – provide information on the partonic structure of the hadron

Observables with ONE identified hadron



$$\sigma^{\text{DIS}}_{\ell p\to\ell' X\,(\text{everything})}$$

Identified initial-state hadron-proton!

Cross section is infrared divergent, and nonperturbative!

QCD factorization
(approximation!)

Cross Section $=$ Infrared-Safe $\otimes$ Nonperturbative-distribution

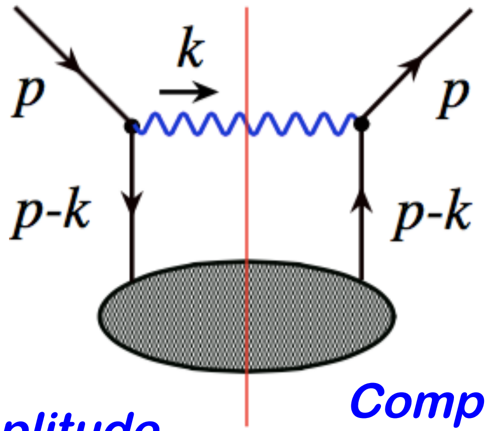
Measured ↑ Hard-probe ↑ Universal-hadron structure ↑

Pinch singularity and pinch surface

☐ “Square” of the diagram with a *“unobserved gluon”*:

“Cut-line” – final-state

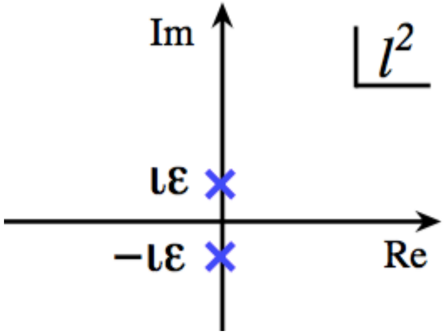
– in a “cut-diagram” notation



$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

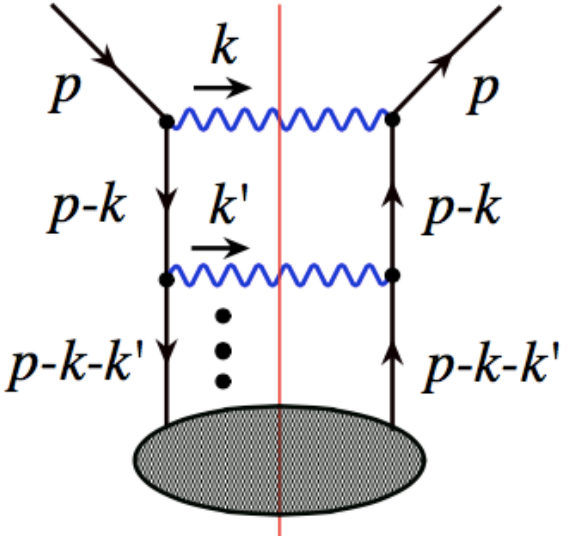
$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



Amplitude

Complex conjugate of the Amplitude



Pinch surfaces

Pinch singularities “perturbatively”

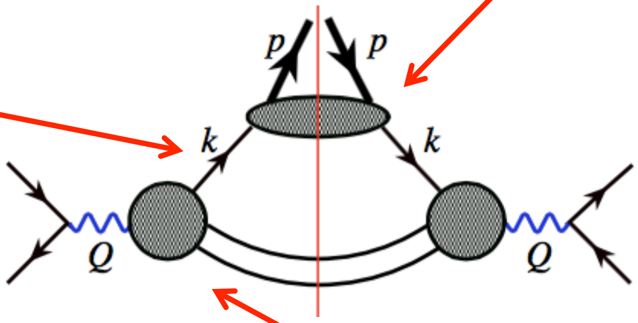
= “surfaces” in k, k’, …

determined by $(p\text{-}k)^2=0$, $(p\text{-}k\text{-}k')^2=0$, …

“perturbatively”

Hard collisions with identified hadron(s)

- **Creation of an identified hadron:**

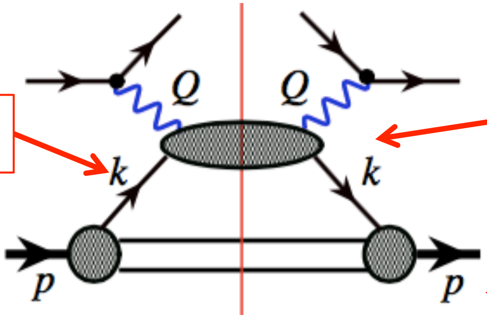


Non-perturbative!

Pinch in k^2

Perturbative!

- **Identified initial hadron:**

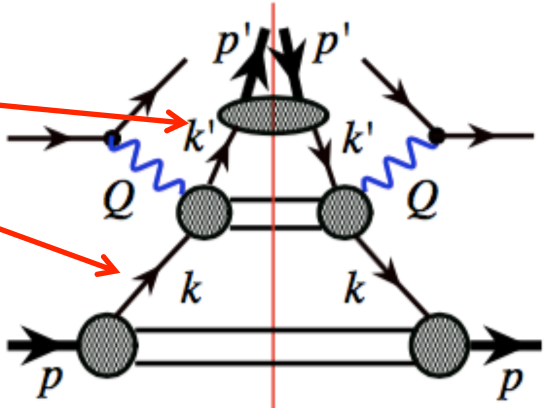


Pinch in k^2

Perturbative!

Non-perturbative!

- **Initial + created identified hadron(s):**

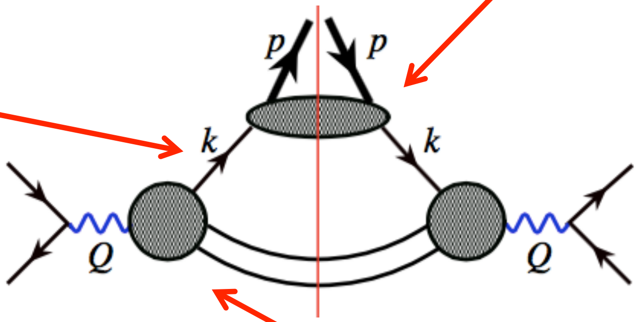


Pinch in both k^2 and k'^2

*Cross section with identified hadron(s)
is NOT perturbatively calculable*

Hard collisions with identified hadron(s)

☐ **Creation of an identified hadron:**

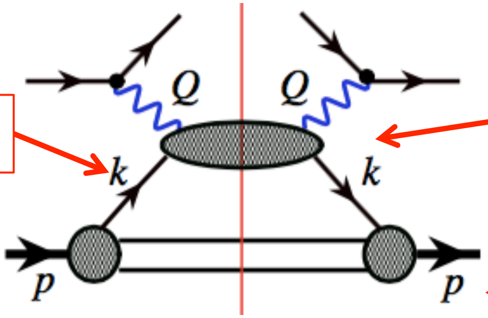


Non-perturbative!

Pinch in k^2

Perturbative!

☐ **Identified initial hadron:**

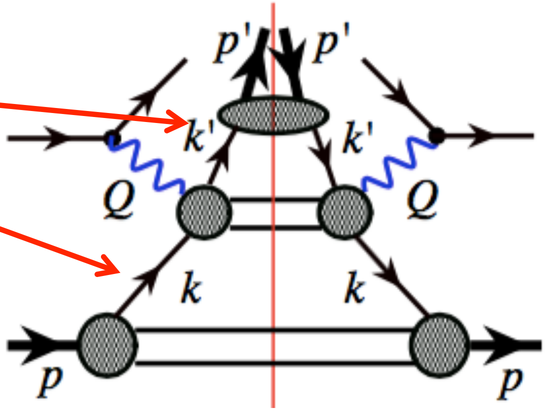


Pinch in k^2

Perturbative!

Non-perturbative!

☐ **Initial + created identified hadron(s):**



Pinch in both k^2 and k'^2

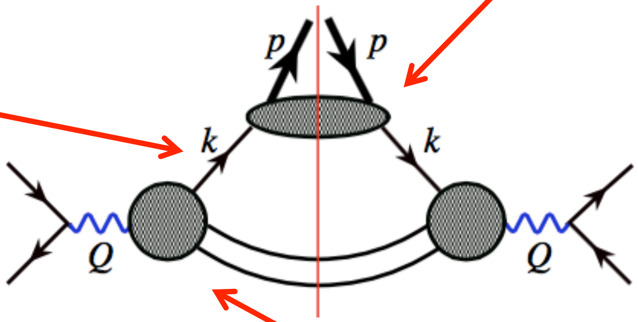
Dynamics at a HARD scale is linked
by partons almost on Mass-Shell

Hard collisions with identified hadron(s)

- Creation of an identified hadron:

Non-perturbative!

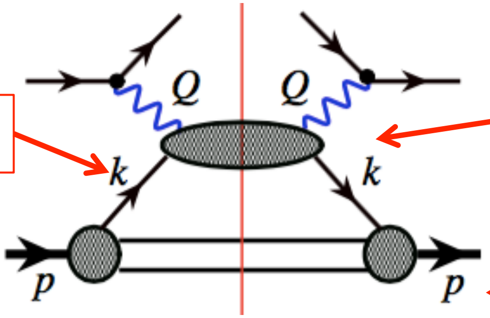
Pinch in k^2



Perturbative!

- Identified initial hadron:

Pinch in k^2

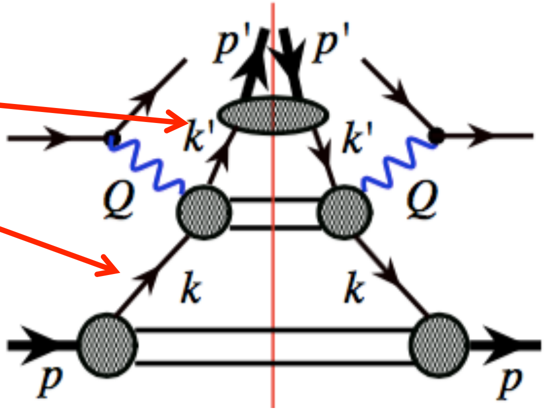


Perturbative!

Non-perturbative!

- Initial + created identified hadron(s):

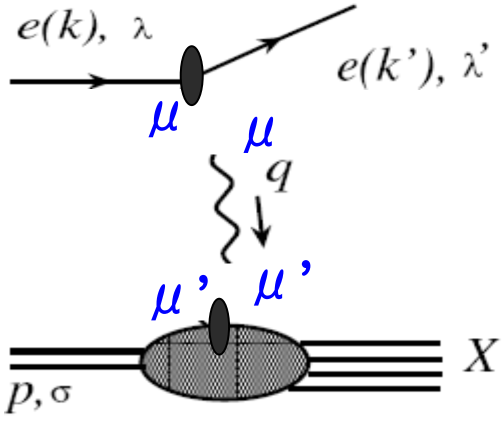
Pinch in both k^2 and k'^2



Quantum interference between dynamics at the HARD and hadronic scales is powerly suppressed!

Inclusive lepton-hadron DIS – one hadron

☐ Scattering amplitude:



$$\mathrm{M}(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_\mu\right]u_\lambda(k)$$

$$* \quad \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$

$$* \quad \left\langle X\left|eJ^{em}_{\mu'}(0)\right|p,\sigma\right\rangle$$

☐ Cross section:

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X}\frac{d^3 l_i}{(2\pi)^3\, 2E_i}\right]\frac{d^3k'}{(2\pi)^3\, 2E'}(2\pi)^4\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

⇒

$$\boxed{E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)}$$

☐ Leptonic tensor:

– known from QED

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}\right)$$

DIS structure functions

❑ **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4 z \; \mathrm{e}^{iq\cdot z} \; \left\langle p,S \middle| J^\dagger_\mu(z) J_\nu(0) \middle| p,S \right\rangle$$

❑ **Symmetries:**

- ✧ **Parity invariance (EM current)** → $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
- ✧ **Time-reversal invariance** → $W_{\mu\nu} = W^*_{\mu\nu}$ real
- ✧ **Current conservation** → $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B,Q^2\right)$$

$$+iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2} g_2\left(x_B,Q^2\right)\right]$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2p\cdot q}$$

❑ **Structure functions – infrared sensitive:**

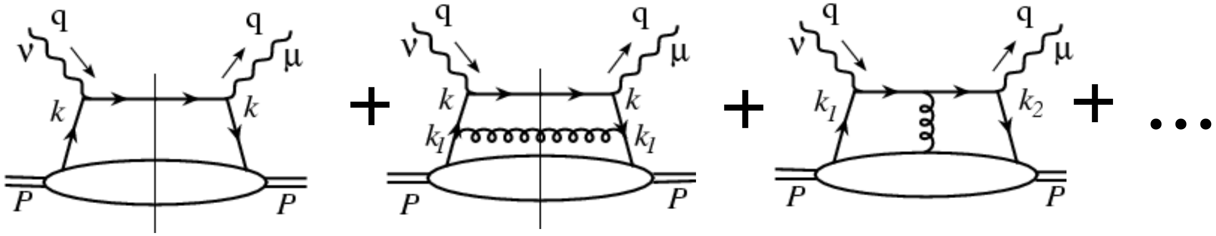
$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

Long-lived parton states

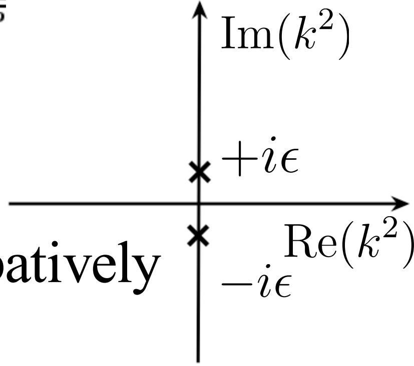
- **Feynman diagram representation of the hadronic tensor:**

$W^{\mu\nu} \propto$



- **Perturbative pinched poles:**

$$\int d^4k\; \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{r_0}\right) \Rightarrow \infty \;\; \text{perturbatively}$$



- **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

Nonperturbative matrix element

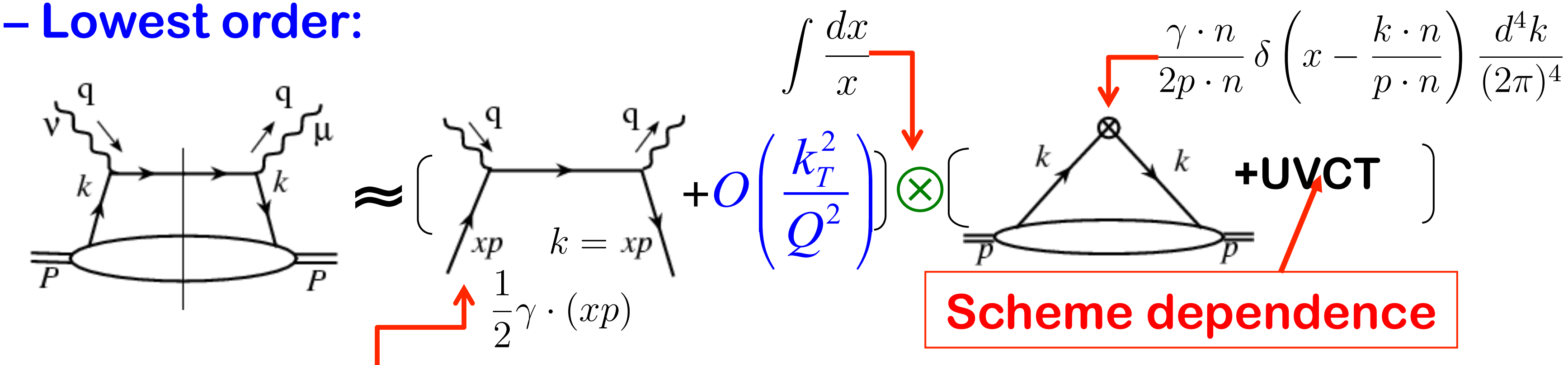
$$\int\frac{dx}{x}d^2k_T\; \mathrm{H}(Q,k^2=0)\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{r_0}\right)+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

Short-distance

Collinear factorization – further approximation

- **Collinear approximation, if** $\boxed{Q \sim xp\cdot n \gg k_T, \sqrt{k^2}}$

  – **Lowest order:**



$$\approx \left[\ \ + O\left(\frac{k_T^2}{Q^2}\right)\right] \otimes \left[\ \ +\text{UVCT} \right]$$

$$\int \frac{dx}{x}$$

$$\frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x-\frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

$$\frac{1}{2}\gamma\cdot(xp)$$

Scheme dependence

Same as an elastic x-section

Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \longrightarrow \infty$, while x_B fixed

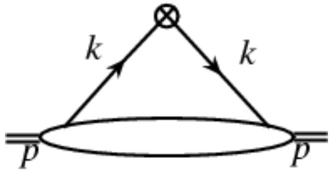
  ⇨ **Feynman's parton model and Bjorken scaling**

$$F_2(x_B,Q^2) = x_B \sum_f e_f^2\,\phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

Spin-½ parton!

- **Corrections:** $\mathcal{O}(\alpha_s) + \mathcal{O}\left(\langle k^2\rangle/Q^2\right)$

Parton distribution functions (PDFs)



❑ **PDFs as matrix elements of two parton fields:**

- **combine the amplitude & its complex-conjugate**

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi} \, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \, \frac{\gamma^+}{2p^+} \, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

$|h(p)\rangle$ **can be a hadron, or a nucleus, or a parton state!**

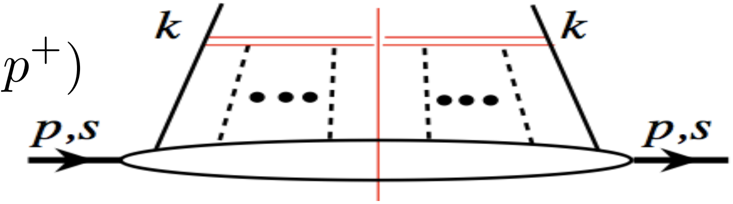
But, it is NOT gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

- **need a gauge link:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi} \, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+} \, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

- **corresponding diagram in momentum space:**

$$\int \frac{d^4k}{(2\pi)^4} \, \delta(x - k^+/p^+)$$



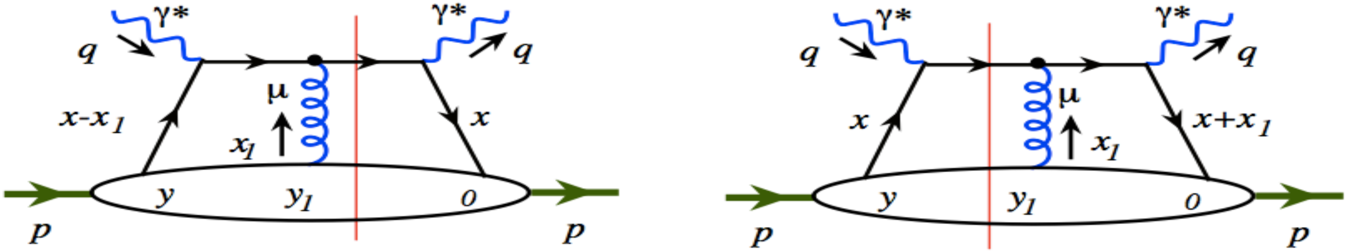
$$+\,\mathrm{UVCT}(\mu^2)$$

μ **-dependence**

Universality – process independence – predictive power

Gauge link – 1st order in coupling "g"

❑ Longitudinal gluon:



❑ Left diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1 p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\,\frac{i((x-x_1-x_B)\gamma\cdot p+(Q^2/2x_Bp^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B+i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{-x_1+i\epsilon}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

❑ Right diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1 p^+ y_1^-}\, n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p+(Q^2/2x_Bp^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B-i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{x_1-i\epsilon}\, \mathrm{e}^{ix_1p^+ y_1^-}\right] \mathcal{M} \;\; = ig\int_{0}^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

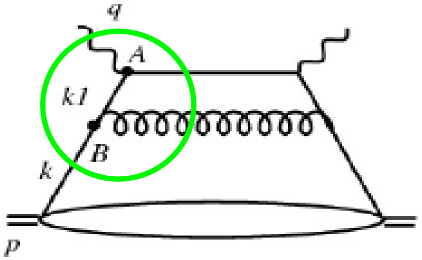
❑ Total contribution:

$$-ig\left[\int_0^\infty - \int_{y^-}^\infty\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$

O(g)-term of the gauge link!

QCD high order corrections

- **NLO partonic diagram to structure functions:**



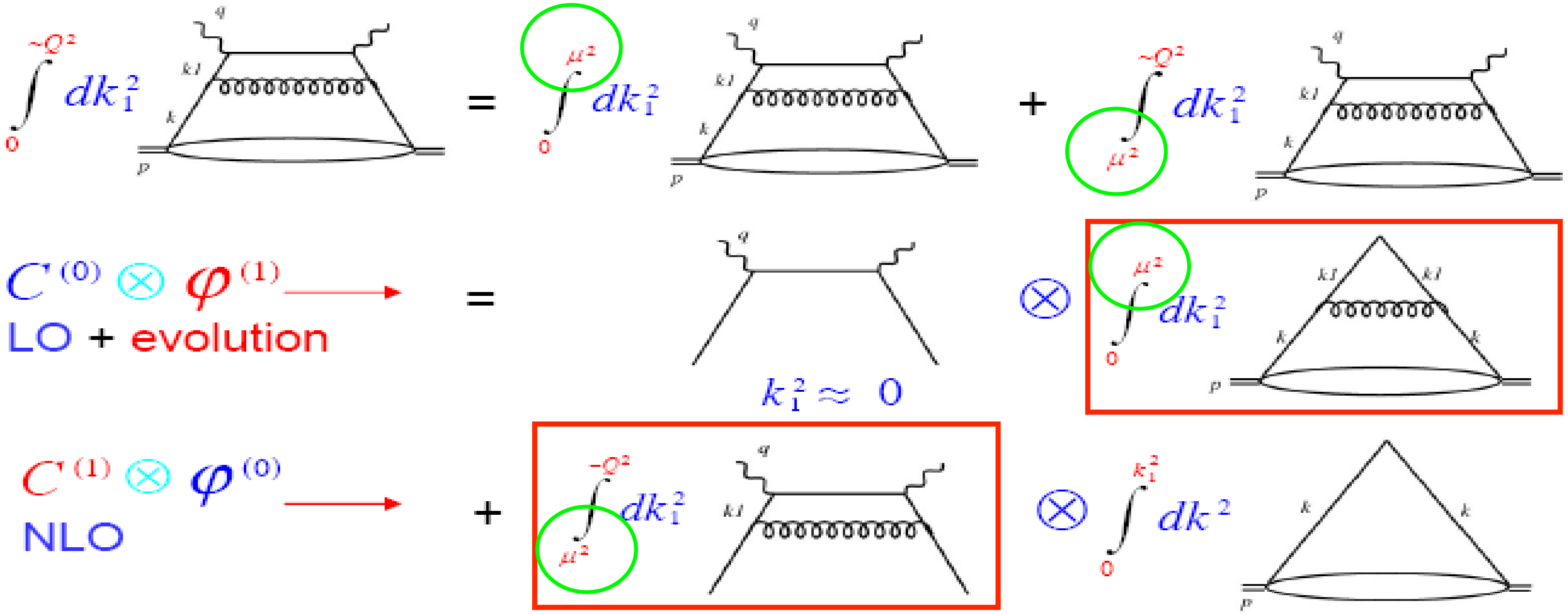
$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by

$$\begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$$

Diagram has both long- and short-distance physics

- **Factorization, separation of short- from long-distance:**



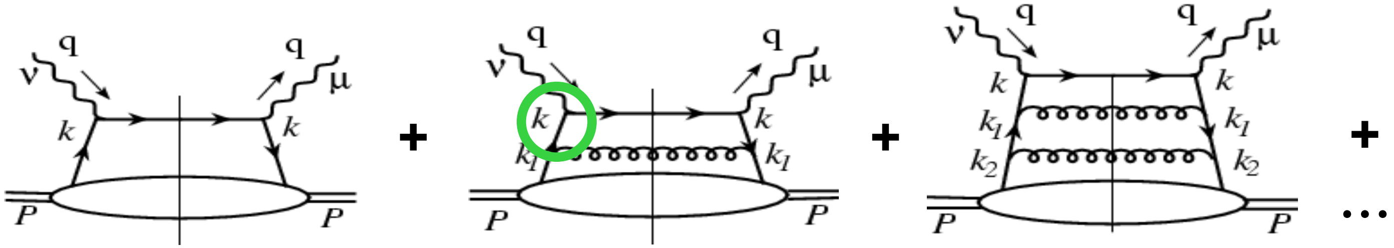
$$\int_0^{\sim Q^2} dk_1^2 \;=\; \int_0^{\mu^2} dk_1^2 \;+\; \int_{\mu^2}^{\sim Q^2} dk_1^2$$

$C^{(0)} \otimes \varphi^{(1)} \longrightarrow$ LO + evolution $=$ $k_1^2 \approx 0$ $\otimes \int_0^{\mu^2} dk_1^2$

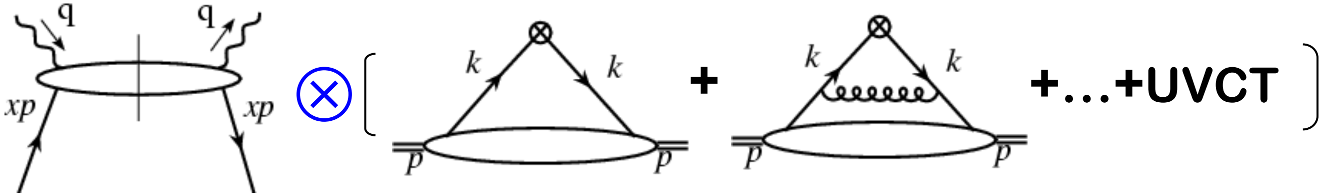
$C^{(1)} \otimes \varphi^{(0)} \longrightarrow$ NLO $+ \int_{\mu^2}^{-Q^2} dk_1^2$ $\otimes \int_0^{k_1^2} dk^2$

QCD leading power factorization

- **QCD corrections: pinch singularities in** $\int d^4 k_i$



- **Logarithmic contributions into parton distributions:**



$$F_2(x_B,Q^2) = \sum_f C_f\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_f\left(x,\mu_F^2\right)+O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

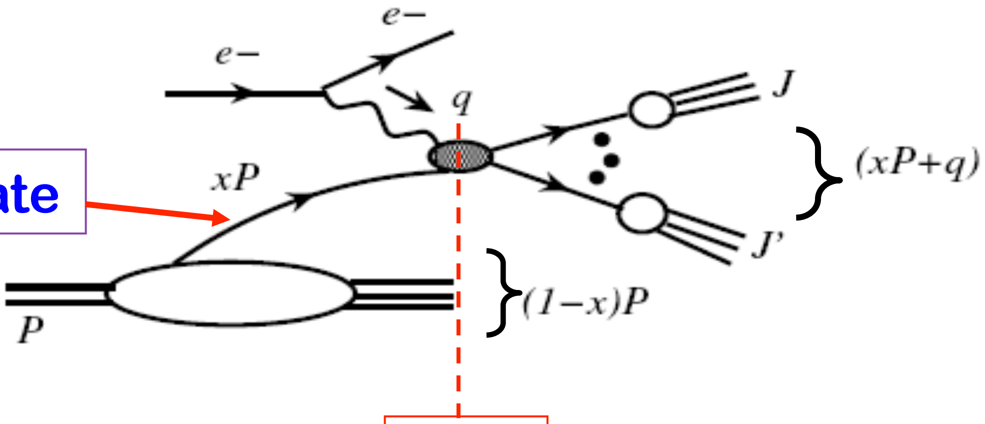
- **Factorization scale:** μ_F^2

→ **To separate the collinear from non-collinear contribution**

Recall: renormalization scale to separate local from non-local contribution

Picture of factorization for DIS

- ❑ **Time evolution:**

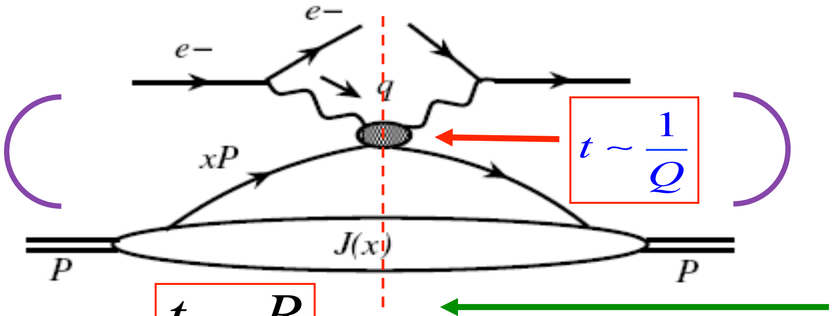


Long-lived parton state

Time: **"Past"** **Now** **"Future"**

- ❑ **Unitarity – summing over all hard jets:**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}\left(\quad \right)$$



$t \sim \frac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

Evolution

- ☐ **Physical cross sections should not depend on the factorization scale**

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

$$F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s)\, \phi_f(x, \mu_F^2)$$

→ **Evolution (differential-integral) equation for PDFs**

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- ☐ **PDFs and coefficient functions share the same logarithms**

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\rm QCD}^2\right)$

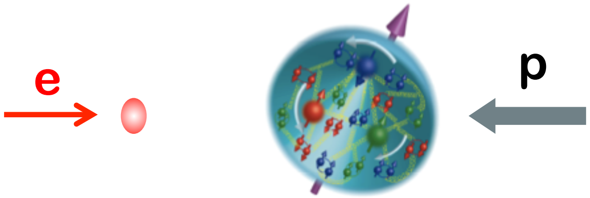
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ **DGLAP evolution equation:**

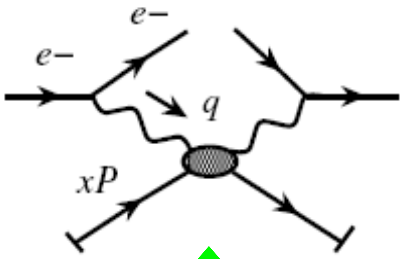
$$\boxed{\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)}$$

From one hadron to two hadrons

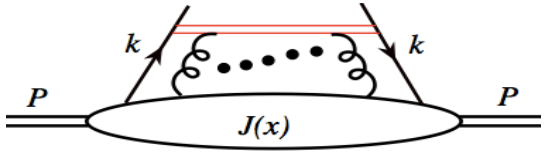
- **One hadron:**



$\sigma_{\text{tot}}^{\text{DIS}} \sim$



$\otimes$



$+ O\left(\frac{1}{QR}\right)$

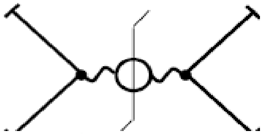
Hard-part
Probe

Parton-distribution
Structure

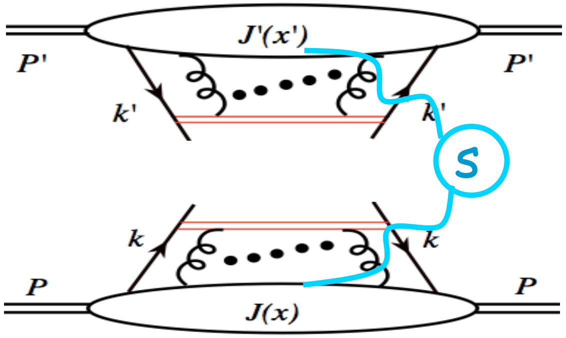
Power corrections
Approximation

- **Two hadrons:**

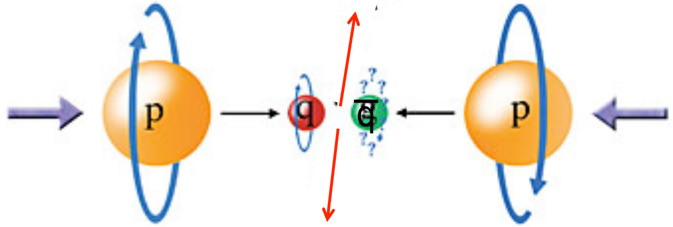
$\sigma_{\text{tot}}^{\text{DY}} \sim$



$\otimes$



$+ O\left(\frac{1}{QR}\right)$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

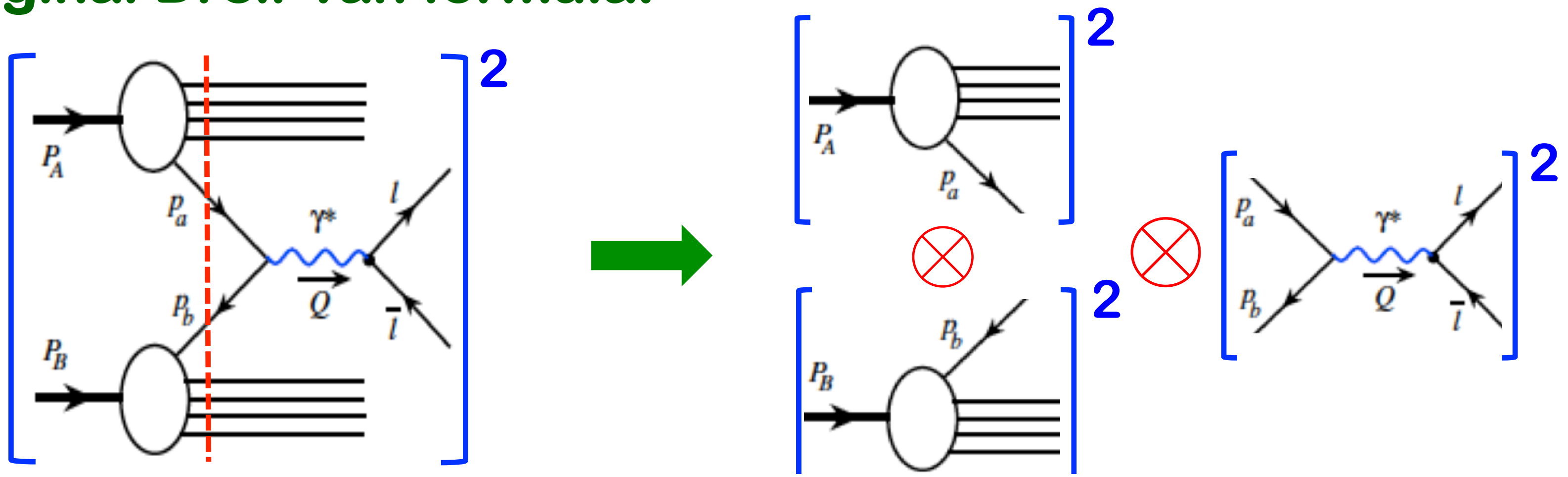
❑ **Drell-Yan mechanism:**

S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X$$ **with** $$q^2 \equiv Q^2 \gg \Lambda_{\mathrm{QCD}}^2 \sim 1/\mathrm{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ **Original Drell-Yan formula:**



$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2dy} = \frac{4\pi\alpha_{em}^2}{3Q^4}\sum_{p,\bar{p}} x_A\phi_{p/A}(x_A)\,x_B\phi_{\bar{p}/B}(x_B)$$

No color yet!

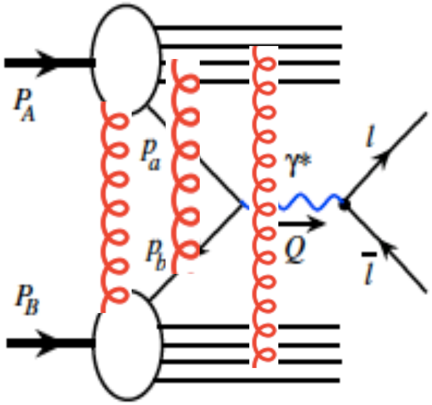
Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}}e^{y} \qquad x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD – factorization

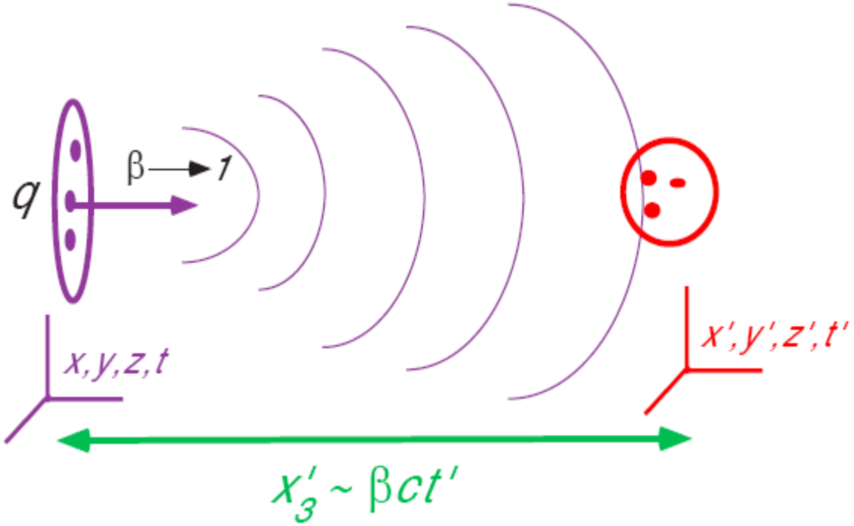
☐ **Beyond the lowest order:**



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

→ Break the Universality of PDFs
Loss the predictive power

☐ **Factorization – power suppression of soft gluon interaction:**



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

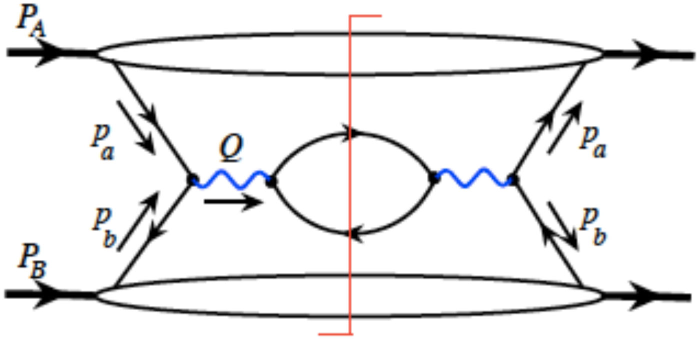
Drell-Yan process in QCD – factorization

Collins, Soper, Sterman, 1988

❑ Factorization – approximation:

- ✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

- ✧ Maintain the universality of PDFs:

  Long-range soft gluon interaction has to be power suppressed

- ✧ Infrared safe of partonic parts:

  Cancelation of IR behavior
  Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

Backup slides for more details

Factorized Drell-Yan cross section

❑ **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^y \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

❑ **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

❑ **Spin dependence:**

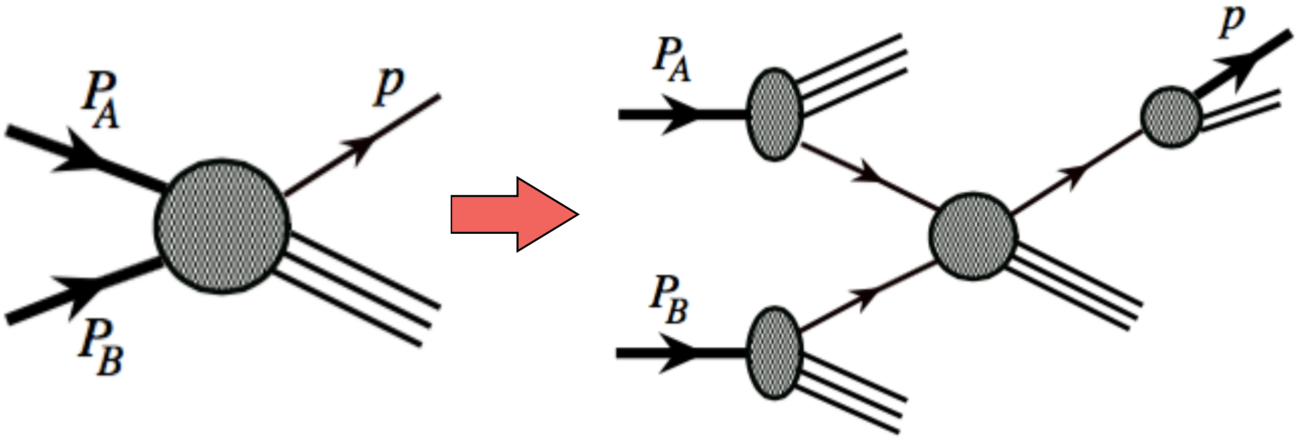
The factorization arguments are independent of the spin states of the colliding hadrons

➡ same formula with polarized PDFs for γ*,W/Z, H^0...

Factorization for more than two hadrons

Nayak, Qiu, Sterman, 2006

☐ Factorization for high p_T single hadron:



$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O $(1/P_T^2)$

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}\left(p_A,p_B,p\right)}{dydp_T^2} = \sum_{a,b,c}\phi_{A\to a}\left(x,\mu_F^2\right)\otimes\phi_{B\to b}\left(x',\mu_F^2\right)$$

$$\otimes\frac{d\hat{\sigma}_{ab\to c+X}\left(x,x',z,y,p_T^2\mu_F^2\right)}{dydp_T^2}\otimes D_{c\to C}\left(z,\mu_F^2\right)$$

- ✧ **Fragmentation function:** $D_{c\to C}\left(z,\mu_F^2\right)$
- ✧ **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

How to calculate the perturbative parts?

❑ **Use DIS structure function F_2 as an example:**

$$\boxed{F_{2h}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)}$$

✧ **Apply the factorized formula to parton states:** $h\rightarrow q$

Feynman diagrams → $F_{2q}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$ ← **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of** α_s:

0th order:

$$F_{2q}^{(0)}(x_B,Q^2)=C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$

$$\boxed{C_q^{(0)}(x)=F_{2q}^{(0)}(x)}\qquad \varphi_{q/q}^{(0)}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order:

$$F_{2q}^{(1)}(x_B,Q^2)=C_q^{(1)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$
$$+\,C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2)=F_{2q}^{(1)}(x,Q^2)-F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

To all orders!

Partonic cross sections – LO

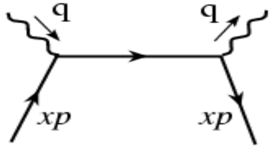
❑ Projection operators for SFs:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x,Q^2\right)$$

$$\boxed{\begin{aligned} F_1(x,Q^2) &= \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \\ F_2(x,Q^2) &= x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \end{aligned}}$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W^{(0)}_{\mu\nu,q} = x g^{\mu\nu} \left[\frac{1}{4\pi} \right.$$



$$\left. \right]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p \gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$\boxed{C_q^{(0)}(x) = e_q^2 x \delta(1-x)}$$

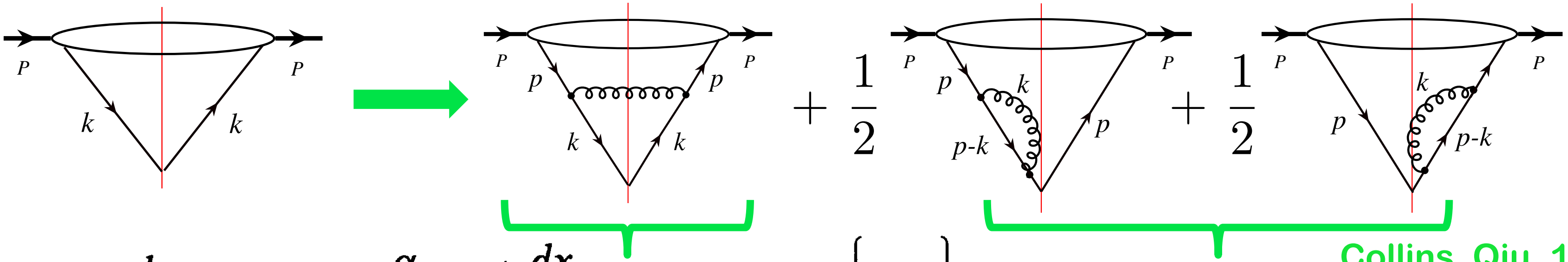
Backup slides for a complete example of NLO calculation in QCD!

Calculation of evolution kernels

❑ **Evolution kernels are process independent**

= Parton distribution functions are universal

❑ **Extract from calculating parton PDFs' scale dependence**



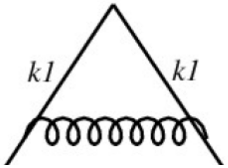
Collins, Qiu, 1989

$$Q^2\frac{d}{dQ^2}q_i(x,Q^2)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}q_i(x_1,Q^2)\gamma_{qq}\left(\frac{x}{x_1}\right)-\frac{\alpha_s}{2\pi}q_i(x,Q^2)\int_0^1 dz\,\gamma_{qq}(z)$$

Change **"Gain"** **"Loss"**

❑ **One loop contribution in dimensional regularization:**

$$\varphi^{(1)}_{q/q}(x,\mu^2)=\left(\frac{\alpha_s}{2\pi}\right)P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\mathrm{UV}}+\left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}}\right\}+\text{UV-CT}$$



$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon\to 0$$

Recall:

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2)=F_{2q}^{(1)}(x,Q^2)-F_{2q}^{(0)}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)}$$

→ **Scheme dependence!**

❑ Common UV-CT terms:

- ✧ **MS scheme:** $\text{UV-CT}\Big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$

- ✧ **$\overline{\text{MS}}$ scheme:** $\text{UV-CT}\Big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right)$

- ✧ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x,Q^2/\mu^2)\Big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\Bigg\{ P_{qq}(x) \ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right)$$

$$+ C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\Bigg]\Bigg\}$$

IR safe as required by the QCD factorization!

Homework (2)

1) Use the criterion on slide 7 to prove that the one-jettiness distribution,

$$\frac{d\sigma_{e^+e^-\to\mathrm{hadrons}}}{d\tau}$$

with the one-jettiness defined as: $\tau = \sum_k \mathrm{min}_{\hat{q}} \left\{ \frac{2\hat{q}\cdot p_k}{Q} \right\}$

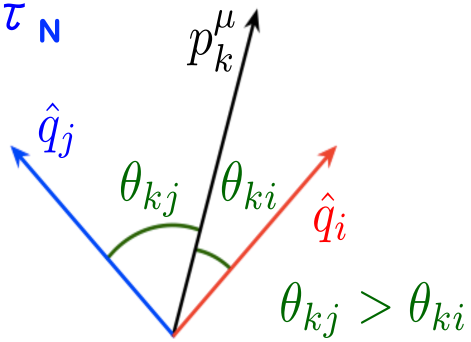
is an infrared safe observable.

Backup slides

N-Jettiness – implementation

❑ Steps for implementation:

- ✧ Use a standard jet algorithms to find N-jets
- ✧ Initial reference vectors = momenta of the N-jets + hadron beam directions
  (reference vectors are the only information used from the jet algorithm)
- ✧ Calculate value for the N-jettiness global event shape: τ_N
  (new reference directions from the minimization)
- ✧ Select events with N narrow well-separated jets and impose veto on additional jets



❑ New "jet" momenta = sum of momenta in jet regions

$$P_i^\mu = \sum_k p_k^\mu \prod_{j \neq i} \theta(\hat{q}_j \cdot p_k - \hat{q}_i \cdot p_k)$$

❑ N-jettiness momentum = sum of jettiness from each region:

$$\mathcal{T}_N = \sum_i \mathcal{T}_N^i \equiv \sum_i 2\hat{q}_i \cdot P_i$$

❑ Dependence on Jet algorithms is power suppressed

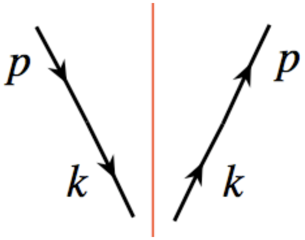
PDFs of a parton

❑ Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\mathrm{parton}(p)\rangle$ ➡ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

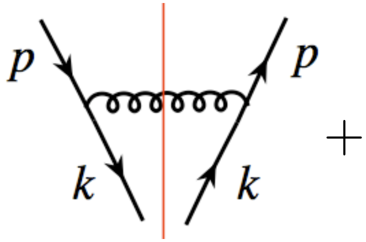
❑ Lowest order quark distribution:



✧ From the operator definition:

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

❑ Leading order in α_s quark distribution:



$+\ldots$

✧ Expand to $(g_s)^2$ – logarithmic divergent:

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right] + \mathrm{UVCT}$$

UV and CO divergence

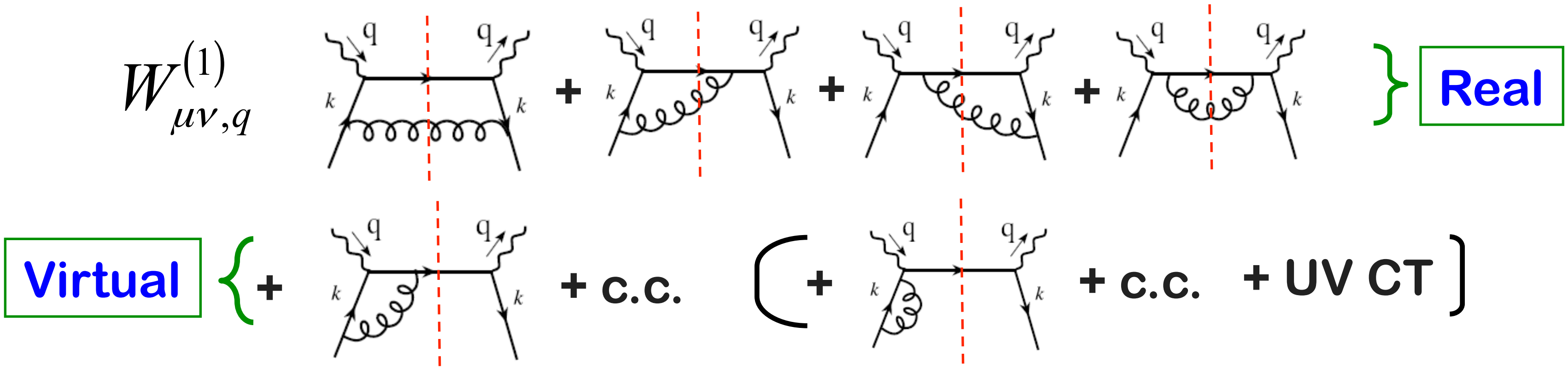
NLO coefficient function – complete example

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

❑ **Projection operators in n-dimension:** $g_{\mu\nu}g^{\mu\nu} = n \equiv 4-2\varepsilon$

$$\boxed{\left(1-\varepsilon\right)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}}$$

❑ **Feynman diagrams:**



❑ **Calculation:** $-g^{\mu\nu}W^{(1)}_{\mu\nu,q}$ and $p^\mu p^\nu W^{(1)}_{\mu\nu,q}$

Contribution from the trace of $W_{\mu\nu}$

- ❑ **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- ❑ **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- ❑ **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ln(1-z) f(1)$$

❑ One loop contribution to the trace of W$_{\mu\,\nu}$:

$$-g^{\mu\nu}W^{(1)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\Bigg\{-\frac{1}{\varepsilon}P_{qq}(x) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)$$

$$+C_F\left[\left(1+x^2\right)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x)\right.$$

$$\left. +3-x-\left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\Bigg\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

❑ One loop contribution to $p^{\mu} p^{\nu}$ $W_{\mu\nu}$:

$$p^{\mu} p^{\nu} W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^{\mu} p^{\nu} W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi} \frac{Q^2}{4x}$$

❑ One loop contribution to F_2 of a quark:

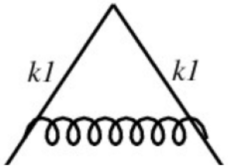
$$F_{2q}^{(1)}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi} \left\{ \left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}} P_{qq}(x)\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right) + P_{qq}(x) \ln\left(\frac{Q^2}{\mu^2}\right) \right.$$

$$\left. + C_F \left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x) \right] \right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi_{q/q}^{(1)}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x) \left\{ \left(\frac{1}{\varepsilon}\right)_{\mathrm{UV}} + \left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}} \right\} + \text{UV-CT}$$



– in the dimensional regularization

$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

➡

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

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$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

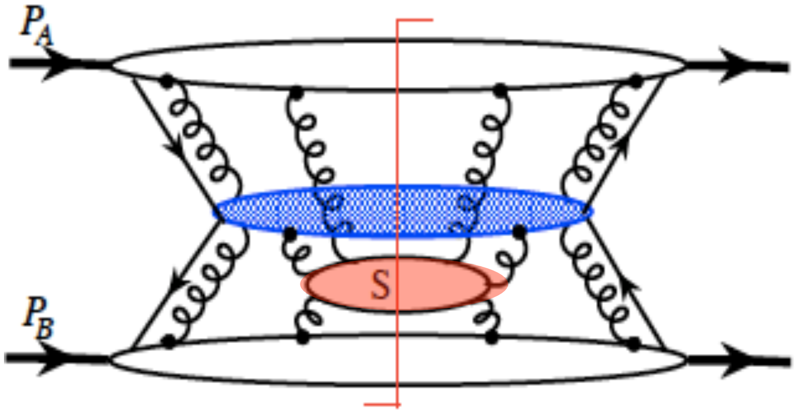
$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x\frac{\alpha_s}{2\pi}\left\{P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2_{\overline{\text{MS}}}}\right)\right.$$

$$\left. +C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right]\right\}$$

IR safe as required by the QCD factorization!

Drell-Yan process in QCD – factorization

❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

Collinear:

✧ lines collinear to A and B

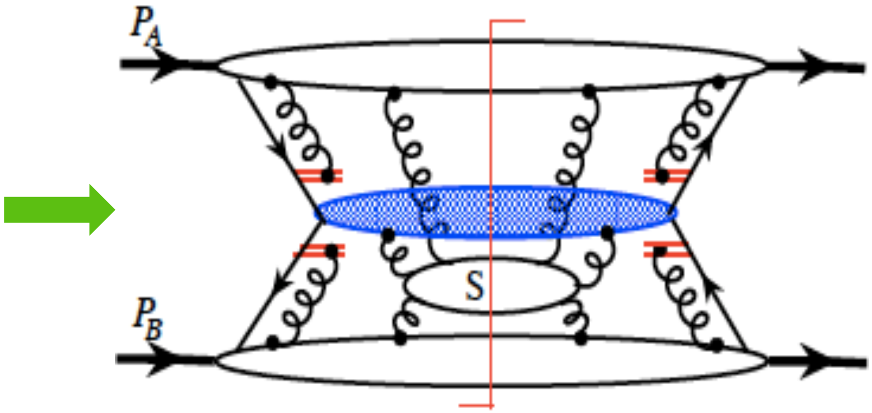
✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$

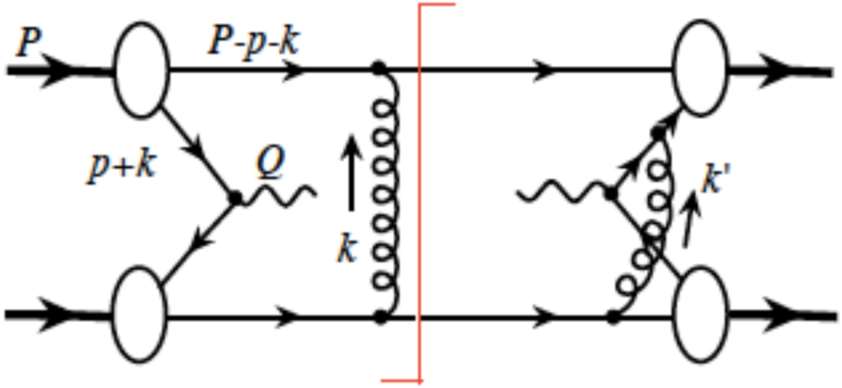
✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ **Trouble with soft gluons:**



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

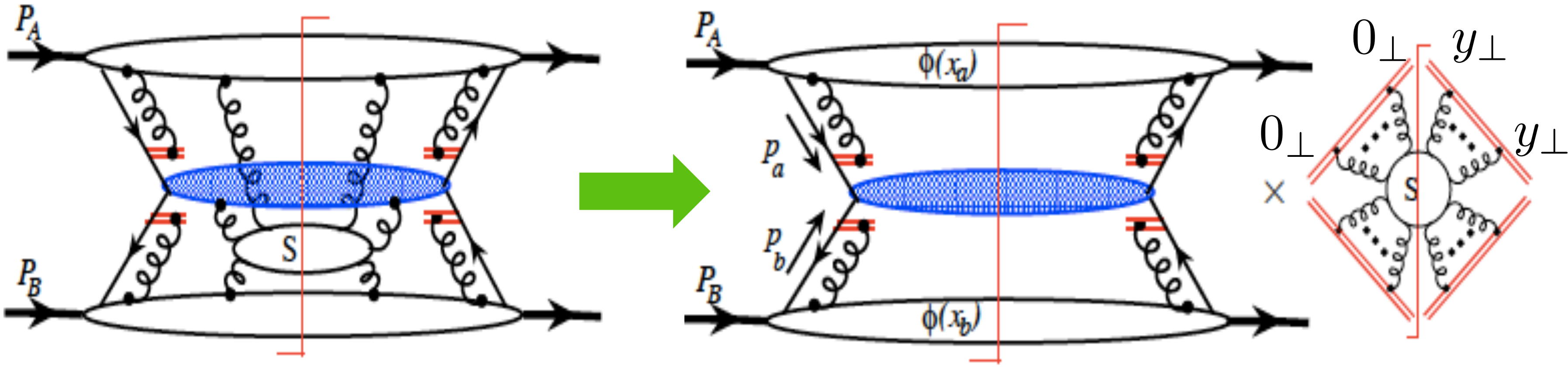
- ✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

- ✧ The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction: $k^\pm \sim M^2/Q \ll k_\perp \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD – factorization

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^\pm$ integration out of the trapped soft region

✧ Eikonal approximation → soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity → soft factor = 1

All identified leading integration regions are factorizable!