Introduction
to
Quantum Chromodynamics (QCD)

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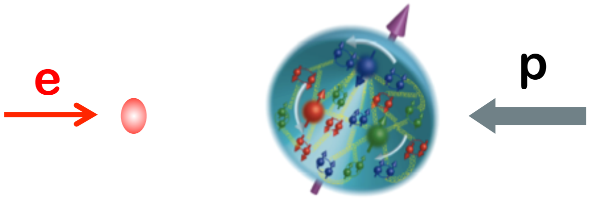
August 16 – 19, 2018
Four Lectures

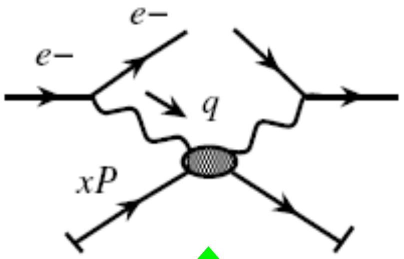
The 3rd WHEPS, August 16-24, 2018, Weihai, Shandong

From one hadron to two hadrons

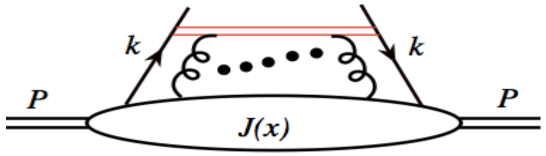
- **One hadron:**



$\sigma^{\rm DIS}_{\rm tot} \sim$



$\otimes$



$+\, O\left(\frac{1}{QR}\right)$

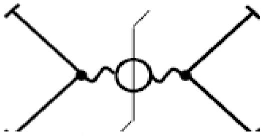
Hard-part
Probe

Parton-distribution
Structure

Power corrections
Approximation

- **Two hadrons:**

$\sigma^{\rm DY}_{\rm tot} \sim$



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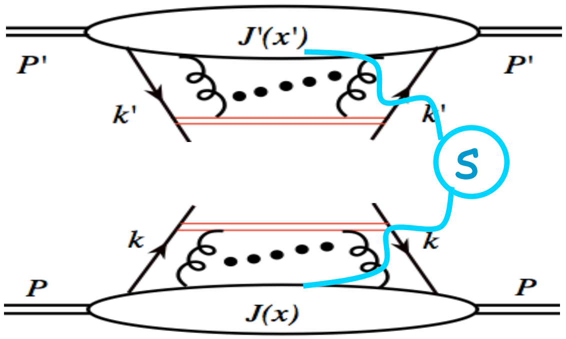
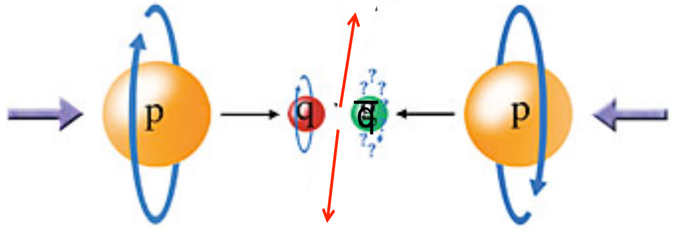


$+\, O\left(\frac{1}{QR}\right)$

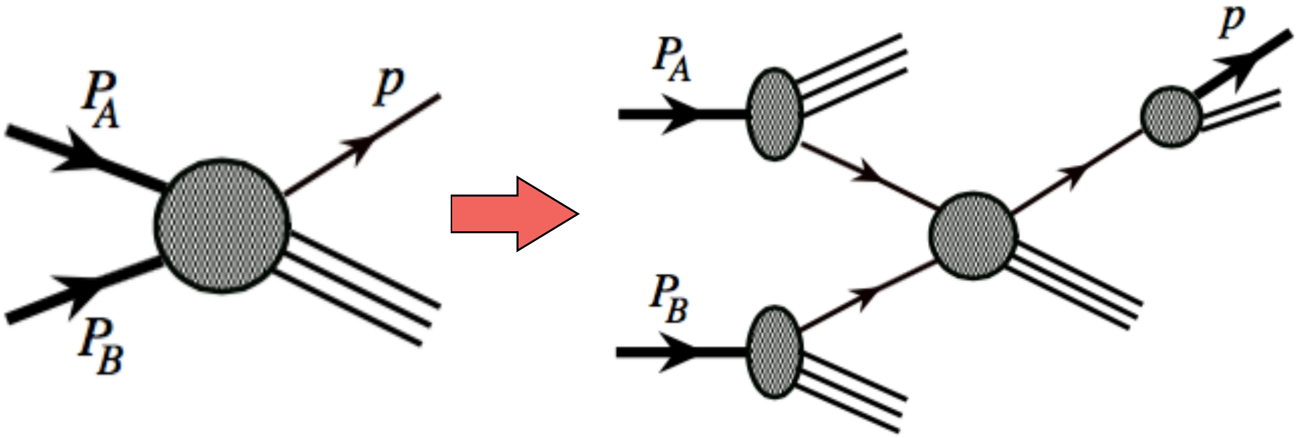


Predictive power:
Universal Parton Distributions

Factorization for more than two hadrons

❑ Factorization for high p$_T$ single hadron:

Nayak, Qiu, Sterman, 2006



$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O (1/P$_T^2$)

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}\left(p_A, p_B, p\right)}{dydp_T^2} = \sum_{a,b,c} \phi_{A\to a}\left(x, \mu_F^2\right) \otimes \phi_{B\to b}\left(x', \mu_F^2\right)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}\left(x, x', z, y, p_T^2 \mu_F^2\right)}{dydp_T^2} \otimes D_{c\to C}\left(z, \mu_F^2\right)$$

✧ **Fragmentation function:** $D_{c\to C}\left(z, \mu_F^2\right)$

✧ **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

How to calculate the perturbative parts?

❑ **Use DIS structure function F$_2$ as an example:**

$$\boxed{F_{2h}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)}$$

✧ **Apply the factorized formula to parton states:** $h\rightarrow q$

Feynman diagrams →

$$F_{2q}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$

← **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of** α_s:

0th order:

$$F_{2q}^{(0)}(x_B,Q^2)=C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$

$$\Longrightarrow\quad\boxed{C_q^{(0)}(x)=F_{2q}^{(0)}(x)}\qquad\varphi_{q/q}^{(0)}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order:

$$F_{2q}^{(1)}(x_B,Q^2)=C_q^{(1)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$
$$+C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

$$\Longrightarrow\quad\boxed{C_q^{(1)}(x,Q^2/\mu^2)=F_{2q}^{(1)}(x,Q^2)-F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

To all orders!

Partonic cross sections – LO

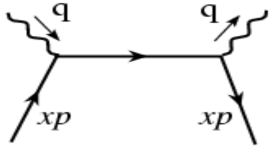
- **Projection operators for SFs:**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x,Q^2\right)$$

$$\boxed{\begin{aligned} F_1(x,Q^2) &= \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \\ F_2(x,Q^2) &= x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \end{aligned}}$$

- **0th order:**

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W^{(0)}_{\mu\nu,q} = x g^{\mu\nu} \Bigg[\frac{1}{4\pi}$$



$$\Bigg]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p \gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$\boxed{C_q^{(0)}(x) = e_q^2 x\delta(1-x)}$$

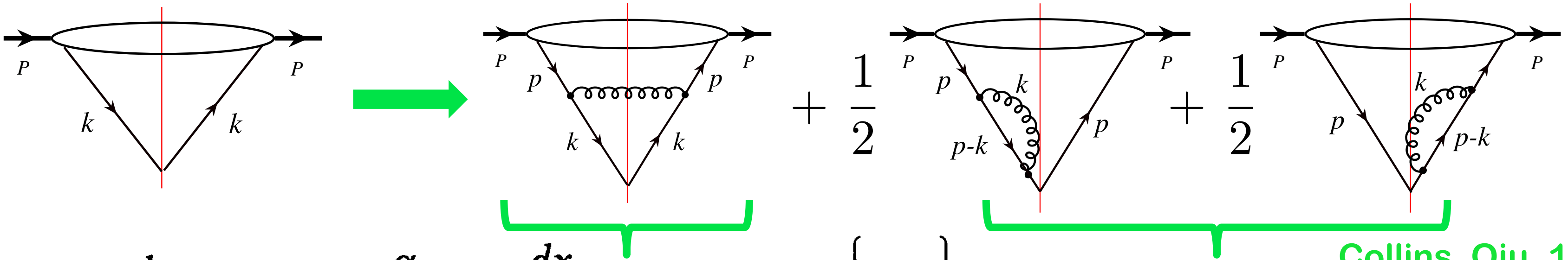
Backup slides for a complete example of NLO calculation in QCD!

Calculation of evolution kernels

- ☐ **Evolution kernels are process independent**

  = Parton distribution functions are universal

- ☐ **Extract from calculating parton PDFs' scale dependence**



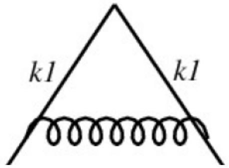
Collins, Qiu, 1989

$$Q^2\frac{d}{dQ^2}q_i(x,Q^2)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}q_i(x_1,Q^2)\gamma_{qq}\left[\frac{x}{x_1}\right]-\frac{\alpha_s}{2\pi}q_i(x,Q^2)\int_0^1 dz\,\gamma_{qq}(z)$$

Change **"Gain"** **"Loss"**

- ☐ **One loop contribution in dimensional regularization:**

$$\varphi^{(1)}_{q/q}(x,\mu^2)=\left(\frac{\alpha_s}{2\pi}\right)P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\text{UV}}+\left(-\frac{1}{\varepsilon}\right)_{\text{CO}}\right\}+\text{UV-CT}$$



$$\Rightarrow\quad\infty\quad\text{as}\quad\varepsilon\to 0$$

Recall:

$$\boxed{C^{(1)}_q(x,Q^2/\mu^2)=F^{(1)}_{2q}(x,Q^2)-F^{(0)}_{2q}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)}$$

→ **Scheme dependence!**

❑ Common UV-CT terms:

- ✧ **MS scheme:** $\text{UV-CT}\big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$

- ✧ **$\overline{\text{MS}}$ scheme:** $\text{UV-CT}\big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right)$

- ✧ **DIS scheme: choose a UV-CT, such that** $C_q^{(1)}(x,Q^2/\mu^2)\big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x)\ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right)\right.$$

$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

IR safe as required by the QCD factorization!

Global QCD analyses – Testing QCD

❑ **Factorization for observables with identified hadrons:**

✧ Factorized cross sections (DIS):

$$\boxed{F_2(x_B,Q^2)=\Sigma_f C_f(x_B/x,\mu^2/Q^2)\otimes f(x,\mu^2)}$$

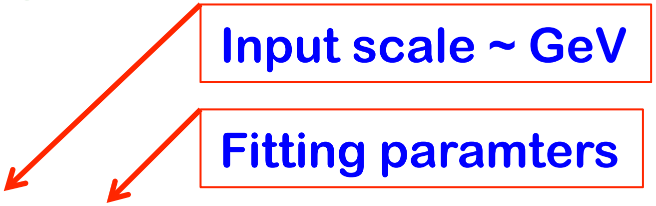
✧ DGLAP Evolution:

$$\boxed{\frac{\partial f(x,\mu^2)}{\partial \ln\mu^2}=\Sigma_{f'}P_{ff'}(x/x')\otimes f'(x',\mu^2)}$$

✧ Adding more observables:
Factorized cross section with multiple-hadrons (next lecture)

Testing QCD: Universal PDFs for all cross sections?

❑ **Input for QCD Global analysis/fitting:**



Input scale ~ GeV

Fitting paramters

✧ World data with “Q” > 2 GeV

✧ PDFs at an input scale: $\phi_{f/h}(x,\mu_0^2,\{\alpha_j\})$

Global QCD analysis – Testing QCD

Input DPFs at Q_0

$\varphi_{f/h}\left(x,\{a_j\}\right)$

Vary $\{a_j\}$

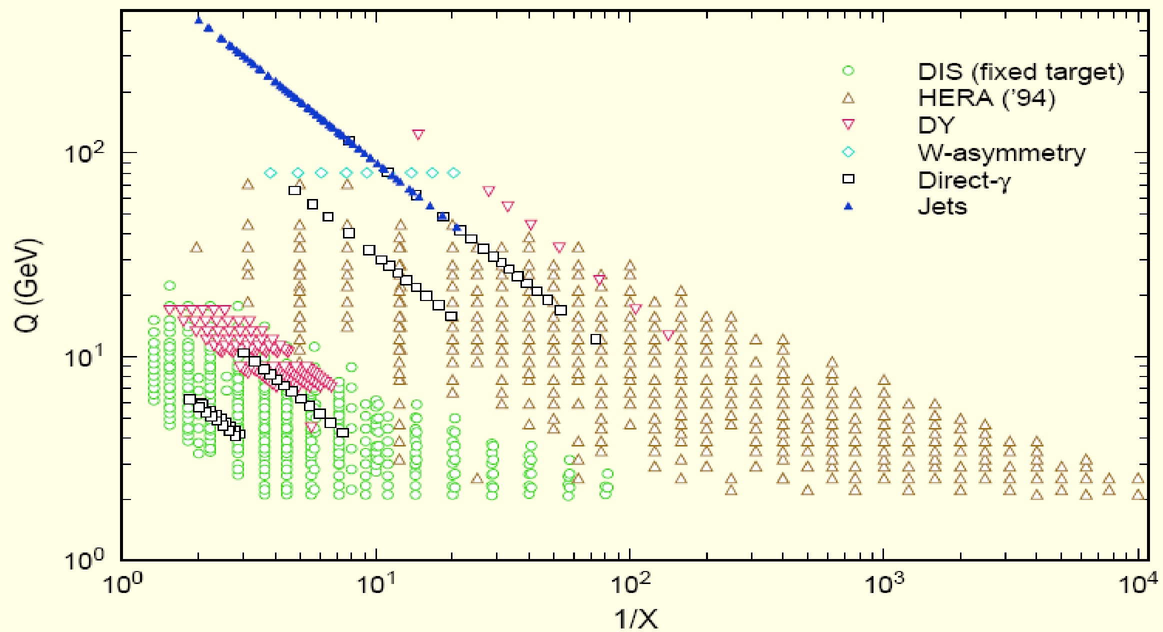
Minimize Chi^2

DGLAP

$\varphi_{f/h}(x)$ **at $Q>Q_0$**

QCD calculation

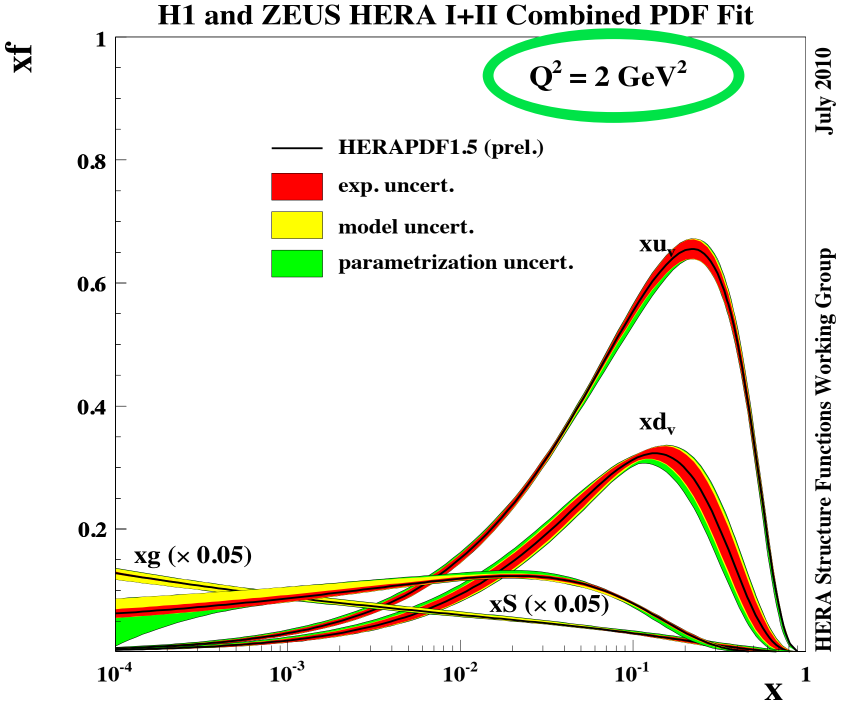
Comparison with Data at various x and Q

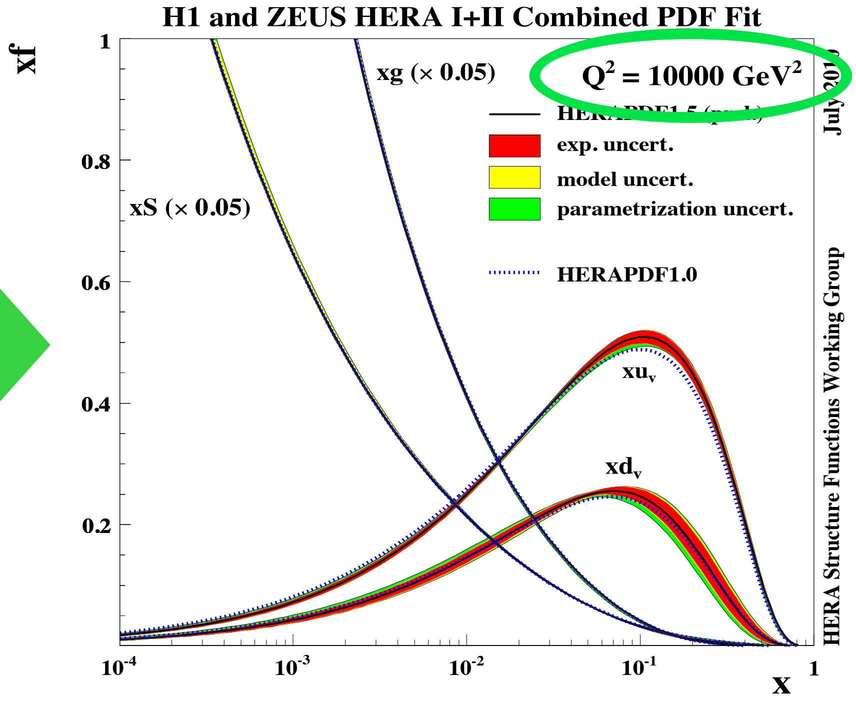


Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs

PDFs from DIS alone

❑ Q^2-dependence is a prediction of pQCD calculation:





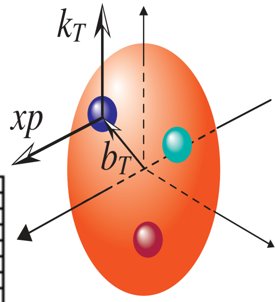
❑ Physics interpretation of PDFs:

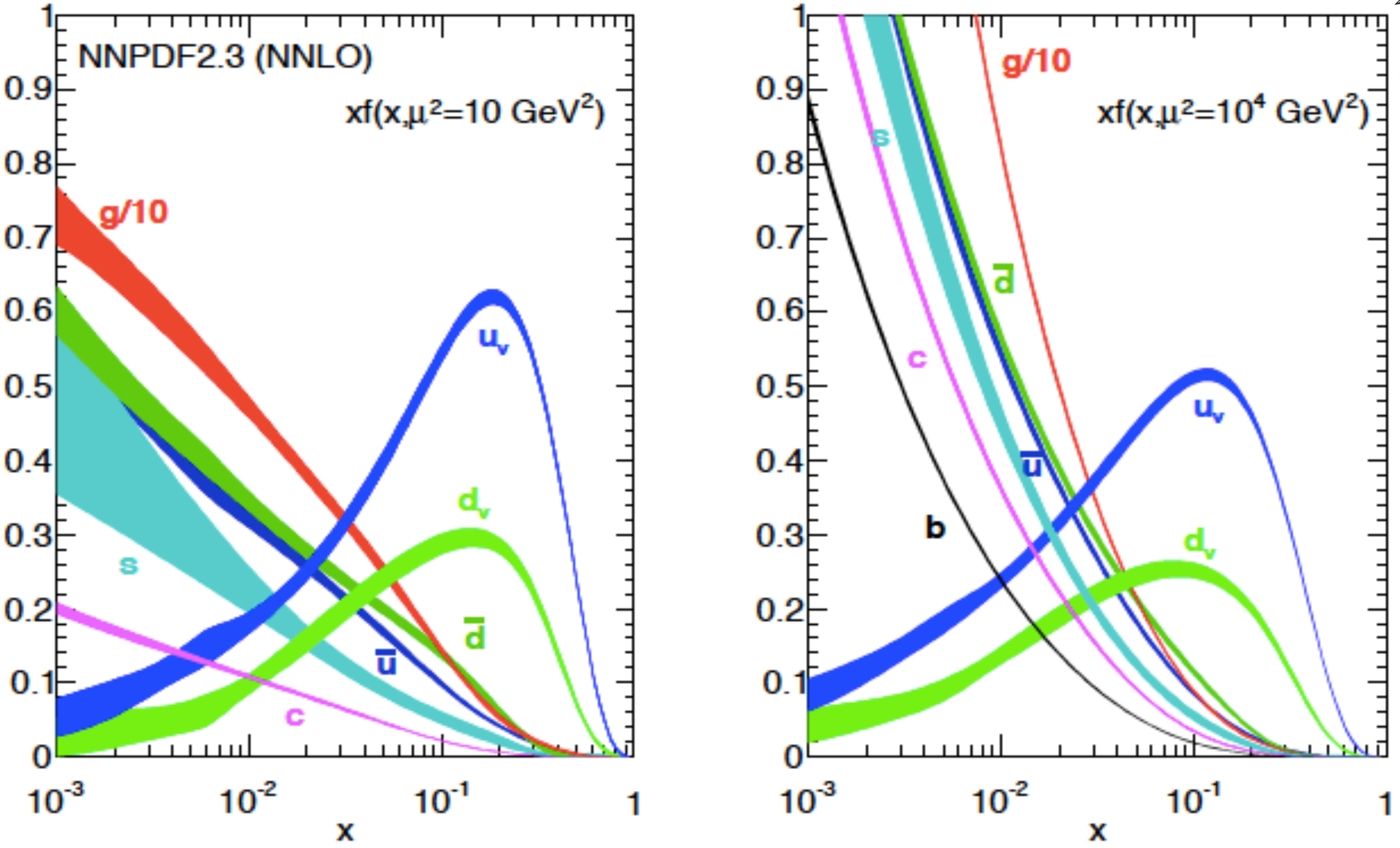
$f(x,Q^2)$: **Probability density to find a parton of flavor "f" carrying momentum fraction "x", probed at a scale of "Q^2"**

- ✧ **Number of partons:** $\int_0^1 dx\, u_v(x,Q^2) = 2, \quad \int_0^1 dx\, d_v(x,Q^2) = 1$
- ✧ **Momentum fraction:** $\langle x(Q^2)\rangle_f = \int_0^1 dx\, x\, f(x,Q^2)$ $\longrightarrow$ $\sum_f \langle x(Q^2)\rangle = 1$

PDFs from Global Fitting

- **Modern sets of PDFs @NNLO with uncertainties:**

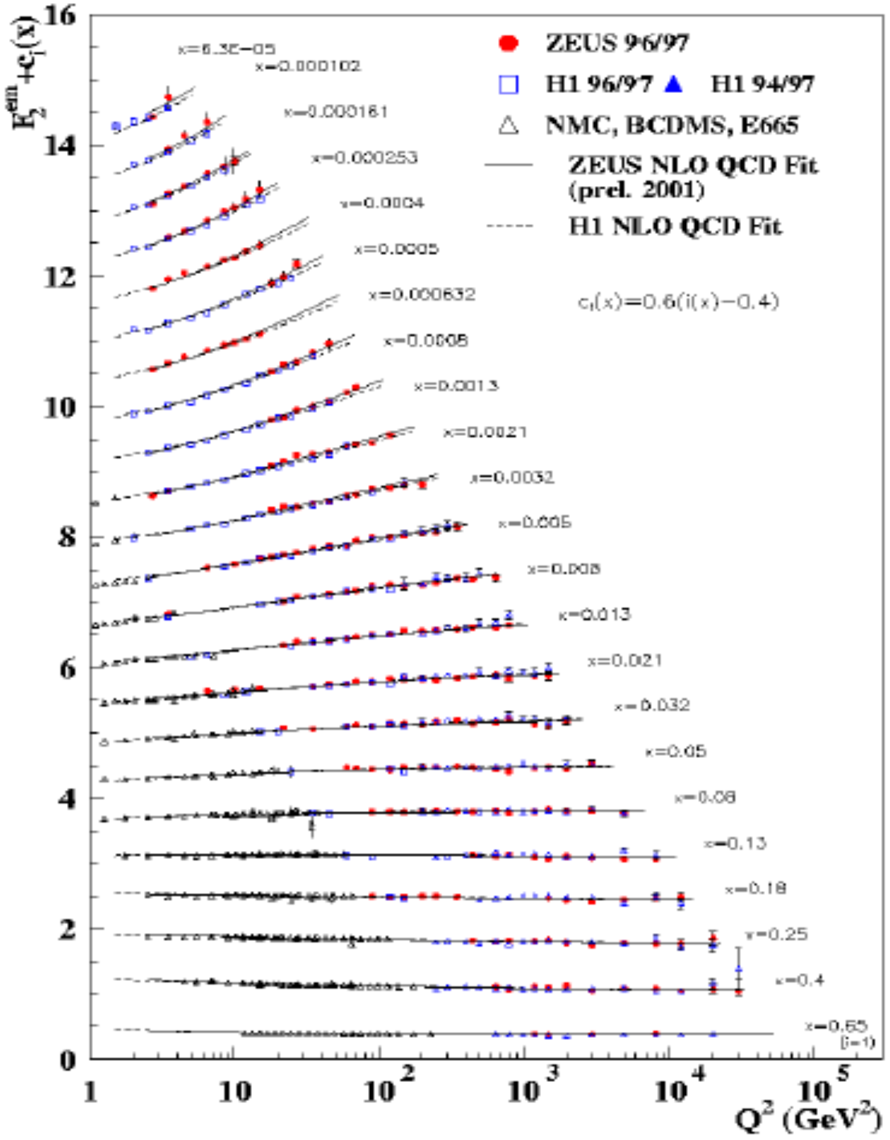


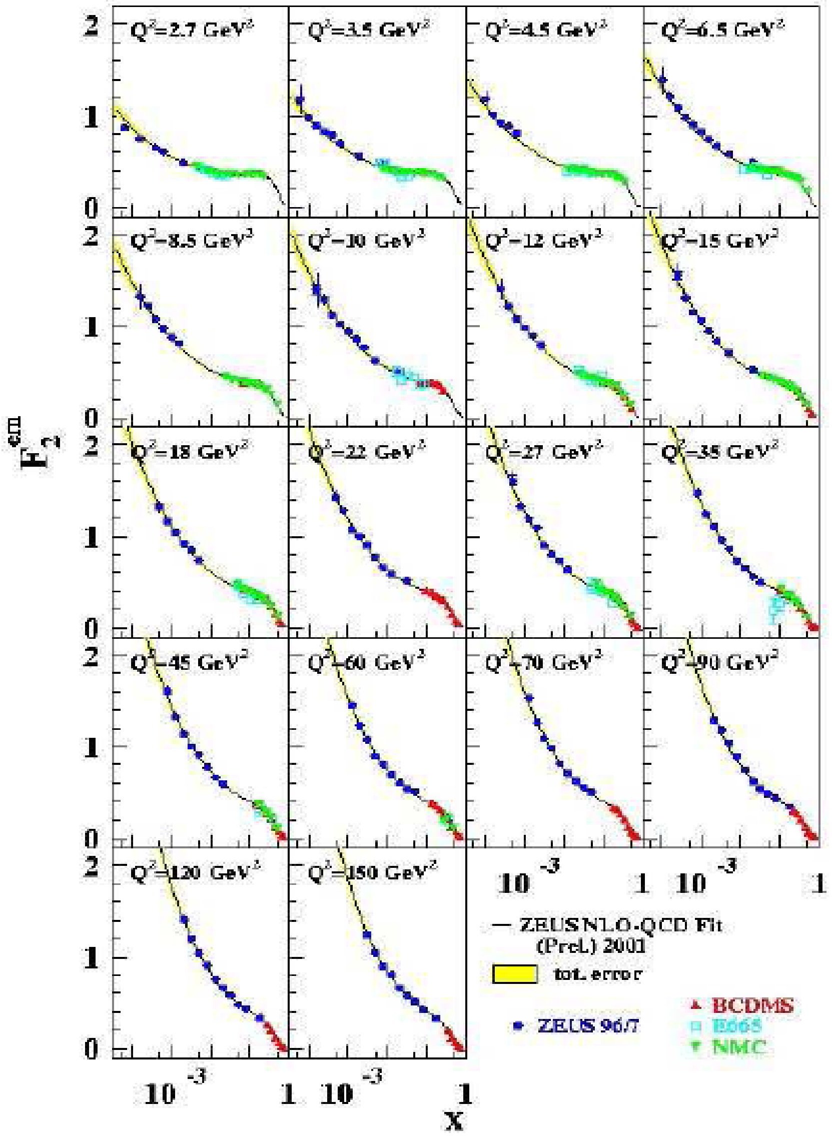


K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

Consistently fit almost all data with Q > 2GeV

Scaling and scaling violation

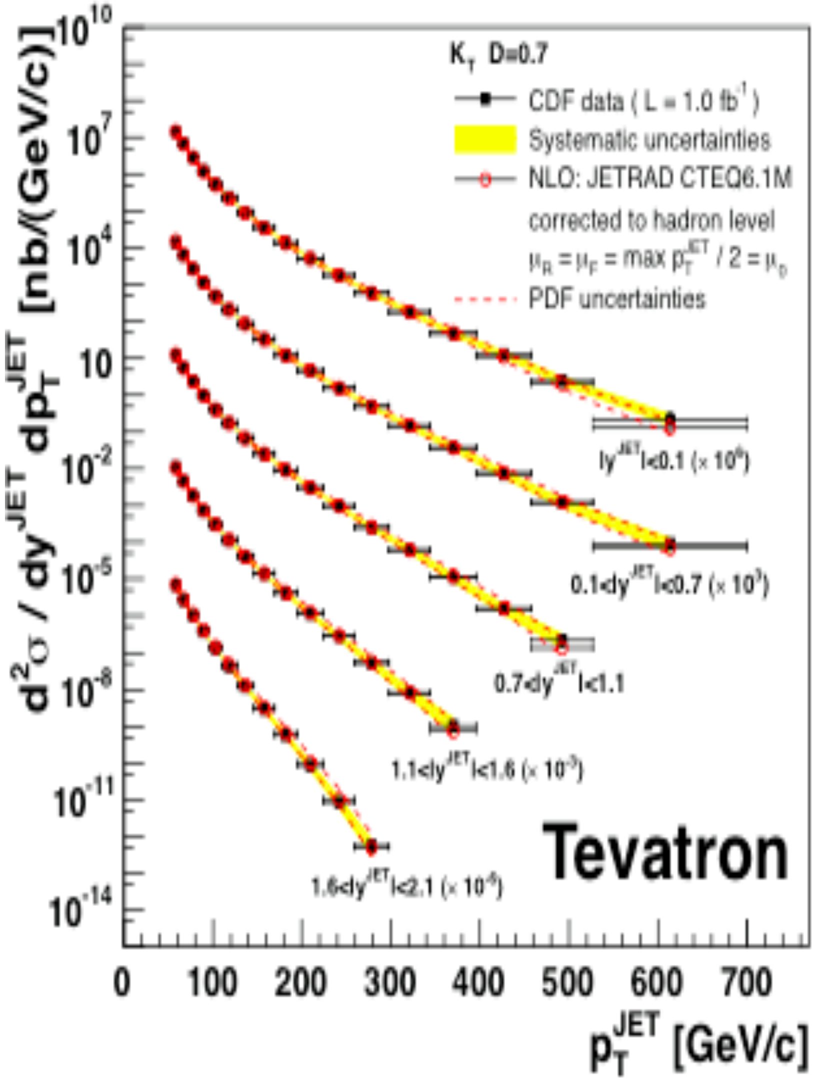


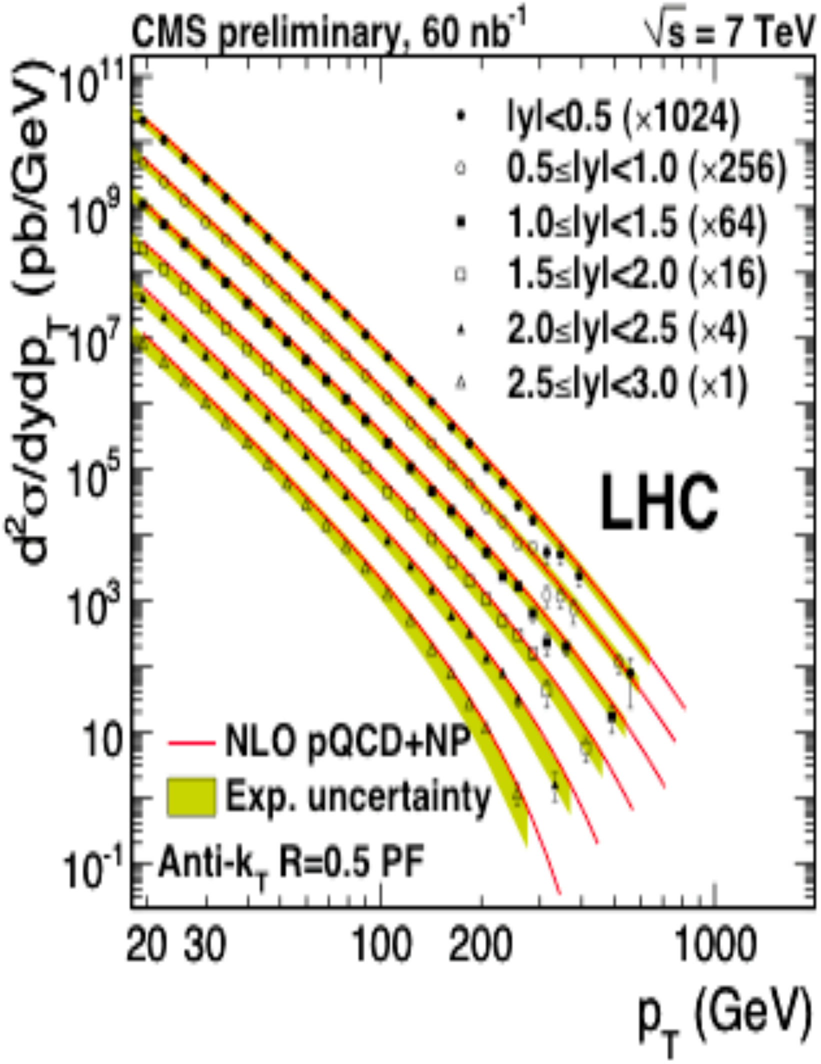


Q^2-dependence is a prediction of pQCD calculation

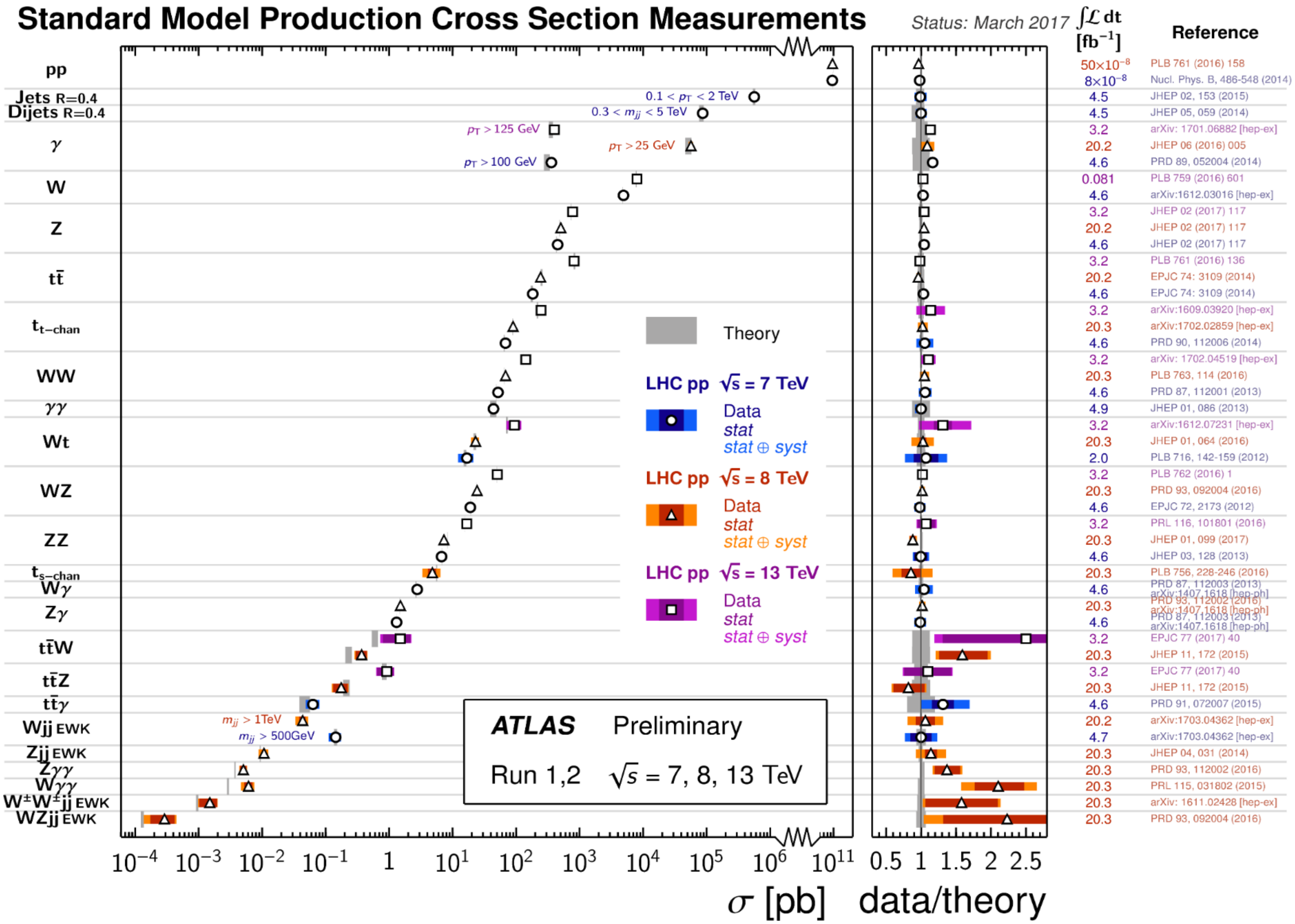
Hadronic Jet Production

- **Predictions with extracted PDFs:**



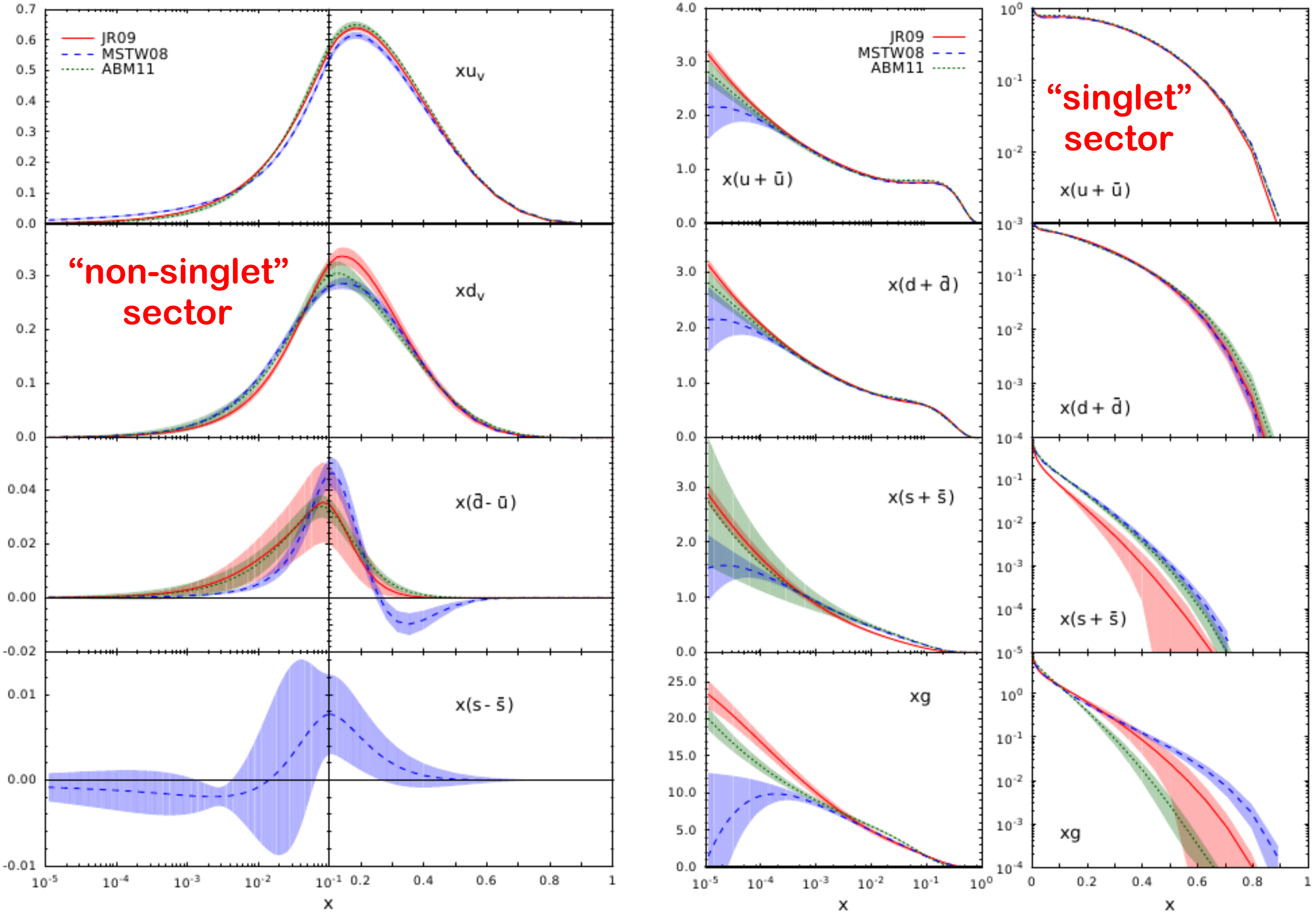


The great success of the SM physics



SM: Electroweak processes + QCD perturbation theory works!

Uncertainties of PDFs

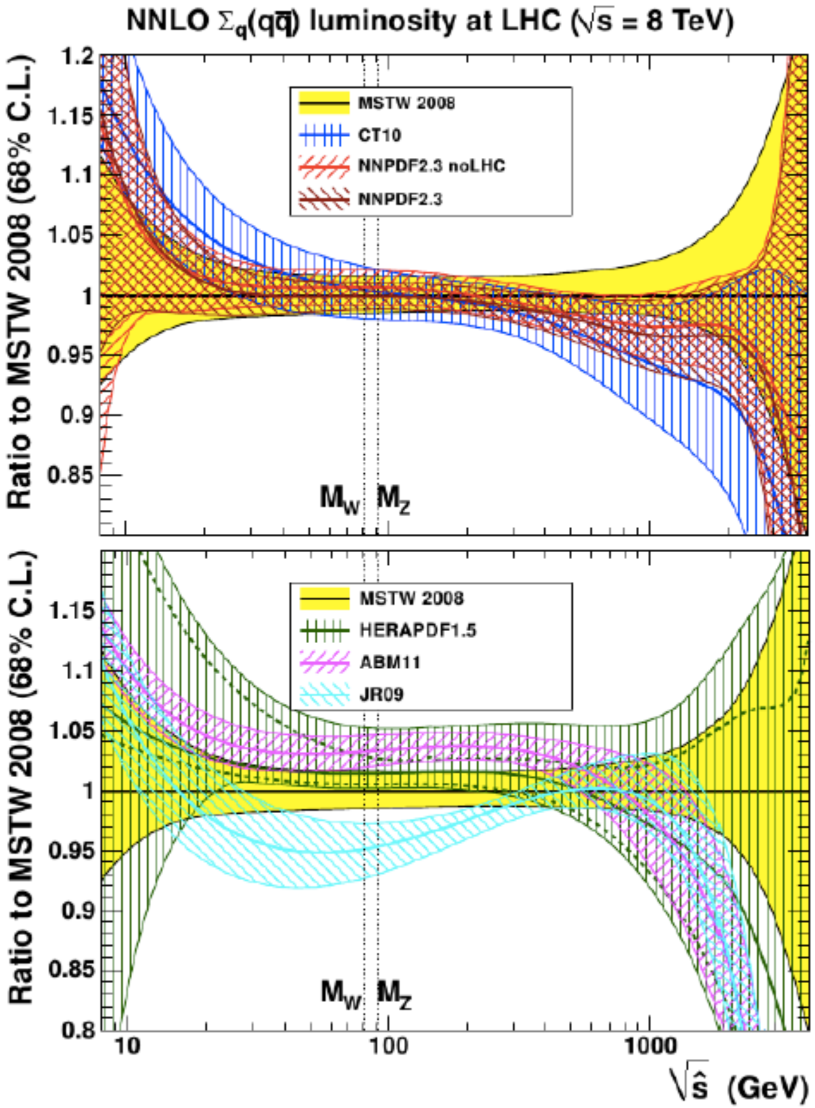


“non-singlet” sector

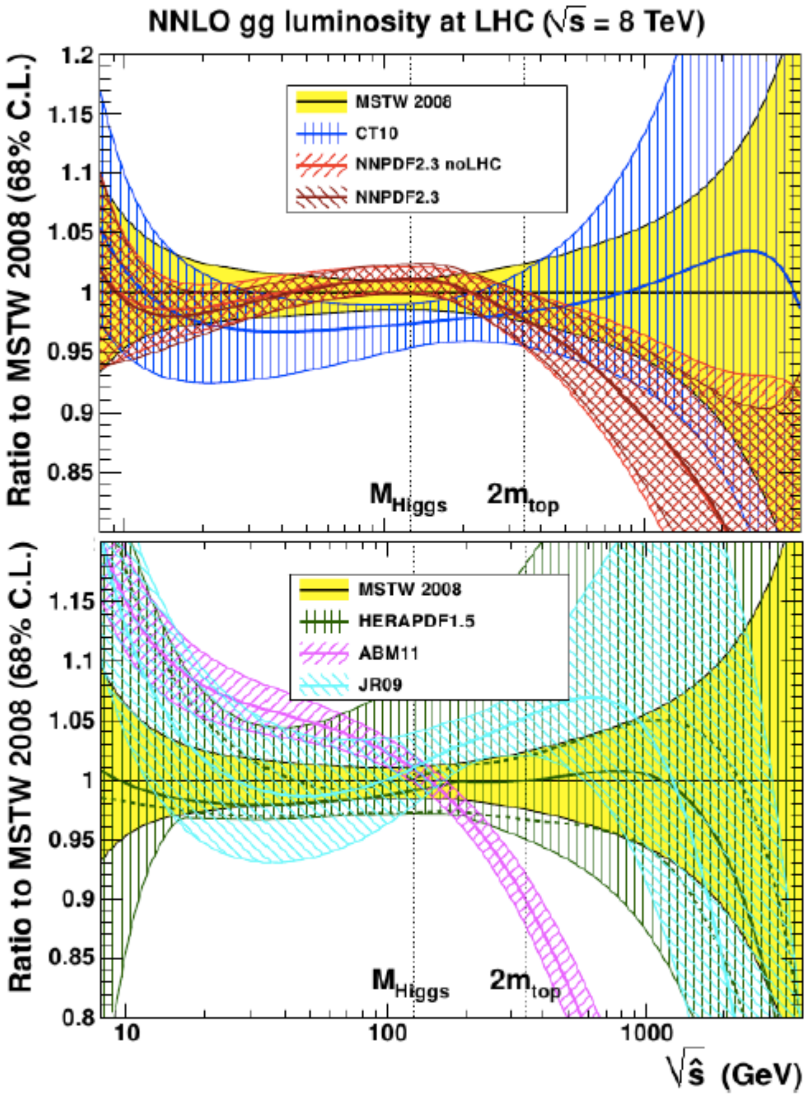
“singlet” sector

Partonic luminosities

q - qbar



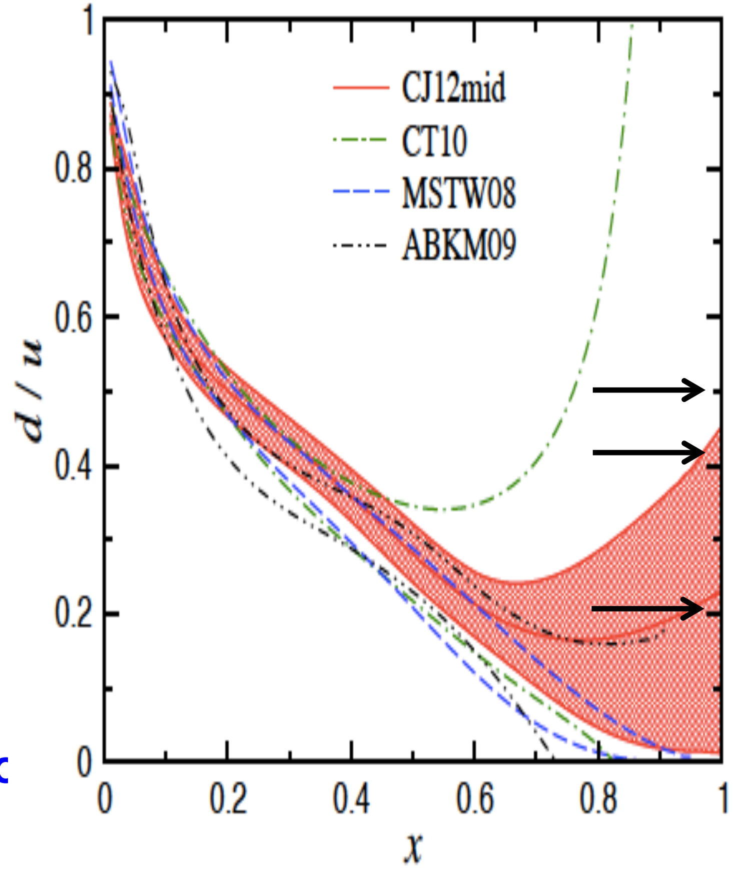
g - g



PDFs at large x

❑ **Testing ground for hadron structure at x ➔1:**

- ✧ $d/u \to 1/2$ **SU(6) Spin-flavor symmetry**
- ✧ $d/u \to 0$ **Scalar diquark dominance**
- ✧ $d/u \to 1/5$ **pQCD power counting**
- ✧ $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$ **Local quark-hadrc duality**



PDFs at large x

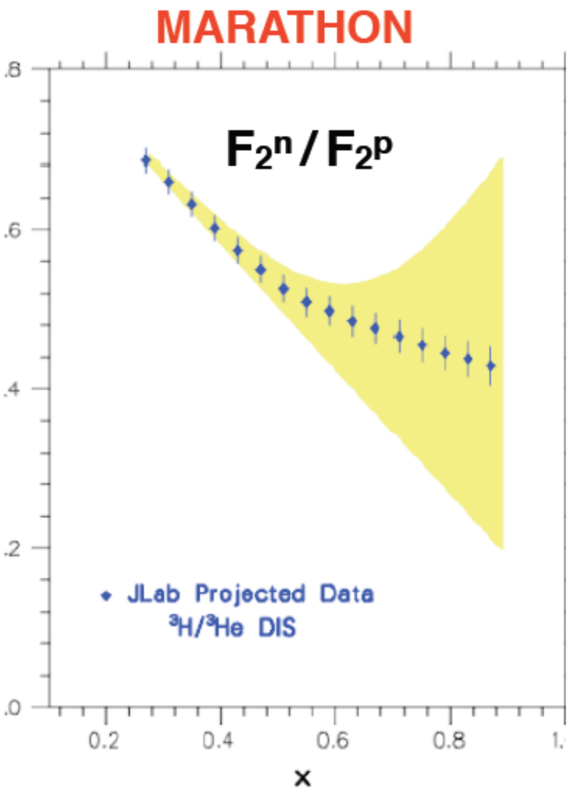
- **Testing ground for hadron structure at x → 1:**

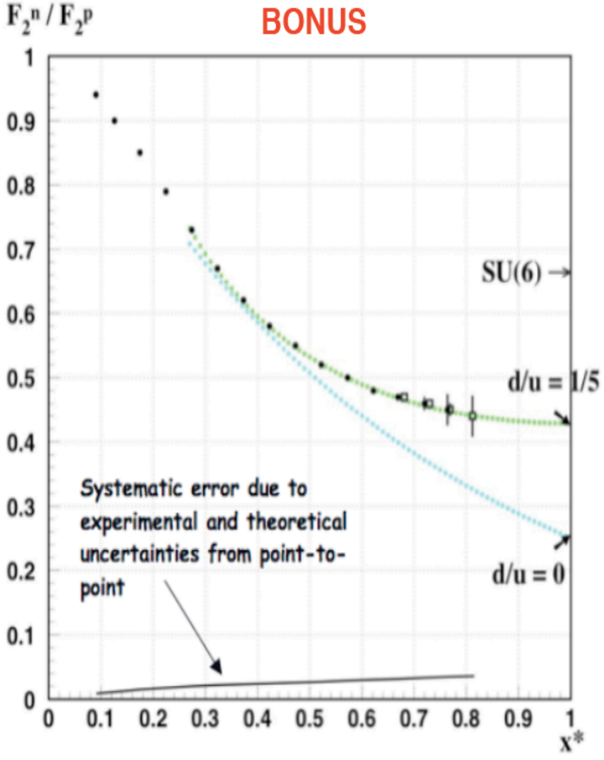
$d/u \to 1/2$	**SU(6) Spin-flavor symmetry**	$\Delta u/u \to 2/3$ $\Delta d/d \to -1/3$
$d/u \to 0$	**Scalar diquark dominance**	$\Delta u/u \to 1$ $\Delta d/d \to -1/3$
$d/u \to 1/5$	**pQCD power counting**	$\Delta u/u \to 1$ $\Delta d/d \to 1$
$d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$	**Local quark-hadron duality**	$\Delta u/u \to 1$ $\Delta d/d \to 1$

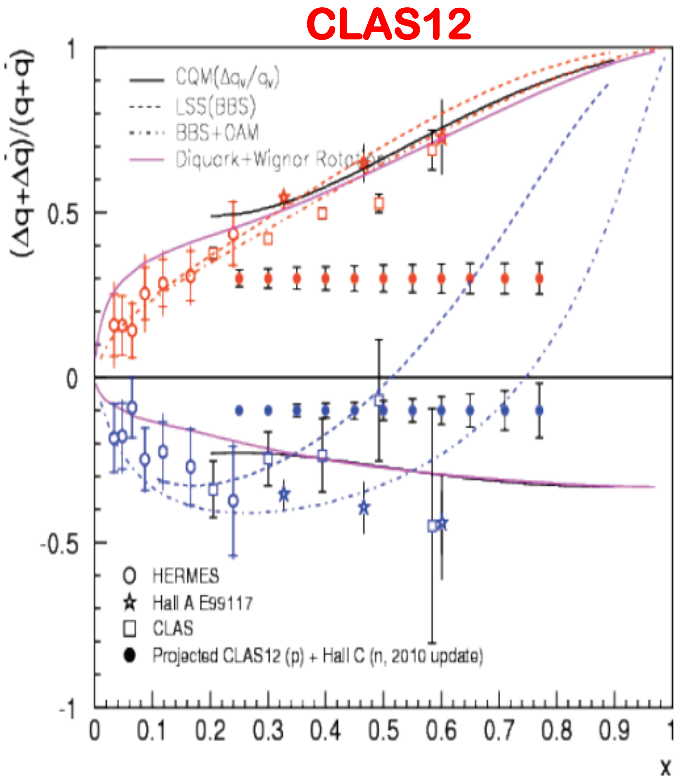
What data try to say?

Future large-x experiments

❑ At JLab 12GeV:







Plus many more JLab experiments:

E12-06-110 (Hall C on ^{3}He), E12-06-122 (Hall A on ^{3}He),

E12-06-109 (CLAS on NH_3, ND_3), …

and Fermilab E906, …

Can lattice QCD help?

Calculate partonic structure in lattice QCD?

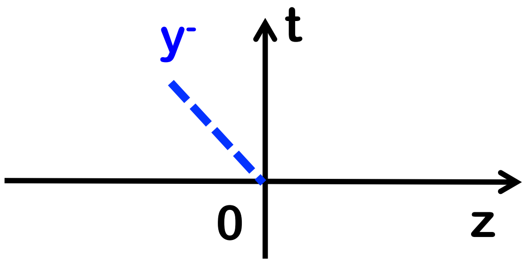
❑ **Answer: Not directly!**

Large momentum transfer & collinear approximation

➡ Operators on light-cone

➡ Can't be calculated in lattice QCD

$\mathcal{F.T.}\ \langle p|\overline{\psi}(0)\gamma^{+}\psi(y^{-})|p\rangle$



❑ **Physical observables in the path integral:**

$$\langle \mathcal{O}\rangle \equiv \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\overline{\psi}\,\mathcal{D}\psi\,\mathcal{O}(\overline{\psi},\psi,A)\,e^{iS(\overline{\psi},\psi,A)} \qquad \text{with} \quad Z=\int \mathcal{D}A\,\mathcal{D}\overline{\psi}\,\mathcal{D}\psi\,e^{iS(\overline{\psi},\psi,A)}$$

❑ **Lattice QCD in Euclidian Space:** $t \to i\,t_E$

$$\langle \mathcal{O}\rangle \equiv \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\overline{\psi}\,\mathcal{D}\psi\,\mathcal{O}_E(\overline{\psi},\psi,A)\,e^{-S_E(\overline{\psi},\psi,A)}$$

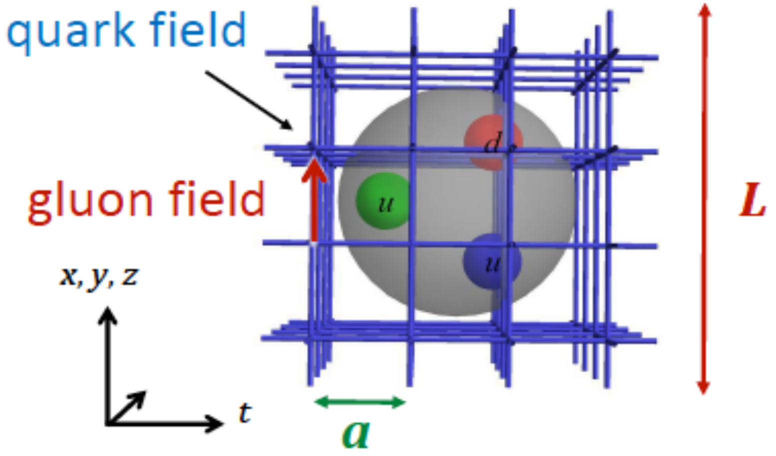
IF $\mathcal{O}(\overline{\psi},\psi,A)$ **depends on time,**

$$\mathcal{O}(\overline{\psi},\psi,A) \neq \mathcal{O}_E(\overline{\psi},\psi,A)\ !$$

➡ **Lattice QCD calculable observables:**

✧ Time-independent operators

✧ Has a reliable continuous limit



Hadron’s partonic structure from Lattice QCD

❑ Quasi-PDFs:

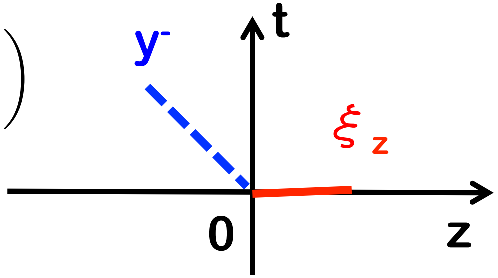
Ji, arXiv:1305.1539

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle + \mathrm{UV}$$

Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y}\, Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

- ✧ Gluon & quark flavor mixing contribution beyond LO!
- ✧ Power UV divergence!
- ✧ Colliner factorization?



$$\mu^2\frac{d}{d\mu^2}\tilde{q}(x,\mu^2,P_z) \neq \mathrm{DGLAP}$$

❑ Pseudo-PDFs:

Radyushkin, 2017

$$\mathcal{M}^\alpha(\nu = p\cdot\xi,\xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$

$$\equiv 2p^\alpha\mathcal{M}_p(\nu,\xi^2) + \xi^\alpha(p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha\mathcal{M}_p(\nu,\xi^2)$$

$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi}\, e^{ix\,\nu}\, \frac{1}{2p^+}\mathcal{M}^+(\nu,\xi^2)$$ **with** $\xi^2 < 0$

Off-light-cone extension of PDFs: $f(x) = \mathcal{P}(x,\xi^2=0)$ **with** $\xi^\mu = (0^+,\xi^-,0_\perp$

❑ Other approaches, …

“OPE without OPE” (Chambers et al. 2017), Hadronic tensor (Liu et al. 1994, …), …

Hadron’s partonic structure from Lattice QCD

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

☐ Good “Lattic cross sections”:

= Single hadron matrix element:

$\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{\mathcal{O}_n(\xi)\} | P \rangle$ with $\omega \equiv P\cdot\xi,\ \xi^2 \neq 0,\ \text{and}\ \xi_0 = 0;$ and

1) can be calculated in lattice QCD with precision,
   has a well-defined continuum limit (UV+IR safe perturbatively), and
2) can be factorized into universal matrix elements of quarks and gluons
   with controllable approximation

Collaboration between lattice QCD and perturbative QCD!

☐ Current-current correlators:

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}^{-1}\, Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

with

d_j : Dimension of the current
Z_j : Renormalization constant of the current

Sample currents:

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q \gamma\cdot\xi \psi_q](\xi),$$

$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q \gamma\cdot\xi \psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4}F^c_{\mu\nu}F^c_{\mu\nu}](\xi), \ldots$$

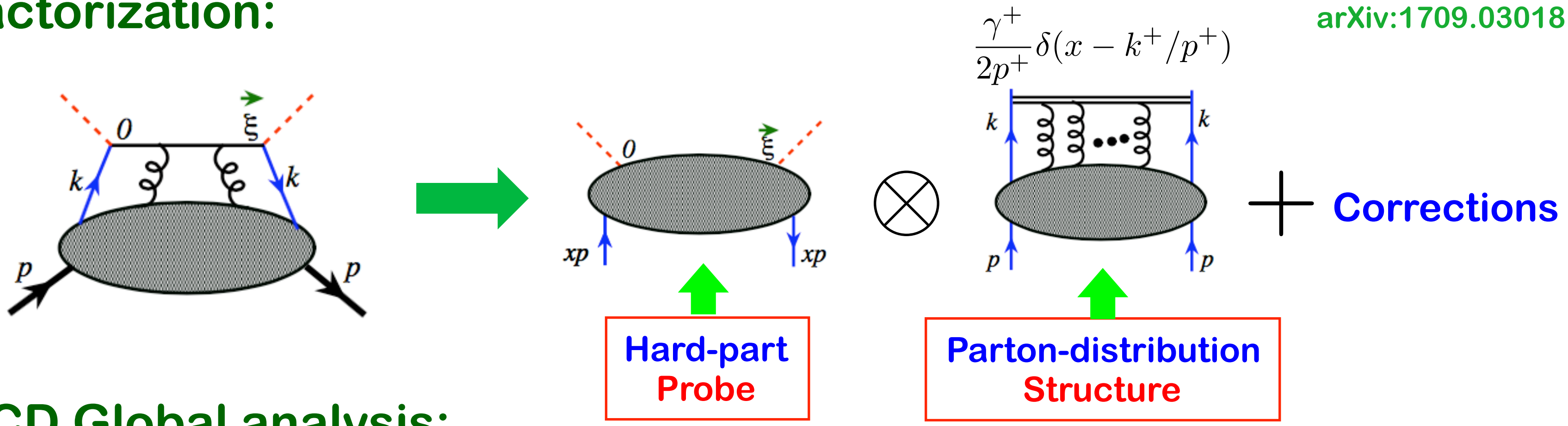
☐ Quasi- and pseudo-PDFs:

$$\mathcal{O}_q(\xi) = Z_q^{-1}(\xi^2)\overline{\psi}_q(\xi)\,\gamma\cdot\xi\,\Phi(\xi,0)\,\psi_q(0) \qquad \Phi(\xi,0) = \mathcal{P}e^{-ig\int_0^1 \xi\cdot A(\lambda\xi)\,d\lambda}$$

Hadron's partonic structure from Lattice QCD

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ **Factorization:**



Corrections

Hard-part
Probe

Parton-distribution
Structure

❑ **QCD Global analysis:**

$$\sigma_n(\omega,\xi^2,P^2)=\sum_a\int_{-1}^{1}\frac{dx}{x}\,f_a(x,\mu^2)\times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2)+O(\xi^2\Lambda_{\rm QCD}^2)$$

with $f_a(x,\mu^2)=-f_a(-x,\mu^2)$

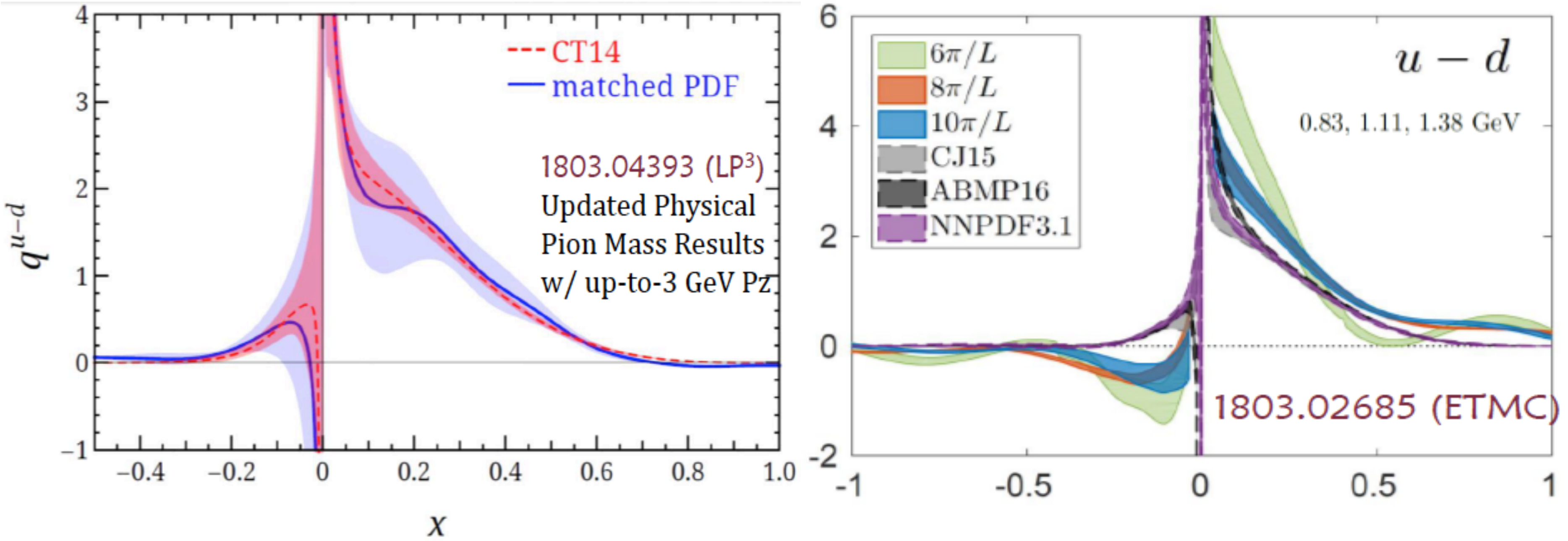
❑ **Complementarity and advantages:**

- ✧ Complementary to existing approaches for extracting PDFs,
- ✧ Quasi-PDFs and pseudo-PDFs are special cases,
- ✧ Have tremendous potentials:

Neutron PDFs, … (no free neutron target!)
Meson PDFs, such as pion, …
More direct access to gluons – gluonic current, quark flavor, …

Numerical results from Lattice QCD

- ❑ **Two collaborations have performed the calculation:**

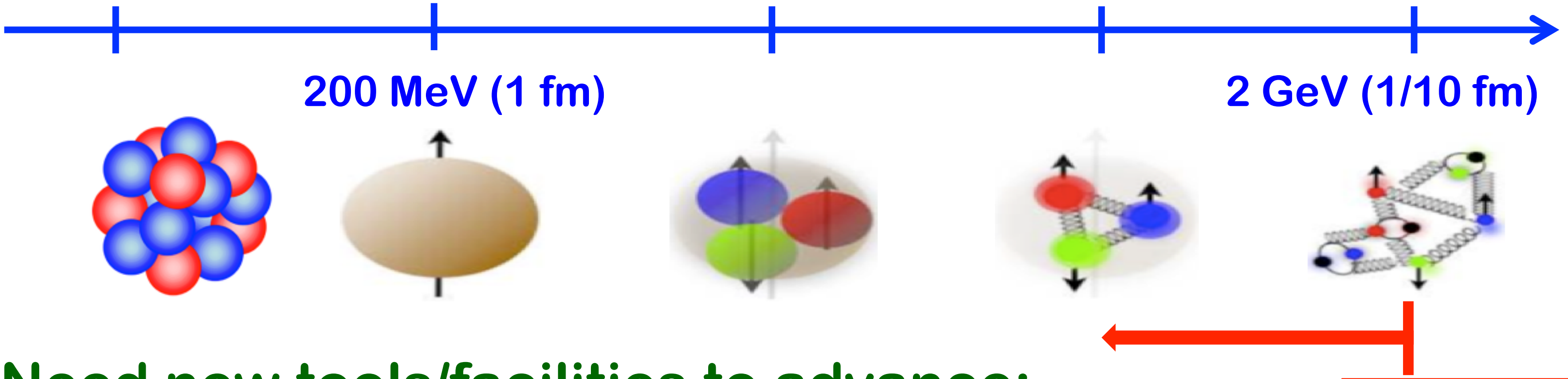


- ✧ Results are encouraging (|x| > 1 region)
- ✧ Differences between two needs to be resolved
- ✧ More detailed studies are still needed
- ✧ BNL/Stony Brook is also trying to verify the results
- ✧ JLab/WM is improving its Pseudo-PDFs calculation
- ✧ JLab is calculating “two-current correlator” of Pion and Keon

Next QCD Frontier: Femto-Science

- **QCD landscape of nucleon and nuclei?**

QCD is successful!



Color Confinement

Asymptotic freedom

Q (GeV)

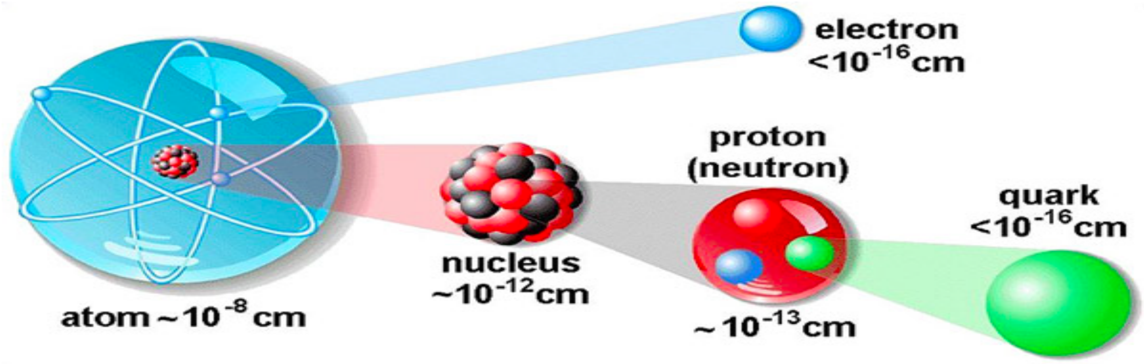
200 MeV (1 fm)

2 GeV (1/10 fm)

Probing scale

More interesting regime of QCD

- **Need new tools/facilities to advance:**



Femto-science
(0.1-10 fm)

Probes with 2 scales:

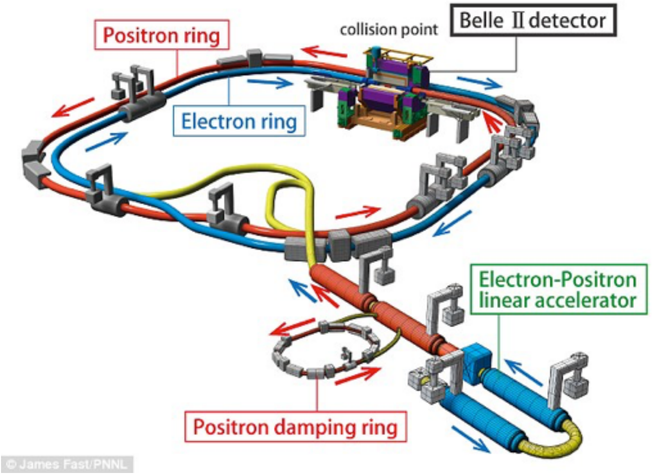
✧ Large scale for particle nature

✧ Small scale for confined motion and imaging

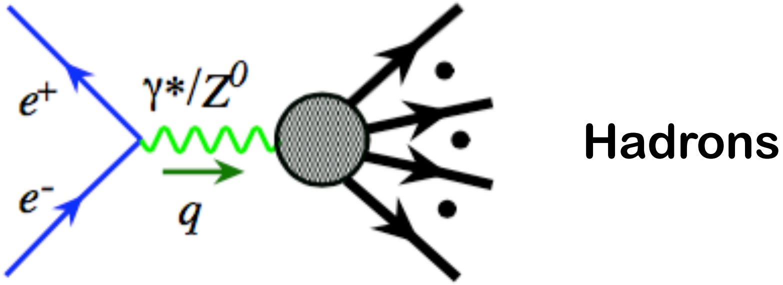
Need a facility to be able to explore/see the structure and dynamics !

Hard probes from high energy collisions

☐ Lepton-lepton collisions:

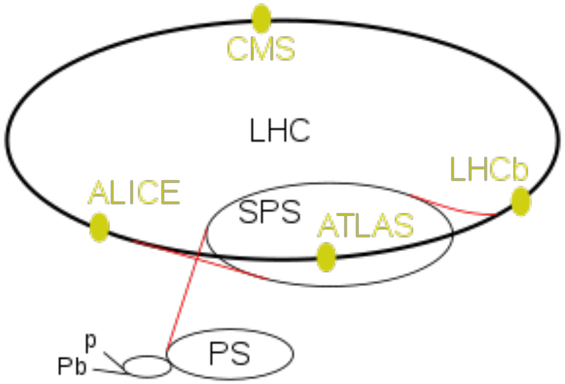


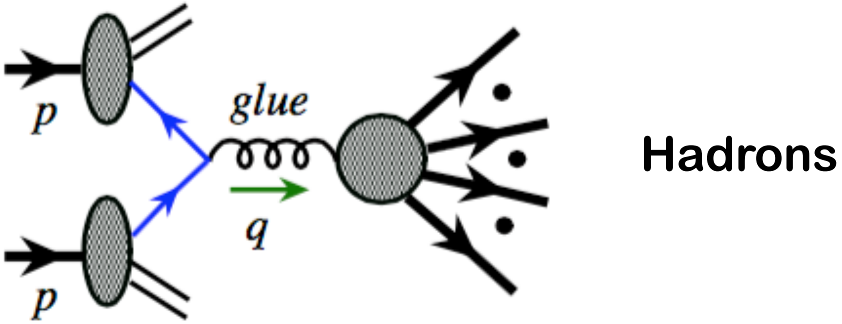
© James Fast/PNNL



- ✧ No hadron in the initial-state
- ✧ Hadrons are emerged from energy
- ✧ Not ideal for studying hadron structure

☐ Hadron-hadron collisions:





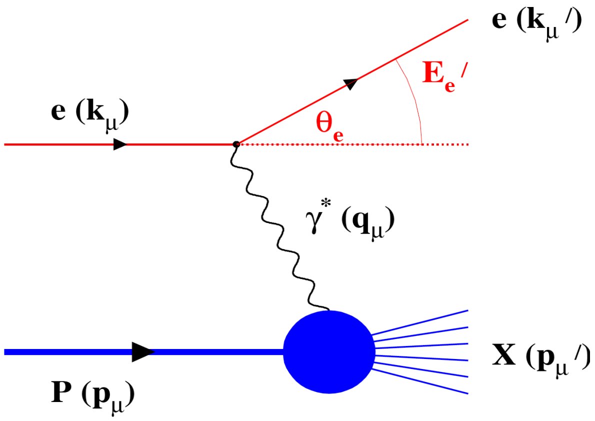
- ✧ Hadron structure – motion of quarks, …
- ✧ Emergence of hadrons, …
- ✧ Initial hadrons broken – collision effect, ..

☐ Lepton-hadron collisions:

Hard collision without breaking the initial-state hadron – spatial imaging, …

Why a lepton-hadron facility is special?

- **Many complementary probes at one facility:**



Q^2 → Measure of resolution

y → Measure of inelasticity

x → Measure of momentum fraction of the struck quark in a proton

$$Q^2 = S \times y$$

Inclusive events: e+p/A → e'+X

Detect only the scattered lepton in the detector

(Modern Rutherford experiment!)

Semi-Inclusive events: e+p/A → e'+h(π,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

(Initial hadron is broken – confined motion! – cleaner than h-h collisions)

Exclusive events: e+p/A → e'+ p'/A'+ h(π,K,p,jet)

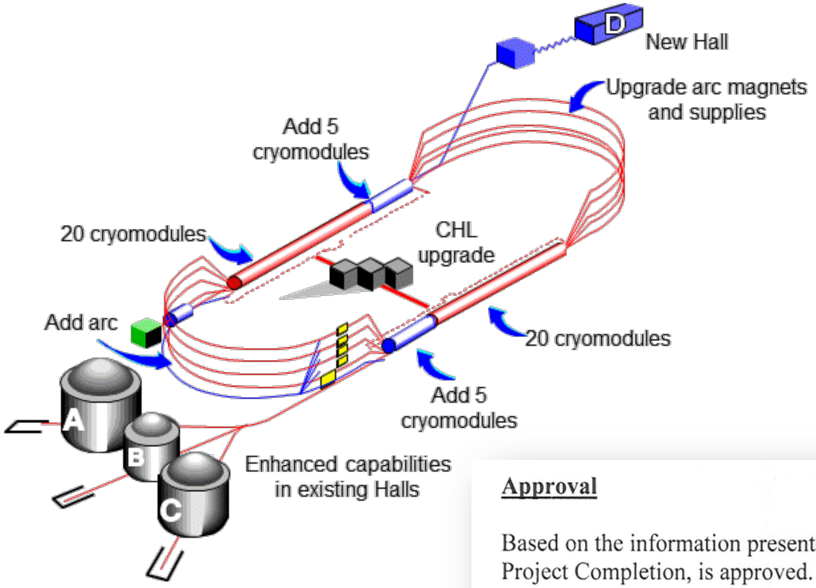
Detect every things including scattered proton/nucleus (or its fragments)

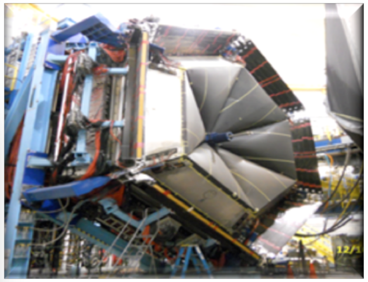
(Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)

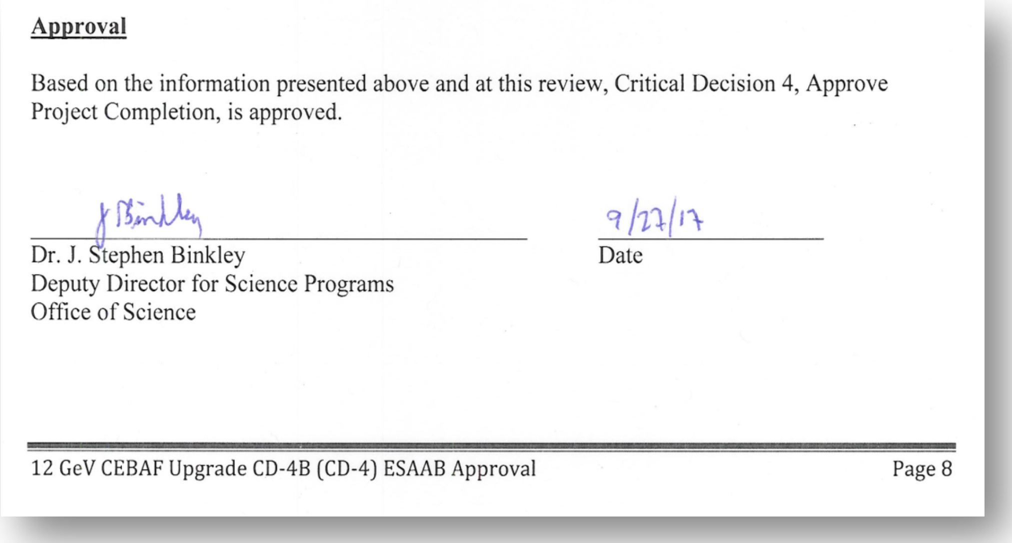
Jefferson Lab @ 12 GeV

❑ Lepton-hadron facility:

12 GeV CEBAF Upgrade Project was completed with one run, ready for more







Approval

Based on the information presented above and at this review, Critical Decision 4, Approve Project Completion, is approved.

Dr. J. Stephen Binkley
Deputy Director for Science Programs
Office of Science

9/27/17
Date

12 GeV CEBAF Upgrade CD-4B (CD-4) ESAAB Approval   Page 8

With an extremely high luminosity ~ 10^{38} [cm^{-2} s^{-1}]

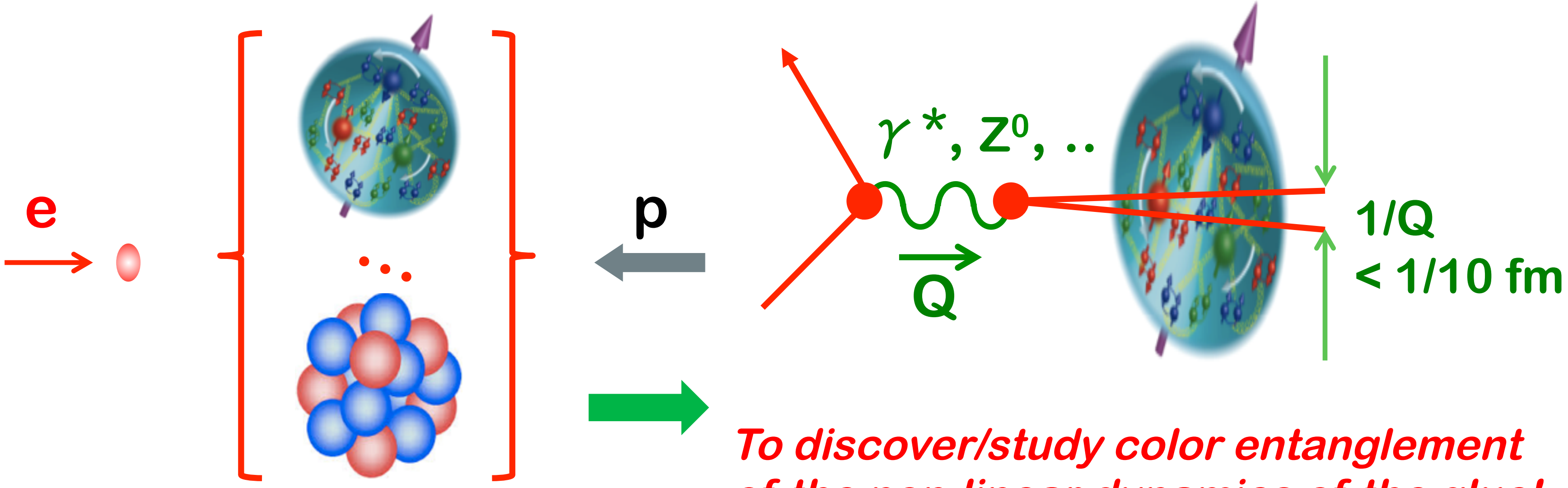
The Electron-Ion Collider (EIC) – the Future!

☐ **A sharpest "CT"** – "imagine" quark/gluon structure without breaking the hadron

- "cat-scan" the nucleon and nuclei with a better than 1/10 fm resolution
- "see" proton "radius" of quark/gluon density comparing with the radius of EM charge density

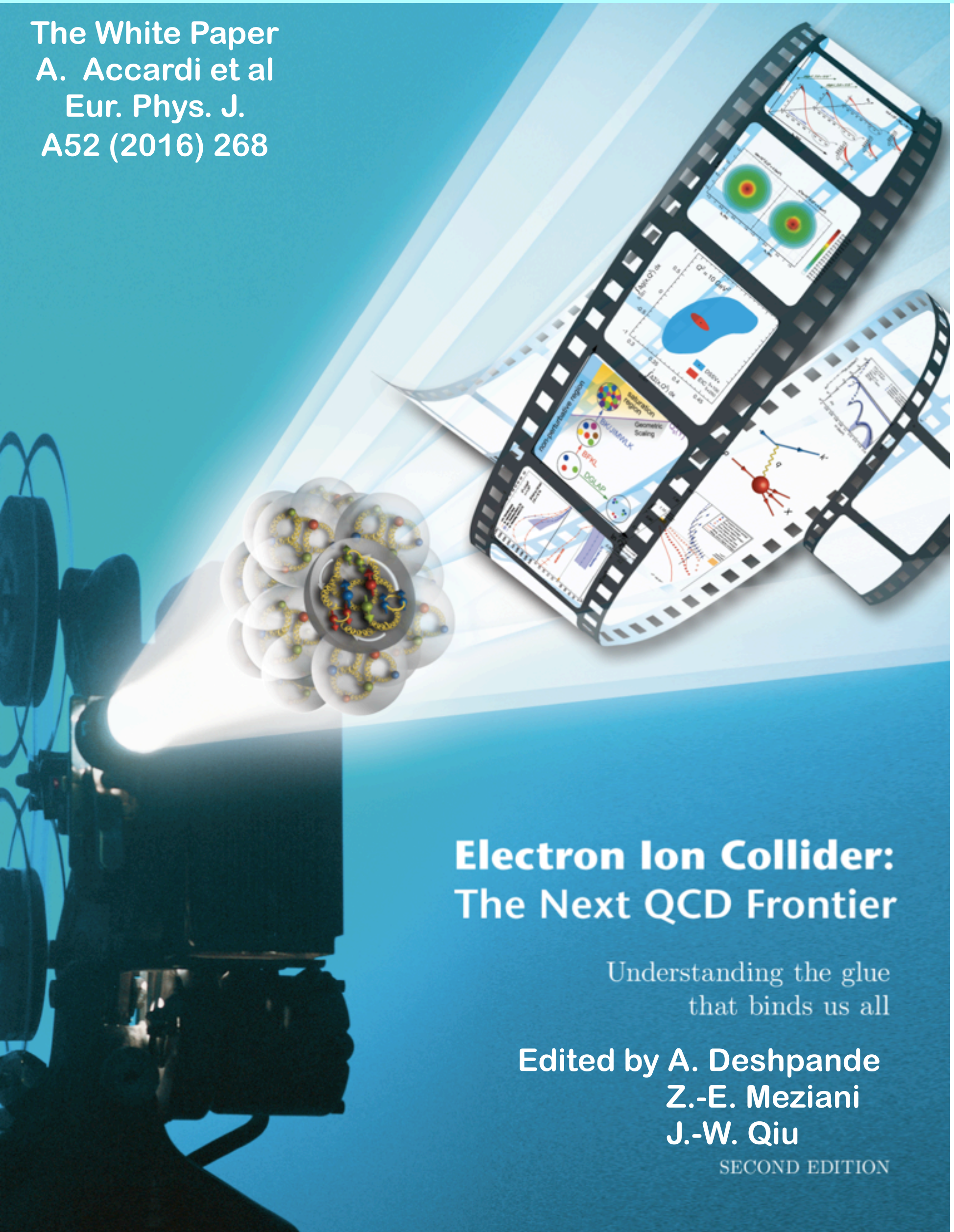
➡ *To discover color confining radius, hints on confining mechanism!*

☐ **A giant "Microscope"** – "see" quarks and gluons by breaking the hadron



➡ *To discover/study color entanglement of the non-linear dynamics of the glue!*

US EIC – Two Options of Realization



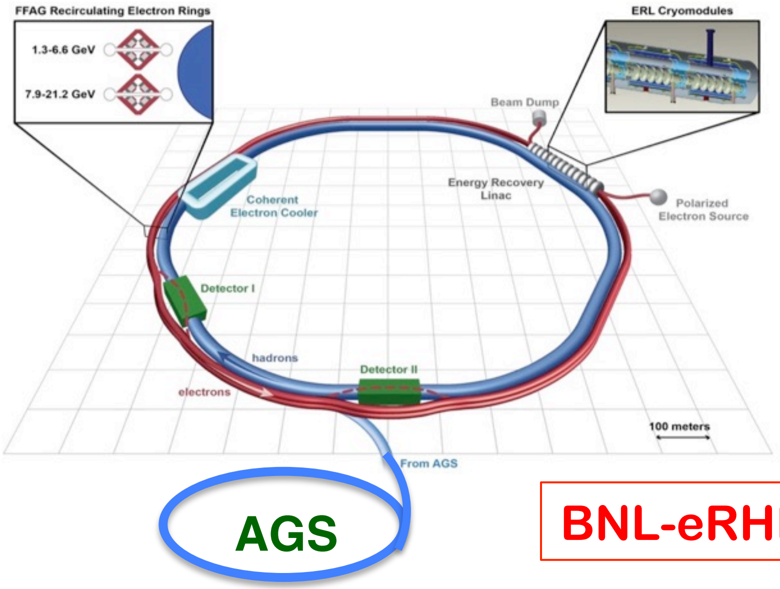
The White Paper
A. Accardi et al
Eur. Phys. J.
A52 (2016) 268

Electron Ion Collider:
The Next QCD Frontier

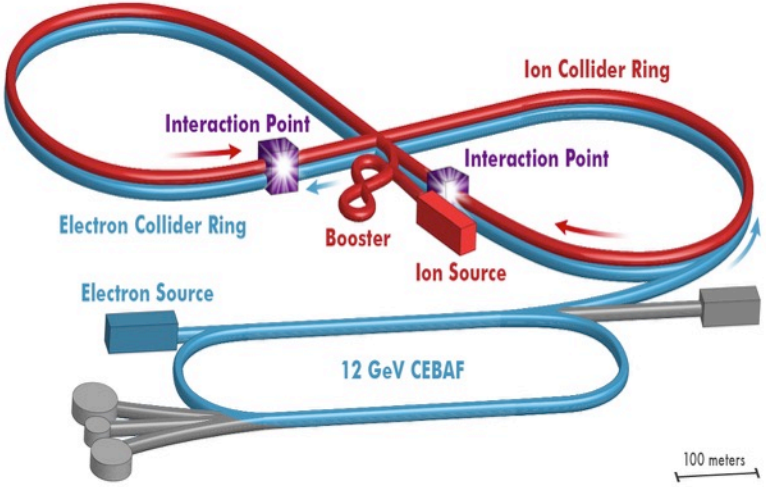
Understanding the glue
that binds us all

Edited by A. Deshpande
Z.-E. Meziani
J.-W. Qiu

SECOND EDITION

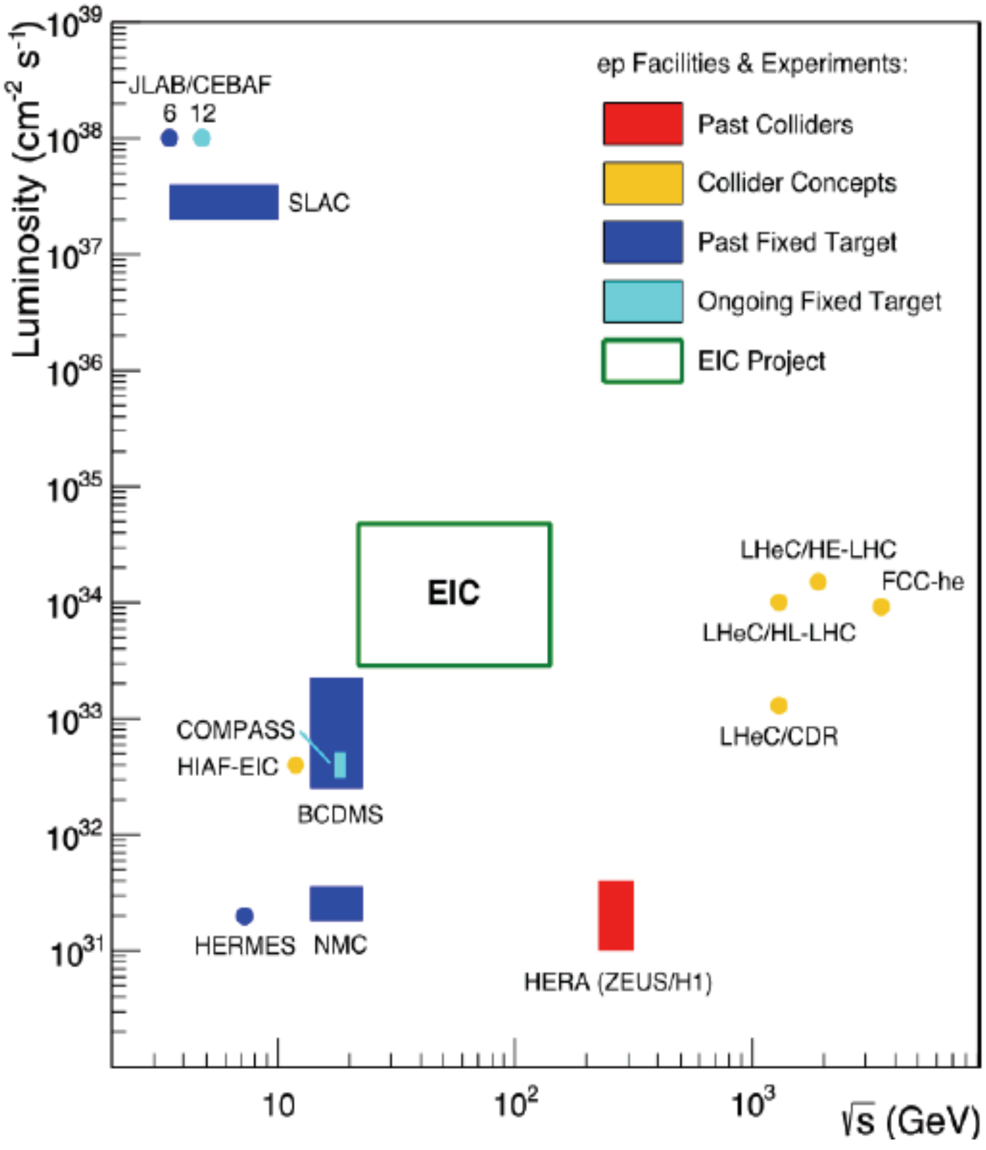


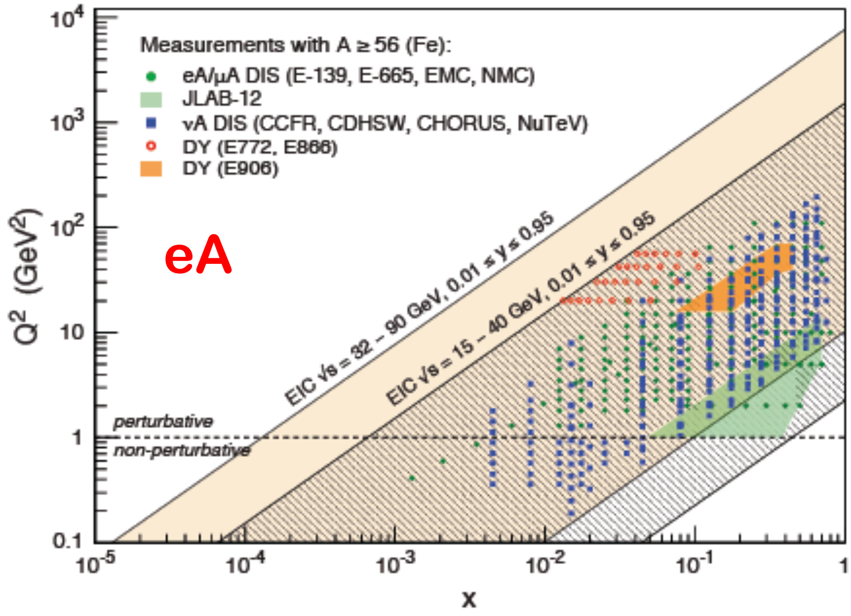
BNL-eRHIC

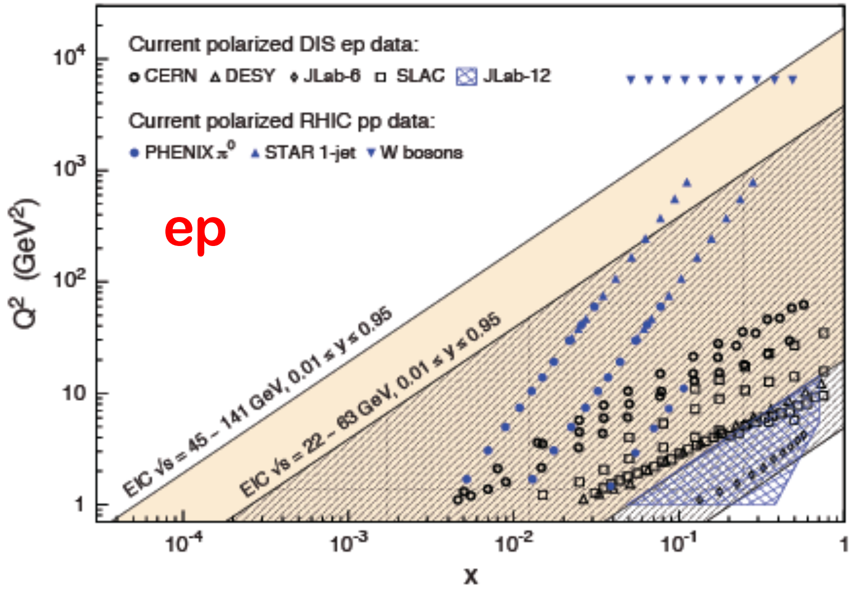


JLab-JLEIC

US EIC – Luminosity & kinematics coverage



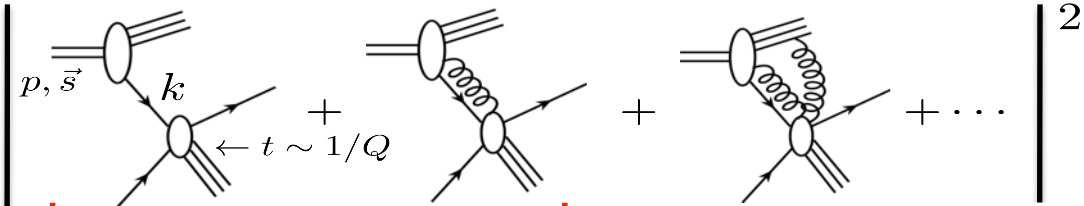




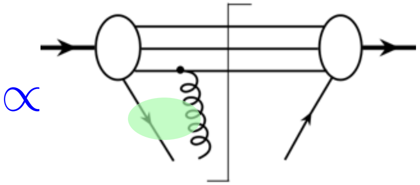
US-EIC – can do what HERA could not do

- **Quantum imaging:**
  - ✧ HERA discovered: 15% of e-p events is diffractive – Proton not broken!
  - ✧ US-EIC: 100-1000 times luminosity – *Critical for 3D tomography!*
- **Quantum interference & entanglement:**
  - ✧ US-EIC: Highly polarized beams – *Origin of hadron property: Spin, …*
    Direct access to chromo-quantum interference!

$\sigma(Q, \vec{s}) \propto$

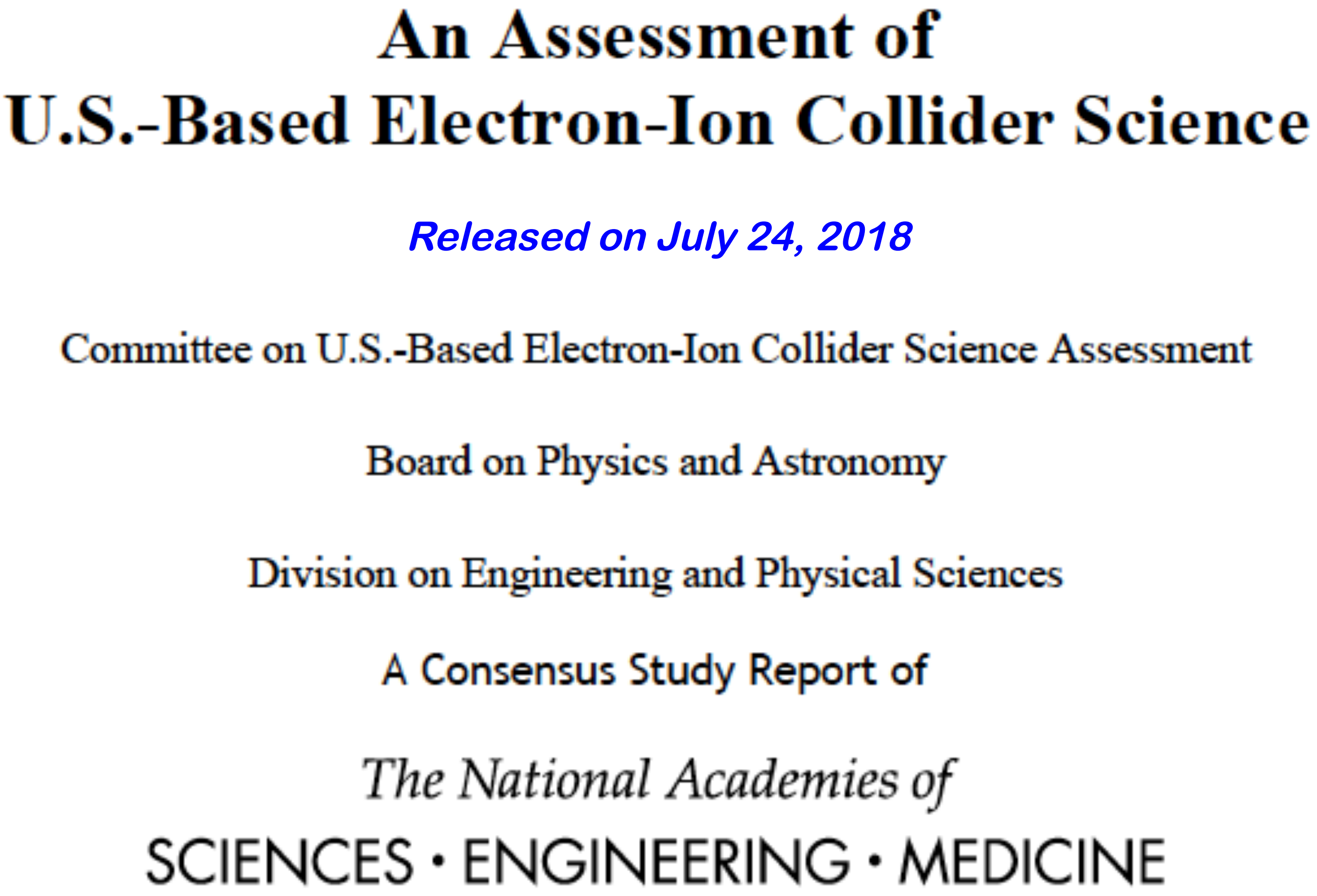


$\sigma(s) - \sigma(-s)$ ➡ Quantum interference ➡ $T^{(3)}(x, x) \propto$



- **Nonlinear quantum dynamics:**
  - ✧ US-EIC: Light-to-heavy nuclear beams – *Origin of nuclear force, …*
    Catch the transition from chromo-quantum fluctuation
    to chromo-condensate of gluons, …
    Emergence of hadrons (femtometer size detector!),
    – "a new controllable knob" – Atomic weight of nuclei

Report from The US National Academy



An Assessment of U.S.-Based Electron-Ion Collider Science

Released on July 24, 2018

Committee on U.S.-Based Electron-Ion Collider Science Assessment

Board on Physics and Astronomy

Division on Engineering and Physical Sciences

A Consensus Study Report of

The National Academies of

SCIENCES • ENGINEERING • MEDICINE

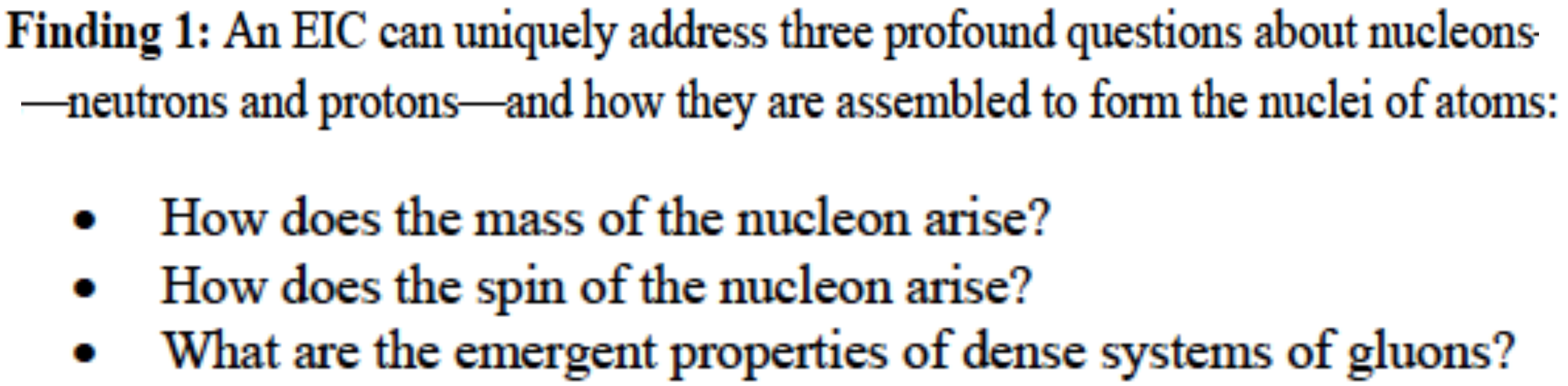
The study is supported by funding from the DOE Office of Science.

(Further information can be found at: https://www.nap.edu/25171)

Report from The US National Academy

The National Academies of Sciences, Engineering, and Medicine was asked by the U.S. Department of Energy to assess the scientific justification for building an Electron-Ion Collider (EIC) facility. The unanimous conclusion of the Committee is that an EIC, as envisioned in this report, would be...

... a unique facility in the world that would answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics.



Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

Emergent Hadron Properties from QCD

- **Mass – intrinsic to a particle:**

  = Energy of the particle when it is at the rest

  - ✧ QCD energy-momentum tensor in terms of quarks and gluons

$$T^{\mu\nu} = \tfrac{1}{2}\,\overline{\psi}\, i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \tfrac{1}{4}\, g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

  - ✧ Proton mass:

$$m = \left.\frac{\langle p|\int d^3x\, T^{00}\,|p\rangle}{\langle p|p\rangle}\right|_{\text{Rest frame}} \sim \text{GeV}$$

- **Spin – intrinsic to a particle:**

  = Angular momentum of the particle when it is at the rest

  - ✧ QCD angular momentum density in terms of energy-momentum tensor

$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \qquad\qquad J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

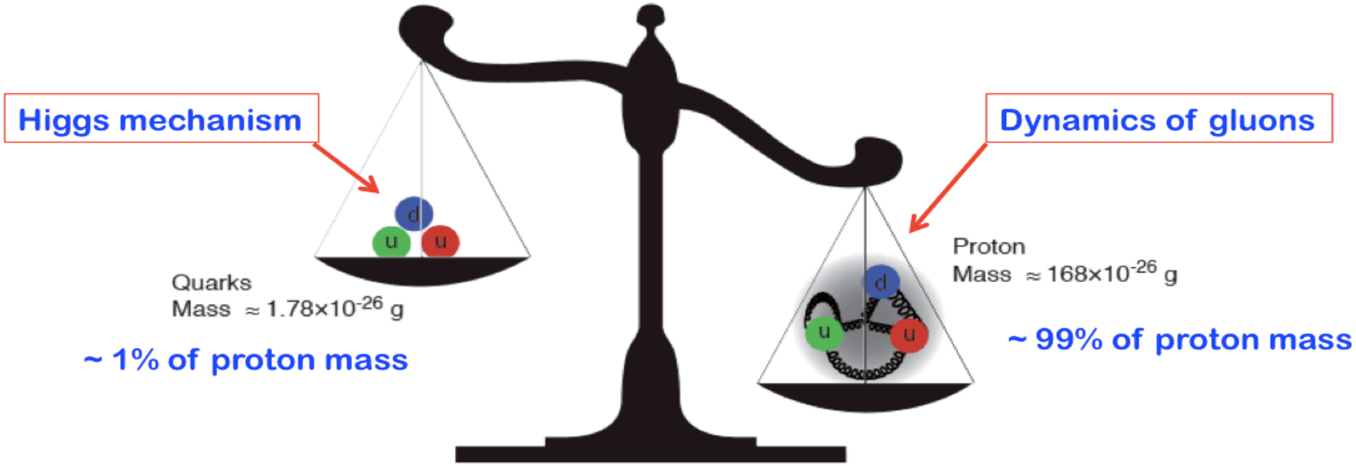
  - ✧ Proton spin:

$$S(\mu) = \sum \langle P, S|\hat{J}^z_f(\mu)|P, S\rangle = \frac{1}{2}$$

If we do not understand proton mass & spin, we do not know QCD!

The Proton Mass

- **Nucleon mass – dominates the mass of visible world:**



Higgs mechanism is not enough!!! *"Mass without mass!"*

- **How does QCD generate the nucleon mass?**

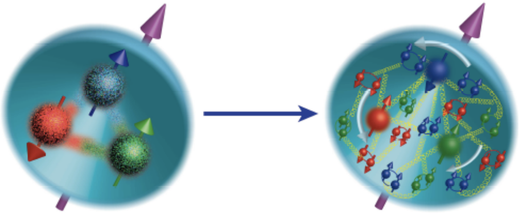
> "… The vast majority of the nucleon's mass is due to quantum fluctuations of quark-antiquark pairs, the gluons, and the energy associated with quarks moving around at close to the speed of light. …"

REACHING FOR THE HORIZON

The 2015 Long Range Plan for Nuclear Science

How to quantify and verify this, theoretically and experimentally?

The Proton Mass: from Models to QCD



❑ **Dynamical scale:**

✧ Asymptotic freedom ⟷ confinement:

➡ A dynamical scale, $\Lambda_{\rm QCD}$, consistent with $\frac{1}{R} \sim 200\ \text{MeV}$

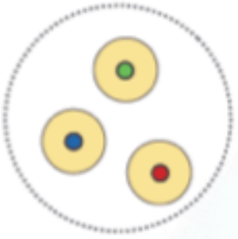
❑ **Bag model:**

✧ Kinetic energy of three quarks: $K_q \sim 3/R$

✧ Bag energy (bag constant B): $T_b = \frac{4}{3}\pi R^3\, B$

✧ Minimize $K_q + T_b$: $M_p \sim \frac{4}{R} \sim \frac{4}{0.88 fm} \sim 912 \text{MeV}$

❑ **Constituent quark model:**



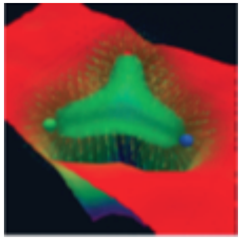
✧ Spontaneous chiral symmetry breaking:

Massless quarks gain ~300 MeV mass when traveling in vacuum

➡ $M_p \sim 3\, m_q^{\rm eff} \sim 900\ \text{MeV}$

C. Roberts'talk

❑ **Lattice QCD:**



✧ With "heavy" (or slow moving) quarks

Energy concentrated in the gluon junction!

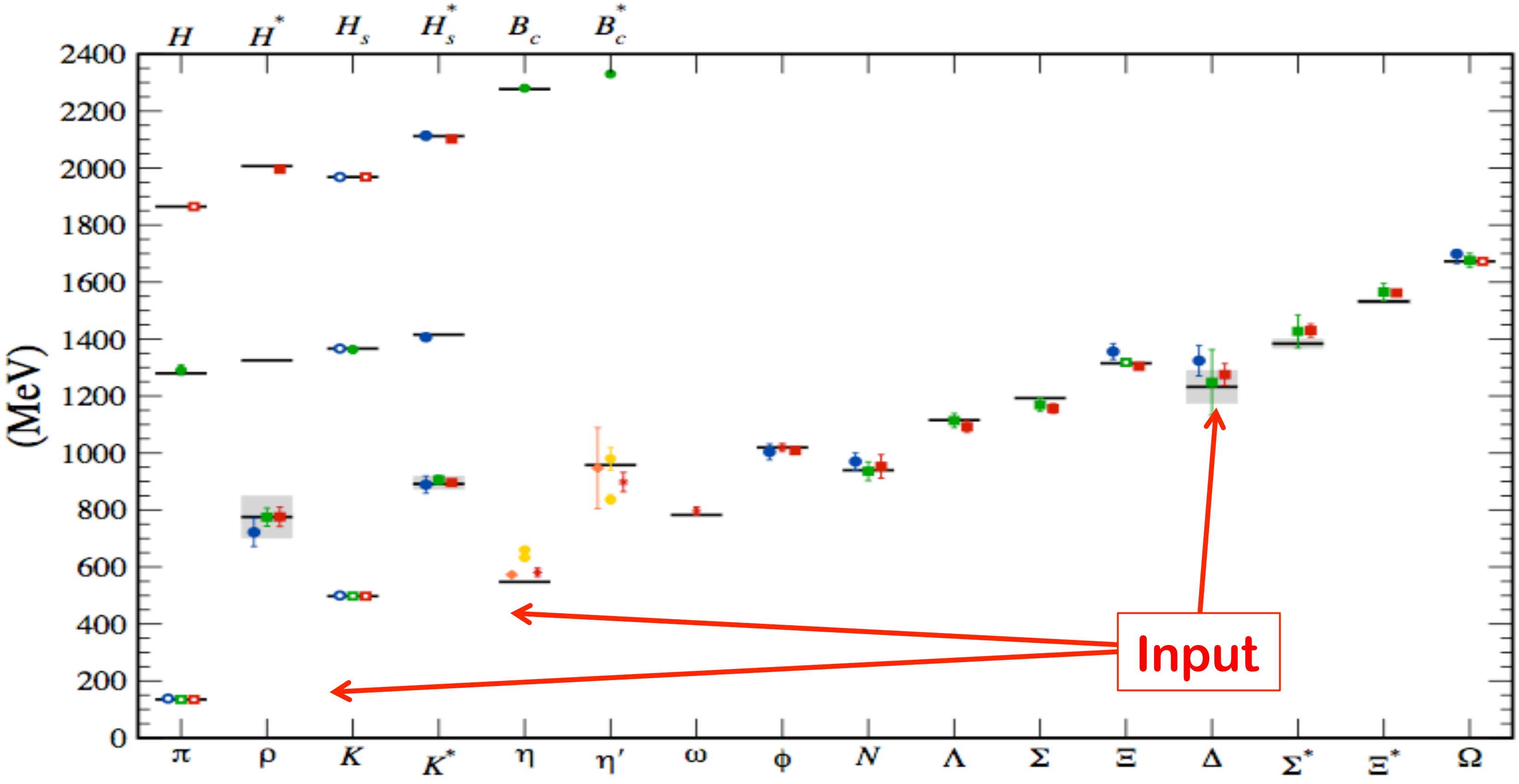
➡ Gluon radius < Charge Radius

EIC!

Mass scale: Lattice space – "a"

The Proton Mass: Lattice QCD

- **Hadron mass from Lattice QCD calculation:**



How does QCD generate this? The role of quarks vs. that of gluons?

Decomposition – Sum Rules

❑ Decomposition of QCD energy-momentum tensor:

$$T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}}$$

Traceless term: $$\overline{T^{\mu\nu}} \equiv T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \ \alpha}$$

Trace term: $$\widehat{T^{\mu\nu}} \equiv \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \ \alpha}$$

with $$T^{\alpha}_{\ \ \alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q (1+\gamma_m) \overbrace{\overline{\psi}_q \psi_q}^{\text{Vacuum expectation breaks chiral symmetry}}$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$$

✧ **Invariant hadron mass (in any frame):**

$$\langle p | \, T^{\mu\nu} \, | p \rangle \propto p^{\mu} p^{\nu} \quad \Rightarrow \quad \langle p | \, T^{\mu\nu} \, | p \rangle (g_{\mu\nu}) \propto p^{\mu} p^{\nu} (g_{\mu\nu}) = m^2$$

$$\Rightarrow \quad m^2 \propto \langle p | T^{\alpha}_{\ \ \alpha} \, | p \rangle$$

Hadron mass: Gluon quantum effect + Chiral symmetry breaking!

✧ **Proton mass sum rule(s):**

Useful only if the individual term can be measured independently

It is not a focus of my lectures, backup slides for other decompositions

The Proton Mass

- ☐ **Three-pronged approach to explore the origin of hadron mass**
  - ✧ Lattice QCD
  - ✧ Mass decomposition – roles of the constituents
  - ✧ Model calculation – approximated analytical approach

The Proton Mass

At the heart of most visible matter.

Temple University, March 28-29, 2016

https://phys.cst.temple.edu/meziani/proton-mass-workshop-2016/

http://www.ectstar.eu/node/2218

A true international effort!

ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

TRENTO, ITALY

Institutional Member of the European Expert Committee NUPECC

Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum, London

The Proton Mass: At the Heart of Most Visible Matter

Trento, April 3 - 7, 2017

Homework (3)

1) **Derive the one-loop contribution to quark distribution within a quark,**

$$\varphi^{(1)}_{q/q}(x,\mu^2)=\left(\frac{\alpha_s}{2\pi}\right)P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV}+\left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\}+\text{UV-CT}$$

on the slide 6. Additional information from the backup slide 43 might be helpful.

Backup slides

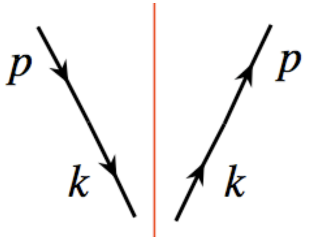
PDFs of a parton

❑ Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \ \Rightarrow |\text{parton}(p)\rangle$ ⇨ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

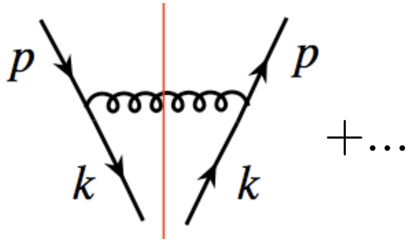
❑ Lowest order quark distribution:



✧ From the operator definition:

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right] \delta\left(x - \frac{k^+}{p^+}\right) (2\pi)^4\delta^4(p-k) \\ &= \delta_{qq'}\ \delta(1-x)\end{aligned}$$

❑ Leading order in α_s quark distribution:



✧ Expand to $(g_s)^2$ – logarithmic divergent:

$$\phi^{(1)}_{q/q}(x) = C_F \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] + \text{UVCT}$$

UV and CO divergence

Decomposition – Sum Rules

X. Ji, PRL (1995)

☐ Another proton mass sum rule:

$$M_p = \left.\frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} \qquad \langle P'|P\rangle = 2P^0(2\pi)^3\delta^3(\vec{P}'-\vec{P})$$

✧ **Hamiltonian:**

$$H_{\text{QCD}} = \int d^3\vec{x}\; T^{00}(0,\vec{x})$$

✧ **QCD energy-momentum tensor:** $T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}}$

"Traceless" term

$$\langle P|\overline{T}^{\mu\nu}|P\rangle = (P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu})/M_p$$

$$\left.\frac{\langle P|\int d^3x\,\overline{T}^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \frac{3}{4}M_p$$

"Trace" term

$$\langle P|\widehat{T}^{\mu\nu}|P\rangle = \frac{1}{4}M_p\, g^{\mu\nu}$$

$$\left.\frac{\langle P|\int d^3x\,\widehat{T}^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \frac{1}{4}M_p$$

✧ **Role of quarks and gluons:**

$$\overline{T^{\mu\nu}} = \overline{T_q^{\mu\nu}} + \overline{T_g^{\mu\nu}} \qquad \widehat{T^{\mu\nu}} = \widehat{T_m^{\mu\nu}} + \widehat{T_a^{\mu\nu}}$$

$$\Rightarrow\quad M_p = \left.\frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \underbrace{M_q}_{\text{Quark Energy}} + \underbrace{M_g}_{\text{Gluon Energy}} + \underbrace{M_m}_{\text{Quark Mass}} + \overbrace{M_a}^{\text{Trace Anomaly}}$$

Decomposition – Sum Rules

❑ Roles of quarks and gluons:

✧ **Quark energy contribution:**

$$H_q = \int d^3\vec{x}\; \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi, \qquad M_q = \left.\frac{\langle P|H_q|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (a-b)\,\frac{3}{4}\,M_p$$

✧ **Gluon energy contribution:**

$$H_g = \int d^3\vec{x}\; \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) \qquad M_g = \left.\frac{\langle P|H_g|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-a)\,\frac{3}{4}\,M_p$$

✧ **Quark mass contribution:**

$$H_m = \int d^3\vec{x}\; \bar{\psi} m \psi \qquad M_m = \left.\frac{\langle P|H_m|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = b\,M_p$$

✧ **Trace anomaly contribution:**

$$H_a = \int d^3\vec{x}\; \frac{9\alpha_s}{16\pi}\,(\mathbf{E^2}-\mathbf{B^2}) \qquad M_a = \left.\frac{\langle P|H_a|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-b)\,\frac{1}{4}\,M_p$$

❑ Two independent parameters:

✧ **Quark momentum fraction:**

$$a(\mu^2) = \frac{1}{P^+P^+}\langle P|i\overline{\psi}\gamma^{(+}D^{+)}\psi|P\rangle = \sum_q \int_0^1 x\left[q(x,\mu^2)+\overline{q}(x,\mu^2)\right]dx$$

✧ **Chiral symmetry breaking:**

$$b(\mu^2) = \frac{1}{2M_p^2}\langle P|m(1+\gamma_m)\overline{\psi}\psi|P\rangle$$

Related to meson-nucleon – σ terms

$$\sigma_{\pi N} = \widehat{m}\langle|\overline{u}u+\overline{d}d|N\rangle$$
$$\widehat{m} = (m_u+m_d)/2$$

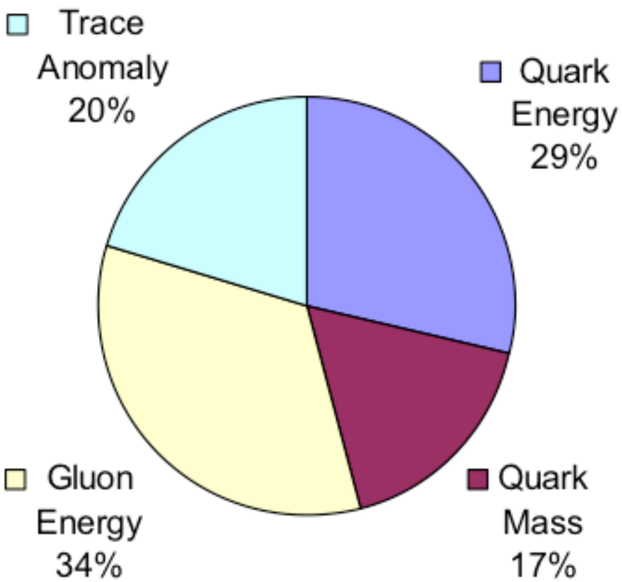
Decomposition – Sum Rules

❑ **Lattice QCD calculation:**

Calculate a(μ²) and b(μ²),
Use the sum rule to fix the others

Not a test!

→ Calculate or measure the Trace Anomaly independently!



❑ **Measure the Trace Anomaly?**

$$T^{\alpha}_{\alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q (1+\gamma_m)\overline{\psi}_q \psi_q$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$$

→ *At the chiral limit, the entire mass is from gluons!*

How to probe the distribution of mass inside the proton?

Need a dilaton field!