Introduction to Quantum Chromodynamics (QCD)

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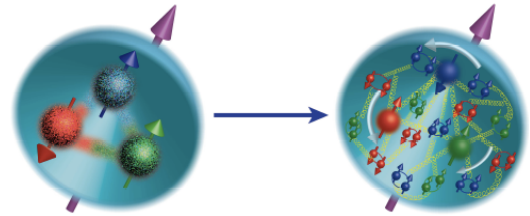
August 16 – 19, 2018

Four Lectures

The Proton Mass: from Models to QCD



- **Dynamical scale:**
  - ✧ Asymptotic freedom ⟷ confinement:
  - ➡ A dynamical scale, Λ_{QCD}, consistent with $\frac{1}{R} \sim 200\ \mathrm{MeV}$

- **Bag model:**

  - ✧ Kinetic energy of three quarks: $K_q \sim 3/R$
  - ✧ Bag energy (bag constant B): $T_b = \frac{4}{3}\pi R^3\, B$
  - ✧ Minimize $K_q + T_b$: $M_p \sim \frac{4}{R} \sim \frac{4}{0.88 fm} \sim 912\mathrm{MeV}$

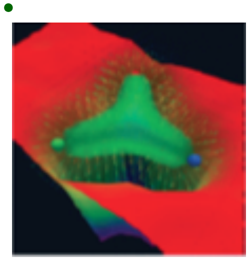
- **Constituent quark model:**

  - ✧ Spontaneous chiral symmetry breaking:

    Massless quarks gain ~300 MeV mass when traveling in vacuum

    ➡ $M_p \sim 3\, m_q^{\mathrm{eff}} \sim 900\ \mathrm{MeV}$ C. Roberts' talk

- **Lattice QCD:**



  - ✧ With "heavy" (or slow moving) quarks

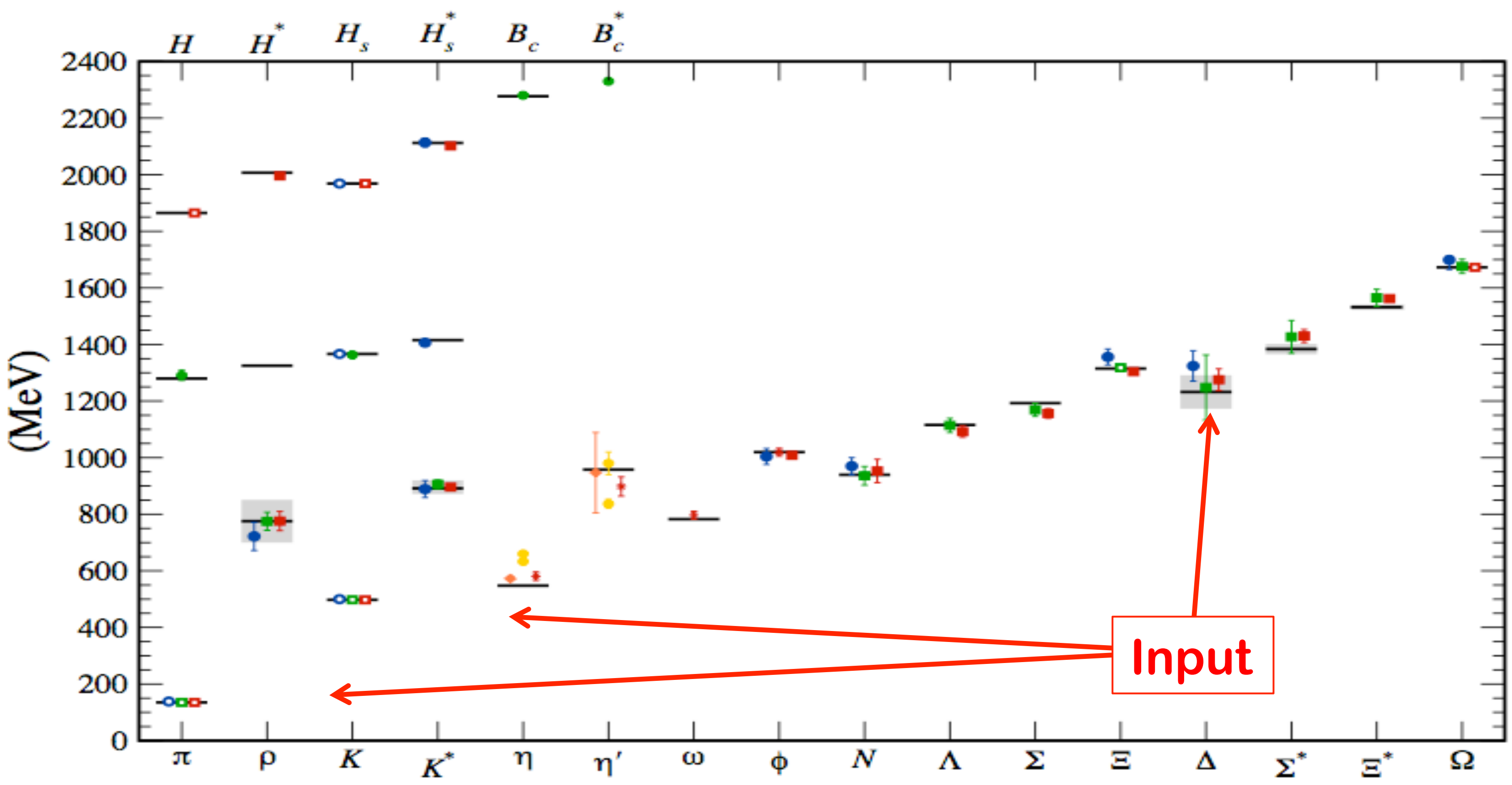
    Energy concentrated in the gluon junction!

    ➡ Gluon radius < Charge Radius EIC!

    Mass scale: Lattice space – "a"

The Proton Mass: Lattice QCD

❑ **Hadron mass from Lattice QCD calculation:**



How does QCD generate this? The role of quarks vs. that of gluons?

Decomposition – Sum Rules

❑ Decomposition of QCD energy-momentum tensor:

$$T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}}$$

Traceless term: $\overline{T^{\mu\nu}} \equiv T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha}$

Trace term: $\widehat{T^{\mu\nu}} \equiv \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha}$

Vacuum expectation breaks chiral symmetry

with $$T^{\alpha}_{\ \alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q (1+\gamma_m) \overbrace{\overline{\psi}_q \psi_q}^{\text{Vacuum expectation breaks chiral symmetry}}$$

QCD trace anomaly $\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$

✧ **Invariant hadron mass (in any frame):**

$$\langle p| T^{\mu\nu} |p\rangle \propto p^\mu p^\nu \quad \Rightarrow \quad \langle p| T^{\mu\nu} |p\rangle (g_{\mu\nu}) \propto p^\mu p^\nu (g_{\mu\nu}) = m^2$$

$$\Rightarrow \quad m^2 \propto \langle p| T^{\alpha}_{\ \alpha} |p\rangle$$

Hadron mass: Gluon quantum effect + Chiral symmetry breaking!

✧ **Proton mass sum rule(s):**

Useful only if the individual term can be measured independently

It is not a focus of my lectures, backup slides for other decompositions

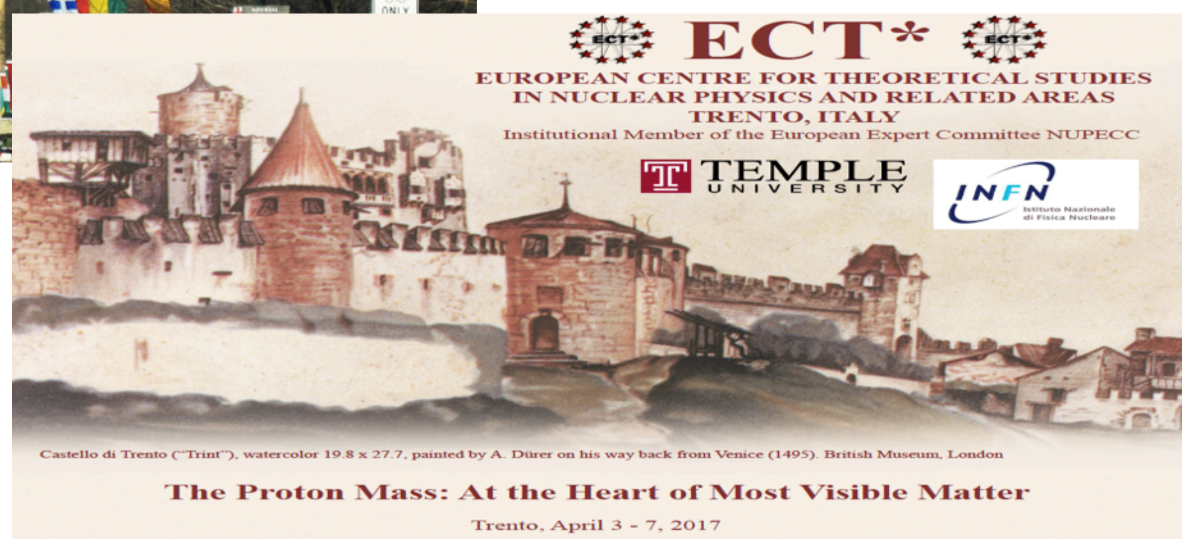
The Proton Mass

- ❑ **Three-pronged approach to explore the origin of hadron mass**
  - ✧ **Lattice QCD**
  - ✧ **Mass decomposition – roles of the constituents**
  - ✧ **Model calculation – approximated analytical approach**

https://phys.cst.temple.edu/meziani/proton-mass-workshop-2016/

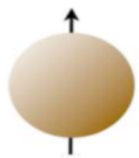
http://www.ectstar.eu/node/2218

A true international effort!



The Proton Spin

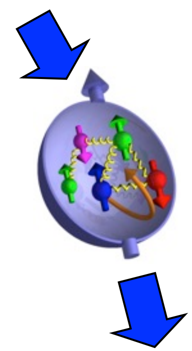
- **The sum rule:** $$S(\mu) = \sum_f \langle P, S | \hat{J}^z_f(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$
  - **Many possibilities of decompositions – connection to observables?**
  - **Intrinsic properties + dynamical motion and interactions**
- **An incomplete story:**



Proton Spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …



Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$
$$\sim 30\%$$

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$
$$\sim 40\% (\text{with RHIC data})$$

Orbital Angular Momentum
of quarks and gluons
Little known

Net effect of partons' transverse motion?

Dual roles of proton spin: property vs. tool!

Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

❑ **Cross section:**

Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

❑ **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ **– Positive definite**

❑ **Asymmetries or difference of cross sections:**

– Not necessary positive!

- **both beams polarized** A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

- **one beam polarized** A_L, A_N

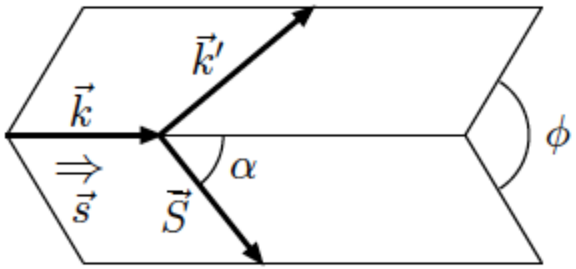
$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Polarized deep inelastic scattering

- **Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) \;-\; \mathcal{W}^{\mu\nu}(P,q,-S)$$



✧ **Define:** $\angle(\hat{k},\hat{S}) = \alpha$**,**
and lepton helicity λ

✧ **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times\left\{\cos\alpha\left\{\left[1-\frac{y}{2}-\frac{m^2x^2y^2}{Q^2}\right] g_1(x,Q^2) \;-\; \frac{2m^2x^2y}{Q^2}\, g_2(x,Q^2)\right\}\right.$$

$$\left. -\sin\alpha\cos\phi\,\frac{2mx}{Q}\sqrt{\left(1-y-\frac{m^2x^2y^2}{Q^2}\right)}\,\left(\frac{y}{2}\, g_1(x,Q^2) \;+\; g_2(x,Q^2)\right)\right\}$$

✧ **Spin orientation:**

$$\alpha \;=\; 0 \;:\; \Rightarrow \quad g_1$$

$$\alpha \;=\; \pi/2 : \;\Rightarrow \quad y\, g_1 \;+\; 2\, g_2$$, suppressed m/Q

Basics for spin observables

❑ Factorized cross section:

More in backup slides!

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

❑ Parity and Time-reversal invariance:

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

❑ IF: $\langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

Operators lead to the "+" sign ⟹ spin-averaged cross sections

Operators lead to the "-" sign ⟹ spin asymmetries

❑ Example:

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \ \Rightarrow\ q(x)$$

Quark helicity: $\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \ \Rightarrow\ \Delta q(x)$

Transversity: $\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \ \Rightarrow\ \delta q(x) \rightarrow h(x)$

Gluon helicity: $\mathcal{O}(\psi,A^\mu) = \dfrac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow\ \Delta g(x)$

Polarized quark helicity distributions

✧ **General expansion of $\phi(x)$:**

must have general expansion in terms of $P\!\!\!/$, $n\!\!\!/$, $s\!\!\!/$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{||}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_\perp\right]$$

✧ **3-leading power quark parton distribution:**

$$q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

$$\Delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

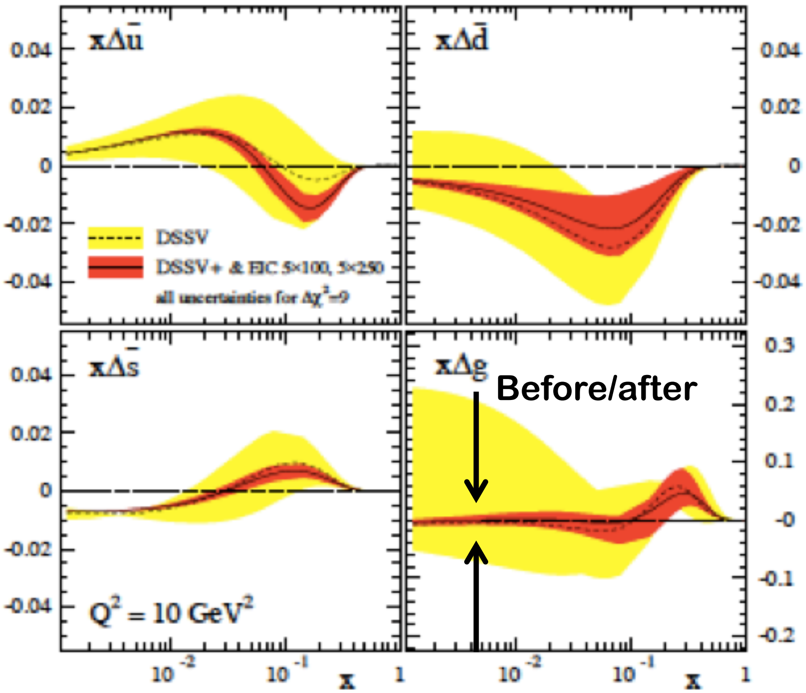
$$\delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

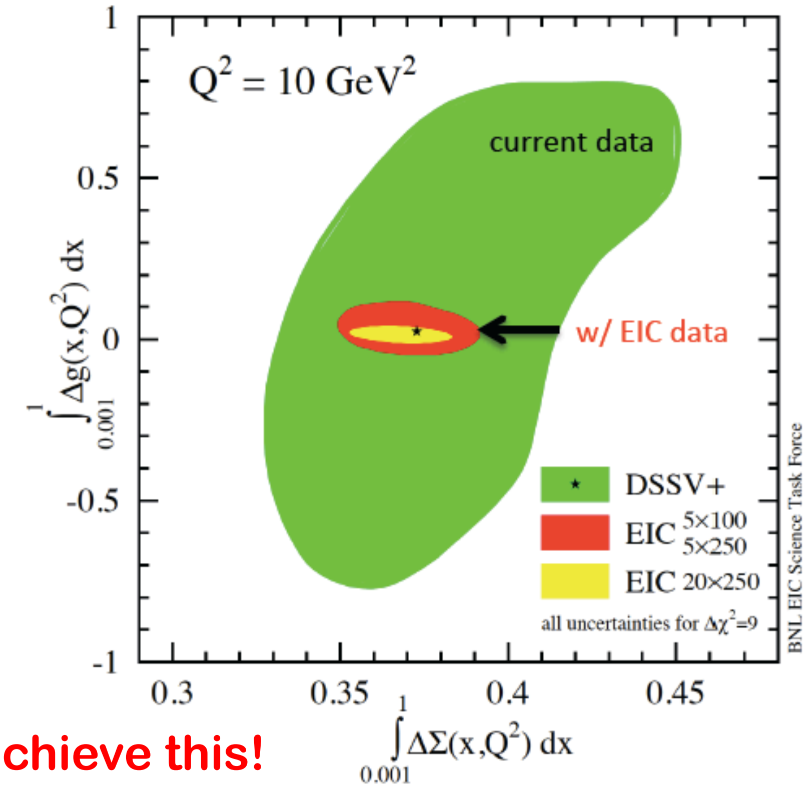
"unpolarized" – "longitudinally polarized" – "transversity"

The Proton Spin

- One-year of running at EIC:

Wider Q^2 and x range including low x at EIC!





No other machine in the world can achieve this!

- Ultimate solution to the proton spin puzzle:
  - ✧ *Precision measurement of $\Delta g(x)$ – extend to smaller x regime*
  - ✧ *Orbital angular momentum contribution – measurement of TMDs & GPDs!*

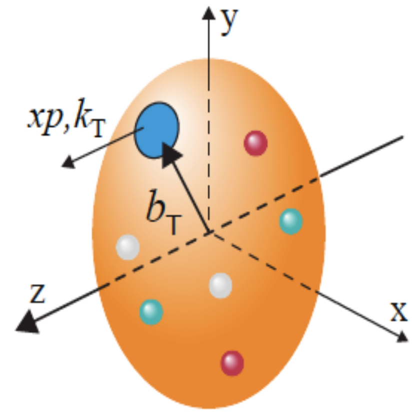
Two-momentum-scale observables

❑ Cross sections with two-momentum scales observed:

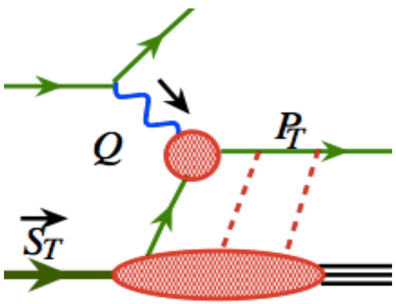
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

✧ Hard scale: Q_1 localizes the probe to see the quark or gluon d.o.f.

✧ "Soft" scale: Q_2 could be more sensitive to hadron structure, e.g., confined motion

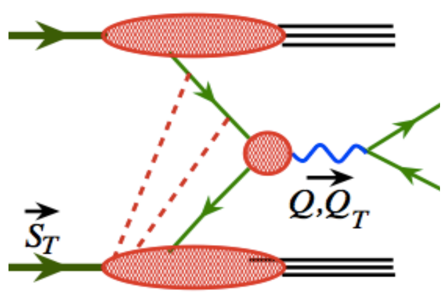


❑ Two-scale observables with the hadron broken:



SIDIS: $Q \gg P_T$

\+



DY: $Q \gg P_T$

\+ Two-jet momentum imbalance in SIDIS, …

✧ Natural observables with TWO very different scales

✧ TMD factorization: partons' confined motion is encoded into TMDs

Two-momentum-scale observables

❑ Cross sections with two-momentum scales observed:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

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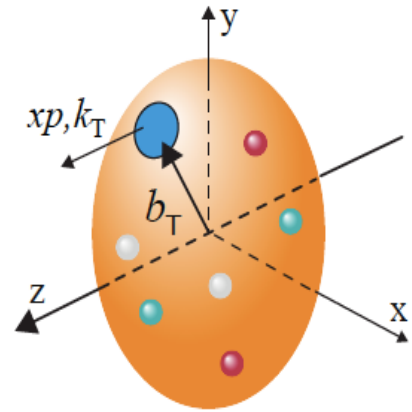
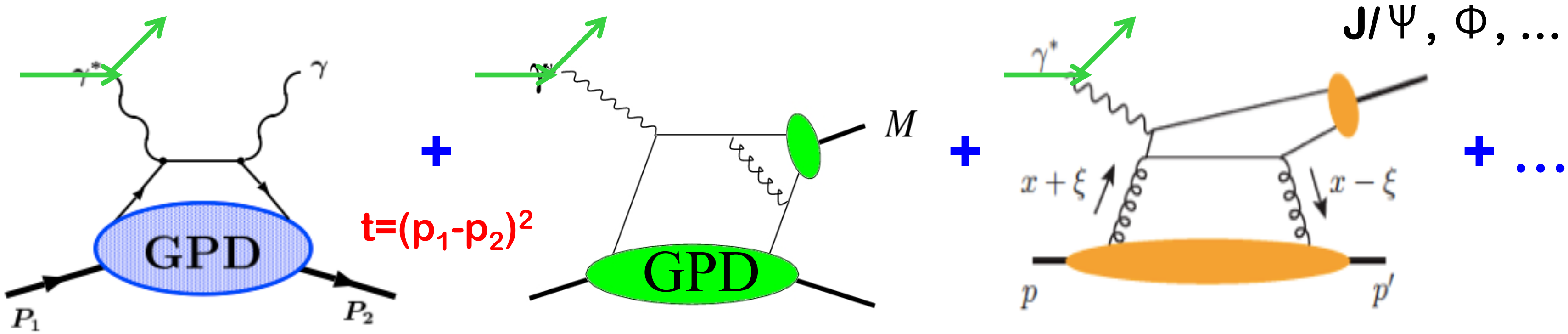


❑ Two-scale observables with the hadron unbroken:



DVCS: $Q^2 \gg |t|$ DVEM: $Q^2 \gg |t|$ EHMP: $Q^2 \gg |t|$

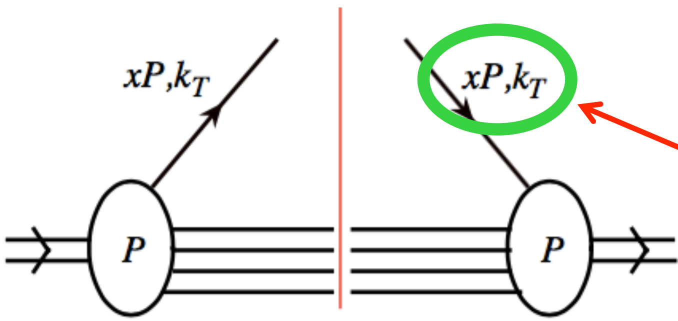
✧ Natural observables with TWO very different scales

✧ GPDs: Fourier Transform of t-dependence gives spatial b_T-dependence

How to quantify the hadron structure?

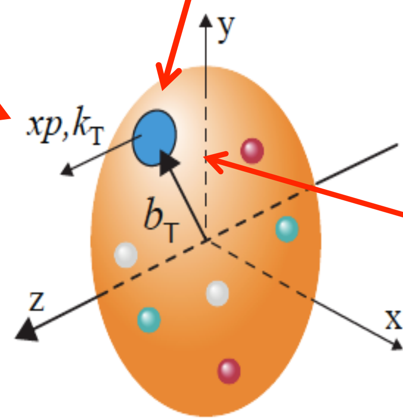
❑ Encoded in TMDs and GPDs:

TMDs

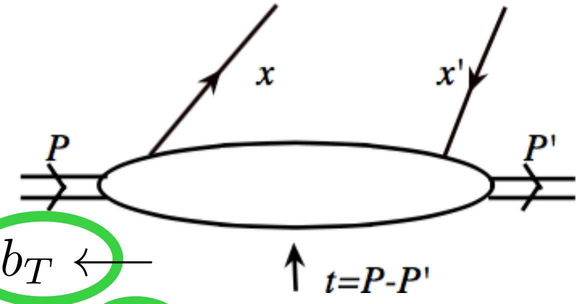


$\propto \langle P|\overline{\psi}\ [\mathrm{Spin}]\ \psi|P\rangle$

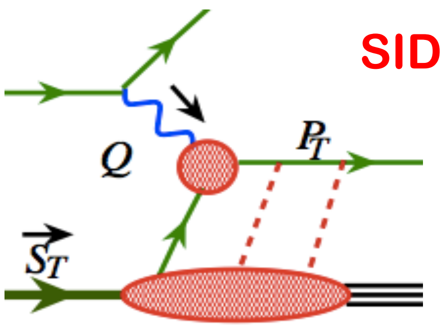
$\boxed{\sim 1/Q}$



GPDs



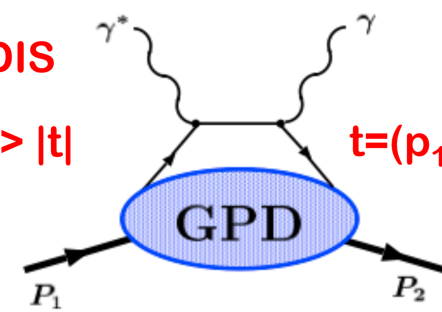
$\propto \langle P'|\overline{\psi}\ [\mathrm{Spin}]\ \psi|P\rangle$



SIDIS: $Q \gg P_T \sim k_T$

Exclusive DIS

DVCS: $Q^2 \gg |t|$



$t=(p_1-p_2)^2$

Two-scales observables

Cross sections ***Amplitudes***

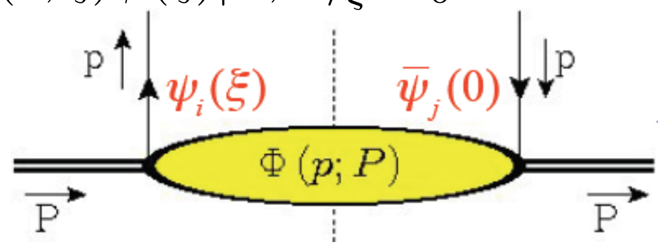
- ✧ ***Confined transverse motion***
- ✧ ***Confined spatial distribution – imaging***

Definition of TMDs

❑ Non-perturbative definition:

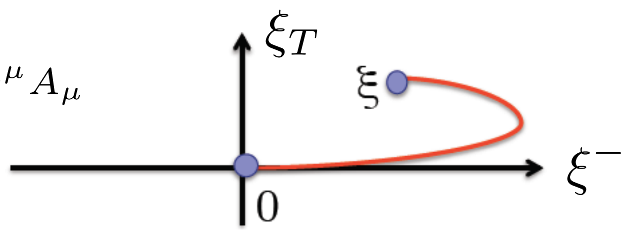
✧ **In terms of matrix elements of parton correlators:**

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{i\,p\cdot\xi} \langle P, S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P, S\rangle_{\xi^+=0}$$



✧ **Depends on the choice of the gauge link:**

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ **Decomposes into a list of TMDs:**

$$\Phi^{[U]}(x, p_T; n) = \Big\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp[U]}(x, p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T)\gamma_5$$
$$+ h_{1T}^{[U]}(x, p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x, p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x, p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

❑ Gives "unique" TMDs, IF we knew proton wave function!

But, we do NOT know proton wave function (calculate it on lattice?)

Like PDFs, TMDs are NOT direct physical observables!

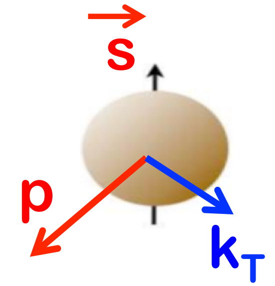
TMDs: confined motion & spin correlation

❑ Power of spin – many more correlations:

		Quark Polarization		
		Un-Polarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity $h_{1T}^\perp$ = −

Nucleon Spin   Quark Spin

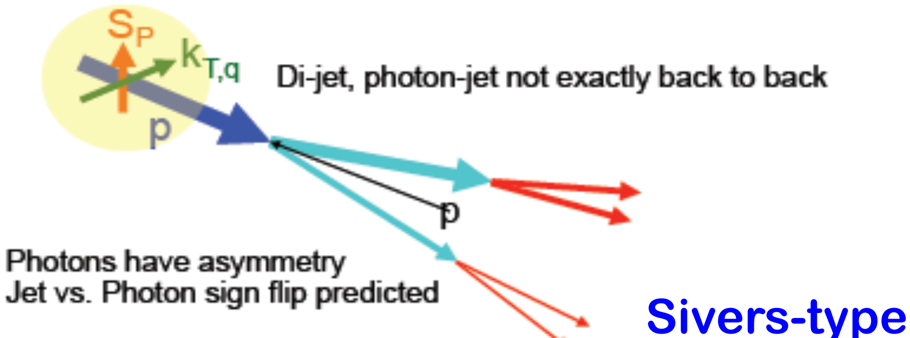
Similar for gluons



Require two Physical scales

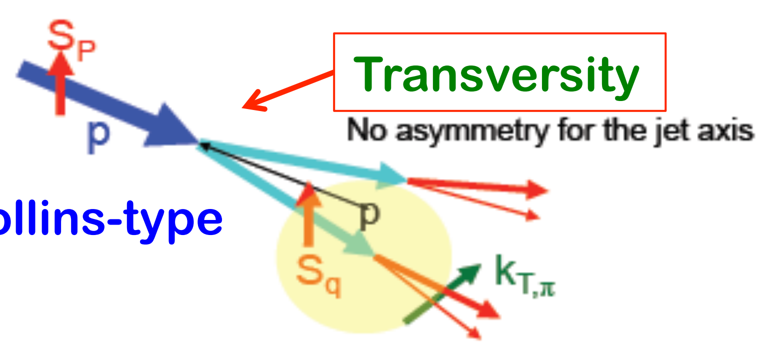
More than one TMD contribute to the same observable!

❑ A_N – single hadron production:



Sivers-type

Collins-type

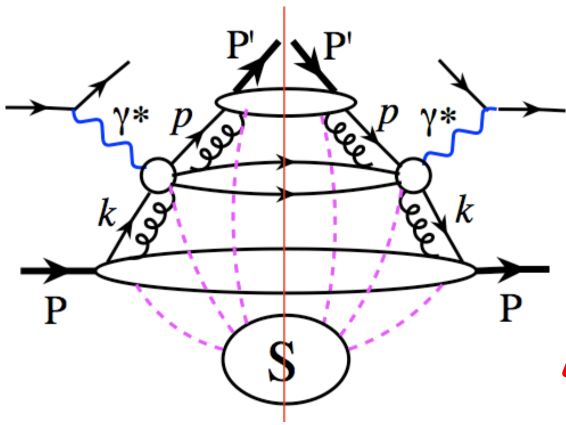


TMDs extracted from data

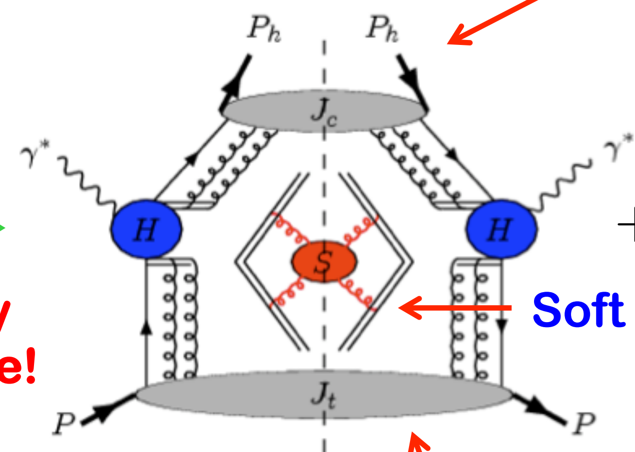
❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:

TMD fragmentation



Theory Advance!



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

$$\sigma_{\mathrm{SIDIS}}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula

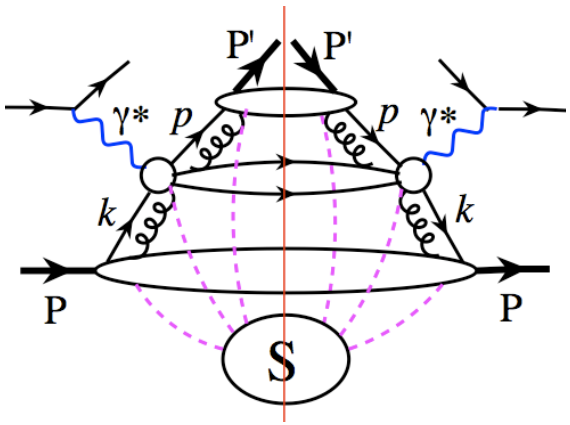
- ✧ **Depending on the perturbatively calculated** $\hat{H}(Q;\mu)$
  perturbative orders, renormalization, factorization schemes, …
- ✧ **Depending on the approximation of neglecting the power corrections, ..**

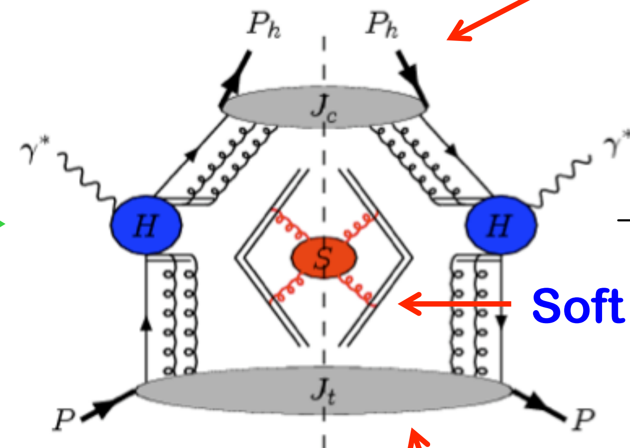
Importance of lattice QCD calculations, …

TMDs extracted from data

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:





$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}, \frac{\langle p^2\rangle}{Q^2}\right)$$

- **Low P_{hT} – TMD factorization:**

$$\sigma_{\mathrm{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

- **High P_{hT} – Collinear factorization:**

$$\sigma_{\mathrm{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

- **P_{hT} Integrated - Collinear factorization:**

$$\sigma_{\mathrm{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

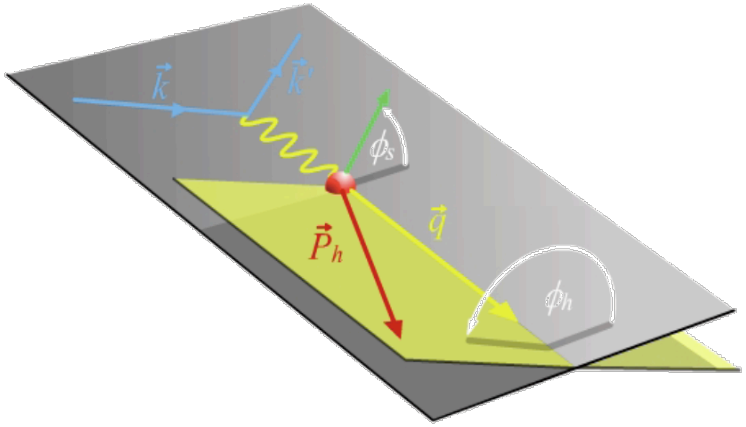
SIDIS is the best for probing TMDs

- Naturally, two scales & two planes:

$$A_{UT}(\varphi_h^l, \varphi_S^l) = \frac{1}{P}\frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}$$

$$= A_{UT}^{Collins}\sin(\phi_h + \phi_S) + A_{UT}^{Sivers}\sin(\phi_h - \phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h - \phi_S)$$



- Separation of TMDs:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

Collins frag. Func. from e^+e^- collisions

Hard, if not impossible, to separate TMDs in hadronic collisions

Using a combination of different observables (not the same observable): jet, identified hadron, photon, …

Evolution equations for TMDs

J.C. Collins, in his book on QCD

❑ TMDs in the b-space:

$$\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; y_P - (-\infty))\sqrt{\frac{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; y_s, -\infty)}} Z_F Z_2$$

❑ Collins-Soper equation:

Renormalization of the soft-factor

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F)$$

$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)}$$

$$\tilde{K}(b_T; \mu) = \frac{1}{2}\frac{\partial}{\partial y_s} \ln\left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)}\right)$$

Introduced to regulate the rapidity divergence of TMDs

❑ RG equations:

Wave function Renormalization

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

Evolution equations are only valid when $b_T << 1/\Lambda_{QCD}$!

$$\frac{d\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F).$$

❑ Momentum space TMDs:

Need information at large b_T for all scale μ !

$$F_{f/P^\uparrow}(x, \mathbf{k}_\mathrm{T}, S; \mu, \zeta_F) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b}_T\, e^{i\mathbf{k}_T\cdot\mathbf{b}_T}\, \tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu, \zeta_F)$$

Evolution equations for Sivers function

Aybat, Collins, Qiu, Rogers, 2011

- Sivers function:

$$F_{f/P^\uparrow}(x,k_T,S;\mu,\zeta_F) = F_{f/P}(x,k_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

- Collins-Soper equation:

Its derivative obeys the CS equation

$$\frac{\partial \ln \tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F)}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)$$

$$\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x,b_T;\mu,\zeta_F)}{\partial b_T}$$

- RG equations:

$$\frac{d\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad\Longrightarrow\quad \frac{\partial\gamma_F(g(\mu);\zeta_F/\mu^2)}{\partial\ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

- Sivers function in momentum space:

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T J_1(k_T b_T)\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F)$$

JI, Ma, Yuan, 2004
Idilbi, et al, 2004,
Boer, 2001, 2009,
Kang, Xiao, Yuan, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012,
Sun, Yuan 2013, …

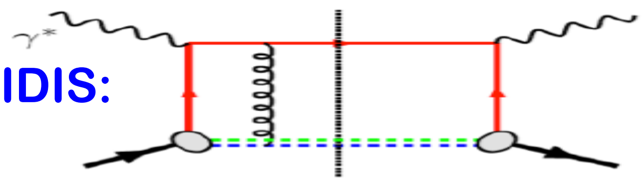
Modified universality for TMDs

❑ Definition:

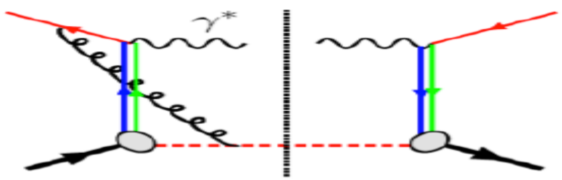
$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

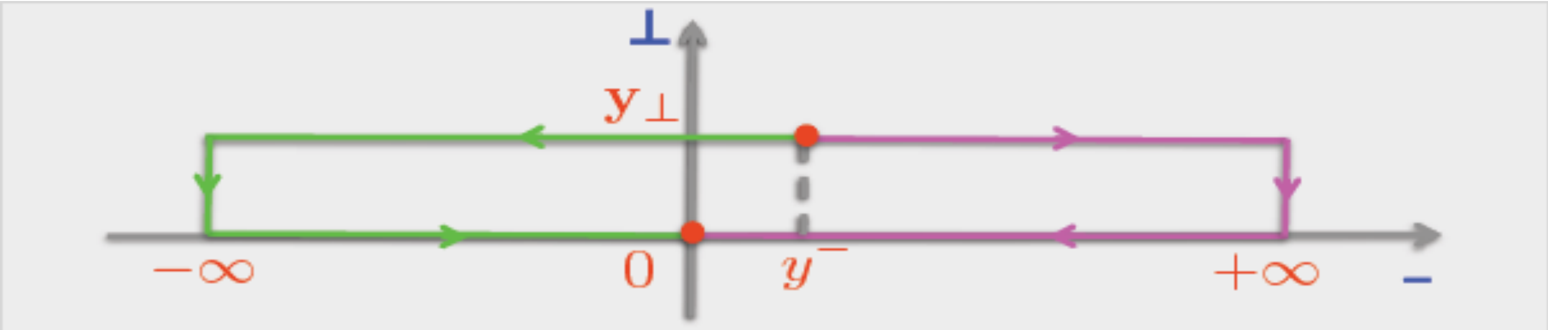
❑ Gauge links:

SIDIS:



DY:





❑ Process dependence:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- Parity – Time reversal invariance:

$$f_{q/h^{\uparrow}}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^{\uparrow}}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- Definition of Sivers function:

$$f_{q/h^{\uparrow}}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^{\uparrow}}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- Modified universality:

$$\Delta^N f_{q/h^{\uparrow}}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^{\uparrow}}^{\rm DY}(x,k_\perp)$$

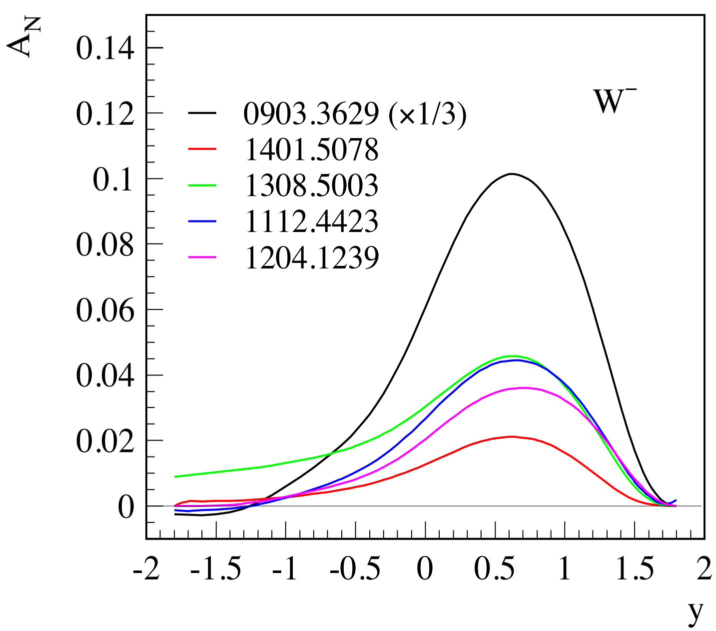
The spin-averaged part of this TMD is process independent,

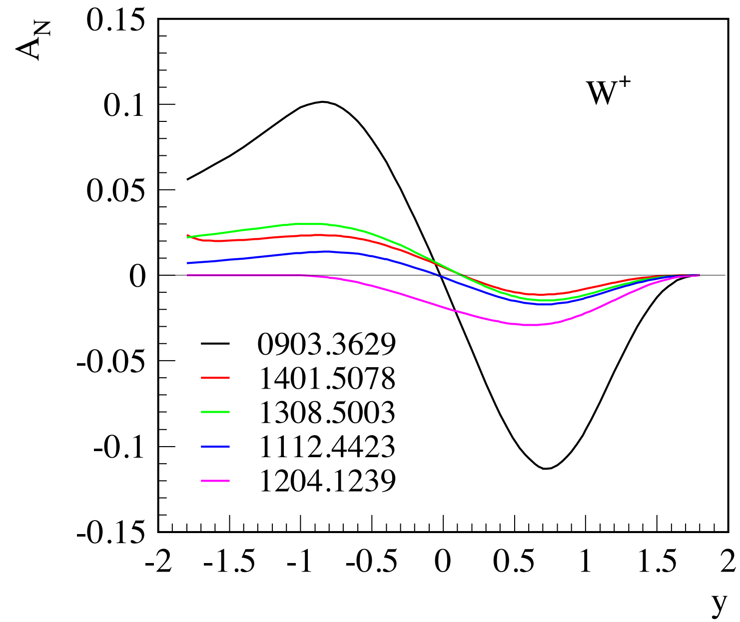
but, spin-averaged Boer-Mulder's TMD requires the sign change!

Same PT symmetry examination needs for TMD gluon distributions!

"Predictions" for A_N of W-production at RHIC?

- **Sivers Effect:**
  - Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons
  - QCD Prediction: Sign change of Sivers function from SIDIS and DY
- **Current "prediction" and uncertainty of QCD evolution:**

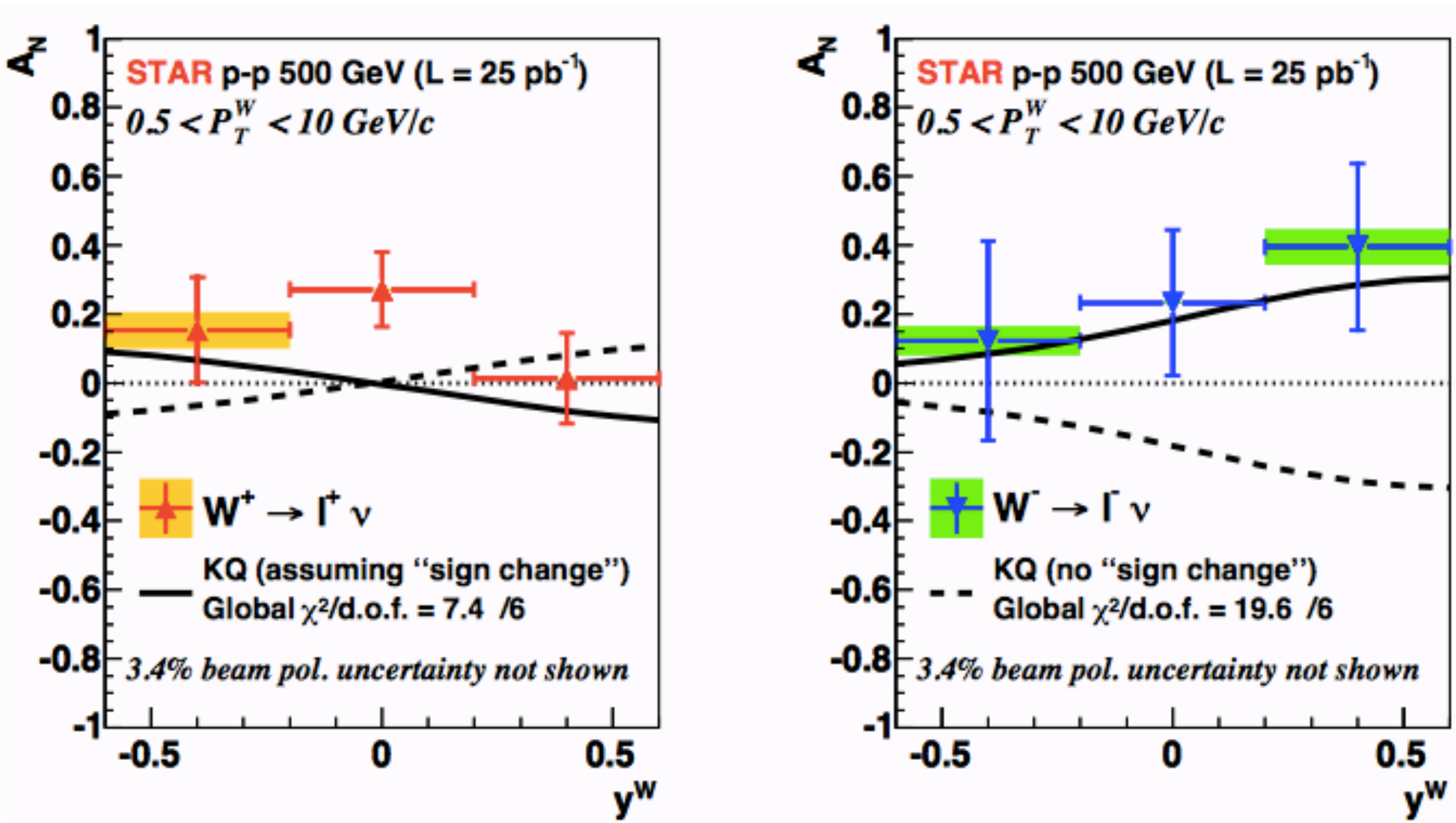




TMD collaboration proposal: Lattice, theory & Phenomenology

RHIC is the excellent and unique facility to test this (W/Z – DY)!

Hint of the sign change: A_N of W production



Data from STAR collaboration on A_N for W-production are consistent with a sign change between SIDIS and DY

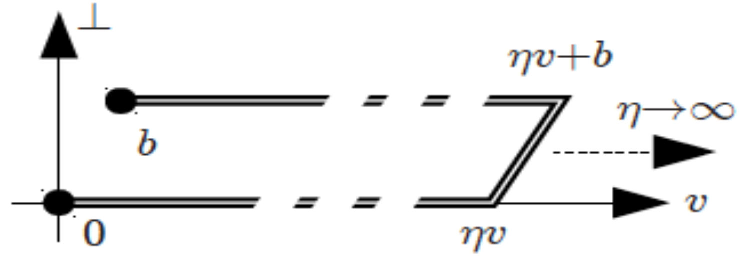
STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)

Hint of the TMD sign change from lattice QCD

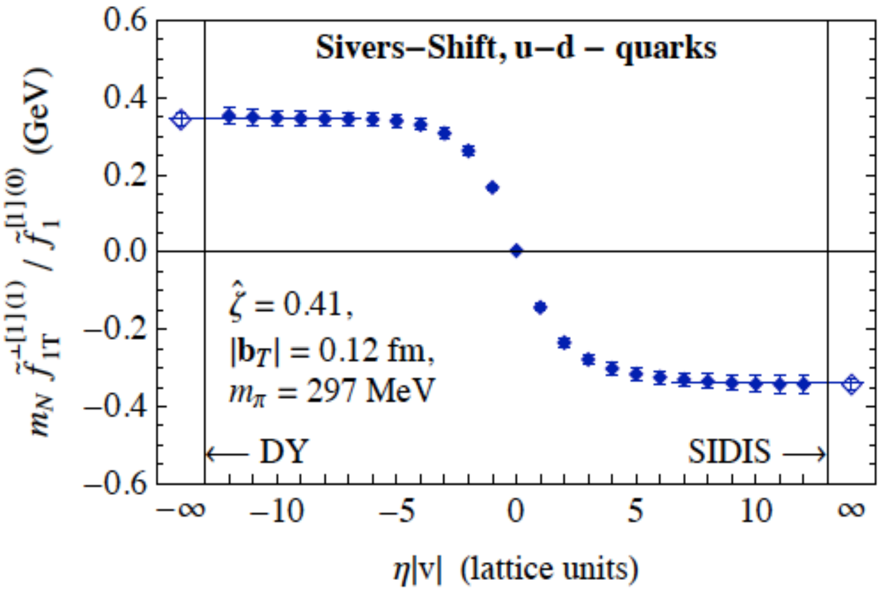
M. Engelhardt

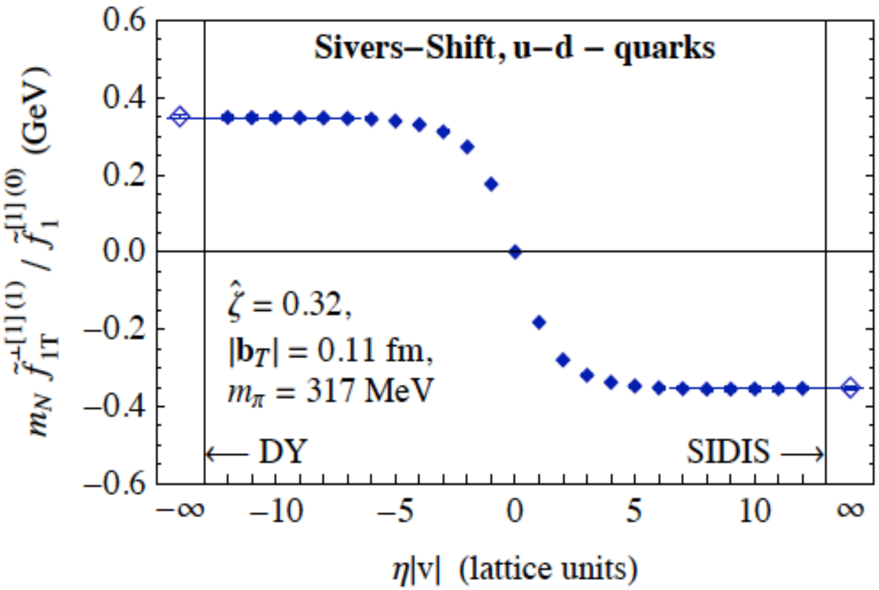
- Gauge link for lattice calculation:

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$



- Normalized moment of Sivers function – at given b_T:





Hint of the TMD sign change from lattice QCD

M. Engelhardt

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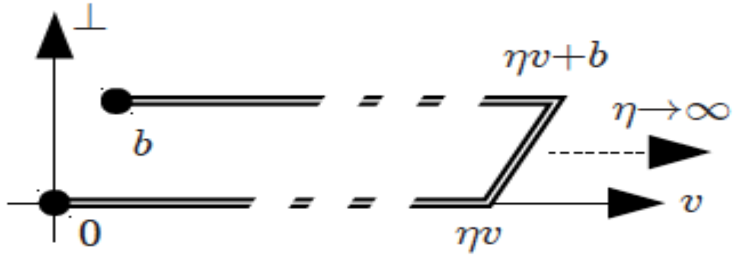
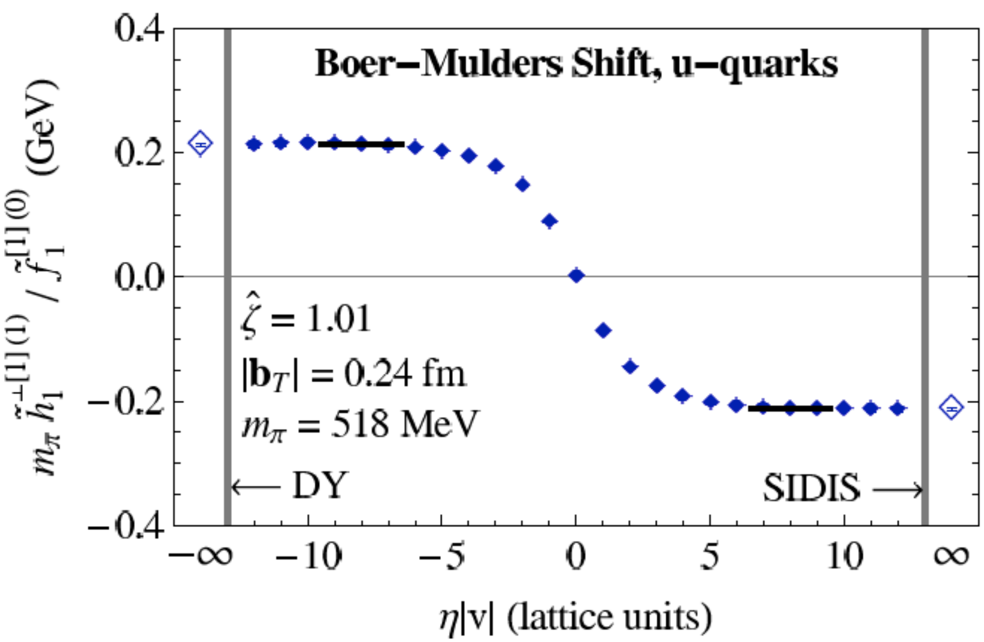


- Normalized moment of Boer-Mulders function – at given b_T:



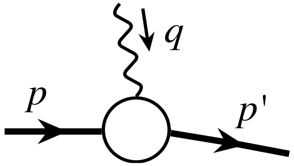
Proton’s radius in color distribution?

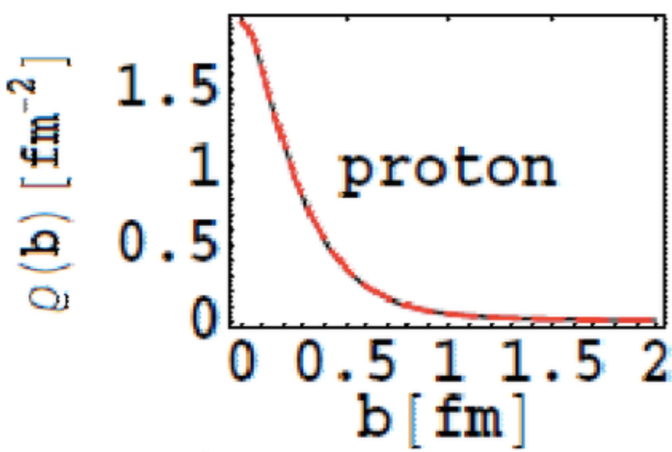
- The “big” question:

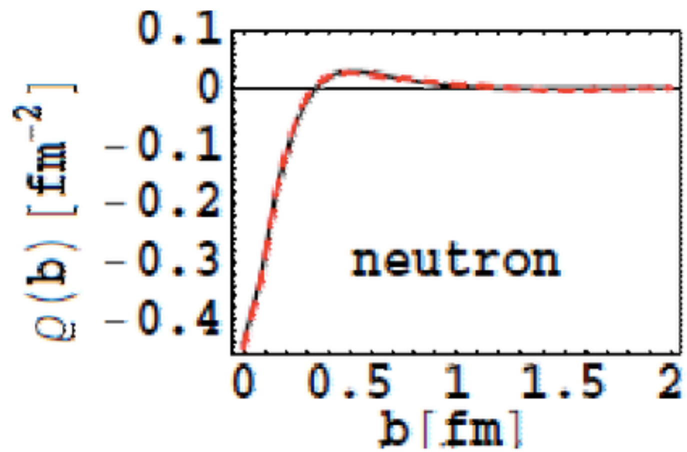
  How color is distributed inside a hadron? (clue for color confinement?)

- Electric charge distribution:

  Elastic electric form factor ➡ Charge distributions

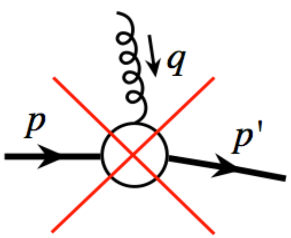






- But, NO color elastic nucleon form factor!

  Hadron is colorless and gluon carries color



➡ *Parton density’s spatial distributions – a function of x as well (more “proton”-like than “neutron”-like?) – GPDs*

GPDs – its role in solving the spin puzzle

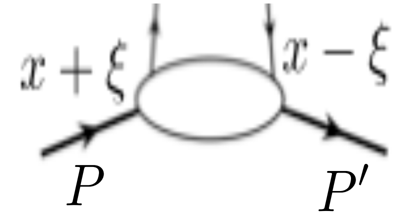
- **Quark "form factor":**

Exchanging vacuum Q.#.
"Density current"

$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P' | \bar{\psi}_q(\lambda/2) \frac{\gamma\cdot n}{2P\cdot n} \psi_q(-\lambda/2) | P \rangle$$

$$\equiv \; H_q(x,\xi,t,\mu^2)\, [\bar{\mathcal{U}}(P')\gamma^\mu \mathcal{U}(P)]\, \frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2) \left[\bar{\mathcal{U}}(P') \frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M} \mathcal{U}(P)\right] \frac{n_\mu}{2P\cdot n}$$



with $\xi = (P'-P)\cdot n/2$ and $t = (P'-P)^2 \;\Rightarrow\; -\Delta_\perp^2$ if $\xi \to 0$

$\tilde{H}_q(x,\xi,t,Q)$, $\tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

- **Total quark's orbital contribution to proton's spin:** Ji, PRL78, 1997

$$J_q = \frac{1}{2} \lim_{t\to 0} \int dx\, x\, [H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$= \frac{1}{2}\Delta q + L_q$$

- **Connection to normal quark distribution:**

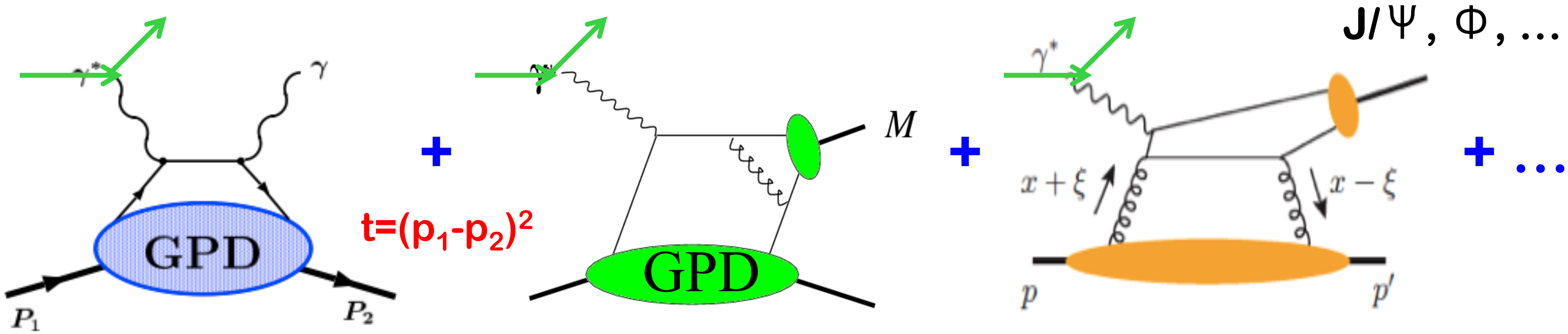
$H_q(x,0,0,\mu^2) = q(x,\mu^2)$ **The limit when** $\xi \to 0$

Exclusive DIS: Hunting for GPDs

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

- ❑ **Experimental access to GPDs:**
  - ✧ **Diffractive exclusive processes – high luminosity:**

    DVCS: Deeply virtual Compton Scattering
    DVEM: Deeply virtual exclusive meson production



DVCS: $Q^2 >> |t|$ DVEM: $Q^2 >> |t|$ EHMP: $Q^2 >> |t|$

  - ✧ **No factorization for hadronic diffractive processes – EIC is ideal**

- ❑ **Much more complicated – (x, ξ, t) variables:**

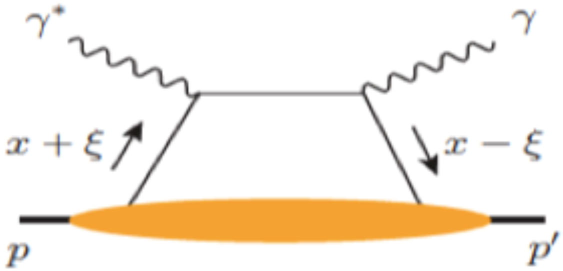
  Challenge to derive GPDs from data

- ❑ **Great experimental effort:**

  HERA, HERMES, COMPASS, JLab → JLab12, COMPASS-II, EIC

Deep virtual Compton scattering

The LO diagram:



$$\xi = Q^2/(2\bar{P}\cdot q)$$
$$P' = P + \Delta$$

Scattering amplitude:

$$T^{\mu\nu}(P,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int dx\left(\frac{1}{x-\xi/2+i\epsilon}+\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}U(P) + E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta\int dx\left(\frac{1}{x-\xi/2+i\epsilon}-\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}\gamma_5U(P) + \tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5U(P)\right]$$

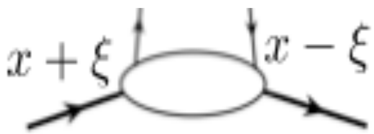
GPDs:

$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P)$$
$$+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P)+\ldots$$

$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5U(P)$$
$$+\tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P)+\ldots$$

What can GPDs tell us?

- **GPDs of quarks and gluons:**

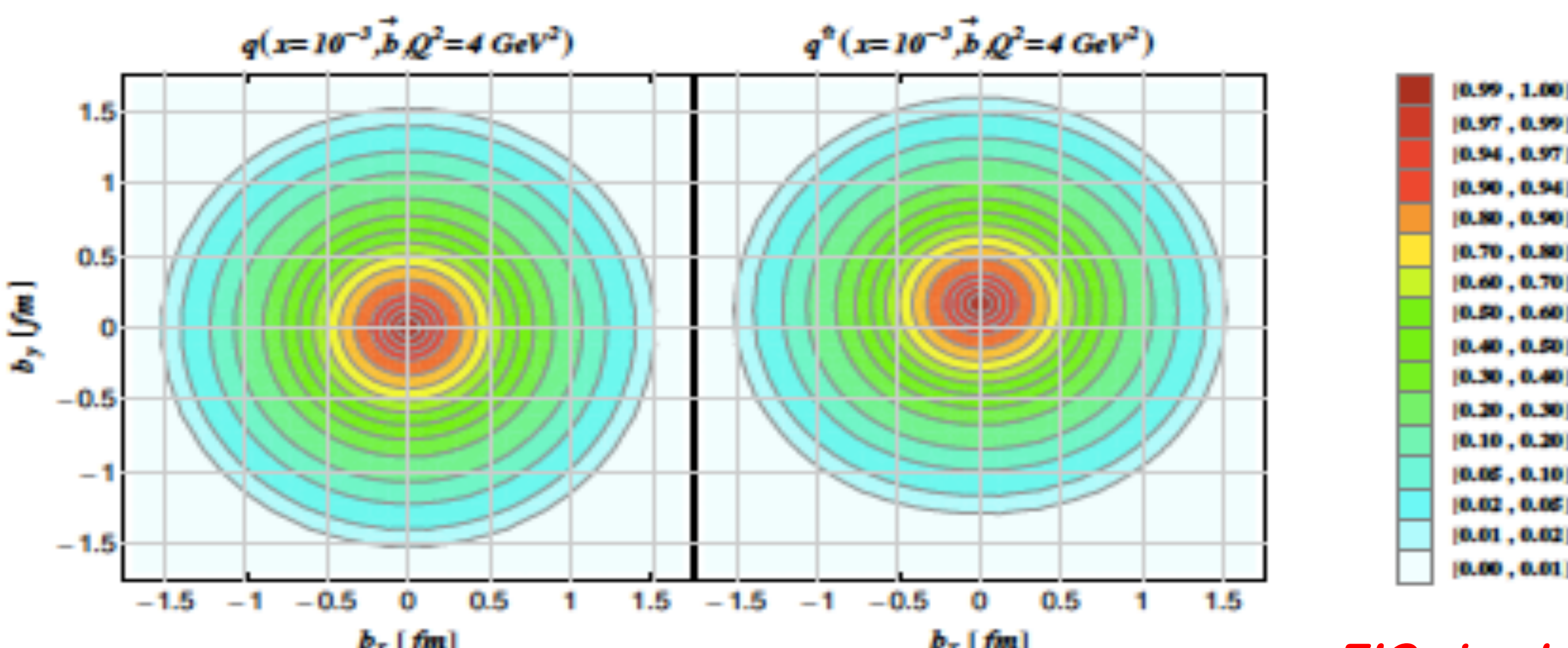


$$H_q(x,\xi,t,Q), \quad E_q(x,\xi,t,Q),$$
$$\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$$

Evolution in Q – gluon GPDs

- **Imaging ($\xi \to 0$):** $q(x,b_\perp,Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp\cdot b_\perp} H_q(x,\xi=0,t=-\Delta_\perp^2,Q)$

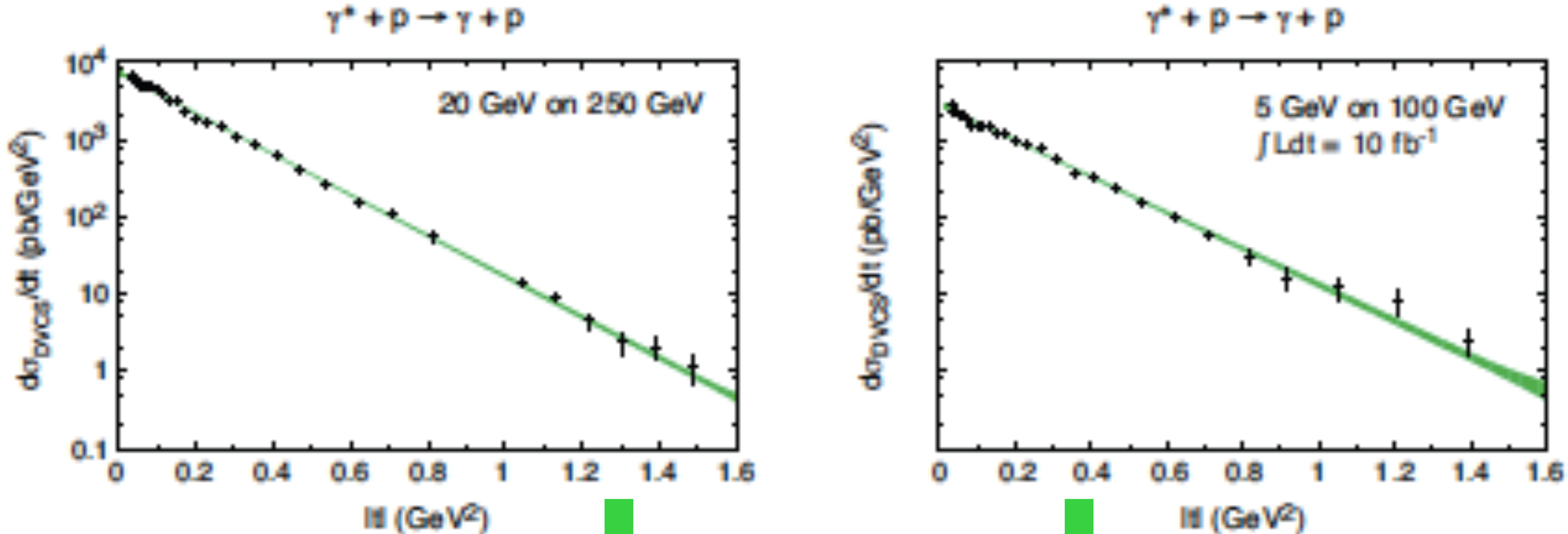
- **Influence of transverse polarization – shift in density:**



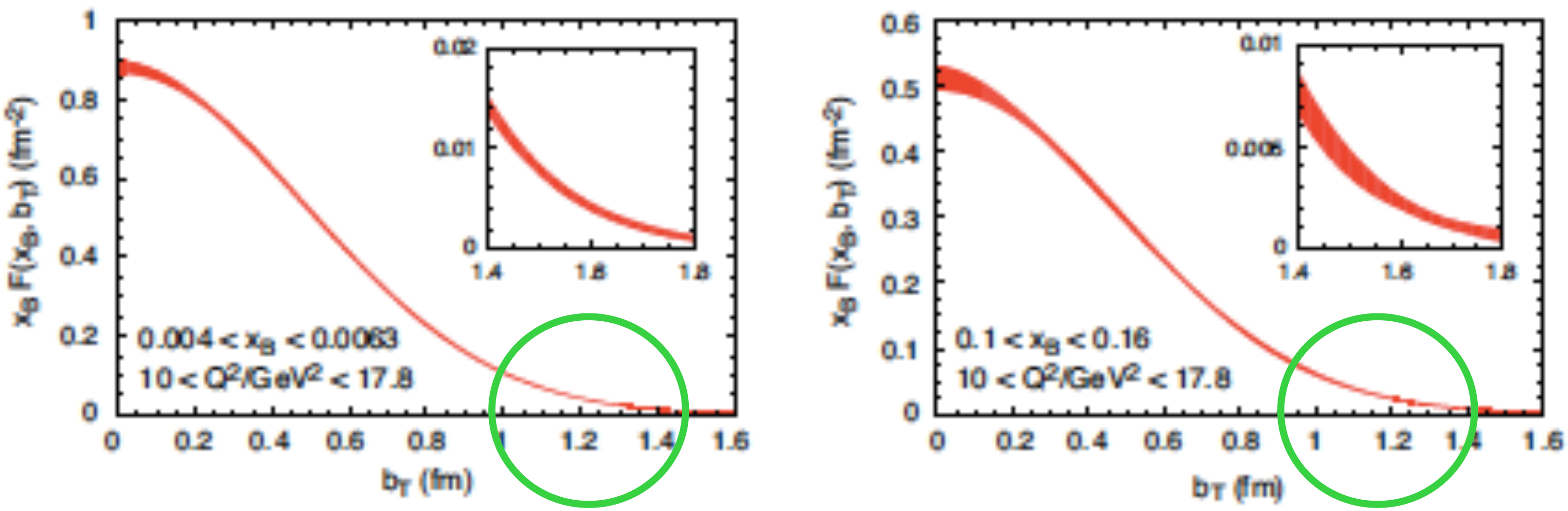
EIC simulation

DVCS @ EIC

- Cross Sections:



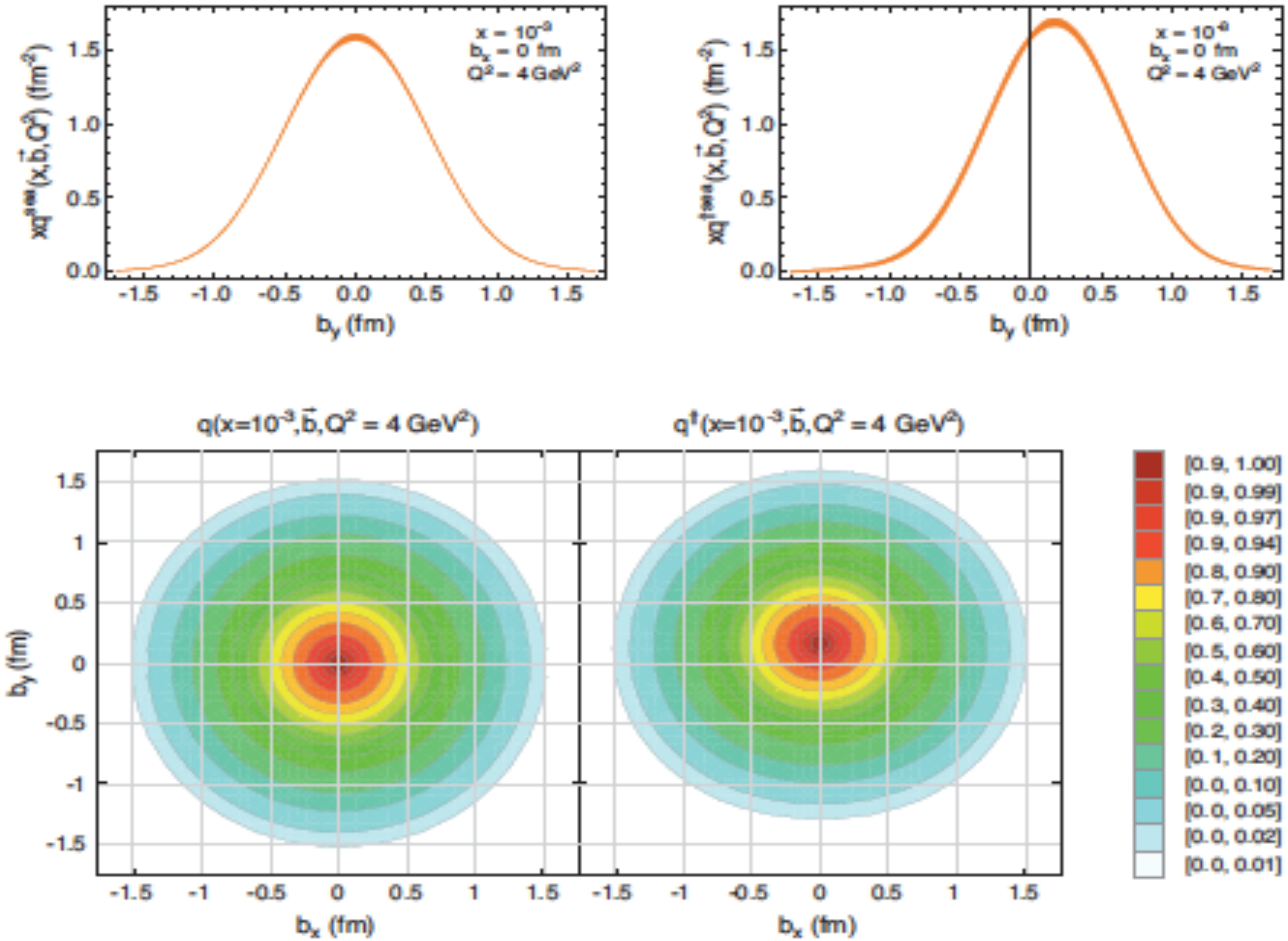
- Spatial distributions:



Proton radius of quark density (x)!

Polarized DVCS @ EIC

❑ **Spin-motion correlation:**

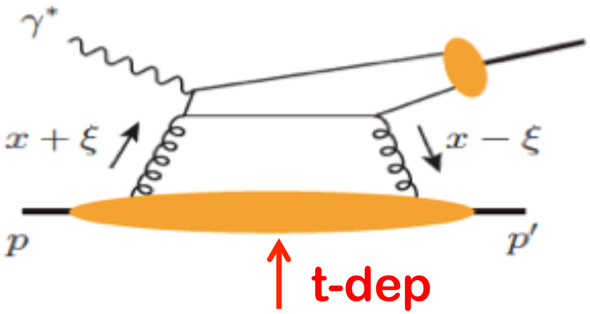


Spatial distribution of gluons

- ❑ **Exclusive vector meson production:**

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



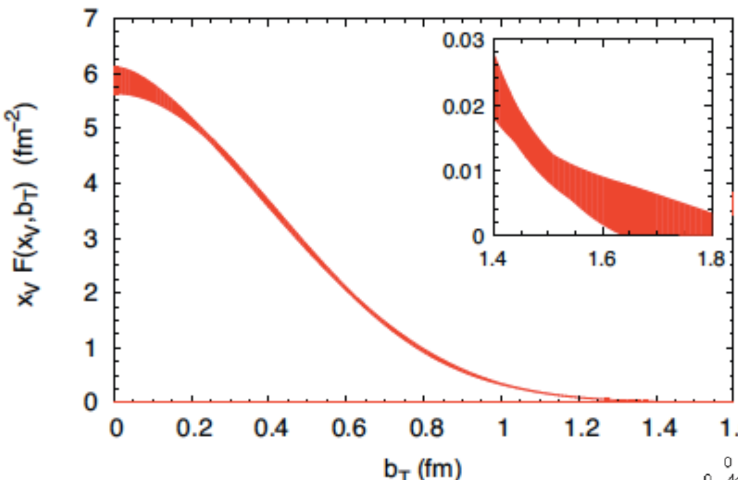
J/Ψ, Φ, …

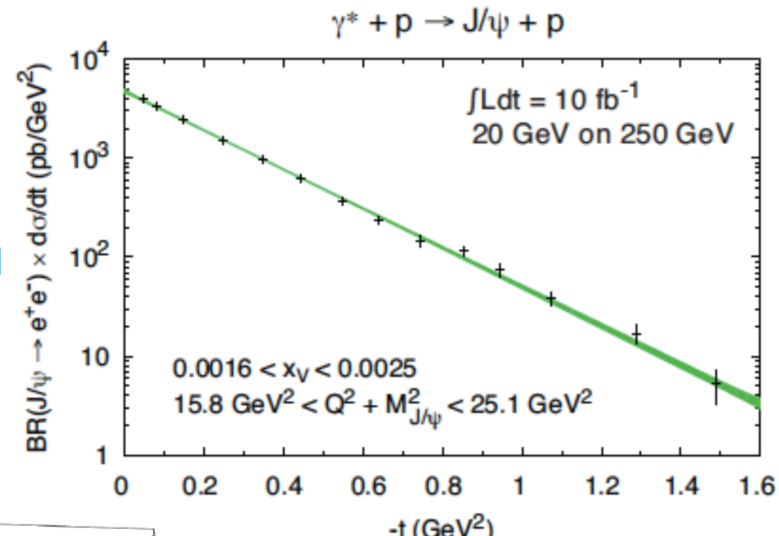
✧ Fourier transform of the t-dep

Spatial imaging of glue density

✧ Resolution ~ 1/Q or 1/M_Q

- ❑ **Gluon imaging from simulation:**

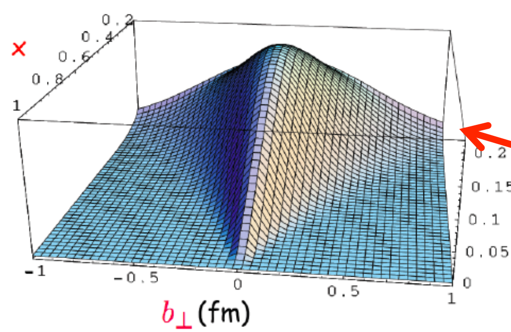




Only possible at the EIC

Proton radius of glue?

Radius as a function of x!



How spread at small-x?

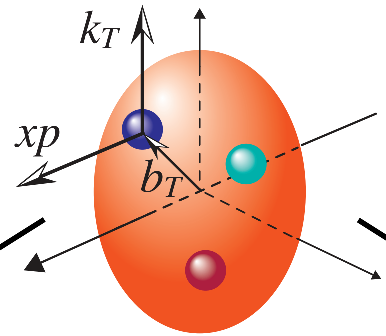
Color confinement

Unified view of nucleon structure

❑ Wigner distributions:

Momentum Space

Coordinate Space

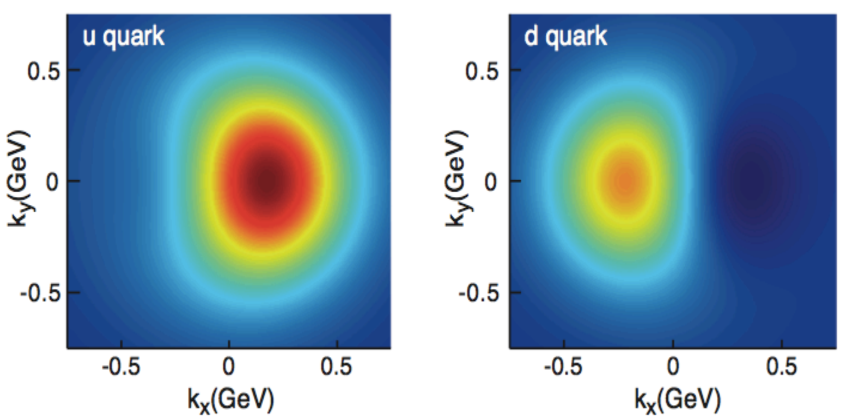


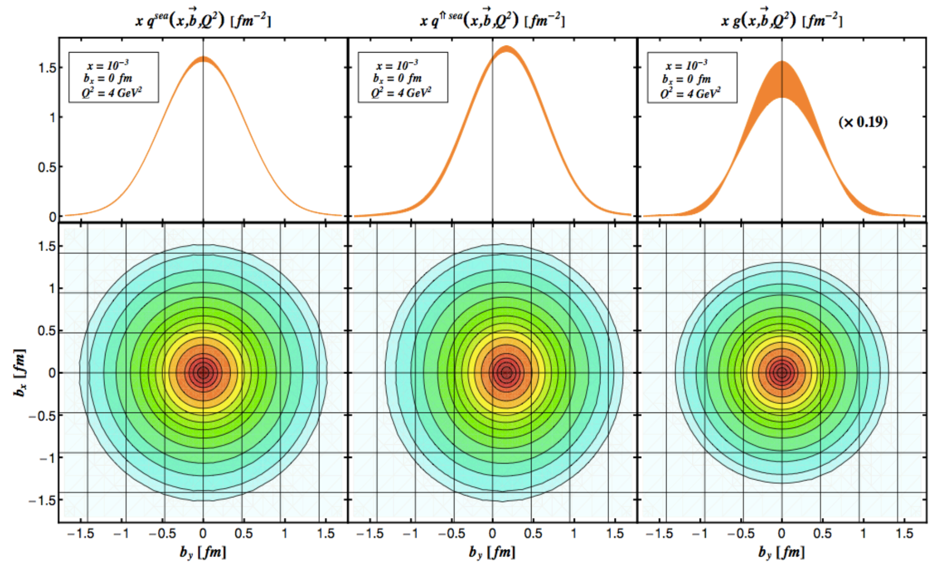
$\int d^2b_T$ $\quad$ $\int d^2k_T$

TMDs $\quad$ *GPDs*

Confined motion $\quad$ $f(x,k_T)$ $\quad$ Two-scales observables $\quad$ $f(x,b_T)$ $\quad$ Spatial distribution

Sivers Functions





Position r x Momentum p → Orbital Motion of Partons

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Lorce, Pasquini, …

✧ compensated by difference between gluon OAM density

✧ represented by different choice of gauge link for OAM Wagner distribution

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x,\vec{b},\vec{k}_T) \left\{W_q(x,\vec{b},\vec{k}_T)\right\}$$

with

$$\mathcal{W}_q \left\{W_q\right\}(x,\vec{b},\vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$

$$\times \langle P'|\, \overline{\psi}_q(0)\, \frac{\gamma^+}{2} \underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}} \psi(y)\, |P\rangle_{y^+=0}$$

JM: "staple" gauge link

Ji: straight gauge link

between 0 and $y=(y^+=0, y^-, y_T)$

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

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$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

  - ✧ generated by a "torque" of color Lorentz force

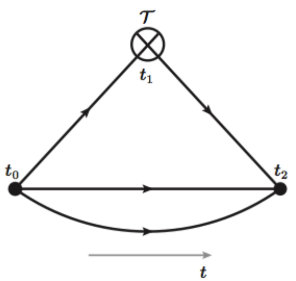
$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0)\frac{\gamma^+}{2}\int_{y^-}^{\infty} dz^- \Phi(0,z^-) \times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{"Chromodynamic torque"}} \Phi(z^-,y)\psi(y)|P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

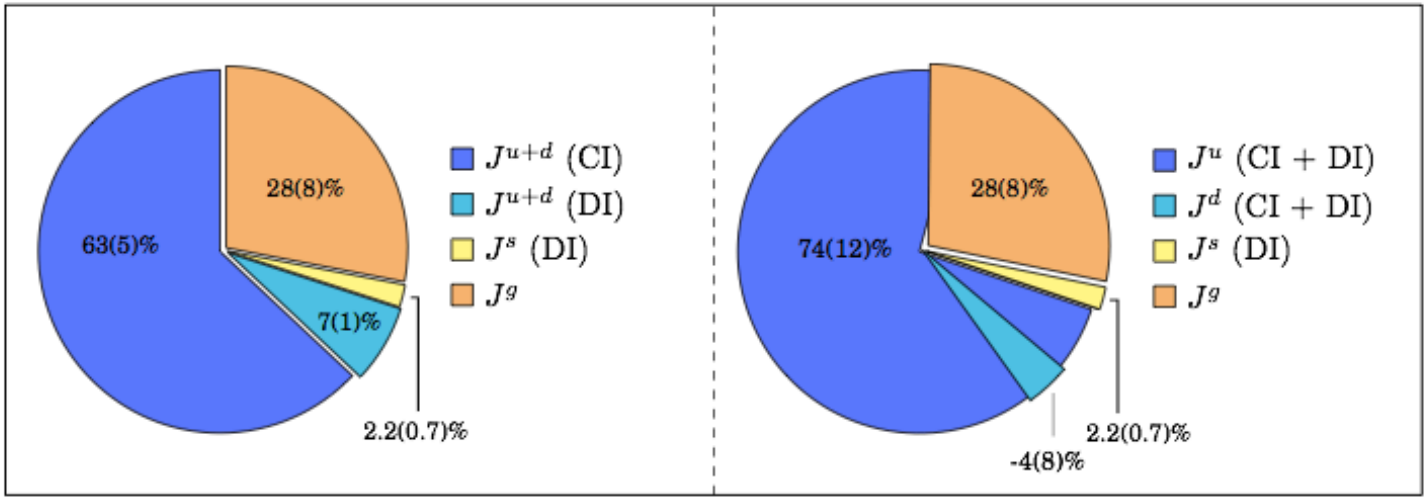
Nucleon spin and OAM from lattice QCD

❑ χQCD Collaboration:

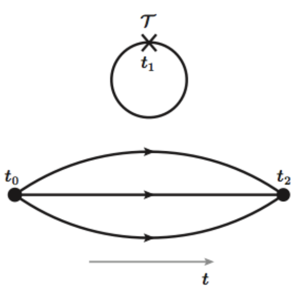
[Deka *et al.* arXiv:1312.4816]



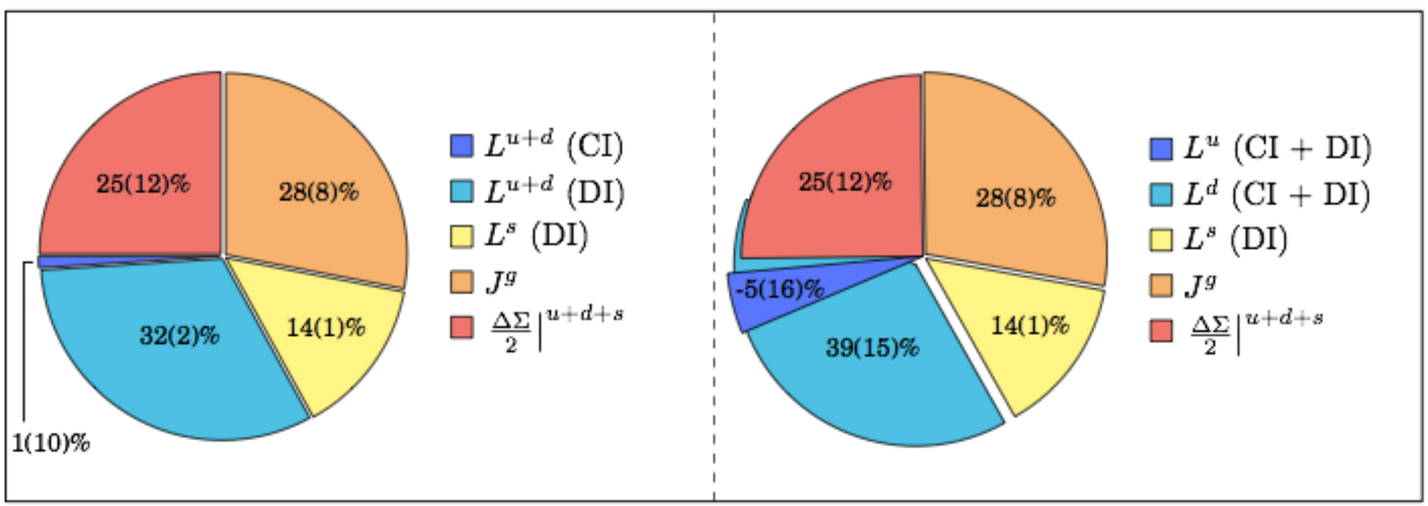
Connected Interaction (CI)



(b)



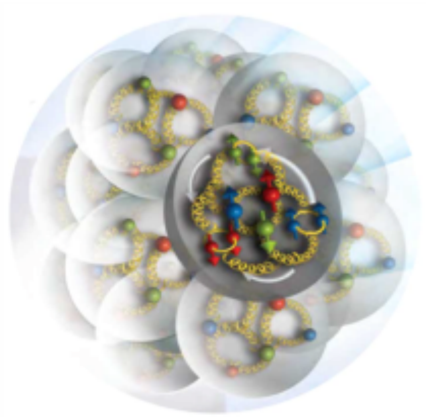
Disconnected Interaction (DI)

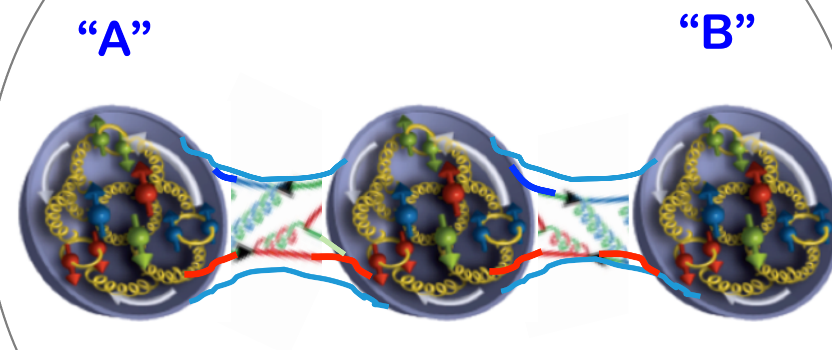


(c)

3D Imaging & origin of nuclear force

- **Nature of nuclear force:**





If we only see
quarks and gluons, …

What does the nucleus look like?

- **Range of color force:**

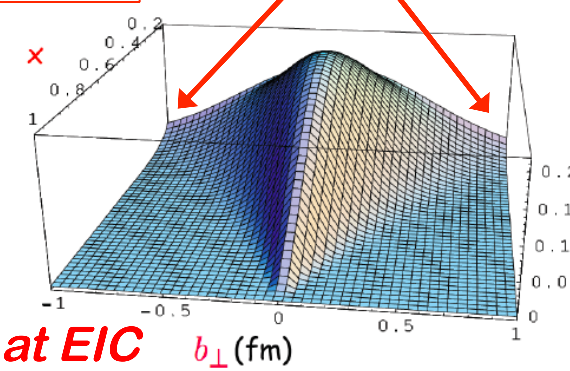
Does glue color of nucleon "A" correlated or entangled with glue color of nucleon "B"?

If it does, what is the strength of such correlation?

Can a large nucleus look like a big proton at small-x? the range of color correlation?

Imaging gluon density

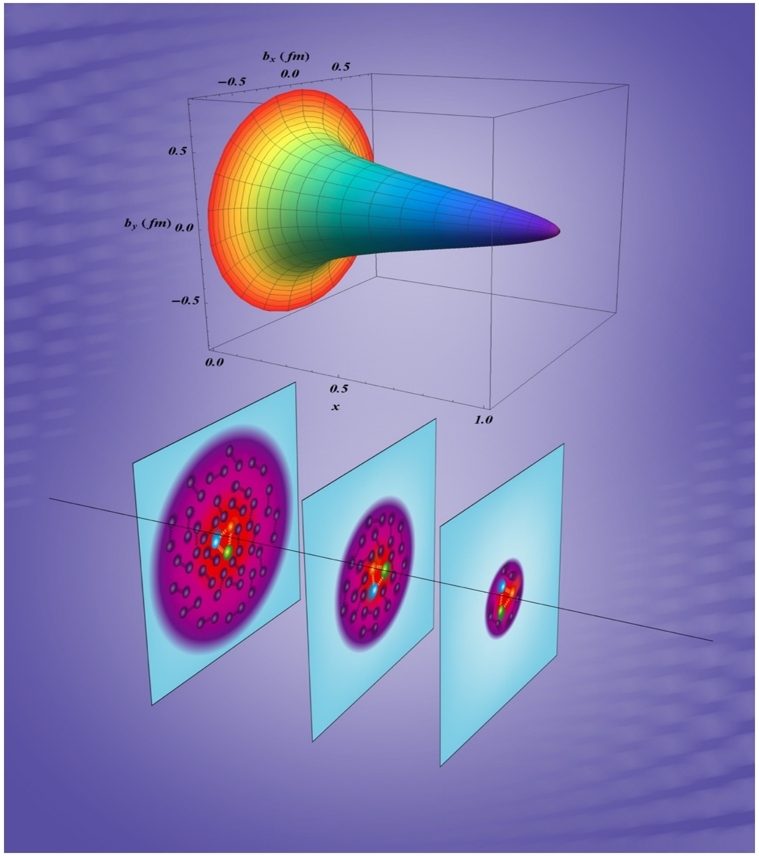
How far does glue density spread?



Only possible at EIC

Paradigm shift: 3D imaging of the "Proton"

- ❑ **This is transformational!**

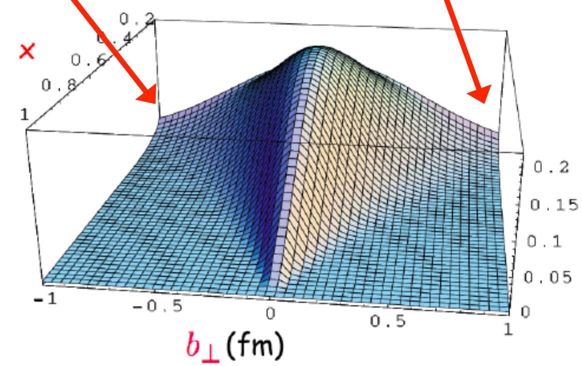


JLab12 – valence quarks,
EIC – sea quarks and gluons

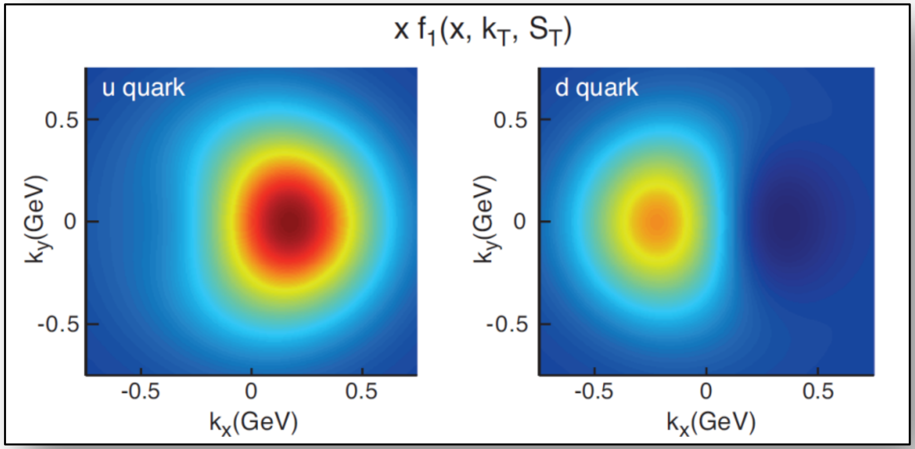
✧ **How color is confined?**

How far does glue density spread?

How fast does glue density fall?

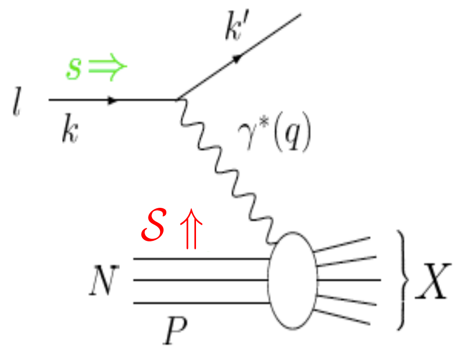


✧ **Why there is preference in motion?**



Homework (4)

1) 50 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance [N. Christ, T.D. Lee, Phys. Rev. 143, 1310 (1966)]



They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions.

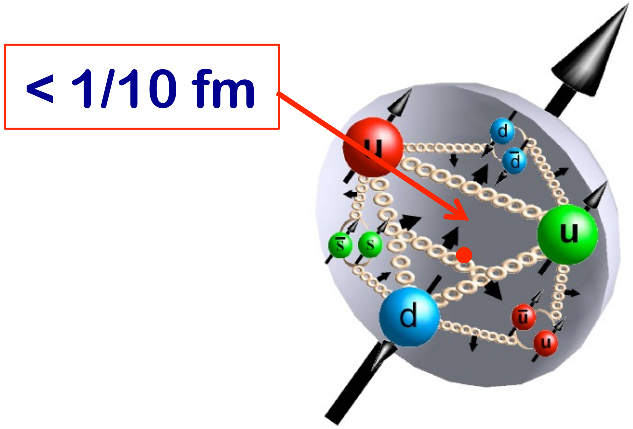
Use the parity and time-reversal invariance to prove that A_N = 0 for inclusive DIS.

Useful hints:

- ✧ Inclusive DIS is given by: $\sigma(\vec{s}_\perp) \propto L^{\mu\nu} W_{\mu\nu}(\vec{s}_\perp)$
- ✧ PT invariance: $\langle p, \vec{s}|\mathcal{O}(\psi, A^\mu)|p, \vec{s}\rangle = \langle p, -\vec{s}|\mathcal{PTO}^\dagger(\psi, A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p, -\vec{s}\rangle$

Summary

- QCD has been extremely successful in interpreting and predicting high energy experimental data!



- But, we still do not know much about hadron structure – work just started!
- Cross sections with large momentum transfer(s) and identified hadron(s) are the source of structure information
- QCD factorization is necessary for any controllable "probe" for hadron's quark-gluon structure!
- EIC is a ultimate QCD machine, and will provide answers to many of our questions on hadron structure, in particular, the confined transverse motions (TMDs), spatial distributions (GPDs), and multi-parton correlations, …

Thank you!

Backup slides

Some fundamentals about spin

❑ Spin in non-relativistic quantum mechanics:

✧ **Spin as an intrinsic angular momentum of the particle**

– **three spin vector:**

$$\vec{\mathcal{S}} = (\mathcal{S}_x\, ,\, \mathcal{S}_y\, ,\, \mathcal{S}_z\,)$$

– **angular momentum algebra:**

$$[\mathcal{S}_i\, ,\, \mathcal{S}_j\,] = i\epsilon_{ijk}\,\mathcal{S}_k \qquad\qquad \epsilon_{123} = +1$$

$$\Longrightarrow \qquad \left[\vec{\mathcal{S}}^{\,2},\, \mathcal{S}_j\right] = 0$$

✧ $\vec{\mathcal{S}}^{\,2},\, \mathcal{S}_z$ **Have set of simultaneous eigenvectors:** $|\, S,\, m\rangle$

$$\vec{\mathcal{S}}^{\,2}\,|\,S,\, m\rangle = S(S+1)\,\hbar^2\,|\,S,\, m\rangle \qquad S = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots$$

$$\mathcal{S}_z\,|\,S,\, m\rangle = m\,\hbar\,|\,S,\, m\rangle \qquad -S \le m \le S$$

✧ **Spin d.o.f. are decoupled from kinematic d.o.f.**

$$\Psi_{\mathsf{Schr}}(\vec{r}) \longrightarrow \Psi_{\mathsf{Schr}}(\vec{r}) \times \chi_m$$

where χ_m is a (2S+1) – component "spinor"

Some fundamentals about spin

- **Spin–1/2:**
  - ✧ **Two component spinors:** $$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$
  - ✧ **Operators could be represented by Pauli-matrices:**

$$\mathcal{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathcal{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \mathcal{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  - ✧ **Eigenstates to $\vec{\mathcal{S}}^{\,2}$ and $\mathcal{S}_z$:**

$$\chi_z^{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_z^{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

  - ✧ **Eigenvalues:**

$$\mathcal{S}_z\,\chi_z^{\uparrow} = +\frac{1}{2}\chi_z^{\uparrow} \qquad \mathcal{S}_z\,\chi_z^{\downarrow} = -\frac{1}{2}\chi_z^{\downarrow}$$

Particles in these states are "polarized in z-direction"

Some fundamentals about spin

- **General superposition:** $$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\,\chi_z^{\uparrow} + b\,\chi_z^{\downarrow} \qquad (\chi^{\dagger}\chi = 1)$$

$$\Rightarrow \quad \langle \mathcal{S}_z \rangle = \chi^{\dagger} \mathcal{S}_z \chi = \left(+\frac{1}{2}\right)|a|^2 + \left(-\frac{1}{2}\right)|b|^2 = \frac{1}{2}\left[|a|^2 - |b|^2\right]$$

- **Example:** $a = b = 1/\sqrt{2}$ $$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \langle \mathcal{S}_z \rangle = 0$$

- **Notice:** $$\langle \mathcal{S}_x \rangle = \chi^{\dagger} \mathcal{S}_x \chi = +\frac{1}{2}$$

- **Eigenstate to $\mathcal{S}_x$:** $$\chi_x^{\uparrow} = \frac{1}{\sqrt{2}}\left[\chi_z^{\uparrow} + \chi_z^{\downarrow}\right]$$

- **Arbitrary direction $\vec{n}$ with $|\vec{n}| = 1$:**

$$\mathcal{S}_n = \vec{n}\cdot\vec{\mathcal{S}} = n_x\mathcal{S}_x + n_y\mathcal{S}_y + n_z\mathcal{S}_z = \frac{1}{2}\begin{pmatrix} n_z & n_x - i n_y \\ n_x + i n_y & -n_z \end{pmatrix}$$

A state that is an eigenstate to this operator: "polarized in $\vec{n}$- direction"

$\vec{n}$ **= Polarization vector** **Eigenvalues =** $\pm 1/2$

Some fundamentals about spin

❑ Spin in the relativistic theory:

Physics is invariant under Lorentz transformation:
boost, rotations, and translations in space and time

✧ **Poincare group – 10 generators:** $\mathcal{P}^{\mu}\,, \quad \mathcal{M}^{\mu\nu}$

✧ **Pure rotations:** $\mathcal{J}_i = -\frac{1}{2}\epsilon_{ijk}\,\mathcal{M}^{jk}$, **pure boosts:** $\mathcal{K}_i = \mathcal{M}^{i0}$

Total angular momentum: $[\,\mathcal{J}_i\,, \mathcal{J}_j\,] = i\,\epsilon_{ijk}\,\mathcal{J}_k$

✧ **Two group invariants (fundamental observables):**

$$\mathcal{P}_\mu\,\mathcal{P}^\mu = \mathcal{P}^2 = m^2$$

$\mathcal{W}_\mu\,\mathcal{W}^\mu$ where $\mathcal{W}_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,\mathcal{M}^{\nu\rho}\,\mathcal{P}^\sigma$ Pauli-Lubanski

✧ **Fact:** $[\,\mathcal{W}_\mu\,, \mathcal{W}_\nu\,] = i\,\epsilon_{\mu\nu\rho\sigma}\,\mathcal{W}^\rho\,\mathcal{P}^\sigma \Longrightarrow [\,\mathcal{W}^i\,, \mathcal{W}^j\,] = i\,m\,\epsilon_{ijk}\,\mathcal{W}^k$

If acting on states at the rest

✧ **Spin:** $\mathcal{S}_i \equiv \frac{1}{m}\mathcal{W}^i = \mathcal{J}_i$

Note: $\mathcal{W}_\mu\,\mathcal{W}^\mu$ **has eigenvalues** $m^2\,\mathcal{S}\,(\mathcal{S}+1)$

Some fundamentals about spin

✧ **Recall: constructed eigenstates to $\vec{S}^2$ and $\vec{n}\cdot\vec{S}$:**

$$\mathcal{W}_\mu \mathcal{W}^\mu \,|p,\, S\rangle \;=\; m^2 S(S+1)\,|p,\, S\rangle \qquad S = \tfrac{1}{2}$$

$$-\frac{W\cdot n}{m}\,|p,\, S\rangle \;=\; \pm\frac{1}{2}\,|p,\, S\rangle \qquad W^\mu = \mathcal{W}^\mu\big|_{\mathrm{at\ \ rest}}$$

✧ **"Polarization operator":**

$$\mathcal{P} \equiv -\frac{W\cdot n}{m}$$

✧ **"Covariant polarization vector":** n^μ **with** $n^2 = -1, \quad n\cdot p = 0$

✧ **For Dirac particles:** $\mathcal{P} = \dfrac{1}{2}\gamma_5\,\gamma_\mu n^\mu$

✧ **Transverse polarization:**

➡ **Projection operators to project out the eigenstates of $\mathcal{P}$:**

$$\tfrac{1}{2}\,(\,\mathbb{1} \pm \gamma_5\,\not{n}\,)$$

✧ **Longitudinal polarization:** $\vec{n} = \vec{p}/|\vec{p}|\,, \quad n^0 = 0$

➡ $\mathcal{P} = \dfrac{1}{2}\gamma_5\gamma_\mu n^\mu = \dfrac{\vec{J}\cdot\vec{p}}{|\vec{p}|}$ **with eigenvalues** $\pm\dfrac{1}{2}$

$$\frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}\,u_\pm(p) \;=\; \pm\frac{1}{2}u_\pm(p) \;\equiv\; \frac{\lambda}{2}u_\pm(p) \qquad \rightsquigarrow\ \lambda \text{ "helicity"}$$

Massless particle: $\dfrac{\vec{J}\cdot\vec{p}}{|\vec{p}|} \to \gamma_5$ helicity = chirality

Some fundamentals about spin

✧ **Transverse polarization:** $n^\mu = (0, \vec{n}_\perp, 0)$ (for $\vec{p}$ in z direction)

➡ $\mathcal{P} = \gamma_0 \vec{J} \cdot \vec{n} = \gamma_0 \, J_\perp \neq J_\perp$

✧ **Transversity, not "transverse spin", has the eigenvalue:** $\pm\frac{1}{2}$

$$\gamma_0 \, J_\perp \, u_{\uparrow\downarrow}(p) = \pm\frac{1}{2} u_{\uparrow\downarrow}(p)$$

with spinors: $u_\uparrow^{(x)} = \frac{1}{\sqrt{2}}\left[u_+ + u_-\right]$

Same as in non-relativistic theory

➡ **Transverse polarization, or transversity, not "transverse spin", is invariant under the "boosts along $\vec{p}$ "**

✧ **Projection operator with both longitudinal and transverse components:**

$$\frac{1}{2}\not{p}\left[\, \mathbb{1} - s_{\|}\gamma_5 + \gamma_5 \not{s}_\perp \right]$$ **at high energy**

with $s_{\|} \sim \lambda, \quad s_\perp \sim n_\perp$

Some fundamentals about spin

❑ Back to Spin–1/2:

✧ **A free spin-1/2 particle obeys Dirac equation**

$$(\not{p} - m)\ u(p) = 0 \qquad \text{where } \not{p} = \gamma_\mu p^\mu$$

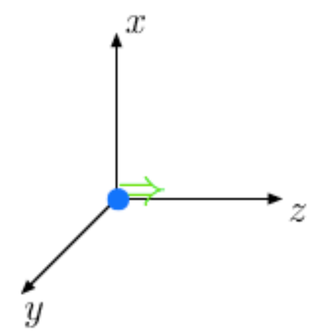
with 4-component solutions:

$$\Psi(x) = \begin{cases} e^{-i\,p\cdot x}\ u(p) & \text{positive energy} \rightarrow \text{particle} \\ e^{+i\,p\cdot x}\ v(p) & \text{negative energy} \rightarrow \textit{anti}\text{particle} \end{cases}$$

Each with "two" solutions: "spin up/down"

✧ **If it is at rest,**

$$u^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^- = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



They are eigenstates to the spin operator $\mathcal{S}_z$:

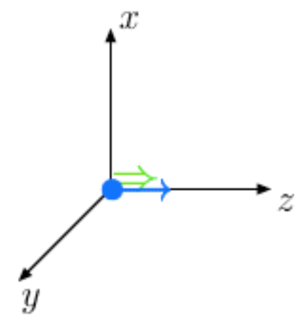
$$\mathcal{S}_z\, u^\pm = \pm\frac{1}{2}\, u^\pm$$

"polarized in z-direction"

Some fundamentals about spin

✧ **Boost the particle to momentum** $p = (E,\, 0,\, 0,\, p_z)$

$$u^{+} = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix} \qquad u^{-} = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_z}{E+m} \end{pmatrix}$$



✧ **Eigenstates of the helicity operator:**

$$\frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}\, u^{\pm} = \pm\frac{1}{2}\, u^{\pm}$$

✧ **Also eigenstates of the Pauli-Lubanski (polarization) operator:**

$$\frac{1}{2}\gamma_5 \not{n}\, u^{\pm} = \pm\frac{1}{2}\, u^{\pm}$$

where the polarization vector $n = (p_z,\, 0,\, 0,\, E)/m$

✧ **At high energy,** $E \approx p_z$ **also become eigenstates to chirality** γ_5 **:**

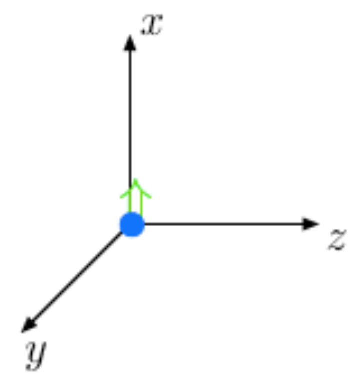
$$\gamma_5\, u^{\pm} = \pm\frac{1}{2}\, u^{\pm}$$

Some fundamentals about spin

❑ Back to rest frame:

✧ Construct eigenstates to the spin operator $\mathcal{S}_x$:

$$\mathcal{S}_x\, u^{\uparrow} = +\frac{1}{2}\, u^{\uparrow} \qquad \mathcal{S}_x\, u^{\downarrow} = -\frac{1}{2}\, u^{\downarrow}$$



with $u^{\uparrow} = \frac{1}{\sqrt{2}}\left[\, u^{+} + u^{-}\,\right] \qquad u^{\downarrow} = \frac{1}{\sqrt{2}}\left[\, u^{+} - u^{-}\,\right]$

"polarized along x-direction"

✧ Boost the particle to momentum $p = (E,\, 0,\, 0,\, p_z)$

$$\Rightarrow \qquad u^{\uparrow} = \frac{N}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ \frac{p_z}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad u^{\downarrow} = \frac{N}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ \frac{p_z}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}$$

Still has

$u^{\uparrow} = (u^{+} + u^{-})/\sqrt{2}$

✧ Still the eigenstates of the Pauli-Lubanski (polarization) operator:

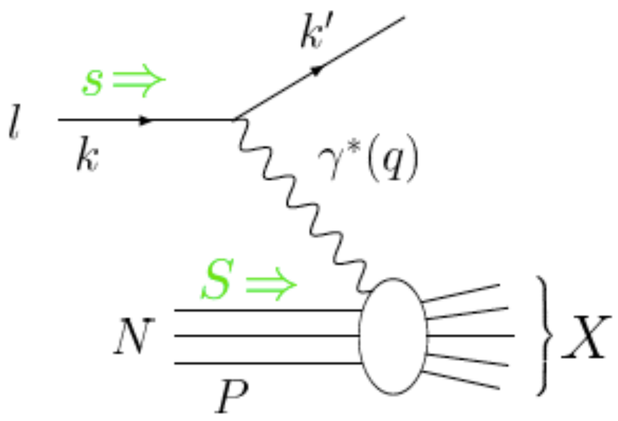
$$\frac{1}{2}\gamma_5\, \not{n}\, u^{\uparrow\downarrow} = \pm\frac{1}{2}\, u^{\uparrow\downarrow} \qquad \text{where} \quad n = (0,\, 1,\, 0,\, 0)$$

✧ But, no longer eigenstates of the transverse-spin operator:

$$\mathcal{S}_x\, u^{\uparrow} \neq +\frac{1}{2}\, u^{\uparrow}$$

Polarized deep inelastic scattering

❑ DIS with polarized beam(s):



"Resolution" $Q \equiv \sqrt{-q^2}$

$$\frac{\hbar}{Q} = \frac{2\times 10^{-16}\mathrm{m}}{Q/\mathrm{GeV}} \lesssim 10^{-16}\mathrm{m} = 1/10\mathrm{fm}$$

"Inelasticity" – known as Bjorken variable

$$x_B = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ **Recall – from lecture 2:**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

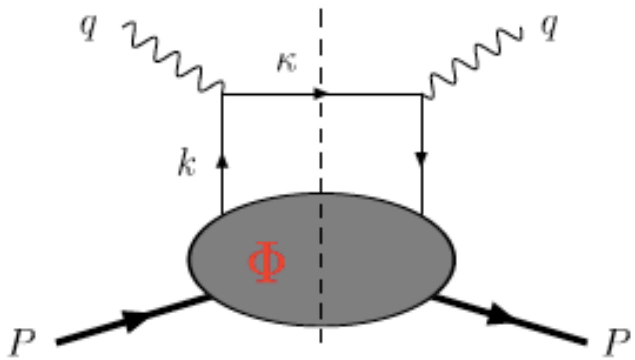
$$+ iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B, Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2} g_2\left(x_B, Q^2\right)\right]$$

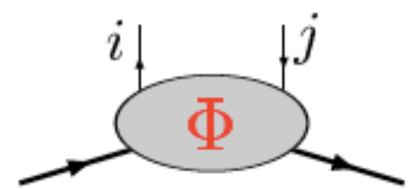
✧ **Polarized structure functions:**

$$g_1(x_B, Q^2),\ \ g_2(x_B, Q^2)$$

Polarized deep inelastic scattering

❑ Systematics polarized PDFs – LO QCD:





✧ Two-quark correlator:

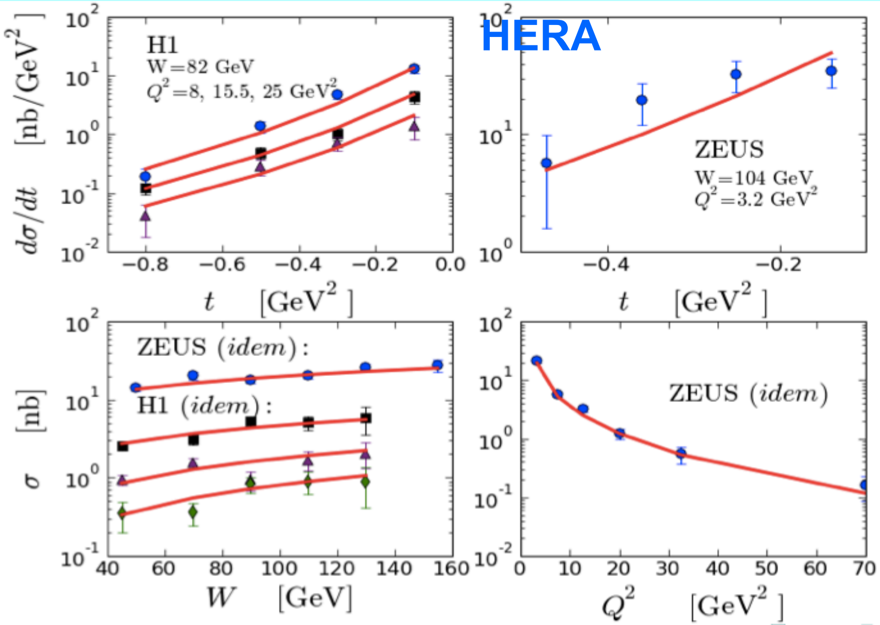
$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X)\, \langle PS|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|PS\rangle$$

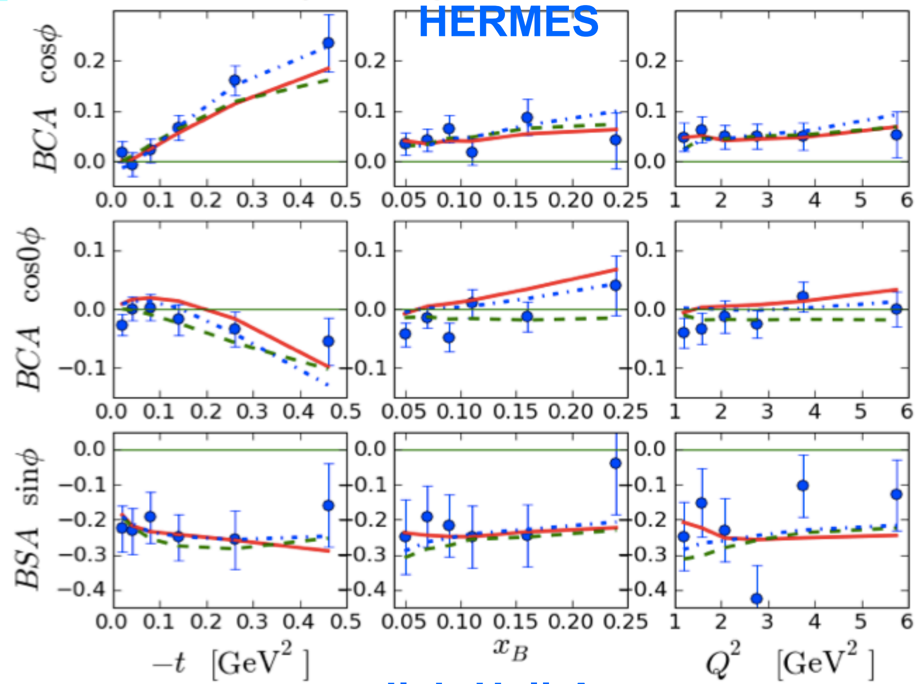
$$= \int d^4z\, e^{ik\cdot z} \langle PS|\, \bar{\psi}_j(0)\, \psi_i(z)\, |PS\rangle$$

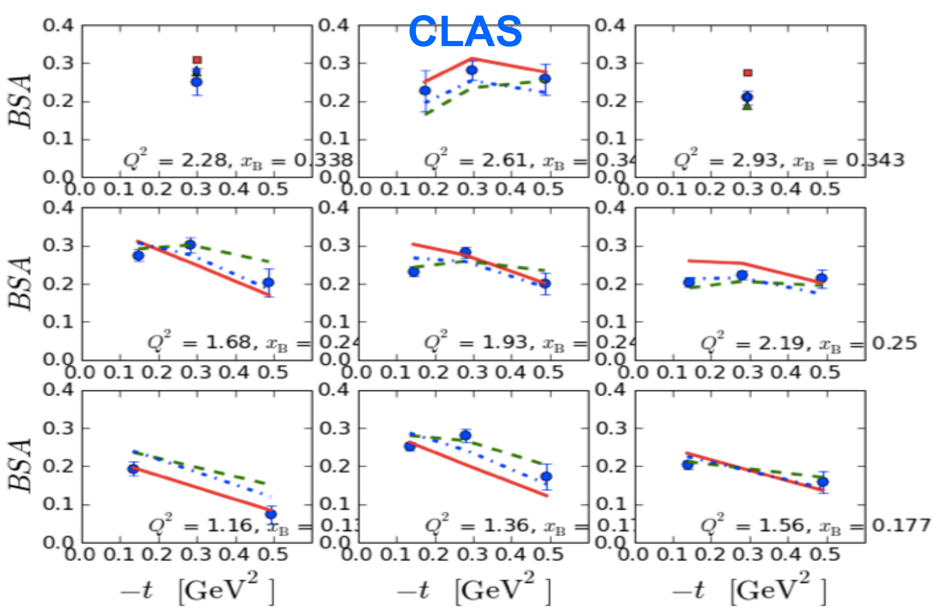
✧ Hadronic tensor (one –flavor):

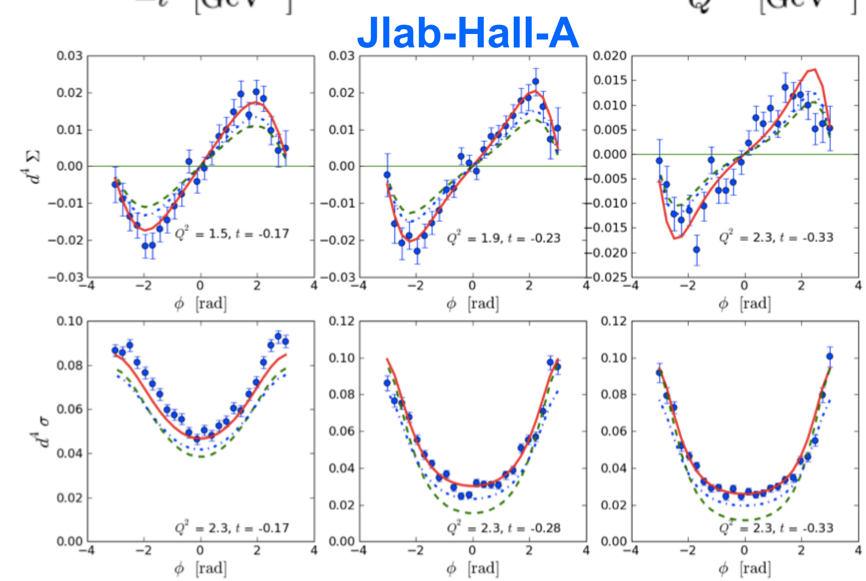
$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4}\, \delta\left((k+q)^2\right)\, \mathrm{Tr}\left[\Phi\, \gamma^\mu (\not{k}+\not{q})\gamma^\nu\right]$$

GPDs: just the beginning





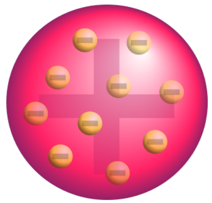




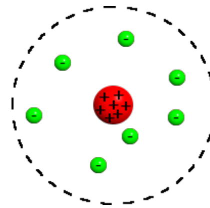
Why 3D hadron structure?

❑ Rutherford's experiment – atomic structure (100 years ago):

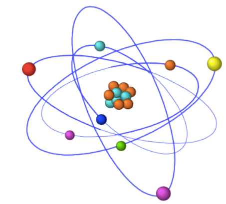
Atom:



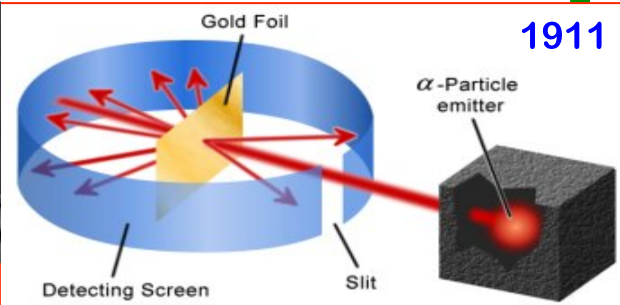
J.J. Thomson's
plum-pudding model



Rutherford's
planetary model



Modern model
Quantum orbitals



Discovery of nucleus
A localized
charge/force center
A vast
"open" space

Discovery of
Quantum Mechanics,
and
the Quantum World!

❑ Completely changed our "view" of the visible world:

- ✧ Mass by "tiny" nuclei – *less than 1 trillionth in volume of an atom*
- ✧ Motion by quantum probability – *the quantum world!*
- ✧ Provided infinite opportunities to improve things around us, …

What would we learn from the hadron structure in QCD, …?