Lecture Plan

Lecture 1: Hadrons as laboratory for QCD:

- **Introduction to QCD**
- **Bare vs effective effective quarks and gluons**
- **Phenomenology of Hadrons**

Lecture 2: Complex analysis

Lecture 3: Phenomenology of hadron reactions

- **Kinematics and observables**

Lecture 4: How to extract resonance information from the data

- **Partial waves and resonances**
- **Properties of reaction amplitudes**
- **Space time picture of Parton interactions and Regge phenomena**
- **Higher states and duality**

Relativistic S-matrix fundamentals

Causality: Determines domain of analyticity of reaction amplitudes as function of kinematical variables.

Unitarity: Determines singularities.

Crossing: Dynamical relation, aka reaction amplitudes in the exchange channel (forces) are analogous to amplitude in the direct channel (resonance)

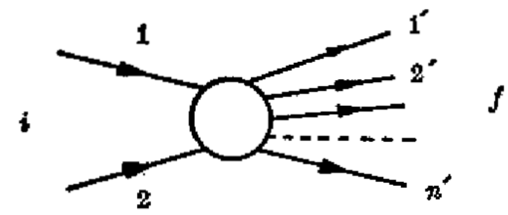
These principles constrain the amplitude on the physical sheet. But on the unphysical sheet, there poles and other singularities, i.e. triangle singularity brains points, that arise from the underlying dynamics. Thus in reality it is the unphysical sheet which is of interest.

Amplitude analysis = make hypothesis about these singularities and use analytical continuation to obtain the amplitude on the physical sheet where you fit to data.

S-matrix properties (in relativistic theory)

- Related to transition probability

$$P_{fi} = |\langle f|S|i\rangle|^2 = \langle i|S^\dagger|f\rangle\langle f|S|i\rangle$$



- Conservation of Probability = Unitarity

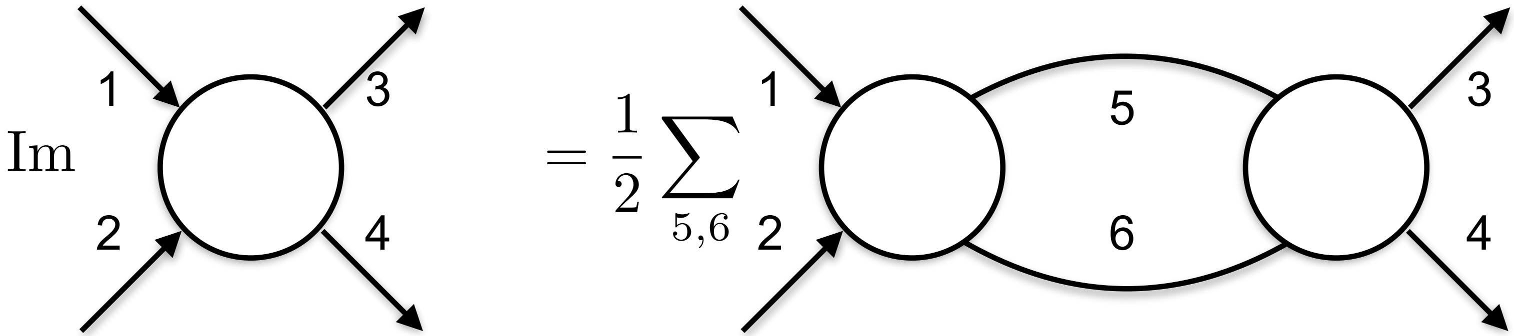
$$\sum_f P_{fi} = 1$$

$$S^\dagger S = I$$

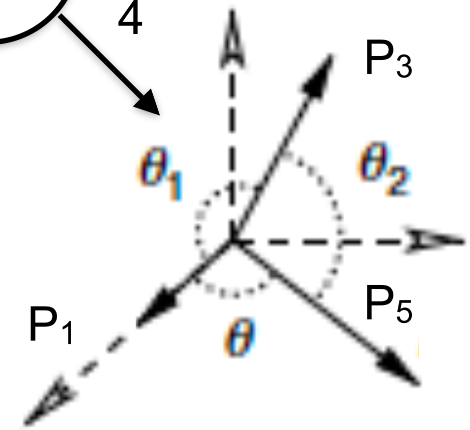
$$2ImT_{ft} = \sum_n 2\pi\delta(E_i - E_n)T^*_{fn}T_{ni}$$

How unitarity constrains singularities: simple example

$$2 Im T_{ft} = \sum_n 2\pi\delta(E_i - E_n) T^*_{fn} T_{ni}$$



Consider elastic scattering of spineless particles



$$Im A(s,t) = \frac{\rho(s)}{16\pi}\int \frac{d\Omega}{4\pi} A(s,\cos\theta_1) A^*(s,\cos\theta_2)$$

$$\rho(s) = 2k_{cm}(s)/\sqrt{s}$$

At fixed s, this is a complicated, integral relation w.r.t momentum transfer, t
It is simplified (diagonalized) by expanding A(s,t) in partial waves

$$A(s,t) = 16\pi \sum_{l=0}^{\infty} (2l+1) f_l(s) P_l(\cos\theta) \qquad Im f_l(s) = \rho(s)|f_l(s)|^2$$

How unitarity constrains singularities

Properties of the partial wave, $f_l(s)$ (for fixed l as function of s):

- $f_l(s)$ is real for s below threshold
- Im $f_l(s)$ is finite above threshold.
- $f_l(s)$ is analytical (since A(s,t) is)

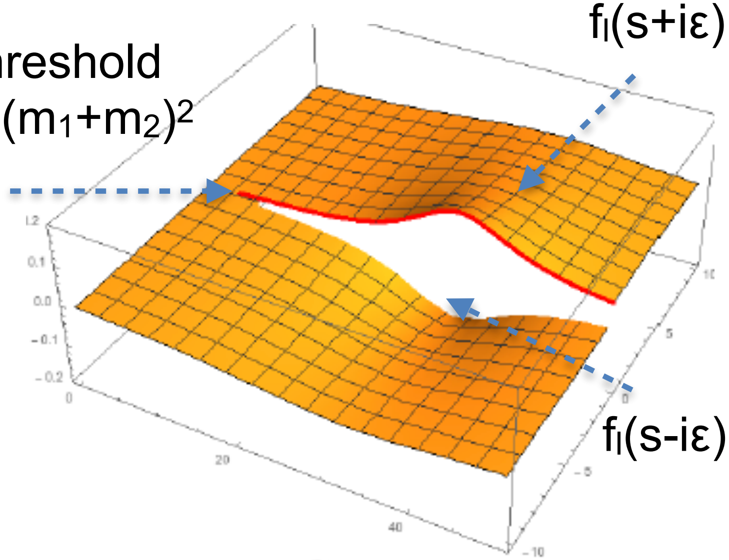
$$f_l(s) = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta P_l(\cos\theta) A(s, t(s,\cos\theta))$$

for simplicity ignore singularities in t

→ Reflection theorem (Calculus 101): $f_l(s^*) = f_l(s^*)$

Threshold
$s=(m_1+m_2)^2$

$f_l(s+i\varepsilon)$



$f_l(s-i\varepsilon)$

$$\frac{1}{2i}[f_l(s+i\epsilon) - f_l(s-i\epsilon)] = \rho(s) f_l(s+i\epsilon) f_l(s-i\epsilon)$$

Lets look for a function, $f_{lI}(s)$ that, for s-iε is equal to $f_l(s+i\varepsilon)$. Theorem of analytical continuation implies there is only one such function

Second sheet

$$f(s+i\epsilon) = \frac{f(s-i\epsilon)}{1-2i\rho(s)f(s-i\epsilon)}$$

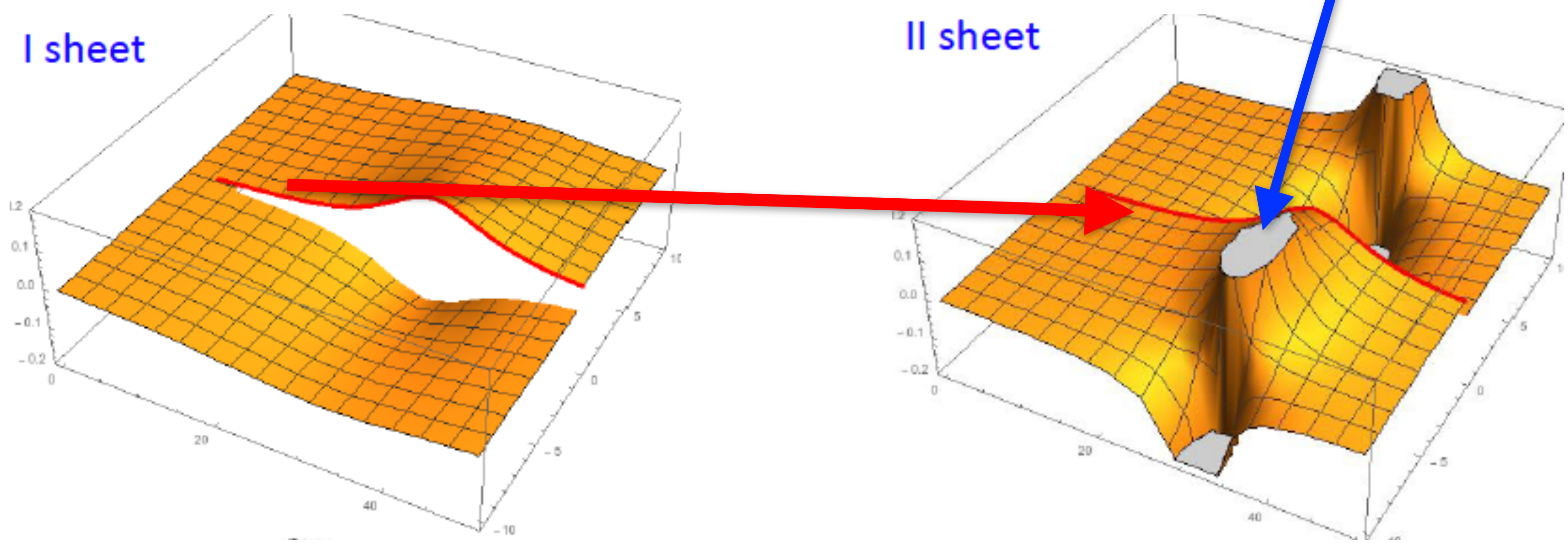
Singularity = Resonance at complex s when

$$f(s) = \frac{1}{2i\rho(s)}$$

Define for Im s < 0 $$f_{II}(s) = \frac{f(s)}{1-2i\rho(s)f(s)}$$

$$f_{II}(s-i\epsilon) = f(s+i\epsilon)$$

This is analytical continuation of f(s) below the real axis

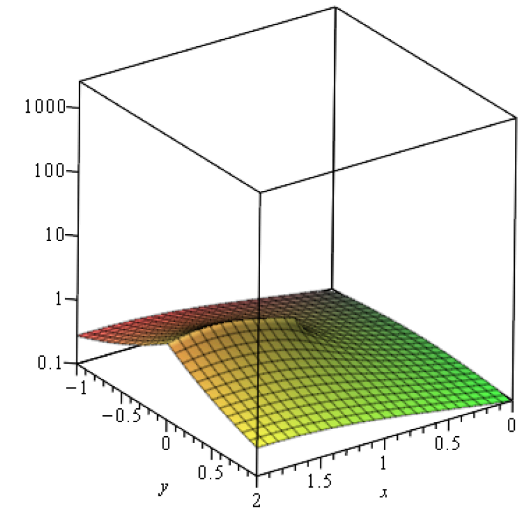


Breit-Wigner

$$f(s) = \frac{g^2\sqrt{s_{tr} - s}}{m^2 - s + g^2\sqrt{s_{tr} - s}}$$

when Im s < 0

$$m^2 - s + ig^2\sqrt{s - s_{tr}}$$

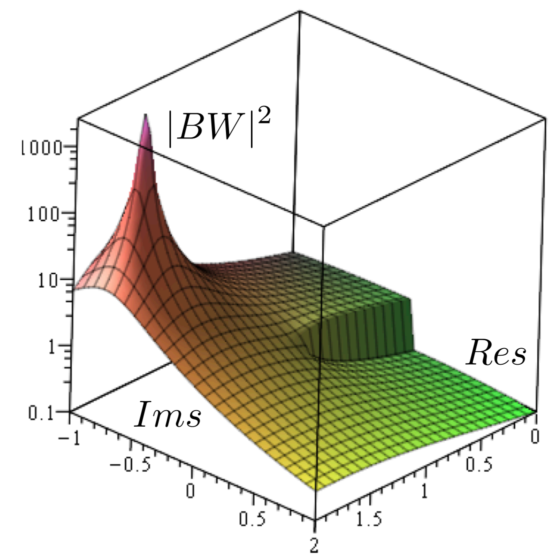


$$\rho(s) = \sqrt{s - s_{tr}}$$

$$f_{II}(s) = \frac{f(s)}{1 - 2i\rho(s)f(s)} = \frac{g^2\sqrt{s_{tr} - s}}{m^2 - s + g^2\sqrt{s_{tr} - s} - 2ig^2\sqrt{s - s_{tr}}}$$

when Im s < 0

$$= \frac{g^2\sqrt{s_{tr} - s}}{m^2 - s - ig^2\sqrt{s - s_{tr}}}$$



Phenomenology of hadron interaction

- Evidence for resonance scattering : connection to QCD bound states.
- Kinematical range for resonance scattering.
- Features of high energy scattering : physics of cross channels
- Space-time interpretation of high and low energy scattering
- Dual models

$$\sigma_{a+b\to a+b} \propto \int \frac{dt}{s^2} |A(s,t)|^2$$

$$\sigma_{a+b\to X} \propto \frac{ImA(s,0)}{s}$$ from unitarity

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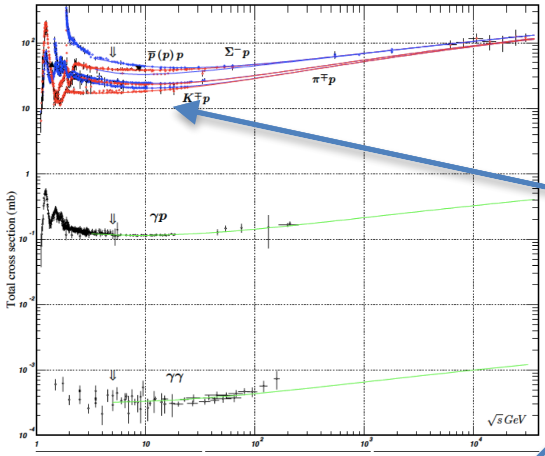
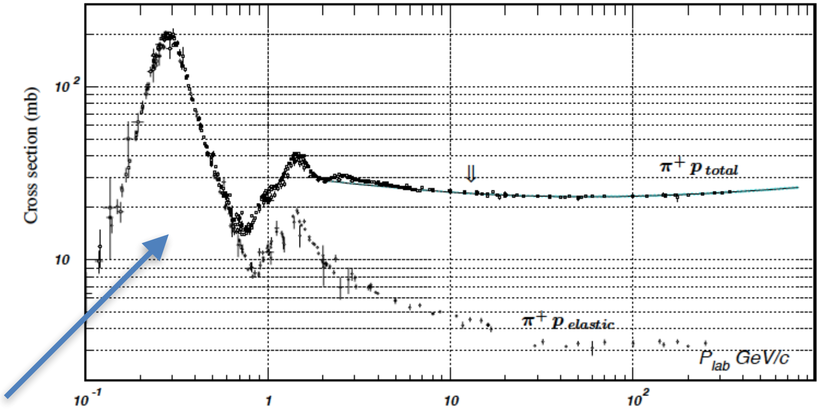
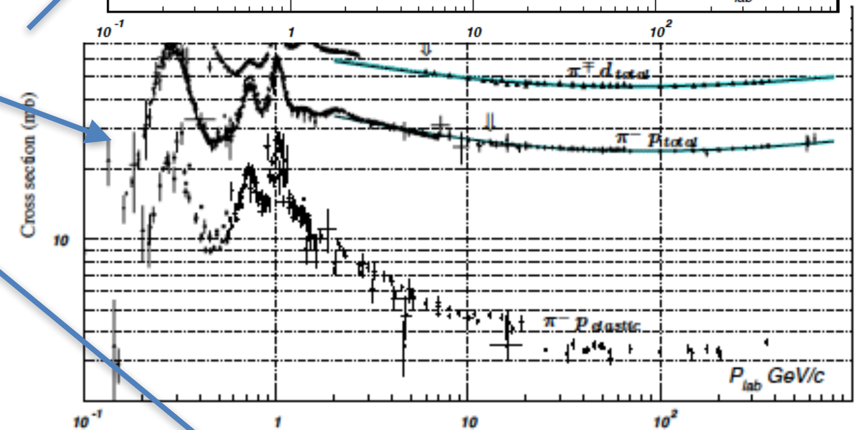
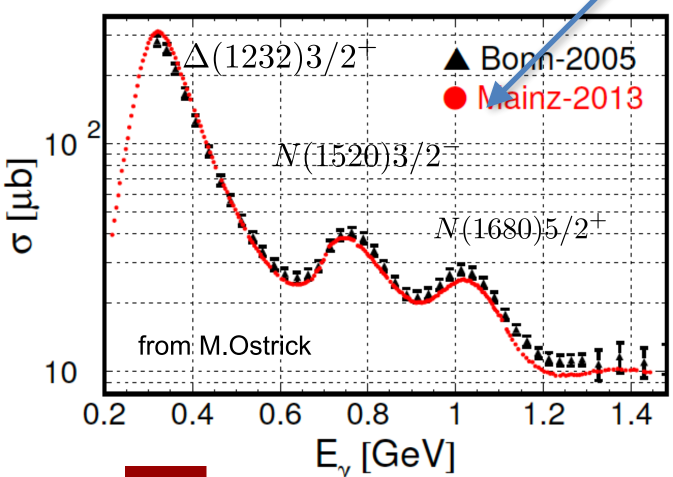


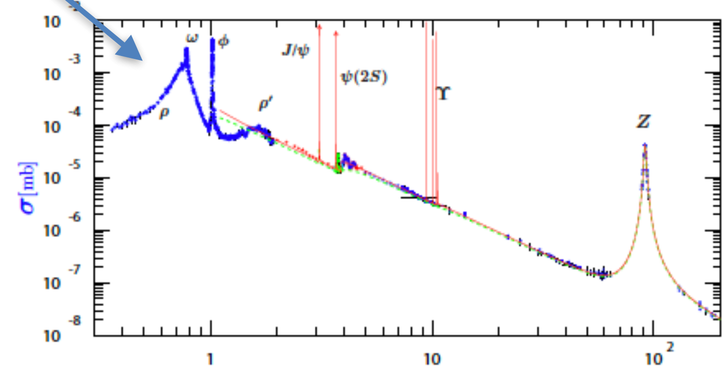


Resonance scattering





$$A(s,t) \sim \frac{1}{m_r^2 - s}$$

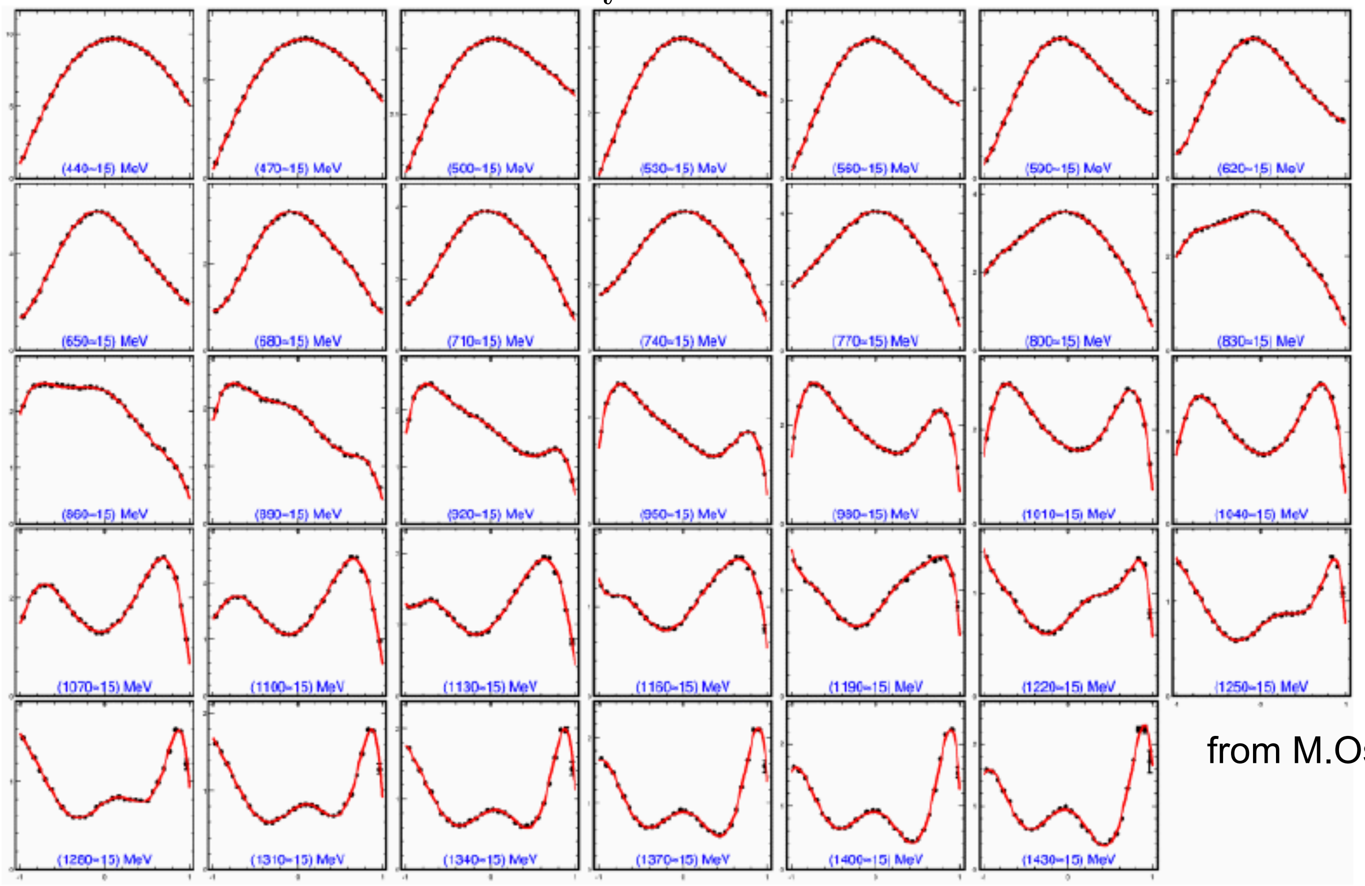


Resonance Scattering : look at angular distribution

$$\frac{d\sigma}{dt} \propto \frac{|A(s,t)|^2}{s^2}$$

Angular distribution: a few “wiggles”

$$A(s,t) = \sum_l (2l+1) f_l(s) P_l(z_s(t))$$



from M.Ostrick

more pronounced forward/backward peaks as energy increases

Resonance scattering

- Due to confinement, we expect an infinite number of resonances (poles at positive energy — recall the potential shell example) of arbitrary large mass and spin.

- String/flux tube breaking leads to screening of color charge and these poles decay. As mass increases they coach to multi-particle final states. The poles are still there, but dive deeper into to complex plane and are more difficult to identify. However, when making a model it makes more sense to parametrize amplitude with BW resonances as compared to some arbitrary background functions.

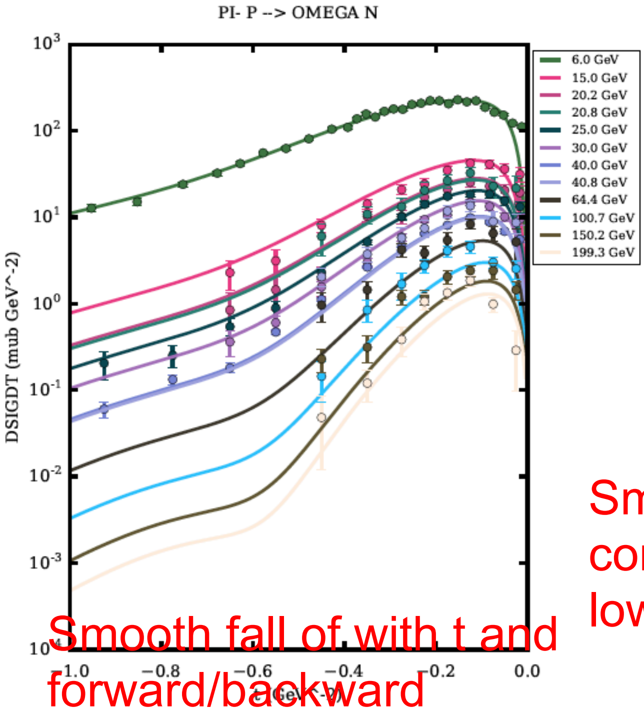
$$p = l/r$$

- For l_{max} ~ 5 and interaction range r_0 ~0.5fm this gives p_{lab} <~ 10/fm ~ 2GeV, [or W ~ (2 P_{lab} m_p)$^{1/2}$ ~ 2GeV]

- For resonance scattering

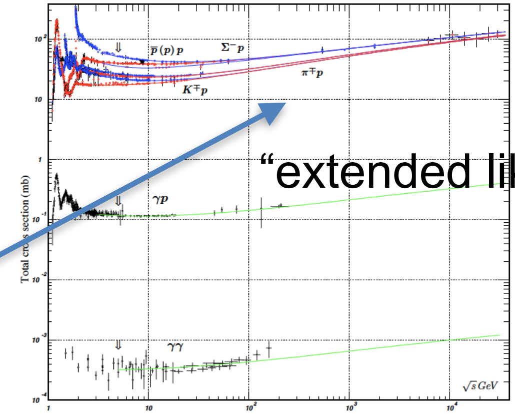
$$A(s,t) = \sum_l (2l+1) f_l(s) P_l(z_s(t)) \longrightarrow \qquad A(s,t) \sim \frac{P_{l_R}(z_s(t))}{s - s_R}$$

Scattering at High energies



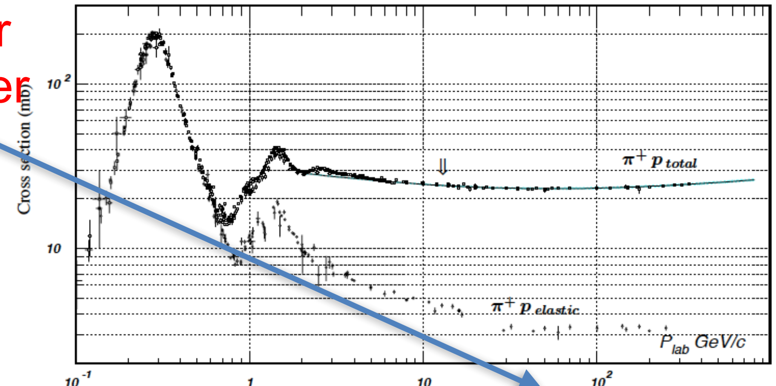
$$\sigma_{a+b\to X} = \frac{1}{s} Im A_{ab\to ab}(s,0)$$

$$\frac{d\sigma}{dt}(s) = \frac{1}{s^2}|A(s,t)|^2$$

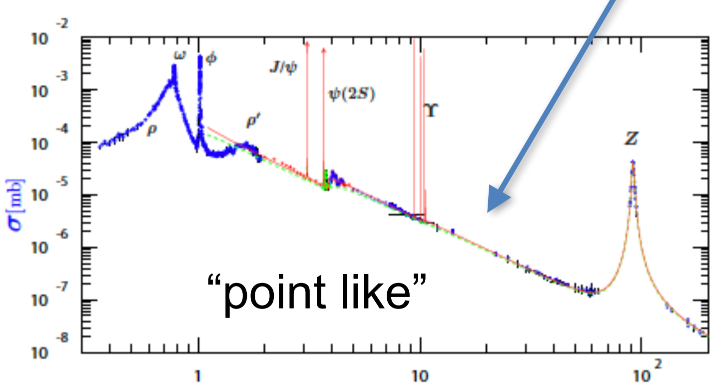


"extended like"

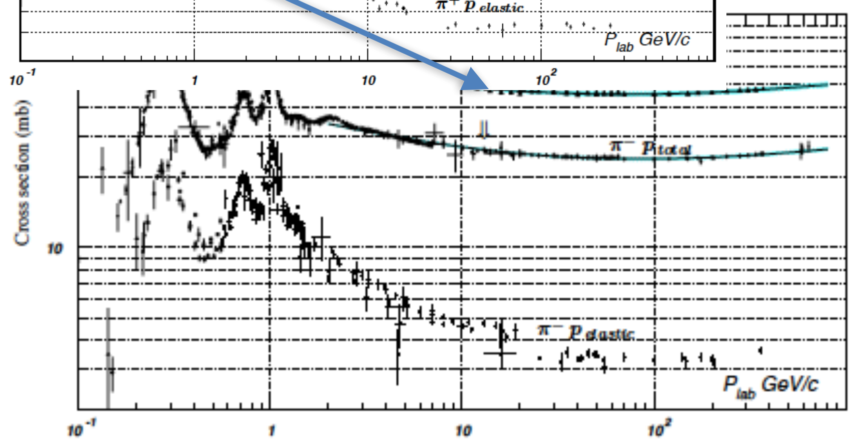
Smooth behavior constant or power low fall off



Smooth fall of with t and forward/backward peking

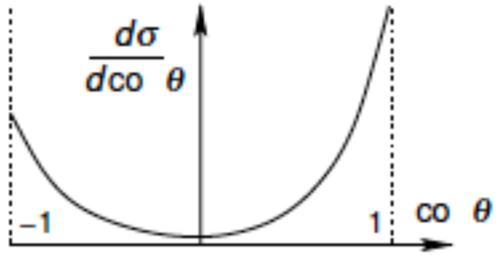


"point like"



Scattering at high energies

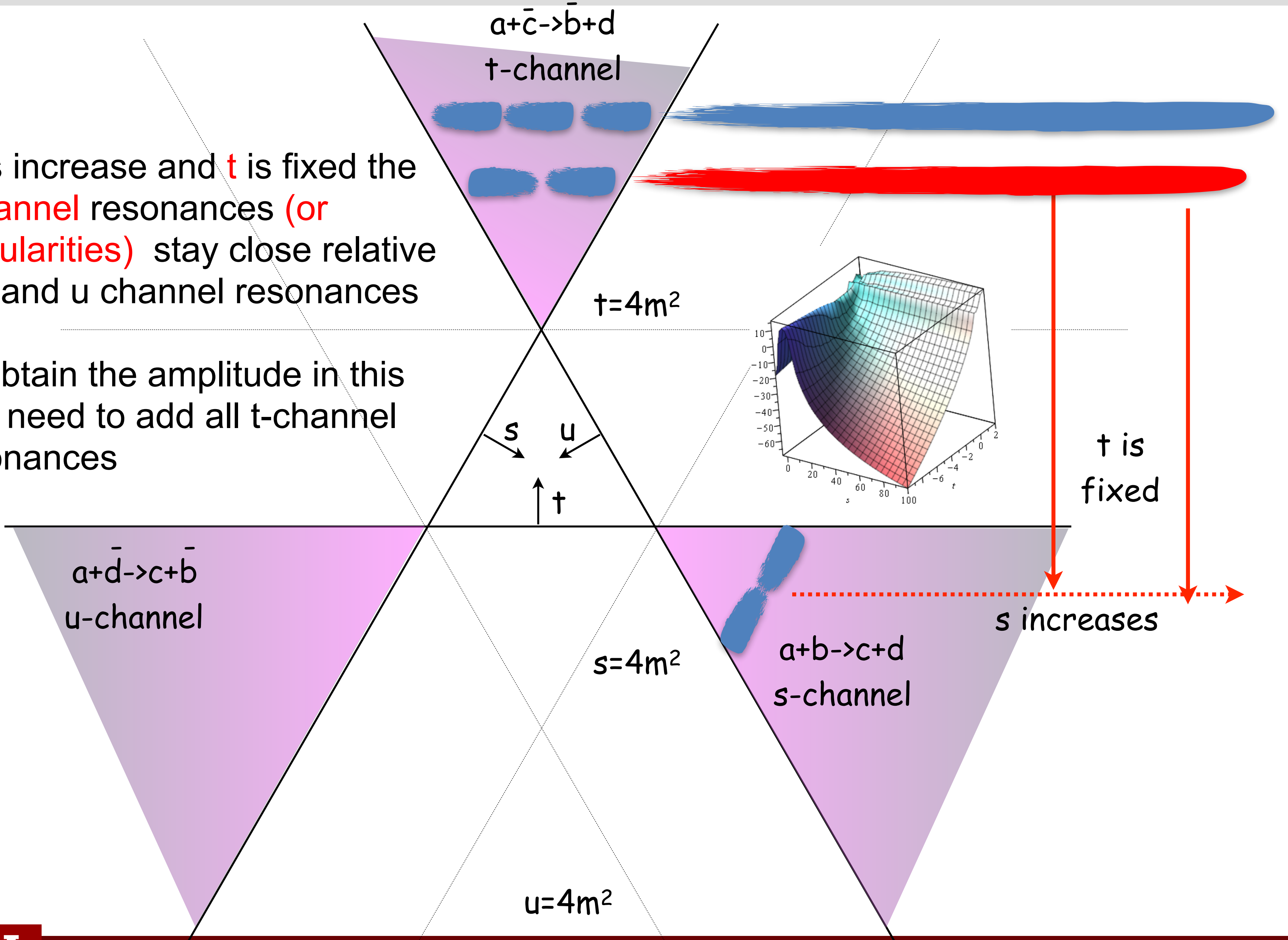
- s-dependence:
  - •many intermediate particles can be produced, unitarity becomes complicated and less useful.
- t-dependence:
  - •high partial waves become important, several Legendre functions are needed.
- There is universality in both s and t-dependencies: smooth (constant or falling s-dependence), and forward/(backward) peaking in t. The universality hints into importance of t/(u) channel singularities.



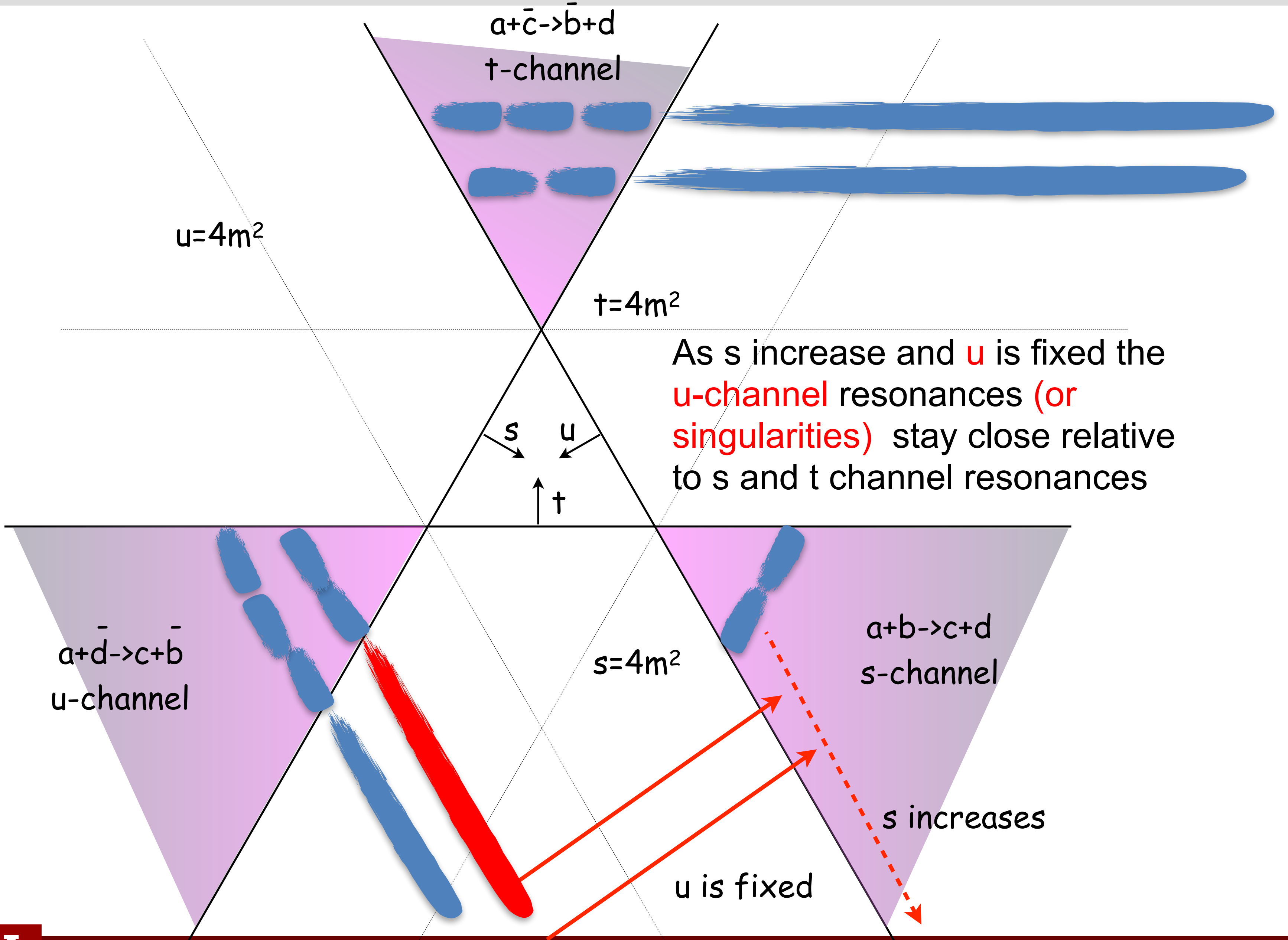
From t-channel to s-channel (high energy forward scattering)

As s increase and t is fixed the t-channel resonances (or singularities) stay close relative to s and u channel resonances

To obtain the amplitude in this limit need to add all t-channel resonances

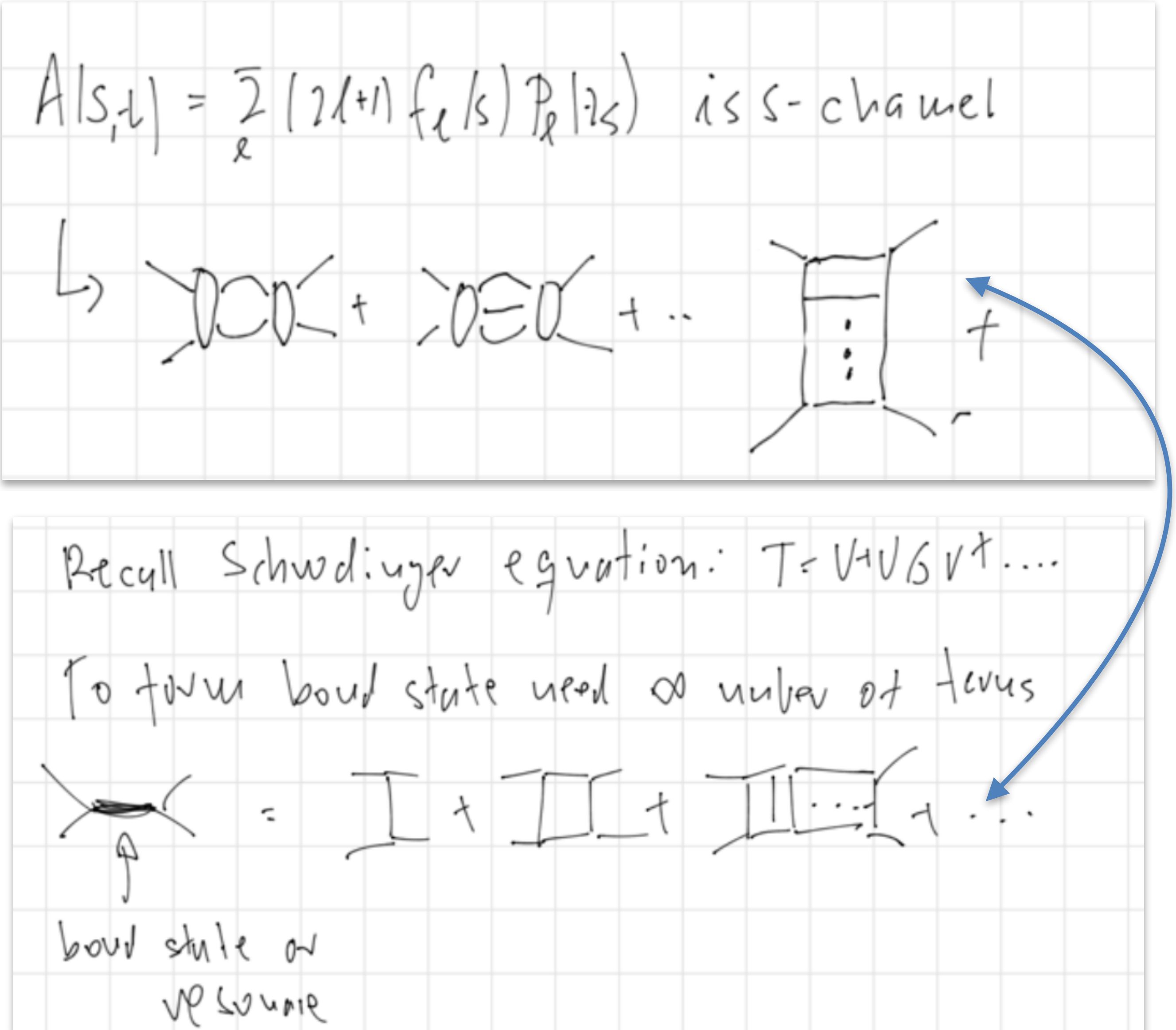


From u-channel to s-channel (high energy backward scattering)



As s increase and u is fixed the u-channel resonances (or singularities) stay close relative to s and t channel resonances

analytical continuation from s to t



Sum of a large number of particle productions at high-s looks like an exchange of various resonances in the t-channel.

Use t-channel partial waves and analytically continue to large-s

$$A(s,t) = \sum_l (2l+1) f_l(s) P_l(\cos\theta)$$

converges if |cosθ|<1 : (e.g. $1+x+x^2+\ldots$ = finite for |x|<1)

$$s = -\frac{t-4m^2}{2}(1-z_t)$$

$$A(s,t) = \sum_l (2l+1) f_l(t) P_l(z_t)$$

"t-channel"

(e.g. what is the value of $1+x+x^2+\ldots$ when x>1 ?

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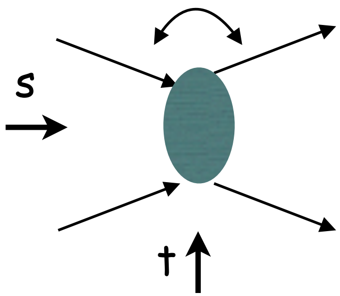
$z = \cos\theta$



$$z = 1 + \frac{2t}{s-4m^2}$$

$$t = -\frac{(1-z)}{2}(s-4m^2) < 0 \text{ for } |z|<1 \text{ and } s > 4m^2$$

"s-channel"

$$s = -\frac{t-4m^2}{2}(1-z_t)$$

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(e.g. what is the value of $1+x+x^2+\ldots$ when $x>1$?

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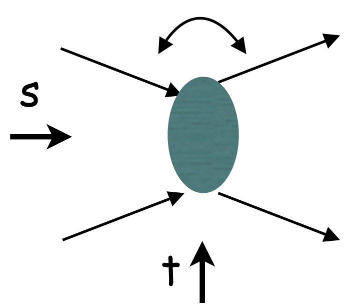

$$z = 1 + \frac{2t}{s-4m^2}$$

$$t = -\frac{(1-z)}{2}(s-4m^2) < 0 \text{ for } |z| < 1 \text{ and } s > 4m^2$$

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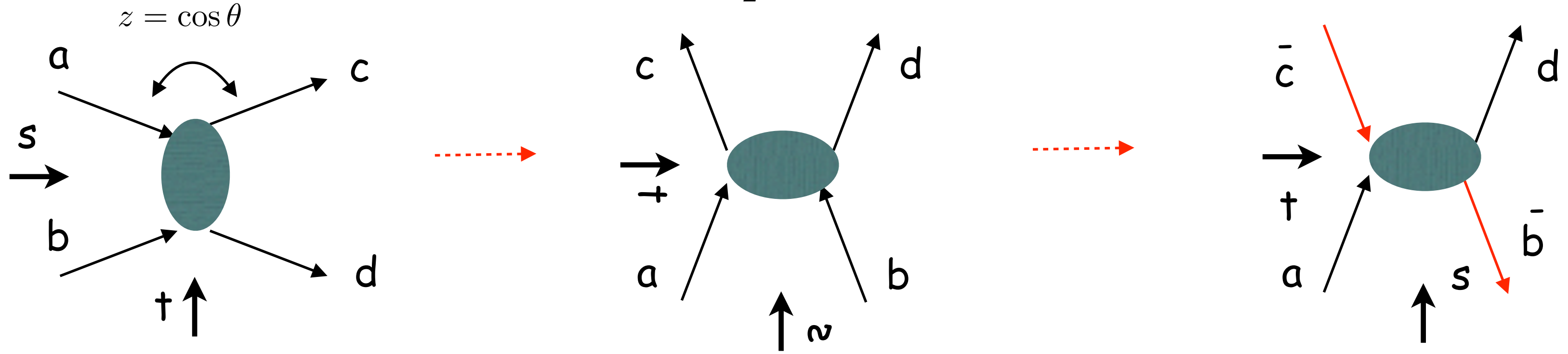
"t-channel"

$$t = -\frac{(1-z)}{2}(s-4m^2) > 4m^2 \text{ for } |z| > 1 \text{ and } s < 0$$

$z = \cos\theta$



a+b -> c+d

(e.g. what is the value of 1+x+x²+... when x>1 ?)

Example of analytical continuation

$$A(s,t) = \sum_l (2l+1) f_l(t) P_l(z_t) \qquad s = -\frac{t-4m^2}{2}(1-z_t)$$

The series converges for $|z_t|<1$ (cosine of scattering angle in the t-channel), i.e. in the t-channel physical region. We want to know A(s,t) for in the s-channel physical region, in particular for large s, with corresponds to $|z_t| >> 1$.

For example, assume $f_l(t) = \dfrac{1}{l-\alpha(t)}$ i.e. it has a pole (resonance) where α(t)=l

$$A(s,t) \sim J(z_t) = \sum_l \frac{z_t^l}{l-\alpha(t)}$$ for α<0 and $|z_t| < 1$ use $$\frac{1}{l-\alpha} = \int_0^\infty dx e^{-x(l-\alpha)}$$

to obtain $$J(z) = \int_0^\infty dx \left[\frac{e^{x\alpha}}{1+ze^{-x}}\right] = z^\alpha \int_0^z \frac{dy}{y^{\alpha+1}(1+y)} \qquad y = ze^{-x}$$

provides analytical continuation for α>0 for large z =z(s) ~ s

$$J(z) = -\frac{z^\alpha \pi}{\sin \pi\alpha} + z^\alpha \int_z^\infty \frac{dy}{y^{\alpha+1}(1+y)} \to -\frac{z^\alpha \pi}{\sin \pi\alpha} \qquad z \to \infty$$

this is analog of

$$f(x) = 1 + x + x^2 + \cdots$$

$$f(x) = \frac{1}{1-x}$$

Reggeon

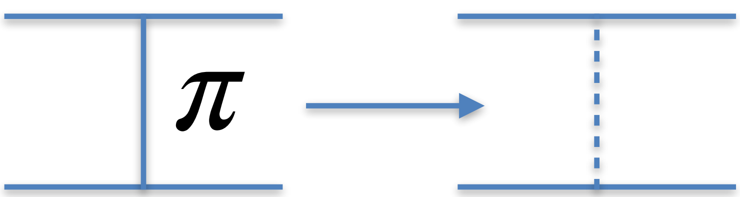
s-channel partial wave expansion $$A(s,t) = \sum_l (2l+1) f_l^{(s)}(s) P_l(\cos\theta_s)$$

t-channel partial wave expansion $$A(s,t) = \sum_l (2l+1) f_l^{(t)}(t) P_l(\cos\theta_t)$$

The amplitude at large-s (in the s-channel physical region) is dominated by a selected, infinite set of t-channel partial waves (t-channel resonances).

This sum is referred to as a Reggeon or a Regge exchange.

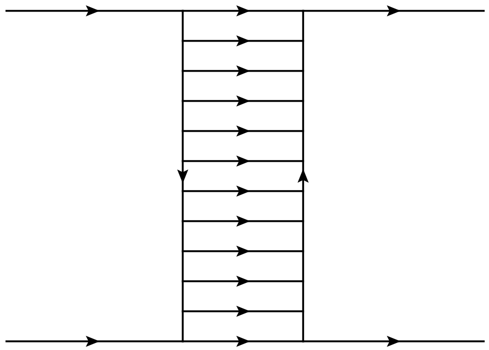
Since Reggeon is a collection of partial waves and partial waves have quantum numbers of resonances, so do Reggeon. They are like special kind of virtual particles. For example in perturbation theory pion we can talk about virtual, single pion exchange. A collection of all pion like exchange becomes a Reggion with pion quantum numbers. “Reggized pion”



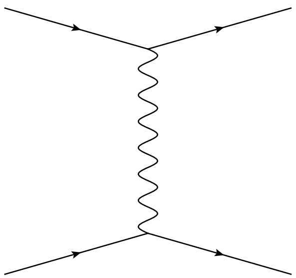
Reggized π

Pomeron vs Reggeons

s-channel: multi-particle production



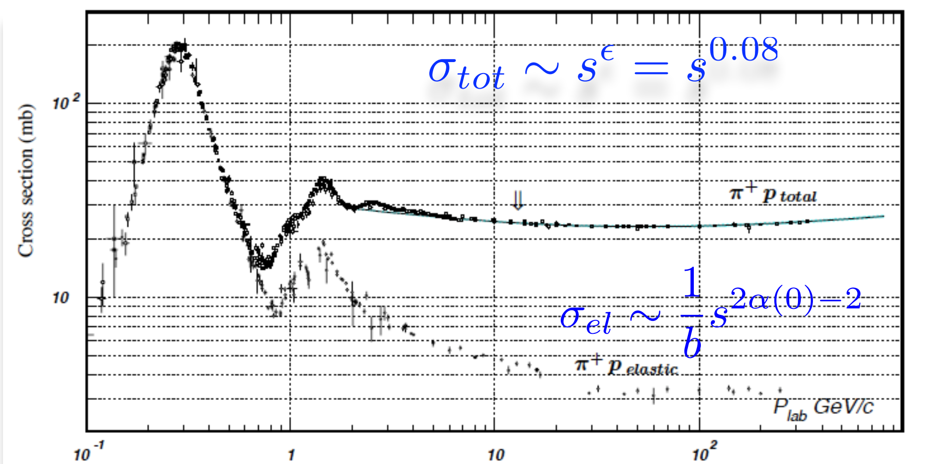
t-channel: collection of resonances: "Regge" exchanges

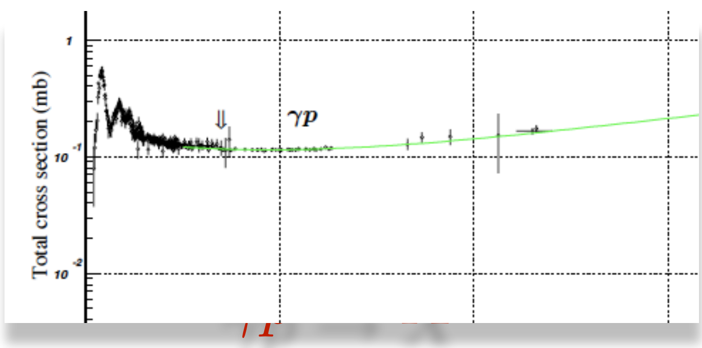


$$A(s, t \sim 0) \sim i s^{\alpha(0)} \sim s\sigma_{tot}$$

* Exchange of t-channel partial wave with quantum numbers of the vacuum is called the Pomeron

(exchange of non-vacuum q.n. falls with energy)

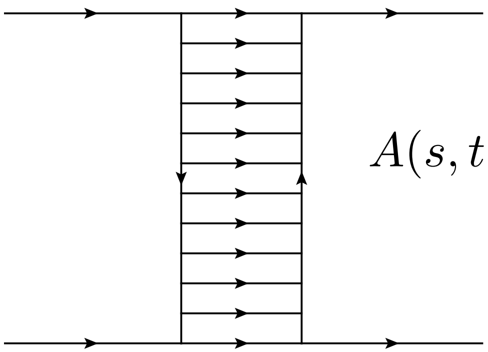




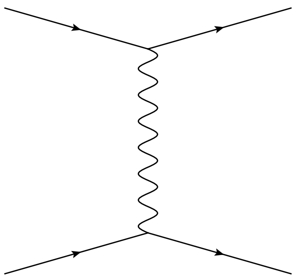
Pomeron vs Reggeons

s-channel: multi-particle production

t-channel: collection of resonances: "Regge" exchanges



$$A(s,t) \propto r(t) s^{\alpha(t)} \quad \texttt{<--} \quad f_l(t) = \frac{r(t)}{l - \alpha(t)}$$



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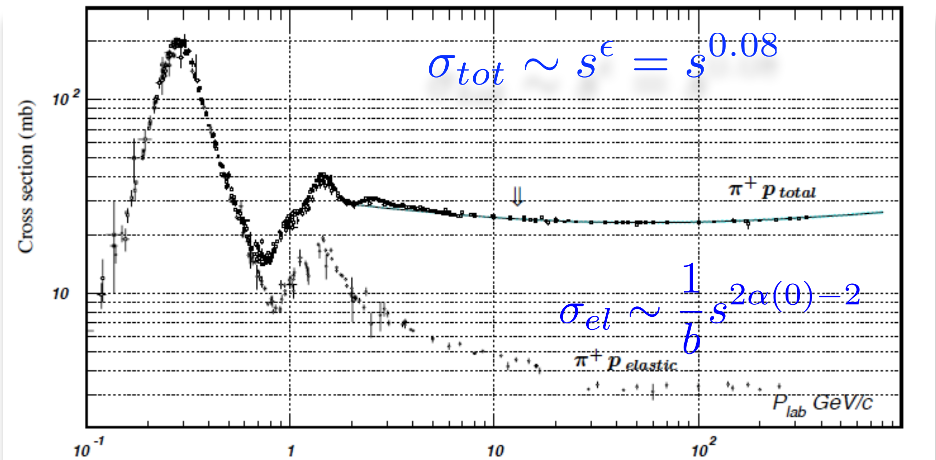

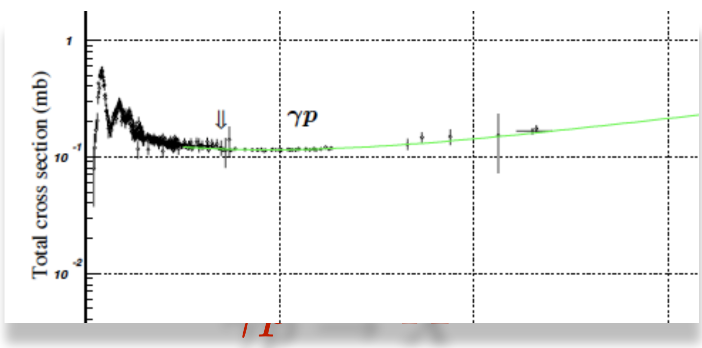

Growing Radius, partons, saturation,…

* Where does to parton model come from

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adding correlated partons is beneficial (expansion not in g^2 but in $g^2 \log s$)

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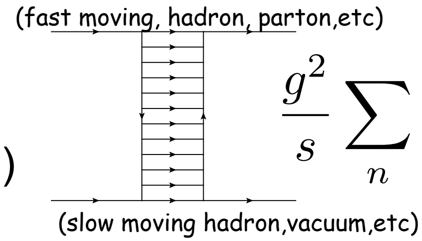
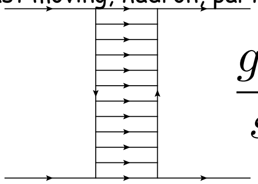


$$\frac{g^2}{s}\sum_n \frac{\beta^{n-1}(t)}{(n-1)!}\log^{n-1} s \to s^{\alpha(-k_\perp^2)}$$

$$\alpha(t) = -1 + \beta(t)$$

Growing Radius, partons, saturation,….

- Where does to parton model come from



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$$\alpha(t) = -1 + \beta(t)$$

adding correlated partons is beneficial (expansion not in g^2 but in $g^2 \log s$)

... and in space-time assuming Pomeron $\alpha(0)=1$

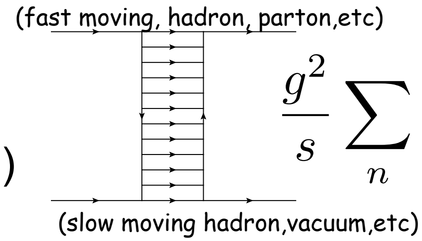
$$A(s,r_\perp) \sim \int d^2k_\perp e^{ik_\perp r_\perp} e^{\alpha(-k_\perp^2)\log s} \sim \frac{1}{\log(s)} e^{-r_\perp^2/\log(s)}$$

hadron swells

Growing Radius, partons, saturation,…

- Where does to parton model come from

adding correlated partons is beneficial (expansion not in g^2 but in $g^2 \log s$)



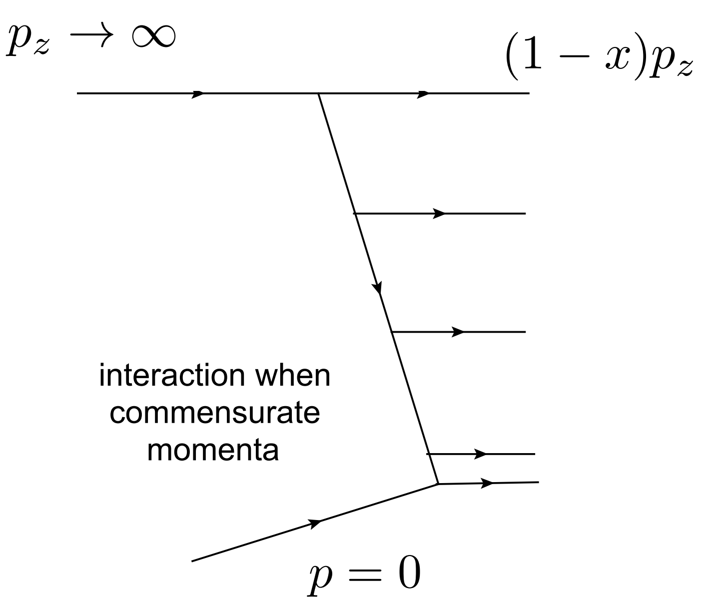
$$\frac{g^2}{s}\sum_n \frac{\beta^{n-1}(t)}{(n-1)!}\log^{n-1} s \to s^{\alpha(-k_\perp^2)}$$

$$\alpha(t) = -1 + \beta(t)$$

... and in space-time assuming Pomeron α(0)=1

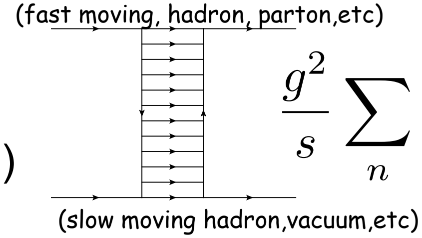
$$A(s, r_\perp) \sim \int d^2k_\perp e^{ik_\perp r_\perp} e^{\alpha(-k_\perp^2)\log s} \sim \frac{1}{\log(s)} e^{-r_\perp^2/\log(s)}$$

hadron swells



Growing Radius, partons, saturation,…

- Where does to parton model come from



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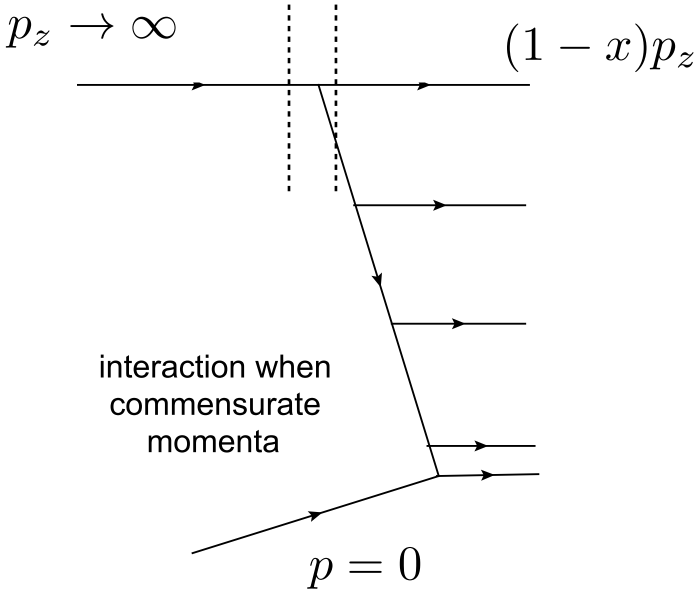
hadron swells

$$\Delta E \sim \frac{\mu_\perp^2}{x(1-x)p_z}$$

- long lived fluctuations finite <x>



Growing Radius, partons, saturation,…

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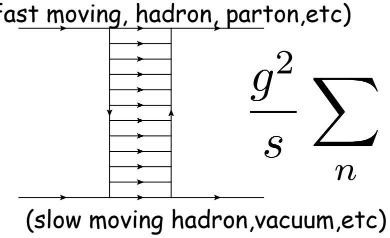

$$\frac{g^2}{s}\sum_n \frac{\beta^{n-1}(t)}{(n-1)!}\log^{n-1} s \to s^{\alpha(-k_\perp^2)}$$

$$\alpha(t) = -1 + \beta(t)$$

... and in space-time assuming Pomeron $\alpha(0)=1$

$$A(s, r_\perp) \sim \int d^2k_\perp e^{ik_\perp r_\perp} e^{\alpha(-k_\perp^2)\log s} \sim \frac{1}{\log(s)} e^{-r_\perp^2/\log(s)}$$

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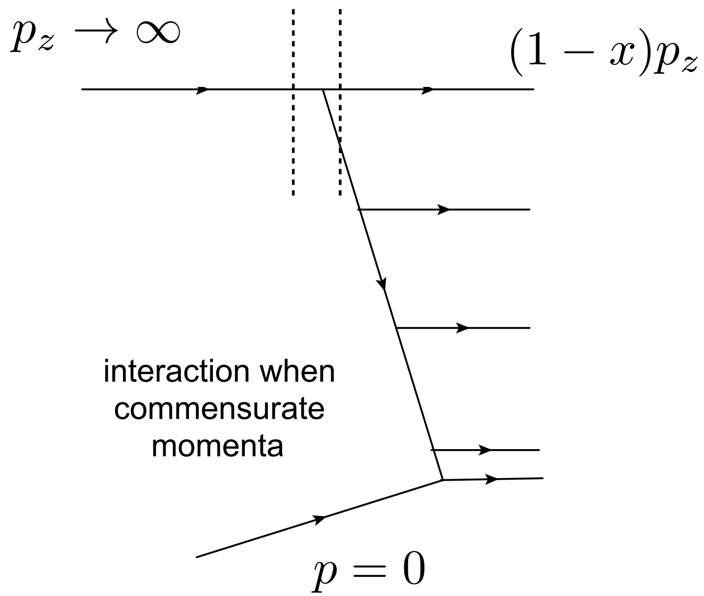


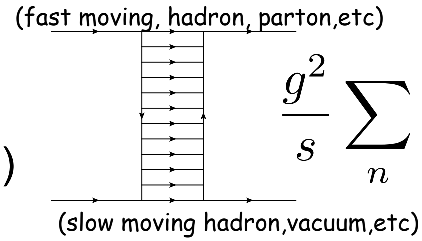
- long lived fluctuations finite <x>

$$\langle x \rangle^{\langle n \rangle} = \frac{p_z}{\mu} \quad \langle n \rangle \sim \log(s)$$

Growing Radius, partons, saturation,…

- Where does to parton model come from

adding correlated partons is beneficial (expansion not in g² but in g² log s)



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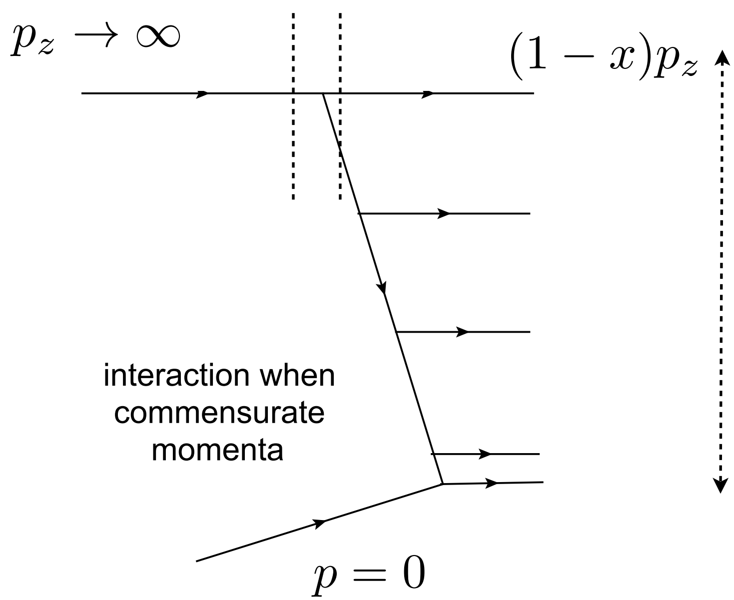

- long lived fluctuations finite <x>

$$\langle x\rangle^{\langle n\rangle} = \frac{p_z}{\mu} \quad \langle n\rangle \sim \log(s)$$

random walk in transverse space

- $$\langle r_\perp\rangle \sim \sqrt{\langle n\rangle \frac{1}{\mu_\perp}} \sim \log^{1/2}(s)$$

Growing Radius, partons, saturation,...

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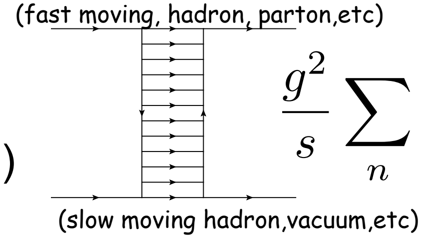

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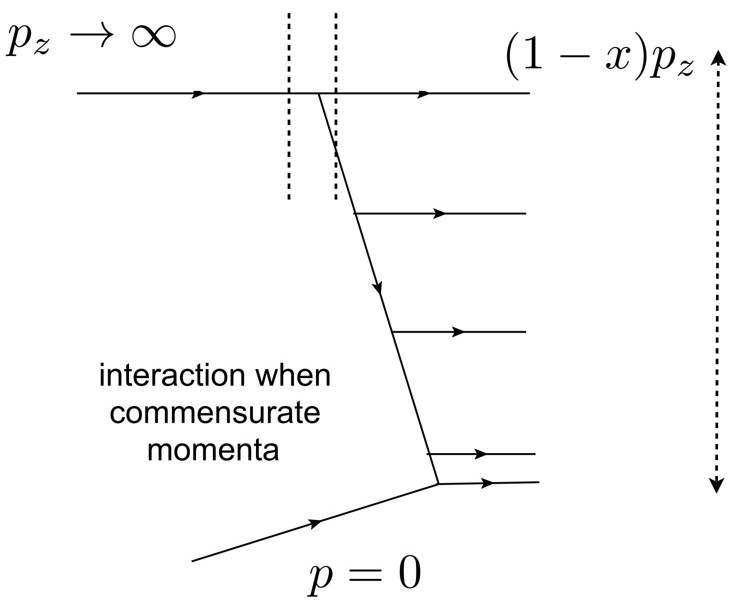

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* large-s behavior universal (Pomeron = vacuum pole, universal mid-rapidity)

Growing Radius, partons, saturation,…

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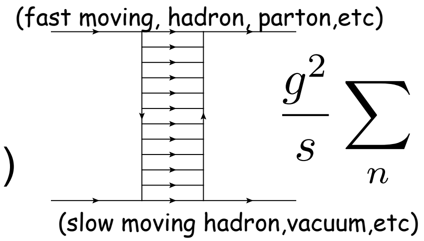

$$\frac{g^2}{s}\sum_n \frac{\beta^{n-1}(t)}{(n-1)!}\log^{n-1} s \to s^{\alpha(-k_\perp^2)}$$

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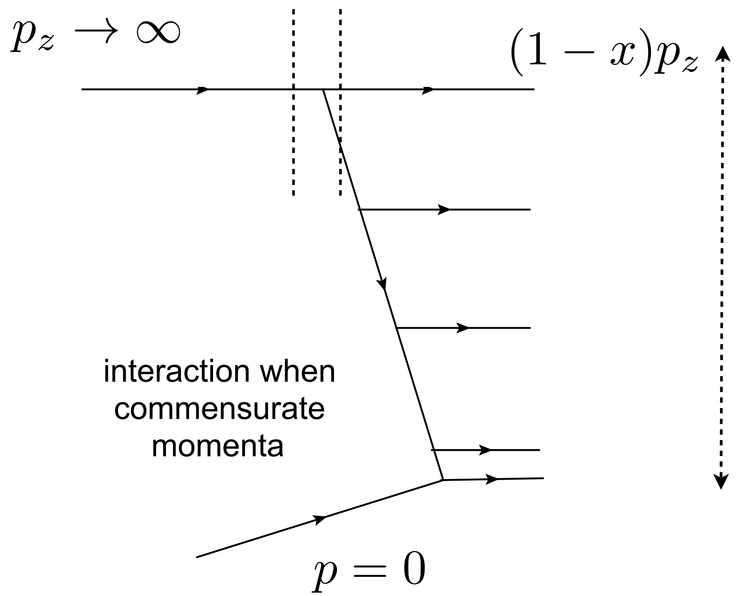

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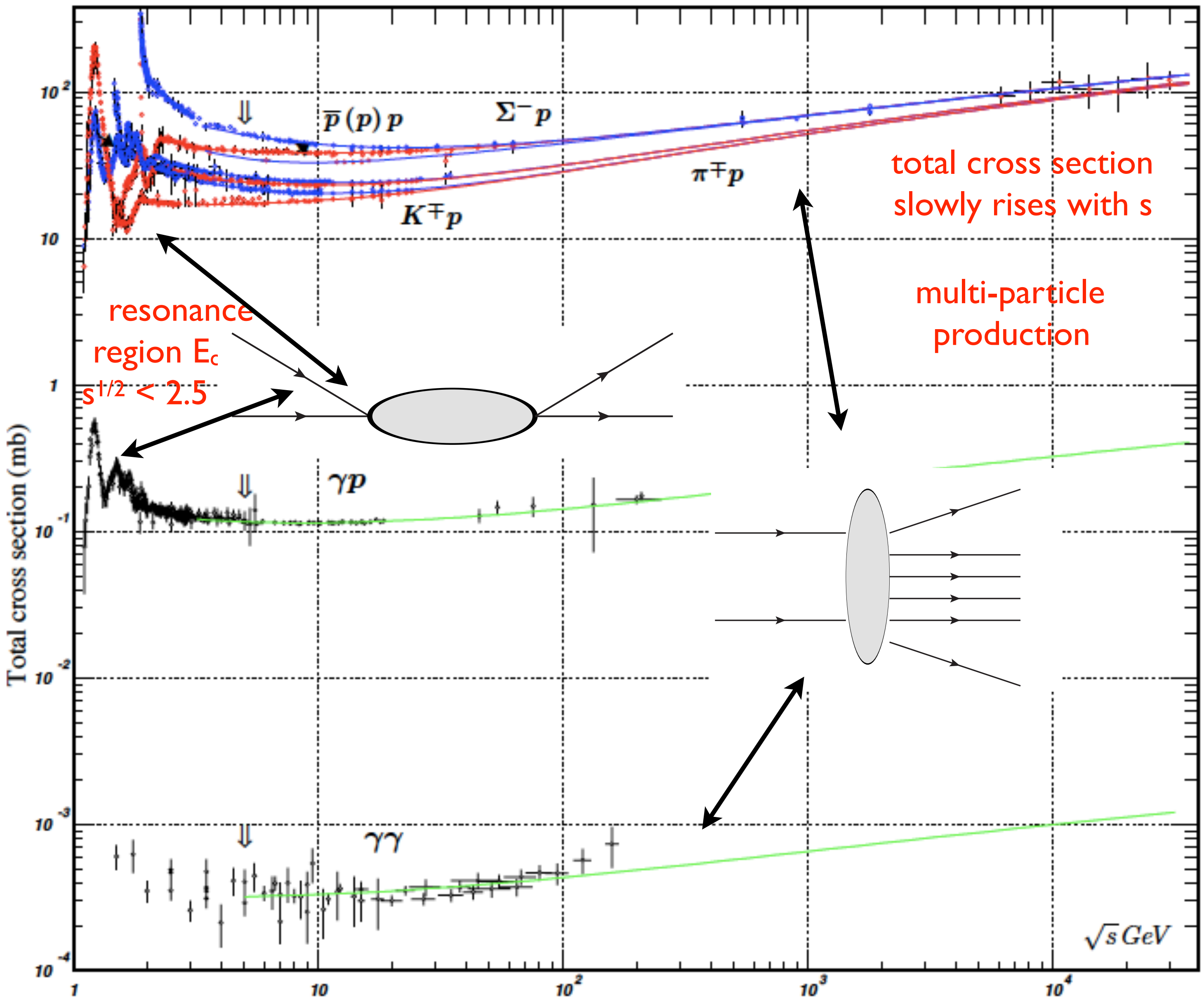
random walk in transverse space

* $$\langle r_\perp\rangle \sim \sqrt{\langle n\rangle \frac{1}{\mu_\perp}} \sim \log^{1/2}(s)$$

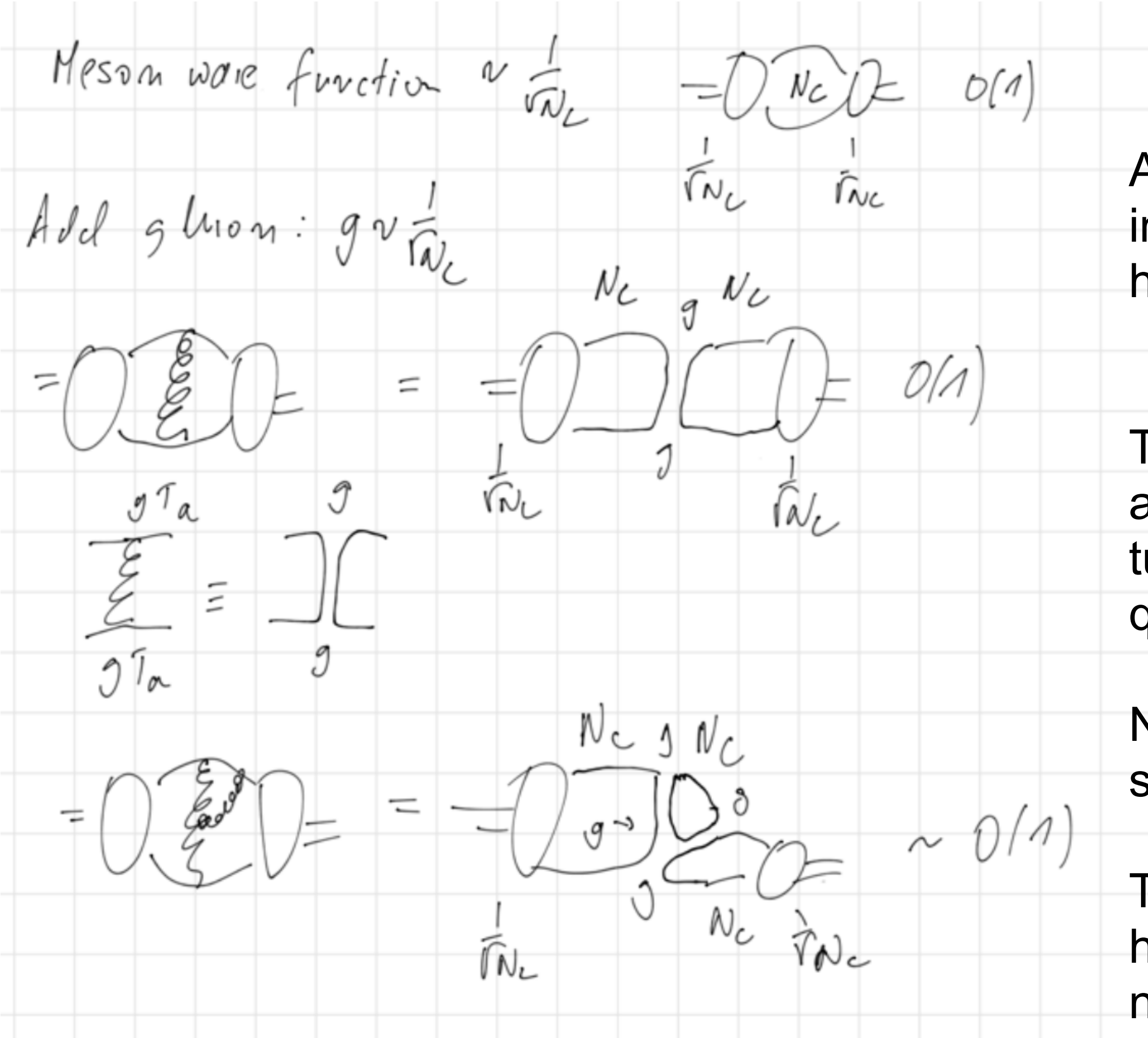
* large-s behavior universal (Pomeron = vacuum pole, universal mid-rapidity)

* it takes "a long time" to develop a low-x parton out of a fast one

Comparing with Experiment



Large N_c



$$N_c \to \infty$$

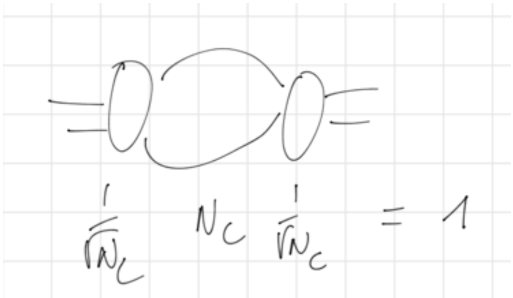
$$g^2 N_C = const.$$

An empty digram represents infinite number of process that happen in a plane !

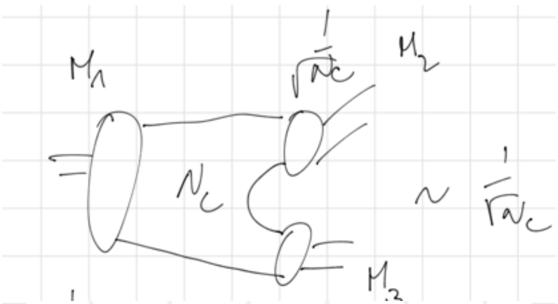
The plane can be intercepted as a world sheet of a string/flux tube connecting the valance quarks

Non planar diagrams are suppressed by 1/Nc

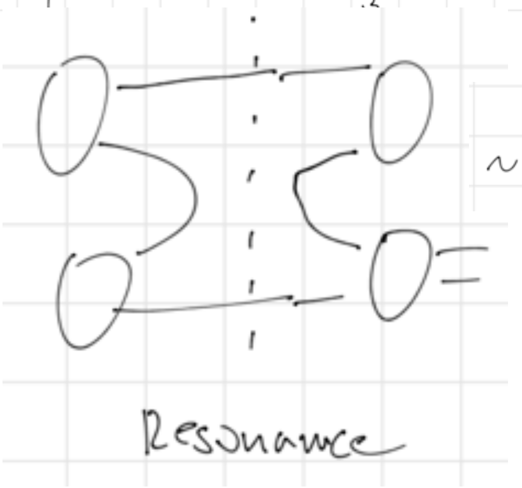
To leading order in 1/Nc hadrons do not decay, that to not scatter.

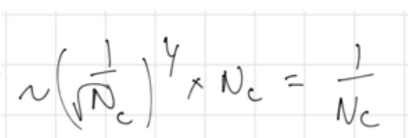


$$\langle M|H|M\rangle = O(1)$$



$$\langle M_1|H|M_2M_3\rangle = g = O(1/\sqrt{N_C})$$





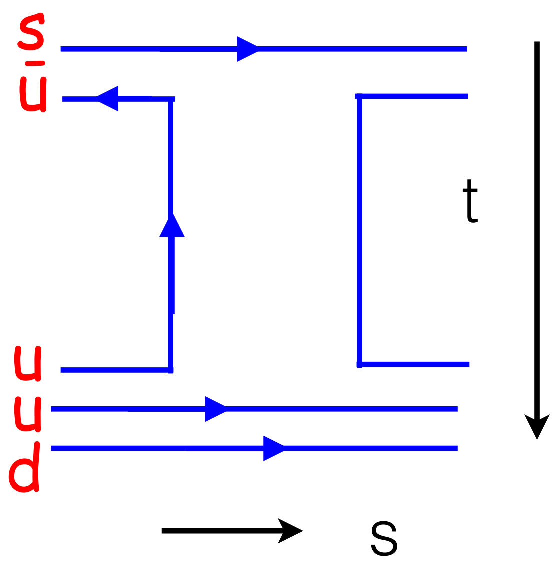
$$\sim \frac{\Gamma}{m^2 - s - i\Gamma} \qquad \Gamma = O(1/N_c) = g^2$$

Dualities

planar diagrams may be considered as either
s-channel or t-channel

K-p

K-p has "normal mesons" in the t-channel
and "normal baryons" in the s channel



Interpretation of what happens in s-channel is dual to what happens in the t-channel :
Mesons require baryons and vice versa

Regge phenomena :
sum of t-channel resonances determines large-s behavior of the s0channel and vice versa.

Does it work ?

In K-p scattering
imaginary parts of a2
and rho add up
In K+p they cancel !

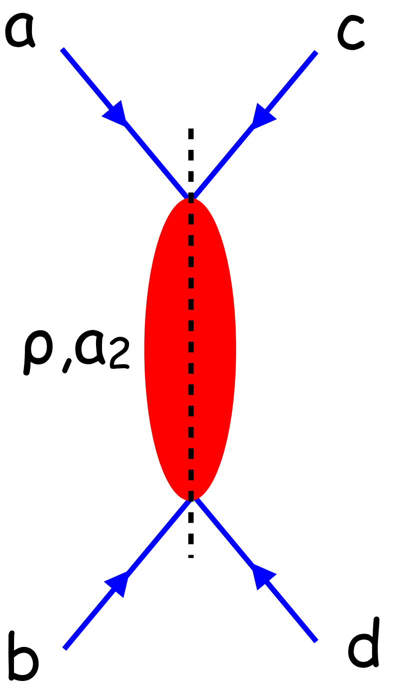
$a_2 \sim 1 + \exp(i\,\pi\,\alpha(t))$

$\rho \sim 1 - \exp(i\,\pi\,\alpha(t))$

$\rho+a_2$

$\rho-a_2$

Does it work ?



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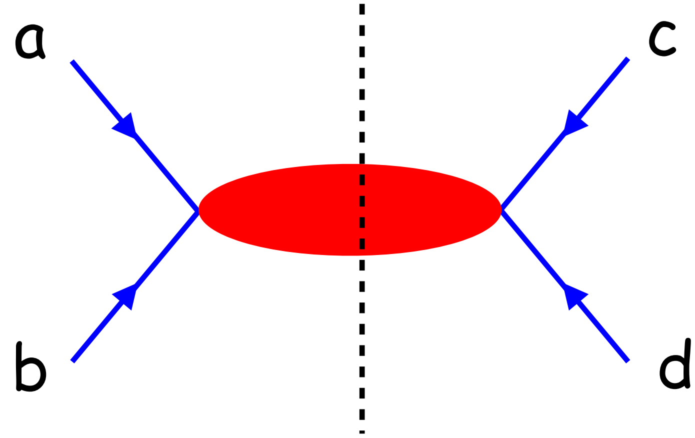

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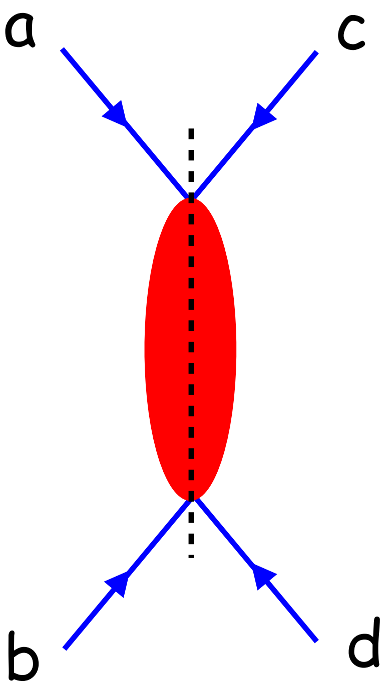

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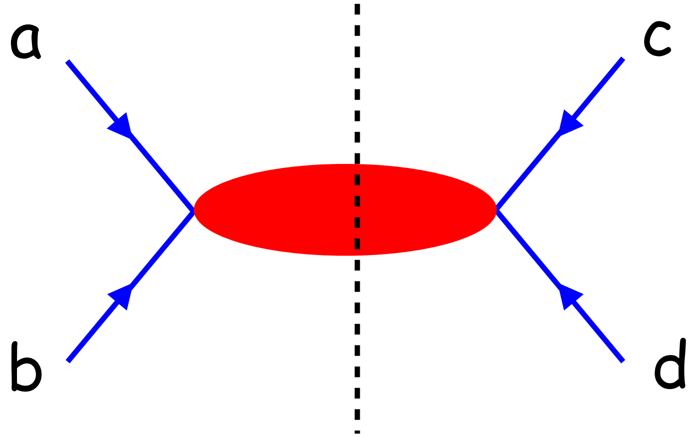

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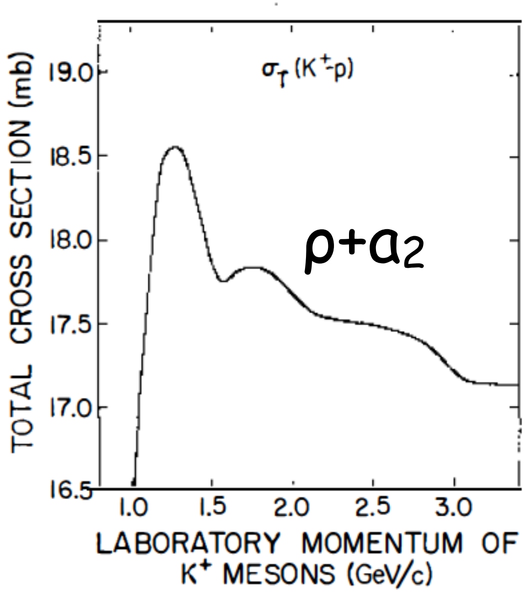
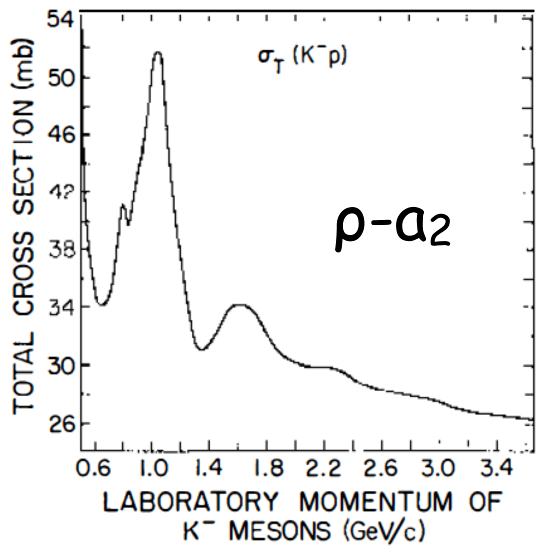




Dolen Horn Schmit duality

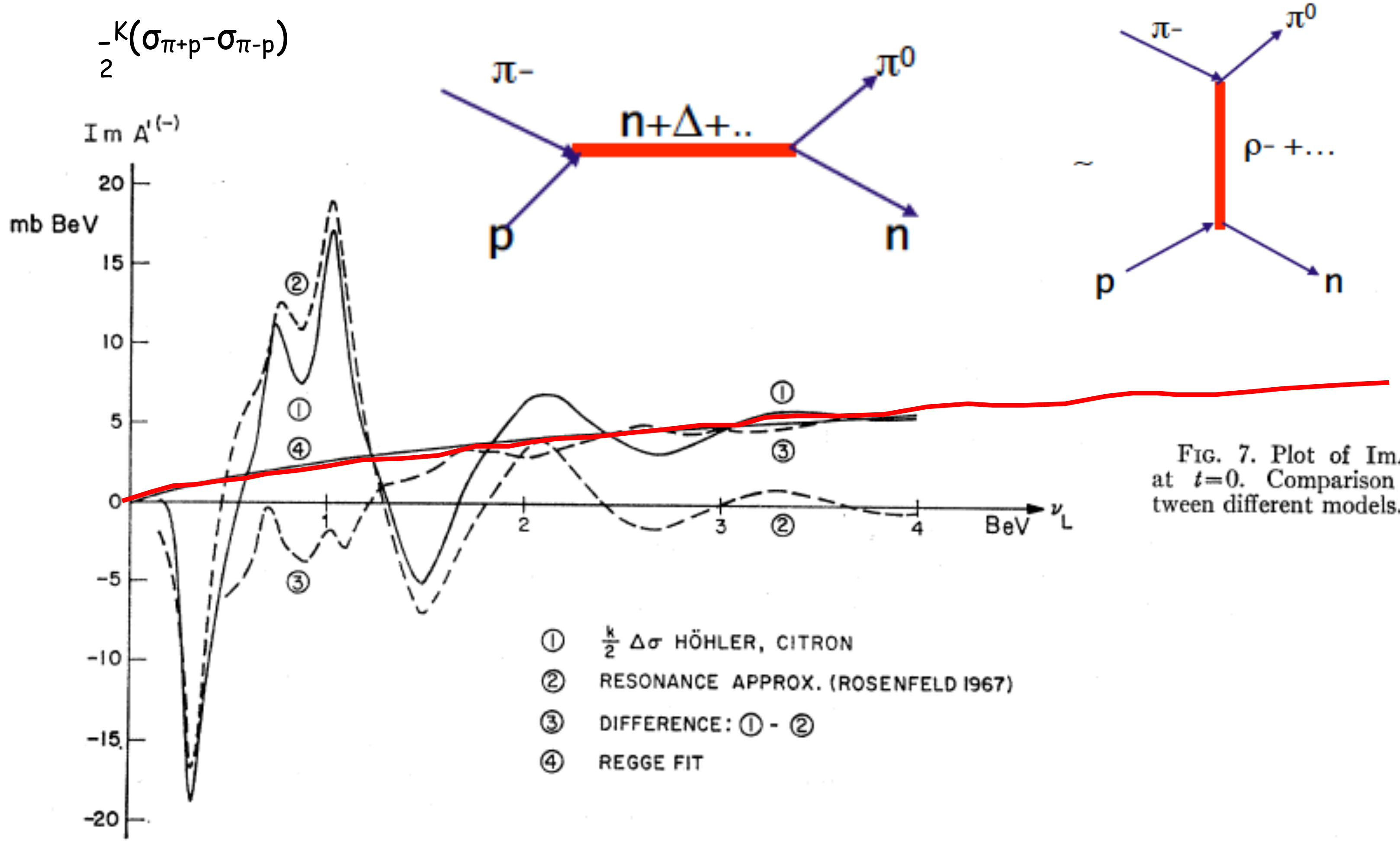
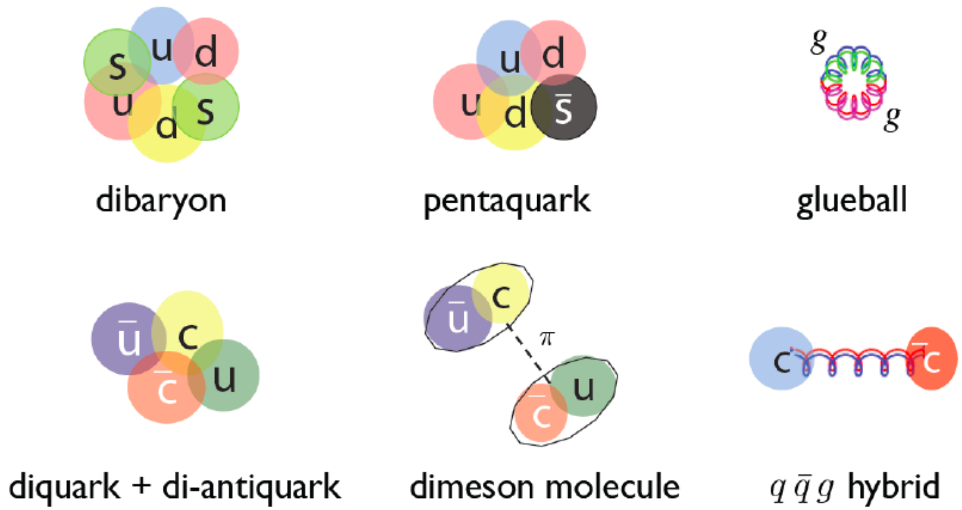


FIG. 7. Plot of Im$A'^{(-)}$ at $t=0$. Comparison between different models.

$$\bar{u}(p_1,\lambda_1)[A(s,t)+(k_1+k_2)_\mu\gamma^\mu B(s,t)]u(p_2,\lambda_2)$$

What about “exotic” hadrons

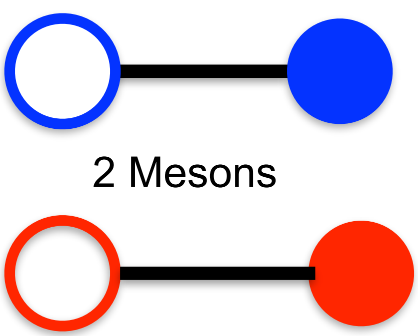


Standard argument for non-existence of multi quark sates: they can fall apart to ordinary mesons and baryons

For example 2 quarks and 2 anti quarks can rearrange into 2 quark-antiquark pairs

But confinement requires quarks are connected by flux tubes and it is possible that certain multi quark configurations are more favorable than “fall apart configurations”

$3 \times \bar{3} = 8 + 1$

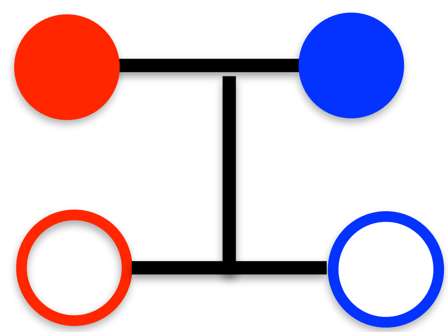


2 Mesons

$3 \times \bar{3} = 8 + 1$

vs

$3 \times 3 = 6 + \bar{3}$



$\bar{3} \times \bar{3} = 6 + 3$

2 di quarks = teraquark

Need to introduce strings

Talk by G.Rossi

Hadronic states → irreducible gauge invariant operators in QCD

Table IIa

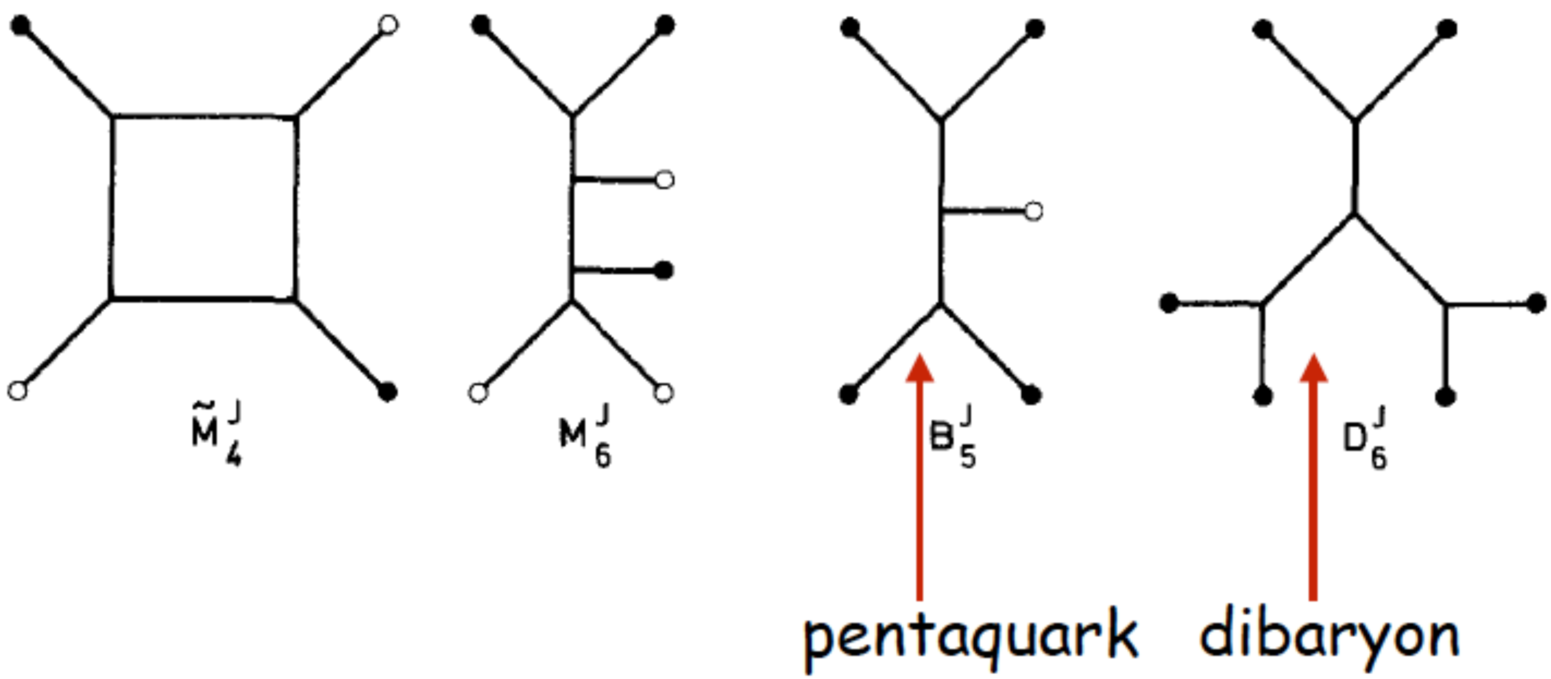
Simplest mesons and baryons : colour structure and string picture

HADRON	GAUGE INVARIANT OPERATOR	STRING PICTURE
$M_2 = q\bar{q}$ meson	$\bar{q}^{j_2}(x_2)\left[P\exp\left(ig\int_{x_1}^{x_2} A_\mu\, dx^\mu\right)\right]_{j_2}^{j_1} q_{j_1}(x_1)$	
M_0 = quarkless meson	$\mathrm{Tr}\left[P\exp\left(ig\oint A_\mu\, dx^\mu\right)\right]$	
$B_3 = qqq$ baryon	$\varepsilon^{j_1j_2j_3}\left[P\exp\left(ig\int_{x_1}^{x} A_\mu\, dx^\mu\right) q(x_1)\right]_{j_1}\left[P\exp\left(ig\int_{x_2}^{x} A_\mu\, dx^\mu\right) q(x_2)\right]_{j_2}\left[P\exp\left(ig\int_{x_3}^{x} A_\mu\, dx^\mu\right) q(x_3)\right]_{j_3}$	

Other states

Other multiquark states

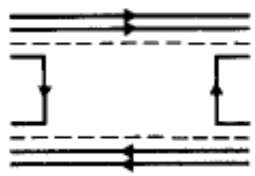
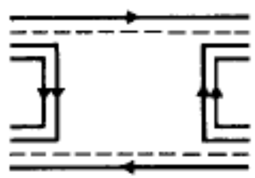
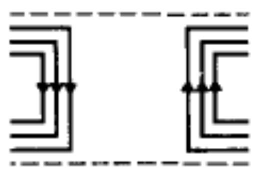
(from G. C. Rossi & GV, Phys. Rep. 1982)



Muti-quark states can be related to ordinary states by duality

s-channel tetraquark are dual to t-channel mesons

Contributions to $B\bar{B}$ scattering ($N_c = 3$)

$B\bar{B} \to B\bar{B}$ Junction duality diagrams annihilation	*s*-channel formation	Multiplicity[(a)]	*t*-channel[(b)] exchange	Slope
1 	M_4^J 	$\bar{n}(s') \simeq \bar{n}_{e^+e^-}(s')$	$s^{\alpha_R - 1} \sim s^{-1/2}$ Regge pole	α'_R
2 	M_2^J 	$\bar{n}(s') \simeq 2\bar{n}_{e^+e^-}(s'/4)$	$s^{2\alpha_R - 2} \sim s^{-1}$ 2-Reggeon cut	$\frac{1}{2}\alpha'_R$
3 	M_0^J 	$\bar{n}(s') \simeq 3\bar{n}_{e^+e^-}(s'/9)$	$s^{3\alpha_R - 3} \sim s^{-3/2}$ 3-Reggeon cut	$\frac{1}{3}\alpha'_R$
4 	Non-resonant two jet background	$\bar{n}(s') \simeq 2\bar{n}_{e^+e^-}(s'/4)$	$s^{\alpha_P - 1} \sim s^0$ Pomeron	$\frac{1}{2}\alpha'_R$

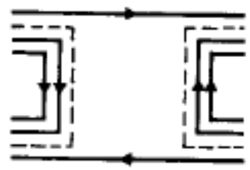
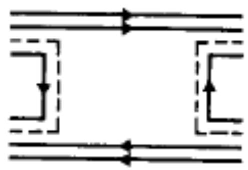
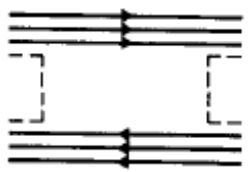
[(a)] s' is the invariant mass of the final state excluding the leading baryons.
[(b)] To estimate the s-behaviour we have taken $\alpha_R = 0.5$.

Muti-quark states can be related to ordinary states by duality

s-channel mesons are dual to t-channel tetra quarks

tetra quarks should form Regge trajectories just like mesons

Contribution to $B\bar{B}$ annihilation ($N_c = 3$)

$B\bar{B} \to B\bar{B}$ Junction duality diagrams annihilation	s-channel formation	Multiplicity	t-channel[a] exchange	Slope
1 	$1q\bar{q}$ – jet	$\bar{n}(s) \simeq \bar{n}_{e^+e^-}(s)$	$s^{\alpha(M_4^J)-1} \sim s^{-3/2}$ Regge pole	$\alpha'(M_4^J) \sim \alpha'_R$
2 	$2q\bar{q}$ – jets	$\bar{n}(s) \simeq 2\bar{n}_{e^+e^-}(s/4)$	$s^{\alpha(M_2^J)-1} \sim s^{-1}$ Regge pole	$\alpha'(M_2^J) \sim \frac{1}{2}\alpha'_R$
3 	$3q\bar{q}$ – jets	$\bar{n}(s) \simeq 3\bar{n}_{e^+e^-}(s/9)$	$s^{\alpha(M_0^J)-1} \sim s^{-1/2}$ Regge pole	$\alpha'(M_0^J) \sim \frac{1}{3}\alpha'_R$
4 	M_0 	$\bar{n}(s) \simeq 2\bar{n}_{e^+e^-}(s/4)$	$s^{2\alpha_B-2} \sim s^{-2}$ 2-Reggeon cut	$\frac{1}{2}\alpha'_R$

[a] To estimate the s-behaviour we have taken $\alpha_B \simeq 0$.

Veneziano Model

$$f(s) = \frac{1}{K^{-1}(s) - i\Gamma(s)}$$

∞ number of poles : confinement

Quadratically spaced radial trajectories

$$K(s) = \sum_{r=1}^{\infty} \frac{g_r^2}{m_r^2 - s} \rightarrow \sum_r \frac{1}{r^2 - s} \sim \frac{\cos(\pi\sqrt{s})}{\sin(\pi\sqrt{s})}$$

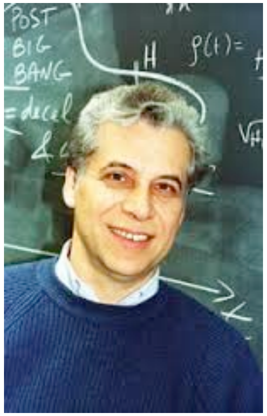
Linearly spaced radial trajectories (Veneziano)

$$K(s) \sim \frac{\Gamma(a-s)}{\Gamma(b-s)}$$

Veneziano amplitude : crossing symmetric:

$$A(s,t) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

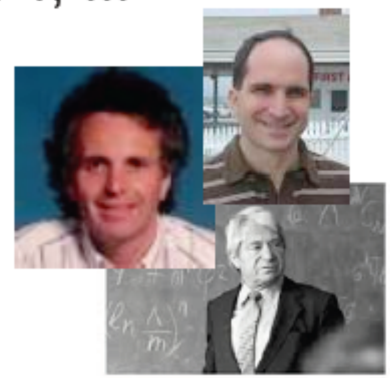
$$\alpha(s) = a + bs$$

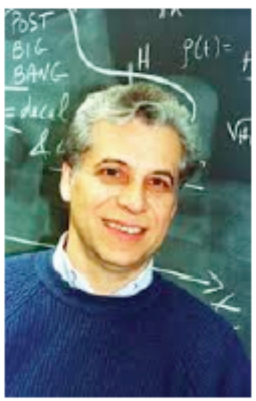


relativistic h.o.

string of relativistic oscillators

QCD, loop representation, large-N_c, AdS/CFT, ...





$\omega \to 3\pi$

string revolution

$$A(s,t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s)-J(t))}$$

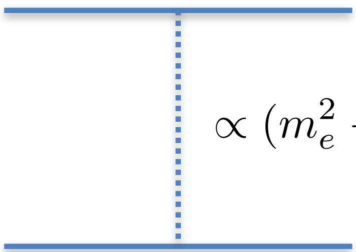
Other effects of partial wave analyticity

Scalar particle scattering 1+2 -> 3 + 4

$$A_l(s) = \int dz_s A(s, t(s, z_s), u(s, z_s)) P_l(\cos\theta)$$

Partial waves have "right hand" singularity (from s) and "left hand" (from t and u)

For example assume equal masses



$$\propto (m_e^2 - t(s, z_s))^{-1}$$

$$t = -\frac{(s - 4m^2)}{2}(1 - z_s)$$

$$A_0(s) \sim \int_{-1}^{1} dz_s \frac{1}{m_e^2 + \frac{(s-4m^2)}{2}(1 - z_s)}$$

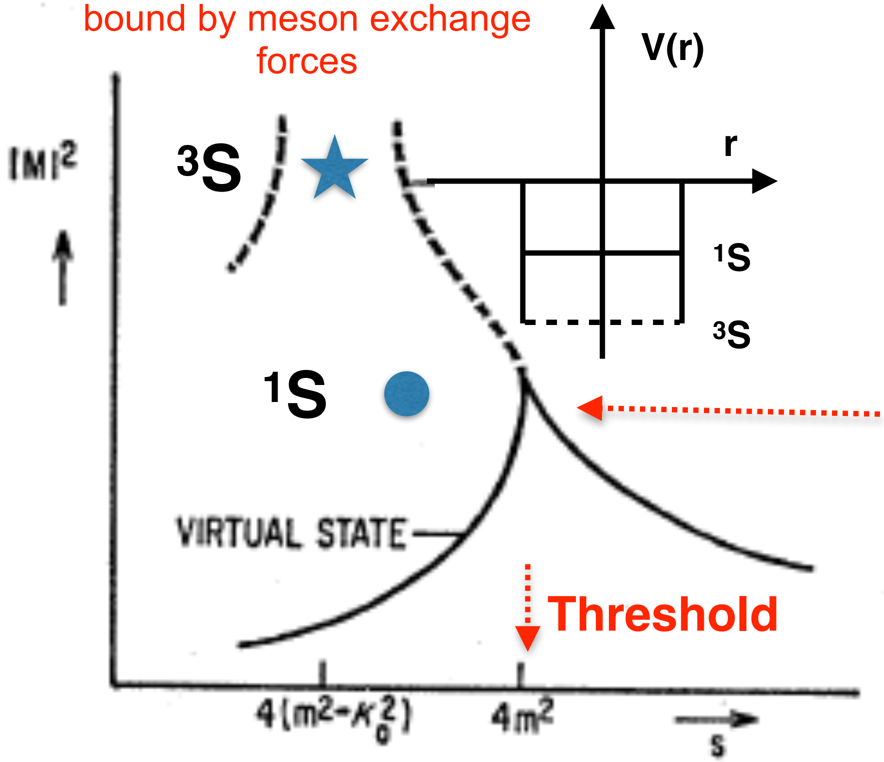
For $s>4m^2$ integral is finite

For $s<4m^2 - m_e^2$ the detonator crosses 0 within integration limi, implying $A_0(s)$ has a cut for negative s

Scalar amplitudes have simple singularity structure, but partial waves a much more complicated. They also have kinematical singularities when spin and/or unequal masse are involved

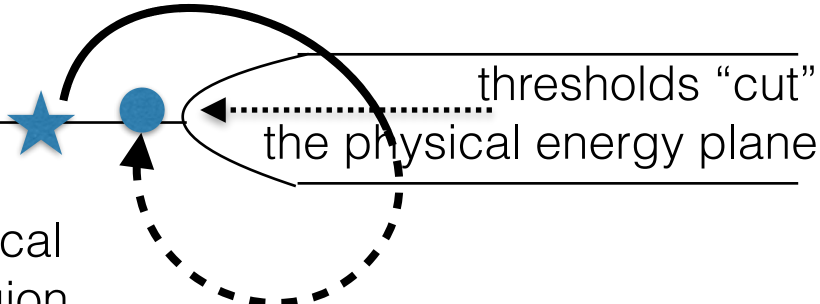
Bound states and Virtual States

- f0(980),
- a0(980),
- a1(1420),
- Lambda(1405),
- XYZ,
- …



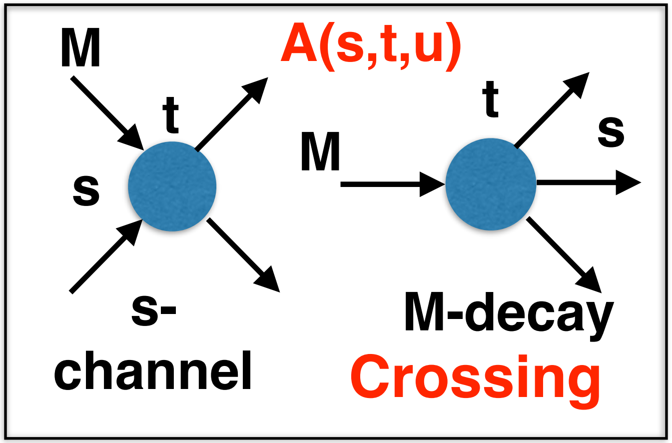
- **Thresholds are "windows" to singularities (particles, visual states, forces") located on the nearby unphysical sheet.**
- **They appear as cusps (if below threshold) or bumps (is above)**

bound state : pole on the physical energy plane



virtual state : pole on "unphysical sheet" closest the physical region

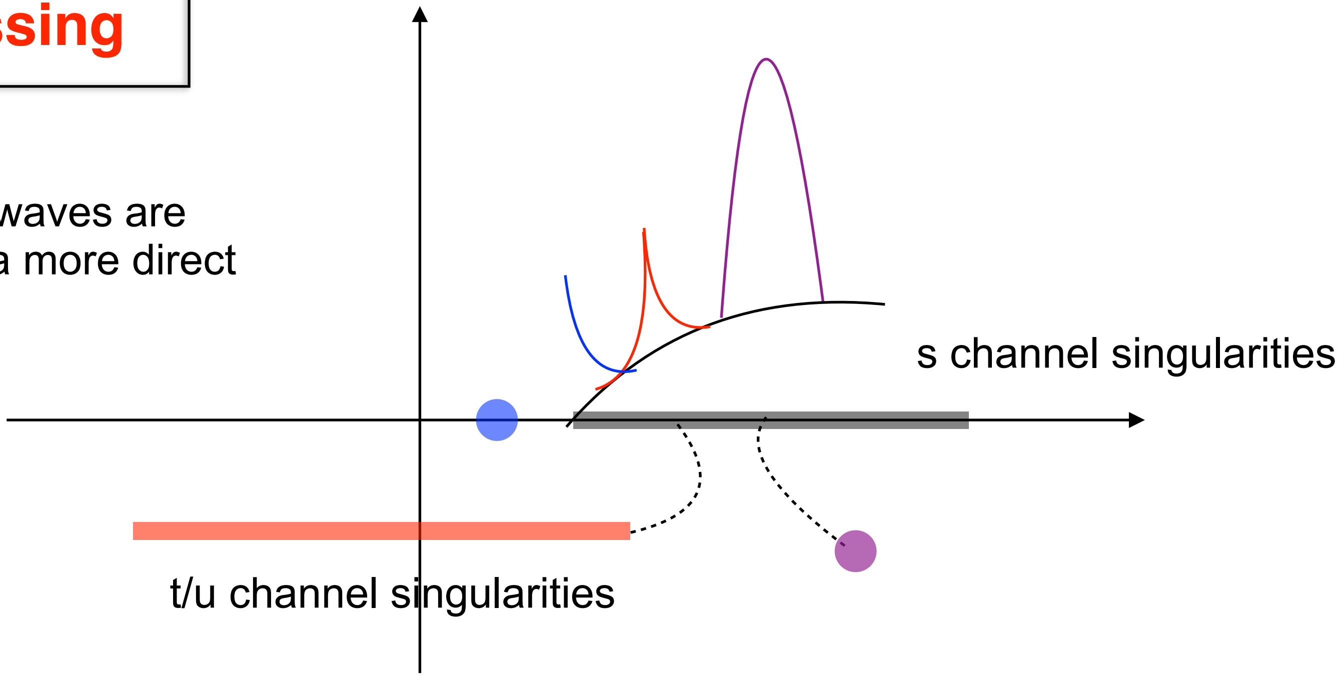
Amplitude singularities



- A(s,t,u) has simple singularity structure. Its connection to particles arises through (complicated) partial waves

$$A_l(s) = \frac{1}{2}\int_{-1}^{1} dz_s A(s, t(s,z), u(s,z))$$

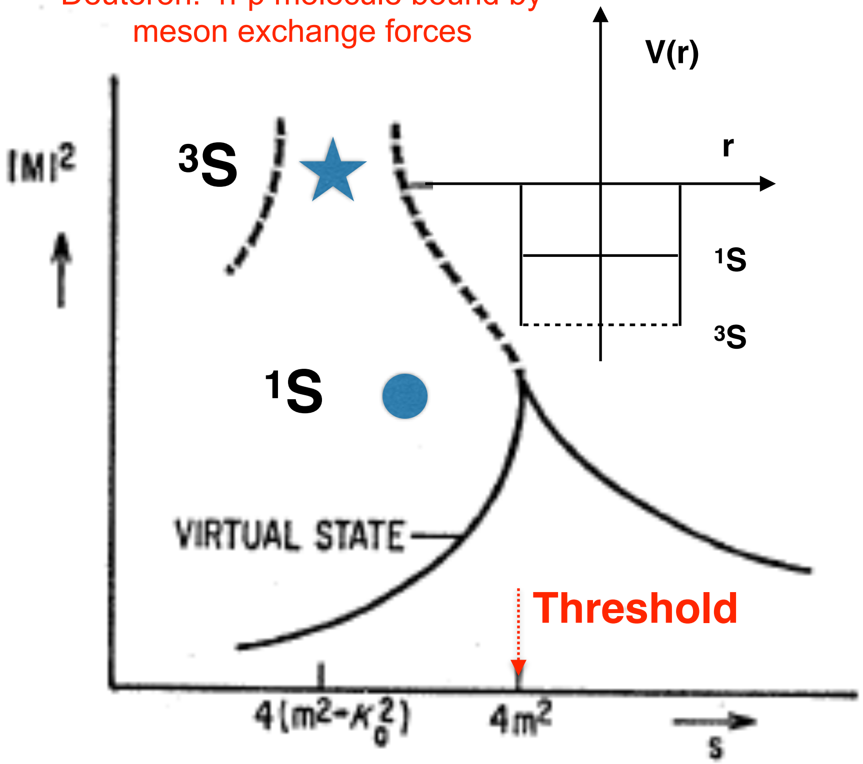
- Singularities of partial waves are complicated but have a more direct physical interpretation



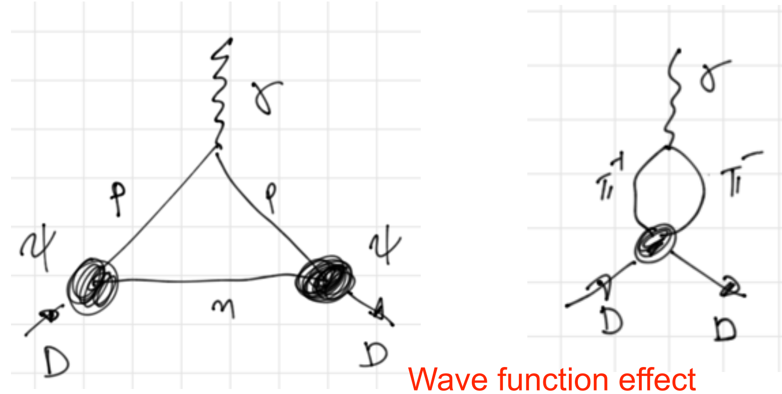
- However, X-sections are given by A(s,t,u) and not by partial waves. In general "bumps" in partial waves are "washed out" and require partial wave analysis.

Well known examples of cusps

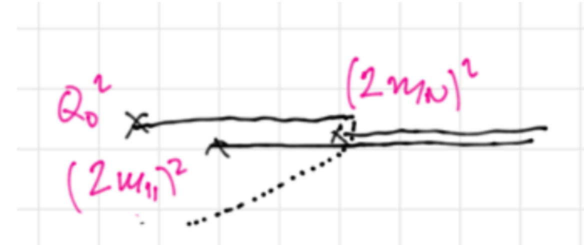
Deuteron: n-p molecule bound by meson exchange forces



$$Q_0 \sim 100 \text{ MeV} < 2m_\pi << 2m_N$$

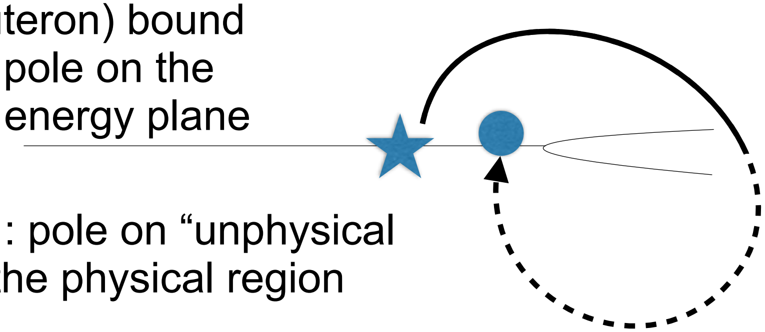


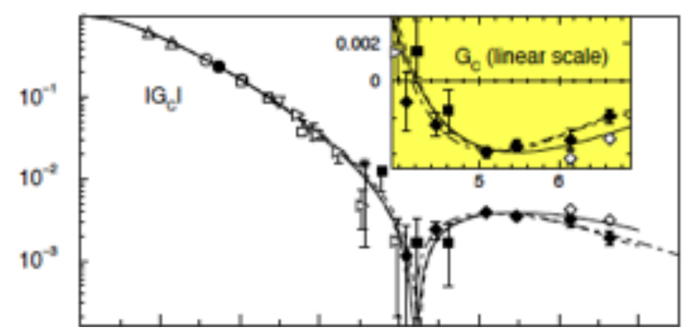
Wave function effect

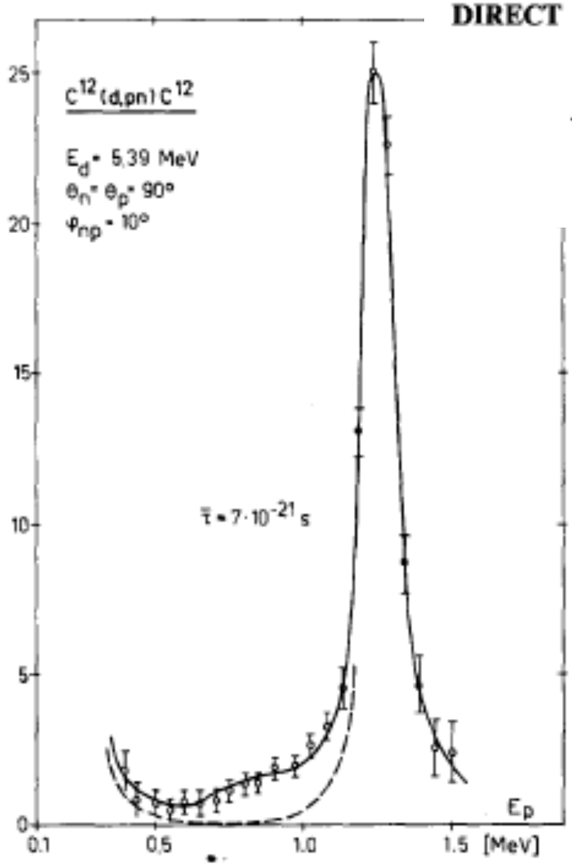


$^3S_1$ (deuteron) bound state : pole on the physical energy plane

$^1S_1$ virtual state : pole on "unphysical sheet" close the physical region







DIRECT DETERMINATION OF A SHORT NUCLEAR LIFETIME ($\approx 10^{-20}$ s) BY THE PROXIMITY SCATTERING METHOD

J. LANG, R. MÜLLER, W. WÖLFLI, R. BÖSCH and P. MARMIER

Laboratorium für Kernphysik, Eidg. Techn. Hochschule, Zürich †

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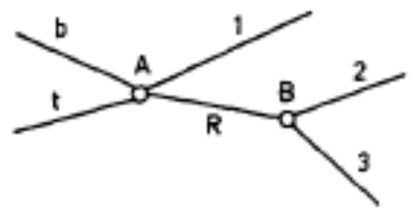
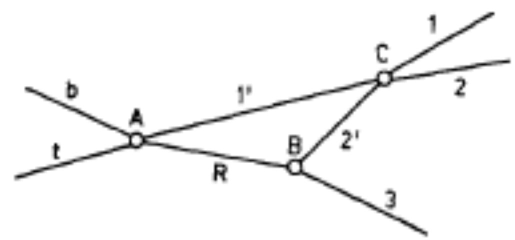
$$b+t \to 1+2+3 \qquad [d+{}^{12}C \to n+p+{}^{12}C],$$



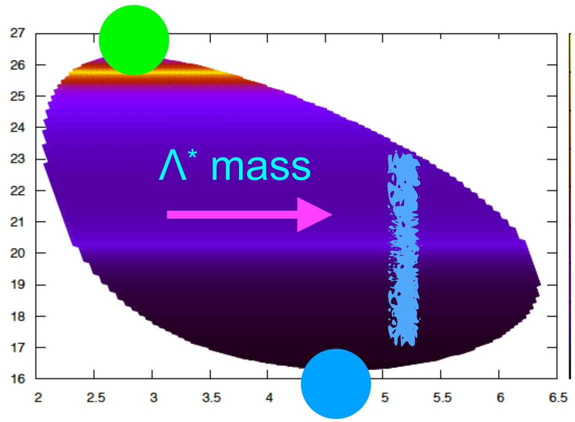


$$b+t \to 1+R, \quad \text{with } Q\text{–value } Q_1 \quad [d+{}^{12}C \to n+{}^{13}N^*, \quad Q_1 = -3.82 \text{ MeV}],$$

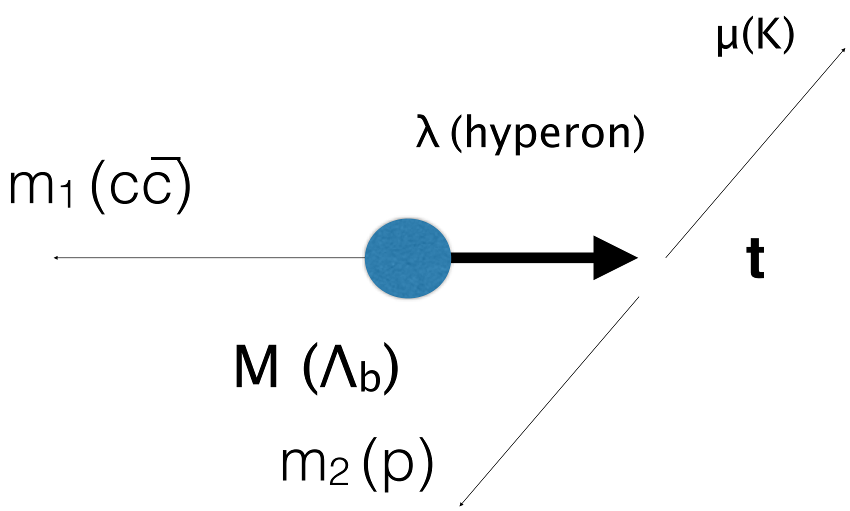
$$R \to 2+3 \quad \text{with } Q\text{–value } Q_2 \quad [{}^{13}N^* \to p+{}^{12}C, \quad Q_2 = 1.59 \text{ MeV}],$$

Classical picture

Coleman-Norton



t-channel resonance can produce s-channel “band” if:



all particles on-shell

m_2 and m_1 collinear

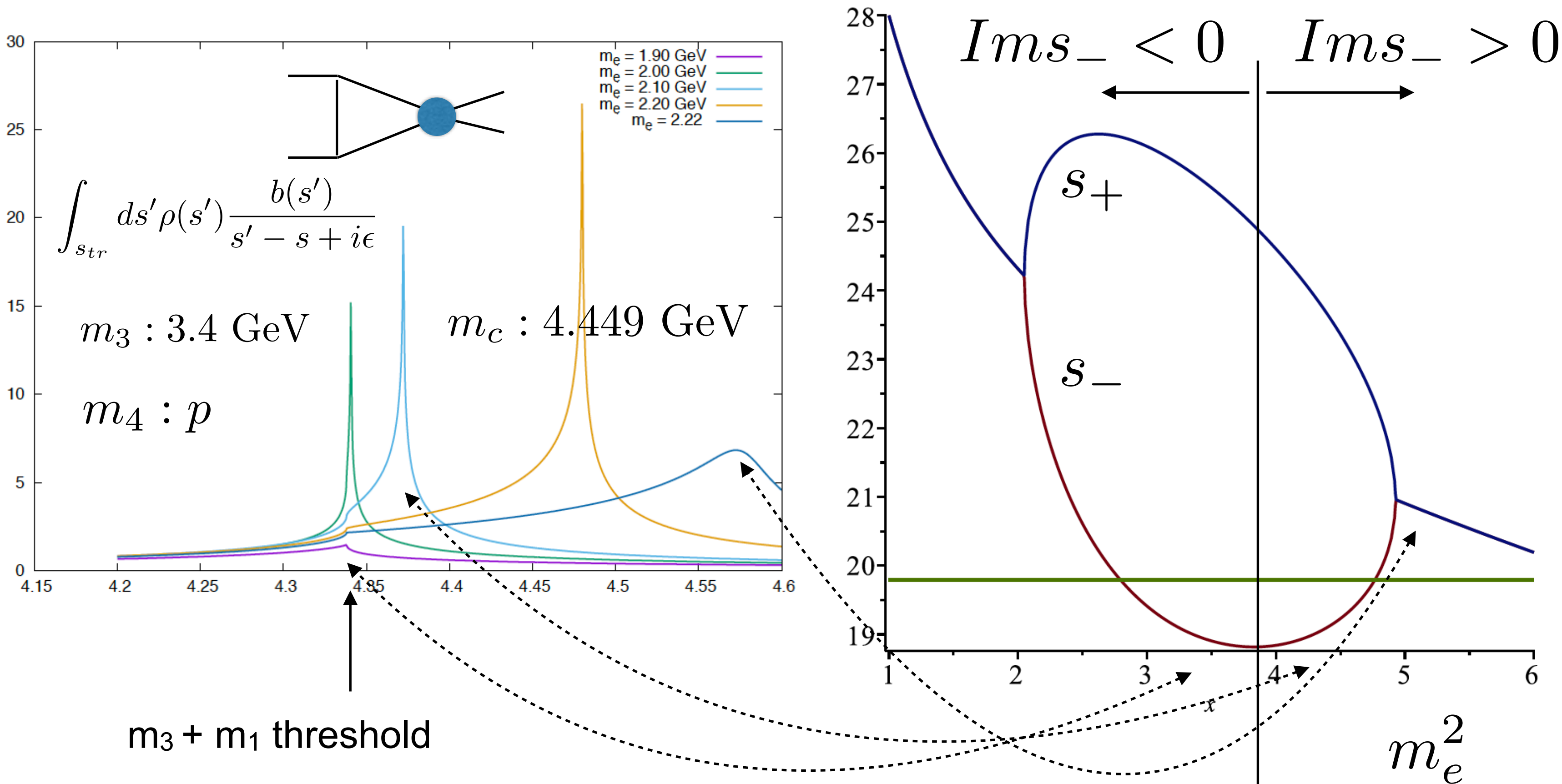
$v(m_2) > v(m_1)$

Example : Pc Kinematics

m_1	m_3
m_2	m_4

$$s_{\pm} = -m_e^2 + p_2^2 + p_3^2 + \frac{(m_e^2 + p_1^2 - p_3^2)(m_e^2 + p_4^2 - p_2^2)}{2m_e^2} \pm \frac{\lambda^{1/2}(m_e^2, p_1^2, p_3^2)\lambda^{1//2}(m_e^2, p_2^2, p_4^2)}{2m_e^2}$$

$m_1 : \Lambda_b \quad m_2 : K$



$m_3 + m_1$ threshold

- Singularities of b(s) are at $s=s_{\pm}$ $\quad b_l(s) = \frac{1}{2}\int_{-1}^{1} dz_s \frac{P_l(z_s)}{m_\Lambda^2 - t(s,z)}$

Summary Lecture 4

- In QCD light quark resonances appear clearly up to ~2GeV But one expected there to be an infinite number of them.
- At higher masses they are harder to find. To help discriminating between various hypotheses one should “consult” with expectations from quark model and duality arguments.
- Duality arguments are consistent with existence of multi quark hadrons.
- Veneziano model and generalizations could be used to implement these ideas in data analysis.
- Unlike non-relativistic theory, besides resonance poles one should work about “left-hand cuts’ (cusps), however, so far there is no unambiguous evidence for them in the data.

Thank you for your attention !