Meson states $|M^0\rangle$, $|\bar{M}^0\rangle$

Flavour operator (strangeness, charm, beauty)

$$\mathcal{F}\,|M^0\rangle = +\,|M^0\rangle \qquad \mathcal{F}\,|\bar{M}^0\rangle = -\,|\bar{M}^0\rangle$$

CP operator

$$CP\,|M^0\rangle = e^{i\xi_{CP}}\,|\bar{M}^0\rangle \qquad CP\,|\bar{M}^0\rangle = e^{-i\xi_{CP}}\,|M^0\rangle$$

CP eigenstates

$$|M_\pm\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\xi_{CP}/2}\,|M^0\rangle \pm e^{i\xi_{CP}/2}\,|\bar{M}^0\rangle\right)$$

Popular choice: $\xi_{CP} = 0$ (also used: $\xi_{CP} = \pi$)

$$\Rightarrow |M_\pm\rangle = \frac{1}{\sqrt{2}}\left(|M^0\rangle \pm |\bar{M}^0\rangle\right)$$

$$CP|M^0\rangle = +\,|\bar{M}^0\rangle \qquad CP|\bar{M}^0\rangle = +\,|M^0\rangle$$

Introduce effective Hamiltonian $\mathcal{H}$ to describe evolution of states in M^0-$\bar{M}^0$ space

$$i\hbar \frac{d}{dt}|\psi\rangle = \mathcal{H}|\psi\rangle,$$

with $\mathcal{H} = \mathcal{M} - \frac{i}{2}\Gamma$, $\mathcal{M}$ & Γ Hermitian,

and $|\psi\rangle = a(t)|M^0\rangle + b(t)|\bar{M}^0\rangle$.

$$\mathcal{H} = \underset{\uparrow \atop \text{strong+EM}}{H_0} + \underset{\nwarrow \atop \text{weak}}{H_{int}} + \sum_n \frac{H_{int}|n\rangle\langle n|H_{int}}{m_0 - E_n}$$

$|M^0\rangle$ and $|\bar{M}^0\rangle$ are eigenstates of H_0 but not of weak Hamiltonian

Element are given by

$$M_{11} = m_0 + \langle M^0|H_{int}|M^0\rangle + \sum_n \mathcal{P}\frac{|\langle n|H_{int}|M^0\rangle|^2}{m_0 - E_n}$$

$$M_{22} = m_0 + \langle \bar{M}^0|H_{int}|\bar{M}^0\rangle + \sum_n \mathcal{P}\frac{|\langle n|H_{int}|\bar{M}^0\rangle|^2}{m_0 - E_n}$$

$$M_{12} = M_{21}^* = \langle M^0|H_{int}|\bar{M}^0\rangle + \sum_n \mathcal{P}\frac{\langle M^0|H_{int}|n\rangle\langle n|H_{int}|\bar{M}^0\rangle}{m_0 - E_n}$$

$\mathcal{P}$ indicates principal part of what is an integral over final states

M_{12} involves virtual final states

$$\Gamma_{11} = 2\pi \sum_n \delta(m_0 - E_n) \left| \langle n | H_{int} | M^0 \rangle \right|^2$$

$$\Gamma_{22} = 2\pi \sum_n \delta(m_0 - E_n) \left| \langle n | H_{int} | \bar{M}^0 \rangle \right|^2$$

partial decay widths

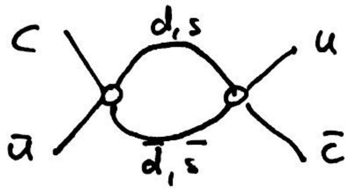
$$\Gamma_{12} = \Gamma_{21}^* = 2\pi \sum_n \delta(m_0 - E_n) \underbrace{\langle M^0 | H_{int} | n \rangle \langle n | H_{int} | \bar{M}^0 \rangle}_{\text{matrix elements involving final states common to } M^0 \text{ and } \bar{M}^0}$$

Theoretical approach:

calculate elements of Γ from measurements of branching fractions.

$\Rightarrow$ currently insufficient experimental input for reliable calculations

Contributions to Γ_{12}



3 diagrams : Γ_{dd} , Γ_{ds} , Γ_{ss}

$\Rightarrow$ with CKM factors $\lambda_i \lambda_j$ $\quad i,j = d, s$

and $\lambda_i = \lambda_{ci} \lambda_{ui}^*$

Can now write

$$\Gamma_{12} = -\left(\lambda_s^2 \Gamma_{ss} + 2 \lambda_s \lambda_d \Gamma_{sd} + \lambda_d^2 \Gamma_{dd} \right)$$

CKM unitarity : $\lambda_d + \lambda_s + \lambda_b = 0$

Eliminate λ_d : $\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2 \Gamma_{sd} + \Gamma_{dd} \right) + 2 \lambda_s \lambda_b \left(\Gamma_{sd} - \Gamma_{dd} \right) - \lambda_b^2 \Gamma_{dd}$

Note : $\lambda_d \simeq \lambda_s \simeq \lambda$ and $\lambda_b \simeq \lambda^5$ $\quad$ where $\lambda \simeq 0.22$

In $SU(3)_F$: $\Gamma_{ss} = \Gamma_{sd} = \Gamma_{dd}$

$\Rightarrow$ first two terms cancel $\quad \leftarrow$ GIM cancellation

$\Rightarrow$ $\Gamma_{12} \simeq -\lambda_b^2 \Gamma_{dd}$ $\quad \leftarrow$ CKM suppression

$\Rightarrow$ extremely small and theoretically difficult

Physical states are eigenstates of $\mathcal{H}$ with eigenvalues

$$\mathcal{H}|M_{1,2}\rangle = \lambda_{1,2}|M_{1,2}\rangle \qquad \lambda_{1,2} \equiv m_{1,2} - i\Gamma_{1,2}/2$$

with masses $m_{1,2}$ and widths $\Gamma_{1,2}$ such that

$$|M_{1,2}(t)\rangle = e^{-im_{1,2}t}e^{-\Gamma_{1,2}t/2}|M_{1,2}(0)\rangle \qquad (1)$$

Define in general

$$|M_1\rangle = p_1|M^0\rangle + q_1|\bar{M}^0\rangle \qquad |M_2\rangle = p_2|M^0\rangle - q_2|\bar{M}^0\rangle$$

with $|p_1|^2 + |q_1|^2 = 1 = |p_2|^2 + |q_2|^2$

CPT implies $q_1/p_1 = q_2/p_2 \equiv q/p$

$$\Rightarrow |M_{1,2}\rangle = p|M^0\rangle \pm q|\bar{M}^0\rangle \qquad (2)$$

and $|M_0\rangle = \frac{1}{2p}\left(|M_1\rangle + |M_2\rangle\right) \qquad |\bar{M}_0\rangle = \frac{1}{2q}\left(|M_1\rangle - |M_2\rangle\right)$ (3)

Note: $\langle M_1|M_2\rangle = |p|^2 - |q|^2$ not generally orthogonal

Inserting (1) in (3) and replacing the initial states by (2),

$$|M^0(t)\rangle = \frac{1}{2p}\Big[\left(e^{-im_1 t}e^{-\Gamma_1 t/2} + e^{-im_2 t}e^{-\Gamma_2 t/2}\right)p\,|M^0\rangle$$
$$+\left(e^{-im_1 t}e^{-\Gamma_1 t/2} - e^{-im_2 t}e^{-\Gamma_2 t/2}\right)q\,|\bar{M}^0\rangle\Big]$$

Introduce $\Delta m \equiv m_2 - m_1$ and $\Delta\Gamma \equiv \Gamma_2 - \Gamma_1$

$$|M^0(t)\rangle = \frac{1}{2}e^{-im_1 t}e^{-\Gamma_1 t/2}\Big[\left(1 + e^{-i\Delta m t}e^{-\Delta\Gamma t/2}\right)|M^0\rangle + \left(1 - e^{-i\Delta m t}e^{-\Delta\Gamma t/2}\right)\frac{q}{p}|\bar{M}^0\rangle\Big]$$

$$= f_+(t)\,|M^0\rangle + \frac{q}{p}f_-(t)\,|\bar{M}^0\rangle \qquad (4)$$

with $f_\pm(t) \equiv \frac{1}{2}e^{-im_1 t}e^{-\Gamma_1 t/2}\left(1 \pm e^{-i\Delta m t}e^{-\Delta\Gamma t/2}\right)$

Equivalently get

$$|\bar{M}^0\rangle = \frac{p}{q}f_-(t)\,|M^0\rangle + f_+(t)\,|\bar{M}^0\rangle \qquad (5)$$

Define $m \equiv \frac{m_1 + m_2}{2}$; $\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$; $x \equiv \frac{\Delta m}{\Gamma}$; $y \equiv \frac{\Delta\Gamma}{2\Gamma}$

Can now calculate

$$P(\overset{(-)}{H}{}^0 \to \overset{(-)}{H}{}^0, t) = |f_+(t)|^2 = \frac{1}{2} e^{-\Gamma t}\left[\cosh(y\Gamma t) + \cos(x\Gamma t)\right]$$

$$P(H^0(t) \to \bar{H}^0) = \left|\frac{q}{p}\right|^2 |f_-(t)|^2 = \frac{1}{2}\left|\frac{q}{p}\right|^2 e^{-\Gamma t}\left[\cosh(y\Gamma t) - \cos(x\Gamma t)\right] \quad (6$$

$$P(\bar{H}^0(t) \to H^0) = \left|\frac{p}{q}\right|^2 |f_-(t)|^2 = \frac{1}{2}\left|\frac{p}{q}\right|^2 e^{-\Gamma t}\left[\cosh(y\Gamma t) - \cos(x\Gamma t)\right]$$

With $x, y < 1\%$ can use Taylor expansion:

$$P(\overset{(-)}{H}{}^0(t) \to \overset{(-)}{H}{}^0) \simeq \frac{1}{2} e^{-\Gamma t}\left(2 - \frac{x^2 - y^2}{2}(\Gamma t)^2\right) \simeq e^{-\Gamma t}$$

$$P(H^0(t) \to \bar{H}^0) \simeq \frac{1}{2}\left|\frac{q}{p}\right|^2 e^{-\Gamma t}\, \frac{x^2 + y^2}{2}(\Gamma t)^2 \qquad (7)$$

$$P(\bar{H}^0(t) \to H^0) \simeq \frac{1}{2}\left|\frac{p}{q}\right|^2 e^{-\Gamma t}\, \frac{x^2 + y^2}{2}(\Gamma t)^2$$