Theoretical Description of the $e^+e^- \to \psi(2S)\,\pi^+\,\pi^-$

Daniel Molnar

Collaborators:
Igor Danilkin and Marc Vanderhaeghen

"10th Workshop on Hadron Physics in China and Opportunities Worldwide"
July, 2018

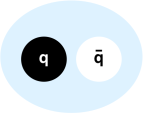
SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

1 Exotic States

Quark Model

- **Mesons:**



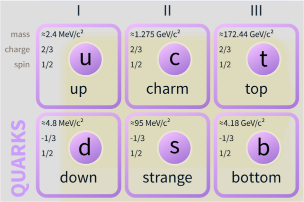
$$P = (-1)^{L+1} \quad C = (-1)^{L+S}$$

L	S	J^{PC}
0	0	0^{-+}
	1	1^{--}
1	0	1^{+-}
	1	$0^{++}\ 1^{++}\ 2^{++}$
2	0	2^{-+}
	1	$1^{--}\ 2^{--}\ 3^{--}$

Forbidden Quantum Numbers:

$$0^{--},\ 0^{+-},\ 1^{-+},\ 2^{+-},\ \cdots$$

Quarks



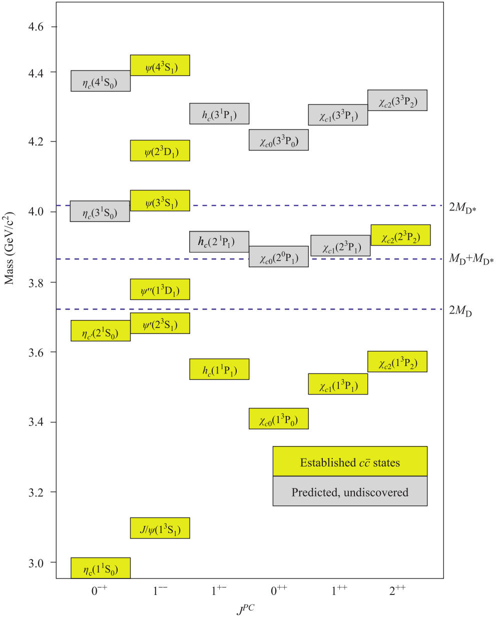
- Heavy Quarks:

$$\frac{d^2}{dr^2}u(r) + 2\mu\left[E - V(r) - \frac{l(l+1)}{2\mu r^2}\right]u(r) = 0$$

- Phenomenological Potential:

$$V(r) \simeq -\frac{\kappa}{r} + br + \text{spin-terms}$$

S. L. Olsen *Front.Phys.* *(2015)*



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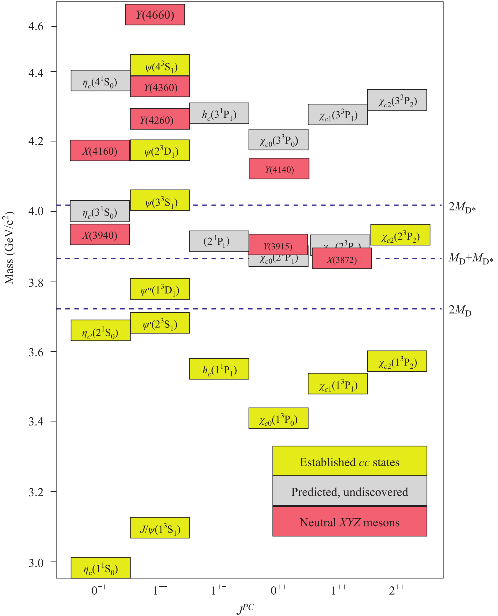

Alternative explanations:

- Hybrids and Glueballs
- Tetraquark
- Molecular state
- Hadrocharmonium
- Kinematics effects

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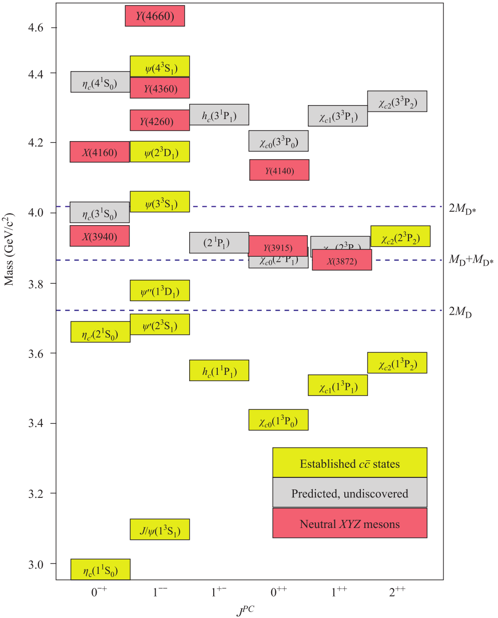

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No Unique Structure

No new physics! All interpretations are based on QCD.

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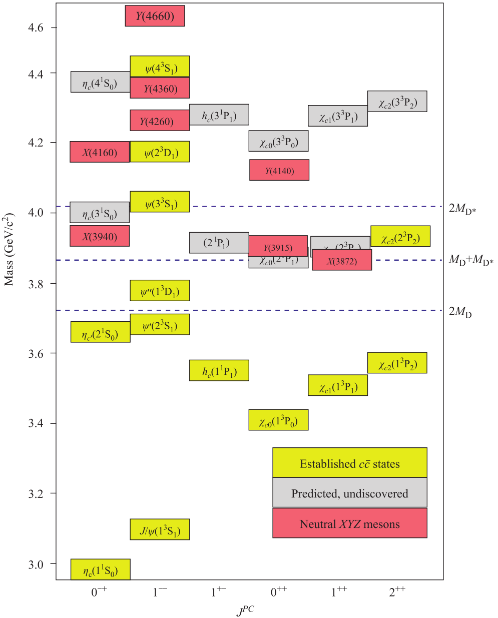

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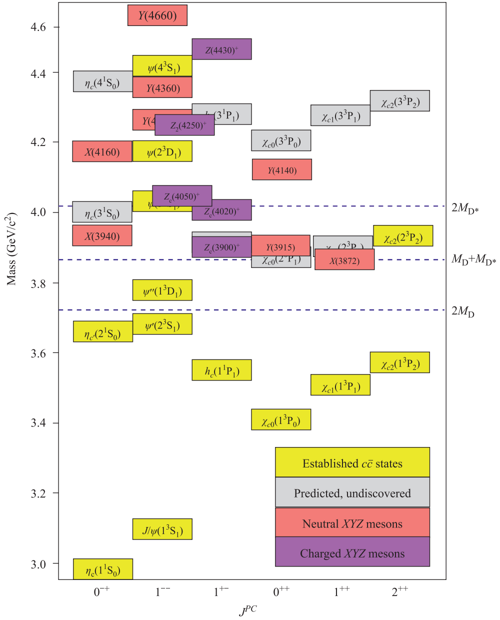
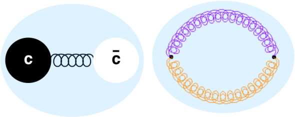
Charged Exotic Mesons

- Confirmed in 2013 by Belle and BESIII
- $c\bar{c} + q_i\bar{q}_j$ $(i \neq j)$

S. L. Olsen *Front.Phys.* (2015)

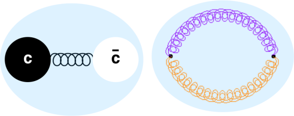


Hybrids and Glueballs



↓ Charged exotic states
↓ Search for the missing J^{PC}
(GlueX and Panda)

Hybrids and Glueballs



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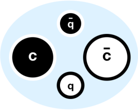
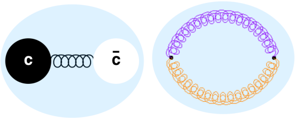
Tetraquark



↓ Too many states predicted

Hybrids and Glueballs



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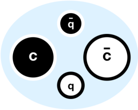

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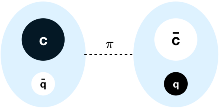
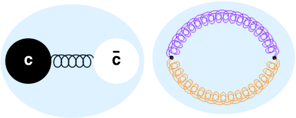
Molecular State



- Not enough to explain radiative decays
- Production at high-p_T

Hybrids and Glueballs



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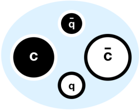

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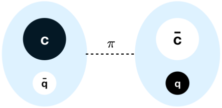

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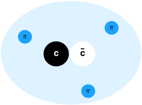
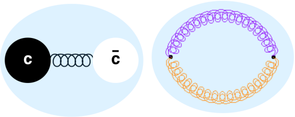
Hadrocharmonium



↓ Decays predominantly into D-mesons

If the force $\begin{cases} \downarrow, & \text{not state} \\ \uparrow, & \text{D-mesons} \end{cases}$

Hybrids and Glueballs



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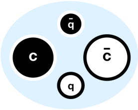

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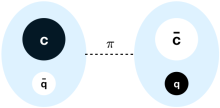
Molecular State



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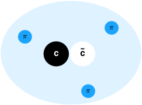

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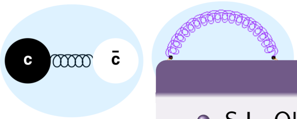
Non-Resonance Interpretation

Threshold effects: Kinematics effects due to the opening of a channel producing a cusp effect.

Triangle singularities: when particles can go simultaneously on shell in a triangle loop.

Both should be a small contributions in the final observable.

Hybrids and Glueballs



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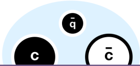
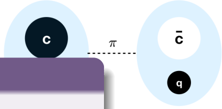
Tetraquark



Molecular State



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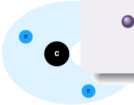
at high-p_T

Hadroch



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Recent Review Papers

- S.L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90, 15003 (2018)
- R.F. Lebed, R.E. Mitchell, E. S. Swanson, Progress in Particle and Nuclear Physics 93, 143 (2017)
- F-K,Guo *et al.* arXiv:1705.00141(2017)
- H-X. Chen *et al.* Physics Reports 639, 1-121 (2016)
- M.R. Shepherd, J.J. Dudek, R.E. Mitchell, Nature 534 (2016) no.7608, 487-493

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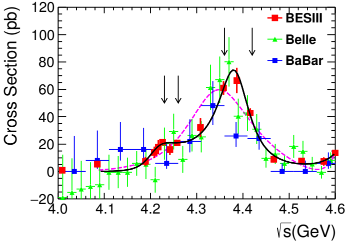
Triangle singularities: when particles can go simultaneously on shell in a triangle loop.

Both should be a small contributions in the final observable.

$e^+e^- \to \psi(2S)\,\pi^+\pi^-$

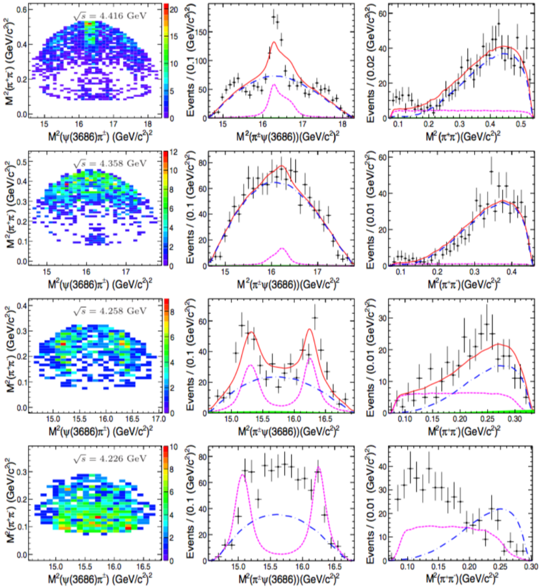
- **Y(4220) + Y(4390)**



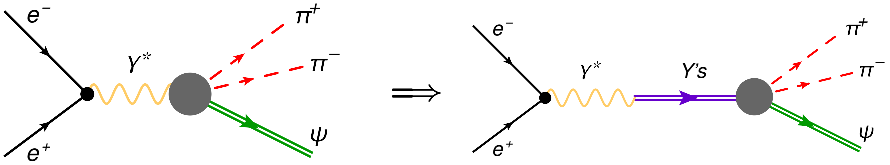
$Z_c(3900) + Z_c(4030)$

- Two charged exotic states!
- No consistent description
- Below $K\bar{K}$ threshold

BESIII PRD (2017)



2 Formalism



Double Differential Cross Section

$$\frac{\partial^2\sigma}{\partial s\partial t} = \frac{1}{3}\frac{e^2}{(2\pi)^3}\frac{1}{2^5}\frac{(q^2+2m_e^2)}{\sqrt{q^2(q^2-4m_e^2)}}\frac{1}{(q^2)^3}\sum_{\lambda_1\lambda_2}|\mathcal{H}_{\lambda_1\lambda_2}|^2$$

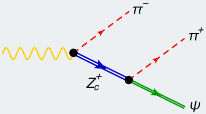
$$\langle\pi\pi\psi(\lambda_2)|\,\mathcal{T}\,|\gamma^*(\lambda_1)\rangle = (2\pi)^4\delta(q-p_\psi-p_{\pi^+}-p_{\pi^-})\,\mathcal{H}_{\lambda_1\lambda_2}$$

Independent Helicity Amplitudes

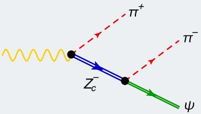
P-symmetry: $\mathcal{H}_{++}$, $\mathcal{H}_{+-}$, $\mathcal{H}_{+0}$, $\mathcal{H}_{0+}$ and $\mathcal{H}_{00}$

Isobar Decomposition

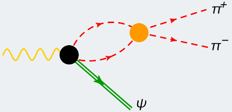
t-channel



u-channel



s-channel



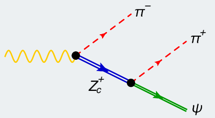
Isobar Decomposition

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) = \sum_{J \text{ even}}^{\infty} (2J+1) h^{(J)}_{\lambda_1\lambda_2}(s) d^{(J)}_{\Lambda,0}(\theta)$$

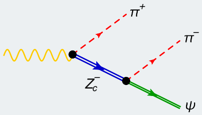
$$\approx \sum_{J \text{ even}}^{J_{max}} (2J+1) h^{(J)}_{\lambda_1\lambda_2}(s) d^{(J)}_{\Lambda,0}(\theta_s) + \sum_{J \text{ even}}^{J_{max}} (2J+1) h^{(J)}_{\lambda_1\lambda_2}(t) d^{(J)}_{\Lambda,0}(\theta_t) + \sum_{J \text{ even}}^{J_{max}} (2J+1) h^{(J)}_{\lambda_1\lambda_2}(u) d^{(J)}_{\Lambda,0}(\theta_u)$$

$$J_{max} = 0 \implies h^0_{\lambda_1\lambda_2}(s) + h^0_{\lambda_1\lambda_2}(t) + h^0_{\lambda_1\lambda_2}(u)$$

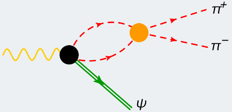
t-channel



u-channel



s-channel

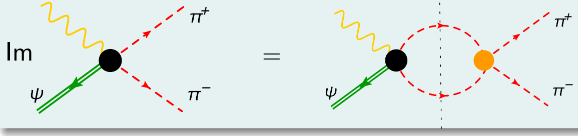


$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) = h^{(0),Z_c}_{\lambda_1\lambda_2}(t) + h^{(0),Z_c}_{\lambda_1\lambda_2}(u) + \underbrace{\bar{h}^{(0)}_{\lambda_1\lambda_2}(s)}_{\text{Rescatt. in s-channel}}$$

$$\mathcal{H}_{+-}(s,t,u) = \mathcal{H}_{+0}(s,t,u) = \mathcal{H}_{0+}(s,t,u) \approx 0$$

$$\sum_{\lambda_1\lambda_2} |\mathcal{H}_{\lambda_1\lambda_2}|^2 = 2|\mathcal{H}_{++}|^2 + |\mathcal{H}_{00}|^2 \approx 3|\mathcal{H}_{++}|^2$$

Unitarity in the s-channel



$$\mathrm{Im}[h_{\gamma^*\to\psi\,\pi\pi}(s)] = h_{\gamma^*\to\psi\,\pi\pi}(s)\ \rho_{\pi\pi(s)}\ t^*_{\pi\pi}(s)$$

- The pions interaction amplitude can be written in terms of the phase shift:

$$t^*_{\pi\pi}(s) = \frac{e^{-i\delta_{\pi\pi}(s)}\sin\delta_{\pi\pi}(s)}{\rho_{\pi\pi}(s)}$$

- Since $h_{\gamma^*\to\psi\,\pi\pi}(s) = |h_{\gamma^*\to\psi\,\pi\pi}(s)|e^{i\phi(s)} \implies \phi(s) = \delta_{\pi\pi}(s)$, Watson Theorem
- Therefore, we can use the $\delta_{\pi\pi}(s)$ through the Omnes Function:

$$\Omega^J_l(s) = exp\left[\frac{s}{\pi}\int_{4m^2}^{\infty}\frac{ds'}{s'}\frac{\delta^J_l(s')}{s'-s}\right]$$

Dispersion Relations

- Partial Wave Helicity Amplitudes have Kinematic Constraints:

$$h^{(0)}_{++}(s) \pm h^{(0)}_{00}(s) \sim (s - (q \pm m_\psi)^2)$$

- Dispersion Relation for

$$\Longrightarrow \quad \frac{h^{(0)}_{++} \pm h^{(0)}_{00} - (h^{(0,Z_c)}_{++} \pm h^{(0,Z_c)}_{00})}{(s - (q \pm m_\psi)^2)} \Omega^{-1}(s)$$

$$\bar{h}^{(0)}_{++} \equiv h^{(0,Resc)}_{++} - h^{(0,Z_c)}_{++} = \Omega(s) \Bigg\{ a(q^2) + b(q^2)s$$

$$- \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} Im[\Omega^{-1}(s')] \left[\left(\frac{1}{s'-s} - \frac{s'-q^2-m_\psi^2}{\lambda(s',q^2,m_\psi^2)} \right) h^{(0,Z_c)}_{++}(s') - \frac{2qm_\psi}{\lambda(s',q^2,m_\psi^2)} h^{(0,Z_c)}_{00}(s') \right] \Bigg\}$$

- 2 subtraction constants to reduce the sensitive to high energy.

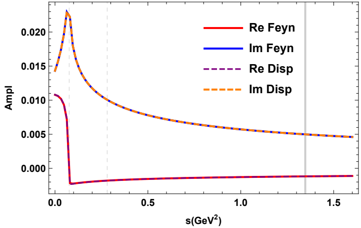
Anomalous Threshold

- Depending on the kinematics new nonphysical singularities might appear.
- Anomalous singularity position: $s_{\text{anomalous}} = \sum_i m_i^2 - 2m_z^2$
- The anomalous piece that emerges because the anomalous branch point moves onto the first Riemann sheet distorting the integration contour. Effectively, that can be written as

$$\int_{-1}^{1} \frac{dz}{t - m_z^2} = -\frac{2}{k(s)} log\left(\frac{X(s)+1}{X(s)-1}\right) - i\frac{4\pi}{k(s)}\theta(s_1 < s < s_{\text{anomalous}})$$

Cross-Check

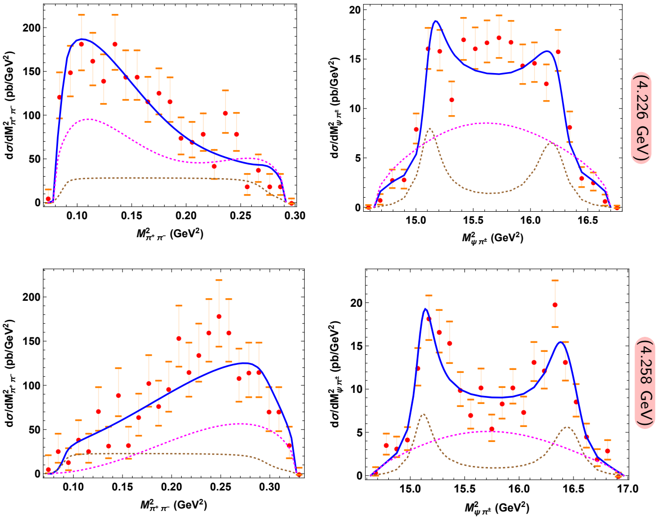
- The modified dispersion relation with the additional anomalous piece should be the same as the scalar triangle loop calculated via traditional method.



S. Mandelstam, PRL (1960); W. Lucha *et al*, PRD (2007); M. Hoferichter *et al*, Mod. Phys. Conf. Ser. (2014)

3 $\psi(2S)\ \pi^+\pi^-$

$Z_c(3900)$



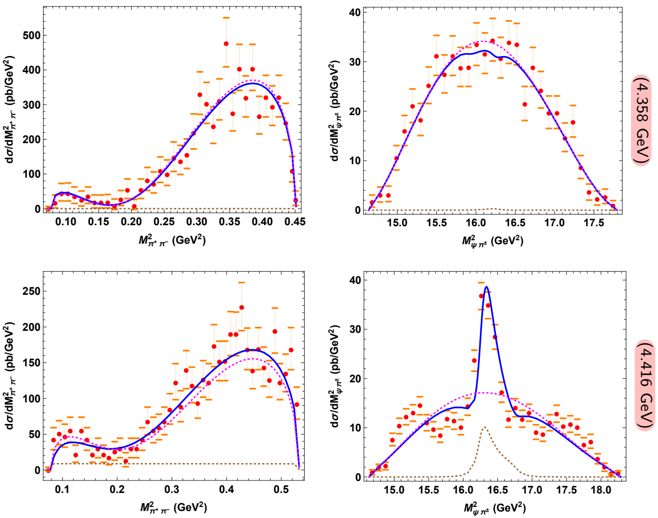
$\chi^2_{red} = 1.27$

- $a(q^2)$ and $b(q^2)$ are complex numbers
- Global normalization: $N(q^2, C_{\gamma^* Z_c \pi}, C_{Z_c \psi \pi})$

$\chi^2_{red} = 0.96$

Data is normalized using the total cross section

$Z_c(4024)$



$\chi^2_{red} = 0.64$

- $a(q^2)$ and $b(q^2)$ are complex numbers
- Global normalization: $N(q^2, C_{\gamma^* Z_c \pi}, C_{Z_c \psi \pi})$

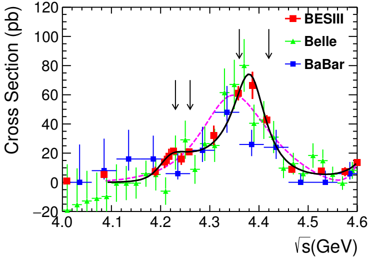
$\chi^2_{red} = 1.97$

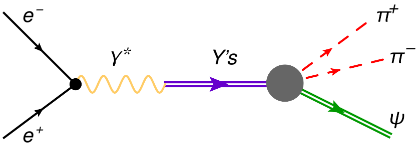
Data is normalized using the total cross section

4 Perspectives & Summary

$\psi(2S)\ \pi^+\pi^-$ Total Cross Section

- **Y(4220) + Y(4390)**



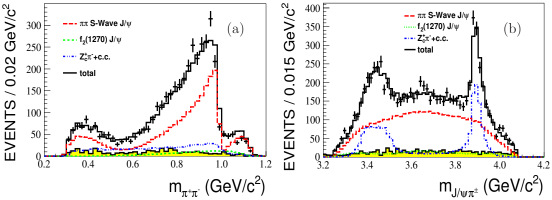


$$\sigma = \frac{1}{3}\frac{e^2}{(2\pi)^3}\frac{1}{2^5}\frac{(q^2+2m_e^2)}{\sqrt{q^2(q^2-4m_e^2)}}\frac{1}{(q^2)^3}\int_{4m_\pi^2}^{(q-m\psi)^2} ds \int_{t_{min}}^{t_{max}} dt \sum_{\lambda_1\lambda_2} |\mathcal{H}_{\lambda_1\lambda_2}|^2$$

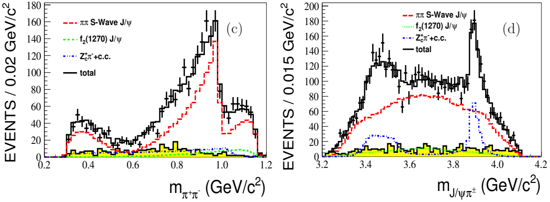
- The global normalization from each fit should contain the information about the total cross section:

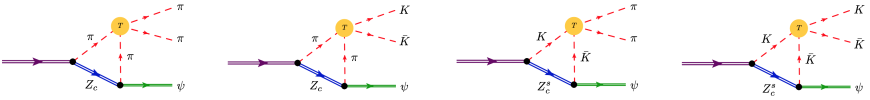
$$N(q^2) = \left| \frac{C_{Y_1 Z_c \pi} C_{Z_c \psi \pi}}{q^2 - M_{Y_1}^2 + i M_{Y_1}\Gamma_{Y_1}(q^2)} + \frac{C_{Y_2 Z_c \pi} C_{Z_c \psi \pi}}{q^2 - M_{Y_2}^2 + i M_{Y_2}\Gamma_{Y_2}(q^2)} e^{i\phi_2} \right|^2$$

$e^+e^- \to J/\psi\ \pi^+\pi^-$



BESIII PRL (2017)





Information about the strange partner of $Zc(3900)$

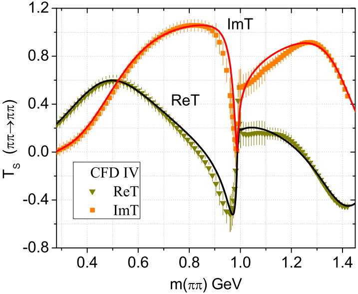
Coupled-Channel Final State Interaction

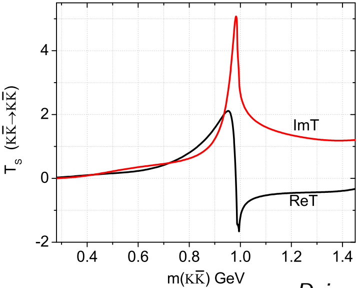
- $\mathbf{M_{K\bar{K}}} \approx 0.987$ GeV

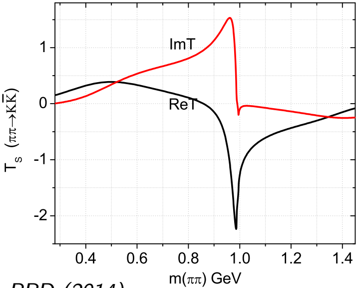
Coupled-Channel Unitarity

$$Im\mathcal{T}_{ij} = \sum_k \rho_k \mathcal{T}^*_{ik}\mathcal{T}_{kj}$$

$$Im\mathcal{A}_i = \sum_j \rho_j \mathcal{A}_j \mathcal{T}^*_{ji}$$







Dai and Pennington PRD (2014)

Summary

- Investigating the decays of exotic states is essential to understand their nature.

$e^+e^- \to \psi(2S)\,\pi^+\,\pi^-$

- The model independent $\pi\pi$-FSI is taken into account using the dispersion theory as well as explicit charged intermediate exotic states a simultaneous description of the invariant mass distributions is given for different e^+e^- center of mass energy regions;
- The $\pi\pi$-FSI seems to be the main mechanism to describe the $\pi\pi$-line shape for all the energies;
- The exotic state $Zc(3900)$ is sufficient to explain the data for the lowest energies, whereas for the energies around 4.4 GeV a heavier charged state $Zc(4024)$ clearly would explain the enhancement in the experimental data.

Thank you for listening!

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