Hybrid states in the framework of Dyson-Schwinger equations

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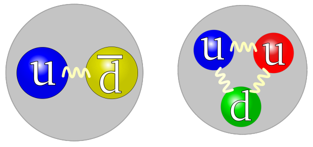
Spectrum of light hadrons

- The well-known light hadron is simple
  - It is qualitatively matches the constituent quark model by Gell-Mann and Zweig (1964).
    - Mesons built from a constituent-quark-antiquark ($Q\bar{Q}$) pair
    - Baryons constituted from three constituent quarks (QQQ)



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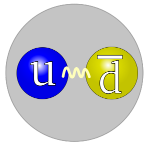

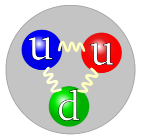

  where Q is u, d, s-quarks

- Gell-Mann and Zweig also raised possibility of multi-quark state
  - Tetraquark: $QQ\bar{Q}\bar{Q}$
  - Pentaquark: $Q\bar{Q}QQQ$

No candidate were then known, and they didn't know gluon

After ~50 years, in heavy quark systems, that now has changed

X, Y, Z,… pentaquark appears.

Spectrum of light hadrons

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- A new possibility appears, that is a system with valence gluon
  - hybrid/exotic meson - $GQ\bar{Q}$
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- There are a few plausible hybrid-meson candidates below 2 GeV
  - Dedicated searches for such states are underway at modern facilities (e.g. COMPASS @ CERN, GlueX @ JLab)

Model studies of hybrids

- Numerous models have employed to study spectrum of light hybrid mesons
  - Approaches are distinguished by their treatment/definition of constituent gluon
  - Their spectrum disagree each other

Table 1: Quantum numbers and approximate masses of non-strange hybrids in various models. The LQCD masses in parentheses are given for a pion mass of 396 MeV$/c^2$. $J^{PC}_{q\bar{q}'}$ and J^{PC}_g denote the quantum numbers of the $q\bar{q}'$ pair and the gluon, respectively, which couple to give J^{PC} of the hybrid. Bold numbers indicate spin-exotic quantum numbers. [B. Ketzer, Pos QNP2012(2012)]

Model	$J^{PC}_{q\bar{q}'}$	J^{PC}_g	J^{PC}	Mass (GeV$/c^2$)
Bag [1,2]	0^{-+}	1^{+-} (TE)	1^{--}	~ 1.7
	1^{--}	1^{+-} (TE)	$(0,\mathbf{1},2)^{-+}$	$\sim 1.3, 1.5, 1.9$
	0^{-+}	1^{--} (TM)	1^{+-}	heavier
	1^{--}	1^{--} (TM)	$(0,1,2)^{++}$	heavier
Flux tube [3,4]	0^{-+}	1^{+-}	1^{--}	1.7-1.9
	1^{--}	1^{+-}	$(0,\mathbf{1},2)^{-+}$	1.7-1.9
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	1^{--}	1^{-+}	$(\mathbf{0},1,\mathbf{2})^{+-}$	1.7-1.9
Constituent gluon [5,6]	0^{-+}	1^{--}	1^{+-}	1.3-1.8 / 2.1
	1^{--}	1^{--}	$(0,1,2)^{++}$	1.3-1.8 / 2.2
	1^{+-}	1^{--}	$(0,\mathbf{1},2)^{-+}$	1.8-2.2 / 2.2
	$(0,1,2)^{++}$	1^{--}	$1^{--}, (\mathbf{0},1,2)^{--}, (1,2,3)^{--}$	1.8-2.2 / 2.3
Constituent gluon / LQCD [7,8]	0^{-+}	1^{+-}	1^{--}	(2.3)
	1^{--}	1^{+-}	$(0,\mathbf{1},2)^{-+}$	(2.1, 2.0, 2.4)
	1^{+-}	1^{+-}	$(0,1,2)^{++}$	(> 2.4)
	$(0,1,2)^{++}$	1^{+-}	$1^{+-}, (\mathbf{0},1,\mathbf{2})^{+-}, (1,\mathbf{2},3)^{+-}$	(> 2.4)

[1]M. S. Chanowitz and S. R. Sharpe, Nucl. Phys. B 222, 211 (1983).

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Model studies of hybrids

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- Development of a reliable continuum method for calculating hybrid meson properties would be valuable
    - For interpretation of empirical observations
    - Provide insights into results obtained via the numerical simulation of LQCD

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Bound state problem

L	S	J^{PC}
0	0	0^{-+}
0	1	1^{--}
1	0	1^{+-}
1	1	0^{++}
1	1	1^{+-}

- $Q\bar{Q}$ mesons in quantum mechanics can't possess exotic quantum numbers: JPC=0+-, 0--,1-+ , etc.

0++	0+-	0-+	0--
1++	1+-	1-+	1--
2++	2+-	2-+	2--
3++	3+-	3-+	3--

- Nevertheless, exotic quantum numbers are allowed in relativistic two-body bound state

Si-xue Qin, et al., Phys.Rev. C85 (2012) 035202

ω	0.4	0.5	0.6
m_{0--}	0.814	0.940	1.053
m_{0+-}	1.186	1.252	1.323
m_{1-+}	1.234	1.277	1.318

- Studies of exotic mesons using simple truncation for Bethe-Salpeter kernel produce unrealistic spectra
- More sophisticated kernel can not remedied, it signal that exotic may contain explicit valence gluon degree of freedom

New perspective on hybrid mesons

- Can one produce sound treatment of hybrids using Poincaré-covariant Faddeev equation?
  - Treat these systems as bound states of valence-gluon, -quark and antiquark.
  - Each constituent is massive in their infrared

    region

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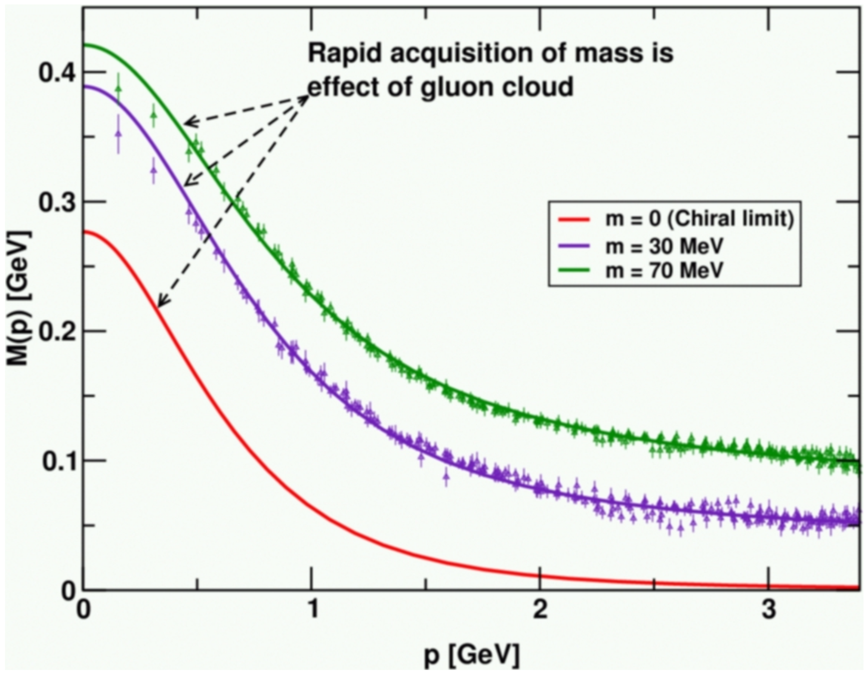
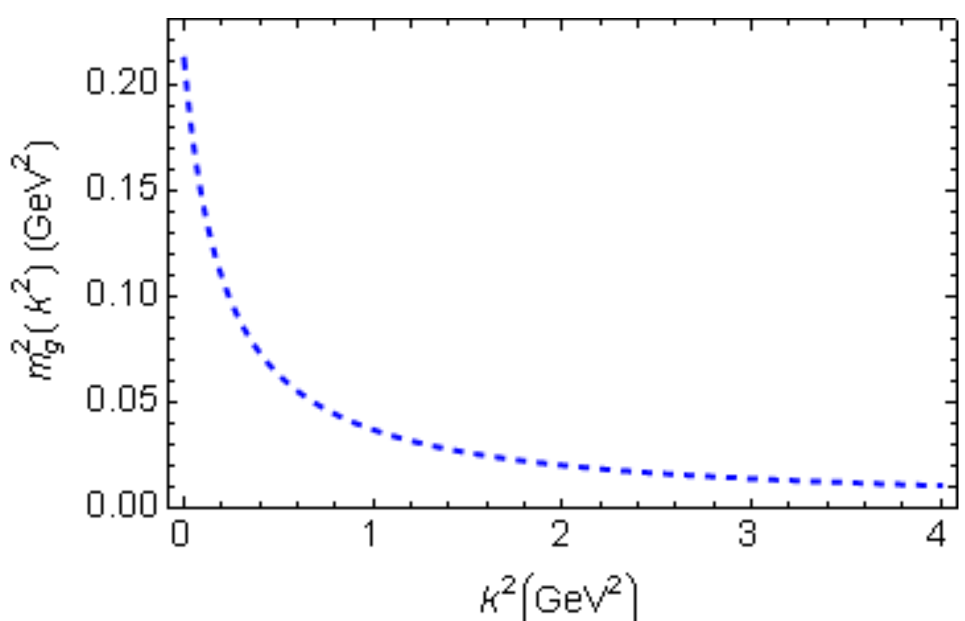


- Recall DSEs for quark propagator and gluon propagator
  - Quark is massive in its infrared region
  - Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

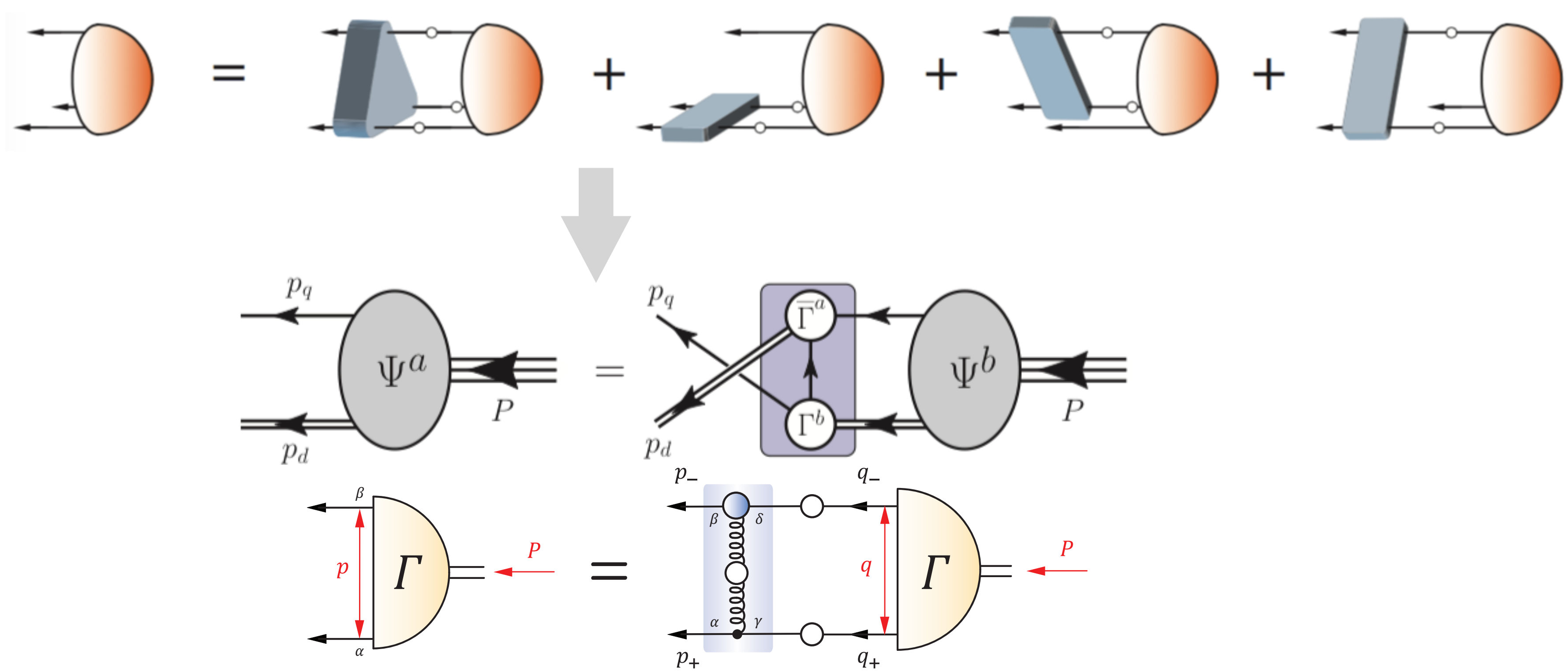
$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$



  - It implies gluon is massive in it's infrared region

Hints from baryons

- Baryon is a bound state of three valence quarks
  - The anti-triplet coloured diquark correlations play in simplifying the baryon three body problem



  - The spectrum obtained from quark-diquark picture is almost same as full 3-body Faddeev equation

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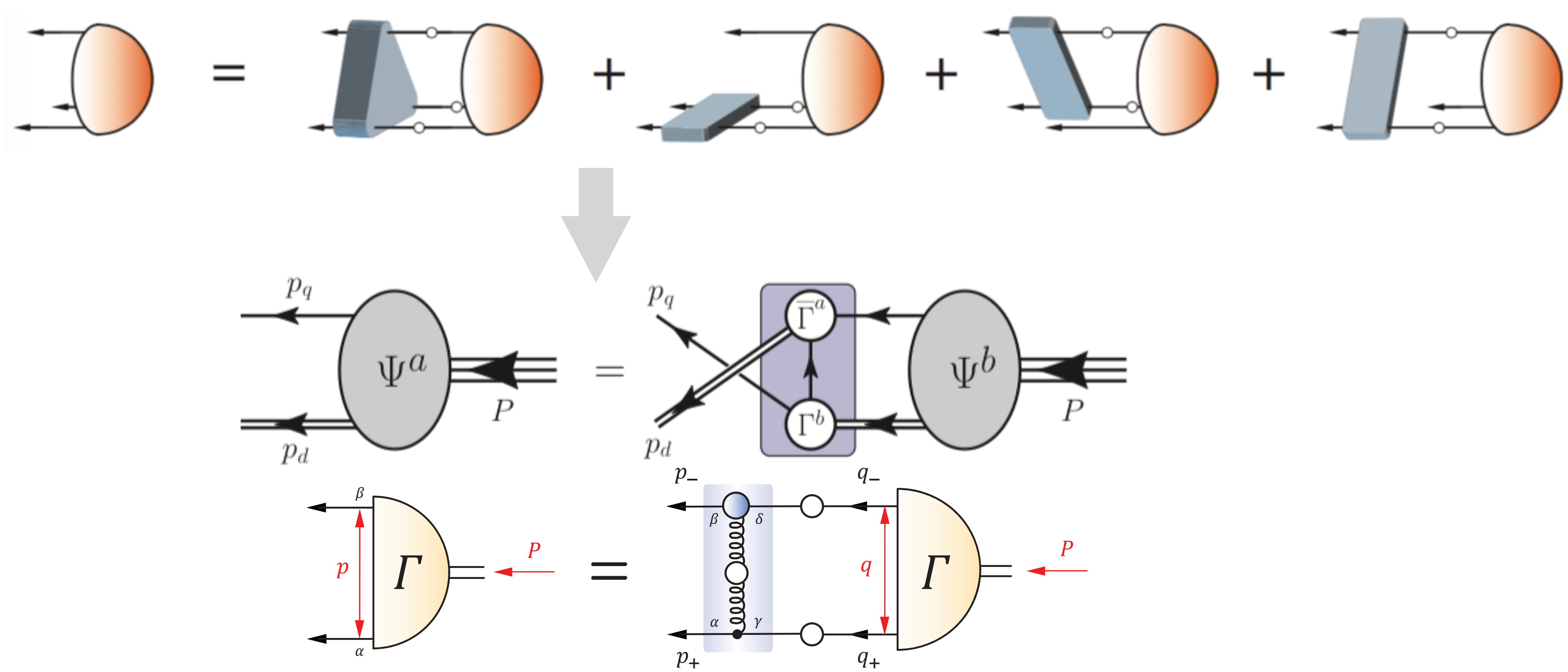


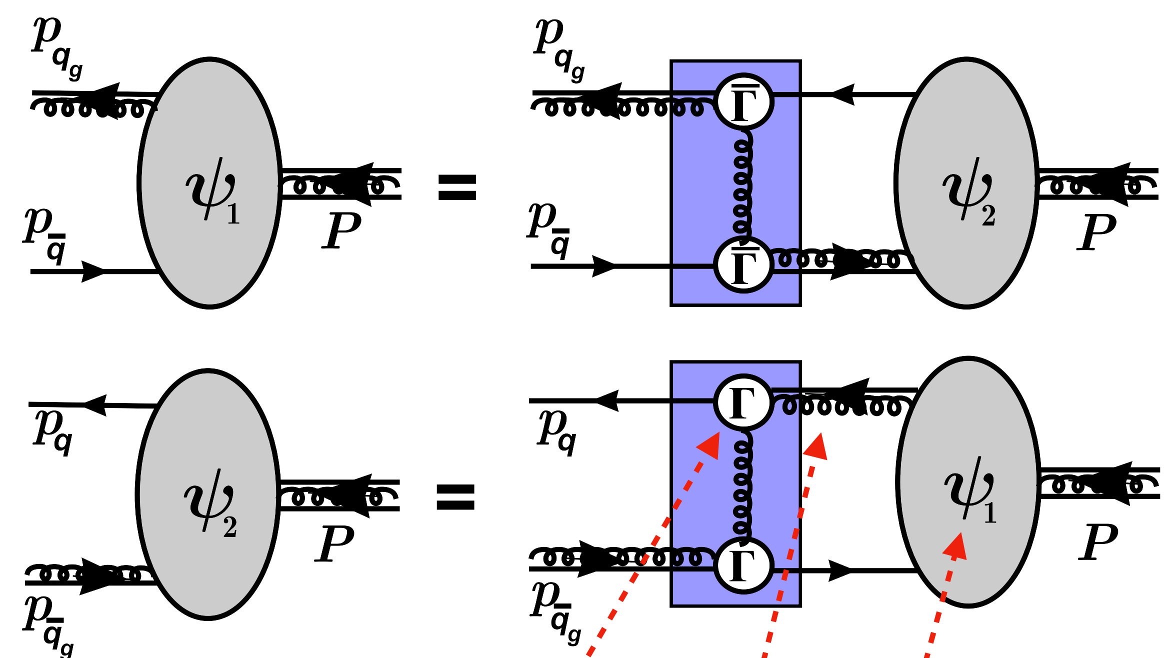
  - The spectrum obtained from quark-diquark picture is almost same as full 3-body Faddeev equation

- Can hybrid states be solved in this way?

The idea towards hybrids

- **Suppose** strong q_g and $\bar{q}_g$ correlation exist, then
  - Hybrids explained by coupled channel Faddeev-like bound state equation

    $\Psi = \Psi_1 + \Psi_2$, where Ψ_1 is Faddeev amplitude for $q_g\bar{q}$ and Ψ_2 is that for $q\bar{q}_g$



$$\Psi_1 = \Gamma^a_\mu(l;p_{qg})\, S_{gq}(p_{qg})\, \psi_1(p;P)$$

gq correlation amplitude

gq correlation propagator

bystander+correlation Faddeev amplitude

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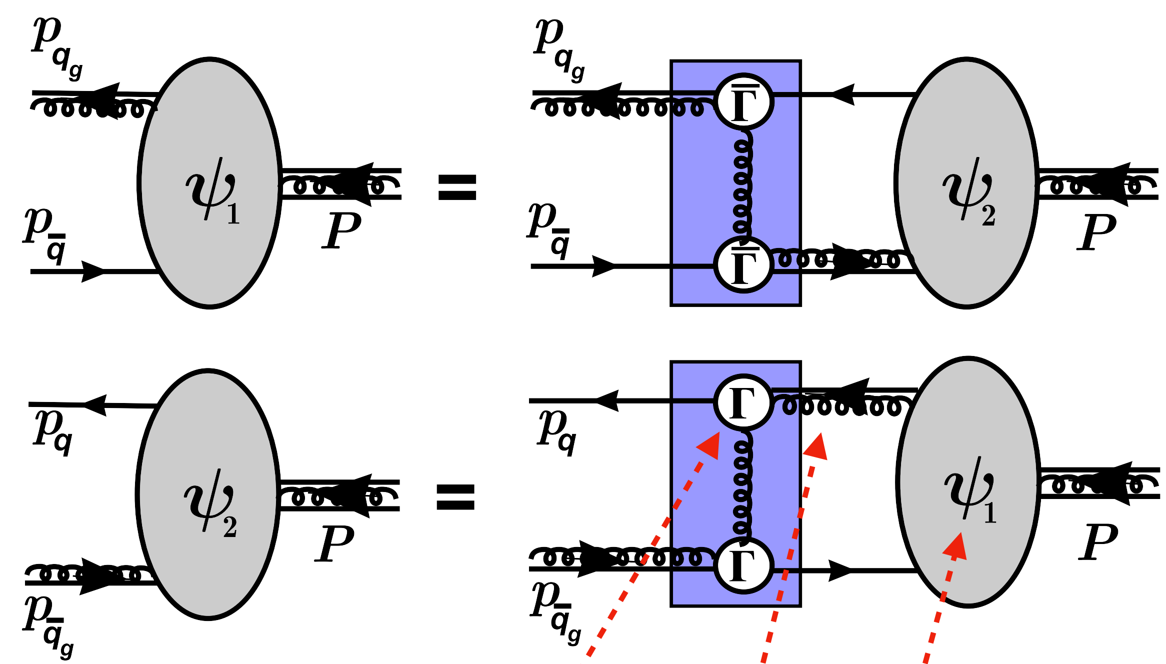
- **Challenge:**
  - confirm existence of tight gluon-quark correlations
  - determine their properties



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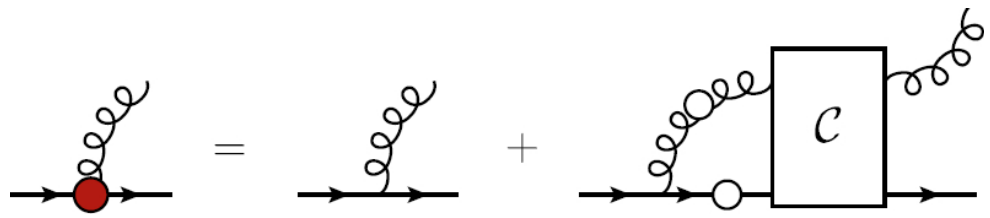
Gluon-quark correlations

- Using rainbow-ladder truncation for gluon-quark Bethe-Salpeter equation, and search for a pole solution



$$t^a\Gamma_\mu(p;Q) = -\int_{d\ell} \mathcal{G}(k^2)t^d\gamma_\rho S(\ell_+) \quad \text{valence quark}$$
$$\times\, t^c\Gamma_\lambda(\ell;Q)D_{\lambda\tau}(\bar{\ell}_-)\, f_{3g}(k^2)\, {}_0V^{bca}_{\rho\tau\mu}(k,\bar{\ell},-\bar{p}_-)$$

bare 3-gluon vertex

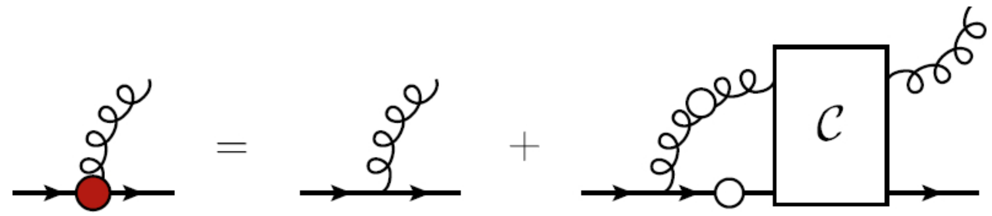
valence gluon

3g vertex dressing factor

continuum & lattice: 3g vertex greatly suppressed on k^2<1 GeV^2

- The gluon infrared mass ~1/2 m_{proton}
- The quark infrared mass ~1/3 m_{proton}
- The pole of gluon-quark corralation located at m_{qg} ~m_{proton}~1.0 GeV.

Gluon-quark correlations

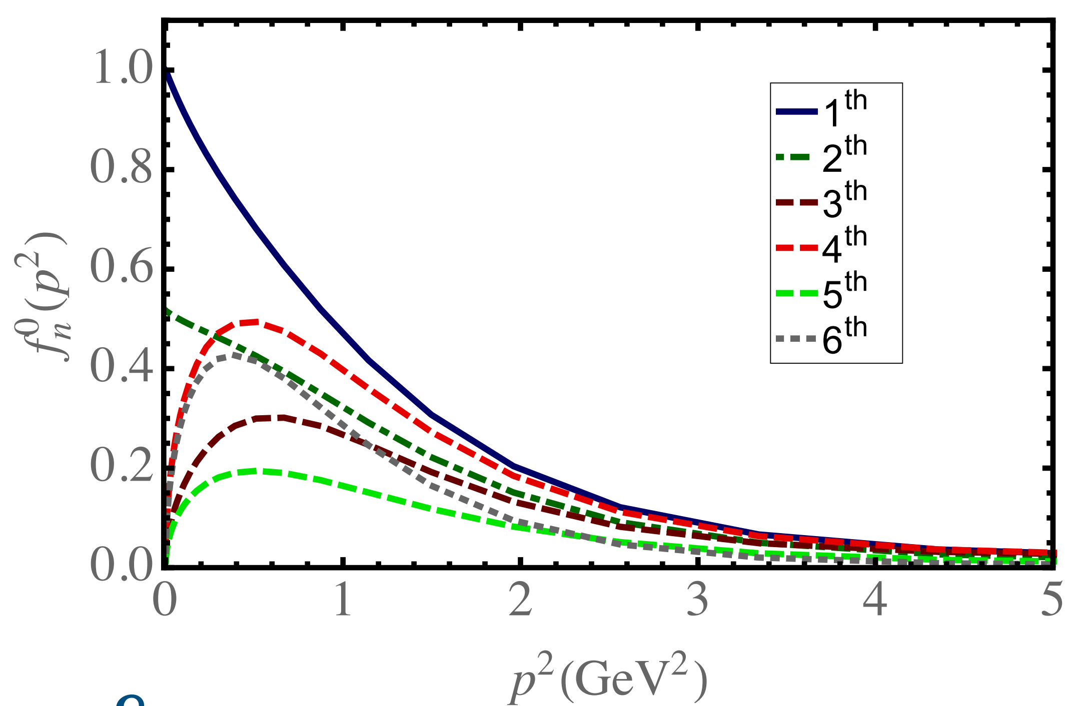


- [gq] correlation behave like a dressed quark
  - Colour-triplet fermion-like object
  - Propagator takes the standard form

$S_{gq}(p) = - i\, \gamma\cdot p\, \sigma_V(p^2) + \sigma_S(p^2)$

$$\sigma_V(s) = \mathcal{E}(s, s_V)\,,\ \sigma_S(s) = \frac{m_{gq}}{s}[1 - s_S \mathcal{E}(s, s_S)]$$

E(s,s0) = (1-Exp[-s/s0])/s



- The behavior of [gq] propagator
  - free-particle like in UV
  - infrared behavior is controlled by s_V & s_S

Hybrid spectrum in Rainbow-Ladder

JPC	0-+	1-+	1--	0+-	0--
m(GeV)RL	1.21(5)	1.78(7)	1.60(6)	1.71(7)	1.72(2)
$LQCD_R$-16^3	1.72(2)	1.73(2)	1.84(2)	2.03(1)	
$LQCD_R$-20^3	1.69(2)	1.72(2)	1.77(6)	1.99(2)	
$LQCD_R$-16^3	2.14(1)	2.15(2)	2.26(2)	2.45(1)	
$LQCD_R$-20^3	2.12(2)	2.16(2)	2.21(6)	2.43(2)	

LQCD. Row 4,5: m_π>0.4 GeV…Dudek, et al. ePrint: arXiv:1004.4930 [hep-ph]
These simulations overestimate mass of pion's first radial excitation by $\delta\pi_1$ = 0.43 GeV

LQCD. Row 2,3 = Row 4,5 - $\delta\pi_1$

- Bound states exist in all changels
- 0-+ and 1-- hybrids are structurally distinct from those accessible using the 2-body Bethe-Salpeter equation in these channel

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- In comparison with LQCD predictions:
  - all states are too light, especially 0-+, and 1^{-+}-1^{--} ordering is reversed.
  - wide variations of model parameters do not alter this outcome.

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We must reconsider each element in our formulation of hybrid mesons.

Hybrid spectrum

- Mismatch between RL-direct and LQCD results
- RL truncation can be improved
  - [gq] correlation amplitude is computed in RL truncation
  - RL truncation underestimates DCSB in bound state amplitudes
- Consequently, anomalous chromomagnetic moment (ACM) associated with this correlation is greatly underestimated
  - ACM enhencement essential to explain a_1-ρ splitting.
- Introduce a correction factor
  - Multiplication of ACM term by constant κ_{gq}

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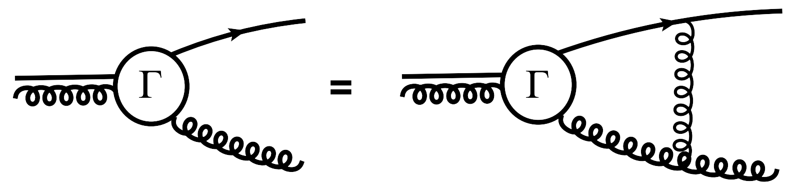
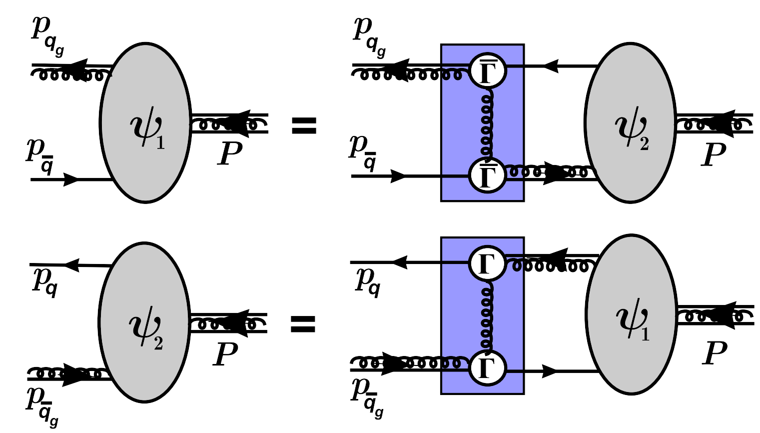
- Can any value of k_{gq} yield match with LQCD?

Hybrid spectrum

- The RL truncation underestimated contributions from angular momentum.





$$\Gamma_\mu(p;Q) = \sum_{i=1}^{6} g_i(p;Q)\, t^i_\mu(p,Q)\,,$$

$$t^1 = \gamma_\mu\,,\ t^2 = i\hat{p}_\mu\,,\ t^3 = \hat{Q}_\mu\,,$$

$$t^4 = i\gamma\cdot\hat{p}\,\hat{Q}_\mu\,, t^5 = i\gamma_\mu\gamma\cdot\hat{p}\,,\ t^6 = \gamma\cdot\hat{p}\,\hat{p}_\mu$$

- We find t^5 raised by 2.5, and omit the spin-independent coupling t^3,the hybrid spectrum will be significantly changed.

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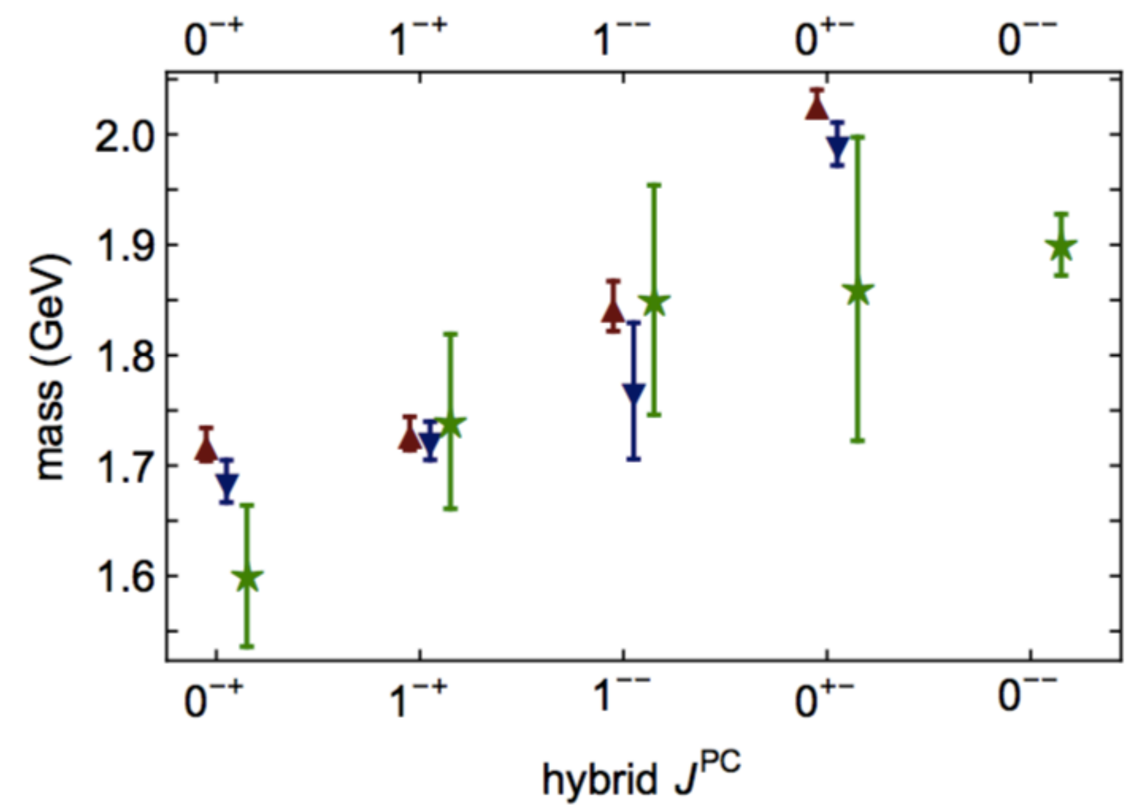
Hybrid spectrum

- Beyond RL spectrum agreement with refined spectrum of LQCD
- Agreement is non-trivial
- Magnitude of our results set by
  - infrared values of the running gluon and quark masses
  - π and ρ meson properties
  - unrelated to hybrid channels



- 0-- state deserves special attention
- LQCD predict lightest 0-- state above m_ρ+2GeV
- We confirm 0-- is ground-state heaviest hybrid, but probably too light.
  - Large angular momentum
  - DCSB-enhancement
  - Simple corrected RL truncation may not be adequate.

Summary

- We introduced a novel approach to the valence-gluon+quark+antiquark bound-state problem in quantum field theory
- Strong correlations exist in $[q_g=qg]$ & $[\bar{q}_g=g\bar{q}]$, and hence that a simpler, coupled pair of effectively two-body equations can provide the basis for a realistic description of hybrid mesons
- It reproduce the mass and ordering of ground-state light-quark hybrids obtained via LQCD
- It should serve as a guide for subsequent continuum treatments of the hybrid-meson three-body problem
- The hybrids wave functions can be used to predict a range of hybrid decays and other processes