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Comined estimation for multi-measurements of branching ratio

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Introduction

- Measurements of branching ratios of resonances are essential in high energy physics experiments.
- Usually, for a particular decay channel of a resonance, different experiments may carry out their respective measurements of its branching ratio.
- Combining these results of a branching ratio based on certain statistical methods will usually lead to a better precision than any individual measurement.
- Then, how?

Introduction

For multi-measurements for a branching ratio, measurements are expressed as $x_i \pm \sigma_i, i = 1, \cdots, I$.

Assuming the measurements follow the normal distribution, the combined estimate of these I independent measurements for the quantity can be expressed as $\mu \pm \sigma$, where

$$\mu = \frac{\sum_{i=1}^{I} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{I} \frac{1}{\sigma_i^2}}, \sigma = \frac{1}{\sqrt{\sum_{i=1}^{I} \frac{1}{\sigma_i^2}}}$$

Introduction ---- Example

$$\Psi' \to \eta J/\Psi, \eta \to \gamma\gamma, J/\Psi \to e^+ e^-$$

$$\Psi' \to \eta J/\Psi, \eta \to \gamma\gamma, J/\Psi \to \mu^+ \mu^-$$

$$B(R \to X) = \frac{N_S(R \to X \to Y)}{N_R \varepsilon(R \to X \to Y) \cdot BR(X \to Y)}$$

Ψ' $\quad$ $\eta J/\Psi$ $\quad$ $\gamma\gamma e^+ e^-, \gamma\gamma\mu^+ \mu^-$

statistical error source

$$B_i = \frac{N_{is}}{N_{Ri}\varepsilon_i BR_i} = \frac{N_{is}}{A_i}, i = 1(\gamma\gamma e^+ e^-), 2(\gamma\gamma\mu^+ \mu^-)$$

systematic error source

Introduction ---- Example

$$\Psi' \to \eta J/\Psi, \eta \to \gamma\gamma, J/\Psi \to e^+e^-$$

$$\Psi' \to \eta J/\Psi, \eta \to \gamma\gamma, J/\Psi \to \mu^+\mu^-$$

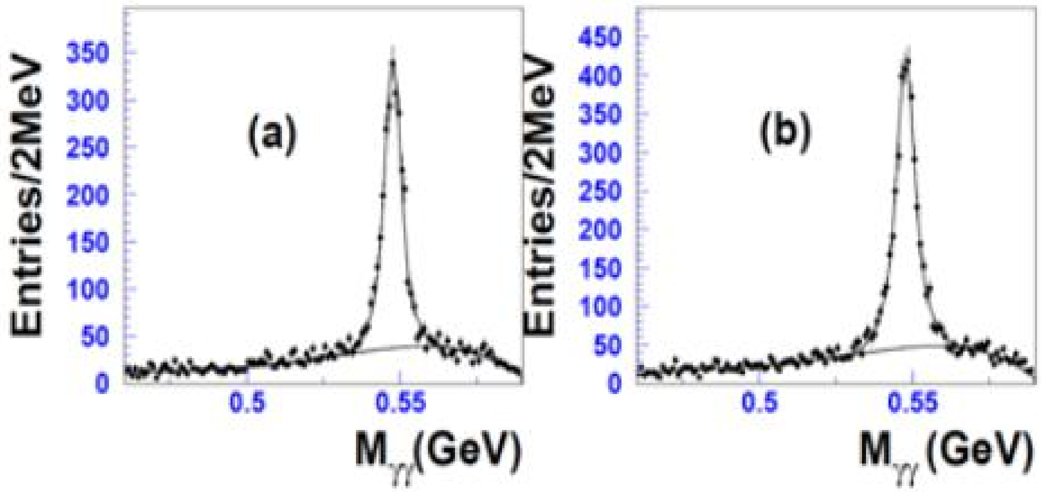

Fig. 1 $\gamma\gamma$ invariant mass spectrum of $\psi' \to \eta J/\psi$ candidate events, (a) $\gamma\gamma e^+e^-$ channel, (b) $\gamma\gamma\mu^+\mu^-$ channel.

$BF = 2.91 \pm 0.12$ $BF = 3.06 \pm 0.14$

Individual observed spectra as function of a same kinematic variable

- **Histograms with same binning**

Poisson variable with expectation λ_j

Number of events in bin j is: $\boxed{n_j} = \sum_{i=1}^{I} n_{ij}, i = 1, \cdots, I, j = 1, \cdots, J$

Number of events in the i^{th} experiments is: $N_i = \sum_{j=1}^{J} n_{ij}$

Total number of events of the I experiments is: $N = \sum_{i=1}^{I} N_i$

Individual observed spectra as function of a same kinematic variable

Joint likelihood function of observing n_j is: $L(n_1, \cdots, n_J) = \prod_{j=1}^{J} \frac{1}{n_j!} \lambda_j^{n_j} e^{-\lambda_j}$

where λ_j is: $\lambda_j = \lambda \int_{\Delta m_j} f(m \mid \theta) dm$

λ is the expectation of total number of events N: $\lambda = \sum_{j=1}^{J} \lambda_j$

Combined pdf $f(m|\theta)$ is: $f(m \mid \theta) = \sum_{i=1}^{I} \frac{N_i}{N} [\underbrace{w_{is}}_{\text{ratio of the signal events to the total observed events}} f_{is}(m \mid \theta_{is}) + (1 - w_{is}) f_{ib}(m \mid \theta_{ib})]$

Individual observed spectra as function of a same kinematic variable

Total number of signal events is: $N_s = \sum_{i=1}^{I} N_{is} = \sum_{i=1}^{I} w_{is} N_i$

$A_i = N_{Ri}\varepsilon_i BR_i$ and $N_{is} = w_{is} N_i$, so $w_{is} = A_i B / N_i$

then $$f(m|\theta) = \sum_{i=1}^{I} \frac{N_i}{N} [\frac{A_i}{N_i} B f_{is}(m|\theta_{is}) + (1 - \frac{A_i}{N_i} B) f_{ib}(m|\theta_{ib})]$$

Individual observed spectra as function of a same kinematic variable

Log function is: $$\ln L = \sum_{j=1}^{J} n_j \ln \lambda_j - \lambda$$

Likelihood equation is: $$\frac{\partial \ln L}{\partial \theta}\Big|_{\theta=\hat{\theta}} = \frac{\partial}{\partial \theta}\Big[\sum_{j=1}^{J} n_j \ln \lambda_j - \lambda\Big]\Big|_{\theta=\hat{\theta}} = 0$$

Parameter θ in the joint likelihood function contain:

$$\theta = \{B, \lambda, \theta_s, \theta_b\}, \theta_s = \{\theta_{1s}, \cdots, \theta_{Is}\}, \theta_b = \{\theta_{1b}, \cdots, \theta_{Ib}\}$$

Individual observed spectra as function of different kinematic variable

- **Histograms with different binning**

Number of events in bin j_i is: n_{ij_i}

Joint likelihood function of observing n_{ij_i} is: $L_i(n_{i1},\cdots,n_{iJ_i}) = \prod_{j_i=1}^{J_i} \frac{1}{n_{ij_i}!} \lambda_{ij_i}^{n_{ij_i}} e^{-\lambda_{ij_i}}$

where λ_{ij_i} is: $\lambda_{ij_i} = \lambda_i \int_{\Delta m_{j_i}} f_i(m_i \mid \theta_i) dm_i$

λ_i is the expectation of the number of events N_i : $\lambda_i = \sum_{j_i=1}^{J_i} \lambda_{ij_i}$

Individual observed spectra as function of different kinematic variable

Pdf of variable m_i is: $$f_i(m_i \mid \theta_i) = w_{is} f_{is}(m_i \mid \theta_{is}) + (1 - w_{is}) f_{ib}(m_i \mid \theta_{ib})$$

$$\downarrow$$

$$f_i(m_i \mid \theta_i) = \frac{A_i}{N_i} B f_{is}(m_i \mid \theta_{is}) + (1 - \frac{A_i}{N_i} B) f_{ib}(m_i \mid \theta_{ib})$$

Parameter θ in the joint likelihood function contain:

$$\theta = \{B, \lambda, \theta_s, \theta_b\}, \lambda = \{\lambda_1, \cdots, \lambda_I\}, \theta_s = \{\theta_{1s}, \cdots, \theta_{Is}\}, \theta_b = \{\theta_{1b}, \cdots, \theta_{Ib}\}$$

Individual observed spectra as function of different kinematic variable

Joint likelihood function for I experiments is: $$L = \prod_{i=1}^{I} L_i$$

Log function is: $$\ln L = \sum_{i=1}^{I} [\sum_{j_i=1}^{J_i} (n_{ij_i} \ln \lambda_{ij_i}) - \lambda_i]$$

In iterative procedure of the maximum lnL calculation, initial value of $\lambda = \{\lambda_1, \cdots, \lambda_I\}$ can be taken as $\{N_1, \cdots, N_I\}$, initial value of B can be the weighted average of all individual results B_i, while the initial values of θ_s and θ_b use the resultant values from each individual experiment.

Some other parts in this paper

- Method to deal with **unbinned case** for both **a same kinematic variable** and **different kinematic variables**

- Method to deal with **credible interval** and **upper limit** **with** or **without inclusion of systematic error**

- Test with Toy Monte Carlo data

Thanks!