Leading twist light-cone distribution amplitudes of vector meson from large momentum effective theory

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## OUTLINE

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### Motivation:

- Light-cone distribution amplitude (LCDA):
  - Characterize the structure of hadron;
  - Essential for analysis of exclusive processes in factorization theorem;
  - Crucial in perturbative QCD approach (PQCD), light-cone sum rule (LCSR).....
  - **Non-perturbative**, can not be calculated with QCD perturbation theory.
- Large momentum effective theory (LaMET):
  - Calculate the equal-time correlators (quasi quantities) instead of the light-cone ones;
  - The matrix elements defined by these equal-time correlators can be **simulated on the lattice**.

#### Large momentum effective theory (LaMET):

• The quasi and light-cone quantities have the **same** infrared (IR) behavior, the ultraviolet

(UV) behaviors are **different**, the difference is involved in the matching coefficient.

- Under the large  $P_z$  limit, the quasi observables can be **factorized as** the convolution of **perturbatively calculable coefficients** and the **standard light-cone observables**.
- Quasi observables can be extracted from **lattice simulation**. So this work gives the

feasibility of evaluating LCDAs from the first principle of QCD.

### LCDAs and Quasi DAs

#### LCDA

#### Quasi DA

Non-perturbative, LCDAs can not be<br/>calculated with QCD perturbation theory.Time-independent spacelike correlation, can<br/>be simulated on a Euclidean lattice.Light-cone coordinateCartesian coordinateWilson line is a light-like straight lineWilson line is a light-like straight lineGauge vector  $n^{\mu} = (1,0,0)$  along the  $n_{+}$  directionGauge vector  $n_{z}^{\mu} = (0,0,0,1)$  along the z-direction

#### Defined as the ratio of the non-local and local matrix elements:

$$\phi_V^{\Gamma}(x,\mu) = \frac{\langle V, P, \epsilon^* | O_V^{\Gamma}(x) | 0 \rangle}{\langle V, P, \epsilon^* | \mathcal{O}_V^{\Gamma}(0) | 0 \rangle}$$

$$\widetilde{\phi}_{V}^{\Gamma}(x,\mu) = \frac{\langle V, P, \epsilon^{*} | \widetilde{O}_{V}^{\Gamma}(x) | 0 \rangle}{\langle V, P, \epsilon^{*} | \widetilde{O}_{V}^{\Gamma}(0) | 0 \rangle}$$

#### One loop results in the ultraviolet cut-off scheme:

The distribution amplitude up to one loop level:

 $\widetilde{\phi}_V^{\Gamma}(x, P_z, \Lambda) = \widetilde{\phi}_V^{\Gamma(0)}(x, P_z) + \widetilde{\phi}_V^{\Gamma(1)}(x, P_z, \Lambda) + \mathcal{O}(\alpha_s^2) \qquad \phi_V^{\Gamma(0)}(x) = \widetilde{\phi}_V^{\Gamma(0)} = \delta(x - x_0)$ 

The matrix element  $\widetilde{O}_V^{\Gamma}(x)$ , up to one loop level:

$$\langle V, P, \epsilon^* | \widetilde{O}_V^{\Gamma}(x) | 0 \rangle = \langle V, P, \epsilon^* | \widetilde{O}_V^{\Gamma}(x) | 0 \rangle^{(0)} (1 + \delta Z_F^{(1)}) + \langle V, P, \epsilon^* | \widetilde{O}_V^{\Gamma}(x) | 0 \rangle^{(1)} + \mathcal{O}(\alpha_s^2)$$

Local matrix elements are also corrected at one loop:

$$\langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\Gamma}(0) | 0 \rangle = \langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\Gamma}(0) | 0 \rangle^{(0)} (1 + \delta Z_F^{(1)} + \delta \widetilde{Z}_V^{\Gamma(1)}) + \mathcal{O}(\alpha_s^2)$$

Then

$$\widetilde{\phi}_{V}^{\Gamma(0)}(x) = \frac{\langle V, P, \epsilon^{*} | \mathcal{O}_{V}^{\Gamma}(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(0) | 0 \rangle^{(0)}}, \qquad \text{local matrix elements, not contribute to the DAs.}$$

$$\widetilde{\phi}_{V}^{\Gamma(1)}(x) = \frac{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(x) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(0) | 0 \rangle^{(0)}} - \frac{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(0) | 0 \rangle^{(0)}} \int dy \frac{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(y) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^{*} | \widetilde{\mathcal{O}}_{V}^{\Gamma}(0) | 0 \rangle^{(0)}}.$$

Quarks' self energy cancels between the non-local and

#### Transverse distribution amplitudes

$$\begin{split} \phi_{V}^{\perp(1)}(x,\Lambda)\Big|_{\text{Fig. 1}(a)} &= \tilde{\phi}_{V}^{\perp(1)}(x,P_{z},\Lambda)\Big|_{\text{Fig. 1}(a)} = 0 \end{split} \qquad (a) \qquad (b) \qquad (c) \\ \phi_{V}^{\perp(1)}(x,\Lambda)\Big|_{\text{Fig. 1}(b)} &= \frac{\alpha_{s}C_{F}}{2\pi} \begin{cases} \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{m_{g}^{2}x}{\Lambda^{2}x_{0}}\right]_{+}, & 0 < x < x_{0}, \\ 0, & \text{others} \end{cases} \qquad (c) \\ \phi_{V}^{\perp(1)}(x,P_{z},\Lambda)\Big|_{\text{Fig. 1}(b)} &= \frac{\alpha_{s}C_{F}}{2\pi} \begin{cases} \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{x}{\Lambda^{2}x_{0}}-\frac{1}{2(x-x_{0})}\right]_{+}, & 0 < x < x_{0}, \\ \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{m_{g}^{2}x}{2\pi}+\frac{1}{2(x-x_{0})}\right]_{+}, & 0 < x < x_{0} \end{cases} \qquad (c) \\ \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{m_{g}^{2}x}{2\pi}+\frac{1}{2(x-x_{0})}\right]_{+}, & 0 < x < x_{0} \end{cases} \qquad (c) \\ \ln \text{ Fig. 1}(b, c), \text{ one end of this internal gluon is attached to the Wilson line, thus there is an eikonal propagator, which is proportional to  $1/(x - x_{0})$   $\tilde{\phi}_{V}^{\perp(1)}(x,\Lambda)\Big|_{\text{Fig. 1}(c)} &= \tilde{\phi}_{V}^{\perp(1)}(x,\Lambda)\Big|_{\text{Fig. 1}(b)}(x_{0} \to 1 - x_{0}, x \to 1 - x) \end{cases}$$$

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### **Transverse distribution amplitudes**

Wilson line's self energy corrections, which is proportional to  $n^2$ , n is the gauge vector in Wilson line.

$$\begin{split} \phi_V^{\perp(1)}(x,\Lambda) \Big|_{\text{Fig. 1}(d)} &= 0 \\ \widetilde{\phi}_V^{\perp(1)}(x,P_z,\Lambda) \Big|_{\text{Fig. 1}(d)} &= \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[ \frac{1}{x-x_0} + \frac{\Lambda}{(x-x_0)^2 P_z} \right]_+, & x < x_0 \\ \left[ -\frac{1}{x-x_0} + \frac{\Lambda}{(x-x_0)^2 P_z} \right]_+, & x > x_0 \end{cases} \\ \end{split}$$
Linear divergence!

**Power divergence** in Wilson line's self energy can be canceled by introducing a "**mass counter term**" of the Wilson line. To do this, we replace the operator in this figure by the "improved" operator:

$$\widetilde{O}_V^{\Gamma \text{ imp.}}(x) = \int \frac{dz}{2\pi} e^{-ixzP_z - \delta m|z|} \widetilde{\mathcal{O}}^{\Gamma}(z)$$

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the mass counter term  $\delta_m$  of the Wilson line can be extracted by using the static quark potential. Perturbative calculation shows that the contribution from  $\delta_m$  cancels the linearly divergent term in  $\tilde{\phi}_V^{\perp(1)}$ 

### Longitudinal distribution amplitudes

$$\phi_V^{\parallel(1)}(x,\Lambda)\Big|_{\text{Fig. 1}(a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[ -\frac{x}{x_0} \ln \frac{m_g^2 x}{\Lambda^2 x_0} \right]_+, & 0 < x < x_0 \\ \left[ -\frac{x-1}{x_0-1} \ln \frac{m^2(x-1)}{\Lambda^2(x_0-1)} \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases}$$

$$\left(\left[\frac{x-1}{x_0-1}\ln\frac{x-1}{x-x_0} - \frac{x}{x_0}\ln\frac{x}{x-x_0}\right]_+, \quad x < 0\right)$$

$$\widetilde{\phi}_{V}^{\parallel(1)}(x, P_{z}, \Lambda) = \frac{\alpha_{s} C_{F}}{2\pi} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \end{bmatrix}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{2\pi} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \end{bmatrix}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{2\pi} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{2\pi} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{2\pi} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{m_{g}^{2}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{x - x_{0}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{x - x_{0}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}} \ln \frac{x - x_{0}}{4P_{z}^{2} x_{0}(x_{0} - x)} - \frac{x - 1}{x_{0} - 1} \ln \frac{x - x_{0}}{x - 1} \right\}_{+}, \quad 0 < x < x_{0} \\ = \frac{\alpha_{s} C_{F}}{4} \left\{ \begin{bmatrix} -\frac{x}{x_{0}$$

Fig. 1(a) 
$$2\pi \left[ \left[ -\frac{x-1}{x_0-1} \ln \frac{m_g^2}{4P_z^2(x_0-1)(x_0-x)} - \frac{x}{x_0} \ln \frac{x-x_0}{x} \right]_+, \quad x_0 < x < 1 \\ \left[ \frac{x}{x_0} \ln \frac{x}{x-x_0} - \frac{x-1}{x_0-1} \ln \frac{x-1}{x-x_0} \right]_+, \quad x > 1 \end{cases} \right]$$

$$\phi_V^{\parallel(1)}(x,\Lambda)\Big|_{\text{Fig. 1}(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2 x}{\Lambda^2 x_0}\right]_+, & x_0 < x < 1\\ 0, & \text{others} \end{cases}$$

### Longitudinal distribution amplitudes

$$\begin{split} \widetilde{\phi}_{V}^{\parallel(1)}(x,P_{z},\Lambda)\Big|_{\text{Fig. 1(b)}} &= \frac{\alpha_{s}C_{F}}{2\pi} \begin{cases} \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{x}{x-x_{0}} - \frac{1}{2(x-x_{0})}\right]_{+}, & x < 0\\ \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{m_{g}^{2}}{4P_{z}^{2}x_{0}(x_{0}-x)} + \frac{2x-x_{0}}{2x_{0}(x-x_{0})}\right]_{+}, & 0 < x < x_{0}\\ \left[\frac{x}{x_{0}(x-x_{0})}\ln\frac{x-x_{0}}{x} + \frac{1}{2(x-x_{0})}\right]_{+}, & x > x_{0} \end{cases}$$

$$\begin{split} \phi_V^{\parallel(1)}(x,\Lambda)\Big|_{\mathrm{Fig.1(c)}} &= \phi_V^{\parallel(1)}(x,\Lambda)\Big|_{\mathrm{Fig.1(b)}}(x_0 \to 1 - x_0, x \to 1 - x)\\ \tilde{\phi}_V^{\parallel(1)}(x,P_z,\Lambda)\Big|_{\mathrm{Fig.1(c)}} &= \tilde{\phi}_V^{\parallel(1)}(x,P_z,\Lambda)\Big|_{\mathrm{Fig.1(b)}}(x_0 \to 1 - x_0, x \to 1 - x) \end{split}$$

$$\left. \phi_V^{\parallel(1)}(x,\Lambda) \right|_{\text{Fig. 1}(d)} = 0$$

$$\left. \widetilde{\phi}_V^{\parallel(1)}(x, P_z, \Lambda) \right|_{\text{Fig. 1}(d)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[ \frac{1}{x - x_0} + \frac{\Lambda}{(x - x_0)^2 P_z} \right]_+, & x < x_0 \\ \left[ -\frac{1}{x - x_0} + \frac{\Lambda}{(x - x_0)^2 P_z} \right]_+, & x > x_0 \end{cases}$$

#### THE MATCHING EQUATION

In LaMET, the quasi-DAs can be factorized as

$$\widetilde{\phi}^{\Gamma}(x, P_z, \Lambda) = \int_0^1 dy Z_{\Gamma}(x, y, P_z, \Lambda) \phi^{\Gamma}(y, \Lambda)$$

The matching coefficient  $Z_{\Gamma}$  is the perturbatively calculable function, can be expanded into the series of  $\alpha_s$  as

$$Z_{\Gamma}(x, y, P_z, \Lambda) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n Z_{\Gamma}^{(n)}(x, y, P_z, \Lambda)$$
$$= \delta(x - y) + \frac{\alpha_s}{2\pi} Z_{\Gamma}^{(1)}(x, y, P_z, \Lambda) + \mathcal{O}(\alpha_s^2)$$

one loop correction to the matching coefficient can be related to the difference between LCDA and quasi-DAs at one loop level:

$$\frac{\alpha_s}{2\pi} Z_{\Gamma}^{(1)}(x, x_0, P_z, \Lambda) = \widetilde{\phi}^{\Gamma(1)}(x, P_z, \Lambda) - \phi^{\Gamma(1)}(x, \Lambda).$$

Then the one loop corrections to the matching coefficients  $Z_{\perp}^{(1)}$ :

$$Z_{\perp}^{(1)}(x,y,P_z,\Lambda) = C_F \begin{cases} \left[\frac{x}{y(x-y)}\ln\frac{x}{x-y} - \frac{x-1}{(y-1)(x-y)}\ln\frac{x-1}{x-y} + \frac{1}{x-y}\right]_{+}^{+}, & x < 0 < y \\ \left[\frac{x}{y(x-y)}\ln\frac{\Lambda^2}{4P_z^2x(y-x)} - \frac{x-1}{(y-1)(x-y)}\ln\frac{x-1}{x-y} + \frac{x+y}{y(x-y)}\right]_{+}^{+}, & 0 < x < y \\ \left[\frac{x-1}{(y-1)(x-y)}\ln\frac{\Lambda^2}{4P_z^2(1-x)(x-y)} + \frac{x}{y(x-y)}\ln\frac{x-y}{x} + \frac{1}{y-1} + \frac{\Lambda}{(x-y)^2P_z}\right]_{+}^{+}, & y < x < 1 \\ \left[\frac{x}{y(x-y)}\ln\frac{x-y}{x} - \frac{x-1}{(y-1)(x-y)}\ln\frac{x-y}{x-1} - \frac{1}{x-y} + \frac{\Lambda}{(x-y)^2P_z}\right]_{+}^{+}, & y < 1 < x \end{cases}$$

$$\begin{aligned} \text{And for } Z_{\parallel}^{(1)} \\ & \left\{ \begin{array}{l} \left[ \frac{x-1}{y-1} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x-1}{x-y} - \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x}{x-y} \right. \\ & \left. + \frac{1}{x-y} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, \\ & \left\{ \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \ln \frac{4P_z^2(y-x)x}{\Lambda^2} - \frac{x-1}{y-1} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x-y}{x-1} \right. \\ & \left. - \frac{x+y}{y(x-y)} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, \\ & \left[ \frac{x-1}{y-1} \left( 1 - \frac{1}{x-y} \right) \ln \frac{4P_z^2(1-x)(x-y)}{\Lambda^2} - \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x-y}{x} \right. \\ & \left[ \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \ln \frac{4P_z^2(1-x)(x-y)}{\Lambda^2} - \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x-y}{x} \right. \\ & \left. + \frac{1}{y-1} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, \\ & \left[ \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x}{x-y} - \frac{x-1}{y-1} \left( 1 - \frac{1}{x-y} \right) \ln \frac{x-1}{x-y} \right. \\ & \left. - \frac{1}{x-y} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+. \end{aligned}$$

Thus we have **proved the LaMET factorization** for vector meson DAs at one loop level.

As we have discussed before, the **linear divergence** can be subtracted by introducing the mass counter term of Wilson line. Therefore, the improved matching coefficients **only contains the logarithm UV divergence**. The relation between improved matching coefficients and the matching coefficients  $Z_{\perp}^{(1)}$  and  $Z_{\parallel}^{(1)}$  is given by

$$Z_{\Gamma}^{(1),\mathrm{imp.}}(x,y,P_z,\Lambda) = Z_{\Gamma}^{(1)}(x,y,P_z,\Lambda) - C_F \left[\frac{\Lambda}{(x-y)^2 P_z}\right]_+$$

Since LCDAs do not depend on  $P_z$ , we can take derivative with  $\ln P_z$  on both sides of the factorization formula, and get an **evolution equation** of quasi-DAs with  $P_z$ :

$$\frac{d\widetilde{\phi}_V^{\Gamma,\mathrm{imp.}}(x,P_z)}{d\ln P_z} = \frac{\alpha_s}{\pi} \int dy V_{\Gamma}(x,y) \ \widetilde{\phi}^{\Gamma,\mathrm{imp.}}(y,P_z)$$

With the evolution kernels

These evolution of quasi-DAs with  $P_z$  shares the same behavior with the scale evolution of LCDAs.

$$\begin{split} V_{\perp}(x,y) &= \left[\frac{x}{y(x-y)}\theta(y-x)\theta(x)\right]_{+} + \left[\frac{x-1}{(y-1)(x-y)}\theta(x-y)\theta(1-x)\right]_{+},\\ V_{\parallel}(x,y) &= \left[\frac{x}{y}\left(1-\frac{1}{x-y}\right)\theta(y-x)\theta(x)\right]_{+} + \left[\frac{x-1}{y-1}\left(1-\frac{1}{x-y}\right)\theta(x-y)\theta(1-x)\right]_{+} \end{split}$$

#### Matching Coefficients with a Finite Cut-off

- In the  $\Lambda \to \infty$  limit, the  $\mathcal{O}(P_z/\Lambda)$  contributions can be **neglected**.
- However, it is difficult to take too large value of P<sub>z</sub> in lattice simulations, in fact, Λ and xP<sub>z</sub> are of the same order in a practical calculation on the lattice.
- It is valuable to consider the finite  $\Lambda$  corrections to the matching coefficients.

$$\begin{split} Z_{\perp}^{(1)}(x,y,P_{z},\Lambda) = & Z_{\perp}^{(1)}(x,y,P_{z},\Lambda) + \delta Z_{\perp}^{(1)}(x,y,P_{z},\Lambda) \\ Z_{\parallel}^{(1)}(x,y,P_{z},\Lambda) = & Z_{\parallel}^{(1)}(x,y,P_{z},\Lambda) + \delta Z_{\parallel}^{(1)}(x,y,P_{z},\Lambda) \end{split} \qquad \qquad \delta Z_{\Gamma}^{(1)} \rightarrow 0 \text{ when } \Lambda \rightarrow \infty \\ \delta Z_{\perp}^{(1)}(x,y,P_{z},\Lambda) = & C_{F} \left[ \frac{x}{y(x-y)} \left( \ln \frac{\Lambda(x) + P_{z}x}{\Lambda(x-y) + P_{z}(x-y)} + \frac{\Lambda(x-y) - \Lambda(x)}{2P_{z}} \right) + \frac{\Lambda(x-y) - \Lambda(0)}{2(x-y)^{2}P_{z}} \right]_{+} \\ & + (x \rightarrow 1 - x, \ y \rightarrow 1 - y), \\ \delta Z_{\parallel}^{(1)}(x,y,P_{z},\Lambda) = & C_{F} \left[ \frac{x}{y} \ln \frac{\Lambda(x) - P_{z}x}{\Lambda(x-y) - P_{z}(x-y)} + \frac{x}{y(x-y)} \left( \ln \frac{\Lambda(x) + P_{z}x}{\Lambda(x-y) + P_{z}(x-y)} + \frac{\Lambda(x-y) - \Lambda(x)}{2P_{z}} \right) \right) \\ & + \frac{\Lambda(x-y) - \Lambda(0)}{2(x-y)^{2}P_{z}} \right]_{+} \\ & + (x \rightarrow 1 - x, \ y \rightarrow 1 - y), \end{split}$$

#### One loop results in dimensional regularization under $\overline{MS}$ scheme

$$Z_{\perp}^{(1)}(x,y,P_{z},\mu) = C_{F} \begin{cases} \left[\frac{x}{y(x-y)}\ln\frac{x}{x-y} - \frac{x-1}{(y-1)(x-y)}\ln\frac{x-1}{x-y} + \frac{1}{x-y}\right], & x < 0 < y \\ \left[\frac{x}{y(x-y)}\ln\frac{\mu^{2}}{4P_{z}^{2}x(y-x)} - \frac{x-1}{(y-1)(x-y)}\ln\frac{x-1}{x-y} + \frac{x+y}{y(x-y)}\right], & 0 < x < y \\ \left[\frac{x-1}{(y-1)(x-y)}\ln\frac{\mu^{2}}{4P_{z}^{2}(1-x)(x-y)} + \frac{x}{y(x-y)}\ln\frac{x-y}{x} + \frac{1}{y-1}\right]_{+}, & y < x < 1 \\ \left[\frac{x}{y(x-y)}\ln\frac{x-y}{x} - \frac{x-1}{(y-1)(x-y)}\ln\frac{x-y}{x-1} - \frac{1}{x-y}\right]_{+}, & y < 1 < x \end{cases}$$

The  $P_z$  dependence in cut-off and DR schemes are all the same, so the evolution equations and kernels are not changed.

and for  $Z_{\parallel}^{(1)}$ , the result reads

$$Z_{\parallel}^{(1)}(x, y, P_{z}, \mu) = C_{F} \begin{cases} \left[\frac{x-1}{y-1}\left(1+\frac{1}{x-y}\right)\ln\frac{x-1}{x-y} - \frac{x}{y}\left(1-\frac{1}{x-y}\right)\ln\frac{x}{x-y}\right]_{+}, & x < 0 < y \\ \left[\frac{x}{y}\left(1-\frac{1}{x-y}\right)\ln\frac{4P_{z}^{2}(y-x)x}{\mu^{2}} - \frac{x-1}{y-1}\left(1+\frac{1}{x-y}\right)\ln\frac{x-y}{x-1} \right. \\ \left. + \frac{x}{y(x-y)} + \frac{x}{y}\right]_{+}, & 0 < x < y \\ \left[\frac{x-1}{y-1}\left(1-\frac{1}{x-y}\right)\ln\frac{4P_{z}^{2}(1-x)(x-y)}{\mu^{2}} - \frac{x}{y}\left(1-\frac{1}{x-y}\right)\ln\frac{x-y}{x} \right. \\ \left. + \frac{1-x}{(y-1)(x-y)} + \frac{1-x}{1-y}\right]_{+}, & y < x < 1 \\ \left[\frac{x}{y}\left(1-\frac{1}{x-y}\right)\ln\frac{x}{x-y} - \frac{x-1}{y-1}\left(1+\frac{1}{x-y}\right)\ln\frac{x-1}{x-y}\right]_{+}. & y < 1 < x \end{cases}$$

## Summary

- One loop calculation on the leading twist LCDAs and quasi DAs of the vector meson in the framework of large momentum effective theory.
- Based on the perturbative calculation under UV cut-off and DR schemes, we have examined the LaMET factorization and determined the matching coefficients at one loop accuracy.
- The collinear divergence cancels out between LCDAs and quasi-DAs, thus the matching coefficients are free of IR divergence. The linear divergence is subtracted by the mass counter term  $\delta_m$  of Wilson line.
- The results of this work will be useful to extract the vector mesons' LCDAs from the future lattice simulations.

# Thank you!