New Look at the Y-Tetraquarks and Ω_c-Baryons in the Diquark Model

Alexander Parkhomenko

P. G. Demidov Yaroslavl State University, Yaroslavl, Russia

2-nd Workshop on Heavy Quark Physics
Institute of High Energy Physics (IHEP), Beijing, China
April 23-25, 2018

based on the paper EPJC 78 (2018) 029 [arXiv:1708.04650]
by A. Ali, L. Maiani, A. V. Borisov, I. Ahmed, M. J. Aslam, AP, A. Polosa, and A. Rehman

Outline

Introduction

- In 2003, the first exotic hidden-charmed state $X(3872)$ was observed by the Belle Collaboration
- This state was confirmed by BaBar, CDF, D0, BESII, and LHCb collaborations
- Soon after, many new other mesons with masses above the $D\bar{D}$ threshold have been observed
- Searches of new exotic states is one of the main topics of BESIII and LHCb collaborations at present
- Observation of hidden-charmed and hidden-bottom charged mesons was the direct manifestation of tetraquarks
- In addition, LHCb observed two narrow states which have got the interpretation as hidden-charmed pentaquarks

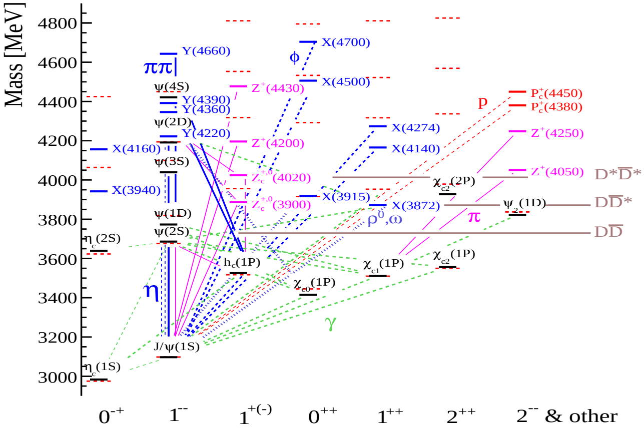
Introduction

Recent review papers on exotic hadrons:

1. A. Esposito, A. Pilloni and A. D. Polosa, "Multiquark Resonances," Phys. Rept. **668**, 1 (2016).
2. H. X. Chen, W. Chen, X. Liu and S. L. Zhu, "The hidden-charm pentaquark and tetraquark states," Phys. Rept. **639**, 1 (2016).
3. R. F. Lebed, R. E. Mitchell and E. S. Swanson, "Heavy-Quark QCD Exotica," Prog. Part. Nucl. Phys. **93**, 143 (2017).
4. A. Ali, J. S. Lange and S. Stone, "Exotics: Heavy Pentaquarks and Tetraquarks," Prog. Part. Nucl. Phys. **97**, 123 (2017).
5. F. K. Guo, C. Hanhart, U. G. Meissner, Q. Wang, Q. Zhao and B. S. Zou, "Hadronic molecules," Rev. Mod. Phys. **90**, 015004 (2018).
6. S. L. Olsen, T. Skwarnicki and D. Zieminska, "Non-Standard Heavy Mesons and Baryons, an Experimental Review," Rev. Mod. Phys. **90**, 015003 (2018).

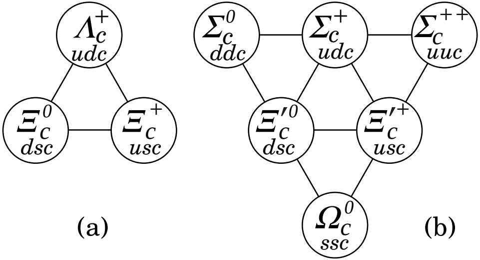
X, Y, Z, P_c and Charmonium States

[S. L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90 (2018) 015003]



Orbitally Excited Ω_c-Baryons

- Charmed baryon multiplets

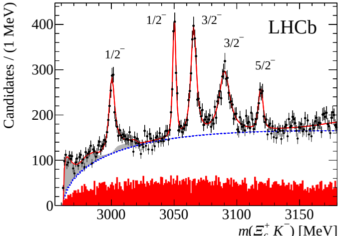


- Observation of 5 narrow excited Ω_c baryons in $\Omega_c \to \Xi_c^+ K^-$
  [LHCb, PRL 118, 182001 (2017)]
- Plausible quantum numbers, J^P, were suggested assuming the diquark model for $\Omega_c (= css) = c\,[ss]$
  [M. Karliner & J. L. Rosner, PRD 95, 114012 (2017)]

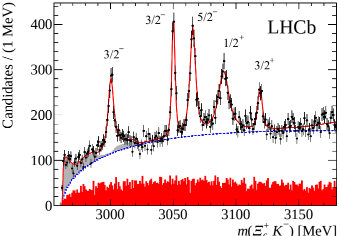
Orbitally Excited Ω_c-Baryons

- Measured masses (in MeV) [LHCb] and plausible J^P quantum numbers, assuming the diquark model for $\Omega_c(= css) = c\,[ss]$ [M. Karliner & J. L. Rosner, PRD 95, 114012 (2017)]

$$
\begin{aligned}
M(\Omega_c(3000)) &= 3000.4 \pm 0.2 \pm 0.1; \quad J^P = 1/2^- \\
M(\Omega_c(3050)) &= 3050.2 \pm 0.1 \pm 0.1; \quad J^P = 1/2^- \\
M(\Omega_c(3066)) &= 3065.6 \pm 0.1 \pm 0.3; \quad J^P = 3/2^- \\
M(\Omega_c(3090)) &= 3090.2 \pm 0.3 \pm 0.5; \quad J^P = 3/2^- \\
M(\Omega_c(3119)) &= 3119.1 \pm 0.3 \pm 0.9; \quad J^P = 5/2^-
\end{aligned}
$$



Adapted from Fig. 2 of arXiv:[1703.04639]

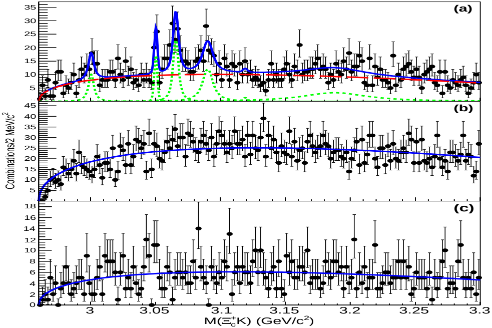


Adapted from Fig. 2 of arXiv:[1703.04639]

Belle Results on Excited Ω_c-Baryons

[J. Yelton et al. [Belle Collab.], Phys.Rev. D97 (2018) 051102]

- Decay mode $\Omega_c \to \Xi_c^+ K^-$

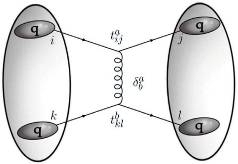


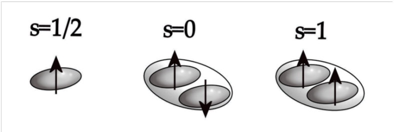
Ω_c^* State Significance	3000 3.9σ	3050 4.6σ	3066 7.2σ	3090 5.7σ	3119 0.4σ	318 2.4
LHCb Mass Belle Mass (with fixed Γ)	$3000.4 \pm 0.2 \pm 0.1$ $3000.7 \pm 1.0 \pm 0.2$	$3050.2 \pm 0.1 \pm 0.1$ $3050.2 \pm 0.4 \pm 0.2$	$3065.5 \pm 0.1 \pm 0.3$ $3064.9 \pm 0.6 \pm 0.2$	$3090.2 \pm 0.3 \pm 0.5$ $3089.3 \pm 1.2 \pm 0.2$	$3119 \pm 0.3 \pm 0.9$ -	3188 ± 3199 ±

Diquark Model of Hadrons

- Quarks q_i^α and diquarks $\mathcal{Q}_{i\alpha}$ are building blocks of baryons
- α is the $SU(3)_C$ index and i is the $SU(3)_F$ index
- Color repres.: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive

$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{3}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } 6}$$





- Interpolating diquark operators for the two spin states

$$\text{Scalar:} \quad 0^+ \quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}^\beta_c \gamma_5 q^\gamma_i - \bar{q}^\beta_{ic}\gamma_5 c^\gamma\right)$$

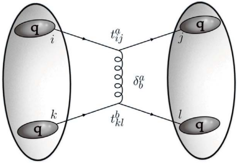
$$\text{Axial-Vector:} \quad 1^+ \quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}^\beta_c \vec{\gamma} q^\gamma_i + \bar{q}^\beta_{ic}\vec{\gamma} c^\gamma\right)$$

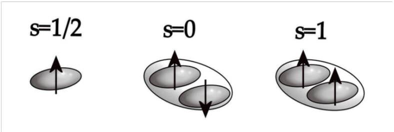
- Colorless combination with the quark results into the baryon

Diquark Model of Hadrons

- Quarks q_i^α and diquarks $\mathcal{Q}_{i\alpha}$ are building blocks of baryons
- α is the $SU(3)_C$ index and i is the $SU(3)_F$ index
- Color repres.: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive

$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{3}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } 6}$$





- Interpolating diquark operators for the two spin states

  Scalar: 0^+ $\quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{ic}^\beta \gamma_5 c^\gamma\right)$

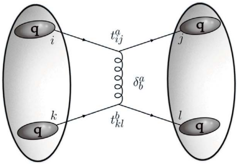
  Axial-Vector: 1^+ $\quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{ic}^\beta \vec{\gamma} c^\gamma\right)$

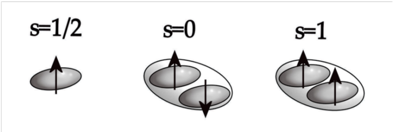
- Colorless combination with the quark results into the baryon

Diquark Model of Hadrons

- Quarks q_i^α and diquarks $\mathcal{Q}_{i\alpha}$ are building blocks of baryons
- α is the $SU(3)_C$ index and i is the $SU(3)_F$ index
- Color repres.: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive

$$t_{ij}^a t_{kl}^a = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{3}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } 6}$$





- Interpolating diquark operators for the two spin states

$$\text{Scalar:} \quad 0^+ \quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{ic}^\beta \gamma_5 c^\gamma\right)$$

$$\text{Axial-Vector:} \quad 1^+ \quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{ic}^\beta \vec{\gamma} c^\gamma\right)$$

- Colorless combination with the quark results into the baryon

Diquark Model of Hadrons

- Diquarks $\mathcal{Q}_{i\alpha}$ and antidiquarks $\bar{\mathcal{Q}}_i^\alpha$ are the building blocks of tetraquarks
- NR limit: States parametrized by Pauli matrices:

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \Gamma^0 & = \sigma_2/\sqrt{2} \\ \text{Axial-Vector:} & 1^+ & \vec{\Gamma} & = \sigma_2\vec{\sigma}/\sqrt{2} \end{array}$$

- (Anti)diquark spin $S_{\mathcal{Q}(\bar{\mathcal{Q}})}$, total angular mom. $J \implies |S_\mathcal{Q}, S_{\bar{\mathcal{Q}}};\ J\rangle$
- Ground-state tetraquarks:

$$|0_\mathcal{Q}, 0_{\bar{\mathcal{Q}}};\ 0_J\rangle = \Gamma^0 \otimes \Gamma^0, \qquad |1_\mathcal{Q}, 1_{\bar{\mathcal{Q}}};\ 0_J\rangle = \frac{1}{\sqrt{3}}\Gamma^i \otimes \Gamma_i$$

$$|0_\mathcal{Q}, 1_{\bar{\mathcal{Q}}};\ 1_J\rangle = \Gamma^0 \otimes \Gamma^i \qquad |1_\mathcal{Q}, 0_{\bar{\mathcal{Q}}};\ 1_J\rangle = \Gamma^i \otimes \Gamma^0$$

$$|1_\mathcal{Q}, 1_{\bar{\mathcal{Q}}};\ 1_J\rangle = \frac{1}{\sqrt{2}}\varepsilon^{ijk}\Gamma_j \otimes \Gamma_k$$

Diquark Model of Hadrons

- Diquarks $\mathcal{Q}_{i\alpha}$ and antidiquarks $\bar{\mathcal{Q}}_i^{\alpha}$ are the building blocks of tetraquarks
- NR limit: States parametrized by Pauli matrices:

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \Gamma^0 & = \sigma_2/\sqrt{2} \\ \text{Axial-Vector:} & 1^+ & \vec{\Gamma} & = \sigma_2\vec{\sigma}/\sqrt{2} \end{array}$$

- (Anti)diquark spin $S_{\mathcal{Q}\,(\bar{\mathcal{Q}})}$, total angular mom. $J \implies \left|S_{\mathcal{Q}}, S_{\bar{\mathcal{Q}}};\ J\right\rangle$
- Ground-state tetraquarks:

$$\left|0_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}};\ 0_J\right\rangle = \Gamma^0 \otimes \Gamma^0, \qquad \left|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\ 0_J\right\rangle = \frac{1}{\sqrt{3}}\Gamma^i \otimes \Gamma_i$$

$$\left|0_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\ 1_J\right\rangle = \Gamma^0 \otimes \Gamma^i \qquad \left|1_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}};\ 1_J\right\rangle = \Gamma^i \otimes \Gamma^0$$

$$\left|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\ 1_J\right\rangle = \frac{1}{\sqrt{2}}\varepsilon^{ijk}\Gamma_j \otimes \Gamma_k$$

Orbitally Excited Ω_c-Baryons in the Diquark Model

- Measured masses (in MeV) [LHCb] and plausible J^P quantum numbers, assuming the diquark model for $\Omega_c (= css) = c\,[ss]$ [M. Karliner & J. L. Rosner, PRD 95, 114012 (2017)]

$$\begin{aligned}
M(\Omega_c(3000)) &= 3000.4 \pm 0.2 \pm 0.1; \quad J^P = 1/2^- \\
M(\Omega_c(3050)) &= 3050.2 \pm 0.1 \pm 0.1; \quad J^P = 1/2^- \\
M(\Omega_c(3066)) &= 3065.6 \pm 0.1 \pm 0.3; \quad J^P = 3/2^- \\
M(\Omega_c(3090)) &= 3090.2 \pm 0.3 \pm 0.5; \quad J^P = 3/2^- \\
M(\Omega_c(3119)) &= 3119.1 \pm 0.3 \pm 0.9; \quad J^P = 5/2^-
\end{aligned}$$

- To get the mass spectrum, effective Hamiltonian is required
- For P states, important to take into account tensor interaction

$$\begin{aligned}
H_{\rm eff} &= m_c + m_{[ss]} + \kappa_{ss}\,(\mathbf{S}_s \cdot \mathbf{S}_s) + \frac{B_{\mathcal{Q}}}{2}\,\mathbf{L}^2 + V_{\rm SD}, \\
V_{\rm SD} &= a_1\,(\mathbf{L} \cdot \mathbf{S}_{[ss]}) + a_2\,(\mathbf{L} \cdot \mathbf{S}_c) + b\,\frac{\langle S_{12} \rangle}{4} + c\,(\mathbf{S}_{[ss]} \cdot \mathbf{S}_c)
\end{aligned}$$

Orbitally Excited Ω_c-Baryons in the Diquark-Quark model

- The term $b\langle S_{12}\rangle/4$represents the tensor interaction

$$\frac{S_{12}}{4} = Q(\mathbf{S}_1, \mathbf{S}_2) = 3\,(\mathbf{S}_1\cdot\mathbf{n})\,(\mathbf{S}_2\cdot\mathbf{n}) - (\mathbf{S}_1\cdot\mathbf{S}_2) = 3\,S_1^i\,S_2^j\,N_{ij}$$

- $\mathbf{S}_1 = \mathbf{S}_{[ss]}$ and $\mathbf{S}_2 = \mathbf{S}_c$ are the spins of the diquark and charmed quark, respectively
- $\mathbf{n} = \mathbf{r}/r$ is the unit vector along the radius vector
- The scalar operator $\langle S_{12}\rangle/4$ in the convolution form contains

$$N_{ij} = n_i\,n_j - \frac{1}{3}\,\delta_{ij}$$

- Can be evaluated between the states with the same fixed value L of the angular momentum operator $\mathbf{L}$
- Required identity can be found in the Landau and Lifshitz book

$$\langle Q(\mathbf{S}_1, \mathbf{S}_2)\rangle_{L=1} = -\frac{3}{5}\left[(\mathbf{L}\cdot\mathbf{S}_1)\,(\mathbf{L}\cdot\mathbf{S}_2) + (\mathbf{L}\cdot\mathbf{S}_2)\,(\mathbf{L}\cdot\mathbf{S}_1) - \frac{4}{3}\,(\mathbf{S}_1\cdot\mathbf{S}_2)\right]$$

Results for the tensor term for Ω_c states in the Diquark-Quark model

- For $S_{[ss]} = 0$, $\langle Q(\mathbf{S}_c, \mathbf{S}_{[ss]})\rangle_{L=1} = 0$
- For $S_{[ss]} = 1$, all three terms are non-zero in $\langle Q(\mathbf{S}_c, \mathbf{S}_{[ss]})\rangle_{L=1}$
- One has to calculate the matrix element ($X = c,\ [ss]$)

$$\langle L, S'; J|\,(\mathbf{L}\cdot\mathbf{S}_X)\,|L, S; J\rangle$$

- After this matrix is calculated, one can use the relation

$$\frac{1}{2}\langle S_{12}\rangle = 2\langle Q(\mathbf{S}_{[ss]}, \mathbf{S}_c)\rangle = \langle Q(\mathbf{S}, \mathbf{S})\rangle - \langle Q(\mathbf{S}_c, \mathbf{S}_c)\rangle - \langle Q(\mathbf{S}_{[ss]}, \mathbf{S}_{[ss]})\rangle$$

- Tensor operator mixes two $J^P = 1/2^-$ and two $J^P = 3/2^-$ states

$$J^P = 1/2^- : \quad \frac{1}{4}\langle S_{12}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & -\sqrt{2} \end{pmatrix}$$

$$J^P = 3/2^- : \quad \frac{1}{4}\langle S_{12}\rangle = \frac{1}{10}\begin{pmatrix} 0 & -\sqrt{5} \\ -\sqrt{5} & 8 \end{pmatrix}$$

$$J^P = 5/2^- : \quad \frac{1}{4}\langle S_{12}\rangle = -\frac{1}{5}$$

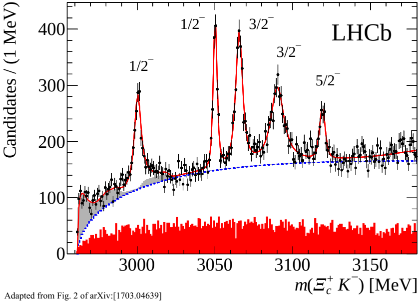
- This was obtained by direct calculations [M. Karliner & J. L. Rosner, PRD 95, 114012 (2017)] and confirmed by us by the other method

Numerical analysis of excited Ω_c states in the Diquark-Quark model

- Coefficients determined from the masses of the Ω_c-baryons (in MeV)

a_1	a_2	b	c	M_0
26.95	25.75	13.52	4.07	3079.94

$$M_0 \equiv m_c + m_{[ss]} + 2\kappa_{ss} + B_{\mathcal{Q}}$$



Adapted from Fig. 2 of arXiv:[1703.04639]

Approaches for Tetraquarks

Quarkonium Tetraquarks:

1. Compact tetraquarks
2. Meson molecule
3. Diquark-onium (diquark-antidiquark system)
4. Hadro-quarkonium
5. Quarkonium-adjoint meson

Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, orbital and tensor inter.

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

with

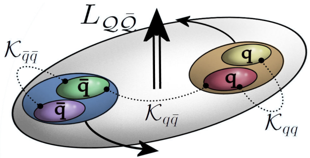
$$H_{SS}^{(qq)} = 2\mathcal{K}_{cq}\left[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})\right]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q)\right] + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}) = 2A_{\mathcal{Q}}(\mathbf{S} \cdot \mathbf{L})$$

$$H_{LL} = \frac{1}{2} B_{\mathcal{Q}} L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)$$

$$H_T = \frac{1}{4} b_Y S_{12} = b_Y\left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})$$



- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

$$H_{\rm eff}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q(\mathbf{L}\cdot\mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q \cdot \mathbf{s}_Q) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}})\right] + b_Y\frac{S_{12}}{4}$$

Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, orbital and tensor inter.

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

constituent mass

with

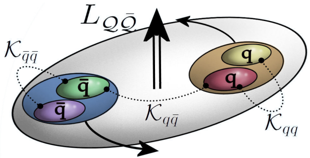
$$H_{SS}^{(qq)} = 2\mathcal{K}_{cq}\left[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})\right]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q)\right] + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}) = 2A_{\mathcal{Q}}(\mathbf{S} \cdot \mathbf{L})$$

$$H_{LL} = \frac{1}{2} B_{\mathcal{Q}} L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)$$

$$H_T = \frac{1}{4} b_Y S_{12} = b_Y\left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})$$



- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

$$H_{\rm eff}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q(\mathbf{L} \cdot \mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q \cdot \mathbf{s}_Q) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}})\right] + b_Y \frac{S_{12}}{4}$$

Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, orbital and tensor inter.

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

qq spin coupling $q\bar{q}$ spin coupling

with

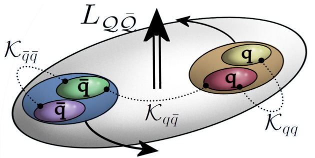
$$H_{SS}^{(qq)} = 2\mathcal{K}_{cq}\left[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})\right]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q)\right] + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}) = 2A_{\mathcal{Q}}(\mathbf{S} \cdot \mathbf{L})$$

$$H_{LL} = \frac{1}{2} B_{\mathcal{Q}} L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)$$

$$H_T = \frac{1}{4} b_Y S_{12} = b_Y\left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})$$



- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

$$H_{\rm eff}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q(\mathbf{L} \cdot \mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q \cdot \mathbf{s}_Q) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}})\right] + b_Y\frac{S_{12}}{4}$$

Hamiltonian for tetraquarks with hidden charm

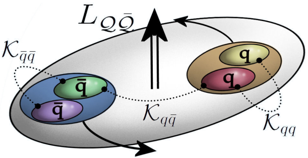
Involves constituent diquark mass, spin-spin, spin-orbit, orbital and tensor inter.

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

LS coupling *LL* coupling

with

$$\begin{aligned}
H_{SS}^{(qq)} &= 2\mathcal{K}_{cq}\left[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})\right] \\
H_{SS}^{(q\bar{q})} &= 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q)\right] \\
&\quad + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}) \\
H_{SL} &= 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}) = 2A_{\mathcal{Q}}\,(\mathbf{S} \cdot \mathbf{L}) \\
H_{LL} &= \frac{1}{2} B_{\mathcal{Q}}\, L_{\mathcal{Q}\bar{\mathcal{Q}}}\left(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1\right) \\
H_T &= \frac{1}{4} b_Y S_{12} = b_Y\left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})
\end{aligned}$$



- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q\,(\mathbf{L} \cdot \mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q \cdot \mathbf{s}_Q) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}})\right] + b_Y \frac{S_{12}}{4}$$

Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, orbital and tensor inter.

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

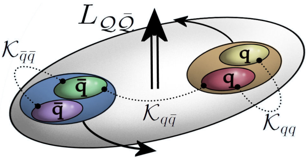
with

$$H_{SS}^{(qq)} = 2\mathcal{K}_{cq}\left[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})\right]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q)\right] + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}) = 2A_{\mathcal{Q}}\,(\mathbf{S} \cdot \mathbf{L})$$

$$H_{LL} = \frac{1}{2} B_{\mathcal{Q}}\, L_{\mathcal{Q}\bar{\mathcal{Q}}}\,(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)$$



$$H_T = \frac{1}{4} b_Y\, S_{12} = b_Y\left[3\,(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})\,(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})$$

- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q\,(\mathbf{L} \cdot \mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q \cdot \mathbf{s}_Q) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}})\right] + b_Y\frac{S_{12}}{4}$$

Low-Lying S-Wave Tetraquark States

- In the $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$ and $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$ bases, the positive parity S-wave tetraquarks are listed below; $M_{00} = 2m_{\mathcal{Q}}$

Label	J^{PC}	$\|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$\|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
X_0	0^{++}	$\|0,0;0,0\rangle_0$	$(\|0,0;0,0\rangle_0 + \sqrt{3}\|1,1;0,0\rangle_0)/2$	$M_{00} - 3\kappa_{qQ}$
X_0'	0^{++}	$\|1,1;0,0\rangle_0$	$(\sqrt{3}\|0,0;0,0\rangle_0 - \|1,1;0,0\rangle_0)/2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$(\|1,0;1,0\rangle_1 + \|0,1;1,0\rangle_1)/\sqrt{2}$	$\|1,1;1,0\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$(\|1,0;1,0\rangle_1 - \|0,1;1,0\rangle_1)/\sqrt{2}$	$(\|1,0;1,0\rangle_1 - \|0,1;1,0\rangle_1)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$\|1,1;1,0\rangle_1$	$(\|1,0;1,0\rangle_1 + \|0,1;1,0\rangle_1)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$\|1,1;2,0\rangle_2$	$\|1,1;2,0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters, $M_{00}(Q)$ and κ_{qQ}, with $Q = c, b$; hence, very predictive
- Some of the states, such as X_0, X_0', X_2, still missing and are being searched for at the LHC

Analysis of Tetraquark Y-States in the Diquark Model

- Effective Hamiltonian for the mass spectrum

$$H_{\rm eff} = 2m_{\mathcal{Q}} + \frac{1}{2} B_{\mathcal{Q}}\, \mathbf{L}^2 + 2a_Y\, (\mathbf{L}\cdot\mathbf{S}) + \frac{1}{4} b_Y\, \langle S_{12}\rangle + 2\kappa_{cq}\, [(\mathbf{S}_q\cdot\mathbf{S}_c) + (\mathbf{S}_{\bar q}\cdot\mathbf{S}_{\bar c})]$$

- There are four $L=1$ and one $L=3$ tetraquark P-wave states with $J^{PC}=1^{--}$ and two $L=1$ states with $J^{PC}=1^{-+}$

Label	J^{PC}	$\vert s_{qQ}, s_{\bar q\bar Q}; S, L\rangle_J$	Mass
Y_1	1^{--}	$\vert 0,0;0,1\rangle_1$	$M_{00} - 3\kappa_{qQ} + B_Q \equiv \tilde M_{00}$
Y_2	1^{--}	$(\vert 1,0;1,1\rangle_1 + \vert 0,1;1,1\rangle_1)/\sqrt{2}$	$\tilde M_{00} + 2\kappa_{qQ} - 2A_Q$
Y_3	1^{--}	$\vert 1,1;0,1\rangle_1$	
Y_4	1^{--}	$\vert 1,1;2,1\rangle_1$	
Y_5	1^{--}	$\vert 1,1;2,3\rangle_1$	$M_{Y_2} + 2\kappa_{qQ} - 14A_Q + 5B_Q - 8b_Y/5$
$Y_2^{(+)}$	1^{-+}	$(\vert 1,0;1,1\rangle_1 - \vert 0,1;1,1\rangle_1)/\sqrt{2}$	$\tilde M_{00} + 2\kappa_{qQ} - 2A_Q$
$Y^{(+)}$	1^{-+}	$\vert 1,1;1,1\rangle_1$	$\tilde M_{00} + \kappa_{qQ} - 2A_Q + b_Y$

- Tensor couplings non-vanishing only for the states with $S_{\mathcal{Q}} = S_{\bar{\mathcal{Q}}} = 1$
- Y_3 and Y_4 are not the mass eigenstates of the Hamiltonian

Analysis of Tetraquark Y-States in the Diquark Model

- Mixing in $Y_3 - Y_4 - Y^{(+)}$ sector is taken into account
- Matrix repres. of $(\mathbf{L}\cdot\mathbf{S}_{[cq]})$ and $(\mathbf{L}\cdot\mathbf{S}_{[\bar{c}\bar{q}]})$ in $\{Y_4, Y_3, Y^{(+)}\}$ basis

$$(\mathbf{L}\cdot\mathbf{S}_{[cq]}) = \frac{1}{2\sqrt{3}}\begin{pmatrix} -3\sqrt{3} & 0 & \sqrt{5} \\ 0 & 0 & 4 \\ \sqrt{5} & 4 & -\sqrt{3} \end{pmatrix}$$

- Tensor contribution to the effective Hamiltonian

$$\frac{1}{4}\langle S_{12}\rangle = \frac{1}{5}\begin{pmatrix} -7 & 2\sqrt{5} & 0 \\ 2\sqrt{5} & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

- Positive parity $Y^{(+)}$-state decouples from Y_3 and Y_4
- Mass eigenvalues

$$M_3 = \widetilde{M}_{00} + 4\kappa_{cq} + E_+, \qquad M_4 = \widetilde{M}_{00} + 4\kappa_{cq} + E_-$$

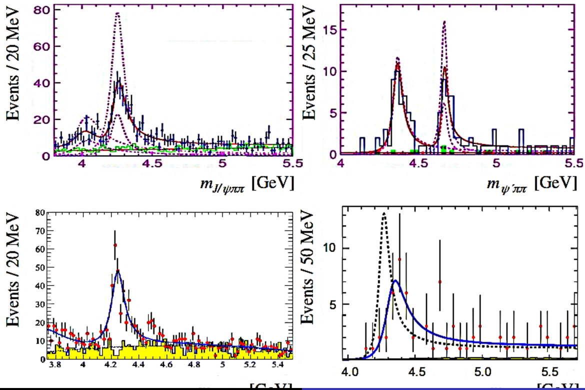
$$E_\pm = \frac{1}{10}\left(-30A_Q - 7b_Y \mp \sqrt{(30A_Q + 7b_Y)^2 + 80b_Y^2}\right)$$

Experimental situation with Y-tetraquark states is rather confusing

- Summary of the Y states observed in Initial State Radiation (ISR) processes in e^+e^- annihilation [BaBar, Belle, CLEO]

$$e^+e^- \to \gamma_{\rm ISR}\ J/\psi\pi^+\pi^-;\ \ \gamma_{\rm ISR}\ \psi'\pi^+\pi^-$$

$$\Longrightarrow\ Y(4008),\ Y(4260),\ Y(4360),\ Y(4660)$$

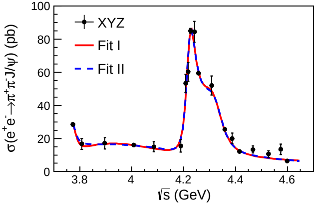


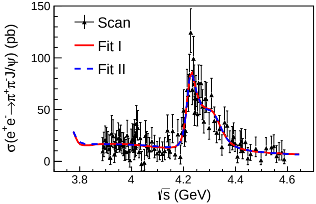
$e^+e^- \to J/\psi\pi^+\pi^-$ cross section at $\sqrt{s} = (3.77 - 4.60)$ GeV

BESIII Collab., PRL 118 (2017) 092001

- $Y(4008)$ is not confirmed;
- $Y(4260)$ is split into 2 resonances: $Y(4220)$ and $Y(4320)$, with $Y(4220)$ probably the same as $Y(4260)$

Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6}$ $(\cdots)$
$\Gamma_{\rm tot}(R_1)$	$476.9^{+78.4}_{-64.8}$ $(\cdots)$
$M(R_2)$	4222.0 ± 3.1 (4220.9 ± 2.9)
$\Gamma_{\rm tot}(R_2)$	44.1 ± 4.3 (44.1 ± 3.8)
$M(R_3)$	4320.0 ± 10.4 (4326.8 ± 10.0)
$\Gamma_{\rm tot}(R_3)$	$101.4^{+25.3}_{-19.7}$ $(98.2^{+25.4}_{-19.6})$



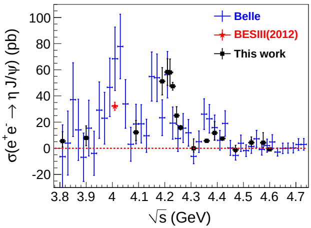


Vector Charmonium States observed at BESIII between 4.2 and 4.4 GeV

- $Y(4260)$ splits into 2 resonances: $Y(4220)$ and $Y(4320)$ or $Y(4390)$ [BESIII Collab., PRD 96 (2017) 032004]

Final State	M_1 (MeV/c^2)	Γ_1 (MeV)	M_2 (MeV/c^2)	Γ_2 (MeV)
$\omega\chi_{c0}$	$4230 \pm 8 \pm 6$	$38 \pm 12 \pm 2$		
$\pi^+\pi^- J/\psi$	$4220.0 \pm 3.1 \pm 1.4$	$44.1 \pm 4.3 \pm 2.0$	$4320.0 \pm 10.4 \pm 7.0$	$101.4^{+25.3}_{-19.7} \pm 10.2$
$\pi^+\pi^- h_c$	$4218.4^{+5.5}_{-4.5} \pm 0.9$	$66.0^{+12.3}_{-8.3} \pm 0.4$	$4391.5^{+6.3}_{-6.8} \pm 1.0$	$139.5^{+16.2}_{-20.6} \pm 0.6$
$\pi^+ D^0 D^{*-} + c.c$	$4224.8 \pm 5.6 \pm 4.0$	$72.3 \pm 9.1 \pm 0.9$	$4400.1 \pm 9.3 \pm 2.1$	$181.7 \pm 16.9 \pm 7.4$
$\pi^+\pi^- \psi(3686)$	$4209.5 \pm 7.4 \pm 1.4$	$80.1 \pm 24.6 \pm 2.9$	$4383.8 \pm 4.2 \pm 0.8$	$84.2 \pm 12.5 \pm 2.1$

- Peaking at the mass 4220 GeV was also observed in the process $e^+e^- \to \eta J/\psi$ at BESIII [PRD 91 (2015) 112005]



Two Experimental Scenarios for the Y States

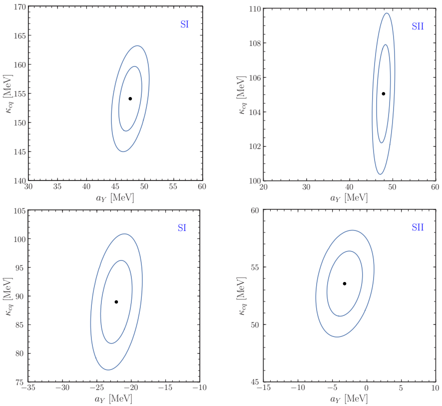
- SI (based on CLEO, BaBar, Belle data): $Y(4008)$, $Y(4260)$, $Y(4360)$, $Y(4660)$
- SII (based on BESIII [PRL 118 (2017) 092001]): $Y(4220)$, $Y(4320)$, $Y(4390)$, with $Y(4660)$ the same as in SI
- Parameters in SI and SII and $\pm 1\sigma$ errors (all in MeV). Here, $c1$ and $c2$ refer to two solutions of the secular equation

	a_Y	b_Y	κ_{cq}	M_{00}
SI (c1)	-22 ± 32	-89 ± 77	89 ± 11	4275 ± 54
SI (c2)	48 ± 23	11 ± 91	159 ± 20	4484 ± 26
SII (c1)	-3 ± 18	-105 ± 32	54 ± 8	4380 ± 25
SII (c2)	48 ± 8	-32 ± 47	105 ± 4	4535 ± 10

- SII (based on BESIII data) is favored, with a_Y and κ_{cq} values similar to Ω_c analysis ($a_1 \simeq 27$ MeV and $\kappa_{cq} \simeq 13.5$ MeV)

Correlations among the parameters

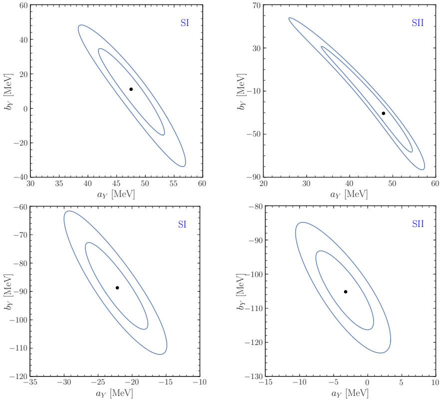
$a_Y - \kappa_{cq}$ Correlations



- SII (based on BESIII data) is favored, with a_Y and κ_{cq} values similar to the Ω_c-baryon analysis

Correlations among the parameters

$a_Y - b_Y$ Correlations



- These parameters are strongly correlated

Energy of the Orbital Excitation

- Fixing $\kappa_{cq} = 67$ MeV (from the S states); fitted the two scenarios $\Longrightarrow$ clear preference for SII, with parameters as follows (in MeV)

Scenario	M_{00}	a_Y	b_Y	$\chi^2_{\rm min}$/n.d.f.
SI	4321 ± 79	2 ± 41	-141 ± 63	12.8/1
SII	4421 ± 6	22 ± 3	-136 ± 6	1.3/1

- SII: $M_{00} \equiv 2m_{\mathcal{Q}} + B_{\mathcal{Q}} \Longrightarrow B_{\mathcal{Q}} = 442$ MeV
- Comparable to the orbital angular momentum excitation energy in charmonia

$$B_{\mathcal{Q}}(c\bar{c}) = M(h_c) - \frac{1}{4}\left[3M(J/\psi) + M(\eta_c)\right] = 457\ \mathrm{MeV}$$

- κ_{cq} and $a_{\rm Y}$ for Y states are similar to the ones in (X, Z) and Ω_c
- Precise data on Y-states are needed to confirm or reject the diquark picture of hedden-charmed tetraquarks

$L = 1$ Multiplet Predictions

J^{PC}	$\|S_Q, S_{\bar{Q}}; S, L\rangle_J$	N_1	$2(L \cdot S)$	$S_{12}/4$	Mass (MeV) best fit	EFG
3^{--}	$\|1,1;2,1\rangle_3$	2	4	$-2/5$	4630	4381
2^{--}	$\|1,1;2,1\rangle_2$	2	-2	$+7/5$	4254	4379
2_a^{--}	$\|\frac{(1,0)+(0,1)}{\sqrt{2}};1,1\rangle_2$	1	+2	0	4398	4315
2^{-+}	$\|1,1;1,1\rangle_2$	2	+2	$-1/5$	4559	4367
2_b^{-+}	$\|\frac{(1,0)-(0,1)}{\sqrt{2}};1,1\rangle_2$	1	+2	0	4398	4315
1^{-+}	$\|1,1;1,1\rangle_1$	2	-2	$+1$	4308	4345
1_b^{-+}	$\|\frac{(1,0)-(0,1)}{\sqrt{2}};1,1\rangle_1$	1	-2	0	4310	4284
0^{-+}	$\|1,1;1,1\rangle_0$	2	-4	-2	4672	4304
0_b^{-+}	$\|\frac{(1,0)-(0,1)}{\sqrt{2}};1,1\rangle_0$	1	-4	0	4266	4269
0_a^{--}	$\|\frac{(1,0)+(0,1)}{\sqrt{2}};1,1\rangle_0$	1	-4	0	4266	4269

- N_1 is the number of “bad” diquarks and antidiquarks
- EFG data are from the paper by Ebert, Faustov & Galkin [EPJC 58 (2008) 399]

Summary

- The assignment of spin-parities $J^P = 1/2^-, 3/2^-, 5/2^-$ to newly observed Ω_c-baryons by the LHCb Collaboration allows to fix all the coefficients in the effective Hamiltonian relevant for the mass spectrum
- Inclusion of the tensor interaction into the Hamiltonian results into the mixture of the states with the same spin-parity, i. e. two with $J^P = 1/2^-$ and two with $J^P = 3/2^-$
- This approach is extended onto the tetraquark states from which the vector Y-tetraquarks are considered
- Two distinct spectra are analyzed based essentially on the BaBar and Belle data (Scenario I) and on the more recent BESIII data (Scenario II). Existing experimental data prefer Scenario II
- Mass predictions for the $L = 1$ multiplet is done and can be checked experimentally
- Allow to test the correctness of the diquark model for the description of multiquark states