kT resummation for Bc decays

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k_T factorization

Why k_T factorization

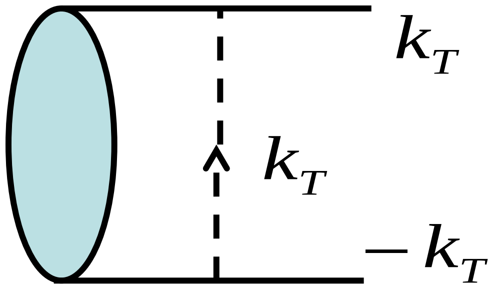
- k_T factorization has been developed for small x physics for some time
- As Bjorken variable $x_B=-q^2/(2p.q)$ is small, parton momentum fraction $x > x_B$ can reach $xp \sim k_T$. k_T is not negligible.
- $xp \sim k_T$ also possible in low q_T spectra, like direct photon and jet production
- In exclusive processes, x runs from 0 to 1. The end-point region is unavoidable
- As end-point region is important (in B decay), need k_T factorization

Dual role of parton k_T

- k_T is not constant like quark mass, but kinematic variable to be integrated out
- Its average order of magnitude depends on processes, $k_T \sim$ 1-2 GeV in B decay
- $\ln k_T$ regarded as "symbolic" IR log from collinear divergence to be subtracted by "symbolic" IR log in hadron wave functions
- $k_T \sim \sqrt{m_b \Lambda_{QCD}}$ same as hard-collinear scale and kept in hard kernel $H(m_b, \sqrt{m_b \Lambda_{QCD}}, k_T)$

TMD wave function

- k_T introduced by gluon emission



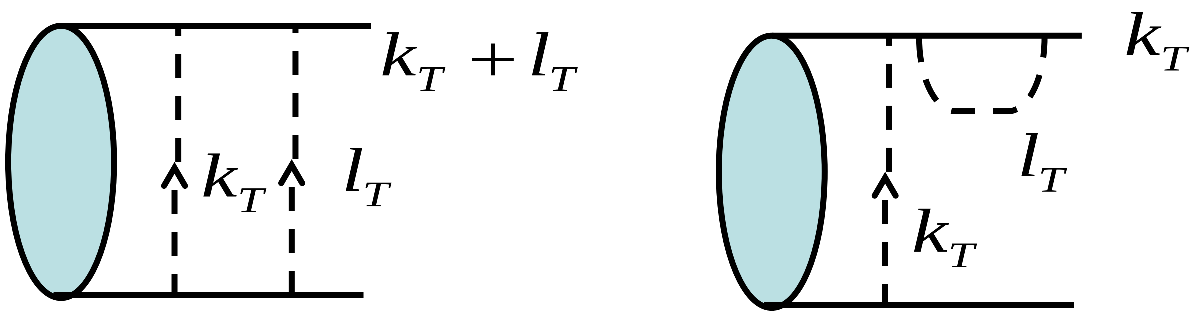
- Parton k_T in $H(x, k_T)$ demands introduction of TMD wave function $\Phi(x, k_T)$ to describe distribution in k_T.
- Amplitude in k_T factorization

$$A = \int dx \int d^2 k_T \Phi(x, k_T) H(x, k_T)$$

k_T resummation

Soft logarithm

- Add one mote gluon ,



$$H(x, k_T + l_T) - H(x, k_T)$$

- Only soft gluon with $l_T << k_T$ cancel exactly
- Soft gluon with $l_T \sim k_T$ generates soft log $\alpha_s \ln(Q/k_T)$

Double logarithm

- Double log from the overlap of collinear and soft enhancements appears in two-scale system. For TMD, they are Q and k_T
- Define leading log (LL) $\alpha_s^n L^{2n}$, next-to-leading log (NLL) $\alpha_s^n L^{2n-1}$, next-to-next-to-leading log (NNLL) $\alpha_s^n L^{2n-2}$
- To improve perturbation, k_T resummation is necessary
- There are different methods available in different gauges up to different accuracy

k_T resummation

- Adopt Collins-Soper-Sterman (CSS) method in axial gauge
- Derived Sudakov factor $\exp[-s(Q,b)]$ up to NLL accuracy

1-loop, process-dependent

$$s(Q,b) = \int_{1/b}^{Q} \frac{d\mu}{\mu} \left[\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\mu)) \right]$$

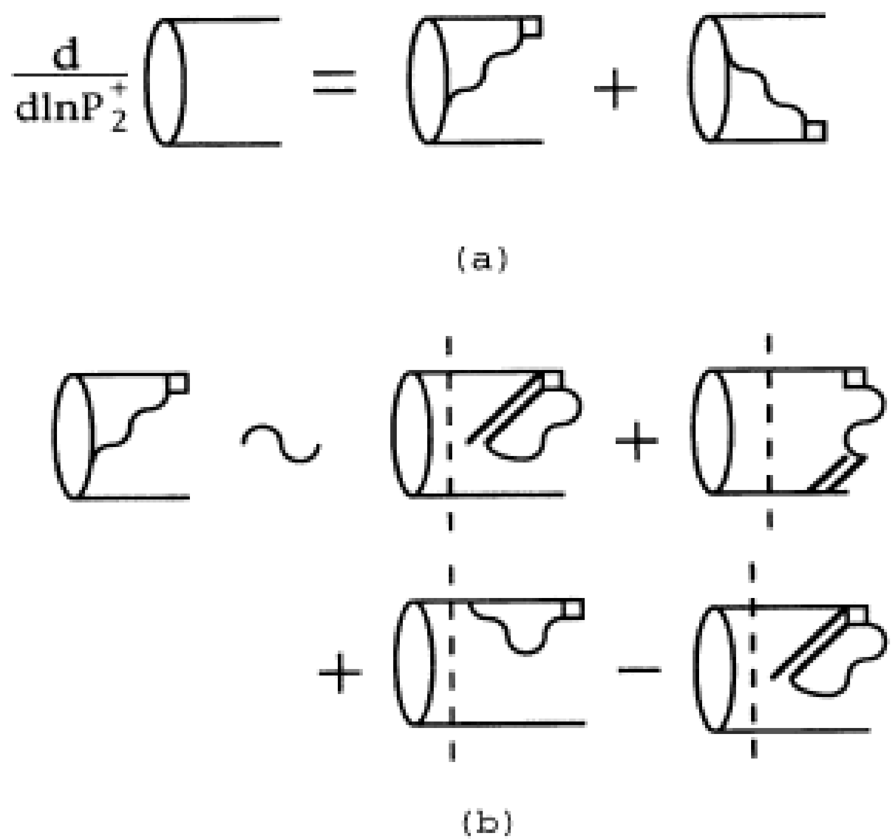
2-loop universal cusp anomalous dimension

- Resultant Sudakov factor describes partial parton k_T distribution

Botts, Sterman 1989

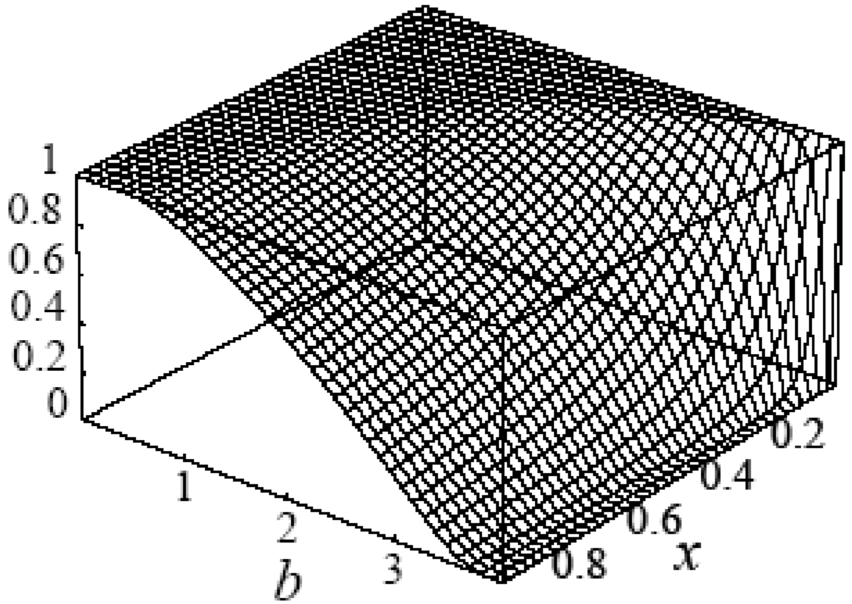
CSS method

- Resummation for heavy-to-light transition

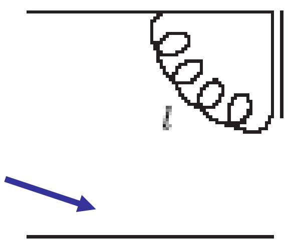


Li, Yu 1995

Sudakov effect



suppression at large b becomes stronger at larger energy



Small x

- Physical picture: large b means large color dipole. Large dipole tends to radiate during hard scattering. No radiation in exclusive processes. Small b configuration preferred.

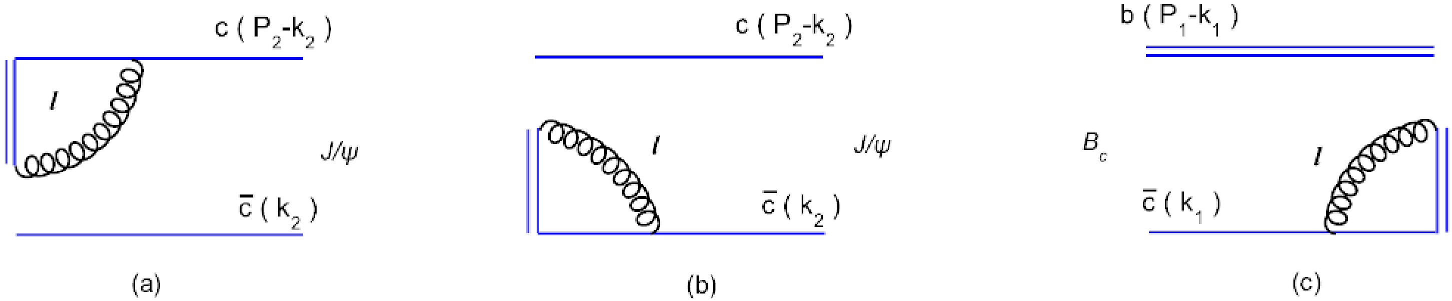
Charm mass effect

Multi-scale processes

- Many PQCD calculations of Bc decays in literature
- Charm mass kept in hard kernels, but neglected in Sudakov factor
- Charmed B decay is a multi-scale process, for which resummation is difficult
- Assume hierarchy $m_b >> m_c >> k_T$ to simplify power counting Kurimoto, Li, Sanda 2002
- If assuming $m_b \sim m_c$, twist expansion on charmed meson side may not hold

Source of double logs

- Apply it to $B_c \to J/\psi$ transition
- Double logs in TMD wave functions come from quark-Wilson-line vertex correction



$$P_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, \mathbf{0_T})$$

$$k_1 = (x_1 P_1^+, x_1 P_1^-, \mathbf{k}_{1T})$$

- Fig.(a) most important, energetic charm provides larger collinear enhancement

Some observations

- Result of Fig.(a)

$$\phi^{(1)} = \frac{\alpha_s}{4\pi} C_F \left[\frac{1}{\epsilon} + \ln \frac{4\pi\mu_f^2}{m_c^2 e^{\gamma_E}} - \ln^2 \frac{\zeta^2}{k_T^2} + \ln^2 \frac{m_c^2}{k_T^2} + \ln \frac{\zeta^2}{m_c^2} + 2 - \frac{2\pi^2}{3} \right]$$

- Soft divergence regularized by k_T, collinear one by m_c
- Replacing m_c by k_T reproduces light meson result
- Partial cancellation implies weaker effect
- Initial scale for RG evolution of TMD is m_c

Resummation for finite mass

The μ_f-independent logarithms in Eq. (1) can be cast into two pieces

$$-\left(\ln^2 \frac{\zeta^2}{k_T^2} - \ln \frac{\zeta^2}{k_T^2}\right) + \left(\ln^2 \frac{m_c^2}{k_T^2} - \ln \frac{m_c^2}{k_T^2}\right),$$

It hints that the Sudakov exponent $s_c(Q,b)$ can be expressed as the difference

$$\begin{aligned} s_c(Q,b) &= s(Q,b) - s(m_c,b), \\ &= \int_{m_c}^{Q} \frac{d\mu}{\mu}\left[\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\mu))\right]. \end{aligned}$$

Sudakov factors

$$S_{B_c} = s_c\left(x_1 P_1^-, b_1\right) + \frac{5}{3}\int_{m_c}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})),$$

$$S_{J/\psi} = s_c\left(x_2 P_2^+, b_2\right) + s_c\left((1-x_2)P_2^+, b_2\right) + 2\int_{m_c}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu}))$$

Numerical analysis

PQCD factorization

wave functions known in literature

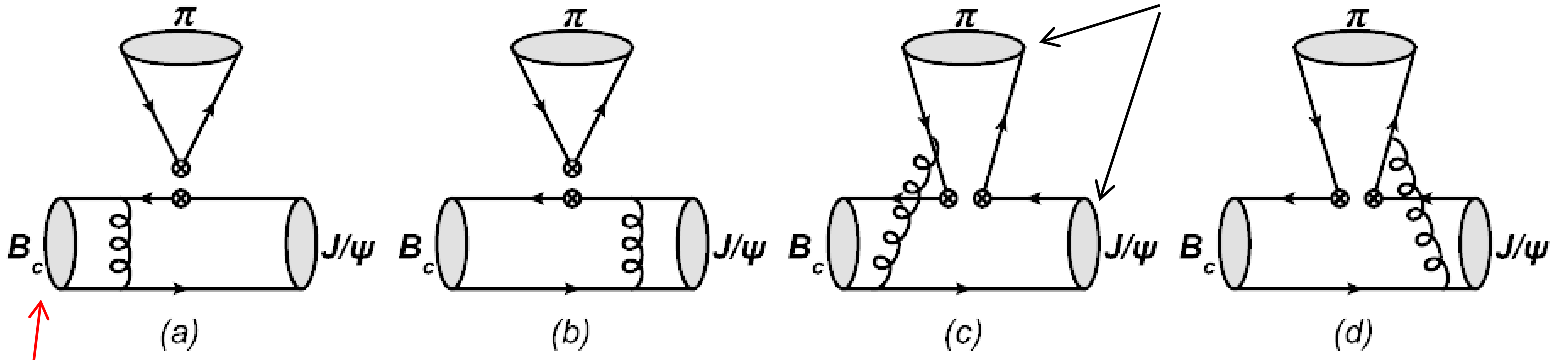


Figure 3: LO diagrams for the $B_c^+ \to J/\psi\pi^+$ decay in the PQCD approach.

$$\phi_{B_c}(x,b) = \frac{f_{B_c}}{2\sqrt{2N_c}} N_{B_c} x(1-x) \exp\left[-\frac{(1-x)m_c^2 + x m_b^2}{8\beta_{B_c}^2 x(1-x)}\right] \times \exp\left[-2\beta_{B_c}^2 x(1-x) b^2\right] ,$$

shape parameter in Gaussian form

Bc wave function

soft gluon exchanged between b and c quarks

$l_s^\mu \sim (\Lambda, \Lambda, \Lambda)$

charm can be off-shell by

$k_1^2 - m_c^2 \approx \mathcal{O}(m_c \Lambda)$

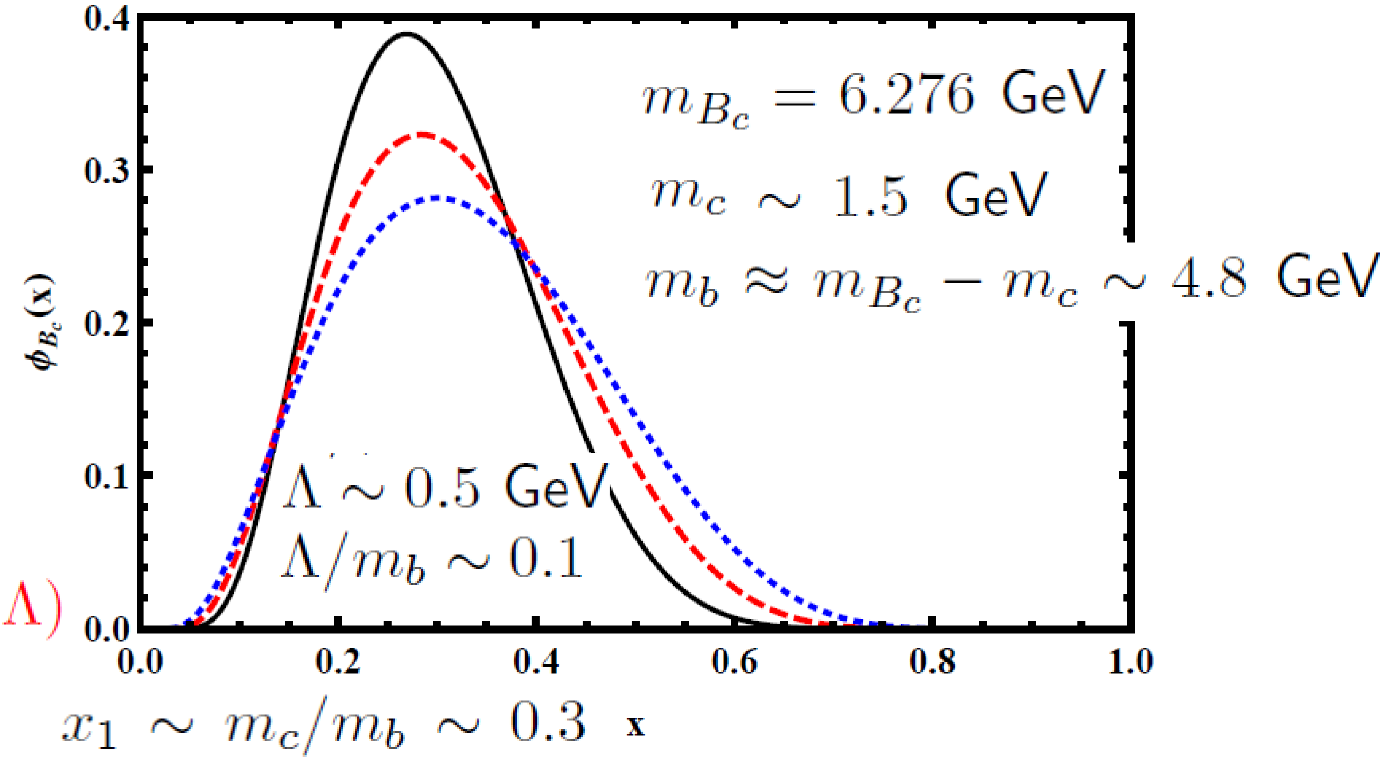


Figure 1: Behavior of $\phi_{B_c}(x)$ for the different shape parameters $\beta_{B_c} = 0.8$ GeV (black-solid curve), 1.0 GeV (red-dashed curve), and 1.2 GeV (blue-dotted curve).

Numerical result

shape parameter	$A_0^{B_c \to J/\psi}(0)$	$Br(B_c^+ \to J/\psi \pi^+)$
$\beta_{B_c} = 0.8$ GeV	$0.488 - i0.095$	2.80×10^{-3}
$\beta_{B_c} = 0.9$ GeV	$0.434 - i0.070$	2.10×10^{-3}
$\beta_{B_c} = 1.0$ GeV	$0.384 - i0.053$	1.60×10^{-3}
$\beta_{B_c} = 1.1$ GeV	$0.341 - i0.039$	1.23×10^{-3}
$\beta_{B_c} = 1.2$ GeV	$0.306 - i0.029$	0.94×10^{-3}

$B_c \to J/\psi$ transition form factor is almost real,
because final state below DD threshold

Br predictions in literature range in $10^{-4} \sim 10^{-2}$

Charm mass effect in resummation is about 10%,
not significantly large, but needed for precise calculation

Summary

(a) The PQCD approach needs essential modifications to meet the measurements for B_c meson decays with good precision;

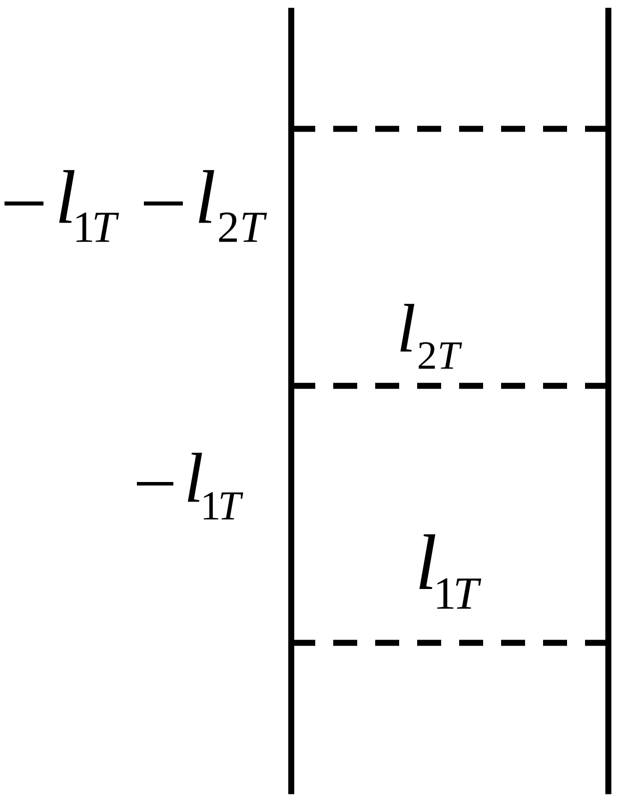
(b) $\phi_{B_c}(x)$ has a peak around the momentum fraction $x \sim 0.3$ with width of order $\Lambda/m_b \sim 0.1$; Gaussian form with $\beta_{B_c} \sim 1$ GeV;

(c) Newly derived $S_c(Q,b)$ for B_c meson decays is obtained by modifying the k_T resummation by including the charm quark mass; only exact at leading-logarithm level, a precise NLL resummation formalism demands a complete one-loop calculation for determining the factor $B(\alpha_s)$.

(d) In iPQCD, largely suppressed strong phase in $A_0^{B_c \to J/\psi}$; $Br(B_c \to J/\psi\pi) \sim 1.6 \times 10^{-3}$; β_{B_c} demands the more precise data of individual BRs or ratios of BRs sensitive to the nonfactorizable emission diagrams.

Back-up slides

k_T ordering



$$\approx$$

$$l_2^+ \approx l_1^+$$

$$l_{2T} >> l_{1T}$$