QCD aspects of the pion-photon form factor

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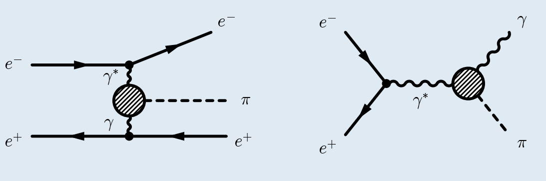
2nd Workshop on Heavy Quark Physics
23. 04. 2018

The pion-photon transition form factor

- Theoretically simplest hadronic matrix element:

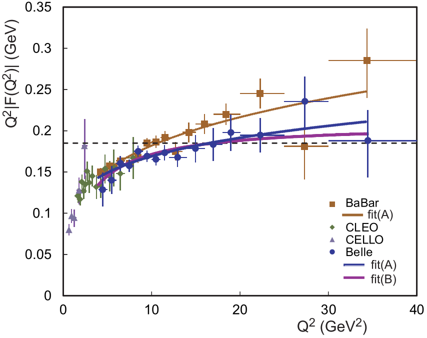
$$\langle \pi(p)|j_\mu^{\rm em}|\gamma(p')\rangle = g_{\rm em}^2\, \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha\, p^\beta\, \varepsilon^\nu(p') F_{\gamma^*\gamma\to\pi^0}(Q^2)\,.$$

  - Related to the axial anomaly at $Q^2 \equiv -(p-p')^2 = 0$.
  - Golden channel to understand the QCD dynamics.
  - e^+e^- collisions:



The BaBar puzzle

- Status of experimental measurements:

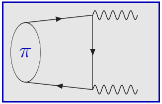


Scaling violation?

Shape of pion wave function?

The onset of QCD factorization?

- Asymptotic limit:



$\longrightarrow$

$$\lim_{Q^2 \to \infty} Q^2 F_{\pi\gamma\gamma^*}(Q^2) = 2 f_\pi$$

Brodsky, Lepage

QCD factorization works perfectly.

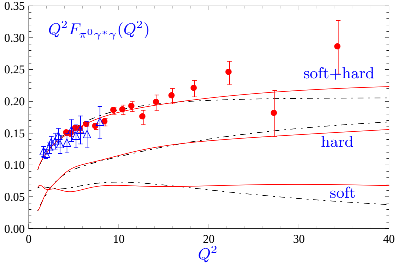
Some popular explanations:

- Non-vanishing pion wave function at the end points (Radyushkin, 2009; Polyakov, 2009).

$$F(Q^2) = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x)}{x Q^2} \Big[1 - \underbrace{\exp\Big(- \frac{x Q^2}{2 \bar{x} \sigma} \Big)}_{\Uparrow} \Big] .$$

from k_T dependence of pion wave function

- Large soft corrections at moderate Q^2 (Agaev, Braun, Offen, Porkert, 2011).

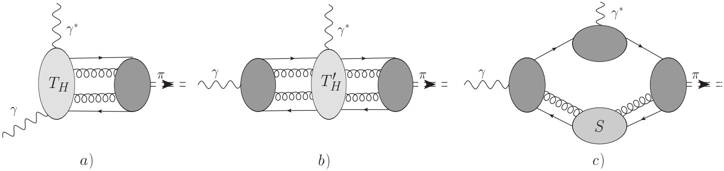


The "hard" and "soft" contributions to the $\pi^0\gamma^*\gamma$ form factor for model I (solid curves) and model III (dash-dotted curves). The experimental data are from BaBar (full circles) and CLEO (open triangles).

- Threshold resummation generates power-like $[x(1-x)]^{c(Q^2)}$ distribution (Li and Mishima 2009). $c(Q^2)$ is around 1 for low Q^2, but small for high Q^2.

The general picture

- Schematic structure of the distinct mechanisms (Agaev, Braun, Offen, Porkert, 2011):



A: hard subgraph that includes both photon vertices $\frac{1}{Q^2} + \frac{1}{Q^4} + \ldots$

B: real photon emission at large distances $\frac{1}{Q^4} + \ldots$

C: Feynman mechanism: soft quark spectator $\frac{1}{Q^4} + \ldots$

- Operator definitions of different terms needed for an unambiguous classification.

Region A: Leading Twist Contribution

- QCD factorization formula:

$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}\,(Q_u^2 - Q_d^2) f_\pi}{Q^2} \int_0^1 dx\, T^\Delta(x, Q^2, \mu)\, \phi_\pi^\Delta(x,\mu)\,.$$

  - Renormalization-scheme dependence due to the IR subtraction.
    $\Rightarrow$ Verify scheme independence of $F_{\gamma^*\gamma\to\pi^0}$ at NLO (this talk).
  - NLO hard function in the NDR scheme (Braaten 1983 + many others):

$$T^{(1)} = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{\bar{x}} \left[-(2\ln\bar{x}+3)\ln\frac{\mu^2}{Q^2} + \ln^2\bar{x} + (-1)\,\frac{\bar{x}\ln\bar{x}}{x} - 9 \right] + (x \leftrightarrow \bar{x}) \right\}.$$

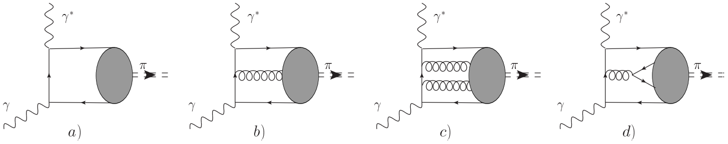
  - Twist-2 pion DA (collinear matrix element):

$$\langle \pi(p) | \bar{\xi}(y)\, W_c(y,0)\, \gamma_\mu\, \gamma_5\, \xi(0) | 0 \rangle = -i f_\pi\, p_\mu \int_0^1 du\, e^{iup\cdot y}\, \phi_\pi(u,\mu) + \mathcal{O}(y^2)\,.$$

    The ERBL evolution implies Gegenbauer expansion.

Region A: Twist-4 Contribution

- Subleading power correction at $\mathcal{O}(1/Q^4)$:



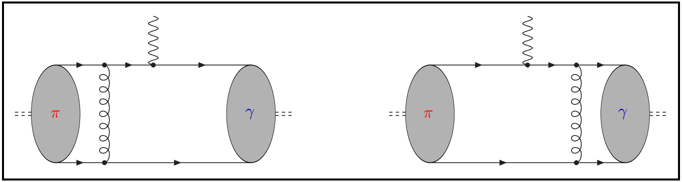
- QCD factorization at tree level (Khodjamirian, 1999):

$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}\,(Q_u^2 - Q_d^2) f_\pi}{Q^2} \left[\int_0^1 dx\, \frac{\phi_\pi(x)}{x} - \frac{80}{9} \frac{\delta_\pi^2}{Q^2} \right], \qquad \delta_\pi^2 \simeq 0.2\,{\rm GeV}^2\,.$$

  - Asymptotic twist-4 contribution.
  - Two-particle and three-particle twist-4 corrections related by the EOM.
  - Four-particle twist-4 correction not included and assumed to be tiny.
  - Asymptotic NLO twist-4 at NLO for future.

Region B: Photon emission at large distances

- Hadronic photon correction:



- QCD factorization at tree level:

$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}\,(Q_u^2 - Q_d^2) f_\pi}{Q^2} \, \frac{16\,\pi\,\alpha_s\,\chi(\mu)\,\langle\bar{q}q\rangle^2}{9 f_\pi^2\, Q^2} \int_0^1 dx\, \frac{\phi^p_{3;\pi}(x)}{x} \int_0^1 dy\, \frac{\phi_\gamma(y)}{\bar{y}^2}\,.$$

Breakdown of QCD factorization due to rapidity singularities.

- QCD calculation of the hadronic photon effect beyond the VMD approximation.

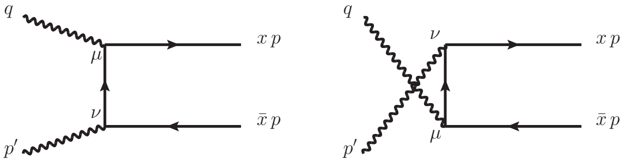
  ⇒ The NLL LCSRs for the long-distance photon effect (this talk).

  ⇒ QCD factorization for the correlation function with a pseudoscalar current.

  ⇒ γ_5 prescription in dimensional regularization.

Part I: Leading twist contribution

- Tree diagrams:



Kinematics:

$$p'_\mu = \underbrace{\frac{n\cdot p'}{2}}_{\mathcal{O}(\sqrt{Q^2})} \bar{n}_\mu \,, \qquad p_\mu = \underbrace{\frac{\bar{n}\cdot p}{2}}_{\mathcal{O}(\sqrt{Q^2})} n_\mu + \underbrace{\frac{n\cdot p}{2}}_{\mathcal{O}(\Lambda^2/\sqrt{Q^2})} \bar{n}_\mu \,.$$

- The four-point partonic matrix element at LO:

$$\begin{aligned}\Pi_\mu &= \langle q(xp)\,\bar{q}(\bar{x}p)|j_\mu^{\rm em}|\gamma(p')\rangle \\ &= -\frac{i\,g_{\rm em}^2\,(Q_u^2-Q_d^2)}{2\sqrt{2}\,n\cdot p}\,\varepsilon^\nu(p')\left\{\frac{\left[\bar{u}(xp)\,\gamma_{\mu,\perp}\,\not{\bar{n}}\,\gamma_{\nu,\perp}\,v(\bar{x}p)\right]}{\bar{x}} - \frac{\left[\bar{u}(xp)\,\gamma_{\nu,\perp}\,\not{\bar{n}}\,\gamma_{\mu,\perp}\,v(\bar{x}p)\right]}{x}\right\} \\ &\quad + \mathcal{O}(\alpha_s^2)\,.\end{aligned}$$

QCD matrix element free of the γ_5 ambiguity, however not the IR subtraction.

Twist-2 factorization at tree level

- Operator matching automatically:

$$\Pi_\mu^{(0)} = -\frac{i\,g_{\rm em}^2\,(Q_u^2 - Q_d^2)}{2\sqrt{2}\,n\cdot p}\,\varepsilon^\nu(p')\left[\frac{1}{\bar{x}'} * \langle O_{A,\mu\nu}(x,x')\rangle^{(0)} - \frac{1}{x'} * \langle O_{B,\mu\nu}(x,x')\rangle^{(0)}\right].$$

- Collinear operators in the momentum space:

$$\begin{aligned} O_{j,\mu\nu}(x') &= \frac{\bar{n}\cdot p}{2\pi}\int d\tau\, e^{i x' \tau \bar{n}\cdot p}\,\bar{\xi}(\tau\bar{n})\,W_c(\tau\bar{n},0)\,\Gamma_{j,\mu\nu}\,\xi(0)\,. \\ \Gamma_{A,\mu\nu} &= \gamma_{\mu,\perp}\,\not{\bar{n}}\,\gamma_{\nu,\perp}\,, \qquad \Gamma_{B,\mu\nu} = \gamma_{\nu,\perp}\,\not{\bar{n}}\,\gamma_{\mu,\perp}\,. \end{aligned}$$

- Matrix elements of the collinear operators:

$$\langle O_{j,\mu\nu}(x,x')\rangle \equiv \langle q(xp)\,\bar{q}(\bar{x}p)|O_{j,\mu\nu}(x')|0\rangle = \bar{\xi}(xp)\,\Gamma_{j,\mu\nu}\,\xi(\bar{x}p)\;\delta(x-x') + \mathcal{O}(\alpha_s)\,.$$

- Operator matching with the collinear operator defining the standard pion DA:

$$\begin{aligned} O_{A,\mu\nu} &= -\left(O_{1,\mu\nu} + O_{2,\mu\nu} + O_{E,\mu\nu}\right), \qquad O_{B,\mu\nu} = -\left(O_{1,\mu\nu} - O_{2,\mu\nu} - O_{E,\mu\nu}\right). \\ \underbrace{\Gamma_{1,\mu\nu}} &= g_{\mu\nu}^{\perp}\,\not{\bar{n}}\,, \qquad \Gamma_{2,\mu\nu} = i\,\varepsilon_{\mu\nu}^{\perp}\,\not{\bar{n}}\,\gamma_5\,, \qquad \underbrace{\Gamma_{E,\mu\nu}} = \not{\bar{n}}\left(\frac{[\gamma_{\mu,\perp},\gamma_{\nu,\perp}]}{2} - i\,\varepsilon_{\mu\nu}^{\perp}\,\not{\bar{n}}\,\gamma_5\right). \end{aligned}$$

wrong projector ($\Gamma_{1,\mu\nu}$) — evanescent operator ($\Gamma_{E,\mu\nu}$)

Twist-2 factorization at tree level

- Operator matching with the evanescent operator:

$$\Pi_\mu = \left[\frac{i\, g_{\rm em}^2\, (Q_u^2 - Q_d^2)}{2\sqrt{2}\, n \cdot p}\, \varepsilon^\nu(p') \right] \sum_i T_i(x') * \langle O_{i,\mu\nu}(x,x') \rangle \,.$$

Expansion $\Downarrow$ at tree level

$$T_1^{(0)}(x') = \frac{1}{x'} - \frac{1}{\bar{x}'} \,, \qquad T_2^{(0)}(x') = T_E^{(0)}(x') = \frac{1}{x'} + \frac{1}{\bar{x}'} \,.$$

- Hard-collinear factorization at tree level:

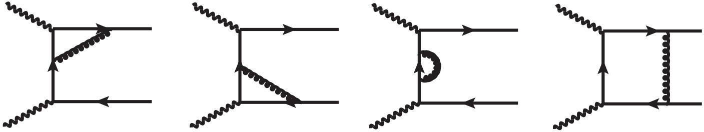
$$F^{\rm LP}_{\gamma^* \gamma \to \pi^0}(Q^2) = \frac{\sqrt{2}\,(Q_u^2 - Q_d^2) f_\pi}{Q^2} \int_0^1 dx\, T_2^{(0)}(x)\, \phi_\pi(x,\mu) + \mathcal{O}(\alpha_s) \,.$$

Evanescent operator does not mix into the physical operator at LO.

$\Rightarrow$ Trivial IR subtraction here.

Twist-2 factorization at NLO

- The four-point partonic matrix element at NLO:



- Extracting the hard contribution with the method of regions:

$$\Pi_\mu^{(1)} = \frac{i\, g_{\rm em}^2\, (Q_u^2 - Q_d^2)}{2\sqrt{2}\, n\cdot p}\, \varepsilon^\nu(p')\, \langle O_{2,\mu\nu}(x,x')\rangle^{(0)} * A_{2,\rm hard}^{(1)}(x') + \ldots .$$

$$A_{2,\rm hard}^{(1)}(x') = \frac{\alpha_s\, C_F}{4\pi} \left\{ \frac{1}{\bar{x}'} \left[-\left(2\ln\bar{x}' + 3\right) \left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{Q^2} \right) + \ln^2\bar{x}' + 7\, \frac{\bar{x}'\ln\bar{x}'}{x'} - 9 \right] + (x' \leftrightarrow \bar{x}') \right\}.$$

Only the hard and collinear regions relevant at leading power in $1/Q^2$.
Independent of the γ_5 prescription!

- However the IR subtraction is not trivial any longer due to the operator mixing.

Twist-2 factorization at NLO

- Expanding the matching equation at NLO:

$$\left[\frac{i\,g_{\rm em}^2\,(Q_u^2-Q_d^2)}{2\sqrt{2}\,n\cdot p}\,\varepsilon^{\nu}(p')\right]\sum_i A_i^{(1)}(x')\ast\langle O_{i,\mu\nu}(x,x')\rangle^{(0)}$$
$$=\left[\frac{i\,g_{\rm em}^2\,(Q_u^2-Q_d^2)}{2\sqrt{2}\,n\cdot p}\,\varepsilon^{\nu}(p')\right]\sum_i\left[T_i^{(1)}(x')\ast\langle O_{i,\mu\nu}(x,x')\rangle^{(0)}+T_i^{(0)}(x')\ast\langle O_{i,\mu\nu}(x,x')\rangle^{(1)}\right].$$

- One-loop renormalized matrix elements of collinear operators:

$$\langle O_{i,\mu\nu}\rangle^{(1)}=\sum_j\left[M_{ij,\rm bare}^{(1)R}+Z_{ij}^{(1)}\right]\ast\langle O_{j,\mu\nu}\rangle^{(0)}.$$

Vanishing bare matrix element $M_{ij,\rm bare}^{(1)R}$ in dimensional regularization $\Rightarrow$

$$T_2^{(1)}=A_2^{(1)}-\sum_i T_i^{(0)}\ast Z_{i2}^{(1)}=\underbrace{A_2^{(1)}-T_2^{(0)}\ast Z_{22}^{(1)}}_{A_{2,\rm hard}^{(1)}}-T_E^{(0)}\ast\underbrace{Z_{E2}^{(1)}}_{\text{operator mixing}}.$$

- The IR finite matrix element of the evanescent operator vanishes (Dugan and Grinstein, 1991).

$$Z_{E2}^{(1)}=-M_{E2}^{(1)\rm off}.$$

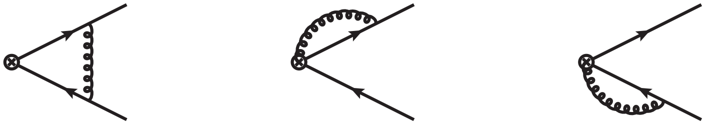
IR singularities regularized by any parameter other than the dimensions of spacetime.

Twist-2 factorization at NLO

- Master formula for the hard function:

$$T_2^{(1)} = A_2^{(1)} - T_2^{(0)} * Z_{22}^{(1)} + T_E^{(0)} * M_{E2}^{(1)\mathrm{off}} = A_{2,\mathrm{hard}}^{(1)} + T_E^{(0)} * M_{E2}^{(1)\mathrm{off}} .$$

- The IR subtraction:



γ_5-scheme dependent subtraction term:

$$T_E^{(0)} * M_{E2}^{(1)\mathrm{off}} \Big|_{\mathrm{NDR}} = \frac{\alpha_s C_F}{2\pi} (-4) \left(\frac{\ln \bar{x}'}{x'} + \frac{\ln x'}{\bar{x}'} \right) .$$

$$T_E^{(0)} * M_{E2}^{(1)\mathrm{off}} \Big|_{\mathrm{HV}} = 0 .$$

$M_{E2}^{(1)\mathrm{off}}$ proportional to the spin-dependent term of the Brodsky-Lepage evolution kernel.

A simple example :

$$\gamma_\alpha \not{n} \gamma_5 \gamma^\alpha = \not{n} \gamma_5 \begin{cases} D-2 , & \text{[NDR scheme]} \\ 6-D . & \text{[HV scheme]} \end{cases}$$

Twist-2 factorization at NLO

- The NLO hard matching coefficient:

$$T_2^{(1)}(x',\mu) = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{\bar{x}'} \left[-\left(2\ln\bar{x}'+3\right) \ln\frac{\mu^2}{Q^2} + \ln^2\bar{x}' + \delta\,\frac{\bar{x}'\ln\bar{x}'}{x'} - 9 \right] + \left(x' \leftrightarrow \bar{x}'\right) \right\}.$$

$\delta = -1$ in NDR scheme and $\delta = +7$ in HV scheme.

- The NLO factorization formula:

$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}(Q_u^2-Q_d^2)f_\pi}{Q^2} \int_0^1 dx \left[T_2^{(0)}(x) + T_2^{(1),\Delta}(x,\mu)\right] \phi_\pi^\Delta(x,\mu) + \mathcal{O}(\alpha_s^2).$$

- Scheme dependence of the pion DA (Melic, Nizic and Passek, 2002):

$$\phi_\pi^{\rm HV}(x,\mu) = \int_0^1 dy\, Z_{\rm HV}^{-1}(x,y,\mu)\, \phi_\pi^{\rm NDR}(y,\mu),$$

$$Z_{\rm HV}^{-1}(x,y,\mu) = \delta(x-y) + \frac{\alpha_s C_F}{2\pi}\, 4 \left[\frac{x}{y}\,\theta(y-x) + \frac{\bar{x}}{\bar{y}}\,\theta(x-y)\right] + \mathcal{O}(\alpha_s^2).$$

$$\Downarrow$$

$$\int_0^1 dx\, T_2^{(0)}(x) \left[\phi_\pi^{\rm HV}(x,\mu) - \phi_\pi^{\rm NDR}(x,\mu)\right] = \frac{\alpha_s C_F}{2\pi}(-4)\int_0^1 dy \left(\frac{\ln\bar{y}}{y} + \frac{\ln y}{\bar{y}}\right) \phi_\pi^{\rm NDR}(x,\mu) + \mathcal{O}(\alpha_s^2).$$

$\Rightarrow$ Scheme independence of $F_{\gamma^*\gamma\to\pi^0}$ at NLO.

Twist-2 factorization at NLL

- RG evolution of the pion LCDA:

$$\mu^2 \frac{d}{d\mu^2} \phi_\pi(x,\mu) = \int_0^1 dy\, V(x,y)\, \phi_\pi(y,\mu)\,, \qquad V(x,y) = \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^{n+1} [V_n(x,y)]_+\,.$$

$$V_0(x,y) = 2\,C_F \left[\frac{1-x}{1-y}\left(1+\frac{1}{x-y}\right)\theta(x-y) + \frac{x}{y}\left(1+\frac{1}{y-x}\right)\theta(y-x)\right].$$

Multiplicative renormalization at LO:

$$\phi_\pi(x,\mu) = 6x\bar{x} \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{V,n}^{(0)}/(2\beta_0)} a_n(\mu_0)\, C_n^{3/2}(2x-1)\,, \qquad a_0(\mu) = 1\,.$$

- NLL resummation needs two-loop evolution kernel (Mikhailov and Radyushkin, 1985; etc):

$$V_1(x,y) = 2\,N_f\, C_F\, V_N(x,y) + 2\,C_F\, C_A\, V_G(x,y) + C_F^2\, V_F(x,y)\,.$$

Triangular evolution matrix (Müller, 1994/1995 + many others):

$$a_n(\mu) = E_{V,n}^{\rm NLO}(\mu,\mu_0)\, a_n(\mu_0) + \frac{\alpha_s(\mu)}{4\pi} \sum_{k=0}^{n-2} E_{V,n}^{\rm LO}(\mu,\mu_0)\, d_{V,n}^{k}(\mu,\mu_0)\, a_k(\mu_0)\,.$$

Construction from the forward anomalous dimensions and the special conformal anomaly matrix.

Twist-2 factorization at NLL

- NLL resummation improved factorization formula:

$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{3\sqrt{2}\,(Q_u^2-Q_d^2)}{Q^2}\, f_\pi \sum_{n=0}^{\infty} a_n(\mu)\, C_n(Q^2,\mu) + \mathcal{O}(\alpha_s^2)\,.$$

NNLO hard kernel in the large β_0 limit (Melic, Müller and Passek-Kumericki, 2003).

- Generating function of the Gegenbauer polynomials:

$$\frac{1}{(1-2\,x\,t+t^2)^{\alpha}} = \sum_{n=0} C_n^{(\alpha)}(x)\, t^n\,.$$

The NLO hard coefficients:

$$C_n(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)\,C_F}{4\pi}\left\{\left[4\,H_{n+1} - \frac{3\,n\,(n+3)+8}{(n+1)(n+2)}\right]\ln\frac{\mu^2}{Q^2} + 4\,H_{n+1}^2 - 4\,\frac{H_{n+1}+1}{(n+1)(n+2)} \right.$$
$$\left. + 2\left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2}\right] + 3\left[\frac{1}{(n+1)} - \frac{1}{(n+2)}\right] - 9\right\}.$$

Part II: Hadronic photon effect

- Breakdown of the direct QCD factorization for the long-distance photon effect
  $\Rightarrow$ Construct the sum rules for the hadronic photon correction.

- Correlation function:

$$G_\mu(p',q) = \int d^4 z\, e^{-iq\cdot z} \langle 0|\mathrm{T}\left\{ j^{\mathrm{em}}_{\mu,\perp}(z), j_\pi(0)\right\}|\gamma(p')\rangle = -g^2_{\mathrm{em}}\,\varepsilon_{\mu\nu\alpha\beta}\, q^\alpha\, p'^\beta\, \varepsilon_\nu(p')\, G(p^2,Q^2)\,.$$

$$j_\pi = \frac{1}{\sqrt{2}}\left(\bar{u}\,\gamma_5\, u - \bar{d}\,\gamma_5\, d\right)\,, \qquad p^2 = (p'+q)^2\,.$$

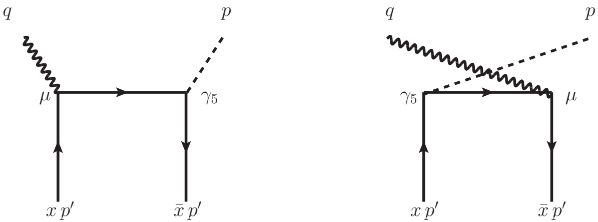
Explicit dependence on $\gamma_5 \Rightarrow$ scheme dependence of the QCD amplitude.

- Task: QCD factorization for the transition form factor $G(p^2,Q^2)$ at NLL.
  Power counting rule:

$$|n\cdot p| \sim \bar{n}\cdot p \sim n\cdot p' \sim \mathcal{O}(\sqrt{Q^2})\,.$$

Hadronic photon effect at tree level

- Tree diagrams:



p for the four-momentum of the pion current and q for the transfer momentum.

- The four-point QCD amplitude at LO:

$$\widetilde{\Pi}_\mu = \int d^4z\, e^{-iq\cdot z} \langle 0|\mathrm{T}\left\{j^{\rm em}_{\mu,\perp}(z), j_\pi(0)\right\}|q(xp')\,\bar{q}(\bar{x}p')\rangle$$

$$= -\frac{i\,g_{\rm em}}{2\sqrt{2}}\,\frac{\bar{n}\cdot p}{Q^2}\left[\frac{1}{x\,r+\bar{x}}+\frac{1}{\bar{x}\,r+x}\right]\sum_{q=u,d}\eta_q\,Q_q\,\bar{q}(\bar{x}p')\,\gamma_5\,\not{n}\,\gamma_{\mu,\perp}\,q(xp'),$$

where $r = -p^2/Q^2$, $\eta_u = 1$ and $\eta_d = -1$.

Hadronic photon effect at tree level

- Operator matching automatically:

$$\widetilde{\Pi}_\mu^{(0)} = -\frac{i\,g_{\rm em}}{2\sqrt{2}}\,\frac{\bar{n}\cdot p}{Q^2}\sum_{q=u,d}\eta_q\,Q_q\left[\frac{1}{x'\,r+\bar{x}'}+\frac{1}{\bar{x}'\,r+x'}\right]*\langle\widetilde{O}_{A,\mu}(x,x')\rangle^{(0)}\,.$$

- (Anti)-collinear operators in the momentum space:

$$\begin{aligned}\widetilde{O}_{j,\mu}(x') &= \frac{\bar{n}\cdot p'}{2\pi}\int d\tau\, e^{i x'\,\tau \bar{n}\cdot p'}\,\bar{\chi}(0)\,W_{\bar{c}}(0,\tau n)\,\widetilde{\Gamma}_{j,\mu}\,\chi(\tau n)\,,\\ \widetilde{\Gamma}_{A,\mu} &= \gamma_5\,\not{n}\,\gamma_{\mu,\perp}\,.\end{aligned}$$

- Matrix element of the (anti)-collinear operators:

$$\langle\widetilde{O}_{j,\mu}(x,x')\rangle \equiv \langle 0|\widetilde{O}_{j,\mu}(x')|q(xp')\,\bar{q}(\bar{x}p')\rangle = \bar{\chi}(\bar{x}p')\,\widetilde{\Gamma}_{j,\mu}\,\chi(xp')\,\delta(x-x')+\mathcal{O}(\alpha_s)\,.$$

- Operator matching with the effective operator defining the photon DA:

$$\begin{aligned}\widetilde{O}_{A,\mu} &= \widetilde{O}_{1,\mu}+\widetilde{O}_{E,\mu}\,,\\ \widetilde{\Gamma}_{1,\mu} &= \frac{n^\alpha}{2}\,\varepsilon^{\perp}_{\mu\nu\alpha\beta}\,\sigma^{\nu\beta}\,, \qquad \underbrace{\widetilde{\Gamma}_{E,\mu}}_{\text{evanescent operator}} = \gamma_5\not{n}\,\gamma_{\mu,\perp}-\frac{n^\alpha}{2}\,\varepsilon^{\perp}_{\mu\nu\alpha\beta}\,\sigma^{\nu\beta}\,.\end{aligned}$$

Hadronic photon effect at tree level

- Operator matching with the evanescent operator:

$$\widetilde{\Pi}_\mu = -\frac{i\,g_{\rm em}}{2\sqrt{2}}\,\frac{\bar{n}\cdot p}{Q^2}\sum_{q=u,d}\eta_q\,Q_q\sum_i \widetilde{T}_i(x') * \langle \widetilde{O}_{i,\mu}(x,x')\rangle\,.$$

Expansion $\Downarrow$ at tree level

$$\widetilde{T}_1^{(0)}(x') = \widetilde{T}_E^{(0)}(x') = \frac{1}{x'\,r+\bar{x}'} + \frac{1}{\bar{x}'\,r+x'}\,.$$

- Leading twist photon DA (Ball, Braun and Kivel, 2002):

$$\begin{aligned}&\langle 0|\bar{\chi}(0)\,W_{\bar{c}}(0,y)\,\sigma_{\alpha\beta}\,\chi(y)|\gamma(p')\rangle \\ &= i\,g_{\rm em}\,Q_q\,\chi(\mu)\,\langle\bar{q}q\rangle(\mu)\,\left[p'_\beta\,\varepsilon_\alpha(p') - p'_\alpha\,\varepsilon_\beta(p')\right]\int_0^1 du\,e^{-iu p'\cdot y}\,\phi_\gamma(u,\mu)\,.\end{aligned}$$

- Hard-collinear factorization at LO:

$$G(p^2,Q^2) = -\frac{Q_u^2 - Q_d^2}{\sqrt{2}\,Q^2}\,\chi(\mu)\,\langle\bar{q}q\rangle(\mu)\int_0^1 dx\,\widetilde{T}_1^{(0)}(x)\,\phi_\gamma(x,\mu) + \mathcal{O}(\alpha_s)\,.$$

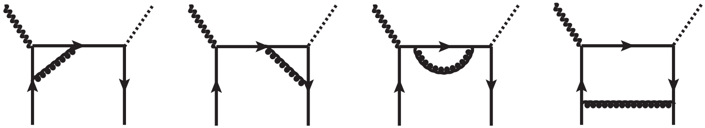
- The tree-level LCSR:

$$F^{\rm NLP}_{\gamma^*\gamma\to\pi^0}(Q^2) = -\frac{\sqrt{2}\,(Q_u^2 - Q_d^2)}{f_\pi\,\mu_\pi(\mu)}\,\chi(\mu)\,\langle\bar{q}q\rangle(\mu)\int_{u_0}^1 \frac{du}{u}\exp\left[-\frac{\bar{u}\,Q^2 + u m_\pi^2}{u M^2}\right]\phi_\gamma(u,\mu) + \mathcal{O}(\alpha_s)\,.$$

Verify the power counting of the hadronic photon correction explicitly.

Hadronic photon effect at NLO

- The four-point partonic matrix element at NLO:



- Extracting the hard contribution with the method of regions:

$$\widetilde{\Pi}_{\mu}^{(1)} = -\frac{i\, g_{\rm em}}{2\sqrt{2}} \, \frac{\bar{n}\cdot p}{Q^2} \sum_{q=u,d} \eta_q \, Q_q \, \langle \widetilde{O}_{1,\mu}(x,x')\rangle^{(0)} * \widetilde{A}_{1,\rm hard}(x') + \ldots .$$

Scheme dependent one-loop QCD amplitude:

$$\widetilde{A}_{1,\rm hard}(x')\big|_{\rm NDR} = \frac{\alpha_s\, C_F}{4\pi} \left\{ \frac{1}{x'\, r+\bar{x}'} \left[\frac{2}{x'\,\bar{x}'\,\bar{r}} \left(\left((x'\, r - \bar{x}')\, \ln(x'\, r+\bar{x}') - x'\, r\, \ln r \right) \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{Q^2} - \frac{1}{2} \ln(x'\, r+\bar{x}') - \frac{1}{2} \ln r \right) \right) - \frac{1}{x'\bar{r}} (\ln r + 3)\, \ln(x'\, r+\bar{x}') - 3 \right] + (x' \leftrightarrow \bar{x}') \right\},$$

$$\widetilde{A}_{1,\rm hard}(x')\big|_{\rm HV} = \widetilde{A}_{1,\rm hard}(x')\big|_{\rm NDR} + \frac{2\,\alpha_s\, C_F}{\pi}\, \widetilde{T}_1^{(0)}(x') \, .$$

Hadronic photon effect at NLO

- Repeating the IR subtraction procedure as before:

$$\widetilde{T}_1^{(1)} = \widetilde{A}_1^{(1)} - \widetilde{T}_1^{(0)} * \widetilde{Z}_{11}^{(1)} + \widetilde{T}_E^{(0)} * \widetilde{M}_{E1}^{(1)\text{off}} = \widetilde{A}_{1,\text{hard}}^{(1)} + \widetilde{T}_E^{(0)} * \widetilde{M}_{E1}^{(1)\text{off}} .$$

- The IR subtraction:

$$\widetilde{T}_E^{(0)} * \widetilde{M}_{E1}^{(1)\text{off}}\Big|_{\text{NDR}} = 0 + \mathcal{O}(\alpha_s^2) , \qquad \widetilde{T}_E^{(0)} * \widetilde{M}_{E1}^{(1)\text{off}}\Big|_{\text{HV}} = \mathcal{O}(\varepsilon\, \alpha_s) .$$

$$\Downarrow$$

$$\widetilde{T}_1^{(1)} = \widetilde{A}_{1,\text{hard}}^{(1)} , \qquad \text{for both NDR and HV} .$$

- Finite renormalization constant $Z_{\text{HV}}^P(\mu)$ for the HV scheme (Larin, 1993):

$$Z_{\text{HV}}^P(\mu) = 1 - \frac{2\, \alpha_s(\mu)\, C_F}{\pi} + \mathcal{O}(\alpha_s^2) .$$

$$\Downarrow$$

$$Z_{\text{HV}}^P(\mu) \left[\widetilde{T}_1^{(0)}(x') + \widetilde{T}_1^{(1)}(x', \mu) \right]_{\text{HV}} = \left[\widetilde{T}_1^{(0)}(x') + \widetilde{T}_1^{(1)}(x', \mu) \right]_{\text{NDR}}$$

A non-trivial check of our results. Also reproduce $H \to J/\psi\, \gamma$ in the $r \to \infty$ limit.

Hadronic photon effect at NLL

- The NLO factorization formula:

$$G(p^2, Q^2) = -\frac{Q_u^2 - Q_d^2}{\sqrt{2}\, Q^2}\, \chi(\mu)\, \langle \bar{q} q \rangle(\mu) \int_0^1 dx \left[\widetilde{T}_1^{(0)}(x) + \widetilde{T}_1^{(1)}(x,\mu) \right]_{\rm NDR} \phi_\gamma(x,\mu) + \mathcal{O}(\alpha_s^2)\,.$$

- RG evolution of the photon LCDA (Lepage and Brodsky, 1979):

$$\mu^2 \frac{d}{d\mu^2} \left[\chi(\mu)\, \langle \bar{q} q \rangle(\mu)\, \phi_\gamma(x,\mu) \right] = \int_0^1 dy\, \widetilde{V}(x,y) \left[\chi(\mu)\, \langle \bar{q} q \rangle(\mu)\, \phi_\gamma(y,\mu) \right],$$

$$\widetilde{V} = \sum_{n=0} \left(\frac{\alpha_s}{4\pi} \right)^{n+1} \widetilde{V}_n\,, \qquad \widetilde{V}_0 = 2\, C_F \left[\frac{\bar{x}}{\bar{y}}\, \frac{1}{x-y}\, \theta(x-y) + \frac{x}{y}\, \frac{1}{y-x}\, \theta(y-x) \right]_+ - C_F\, \delta(x-y)\,.$$

$\Rightarrow$ Factorization-scale independence of $G(p^2, Q^2)$ at one loop.

- NLL resummation needs two-loop evolution kernel (Mikhailove and Vladimirov, 2009 + others):

$$\widetilde{V}_1(x,y) = \frac{N_f}{2}\, C_F\, \widetilde{V}_N(x,y) + C_F\, C_A\, \widetilde{V}_G(x,y) + C_F^2\, \widetilde{V}_F(x,y)\,.$$

$$\chi(\mu)\, \langle \bar{q} q \rangle(\mu)\, b_n(\mu) = E_{T,n}^{\rm NLO}(\mu,\mu_0)\, b_n(\mu_0) + \frac{\alpha_s(\mu)}{4\pi} \sum_{k=0}^{n-2} E_{T,n}^{\rm LO}(\mu,\mu_0)\, d_{T,n}^k(\mu,\mu_0)\, b_k(\mu_0)\,.$$

Hadronic photon effect at NLL

- Dispersion form of the NLL factorization formula:

$$G(p^2,Q^2) = -\frac{\sqrt{2}\,(Q_u^2 - Q_d^2)}{Q^2}\,\chi(\mu)\,\langle\bar{q}q\rangle(\mu)\int_0^\infty \frac{ds}{s-p^2-i0}\left[\rho^{(0)}(s,Q^2)+\frac{\alpha_s\,C_F}{4\pi}\,\rho^{(1)}(s,Q^2)\right].$$

The NLO spectral function:

$$\begin{aligned}\rho^{(1)}(s,Q^2) = 2\int_0^1 du\,\frac{\phi_\gamma(u,\mu)}{\bar{u}}\Big\{\theta\left(u-\frac{Q^2}{Q^2+s}\right)\frac{Q^2}{Q^2+s}\left[\frac{\bar{u}-u}{u}\ln\left(\frac{\mu^2}{u\,s-\bar{u}\,Q^2}\right)+\frac{3}{2}\,\frac{\bar{u}}{u}\right]\\ +\ln\left(\frac{\mu^2}{s}\right)\left[\frac{Q^2}{Q^2+s}-\mathscr{P}\,\frac{\bar{u}\,Q^2}{\bar{u}\,Q^2-u\,s}\right]\Big\}\\ +\frac{Q^2}{Q^2+s}\int_0^1 du\,\theta\left(u-\frac{Q^2}{Q^2+s}\right)\Big\{2\ln\left(\frac{u\,s-\bar{u}\,Q^2}{Q^2}\right)\Big[\ln\left(\frac{\mu^2}{u\,s-\bar{u}\,Q^2}\right)\\ +\ln\left(\frac{\mu^2}{Q^2}\right)+\frac{3}{2}\Big]-\ln^2\left(\frac{\mu^2}{Q^2}\right)+\ln^2\left(\frac{\mu^2}{s}\right)-\frac{\pi^2}{3}+3\Big\}\,\frac{d}{du}\,\phi_\gamma(u,\mu).\end{aligned}$$

- The NLL LCSRs for the hadronic photon effect:

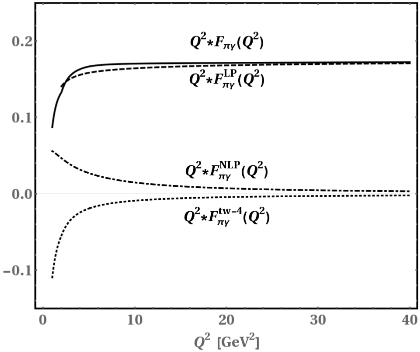
$$\begin{aligned}F^{\rm NLP}_{\gamma^*\gamma\to\pi^0}(Q^2) = -\frac{\sqrt{2}\,(Q_u^2-Q_d^2)}{f_\pi\,\mu_\pi(\mu)\,Q^2}\,\chi(\mu)\,\langle\bar{q}q\rangle(\mu)\int_0^{s_0} ds\exp\left[-\frac{s-m_\pi^2}{M^2}\right]\\ \times\left[\rho^{(0)}(s,Q^2)+\frac{\alpha_s\,C_F}{4\pi}\,\rho^{(1)}(s,Q^2)\right]+\mathscr{O}(\alpha_s^2).\end{aligned}$$

Final expression of the pion-photon form factor

- Adding up different contributions together:

$$F_{\gamma^*\gamma\to\pi^0}(Q^2) = F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) + F^{\rm NLP}_{\gamma^*\gamma\to\pi^0}(Q^2) + F^{\rm tw-4}_{\gamma^*\gamma\to\pi^0}(Q^2)\,.$$

- Breakdown of various contributions:



Input parameters:

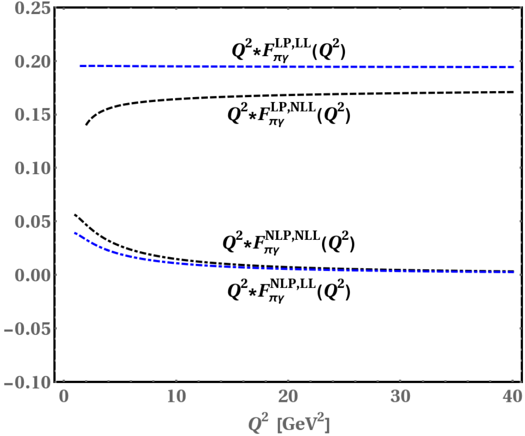
$$\begin{aligned}
a_2(1.0\,{\rm GeV}) &= 0.21^{+0.07}_{-0.06}\,, \\
a_4(1.0\,{\rm GeV}) &= -\left(0.15^{+0.10}_{-0.09}\right)\,, \\
\delta^2_\pi(1.0\,{\rm GeV}) &= 0.20 \pm 0.04\ {\rm GeV}^2\,, \\
\chi(1.0\,{\rm GeV}) &= (3.15 \pm 0.3)\ {\rm GeV}^{-2}\,, \\
b_2(1.0\,{\rm GeV}) &= 0.07 \pm 0.07\,.
\end{aligned}$$

Will discuss the model dependence of the pion DA.

Large cancellation between the hadronic photon correction and the twist-4 effect.
Smallness of the power corrections at large Q^2 numerically.

QCD corrections to the pion-photon form factor

- NLO QCD corrections to both the twist-2 and the hadronic photon contributions:

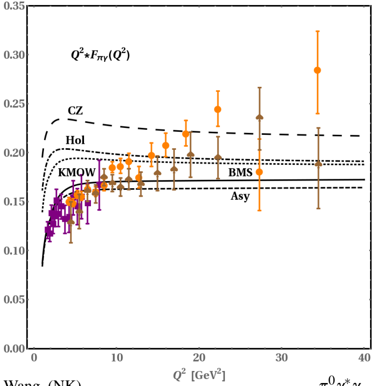


  - $\mathcal{O}$ (25 %) QCD correction to the twist-2 contribution (almost Q^2 independent).
  - (20-30) % QCD correction to the hadronic photon effect, but not relevant at large Q^2.

Model dependence of the pion-photon form factor

- Models of the pion DA:

$$
\begin{aligned}
a_2(1.0\,\mathrm{GeV}) &= 0.21^{+0.07}_{-0.06}\,, \quad a_4(1.0\,\mathrm{GeV}) = -\left(0.15^{+0.10}_{-0.09}\right)\,, && [\mathrm{BMS}, 2004]\,;\\
a_2(1.0\,\mathrm{GeV}) &= 0.17 \pm 0.08\,, \quad a_4(1.0\,\mathrm{GeV}) = 0.06 \pm 0.10\,, && [\mathrm{KMOW}, 2011]\,;\\
a_n(1.0\,\mathrm{GeV}) &= \frac{2n+3}{3\pi}\left(\frac{\Gamma[(n+1)/2]}{\Gamma[(n+4)/2]}\right)^2\,, && [\mathrm{Holographic}]\,,\\
a_2(1.0\,\mathrm{GeV}) &= 0.5\,, \quad a_{n>2}(1.0\,\mathrm{GeV}) = 0\,, && [\mathrm{CZ}]\,.
\end{aligned}
$$



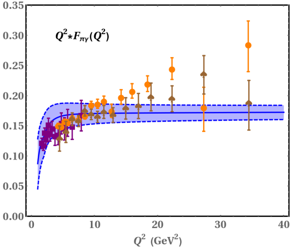
Purple squares from CLEO.
Orange circles from BaBar.
Brown bins from Belle.

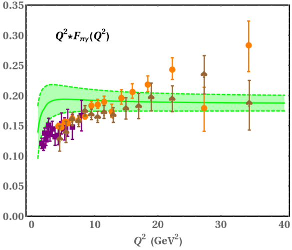
KMOW and Holographic models appear to describe the data at $Q^2 \geq 10\,\mathrm{GeV}^2$.

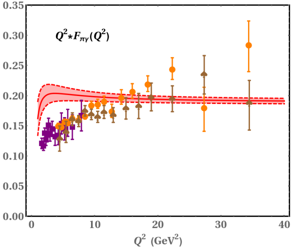
"Soft" physics more important at low Q^2 and the nonperturbative modification of the spectral function works better.

Model dependence of the pion-photon form factor

Theory predictions with BMS, KMOW and Holographic models:







Concluding Remarks

- Scheme independence of the leading twist contribution at one loop explicitly.
  - Operator matching with evanescent operators and the γ_5 prescription.
  - $\mathcal{O}$ (25 %) QCD correction to the twist-2 contribution (almost Q^2 independent).

- Hadronic photon effect at NLO.
  - Construction of the NLL LCSRs due to the violation of the direct QCD factorization.
  - QCD factorization for the correlation function at NLO.
  - Operator matching with both the NDR and HV schemes.
  - (20-30) % QCD correction, but not relevant at large Q^2.

- Predictions with different models of the pion DAs confronted with the data.
  - "Exotic" end-point behavior not needed.
  - More precise data needed (BES III at low Q^2 , Belle II?).

- Future work:
  - A complete NNLO twist-2 calculation.
  - The NLO twist-4 calculation (including the 3-particle DA effect).
  - Yet higher twist contributions (no correspondence between the twist and power expansion).