NNLO QCD corrections to radiative B meson decays

Abdur Rehman

National Centre for Physics, QAU campus, Islamabad

Based on:

M. Misiak, H. Asatrian, R. Boughezal, M. Czakon, T. Ewerth, A. Ferroglia, P. Fiedler, P. Gambino, C. Greub, U. Haisch, T. Huber, M. Kamiński, G. Ossola, M. Poradziński, A. Rehman, T. Schutzmeier, M. Steinhauser and J. Virto
Phys. Rev. Lett. **114** (2015) 22, 221801 [arXiv:1503.01789].

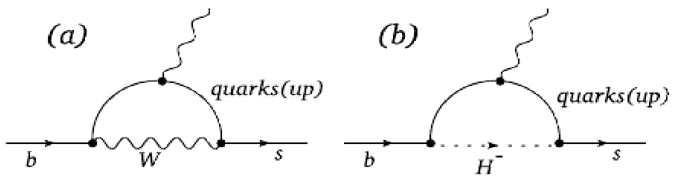
M. Misiak, A. Rehman and M. Steinhauser, Phys. Lett. B**770** (2017) 431 [arXiv:1702.07674].

M. Misiak, A. Rehman and M. Steinhauser, ... in progress.

- **Introduction: radiative B-decays**
- **Interpolation in the charm quark mass**
- **NNLO ($\mathcal{O}(\alpha_s^2)$) counterterms: no interpolation**
- **Bare NNLO calculations: an enterprise**
- **Summary**

Motivation

FCNC process $\Rightarrow$ bounds on beyond-SM physics



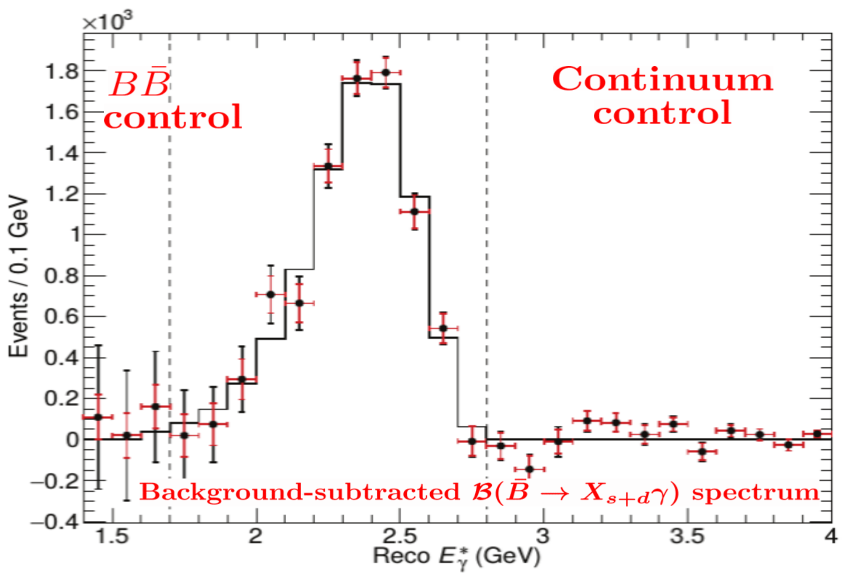
Recent bounds on $M_{H^\pm}$; [arXiv:1702.04571]

in 2HDM II

$$M_{H^\pm} > 580 \text{ GeV at } 95\%\text{C.L.}$$
$$M_{H^\pm} > 440 \text{ GeV at } 99\%\text{C.L.}$$

Photon energy spectrum in inclusive measurements

Belle, [arXiv:1608.02344]

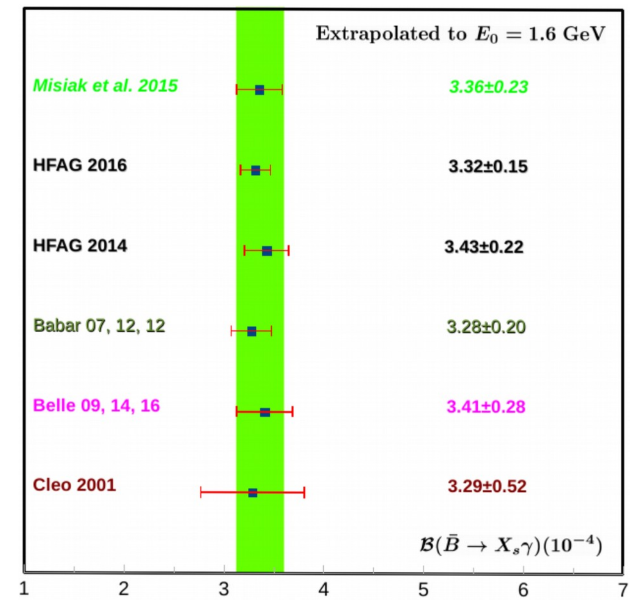


Background grows for smaller E_0.

Branching fraction; HFAG [arXiv:1612.07233]

uncertainty 4.5%

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{Exp}_{E_\gamma > 1.6\text{GeV}} = (3.32 \pm 0.15) \cdot 10^{-4}$$



Belle-II Better accuracy is expected, $\sim 3\%$.

SM Prediction; [arXiv:1503.01789]

uncertainty 6.9%

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{SM}_{E_\gamma > 1.6\text{GeV}} = (3.36 \pm 0.23) \cdot 10^{-4}$$

Interpolation uncertainty is 3%.

Non-perturbative	5%	$\mathcal{O}(\Lambda/m_b)$, [arXiv:1003.5012]
Parametric	2%	$\alpha_s(M_Z)(0.75\%), \mathcal{B}^{Exp}_{SL}(1.49\%), \text{CKM}(0.12\%), \ldots$
Charm mass dependence	3%	$\mathcal{Q}_1,\ \mathcal{Q}_2$ matrix elements
Higher order NNNLO	3%	$\mu_b(2.0\text{GeV}), \mu_c(2.0\text{GeV}), \mu_0(160\text{GeV})$

Theoretical framework

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{8} C_i(\mu_b)\mathcal{Q}_i$$

Operators basis: Chetyrkin, Misiak, Münz, 1996

$$\mathcal{Q}_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L)$$
$$\mathcal{Q}_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$
$$\mathcal{Q}_7 = \frac{e}{16\pi^2} m_b\,(\bar{s}_L\sigma_{\mu\nu}b_R)\,F^{\mu\nu}$$
$$\mathcal{Q}_8 = \frac{g}{16\pi^2} m_b\,(\bar{s}_L\sigma_{\mu\nu}T^a b_R)\,G^{a\mu\nu}$$
$$\mathcal{Q}_3 = (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q)$$
$$\mathcal{Q}_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q)$$
$$\mathcal{Q}_5 = (\bar{s}_L\gamma_{\mu 1}\gamma_{\mu 2}\gamma_{\mu 3} b_L)\sum_q(\bar{q}\gamma^{\mu 1}\gamma^{\mu 2}\gamma^{\mu 3} q)$$
$$\mathcal{Q}_6 = (\bar{s}_L\gamma_{\mu 1}\gamma_{\mu 2}\gamma_{\mu 3} T^a b_L)\sum_q(\bar{q}\gamma^{\mu 1}\gamma^{\mu 2}\gamma^{\mu 3} T^a q)$$

$|C_7| : |C_{1,2}| : |C_8| \simeq 1 : 3 : 1/2$

current-current	photonic dipole	gluonic dipole	penguin
$\mathcal{Q}_{1,2}$	$\mathcal{Q}_7$	$\mathcal{Q}_8$	$\mathcal{Q}_{3,4,5,6}$
$C_{1,2}(m_b) \sim 1$	$C_7(m_b) \sim -0.3$	$C_8(m_b) \sim -0.15$	$C_{3,4,5,6}(m_b) \sim 0.07$

Higher-order EW and/or CKM-suppressed effects ($|V_{ub}V_{us}^*/V_{tb}V_{ts}^*| < 0.02$) bring other operators.

The matrix elements can be effectively evaluated in perturbation theory

$$\Gamma(\bar{B}\to X_s\gamma)_{E_\gamma>E_0} = \Gamma(b\to X_s^p\gamma)_{E_\gamma>E_0} + \begin{pmatrix}\text{Non-perturbative}\\ \sim(\pm 5)\%\\ \text{arXiv : 1003.5012}\end{pmatrix} \quad b\in\bar{B} = B^-(b\bar{u}) \text{ or } \bar{B}^0(b\bar{d})$$

Provided that E_0 is large ($\sim m_b/2$) but not close to endpoint ($m_b - 2E_0 \gg \Lambda_{QCD}$).
$E_0 \sim m_b/3 \simeq 1.6$ GeV is now conventional.

$$\Gamma(b\to X_s^p\gamma)_{E_\gamma>E_0} = N\sum_{i,j=1}^{8} C_i^{\rm eff}(\mu_b)C_j^{\rm eff}(\mu_b)\widetilde{G}_{ij}(E_0,\mu_b)$$

$$K_{ij} \equiv \widetilde{G}_{ij}/G_u^{\rm semi}$$

$N = \frac{G_F^2 m_b^5}{32\pi^3}\left(\frac{\alpha_{em}}{\pi}\right)|V_{tb}V_{ts}^*|^2$

$\mu_b \sim m_b/2$

$\delta = 1 - 2E_0/m_b$

$z = m_c^2/m_b^2$

At NNLO $K_{77}^{(2)}$, $K_{17}^{(2)}$, $K_{27}^{(2)}$ depend on z. The central value of $z \simeq 0.056$.

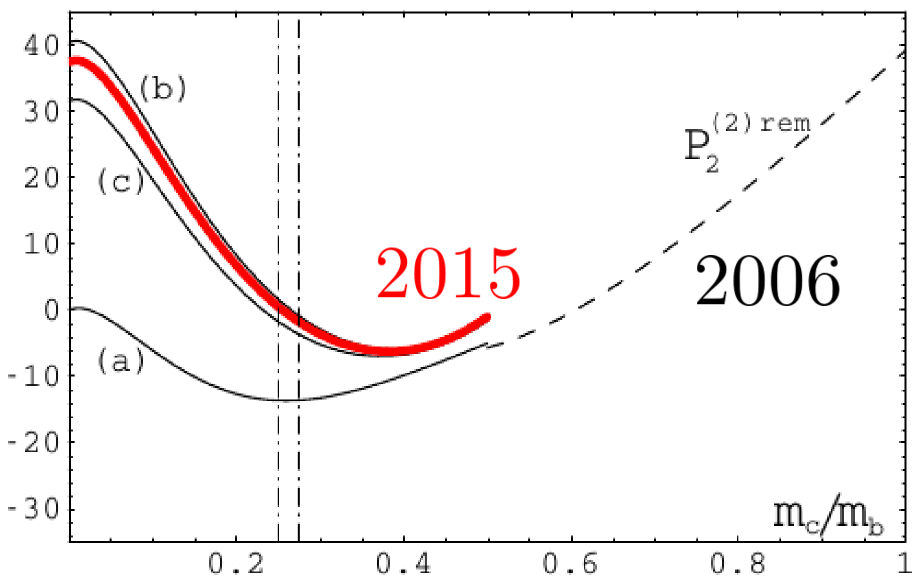
Interpolation in the charm quark mass

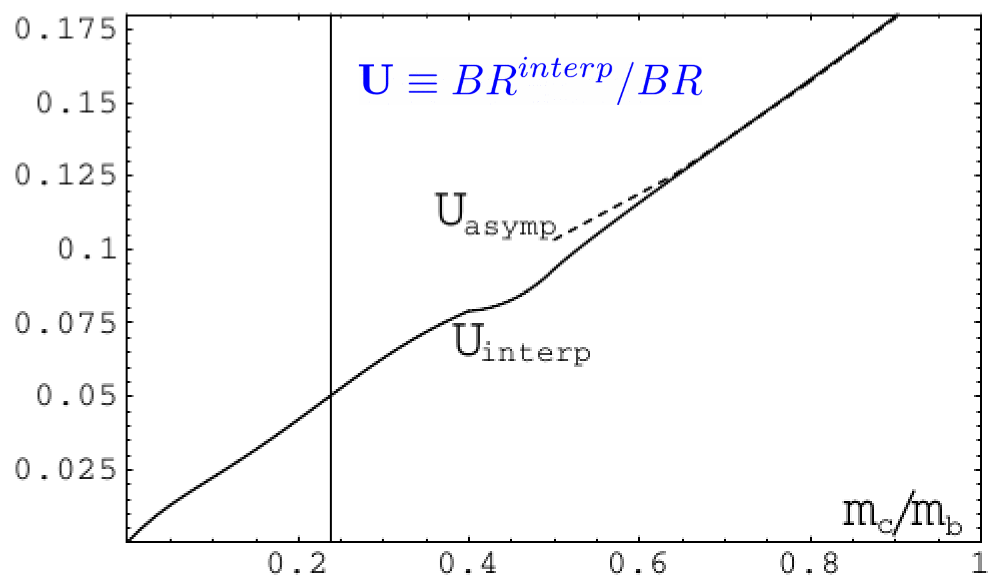
$\frac{G_{27,17}}{m_c = 0}$

$$C_i(\mu_b) = \sum_{n=0} \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^n C_i^{(n)}(\mu_b) \quad \textbf{and} \quad K_{ij} = \sum_{n=0} \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^n K_{ij}^{(n)}$$

$$\sum_{i,j=1}^{8} C_i^{(0)}(\mu_b) C_j^{(0)}(\mu_b) K_{ij}^{(2)}(E_0, \mu_b) \equiv P_2^{(2)} = \underbrace{P_2^{(2)\beta_0}}_{\text{BLM}} + \underbrace{P_2^{(2)\text{rem}}}_{\text{Non-BLM}}$$

- **BLM with arbitrary charm quark mass** — K. Bieri, C. Greub, M. Steinhauser, 2003; Z. Ligeti, M. Luke, A. Manohar, M. Wise, 1999
- **Non-BLM by interpolation in m_c** assuming BLM at $m_c = 0$ — M. Steinhauser, M. Misiak, 2006
- **Non-BLM by interpolation in m_c** with explicit calculation at $m_c = 0$ — M. Czakon, P. Fiedler, T. Huber, M. Misiak, T. Schutzmeier, M. Steinhauser; 2015

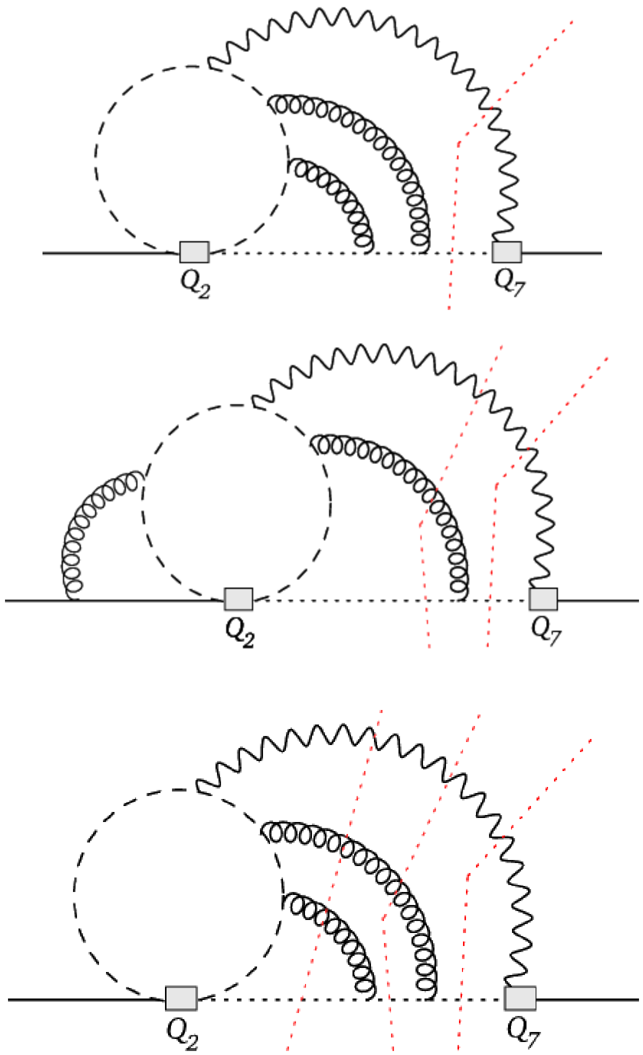




Interpolation uncertainty estimate remains unchanged w.r.t. 2006 (±3%)

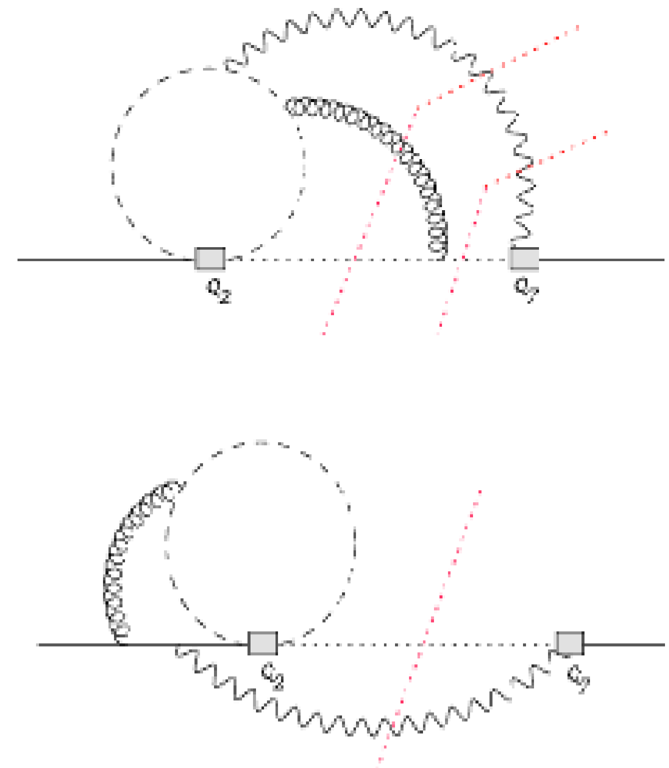
Sample diagrams for bare and counterterm NNLO contributions

Such contributions are obtained from four- and three-loop propagators with unitarity cuts.



$+ \;\cdots\cdots$

$s\gamma, s\gamma g, s\gamma gg, s\gamma q\bar{q}$

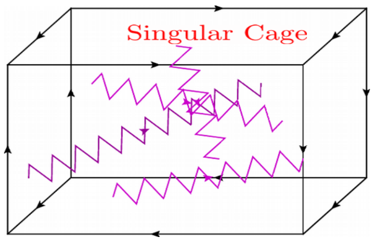


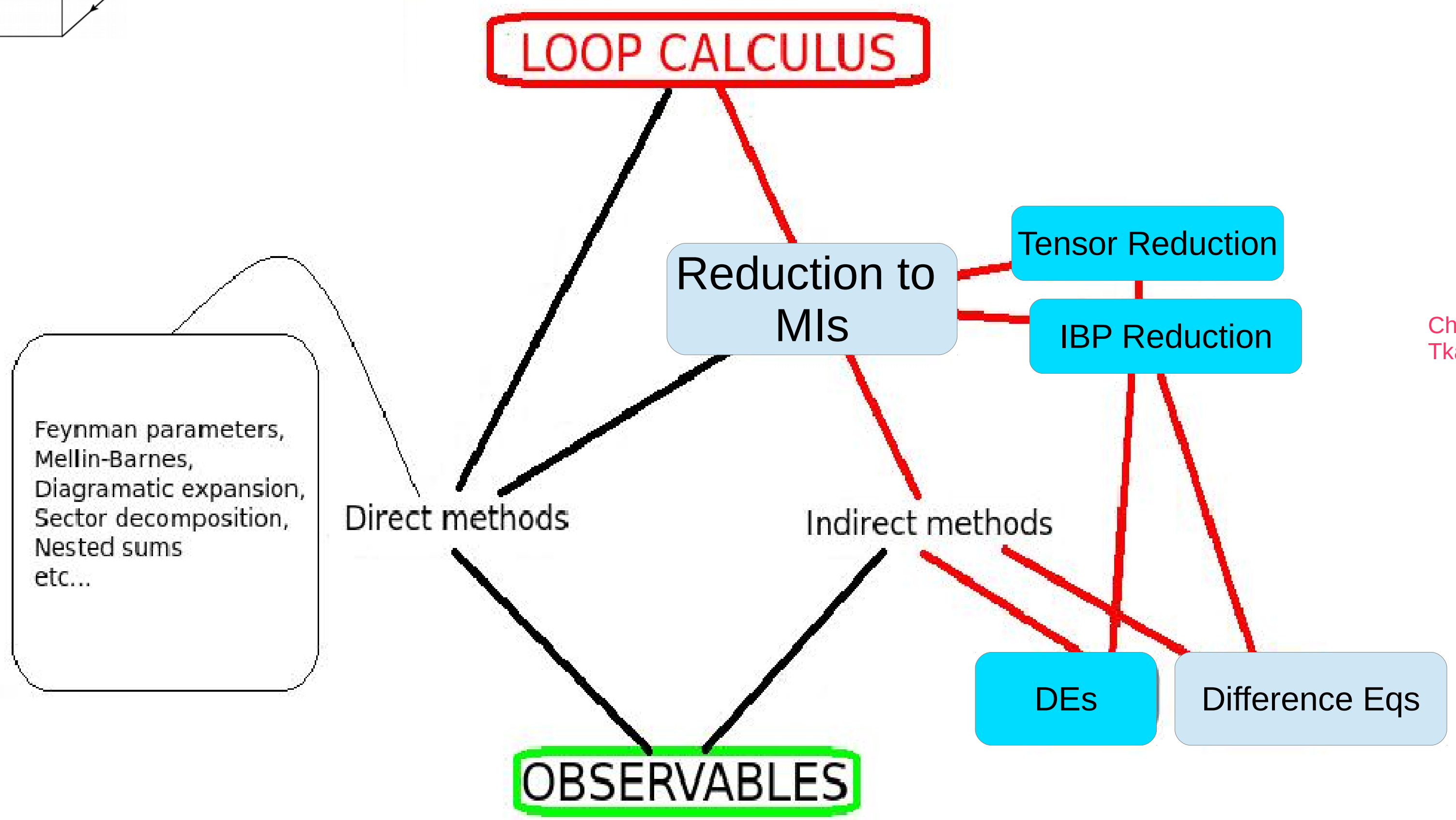
$+ \;\cdots\cdots$

Loop calculations: general methods

Revealing and getting rid of infinities ......

Regularization
+
Renormalization





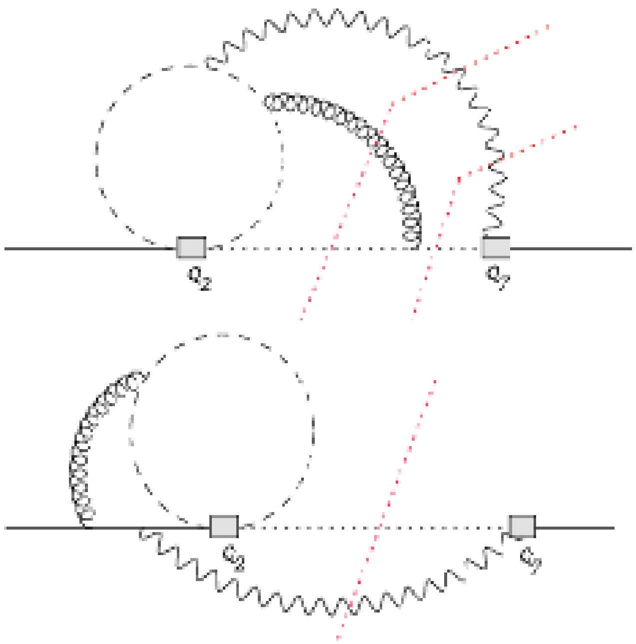
Chetyrkin, Tkachov 81

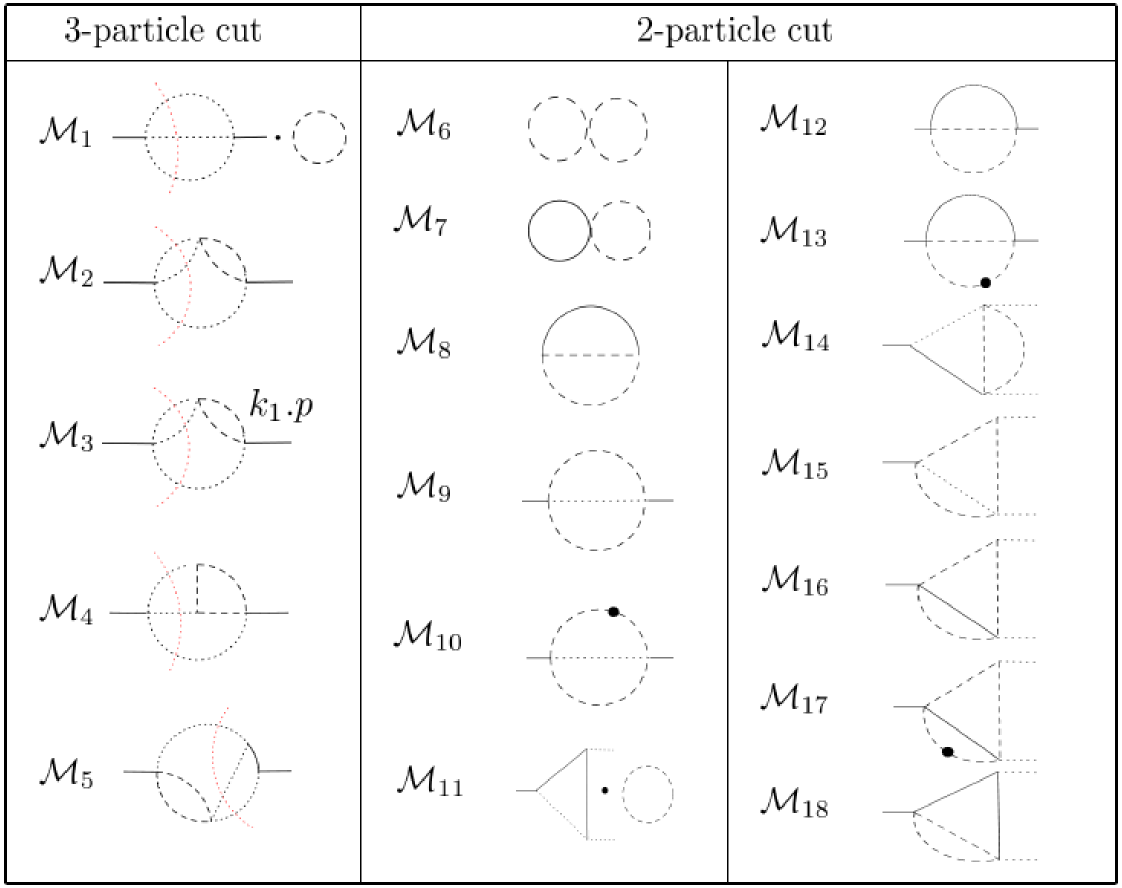
Smirnov 99, Tausk 99

Binoth, Heinrich 00
Moch, Uwer, Weinzierl 01

Laporta 01

Calculational method



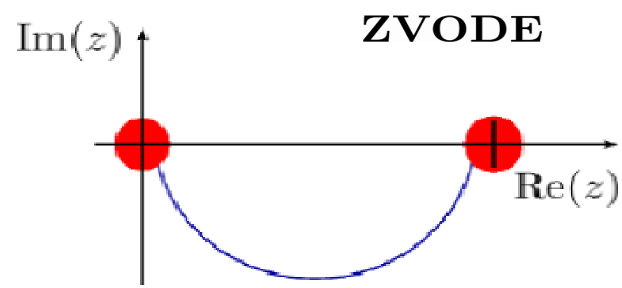


Reduction to master integrals

FIRE [A. V. Smirnov, arXiv:1408.2372]
REDUZE [C. Studerus, arXiv:0912.2546]

$$\frac{\mathrm{d}}{\mathrm{d}z}\mathcal{M}_n(z,\epsilon) = \sum_m R_{nm}(z,\epsilon)\mathcal{M}_m(z,\epsilon)$$

ZVODE



Boundary conditions at large z

MB method, MB.m, M. Czakon
Asymptotic expansions, exp, M. Steinhauser

Higher order terms using power-log ansatz →
$$I_i(w,\epsilon) = \sum_{n,m,k} c_{inmk}\epsilon^n w^m \mathrm{Log}^k(w) \qquad w = 1/z$$

NNLO Counterterms: no interpolation

$\frac{G_{27,17}}{m_c \neq 0}$

Renormalization

$$\widetilde{\alpha}_s \widetilde{G}_{27}^{(1)} + \widetilde{\alpha}_s^2 \widetilde{G}_{27}^{(2)} = Z_b^{OS} Z_m^{OS} \bar{Z}_{77} \Big\{ \widetilde{\alpha}_s^2 s^{3\varepsilon} \widetilde{G}_{27}^{(2)\mathrm{bare}} + (Z_m^{OS} - 1) s^{\varepsilon} \left[\bar{Z}_{24} \hat{G}_{47}^{(0)m} + \widetilde{\alpha}_s s^{\varepsilon} \hat{G}_{27}^{(1)m} \right]$$
$$+ \widetilde{\alpha}_s (Z_G^{OS} - 1) s^{2\varepsilon} \hat{G}_{27}^{(1)3P} + \bar{Z}_{27} Z_m^{OS} \left[\hat{G}_{77}^{(0)} + \widetilde{\alpha}_s s^{\varepsilon} \hat{G}_{77}^{(1)\mathrm{bare}} \right]$$
$$+ \widetilde{\alpha}_s \bar{Z}_{28} s^{\varepsilon} \hat{G}_{78}^{(1)\mathrm{bare}} + \sum_{j=1,\ldots,6,11,12} \bar{Z}_{2j} s^{\varepsilon} \left[\hat{G}_{j7}^{(0)} + \widetilde{\alpha}_s s^{\varepsilon} Z_g^2 \hat{G}_{j7}^{(1)\mathrm{bare}} \right] \Big\}$$
$$+ 2\widetilde{\alpha}_s \left(\frac{\mu_b^2}{\mu_c^2} \right)^{\varepsilon} s^{2\varepsilon} (\bar{Z}_m - 1)\, z \frac{d}{dz} \widetilde{G}_{27}^{(1)\mathrm{bare}} + \mathcal{O}(\widetilde{\alpha}_s^3)$$

$$\hat{G}_{27}^{(1)} = \hat{G}_{27}^{(1)2P} + \hat{G}_{27}^{(1)3P}$$

$$\widetilde{\alpha}_s = \alpha_s / 4\pi = g_s^2 / 16\pi^2$$

$$s = \frac{4\pi \mu_b^2}{m_b^2} e^{\gamma}$$

$\hat{G}_{j7}^{(0)}$ vanish for $j = 1, 2, 11, 12$

$$Q_{11} = (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} T^a c_L)(\bar{c}_L \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} T^a b_L) - 16 Q_1$$
$$Q_{12} = (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} c_L)(\bar{c}_L \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} b_L) - 16 Q_2$$

The $\hat{G}_{ij}$'s correspond to $\widetilde{G}_{ij}$'s once we replace C^{eff}'s with C's.

Now, $m_c = 0$ counterterms are known to all order in ϵ except $\hat{G}_{47}^{(1)}$, [AR, PhD thesis].

$$\hat{G}_{27}^{(1)3P}(z) = g_0(z) + \epsilon\, g_1(z) + \mathcal{O}(\epsilon^2)$$

$$\hat{G}_{7(12)}^{(1)3P}(z) = 0 - \epsilon\, (20\, g_0(z)) + \mathcal{O}(\epsilon^2)$$

$$\hat{G}_{27}^{(1)m,3P}(z) = j_0(z) + \epsilon\, j_1(z) + \mathcal{O}(\epsilon^2)$$

$$\hat{G}_{27}^{(1)2P}(z) = -\frac{92}{81\epsilon} + f_0(z) + \epsilon\, f_1(z) + \mathcal{O}(\epsilon^2)$$

$$\hat{G}_{7(12)}^{(1)2P}(z) = \frac{2096}{81} + \epsilon\, e_1(z) + \mathcal{O}(\epsilon^2)$$

$$\hat{G}_{27}^{(1)m,2P}(z) = -\frac{1}{3\epsilon^2} + \frac{1}{\epsilon} r_{-1}(z) + r_0(z) + \epsilon\, r_1(z) + \mathcal{O}(\epsilon^2)$$

Results: 3-body

for $\delta = 1$

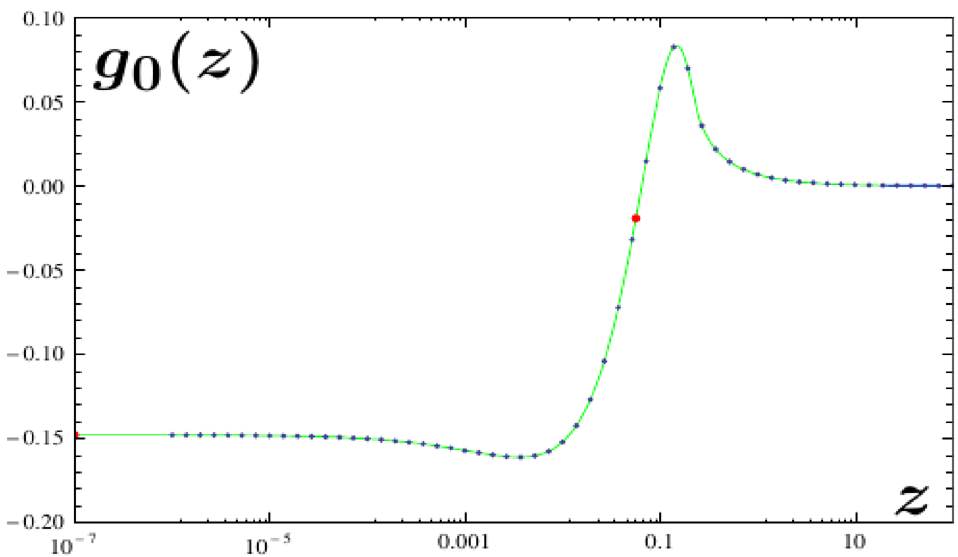
$$\hat{G}_{27}^{(1)3P}(z) = g_0(z) + \epsilon\, g_1(z) + \mathcal{O}(\epsilon^2)$$

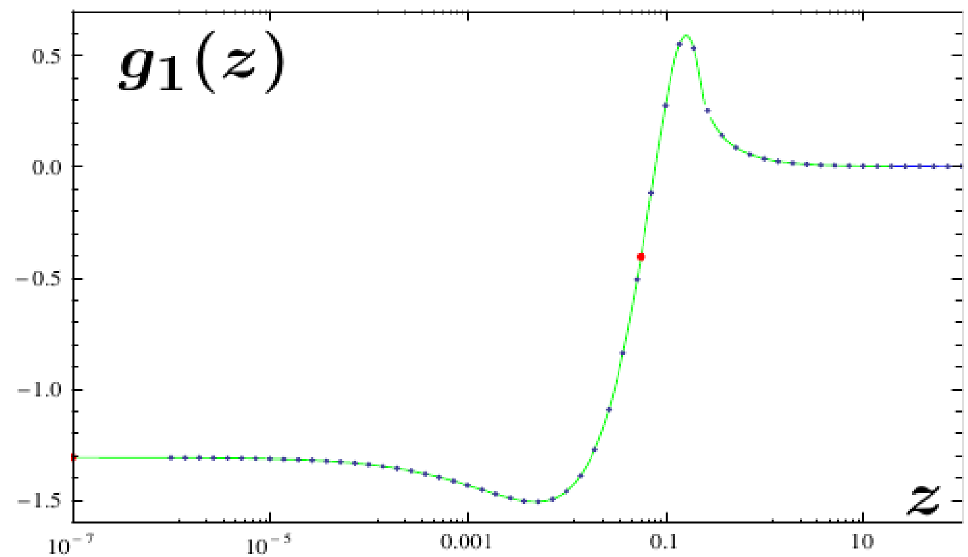
$$\hat{G}_{7(12)}^{(1)3P}(z) = 0 - \epsilon\,(20\, g_0(z)) + \mathcal{O}(\epsilon^2)$$

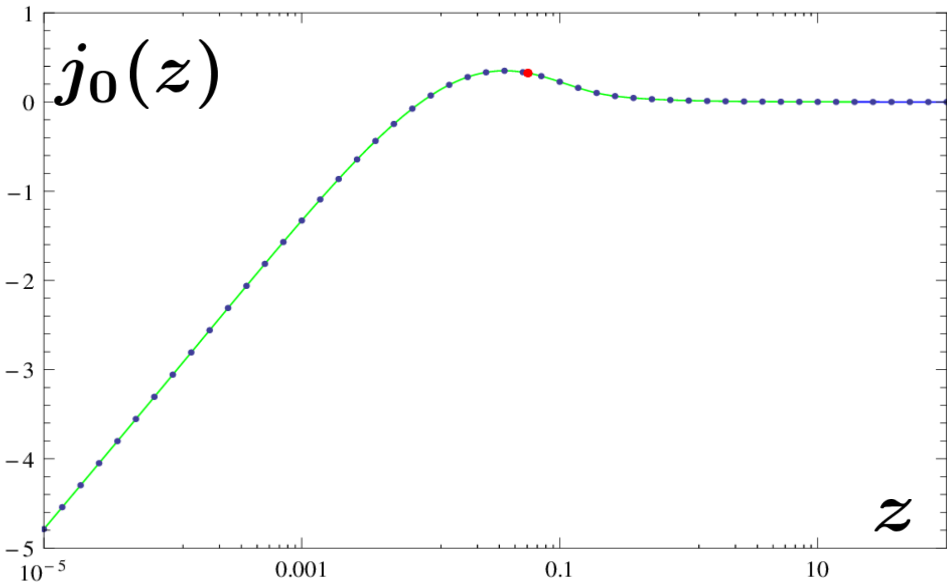
$$\hat{G}_{27}^{(1)m,3P}(z) = j_0(z) + \epsilon\, j_1(z) + \mathcal{O}(\epsilon^2)$$

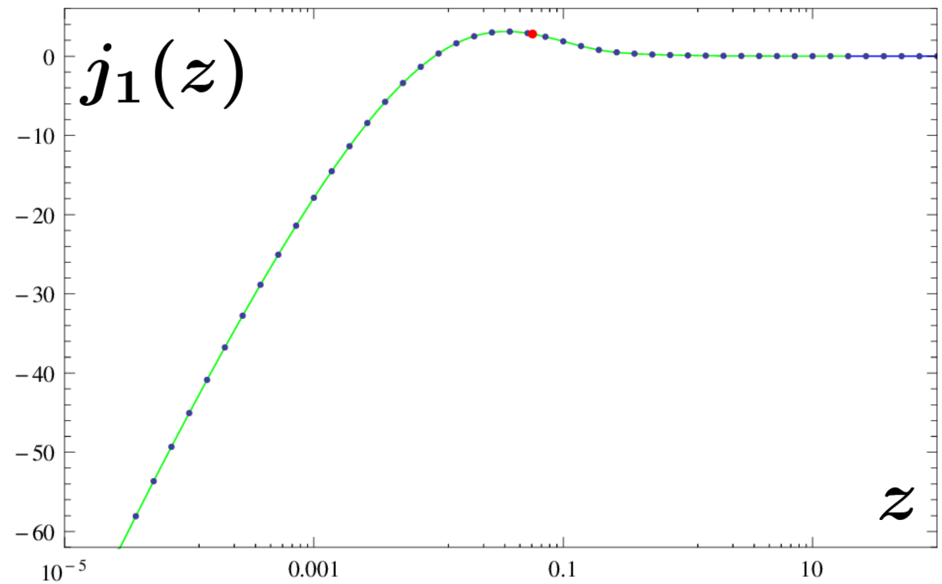
$$g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, s\, L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \le \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, t\, A + \frac{16}{9}z(6z^2-4z+1)\, A^2, & \text{for } z > \frac{1}{4}, \end{cases}$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, and $A = \arctan(1/t)$.









Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.

Boundary condition for the numerical DE's is at $z = 20$.

At $z = 1/4$ we have the charm production threshold, and the DE's have a singular point there.

Agreement with numerical solution of differential equations over wide range of z.

Blue curves: large-z asymptotic expansions above $z = 20$.

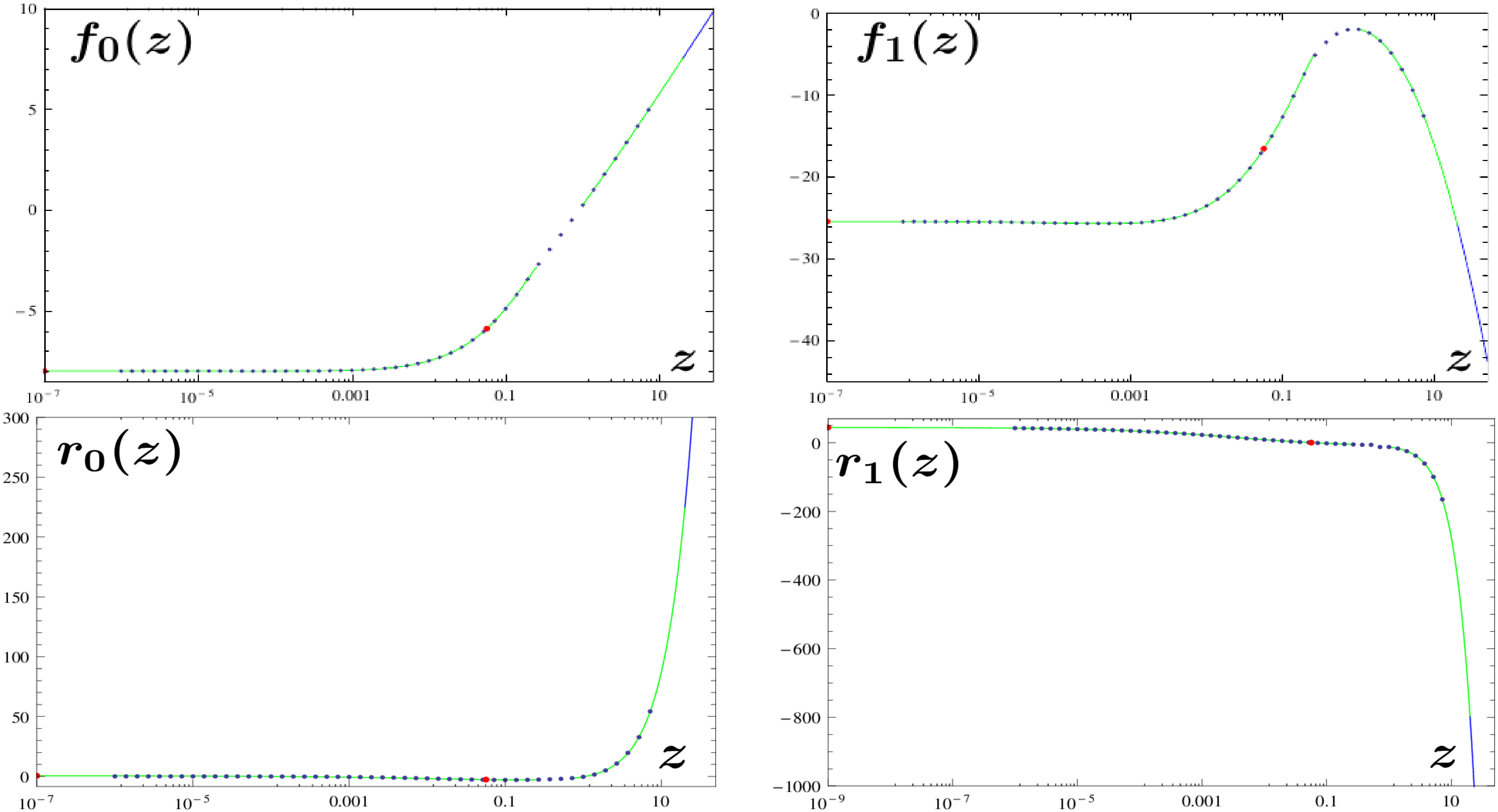
Red dots: Exact $z = 0$ results and numerical results from the DE's at the physical z.

Green curves: known asymptotic expansions either at large z or at small z.

This provides a test of our DE algorithm that is aimed at to be used in bare NNLO calculation where no analytic expansion at small z is going to be available.

Results: 2-body

$$f_0(z) = -\frac{1942}{243} + 2\,\mathcal{R}e\left[a(z) + b(z)\right] \qquad e_1(z) = \frac{39112}{243} - 8\,\mathcal{R}e\left[5\,a(z) + b(z)\right]$$

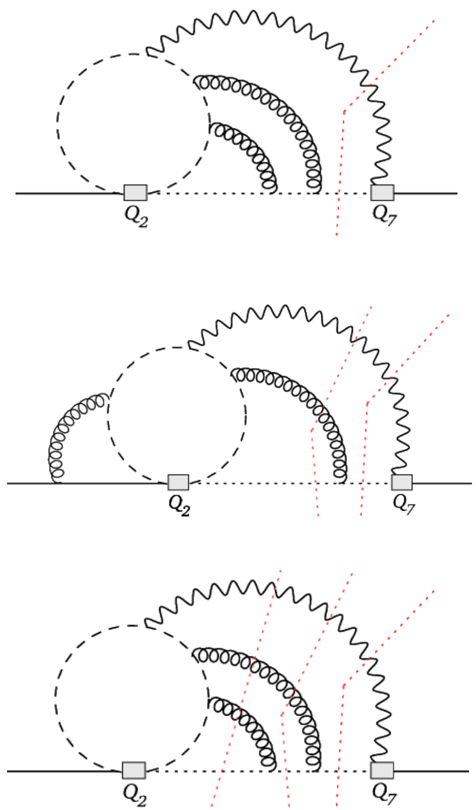


This provides a test of our DE algorithm that is aimed at to be used in bare NNLO calculation where no analytic expansion at small z is going to be available.

Bare NNLO Contribution: an enterprise

$G^{(2)\text{bare}}_{17,27}(z)$

M. Misiak, A. Rehman, M. Steinhauser, $\cdots$ in progress



$+ \;\cdots\cdots$

$s\gamma, s\gamma g, s\gamma gg, s\gamma q\bar{q}$

Required same techniques as used/mentioned above, but problem is much more complex.

Generated diagrams (437 families),

Constructed amplitudes from QGRAF output,

Performed partial fractioning whenever necessary,

Changed routing of momenta for comparison purpose,

Marking all possible unitarity cuts for each class of family,

Performed Dirac algebra with FORM, and performed color algebra,

AR all inputs for IBP reduction agreed with Misiak.

We got four-loop two-scale scalar integrals $\sim 58,5309$.

Ongoing work: Reduction to master integrals with available C++ codes FIRE and REDUZE. Furhter, LiteRed by [R. N. Lee, arXiv:1212.2685] exploits symmetries to speed up the reduction

[For most difficult families: expected need is 100 GB RAM and 1 month CPU]

Future challenges: Calculating three-loop single-scale masters that will appear in the automatized asymptotic expansions required for evaluating boundary conditions at $m_c \gg m_b$.

Summary

- **The $\bar{B} \to X_s\gamma$ process constrains extensions of the SM in a strong manner. At present, its observed branching ratio agrees with the SM well within 1σ.**

- **However, $K_{17}^{(2)}, K_{27}^{(2)}$ are currently included with the help of interpolation in the charm quark mass. Completing the calculation of $K_{17}^{(2)}, K_{27}^{(2)}$ for arbitrary z to remove $\pm 3\%$ uncertainty caused by interpolation is necessary.**

- **A calculation of the counterterm contribution to $K_{17}^{(2)}(z), K_{27}^{(2)}(z)$ has been presented. Such contributions are obtained by evaluating three-loop propagator diagrams with unitarity cuts.**

- **Bare calculations of $K_{17}^{(2)}(z), K_{27}^{(2)}(z)$ for the physical value of m_c are at the level of IBP reduction.**

Backup

Branching ratio; PDG, 2014 $\boldsymbol{\mathcal{B}(\bar{B} \to X_s\gamma)^{Exp}_{E_\gamma>1.6\mathrm{GeV}} = (3.40 \pm 0.21) \cdot 10^{-4}}$

CP-averaged decay rate: $\Gamma_0 = \dfrac{\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^0 \to X_{\bar{s}}\gamma)}{2}, \qquad \Gamma_\pm = \dfrac{\Gamma(B^- \to X_s\gamma) + \Gamma(B^+ \to X_{\bar{s}}\gamma)}{2}$

Isospin-averaged decay rate: $\Gamma = (\Gamma_0 + \Gamma_\pm)/2$

Isospin asymmetry: $\Delta_{0\pm} = (\Gamma_0 - \Gamma_\pm)/(\Gamma_0 + \Gamma_\pm)$

CP- and isospin-averaged branching ratio: $\mathcal{B}(\bar{B} \to X_s\gamma) = \tau_{B^0}\Gamma\left(\dfrac{1 + r_f r_\tau}{1 + r_f} + \Delta_{0\pm}\dfrac{1 - r_f r_\tau}{1 + r_f}\right)$

$$r_f = f^{+-}/f^{00} = 1.059 \pm 0.027 \quad r_\tau = \tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$$

A breakdown of parametric uncertainties in the SM prediction

$\mathcal{B}(\bar{B} \to X_c e\bar{\nu})$	1.50%	m_b^{kin}	0.27%
$\alpha_s(M_Z)$	0.75%	$m_c(\mu_c)$	0.57%
$m_{t,\mathrm{pole}}$	0.19%	m_b/m_q	0.37%
$\lambda = s_{12}$	0.02%	μ_G^2	0.69%
$A = s_{23}/s_{12}^2$	0.01%	μ_π^2	0.03%
$\bar{\rho}$	0.12%	ρ_D^3	0.74%
$\bar{\eta}$	0.01%	ρ_{LS}^3	0.05%

Uncertainties due to the higher-order $\mathcal{O}(\alpha_s^3)$ corrections

$$\frac{\alpha_s(\mu_b)}{\pi} \simeq 0.093, \qquad \left(\frac{\alpha_s(\mu_b)}{\pi}\right)^2 \simeq 0.0087, \qquad \left(\frac{\alpha_s(\mu_b)}{\pi}\right)^3 \simeq 0.00081$$

$$r_{-1}(z) = -1 - \frac{4\pi^2}{81} - 2z + \mathcal{O}(z^8).$$