Prospects of discovering stable double-heavy tetraquarks at a Tera-Z factory

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DESY, Hamburg

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- Current Evidence for Multiquark states X, Y, Z and P_c
- Models for X, Y, Z Mesons
- The Diquark model of Tetraquarks
- Doubly Heavy Tetraquarks - Theoretical Expectations
- Prospects of Discovery at a Tera-Z Factory (CPEC, CERN-ee)
- Summary

$X(3872)$ - the poster Child of the X, Y, Z Mesons

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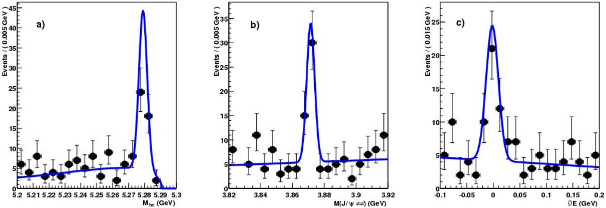
Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays

S.-K. Choi,[5] S. L. Olsen,[6] K. Abe,[7] T. Abe,[7] I. Adachi,[7] Byoung Sup Ahn,[14] H. Aihara,[43] K. Akai,[7] M. Akatsu,[20] M. Akemoto,[7] Y. Asano,[48] T. Aso,[47] V. Aulchenko,[1] T. Aushev,[11] A. M. Bakich,[38] Y. Ban,[31] S. Banerjee,[39] A. Bondar,[1] A. Bozek,[25] M. Bračko,[18,12] J. Brodzicka,[25] T. E. Browder,[6] P. Chang,[24] Y. Chao,[24] K.-F. Chen,[24] B. G. Cheon,[37] R. Chistov,[11] Y. Choi,[37] Y. K. Choi,[37] M. Danilov,[11] L. Y. Dong,[9] A. Drutskoy,[11] S. Eidelman,[1] V. Eiges,[11] J. Flanagan,[7] C. Fukunaga,[45] K. Furukawa,[7] N. Gabyshev,[7] T. Gershon,[7] B. Golob,[17,12] H. Guler,[6] R. Guo,[22] C. Hagner,[50] F. Handa,[42] T. Hara,[29] N. C. Hastings,[7] H. Hayashii,[21] M. Hazumi,[7] L. Hinz,[16] Y. Hoshi,[41] W.-S. Hou,[24] Y. B. Hsiung,[24,*] H.-C. Huang,[24] T. Iijima,[20] K. Inami,[20] A. Ishikawa,[20] R. Itoh,[7] M. Iwasaki,[43] Y. Iwasaki,[7] J. H. Kang,[52] S. U. Kataoka,[21] N. Katayama,[7] H. Kawai,[2] T. Kawasaki,[27] H. Kichimi,[7] E. Kikutani,[7] H. J. Kim,[52] Hyunwoo Kim,[14] J. H. Kim,[37] S. K. Kim,[36] K. Kinoshita,[3] H. Koiso,[7] P. Koppenburg,[7] S. Korpar,[18,12] P. Križan,[17,12] P. Krokovny,[1] S. Kumar,[30] A. Kuzmin,[1] J. S. Lange,[4,33] G. Leder,[10] S. H. Lee,[36] T. Lesiak,[25] S.-W. Lin,[24] D. Liventsev,[11] J. MacNaughton,[10] G. Majumder,[39] F. Mandl,[10] D. Marlow,[32] T. Matsumoto,[45] S. Michizono,[7] T. Mimashi,[7] W. Mitaroff,[10] K. Miyabayashi,[21] H. Miyake,[29] D. Mohapatra,[50] G. R. Moloney,[19] T. Nagamine,[42] Y. Nagasaka,[8] T. Nakadaira,[43] T. T. Nakamura,[7] M. Nakao,[7] Z. Natkaniec,[25] S. Nishida,[7] O. Nitoh,[46] T. Nozaki,[7] S. Ogawa,[40] Y. Ogawa,[7] K. Ohmi,[7] Y. Ohnishi,[7] T. Ohshima,[20] N. Ohuchi,[7] K. Oide,[7] T. Okabe,[20] S. Okuno,[13] W. Ostrowicz,[25] H. Ozaki,[7] H. Palka,[25] H. Park,[15] N. Parslow,[38] L. E. Piilonen,[50] H. Sagawa,[7] S. Saitoh,[7] Y. Sakai,[7] T. R. Sarangi,[49] M. Satapathy,[49] A. Satpathy,[7,3] O. Schneider,[16] A. J. Schwartz,[3] S. Semenov,[11] K. Senyo,[20] R. Seuster,[6] M. E. Sevior,[19] H. Shibuya,[40] T. Shidara,[7] B. Shwartz,[1] V. Sidorov,[1] N. Soni,[30] S. Stanič,[48,†] M. Starič,[12] A. Sugiyama,[34] T. Sumiyoshi,[45] S. Suzuki,[51] F. Takasaki,[7] K. Tamai,[7] N. Tamura,[27] M. Tanaka,[7] M. Tawada,[7] G. N. Taylor,[19] Y. Teramoto,[28] T. Tomura,[43] K. Trabelsi,[6] T. Tsukamoto,[7] S. Uehara,[7] K. Ueno,[24] Y. Unno,[2] S. Uno,[7] G. Varner,[6] K. E. Varvell,[38] C. C. Wang,[24] C. H. Wang,[23] J. G. Wang,[50] Y. Watanabe,[44] E. Won,[14] B. D. Yabsley,[50] Y. Yamada,[7] A. Yamaguchi,[42] Y. Yamashita,[26] H. Yanai,[27] Heyoung Yang,[36] J. Ying,[31] M. Yoshida,[7] C. C. Zhang,[9] Z. P. Zhang,[35] and D. Žontar[17,12]

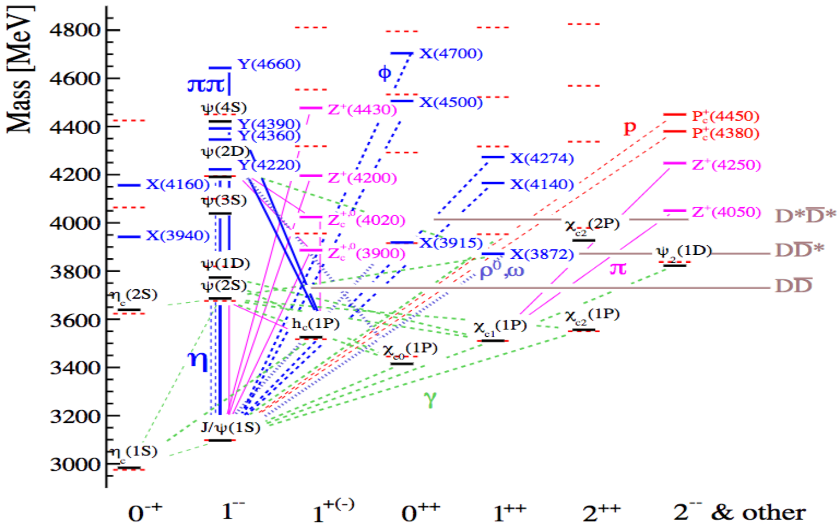
(Belle Collaboration)

- Discovery Mode : $B \to J/\psi\pi^{+}\pi^{-}K$
- $M = 3872.0 \pm 0.6 \pm 0.5$ MeV
- $\Gamma < 2.3$ MeV
- $J^{PC} = 1^{++}$ [LHCb] [PRL110, 22201(2013)]



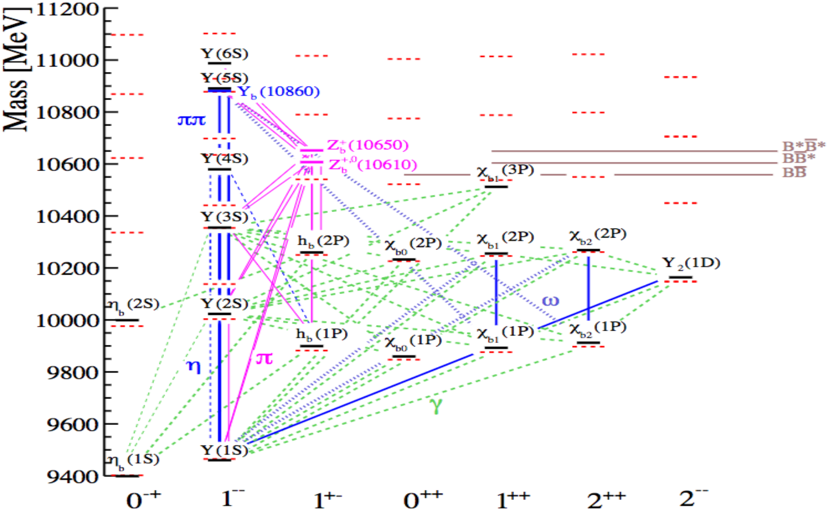
X, Y, Z, P_c and Charmonium States

[S.L. Olsen, T. Skwarnicki, D. Zieminska, arxiv: 1708.04012]



Bottomonium and Bottomonium-like States

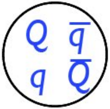
[S.L. Olsen, T. Skwarnicki, D. Zieminska, arxiv: 1708.04012]



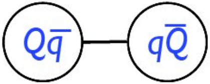
Models for *XYZ* Mesons

Quarkonium Tetraquarks

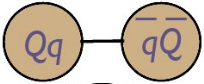
- compact tetraquark



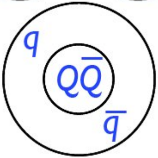
- meson molecule



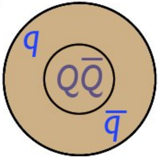
- diquark-onium



- hadro-quarkonium



- quarkonium adjoint meson



Diquark Model of Tetra- and Pentaquarks

Diquarks and Anti-diquarks are the building blocks of Tetraquarks
Color representation: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive; $C_{\bar{3}} = 1/2\, C_3$

Interpolating diquark operators for the two spin-states of diquarks

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \mathcal{Q}_{i\alpha} = & \epsilon_{\alpha\beta\gamma}(\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{i_c}^\beta \gamma_5 c^\gamma) \\ \text{Axial-Vector:} & 1^+ & \vec{\mathcal{Q}}_{i\alpha} = & \epsilon_{\alpha\beta\gamma}(\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{i_c}^\beta \vec{\gamma} c^\gamma) \end{array}$$

α, β, γ: $SU(3)_C$ indices

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α, β, γ: $SU(3)_C$ indices

NR limit: States parametrized by Pauli matrices :

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \Gamma^0 & = \frac{\sigma_2}{\sqrt{2}} \\ \text{Axial-Vector:} & 1^+ & \vec{\Gamma} & = \frac{\sigma_2\vec{\sigma}}{\sqrt{2}} \end{array}$$

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Diquark spin $s_{\mathcal{Q}}$ ➡ tetraquark total angular momentum J:

$$\left|Y_{[bq]}\right\rangle = \left|s_{\mathcal{Q}}, s_{\bar{\mathcal{Q}}};\, J\right\rangle$$

➥ Tetraquarks:

$$\begin{aligned} \left|0_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}};\, 0_J\right\rangle &= \Gamma^0 \otimes \Gamma^0 \\ \left|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\, 0_J\right\rangle &= \frac{1}{\sqrt{3}} \Gamma^i \otimes \Gamma_i \ldots \\ \left|0_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\, 1_J\right\rangle &= \Gamma^0 \otimes \Gamma^i \end{aligned}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\rm eff}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}L^2 + 2A_Q(L\cdot S) + 2\kappa_{qQ}\left[s_q\cdot s_Q + s_{\bar{q}}\cdot s_{\bar{Q}}\right] + b_Y\frac{S_{12}}{4}$$

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constituent mass

with

$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

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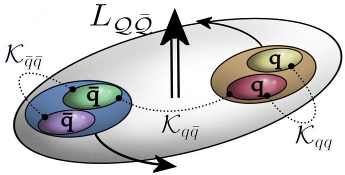
qq spin coupling $q\bar{q}$ spin coupling

with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$



$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

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NR Hamiltonian for Tetraquarks with hidden charm

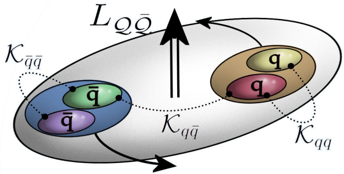
Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

LS coupling   *LL* coupling

with

$$\begin{aligned}
H_{SS}^{(qq)} &= 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})] \\
H_{SS}^{(q\bar{q})} &= 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) \\
&\quad + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}) \\
H_{SL} &= 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}) \\
H_{LL} &= B_{\mathcal{Q}} \frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)}{2}
\end{aligned}$$



$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2} L^2 + 2A_Q (L \cdot S) + 2\kappa_{qQ}[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}] + b_Y \frac{S_{12}}{4}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

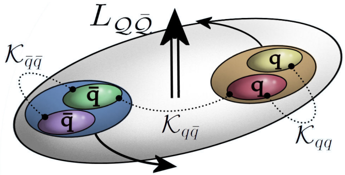
with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L})$$

$$H_{LL} = B_{\mathcal{Q}} \frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)}{2}$$



$$H_T = b_Y \frac{S_{12}}{4} = b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

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$$H_{\text{eff}}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2} L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}\left[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}\right] + b_Y \frac{S_{12}}{4}$$

Low-lying S-Wave Tetraquark States

- In the $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$ and $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$ bases, the positive parity S-wave tetraquarks are listed below; $M_{00} = 2m_{\mathcal{Q}}$

Label	J^{PC}	$\lvert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$\lvert s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
X_0	0^{++}	$\lvert 0,0;0,0\rangle_0$	$\left(\lvert 0,0;0,0\rangle_0 + \sqrt{3}\lvert 1,1;0,0\rangle_0\right)/2$	$M_{00} - 3\kappa_{qQ}$
X_0'	0^{++}	$\lvert 1,1;0,0\rangle_0$	$\left(\sqrt{3}\lvert 0,0;0,0\rangle_0 - \lvert 1,1;0,0\rangle_0\right)/2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$\left(\lvert 1,0;1,0\rangle_1 + \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$\lvert 1,1;1,0\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$\left(\lvert 1,0;1,0\rangle_1 - \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$\left(\lvert 1,0;1,0\rangle_1 - \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$\lvert 1,1;1,0\rangle_1$	$\left(\lvert 1,0;1,0\rangle_1 + \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$\lvert 1,1;2,0\rangle_2$	$\lvert 1,1;2,0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters, $M_{00}(Q)$ and κ_{qQ}, $Q = c, b$, hence very predictive
- Some of the states, such as X_0, X_0', X_2, still missing and are being searched for at the LHC

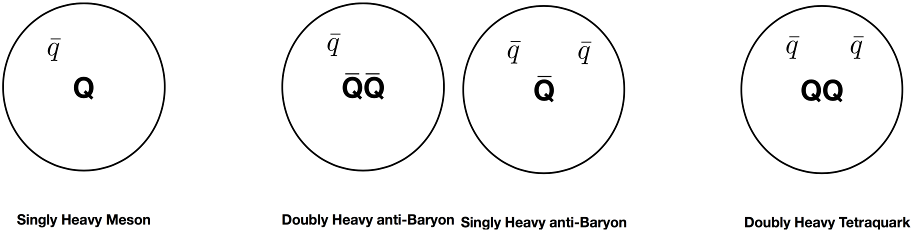
Parameters in the Mass Formula

	charmonium-like	bottomonium-like
M_{00} [MeV]	3957	10630
κ_{qQ} [MeV]	67	22.5

Label	J^{PC}	charmonium-like State	charmonium-like Mass [MeV]	bottomonium-like State	bottomonium-like Mass [MeV]
X_0	0^{++}	—	3756	—	10562
X_0'	0^{++}	—	4024	—	10652
X_1	1^{++}	$X(3872)$	3890	—	10607
Z	1^{+-}	$Z_c^+(3900)$	3890	$Z_b^{+,0}(10610)$	10607
Z'	1^{+-}	$Z_c^+(4020)$	4024	$Z_b^+(10650)$	10652
X_2	2^{++}	—	4024	—	10652

Heavy Quark Symmetry relations involving heavy Mesons, Baryons and Tetraquarks

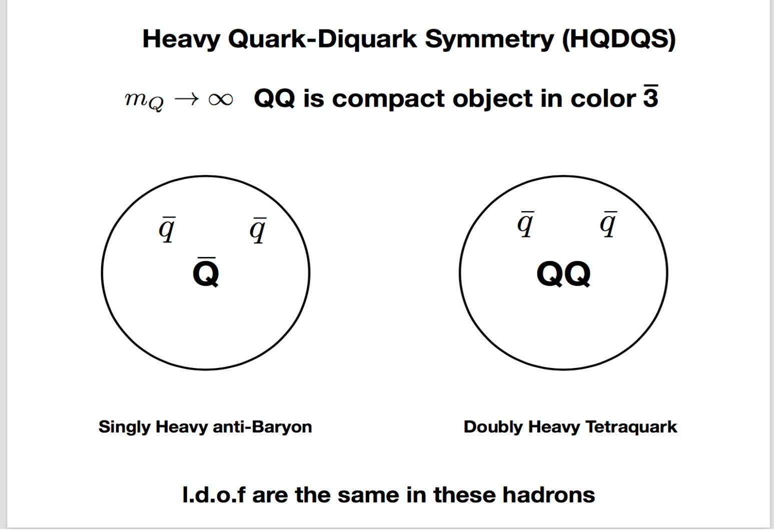
[See Eichten, Mehen, Karliner @ Quarkonium 2017; S-Q. Luo et al., EPJC 77:709 (2017)]



- Heavy quark symmetry relates a singly heavy meson $Q\bar{q}$ and a doubly heavy antibaryon $\bar{Q}\bar{Q}\bar{q}$
- Likewise, it relates a singly heavy antibaryon $\bar{Q}\bar{q}\bar{q}$ and a doubly heavy tetraquark $QQ\bar{q}\bar{q}$

Stable Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[See Eichten, Mehen, Karliner @ Quarkonium 2017; S-Q. Luo et al., EPJC 77:709 (2017)]



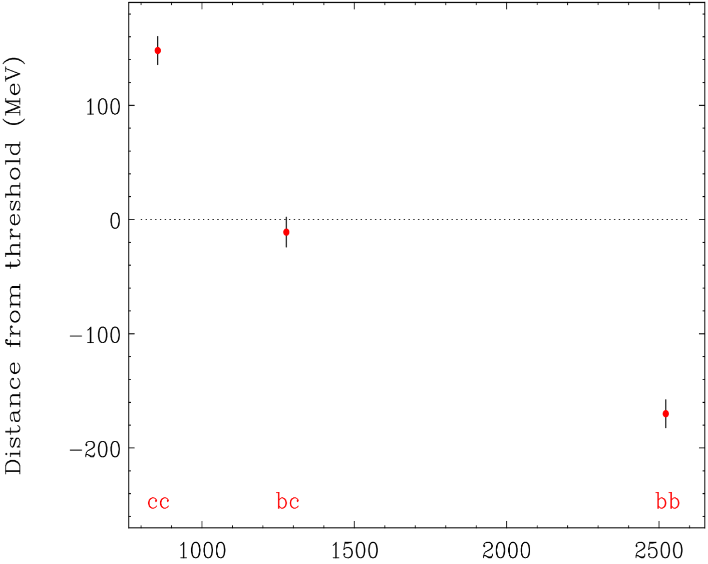
Quark Model Estimates of theTetraquark Masses $M(T(QQ\bar{q}\bar{q}'))$

[M. Karliner, J. Rosner, PRL 119, 202001 (2017)]

- Masses of heavy mesons and baryons successfully reproduced in Quark Models using effective Hamiltonians, incorporating spin-spin, spin-orbit, and tensor contributions
- In particular, masses of the doubly heavy $J = 1/2$ baryons, $M(\Xi_{cc})$, $M(\Xi_{cb})$, $M(\Xi_{bb})$ were estimated in this approach [M. Karliner, J. Rosner, 2014]
- Accurately predicted the mass of the doubly charmed baryon $M(\Xi_{cc}^{++}) = 3627 \pm 12$ MeV, compared to $M(\Xi_{cc}^{++}) = 3621 \pm 0.78$ MeV (LHCb 2017)
- This framework predicts $M(T(bb[\bar{u}\bar{d}])) = 10389 \pm 12$ MeV
- The lowest-lying $J^P = 1^+$ tetraquark $T(bb[\bar{u}\bar{d}])$ has a mass 215 MeV below the BB^* threshold and 170 MeV below the $BB\gamma$ threshold. Thus, $T(bb[\bar{u}\bar{d}])$ is stable against strong decay
- Predict: $M(T(cc[\bar{u}\bar{d}])) = 3882 \pm 12$ MeV, well above th DD^* threshold
- Predict: $M(T(bc[\bar{u}\bar{d}])) = 7133 \pm 13$ MeV; central value about 11 MeV below the BD^* threshold

Distance from Thresholds in MeV for $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[M. Karliner, J. Rosner, PRL 119, 202001 (2017)



Doubly Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[E. Eichten, C. Quigg, PRL 119, 202002 (2017)]

Heavy quark symmetry relations

- In the heavy limit, the color of the core $Q_i Q_j$ is $\bar{3}$ the same as a $\bar{Q}_x$. Hence in leading order of $\mathcal{M}^{-1}$ the light degrees of freedom have the same dynamics in the two systems leading to the following relations

$$
\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y) \,.
\end{aligned}
$$

- Finite mass corrections for all the states in these relations:

$$
\delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j}_\ell}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}}
$$

Stable Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[E. Eichten, C. Quigg, PRL 119, 202002 (2017)]

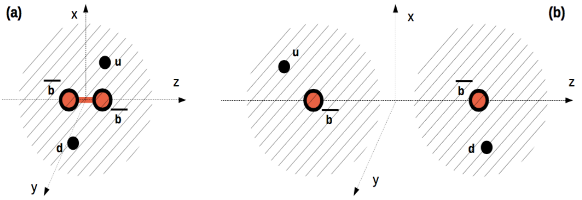
- $T(bb[\bar{u}\bar{d}])$ and $T(bb[\bar{q}\bar{s}])$ are stable against strong decay
- Will decay weakly by charged current interactions

Expectations for ground-state tetraquark masses

State	J^P	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	4146, 4167, 4210	D^+D^0, D^+D^{*0} 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	7461, 7472, 7493	BD/B^*D 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	10482	$B^-\bar{B}^{*0}$ 10603	**−121**
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	10643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10695/10691	**−48**
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	10674, 10681, 10695	B^-B^0, B^-B^{*0} 10559, 10603	115, 78, 136

QCD dynamics of a doubly heavy tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[P. Bicudo et al., Phys.Rev. D95, 142001 (2017)]

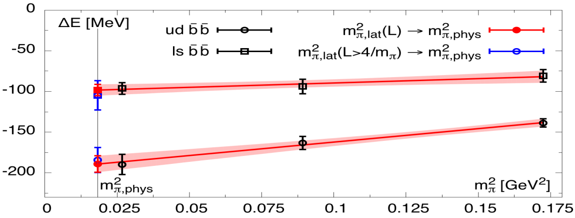


- At very short $\bar{b}\bar{b}$ distances, the interaction is Coulomb-like, given by one-gluon exchange (a)
- At large $\bar{b}\bar{b}$ separations, the light quarks ud screen the interaction, and the four quarks form two rather weakly interacting $B\,B^*$ mesons (b)
- Using this (Born-Oppenheimer) potential, a coupled-channel Schrödinger equation is solved, leading to a bound state, whose mass is estimated as $M(\mathcal{T}_{[\bar{u}\bar{d}]}^{\{bb\}-}) = 10545^{+38}_{-30}$ MeV.

Lattice QCD estimates of $bb\bar{u}\bar{d}$ tetraquark mass using NRQCD

[A. Francis et al., PRL 118, 142001 (2017)]

- Chiral extrapolations of the $ud\bar{b}\bar{b}$ and $qs\bar{b}\bar{b}$ binding energies
- $M(\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]}) = 10415 \pm 10\,\mathrm{MeV}$
- $M(\mathcal{T}^{\{bb\}-}_{[\bar{q}\bar{s}]}) = 10549 \pm 8\,\mathrm{MeV}$
- Both lie below their respective thresholds



Summary of $M(T(bb[\bar{u}\bar{d}]))$

[T. Mehen @Quarkonium 2017]

Stable Doubly Heavy Tetraquarks

quark model M. Karliner, J. Rosner, arXiv:1707.07666

$T_{bb\bar{u}\bar{d}}$ $\quad J^P = 1^+ \quad I = 0 \quad 10389 \pm 12\,\mathrm{MeV}$

$215\,\mathrm{MeV}$ below $B\bar{B}^*$ threshold, stable to strong interaction

heavy quark symmetry E. Eichten, C. Quigg, arXiv:1707.09575

$10468\,\mathrm{MeV}$ $\quad 135\,\mathrm{MeV}$ below threshold

lattice QCD

$189 \pm 10\,\mathrm{MeV}$ below threshold A. Francis, et. al., PRL **118**, 142001 (2017)

$60^{+30}_{-38}\,\mathrm{MeV}$ below threshold P. Bicudo, et. al., PRD **95**, 142001 (2017)

no analogous prediction for $T_{cc\bar{q}\bar{q}}, T_{bc\bar{q}\bar{q}}$ tetraquarks

molecular $T_{cc} = DD^*$ D. Janc, M. Rosina, Few Body Syst. **35** (2004) 175

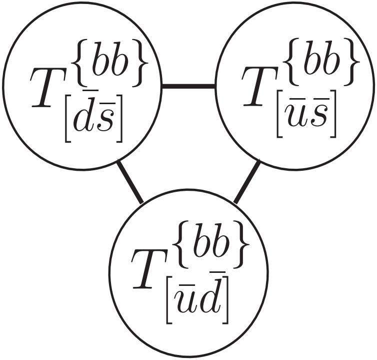
arguments for stability in heavy quark limit

J.-P. Ader, J.-M. Richard, P. Taxil, PRD **25** (1982) 2370

A. Manohar, M. Wise, NPB 399 (1993) 17

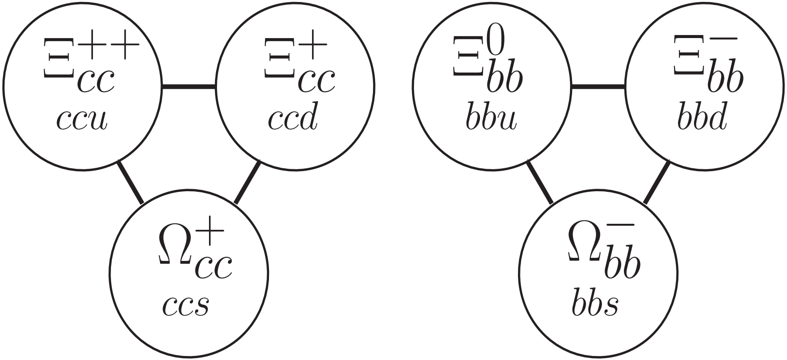
$SU(3)_F$-triplet of stable double-bottom tetraquarks $T(bb\bar{q}\bar{q}')$

$$S_{\{bb\}} = 1, S_{[\bar{q}\bar{q}']} = 0, J^P = 1^+$$



$SU(3)_F$-triplet of double charm & double-bottom baryons

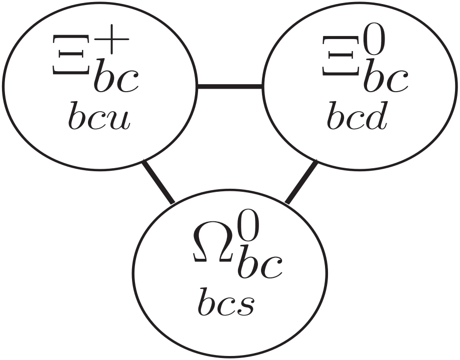
Of these, $\Xi_{cc}^{++} = (ccu)$ has been discovered at the LHCb



Great discovery potential at the LHC & Tera-Z factory!

$SU(3)_F$-triplet of bottom-charmed baryons

So far, the only known bottom-charmed hadrons are B_c and B_c^*



Likewise, Great discovery potential at the LHC & Tera-Z factory!

Prospects of observing Stable Tetraquarks at a Tera-Z Factory

- The partonic process at the Z-factory is:

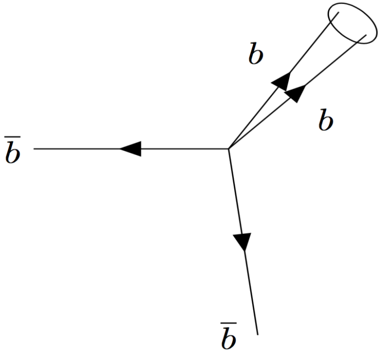
$$e^+e^- \to Z \to b\bar{b}b\bar{b}$$

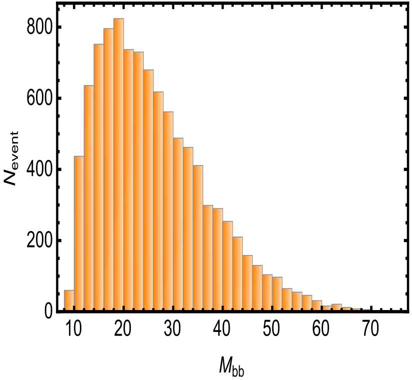
- Measured at LEP: $\mathcal{B}(Z \to b\bar{b}b\bar{b}) = (3.6 \pm 1.3) \times 10^{-4}$
- Unlike the inclusive production of hadrons, such as $Z \to (B, \Lambda_b, \Sigma_b) + X$, which results from the fragmentation of a single-b quark, the production of hadrons with double-b quarks requires the topology in which two b-quarks have to be in a jet:

$$e^+e^- \to Z \to (bb)_{\text{Jet}} + \bar{b}\bar{b}$$

- The $(bb)_{\text{Jet}}$ requires a jet-definition, such as the invariant mass $M(bb)^2_{\text{Jet}} = (p(b_1) + p(b_2))^2$, with Jet-resolution parameter: $M(T^{\{bb\}}_{[\bar{u}\bar{d}]})^2 \leq M(bb)^2_{\text{Jet}} \leq (2m_b + \Delta M)^2$
- For $M(bb)^2_{\text{Jet}} \gg 4M_B^2$, independent fragmentation of the 2 b quarks into two B-hadrons, $(bb)_{\text{Jet}} \to (BB, BB^*, 2\Lambda_b + ...) + X$ dominates, and the probability of a double-b hadron production becomes insignificant, $\mathcal{P}(bb \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X) \ll 1$

Typical topology of $Z \to (bb)_{\text{Jet}} + \bar{b}\bar{b}$





- Double-b-hadrons, such as the tetraquark $T^{\{bb\}}_{[\bar{u}\bar{d}]}$ and double-b baryons Ξ^{q}_{bb}, are the fragmentation products of the $(bb)_{\text{Jet}}$
- They are anticipated to populate low-M_{bb} invariant mass region

Estimates of the $(bb)_{\text{Jet}}$-parameter ΔM

[AA, A. Parkhomenko, Qin Qin, Wei Wang, to be published]

- We model ΔM by using a similar (but not identical) process which involves the fusion $b\bar{c} \to B_c$ from $e^+e^- \to Z \to b\bar{b}c\bar{c}$, leading to $e^+e^- \to Z \to B_c + \bar{b} + c$, using [Z. Yang, X. G. Wu, G. Chen, Q. L. Liao and J. W. Zhang, Phys. Rev. D **85**, 094015 (2012)] and the Monte Carlo generator MadGraph
- Input quark masses (GeV): $(m_b, m_c) = (4.9, 1.5)$ (central); $(m_b, m_c) = (5.3, 1.2)$ (upper), $(m_b, m_c) = (4.8, 1.5)$

Quark masses	$\sigma(B_c b\bar{c})$ [pb]	$\sigma(b\bar{b}c\bar{c})$ [pb]	$f(\bar{b}c \to B_c)$	ΔM [GeV]
central	5.19	64.50	8.05%	2.82
upper	11.41	76.79	14.86%	4.22
lower	2.77	56.75	4.88%	2.22

- This gives an estimate of ΔM, which we use for simulating $e^+e^- \to Z \to (bb)_{\text{Jet}} + \bar{b}\bar{b}$

Estimates of the $(bb)_{\text{Jet}}$-parameter ΔM

[AA, A. Parkhomenko, Qin Qin, Wei Wang, to be published]

- Processes simulated: $e^+e^- \to Z \to b\bar{b}b\bar{b}$ (inclusive $4b$-production)& $e^+e^- \to Z \to T^{\{bb\}}_{[\bar{u}\bar{d}]}\bar{b}\bar{b} + ...$ (semi-inclusive),, with the invariant mass cut $M(T^{\{bb\}}_{[\bar{u}\bar{d}]})^2 \leq M(bb)^2_{\text{Jet}} \leq (2m_b + \Delta M)^2$
- Generated 10^4 showered $e^+e^- \to b\bar{b}b\bar{b}$ events at the Z mass with MadGraph and Pythia6 using NLO
- MadGraph (NLO) yields: $\mathcal{B}(Z \to b\bar{b}b\bar{b}) = 4.26 \times 10^{-4}$ for $m_b = 4.9$ GeV, consistent with the LEP measurements $(3.6 \pm 1.3) \times 10^{-4}$
- This yields the fraction $f((bb)_{\text{Jet}}(\Delta M) \to H_{bb} + X) = (8.8^{+4.6}_{-1.9})\%$
- We assume that the double-b hadron H_{bb} consists of $T^{\{bb\}}_{[\bar{q}\bar{q}']}$ (tetraquark) or $\Xi(bbq)$ (double-b baryon)
- Using heavy quark symmetry, anticipate $\frac{f(bb \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X)}{f(bb \to \Xi_{bb} + X)} = \frac{f_{\Lambda_b}}{f_{B_u} + f_{B_d}} \simeq 0.3$
- Estimate (Preliminary): $\mathcal{B}(Z \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X) = (4.5^{+2.6}_{-1.6}) \times 10^{-6}$

Estimates of the lifetime for $T^{\{bb\}}_{[\bar{u}\bar{d}]}$

[AA, A. Parkhomenko, Qin Qin, Wei Wang, to be published]

- The decay rate of $T^{\{bb\}}$ into an inclusive final state X can be expressed as:

$$\Gamma(T^{\{bb\}}_{[\bar{q}\bar{q}']} \to X) = \frac{1}{2m_{T^{\{bb\}}_{[\bar{q}\bar{q}']}}} \sum_X \int \prod_i \left[\frac{d^3 p_i}{(2\pi)^3 2E_i} \right]$$
$$\times (2\pi)^4 \delta^{(4)}(p_{T^{\{bb\}}_{[\bar{q}\bar{q}']}} - \sum_i p_i) |\langle X | \mathcal{H}^{ew}_{eff} | T^{\{bb\}}_{[\bar{q}\bar{q}']} \rangle|^2$$

- $\mathcal{H}^{ew}_{eff}$ is the effective electroweak Hamiltonian governing the weak decays. Using the optical theorem, one can rewrite the total rate as

$$\Gamma(T^{\{bb\}} \to X) = \frac{1}{2m_{T^{\{bb\}}}} \langle T^{\{bb\}} | \mathcal{T} | T^{\{bb\}} \rangle$$

with the transition operator defined as:

$$\mathcal{T} = \text{Im}\, i \int d^4x T[\mathcal{H}^{ew}_{eff}(x), \mathcal{H}^{ew}_{eff}(0)]$$

Estimates of the lifetime for $T^{\{bb\}}_{[\bar{u}\bar{d}]}$ using heavy quark expansion

- HQE simplifies the inclusive decay widths. Up to dimension 6 :

$$\mathcal{T} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{CKM}|^2 \Big[c_{3,b} \bar{b}b + \frac{c_{5,b}}{m_b^2} \bar{b} g_s \sigma_{\mu\nu} G^{\mu\nu} b + 2\frac{c_{6,b}}{m_b^3} (\bar{b}q)_\Gamma (\bar{q}b)_\Gamma + ... \Big]$$

- At leading order in $1/m_b$, only the $\bar{b}b$ operator contributes:

$$\Gamma(T^{\{bb\}}_{[\bar{q}\bar{q}']}) = \frac{G_F m_b^5}{192\pi^3} |V_{CKM}|^2 c_{3,b} \frac{\langle T^{\{bb\}}_{[\bar{q}\bar{q}']} | \bar{b}b | T^{\{bb\}}_{[\bar{q}\bar{q}']} \rangle}{2 m_{T^{\{bb\}}_{[\bar{q}\bar{q}']}}}$$

- $\frac{\langle T^{\{bb\}}_{[\bar{q}\bar{q}']} | \bar{b}b | T^{\{bb\}}_{[\bar{q}\bar{q}']} \rangle}{2 m_{T^{\{bb\}}_{[\bar{q}\bar{q}']}}}$ corresponds to the bottom-quark number in $T^{\{bb\}}_{[\bar{q}\bar{q}']}$, and is twice the matrix element for B meson and Λ_b baryon
- Hence, expect $\tau(T^{\{bb\}}_{[\bar{u}\bar{d}]}) \simeq 1/2\tau(B)$:

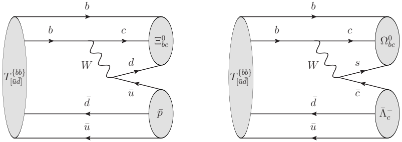
$$\tau(T^{\{bb\}}_{[\bar{q}\bar{q}']}) \sim \frac{1}{2} \times 1.6 \times 10^{-12} s = 800 \times 10^{-15} s$$

Weak Decays of $T^{\{bb\}}_{[\bar{u}\bar{d}]}$

- Effective Weak Hamiltonian

$$\begin{aligned}\mathcal{H}^{(cc)}_{\text{eff}} &= \frac{4G_F}{\sqrt{2}} V_{cb}V^*_{ud}\Big\{C_1\,[\bar{c}_\alpha\gamma_\mu P_L b^\alpha]\,\Big[\bar{d}_\beta\gamma^\mu P_L u^\beta\Big] \\ &\qquad + C_2\,[\bar{c}_\beta\gamma_\mu P_L b^\alpha]\,\Big[\bar{d}_\alpha\gamma^\mu P_L u^\beta\Big]\Big\} \\ &+ \frac{4G_F}{\sqrt{2}} V_{cb}V^*_{cs}\Big\{C_1\,[\bar{c}_\alpha\gamma_\mu P_L b^\alpha]\,\Big[\bar{s}_\beta\gamma^\mu P_L c^\beta\Big] \\ &\qquad + C_2\,[\bar{c}_\beta\gamma_\mu P_L b^\alpha]\,\Big[\bar{s}_\alpha\gamma^\mu P_L c^\beta\Big]\Big\} + \text{h. c.}\end{aligned}$$

- Two-Body Baryonic Decays from $b \to c + d + \bar{u}$ (left panel) and $b \to c + s + \bar{c}$ (right panel)



An order of magnitude estimate

- Involve non-factorizable Amplitudes . For the $J^P = 1^+$ tetraquark, the general form of the decay amplitude is:

$$\mathcal{M}(T^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \Xi^0_{bc}\,\bar{p}) = \bar{v}(p_p)\left[f_1^{\Xi_{bc}\bar{p}}\, q_\mu + f_2^{\Xi_{bc}\bar{p}}\, \gamma_\mu + f_3^{\Xi_{bc}\bar{p}}\, \sigma_{\mu\nu}\frac{q^\nu}{M_T} + g_1^{\Xi_{bc}\bar{p}}\, \gamma_5\, q_\mu + g_2^{\Xi_{bc}\bar{p}}\, \gamma_\mu\gamma_5 + g_3^{\Xi_{bc}\bar{p}}\, \sigma_{\mu\nu}\gamma_5\frac{q^\nu}{M_T}\right] u(p_{\Xi_{bc}})\, \varepsilon^\mu_T(p_T)$$

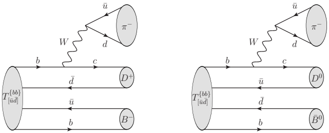
- Inspired by the B meson decay data

$$\mathcal{B}(\overline{B}^0 \to D^+\pi^-) = (2.52 \pm 0.13) \times 10^{-3}$$
$$\mathcal{B}(\overline{B}^0 \to D^+D_s^-) = (7.2 \pm 0.8) \times 10^{-3}$$

- Infer that $\mathcal{B}(T^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \Xi^0_{bc}\,\bar{p})$ and $\mathcal{B}(T^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \Omega^0_{bc}\,\bar{\Lambda}_c^-)$ are of $O(10^{-3})$
- Needs reconstructing the doubly heavy baryons Ξ^0_{bc} and Ω^0_{bc}, such as through $\Xi^0_{bc} \to \Lambda_b K^-\pi^+$, expect the two-body baryonic decay modes of $T^{\{bb\}-}_{[\bar{u}\bar{d}]}$ can have branching fractions of order 10^{-6}

Three-body Mesonic Decay Modes of $T^{\{bb\}-}_{[\bar{u}\bar{d}]}$

- Feynman diagrams due to the b-quark decay $b \to c + d + \bar{u}$



- The factorizable amplitudes of these decays can be written as:

$$\mathcal{M}(T^{\{bb\}-}_{[\bar{u}\bar{d}]} \to B^- \, D^+ \, \pi^-) = i \, \frac{G_F}{\sqrt{2}} \, V_{cb} V^*_{ud} \, a_1^{\rm eff} f_\pi \, p_\pi^\mu$$

$$\times \langle (BD)^0_{J_{BD}} (p_{BD}) \left| \bar{d} \gamma_\mu \, (1 - \gamma_5) \, u \right| T^{\{bb\}-}_{[\bar{u}\bar{d}]} (p_T) \rangle,$$

$$\mathcal{M}(T^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \bar{B}^0 \, D^0 \, \pi^-) = i \, \frac{G_F}{\sqrt{2}} \, V_{cb} V^*_{ud} \, a_1^{\rm eff} f_\pi \, p_\pi^\mu$$

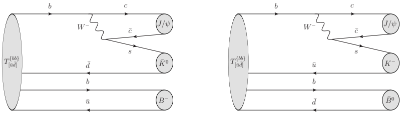
$$\times \langle (BD)^0_{J_{BD}} (p_{BD}) \left| \bar{s} \gamma_\mu \, (1 - \gamma_5) \, c \right| T^{\{bb\}-}_{[\bar{u}\bar{d}]} (p_T) \rangle$$

$$\mathcal{B}(T^{\{bb\}}_{[\bar{u}\bar{d}]} \to B^- D^+ \pi^-) = \mathcal{B}(T^{\{bb\}}_{[\bar{u}\bar{d}]} \to \bar{B}^0 D^0 \pi^-) \sim 0.5 \times 10^{-3}$$

Hidden-Charm final states in $T^{\{bb\}-}_{[\bar{u}\bar{d}]}$ decays

- In some decays hidden-charm mesons, such as J/ψ, ψ', can be produced

$$T^{\{bb\}}_{[\bar{u}\bar{d}]} \to J/\psi \overline{K}^0 B^-,$$
$$T^{\{bb\}}_{[\bar{u}\bar{d}]} \to J/\psi K^- \overline{B}^0$$



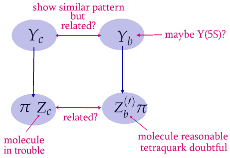
- Their decay branching ratios can be comparable with the $\mathcal{B}(B \to J/\psi K)$:

$$\mathcal{B}(\overline{B}^0 \to J/\psi \overline{K}^0) \quad = \quad (8.73 \pm 0.32) \times 10^{-4}$$

- Expect that the product branching ratios to measure the mass of $T^{\{bb\}-}_{[\bar{u}\bar{d}]}$ are at most of $O(10^{-5})$

Summary

- A new facet of QCD is opened by the discovery of the exotic states X, Y, Z, $\mathbb{P}(4380)$, $\mathbb{P}(4450)$
- Important puzzles remain in the complex:



- What is the nature of $Y_c(4260)$? A tetraquark? or a $c\bar{c}g$ hybrid? Is $Y_c(4260)$ split? How many P states are there? We do expect a tower of radial and orbital excited states in the diquark picture!
- CEPC running as a Z factory will advance the multiquark sector of QCD decisively, in particular, the entire pentaquark spectrum with hidden charm can be measured from the production and decays of the b-baryons
- Also CEPC has great potential to discover doubly heavy tetraquarks, establishing diquarks as a building block in QCD. First results from the simulation at a Tera-Z factory presented here. Likewise, an entire spectrum of doubly-heavy baryons can be measured
- We look forward to decisive experimental results from BESIII, Belle-II, LHC, and the CEPC