5-body contributions to inclusive semileptonic rare B decays

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Outline

- Motivation
- Calculation
  - Amplitudes
  - Phase space integral
  - Results
- Summary and Outlook

Motivation

Motivation

- For $\bar{B} \to X_s \ell^+ \ell^-$, a thorough phenomenological analysis is available in the market [Huber, Hurth & Lunghi, 15']

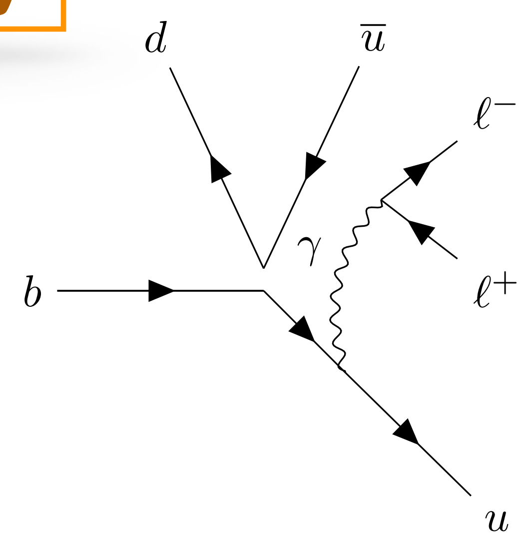
- A similar study for $\bar{B} \to X_d \ell^+ \ell^-$ is in order (for Belle II). What's new?

$P^u_{1,2} = (\bar{d}\gamma^\mu P_L u)(\bar{u}\gamma_\mu P_L b)$ not CKM suppressed for b-d transition ⇨

- new NLO & NNLO QCD corrections [Greub et al, 03; Seidel, 04']
- new power corrections [Ali, Hiller, Handoko & Morozumi 96'; Ligeti & Tackmann 07; Benzke, Turkczyk & Hurth, 17']
- new log-enhanced EM contributions [Huber, Hurth & Lunghi, 07']
- ...
- five-body processes at the quark level (totally new)

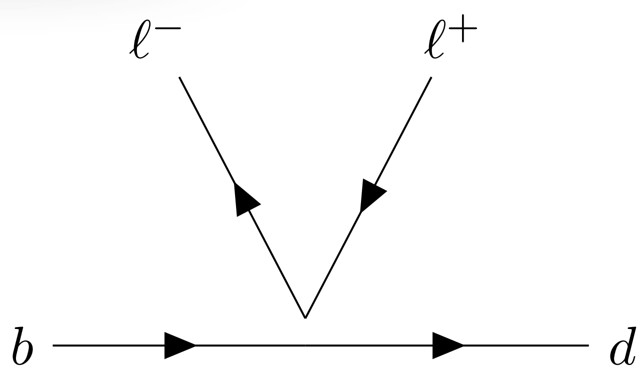
Motivation

5 body



$P^u_{1,2} = (\bar{d}\gamma^\mu P_L u)(\bar{u}\gamma_\mu P_L b)$

3 body



$P_9 = (\bar{d}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell)$

$$|V^*_{ud}V_{ub}|^2\alpha_s^2\kappa^2C^2_{1,2} \quad \text{vs} \quad |V^*_{td}V_{tb}|^2C_9^2$$

$\kappa \equiv \frac{\alpha_e}{\alpha_s}$

- Formally, $\lambda^4\alpha_s^2\kappa^2$ vs $\lambda^4\kappa^2$ $(C_9 \sim \kappa)$, 5-body is necessary to complete NNLO
- Numerically, $\lambda^4\alpha_s^2\kappa^2$ vs $\lambda^4\alpha^2\kappa^2$ $(C_9 \sim \alpha_s\kappa)$, LO!!

→ only suppressed by phase pace

Calculation

Calculation

$$\Gamma = \frac{1}{2m_b} \int |\mathcal{A}|^2 \, \mathrm{dPS}$$

- Write down the decay amplitude
- square it
- and make the phase-space integration

Amplitudes

Amplitudes

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}}(u,d,s,c,b,e,\mu,\tau) - \frac{4G_F}{\sqrt{2}} V_{ud}^* V_{ub} (C_1^u P_1^u + C_2^u P_2^u) + \frac{4G_F}{\sqrt{2}} V_{td}^* V_{tb} \sum_{i=3}^{10} C_i(\mu) P_i,$$

$$P_1^u = (\bar{d}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L),$$

$$P_2^u = (\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L),$$

$$P_3 = (\bar{d}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q}\gamma^\mu q),$$

$$P_4 = (\bar{d}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q}\gamma^\mu T^a q),$$

$$P_5 = (\bar{d}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q),$$

$$P_6 = (\bar{d}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q),$$

$$P_7 = \frac{e}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu},$$

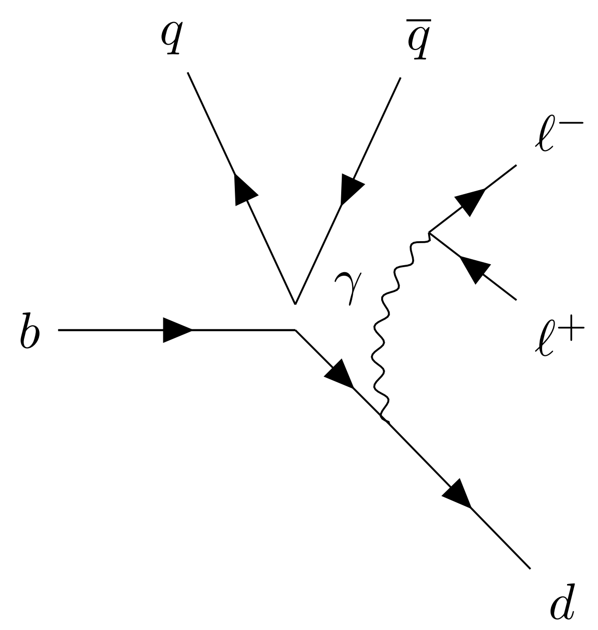
$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu},$$

$$P_9 = (\bar{d}_L \gamma_\mu b_L) \textstyle\sum_l (\bar{l}\gamma^\mu l),$$

$$P_{10} = (\bar{d}_L \gamma_\mu b_L) \textstyle\sum_l (\bar{l}\gamma^\mu \gamma_5 l).$$

Amplitudes

$$P_{1-6}^{(u)} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$



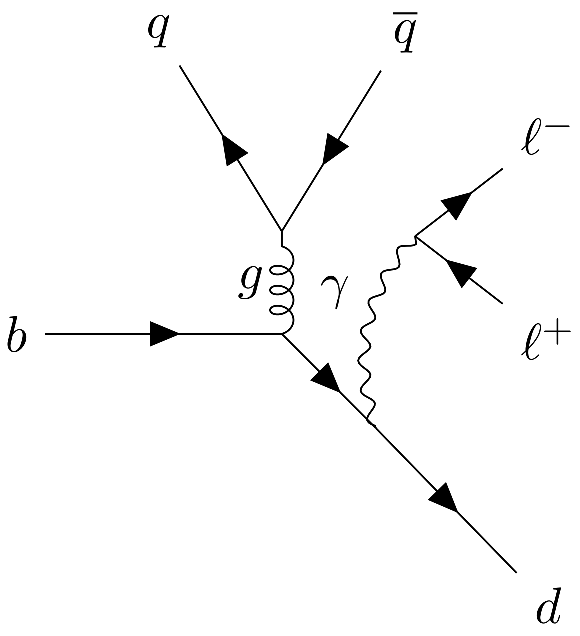
$$4 \times 6$$

Amplitudes

$$P_{1-6}^{(u)} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$

4×6

$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$



4

Amplitudes

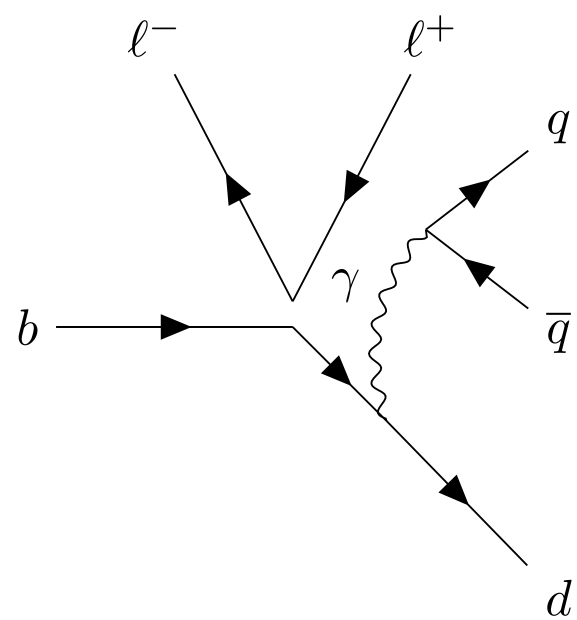
$$P_{1-6}^{(u)} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$

4×6

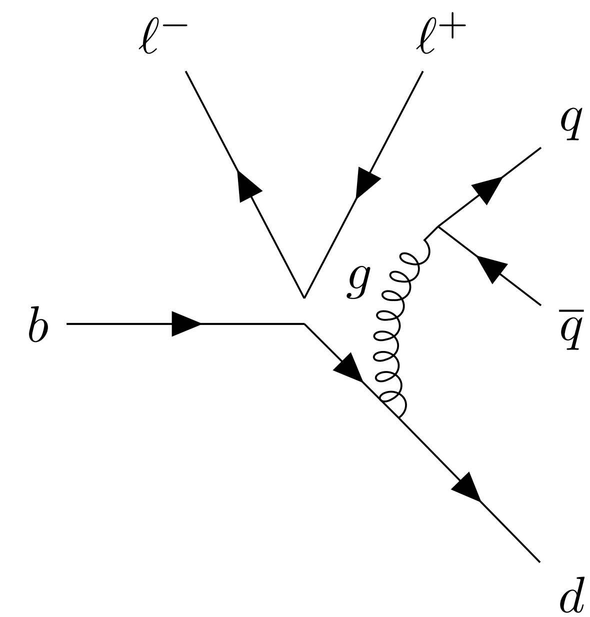
$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

4

$$P_{9,10} = (\bar{d}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$



4×2



2×2

Amplitudes

$$P_{1-6}^{(u)} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$

$$4 \times 6$$

$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

$$4$$

$$P_{9,10} = (\bar{d}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$

$$4 \times 2 + 2 \times 2$$

Amplitudes

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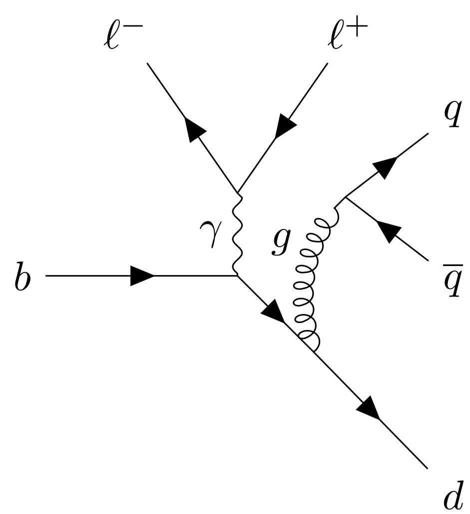

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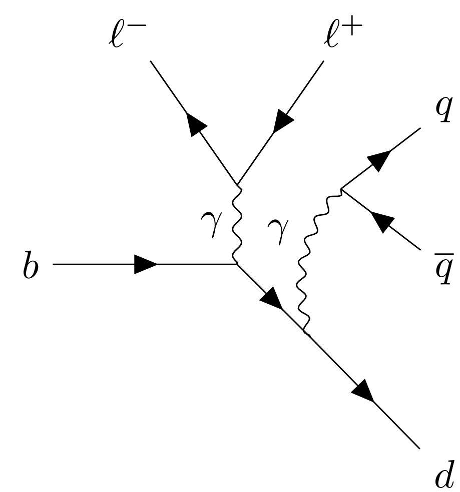

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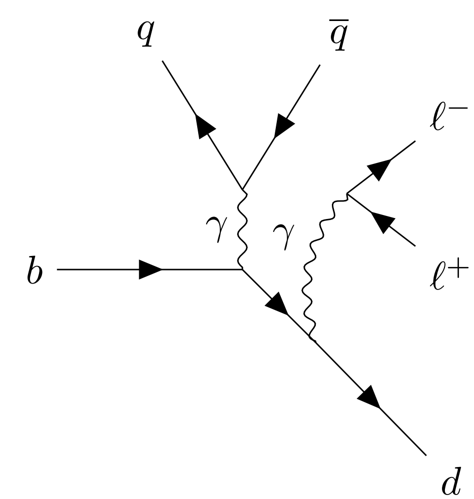


4

Amplitudes

$$P^{(u)}_{1-6} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$ 4×6

$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$ 4

$$P_{9,10} = (\bar{d}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$ $4 \times 2 + 2 \times 2$

$$P_7 = \frac{e}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$ $2 + 4 + 4$

In total, 50

Amplitudes

$$P_{1-6}^{(u)} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$

4×6

$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

4

$$P_{9,10} = (\bar{d}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$

$4 \times 2 + 2 \times 2$

$$P_7 = \frac{e}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$2 + 4 + 4$

In total, 50

- Amplitude square
  - 50*50 = 2500 (only for u)
  - 42*42 (for s)
  - 42*42 (for d)?

Amplitudes

$$P_{1-6}^{(u)} = (\bar{q}\Gamma_1 q)(\bar{d}\Gamma_2 b)$$ 4×6

$$P_8 = \frac{g}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$ 4

$$P_{9,10} = (\bar{d}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu (\gamma_5)\ell)$$ $4 \times 2 + 2 \times 2$

$$P_7 = \frac{e}{16\pi^2} m_b (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$ $2 + 4 + 4$

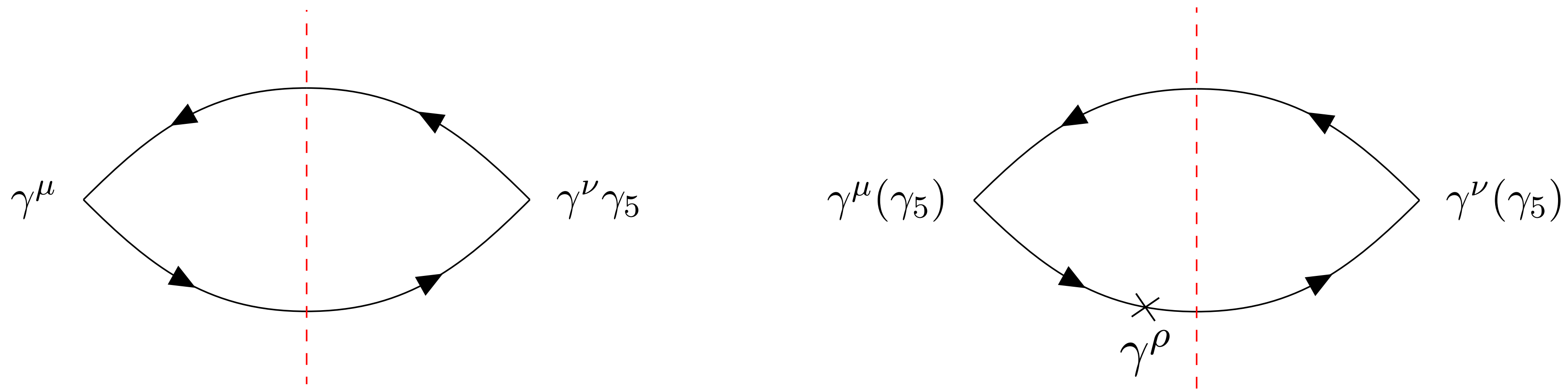
In total, 50

- Amplitude square
  - 50*50 = 2500 (only for u)
  - 42*42 (for s)
  - 42*42 (for d)?
  - 84*84 (for d)

identical particle

Amplitudes

- Rules to reduce terms
  - color conservation
  - what vanishes in the sense of exchanging fermions



  - As = Au(Qu->Qs), Ad(I) = Au(Qu->Qd)
  - Qu + Qd + Qs = 0
  - power counting up to $\alpha_s^3\kappa^3$

Amplitudes

- Power counting

$$\left(\begin{array}{ccccc}
\mathcal{P}_{1-6} & \mathcal{P}_7 & \mathcal{P}_8 & \mathcal{P}_9 & \mathcal{P}_{10} \\
\hline
\alpha_s^2\kappa^2 & \begin{pmatrix}\text{QCD }\alpha_s^3\kappa^2 \\ \text{QED }\alpha_s^3\kappa^3\end{pmatrix} & \alpha_s^3\kappa^2 & \begin{pmatrix}\text{QCD }\alpha_s^3\kappa^2 \\ \text{QED }\alpha_s^3\kappa^3\end{pmatrix} & \begin{pmatrix}\text{QCD }\alpha_s^3\kappa^2 \\ \text{QED }\alpha_s^3\kappa^3\end{pmatrix} \\
\ldots & \times & \times & \times & \times \\
\ldots & \times & \times & \times & \times \\
\ldots & \times & \times & \times & \times \\
\ldots & \times & \times & \times & \times
\end{array}\right)$$

$C_{1-8} \sim 1$

$C_9 \sim 0.04\kappa + 2\alpha_s\kappa + \ldots$

$(\sim \alpha_s\kappa)$

$C_{10} \sim 0.005\kappa^2 - 4\alpha_s\kappa + \ldots$

$(\sim \alpha_s\kappa^2)$

Phase space integral

Phase space integral

- Start from

$$\int d\Phi_5 \equiv \int \prod_{i=1}^{5} \frac{d^3 p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4(p_b - \sum_{i=1}^{5} p_i)$$

- an issue of simplifying the calculation again

$$|\mathcal{A}|^2 = |\mathcal{A}|^2_{\rm I}\left(\{p_i \cdot p_j\}\right) + {\rm Re}\left(\mathcal{A}^2_{\rm II}(\{p_i \cdot p_j\}) i\epsilon_{\mu\nu\rho\sigma} p_i^\mu p_j^\nu p_k^\rho p_l^\sigma\right)$$

Under the phase space int. $\int \prod_{i=1}^{5} \frac{d^3 p_i}{(2\pi)^3 2E_i}$

p_i^0 is even, p_i^{1-3} are odd

➩ ${\rm Re}\left(\mathcal{A}^2_{\rm II}(\{p_i \cdot p_j\}) i\epsilon_{\mu\nu\rho\sigma} p_i^\mu p_j^\nu p_k^\rho p_l^\sigma\right)$ must be odd

➩ $$\int d\Phi_5 \, |\mathcal{A}|^2 = \int d\Phi_5 \, |\mathcal{A}|^2_{\rm I}\left(\{p_i \cdot p_j\}\right)$$

Phase space integral

- The integral formula [Kumar, 69']

$$
\begin{aligned}
\int d\Phi_5 &\equiv \int \prod_{i=1}^{5} \frac{d^3 p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4 (p_b - \sum_{i=1}^{5} p_i) \\
&= \frac{\pi^2 m_b^6}{4(2\pi)^{11}} \int_0^1 ds_3 \int_{s_3}^1 ds_2 \int_{s_2}^1 ds_1 \int_{u_{1-}}^{u_{1+}} du_1 \int_{u_{2-}}^{u_{2+}} du_2 \int_{u_{3-}}^{u_{3+}} du_3 \int_{t_{2-}}^{t_{2+}} dt_2 \int_{t_{3-}}^{t_{3+}} dt_3 \\
&\times \frac{1}{(1-u_1)(1-u_2)(1-u_3)\sqrt{\left(1-\eta_2^2\right)\left(1-\xi_2^2\right)-(\omega_2-\eta_2\xi_2)^2}} \\
&\times \frac{1}{\sqrt{\left(1-{\eta_3}^2\right)\left(1-\xi_3^2\right)-(\omega_3-\eta_3\xi_3)^2}\sqrt{\lambda(1, s_2, -s_1+s_2-u_1+1)}} \\
&\times \frac{1}{\sqrt{\lambda(1, t_2, t_2-u_1-u_2+1)}\sqrt{\lambda(1, s_3, -s_1+s_3-u_1-u_2+2)}},
\end{aligned}
$$

$$
s_n = \frac{(p_b - \sum_{i=1}^{n} p_i)^2}{m_b^2}, \qquad u_n = \frac{(p_b - p_{n+1})^2}{m_b^2}, \qquad t_n = \frac{(p_b - \sum_{i=2}^{n+1} p_i)^2}{m_b^2}.
$$

Phase space integral

$$p_i \cdot p_j \Rightarrow f(s,t,u)$$

$$\frac{p_1 \cdot p_b}{m_b^2} \to \frac{1-s_1}{2}, \quad \frac{p_2 \cdot p_b}{m_b^2} \to \frac{1-u_1}{2}, \quad \frac{p_3 \cdot p_b}{m_b^2} \to \frac{1-u_2}{2}, \quad \frac{p_4 \cdot p_b}{m_b^2} \to \frac{1-u_3}{2},$$

$$\frac{p_1 \cdot p_2}{m_b^2} \to \frac{1-s_1+s_2-u_1}{2}, \quad \frac{p_1 \cdot p_3}{m_b^2} \to \frac{u_1-s_2+s_3-t_2}{2}, \quad \frac{p_1 \cdot p_4}{m_b^2} \to \frac{t_2-s_3-t_3}{2},$$

$$\frac{p_2 \cdot p_3}{m_b^2} \to \frac{1+t_2-u_1-u_2}{2}, \quad p_2 \cdot p_4 = \frac{1-t_2+t_3-u_3}{2} - p_3 \cdot p_4$$

??

An exercise of counting numbers

$$\left(\sum_{i=1}^{5} p_i\right)^2 = m_b^2$$

No. of independent p-products: 4*5/2 - 1 = 9

No. of integral variables: 3*5 - 4 - 3 = 8

One p-product redundant?

$$\delta^4(p_b - \sum_{i=1}^{5} p_i)$$

3 space angles

Phase space integral

One p-product redundant?

The answer is yes.

In the dim-4 space-time, at most 4 independent momenta, so

$$\sum_{i=1}^{5} c_i p_i = 0 \quad \Rightarrow \quad \det \begin{pmatrix} p_1 \cdot p_1 & p_1 \cdot p_2 & p_1 \cdot p_3 & p_1 \cdot p_4 & p_1 \cdot p_5 \\ p_2 \cdot p_1 & p_2 \cdot p_2 & p_2 \cdot p_3 & p_2 \cdot p_4 & p_2 \cdot p_5 \\ p_3 \cdot p_1 & p_3 \cdot p_2 & p_3 \cdot p_3 & p_3 \cdot p_4 & p_3 \cdot p_5 \\ p_4 \cdot p_1 & p_4 \cdot p_2 & p_4 \cdot p_3 & p_4 \cdot p_4 & p_4 \cdot p_5 \\ p_5 \cdot p_1 & p_5 \cdot p_2 & p_5 \cdot p_3 & p_5 \cdot p_4 & p_5 \cdot p_5 \end{pmatrix} = 0$$

$$\Rightarrow \quad p_3 \cdot p_4 = [111 \text{ terms}] \pm \sqrt{[2179 \text{ terms}]}$$

The solution is only the first part, according to [Kumar, 69'], and this is wrong!

Phase space integral

Kumar's method

$$\int d^4p_4\delta\left(p_4^2\right)\delta\left((p_b-p_{1-4})^2\right)\delta\left((p_b-p_{2-4})^2-t_3\right)\delta\left((p_b-p_4)^2-u_3\right)p_4^\mu$$
$$=(\alpha p_{1-3}+\beta p_{2-3}+\gamma p_b)^\mu\times I$$

√

4 delta fix p4 ➪ $$p_4=\alpha p_{1-3}+\beta p_{2-3}+\gamma p_b$$ [Kumar, 69'] X

The key point: the 4 delta don't fix one p4 point, but two (two solutions).

$$p_4(\mathrm{I})+p_4(\mathrm{II})=\alpha p_{1-3}+\beta p_{2-3}+\gamma p_b$$

√

Results

Results

One example (dominant)

$$\left|\langle d\ell^+\ell^- u\bar{u}|P^u_{1,2}|b\rangle\right|^2$$

First, I show the results before phase space integration (without expression of p3*p4 substituted).

```
1/s3^2 8 e^4 QL^2 (-(Qu^2 (1 - u1) (s2 t2 (-s3 + t2 - t3) - s3 t2 t3 + 2 t2^2 t3 - 2 t2 t3^2 + 2 p3p4 t2 (s3 - t2 + 2 t3) + s3^2 u1 - s3 t2 u1 + 2 s3 t3 u1 - 2 t2 t3 u1 + 2 t3^2 u1))/t2^2 - (Qd^2 (1 - u1) (-8 p3p4^2 (s2 - u1) + (s2 - s3) (s2 (t2 - t3) - s3 u1) + p3p4 (4 s2^2 + 4 s3 u1 - 2 s2 (s3 + t2 - 2 t3 + 2 u1))))/s2^2 -
1/(s2 t2) Qd Qu (1 - u1) (8 p3p4^2 (s3 - t2) + s2^2 (t2 - t3) + s2 (t2^2 + t2 (t3 - 2 u1) + (s3 - 2 t3) (t3 - u1)) - 2 p3p4 (s2 (s3 - t2 - 2 t3) + (t2 - 2 t3) (t2 - 2 u1) + s3 (-t2 + 4 t3 + 2 u1)) - s3 (t2 (t3 + u1) - 2 (t3^2 - t3 u1 + u1^2)))) + 1/(-1 + s1 - s3 + u1 + u2)^2
Qd^2 (s2 - s3 + t2 - u1) (4 + 6 p3p4 - 2 s1 + 3 s2 - 2 s3 - s1^2 s3 + s1 s2 s3 + 6 t2 + s1^2 t2 + 3 s3 t2 - 3 t3 - s1^2 t3 - 3 s3 t3 + s1 s3 t3 - 6 u1 - s2 u1 + 2 s3 u1 - 3 s1 s3 u1 + s2 s3 u1 - 5 t2 u1 + 2 s1 t2 u1 + 4 t3 u1 - 2 s1 t3 u1 + s3 t3 u1 + 2 u1^2 - 2 s3 u1^2 + t2 u1^2 - t3 u1^2 - 2 u2 - s2 u2 + 3 s3 u2 - 2 s1 s3 u2 + s2 s3 u2 - 5 t2 u2 +
  2 s1 t2 u2 + 4 t3 u2 - 2 s1 t3 u2 + s3 t3 u2 + 2 u1 u2 - 3 s3 u1 u2 + 2 t2 u1 u2 - 2 t3 u1 u2 - s3 u2^2 + t2 u2^2 - t3 u2^2 - 6 u3 - 5 s2 u3 + s1 s2 u3 + 6 s3 u3 - 2 s1 s3 u3 + s2 s3 u3 - 7 t2 u3 + 3 s1 t2 u3 - s3 t2 u3 + 2 t3 u3 - 2 s1 t3 u3 + 2 s3 t3 u3 + 8 u1 u3 - 2 s1 u1 u3 + s2 u1 u3 - 2 s3 u1 u3 + 3 t2 u1 u3 - 2 t3 u1 u3 -
  2 u1^2 u3 + 2 u2 u3 + s2 u2 u3 - 2 s3 u2 u3 + 3 t2 u2 u3 - 2 t3 u2 u3 - 2 u1 u2 u3 + 2 u3^2 + 2 s2 u3^2 - 2 s3 u3^2 + 2 t2 u3^2 - 2 u1 u3^2 + s1 (-s2 + 3 s3 - 5 t2 + 4 t3 + 2 u1 + 2 u3) + 2 p3p4 (s1 - s3 + u1 + u2) (s1 + u1 + u2 + 2 u3) - 2 p3p4 (4 s1 - 3 s3 + 4 u1 + 4 u2 + 2 u3)) + 1/(-1 + s1 - s2 + s3 - t2 + u1 + u2)^2
Qu^2 (s2 - s3 + t2 - u1) (4 - 6 p3p4 - 2 s1 + 5 s2 + s2^2 - 4 s3 - s1^2 s3 - s2 s3 + s1 s2 s3 + 2 t2 + s1^2 t2 - s2 t2 - s1 s2 t2 + 2 s3 t2 - 2 t2^2 + s1 t2^2 + 3 t3 - s1^2 t3 + 3 s2 t3 + s1 s2 t3 - 3 s3 t3 + s1 s3 t3 + 3 t2 t3 - 3 s1 t2 t3 + 2 s1 t3^2 - 6 u1 - 3 s2 u1 + 2 s3 u1 - s1 s3 u1 + t2 u1 + s2 t2 u1 - s3 t2 u1 +
  t2^2 u1 - 4 t3 u1 - s2 t3 u1 + s3 t3 u1 - t2 t3 u1 + 2 u1^2 - t2 u1^2 + t3 u1^2 - 2 u2 - s2 u2 + 3 s3 u2 - 2 s1 s3 u2 + s2 s3 u2 - 3 t2 u2 + 2 s1 t2 u2 - s2 t2 u2 + t2^2 u2 + 2 t3 u2 - 2 s1 t3 u2 + s2 t3 u2 + s3 t3 u2 - 3 t2 t3 u2 + 2 t3^2 u2 + 2 u1 u2 - s3 u1 u2 - s3 u2^2 + t2 u2^2 - t3 u2^2 + 8 p3p4^2 (s1 + u2) - 6 u3 - 7 s2 u3 +
  s1 s2 u3 - s2^2 u3 + 6 s3 u3 - 2 s1 s3 u3 + s2 s3 u3 - 5 t2 u3 + 3 s1 t2 u3 - s3 t2 u3 + t2^2 u3 - 2 t3 u3 - 2 s1 t3 u3 - 2 s2 t3 u3 + 2 s3 t3 u3 - 2 t2 t3 u3 + 8 u1 u3 - 2 s1 u1 u3 + 3 s2 u1 u3 - 2 s3 u1 u3 + t2 u1 u3 + 2 t3 u1 u3 - 2 u1^2 u3 + 2 u2 u3 + s2 u2 u3 - 2 s3 u2 u3 + 3 t2 u2 u3 - 2 t3 u2 u3 - 2 u1 u2 u3 + 2 u3^2 +
  2 s2 u3^2 - 2 s3 u3^2 + 2 t2 u3^2 - 2 u1 u3^2 + s1 (-s2 + 3 s3 - 3 t2 + 2 t3 + 2 u1 + 2 u3) + p3p4 (-4 s1 - 6 s2 + 6 s3 - 6 t2 + 8 u1 - 4 u2 + 4 u3) + 2 p3p4 (s1^2 - s3 u1 + t2 u1 - u1^2 - s3 u2 + 3 t2 u2 - 4 t3 u2 + u2^2 - s1 (s2 + s3 - 3 t2 + 4 t3 - 2 u2 - 2 u3) - 2 s3 u3 + 2 t2 u3 - 2 u1 u3 + 2 u2 u3 + s2 (u1 - u2 + 2 u3))) -
1/(t2 (1 - s1 + s3 - u1 - u2)) Qd Qu (s3^2 + 6 p3p4 (s3 - t2) - 8 p3p4^2 t2 - s2 t2 + s2^2 t2 - s1 s3 t2 + s1 s2 s3 t2 - 4 t2^2 + 2 s1 t2^2 - s1 s2 t2^2 + s3 t2^2 - 2 t2^3 + s1 t2^3 + 3 s2 t3 + s1 s3 t3 + s3^2 t3 + 6 t2 t3 - 2 s1 t2 t3 + s1 s2 t2 t3 - 5 s3 t2 t3 + s1 s3 t2 t3 + 5 t2^2 t3 - 3 s1 t2^2 t3 + 2 s3 t3^2 -
  2 t2 t3^2 + 2 s1 t2 t3^2 + s3 (2 t2 - 3 t3 - 2 u1) + s1 s2 s3 u1 - s2^2 s3 u1 + 4 s3^2 u1 - 2 s1 s3^2 u1 + s2 s3^2 u1 + 4 t2 u1 - 2 s1 t2 u1 - s1 s2 t2 u1 - 9 s3 t2 u1 + 3 s1 s3 t2 u1 + 8 t2^2 u1 - 3 s1 t2^2 u1 - s2 t2^2 u1 - s3 t2^2 u1 + t2^3 u1 - 6 t3 u1 + 2 s1 t3 u1 + s1 s2 t3 u1 + 8 s3 t3 u1 - 3 s1 s3 t3 u1 - s2 s3 t3 u1 +
  s3^2 t3 u1 - 13 t2 t3 u1 + 5 s1 t2 t3 u1 + s2 t2 t3 u1 + s3 t2 t3 u1 - 3 t2^2 t3 u1 + 2 t3^2 u1 - 2 s1 t3^2 u1 + 2 t2 t3^2 u1 + 5 s3 u1^2 - s1 s3 u1^2 + 3 s2 s3 u1^2 - 3 s3^2 u1^2 - 6 t2 u1^2 + 2 s1 t2 u1^2 - s2 t2 u1^2 + 6 s3 t2 u1^2 - 3 t2^2 u1^2 + 8 t3 u1^2 - 2 s1 t3 u1^2 + s2 t3 u1^2 - 3 s3 t3 u1^2 + 5 t2 t3 u1^2 - 2 t3^2 u1^2 -
  3 s3 u1^3 + 2 t2 u1^3 - 2 t3 u1^3 - s2^2 s3 u2 - 2 s3 t2 u2 + s2 s3 t2 u2 + 2 t2^2 u2 - s2 t2^2 u2 - s3 t2^2 u2 + t2^3 u2 + s3 t3 u2 - 2 s2 s3 t3 u2 - 2 t2 t3 u2 + s2 t2 t3 u2 + 3 s3 t2 t3 u2 - 3 t2^2 t3 u2 - 2 s3 t3^2 u2 + 2 t2 t3^2 u2 + 2 s3 u1 u2 + 3 s2 s3 u1 u2 - 2 s3^2 u1 u2 - 2 t2 u1 u2 - s2 t2 u1 u2 + 5 s3 t2 u1 u2 -
  3 t2^2 u1 u2 + 2 t3 u1 u2 + s2 t3 u1 u2 - 3 s3 t3 u1 u2 + 5 t2 t3 u1 u2 - 2 t3^2 u1 u2 - 3 s3 u1^2 u2 + 2 t2 u1^2 u2 - 2 t3 u1^2 u2 + 8 p3p4^2 (s1 t2 - (s3 - t2) (u1 + u2)) - s2^2 t2 u3 - 2 s3 t2 u3 + s2 s3 t2 u3 + 2 t2^2 u3 - s3 t2^2 u3 + t2^3 u3 + 2 s3 t3 u3 - 4 t2 t3 u3 - 2 s2 t2 t3 u3 + 2 s3 t2 t3 u3 - 2 t2^2 t3 u3 +
  2 s3 u1 u3 + 2 s2 s3 u1 u3 - 2 s3^2 u1 u3 - 2 t2 u1 u3 - s2 t2 u1 u3 + 4 s3 t2 u1 u3 - 3 t2^2 u1 u3 + 4 t3 u1 u3 + 4 s2 t3 u1 u3 - 4 s3 t3 u1 u3 + 6 t2 t3 u1 u3 - 2 s3 u1^2 u3 + 2 t2 u1^2 u3 - 4 t3 u1^2 u3 + s2 (s3^2 + t2^2 + s1 (t2 - t3) + t2 t3 + 3 t2 u1 - 6 t3 u1 + t2 u2 - t3 u2 - s3 (3 t2 - t3 + 4 u1 + u2) - 2 t3 u3) -
  2 p3p4 (s3^2 u1 + 2 s3 t2 u1 - 3 t2^2 u1 - 2 s3 t3 u1 + 4 t2 t3 u1 - s3 u1^2 + 2 t2 u1^2 - 2 t3 u1^2 + s1 (s2 t2 + s3 t2 - 3 t2^2 + 4 t2 t3 - s3 u1 + 2 t2 u1 - 2 t3 u1) + 3 s3 t2 u2 - 3 t2^2 u2 - 4 s3 t3 u2 + 4 t2 t3 u2 - s3 u1 u2 + 2 t2 u1 u2 - 2 t3 u1 u2 + s2 (-2 s3 (u1 + u2) + t2 (u1 + u2 - 2 u3)) + 2 s3 t2 u3 - 2 t2^2 u3 + 2 t2 u1 u3) -
  2 p3p4 (s3^2 + s1 (s3 - t2) + s2 t2 + 5 t2^2 - 4 t2 t3 - 5 t2 u1 + 2 t3 u1 - t2 u2 - 2 t2 u3 + s3 (-4 t2 + 2 t3 + 2 u1 + u2 + 2 u3))) +
(Qd Qu (-s2 + s3 - t2 + u1) (8 - 8 p3p4^2 + s2^2 - 2 s1^2 s3 - s2 s3 + 2 s1 s2 s3 + 2 s1^2 t2 + s2 t2 - s1 s2 t2 + 5 s3 t2 - 2 t2^2 + s1 t2^2 - 2 s1^2 t3 + s2 t3 + s1 s2 t3 - 6 s3 t3 + 2 s1 s3 t3 + 5 t2 t3 - 3 s1 t2 t3 - 2 t3^2 + 2 s1 t3^2 - 4 s2 u1 + 2 s3 u1 - 4 s1 s3 u1 + s2 s3 u1 - 4 t2 u1 + 2 s1 t2 u1 -
    s2 t2 u1 - s3 t2 u1 + t2^2 u1 - 2 s1 t3 u1 + s2 t3 u1 + 2 s3 t3 u1 - 3 t2 t3 u1 + 2 t3^2 u1 + 4 u1^2 - 2 s2 u2 + 6 s3 u2 - 4 s1 s3 u2 + 2 s2 s3 u2 - 8 t2 u2 + 4 s1 t2 u2 - s2 t2 u2 + t2^2 u2 + 6 t3 u2 - 4 s1 t3 u2 + s2 t3 u2 + 2 s3 t3 u2 - 3 t2 t3 u2 + 2 t3^2 u2 + 4 u1 u2 - 4 s3 u1 u2 + 2 t2 u1 u2 -
    2 t3 u1 u2 - 2 s3 u2^2 + 2 t2 u2^2 - 2 t3 u2^2 + 8 p3p4^2 (s1 + u1 + u2) - 2 p3p4 (6 s1 + s2 - 6 s3 + 5 t2 - 4 t3 + 6 u2) - 12 s2 u3 + 2 s1 s2 u3 - s2^2 u3 + 12 s3 u3 - 4 s1 s3 u3 + 2 s2 s3 u3 - 12 t2 u3 + 6 s1 t2 u3 - 2 s3 t2 u3 + t2^2 u3 - 4 s1 t3 u3 - 2 s2 t3 u3 + 4 s3 t3 u3 - 2 t2 t3 u3 + 16 u1 u3 -
    4 s1 u1 u3 + 4 s2 u1 u3 - 4 s3 u1 u3 + 4 t2 u1 u3 - 4 u1^2 u3 + 4 u2 u3 + 2 s2 u2 u3 - 4 s3 u2 u3 + 6 t2 u2 u3 - 4 t3 u2 u3 - 4 u1 u2 u3 + 4 u3^2 + 4 s2 u3^2 - 4 s3 u3^2 + 4 t2 u3^2 - 4 u1 u3^2 + s1 (-2 s2 + 6 s3 - 8 t2 + 6 t3 + 4 u1 + 4 u3) - 2 (2 s1 - 4 s2 + 3 s3 - 4 t2 + 6 u1 + 2 u2 + 6 u3) +
    p3p4 (4 s1^2 - 4 s3 u1 + 6 t2 u1 - 8 t3 u1 - 4 s3 u2 + 6 t2 u2 - 8 t3 u2 + 4 u1 u2 + 4 u2^2 - 2 s2 (u1 + u2 - 2 u3) - 8 s3 u3 + 4 t2 u3 + 8 u2 u3 + s1 (-2 s2 - 4 s3 + 6 t2 - 8 t3 + 4 u1 + 8 u2 + 8 u3)))) / ((1 - s1 + s3 - u1 - u2) (-1 + s1 - s2 + s3 - t2 + u1 + u2)) + 1/(t2 (-1 + s1 - s2 + s3 - t2 + u1 + u2))
Qu^2 (-8 p3p4^2 s3 + s3^2 + 6 p3p4 (s3 - t2) - s2 t2 - s1 s3 t2 + s1 s2 s3 t2 - 4 t2^2 + 2 s1 t2^2 - s1 s2 t2^2 + s3 t2^2 - 2 t2^3 + s1 t2^3 + 3 s2 t3 + s2^2 t3 + s1 s3 t3 + s3^2 t3 + 6 t2 t3 - 2 s1 t2 t3 + s1 s2 t2 t3 - 2 s3 t2 t3 + s1 s3 t2 t3 + t2^2 t3 - 3 s1 t2^2 t3 + 2 t2 t3^2 + 2 s1 t2 t3^2 +
  s3 (2 t2 - 3 t3 - 2 u1) + s1 s2 s3 u1 - s2^2 s3 u1 + 2 s3^2 u1 - 2 s1 s3^2 u1 + s2 s3^2 u1 + 4 t2 u1 - 2 s1 t2 u1 - s1 s2 t2 u1 + s2^2 t2 u1 - 6 s3 t2 u1 + 3 s1 s3 t2 u1 - 2 s2 s3 t2 u1 + 8 t2^2 u1 - 3 s1 t2^2 u1 + 2 s2 t2^2 u1 - s3 t2^2 u1 + t2^3 u1 - 6 t3 u1 + 2 s1 t3 u1 + s1 s2 t3 u1 - s2^2 t3 u1 +
  6 s3 t3 u1 - 3 s1 s3 t3 u1 + s3^2 t3 u1 - 9 t2 t3 u1 + 5 s1 t2 t3 u1 - 2 s3 t2 t3 u1 + t2^2 t3 u1 - 2 t3^2 u1 - 2 s1 t3^2 u1 - 2 s2 t3^2 u1 + 2 s3 t3^2 u1 - 2 t2 t3^2 u1 + 3 s3 u1^2 - s1 s3 u1^2 + 2 s2 s3 u1^2 - s3^2 u1^2 - 6 t2 u1^2 + 2 s1 t2 u1^2 - 3 s2 t2 u1^2 + 3 s3 t2 u1^2 - 3 t2^2 u1^2 +
  8 t3 u1^2 - 2 s1 t3 u1^2 + 3 s2 t3 u1^2 - s3 t3 u1^2 + t2 t3 u1^2 + 2 t3^2 u1^2 - s3 u1^3 + 2 t2 u1^3 - 2 t3 u1^3 - s2^2 s3 u2 - 2 s3 t2 u2 + s2 s3 t2 u2 + 2 t2^2 u2 - s2 t2^2 u2 - s3 t2^2 u2 + t2^3 u2 + s3 t3 u2 - 2 s2 s3 t3 u2 - 2 t2 t3 u2 + s2 t2 t3 u2 + 3 s3 t2 t3 u2 - 3 t2^2 t3 u2 - 2 s3 t3^2 u2 +
  2 t2 t3^2 u2 + 2 s3 u1 u2 + 3 s2 s3 u1 u2 - 2 s3^2 u1 u2 - 2 t2 u1 u2 - s2 t2 u1 u2 + 5 s3 t2 u1 u2 - 3 t2^2 u1 u2 + 2 t3 u1 u2 + s2 t3 u1 u2 - 3 s3 t3 u1 u2 + 5 t2 t3 u1 u2 - 2 t3^2 u1 u2 - 3 s3 u1^2 u2 + 2 t2 u1^2 u2 - 2 t3 u1^2 u2 + 8 p3p4^2 (s1 t2 + (-s3 + t2) u2) - s2^2 t2 u3 - 2 s3 t2 u3 +
  s2 s3 t2 u3 + 2 t2^2 u3 - s3 t2^2 u3 + t2^3 u3 + 2 s3 t3 u3 - 4 t2 t3 u3 - 2 s2 t2 t3 u3 + 2 s3 t2 t3 u3 - 2 t2^2 t3 u3 + 2 s3 u1 u3 + 2 s2 s3 u1 u3 - 2 s3^2 u1 u3 - 2 t2 u1 u3 - s2 t2 u1 u3 + 4 s3 t2 u1 u3 - 3 t2^2 u1 u3 + 4 t3 u1 u3 + 4 s2 t3 u1 u3 - 4 s3 t3 u1 u3 + 6 t2 t3 u1 u3 - 2 s3 u1^2 u3 +
  2 t2 u1^2 u3 - 4 t3 u1^2 u3 + 2 p3p4 (-s3^2 + s3 t2 - 2 t2^2 + s1 (-s3 + t2) + 2 s3 t3 - 2 t2 t3 + s2 (s3 - 2 (t2 + t3)) + 3 t2 u1 + 2 t3 u1 - s3 u2 + t2 u2 - 2 s3 u3 + 2 t2 u3) + s2 (s3^2 - 2 t2^2 + s1 (t2 - t3) + 2 t2 t3 + 2 t3^2 + 5 t2 u1 - 8 t3 u1 + t2 u2 - t3 u2 - s3 (t2 + 3 u1 + u2) - 2 t3 u3) -
  2 p3p4 (s3^2 u1 - s3 t2 u1 + 2 s3 t3 u1 - 2 t2 t3 u1 + s3 u1^2 + 2 t3 u1^2 + s1 (s2 t2 + s3 t2 - 3 t2^2 + 4 t2 t3 - s3 u1 + 2 t2 u1 - 2 t3 u1) + 3 s3 t2 u2 - 3 t2^2 u2 - 4 s3 t3 u2 + 4 t2 t3 u2 - s3 u1 u2 + 2 t2 u1 u2 - 2 t3 u1 u2 + 2 s3 t2 u3 - 2 t2^2 u3 + 2 t2 u1 u3 - s2 (s3 u1 + 2 t3 u1 + 2 s3 u2 - t2 u2 + 2 t2 u3))) -
1/(s2 (1 - s1 + s2 - s3 + t2 - u1 - u2)) Qd Qu (2 s2^2 + s2^3 - s1 s2^2 t2 + s1 s2 t2^2 - s1 s3 t2^2 + s1 s2^2 t3 - s1 s2 s3 t3 - 3 s1 s2 t2 t3 + 2 s1 s3 t2 t3 + 2 s1 s2 t3^2 - 2 s1 s3 t3^2 - 6 p3p4 (2 s2 - s3 + t2 - 2 u1) + 8 p3p4^2 (s2 + t2 - u1) + s1 s2 s3 u1 + 2 s2^2 t2 u1 + s1 s3 t2 u1 - 3 s2 s3 t2 u1 + s3^2 t2 u1 +
  2 s2 t2^2 u1 - s3 t2^2 u1 - 2 s2^2 t3 u1 + s1 s3 t3 u1 + 3 s2 s3 t3 u1 - s3^2 t3 u1 - 2 s2 t2 t3 u1 + s3 t2 t3 u1 - s1 s3 u1^2 - s2 s3 u1^2 + s3^2 u1^2 - 2 s2 t2 u1^2 + 2 s2 t3 u1^2 - s3 t3 u1^2 + s3 u1^3 - s2 (4 s3 + t2 - 3 t3 + 2 u1) + s3 (s3 - 3 t3 + 4 u1) + s2^2 s3 u2 - s2^2 t2 u2 + s2 t2^2 u2 + s2^2 t3 u2 + s2 s3 t3 u2 - 3 s2 t2 t3 u2 +
  2 s2 t3^2 u2 - s2 s3 u1 u2 - s3 t2 u1 u2 + s3 t3 u1 u2 + s3 u1^2 u2 + 8 p3p4^2 (s1 (s2 - s3 - u1) + u1 (s3 - t2 + u1 - u2) + s2 (-u1 + u2)) - s2^3 u3 + s2^2 s3 u3 - s2 s3 t2 u3 + s2 t2^2 u3 - 2 s2^2 t3 u3 + 2 s2 s3 t3 u3 - 2 s2 t2 t3 u3 + 3 s2^2 u1 u3 - 4 s2 s3 u1 u3 + 2 s3^2 u1 u3 + s2 t2 u1 u3 - 2 s3 t2 u1 u3 + 2 s2 t3 u1 u3 -
  2 s2 u1^2 u3 + 2 s3 u1^2 u3 - s2^2 (2 s3 + 2 t2 - 4 t3 + 3 u1 + 2 u3) + s3 (t2 t3 - 2 t3^2 + s1 (-t2 + t3) + s3 (t2 - t3 - 4 u1) + 3 t2 u1 + 2 t3 u1 - 5 u1^2 + t3 u2 - 2 u1 u2 + 2 t3 u3 - 2 u1 u3) + s2 (-3 t2^2 + s1 (t2 - t3) + 4 t2 t3 + 2 t2 u1 - 5 t3 u1 + 2 u1^2 + t2 u2 - t3 u2 - 2 t3 u3 + 2 u1 u3 + s3 (2 t2 - 5 t3 + 8 u1 + u2 + 2 u3)) +
  2 p3p4 (-5 s2^2 + s3^2 + t2^2 + 2 s3 t3 - 2 t2 t3 + s1 (2 s2 - s3 + t2 - 2 u1) - 8 s3 u1 + 6 t2 u1 + 2 t3 u1 - 8 u1^2 - s3 u2 + t2 u2 - 2 u1 u2 - 2 s3 u3 + 2 t2 u3 - 4 u1 u3 + s2 (4 s3 - 4 t2 - 2 t3 + 13 u1 + 2 u2 + 4 u3)) - 2 p3p4 (s1 (s2^2 + s3 (2 t2 - 4 t3 - u1) + s2 (-s3 - 3 t2 + 4 t3 + u1) + u1 (3 t2 - 2 (t3 + u1))) +
    s2^2 (-3 u1 + u2 - 2 u3) + s2 (-2 t3 u1 + 5 u1^2 + 4 t3 u2 + u1 u2 + 6 u1 u3 + s3 (4 u1 + u2 + 2 u3) - t2 (2 u1 + 3 u2 + 2 u3)) - u1 (s3^2 - t2^2 + t2 (2 t3 - u1 - 3 u2 - 4 u3) + s3 (-2 t3 + 3 u1 + u2 + 4 u3) + 2 (t3 (-u1 + u2) + u1 (u1 + u2 + 2 u3))))) -
1/(s2 (1 - s1 + s3 - u1 - u2)) Qd^2 (2 s2^2 + s2^3 - s1 s2^2 t2 + s1 s2 t2^2 - s1 s3 t2^2 + s1 s2^2 t3 - s1 s2 s3 t3 - 3 s1 s2 t2 t3 + 2 s1 s3 t2 t3 + 2 s1 s2 t3^2 - 2 s1 s3 t3^2 - 6 p3p4 (2 s2 - s3 + t2 - 2 u1) + s1 s2 s3 u1 - s2^2 t2 u1 + s1 s3 t2 u1 - s2 s3 t2 u1 + s3^2 t2 u1 + s2 t2^2 u1 - s3 t2^2 u1 +
  s2^2 t3 u1 + s1 s3 t3 u1 - s3^2 t3 u1 - 3 s2 t2 t3 u1 + 2 s3 t2 t3 u1 + 2 s2 t3^2 u1 - 2 s3 t3^2 u1 - s1 s3 u1^2 + 2 s2 s3 u1^2 - s3^2 u1^2 + s3 t2 u1^2 + s3 t3 u1^2 - s3 u1^3 + 8 p3p4^2 (-s2 + s3 + u1) - s2 (4 s3 + t2 - 3 t3 + 2 u1) + s3 (s3 - 3 t3 + 4 u1) + s2^2 s3 u2 -
  s2^2 t2 u2 + s2 t2^2 u2 + s2^2 t3 u2 + s2 s3 t3 u2 - 3 s2 t2 t3 u2 + 2 s2 t3^2 u2 - s2 s3 u1 u2 - s3 t2 u1 u2 + s3 t3 u1 u2 + s3 u1^2 u2 + 8 p3p4^2 (s1 (s2 - s3 - u1) + (s2 - u1) (u1 + u2)) + s2^2 (-2 s3 + t2 + t3 - 3 u1 - 2 u3) - s2^3 u3 + s2^2 s3 u3 - s2 s3 t2 u3 + s2 t2^2 u3 -
  2 s2^2 t3 u3 + 2 s2 s3 t3 u3 - 2 s2 t2 t3 u3 + 3 s2^2 u1 u3 - 4 s2 s3 u1 u3 + 2 s3^2 u1 u3 + s2 t2 u1 u3 - 2 s3 t2 u1 u3 + 2 s2 t3 u1 u3 - 2 s2 u1^2 u3 + 2 s3 u1^2 u3 + s3 (s1 (-t2 + t3) + s3 (t2 - t3 - 2 u1) + 2 t2 u1 - 3 u1^2 + t3 u2 - 2 u1 u2 + 2 t3 u3 - 2 u1 u3) +
  s2 (-2 t2^2 + s1 (t2 - t3) - 2 s3 t3 - 2 t3^2 + 5 s3 u1 - 3 t3 u1 + 2 u1^2 + s3 u2 - t3 u2 + t2 (5 t3 + u2) + 2 s3 u3 - 2 t3 u3 + 2 u1 u3) + 2 p3p4 (-s2^2 + s3^2 + s3 t2 - 2 s3 t3 + s1 (2 s2 - s3 + t2 - 2 u1) - 6 s3 u1 + 8 t2 u1 - 2 t3 u1 - 8 u1^2 - s3 u2 + t2 u2 - 2 u1 u2 - 2 s3 u3 + 2 t2 u3 - 4 u1 u3 + s2 (s3 - 5 t2 + 4 t3 + 9 u1 + 2 u2 + 4 u3))
  2 p3p4 (s1 (s2^2 + s3 (2 t2 - 4 t3 - u1) + s2 (-s3 - 3 t2 + 4 t3 + u1) + u1 (3 t2 - 2 (t3 + u1))) + s2^2 (u1 + u2 - 2 u3) + s2 (4 t3 u1 + u1^2 + 4 t3 u2 + u1 u2 + 6 u1 u3 + s3 (u1 + u2 + 2 u3) - t2 (3 u1 + 3 u2 + 2 u3)) - u1 (s3^2 + s3 (-t2 + 2 t3 + u1 + u2 + 4 u3) - t2 (3 u1 + 3 u2 + 4 u3) + 2 (t3 (u1 + u2) + u1 (u1 + u2 + 2 u3)))))
```

Results

One example (dominant)

$$\left|\langle d\ell^+\ell^- u\bar{u}|P_{1,2}^u|b\rangle\right|^2$$

First, I show the results before phase space integration (without expression of p3*p4 substituted).

Then, let's have a look at the result after the integration, (with only one variable s, the dilepton mass square, unintegrated.)

$$\Big|\langle d\ell^+\ell^- u\bar{u}|P^u_{1,2}|b\rangle\Big|^2$$

$$\tilde{\alpha}_e^2 Q_L^2 \Bigg[\frac{Q_d Q_u}{81}\Bigg(-108\sqrt{4-s}\sqrt{s}\left(4s^2-7s-6\right) f_1(s)\ln(s) + 108\left(4s^3-15s^2+10\right)[f_1(s)]^2$$
$$+216\sqrt{4-s}\sqrt{s}(4s-1)f_1(s) - 72(s+1)\left(4s^2+5s-5\right) f_2(s) - 9\left(12s^3-45s^2-10\right)\ln^2(s)$$
$$+432 f_3(s) - \frac{(s-1)\left(955s^3-425s^2-749s+87\right)}{s} + \frac{12\left(84s^3-15s^2-56s+3\right)\ln(s)}{s}\Bigg)$$
$$+\frac{Q_d^2}{324}\Bigg(216\sqrt{4-s}\sqrt{s}\left(s^2+2s-12\right) f_1(s)\ln(s) - \frac{216\left(s^4-18s^2+16s+6\right)[f_1(s)]^2}{s}$$
$$+\frac{108\left(4s^3+13s^2-110s-60\right) f_1(s)}{\sqrt{4-s}\sqrt{s}} + \frac{18\left(3s^4-18s^2+16s+6\right)\ln^2(s)}{s}$$
$$+\frac{(s-1)\left(259s^3+43s^2+133s-1863\right)}{s} - \frac{6\left(36s^3+477s^2-172s-120\right)\ln(s)}{s}\Bigg)$$
$$+\frac{Q_u^2}{324}\Bigg(\frac{144(s+1)\left(s^3+5s^2+13s-3\right) f_2(s)}{s} + \frac{72\left(18s^2+10s-3\right)\ln^2(s)}{s} + 1728 f_3(s)$$
$$+\frac{(s-1)\left(259s^3+715s^2+6835s-2901\right)}{s} - \frac{12\left(24s^3+348s^2+148s-111\right)\ln(s)}{s}\Bigg)\Bigg]$$

$$f_1(s) = \frac{\pi}{6} - \arctan\sqrt{s/(4-s)}\,,$$
$$f_2(s) = 2\,\mathrm{Li}_2(-s) - \ln^2(s) + 2\,\ln(s)\ln(1+s) + \zeta_2\,,$$
$$f_3(s) = 4\,\mathrm{Li}_3(-s) - 2\,\mathrm{Li}_2(-s)\ln(s) - \frac{1}{6}\ln^3(s) + \zeta_2\,\ln(s) + 3\zeta_3\,,$$

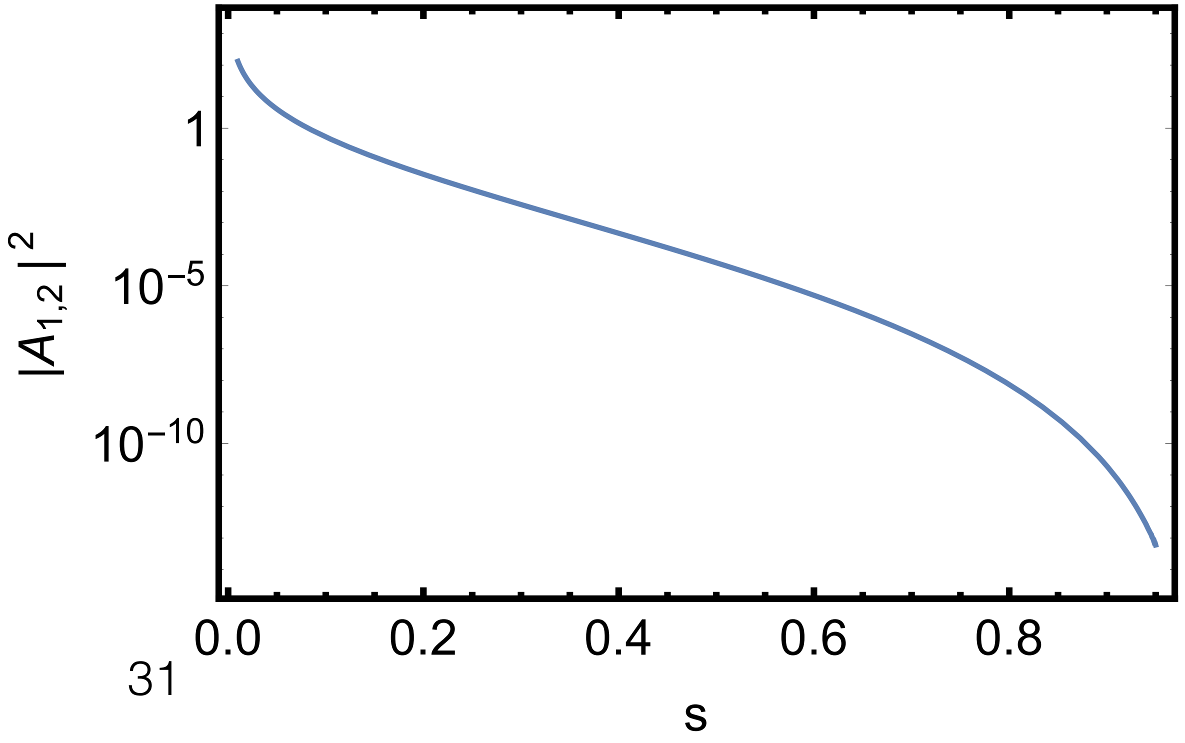
Results

One example (dominant)

$$\left|\langle d\ell^+\ell^- u\bar{u}|P^u_{1,2}|b\rangle\right|^2$$

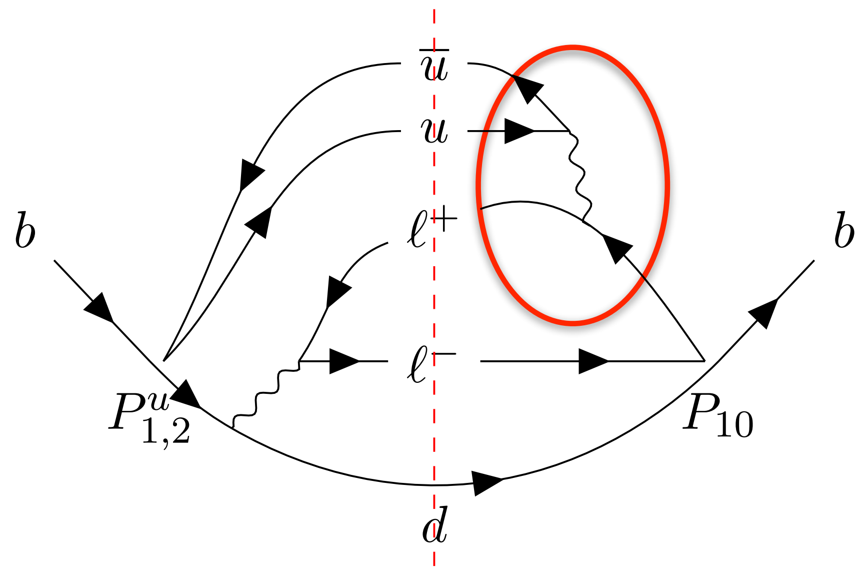
First, I show the results before phase space integration (without expression of p3*p4 substituted).

Then, let's have a look at the result after the integration, (with only one variable s, the dilepton mass square, unintegrated.)



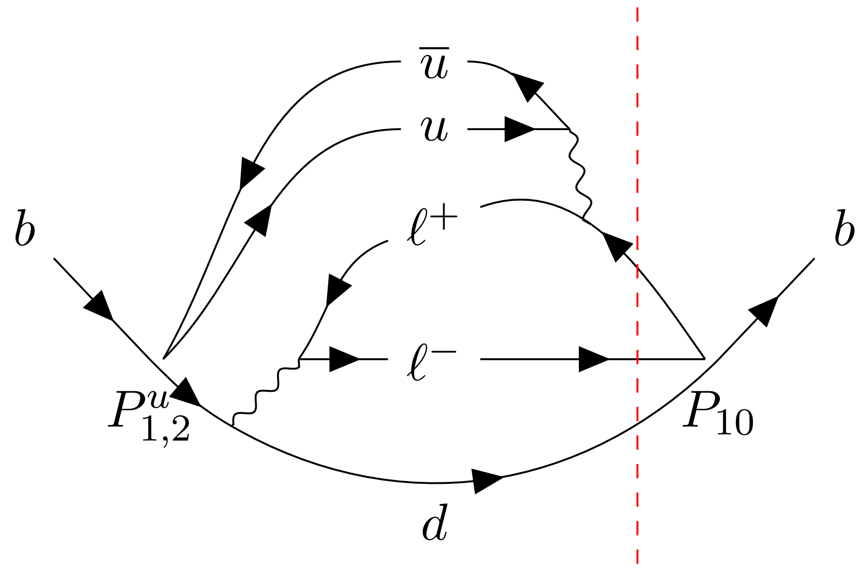
IR divergence

P_{10} interference with others

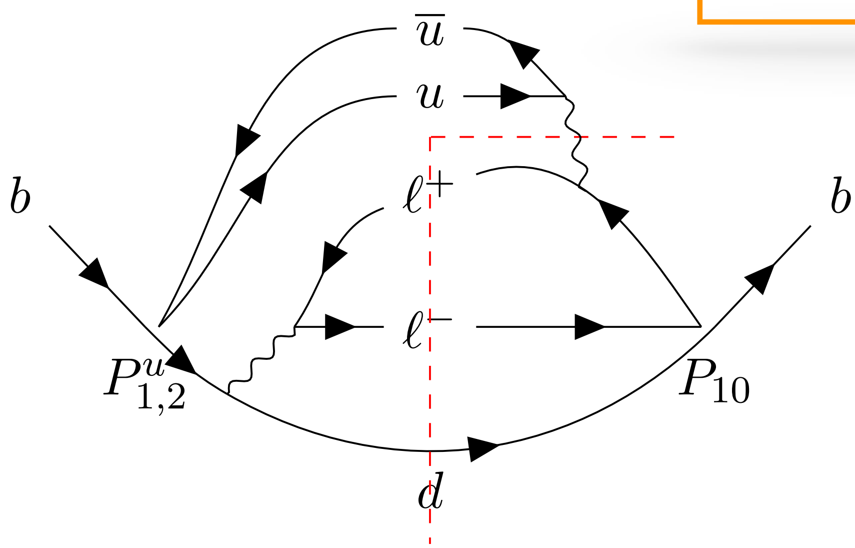


To cancel the divergence

NNLO QED



3 body, 2 loop



4 body, 1 loop

Numerical results (preliminary)

Branching ratio (5 body compared to 3 body)

$M_{\ell\ell}^2$	$[1,\ 6]\ \mathrm{GeV}^2$	$[1,\ 3.5]\ \mathrm{GeV}^2$
$\mathcal{B}(5)\ [10^{-10}]$	9.6	9.3
$\mathcal{B}(3)\ [10^{-8}]$	6.8	3.8
$\mathcal{B}(5/3)$	1.4%	2.4%

Percentage of other contributions

- 1/mb corrections 0.2%
- 1/mc corrections -1%
- log-enhanced QED corrections 5% (e), 2% (muon)

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- Yet IR divergence to be canceled
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Thank you for your attention!