Tools of the Baryonic spectroscopy study

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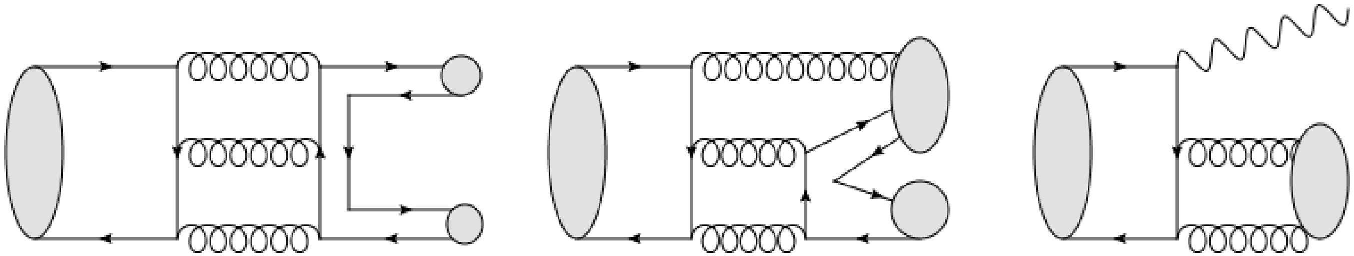
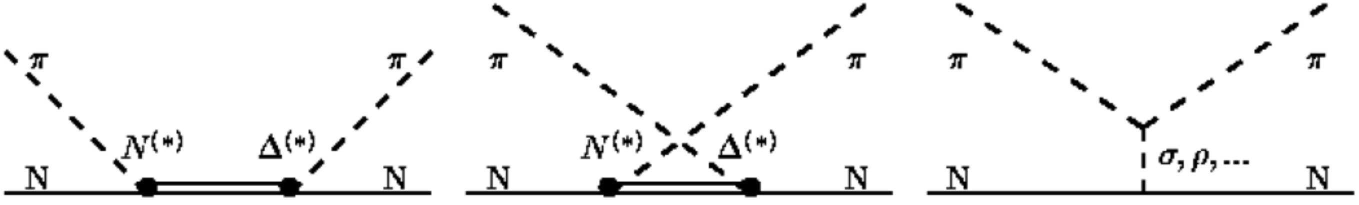
Introduction

- BESIII subjects: hadron spectroscopy and charm physic (~5 billion J/ψ decays)



- Partial wave analysis of πN reaction experiments .

  e.g. GWU group, Yerevan/JLab group(πN), Giessen Group (K-matrix), …

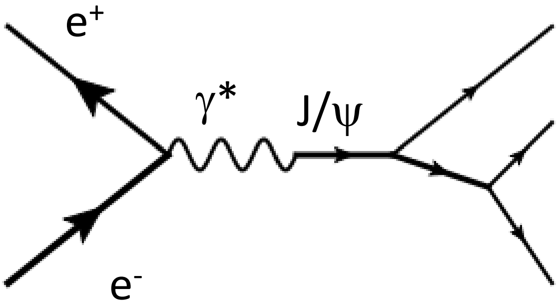


s- 道 u- 道 t- 道

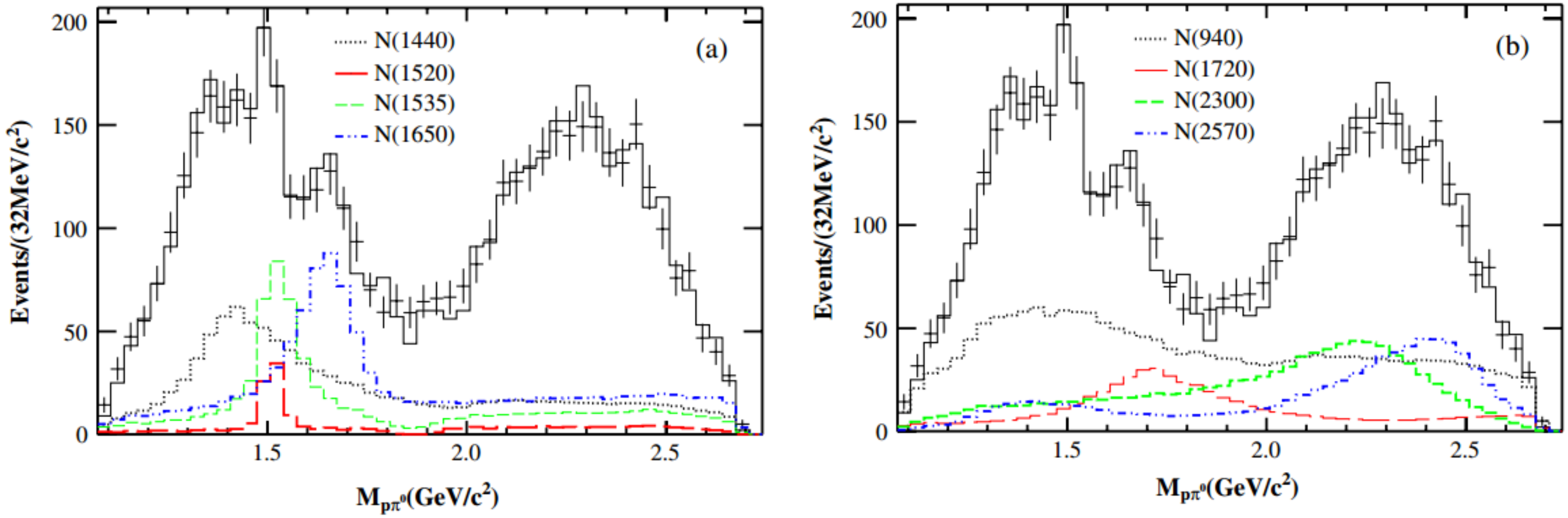
PWA tools at BESIII

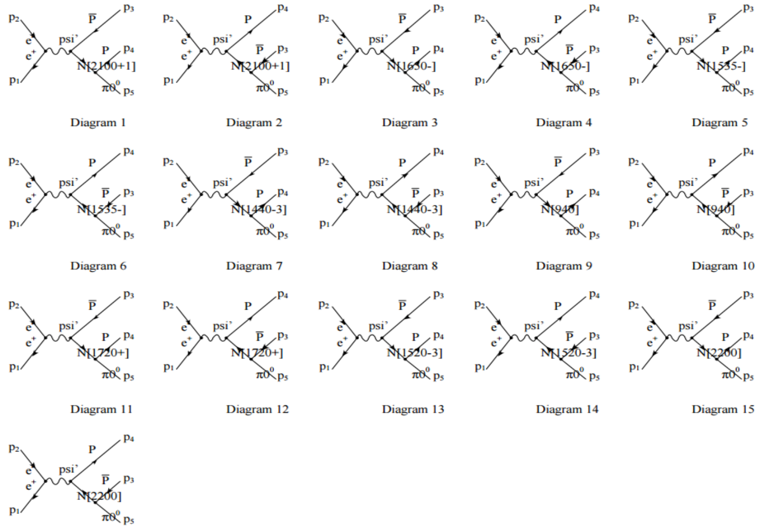
- FDC-PWA [Jian-Xiong Wang, Computer Physics Communications 77 (1993) 263]

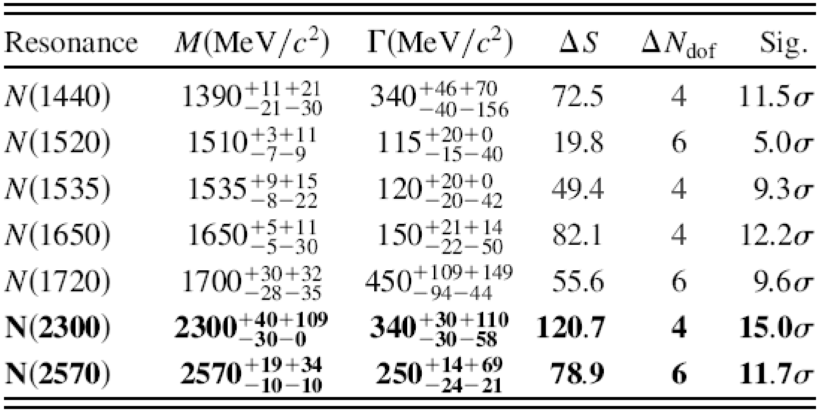
- 在BEPC能区产生了一个包括高自旋态到7/2的有效拉氏量模型及其手册。
- 产生了 J/psi 在BES上所有的 2, 3，4，5，6 体末态衰变过程.
- 被BES实验组用来进行了多个J/Ψ物理过程的拟合和分析，
- Event Generator: 在产生的做分波分析的每一个物理过程的程序中,都有这个过程的事例产生器.其参数需要分波分析拟合或其他方法来定
- FDC-PWA还在发展中，还有更多的内容可以包括在其中。随着更多的物理分析的进行，还将发挥更重要的作用。



$\Psi'\rightarrow p\underline{p}\pi^0$ [BESIII, PRL 110, 022001 (2013)]







Resonance	$M(\mathrm{MeV}/c^2)$	$\Gamma(\mathrm{MeV}/c^2)$	ΔS	ΔN_{dof}	Sig.
$N(1440)$	1390^{+11+21}_{-21-30}	$340^{+46+70}_{-40-156}$	72.5	4	11.5σ
$N(1520)$	1510^{+3+11}_{-7-9}	115^{+20+0}_{-15-40}	19.8	6	5.0σ
$N(1535)$	1535^{+9+15}_{-8-22}	120^{+20+0}_{-20-42}	49.4	4	9.3σ
$N(1650)$	1650^{+5+11}_{-5-30}	150^{+21+14}_{-22-50}	82.1	4	12.2σ
$N(1720)$	1700^{+30+32}_{-28-35}	$450^{+109+149}_{-94-44}$	55.6	6	9.6σ
N(2300)	$\mathbf{2300^{+40+109}_{-30-0}}$	$\mathbf{340^{+30+110}_{-30-58}}$	**120.7**	**4**	$\mathbf{15.0\sigma}$
N(2570)	$\mathbf{2570^{+19+34}_{-10-10}}$	$\mathbf{250^{+14+69}_{-24-21}}$	**78.9**	**6**	$\mathbf{11.7\sigma}$

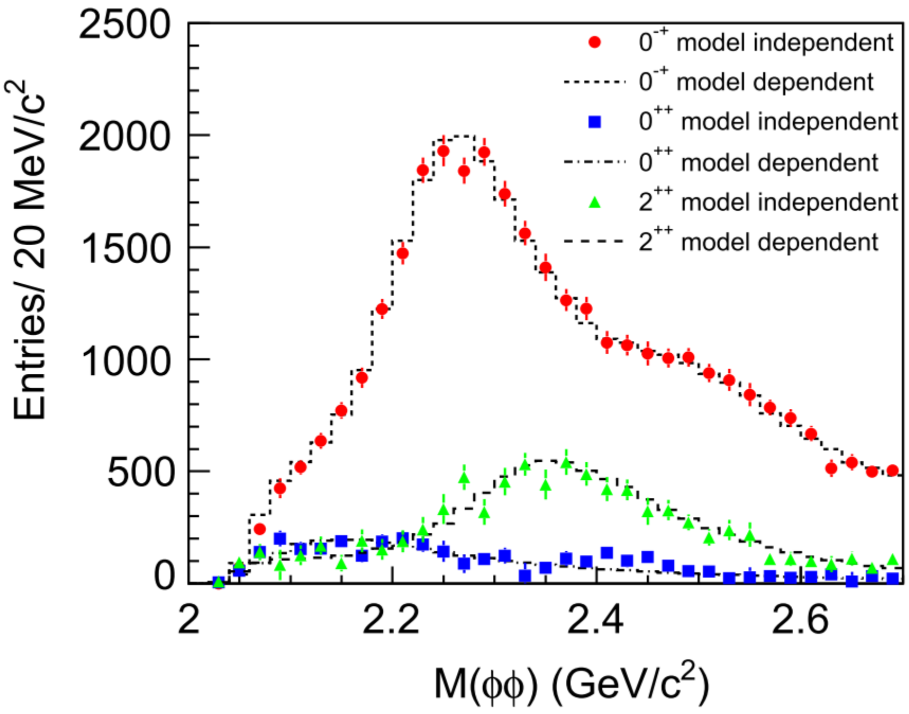
GPUPWA at BESIII

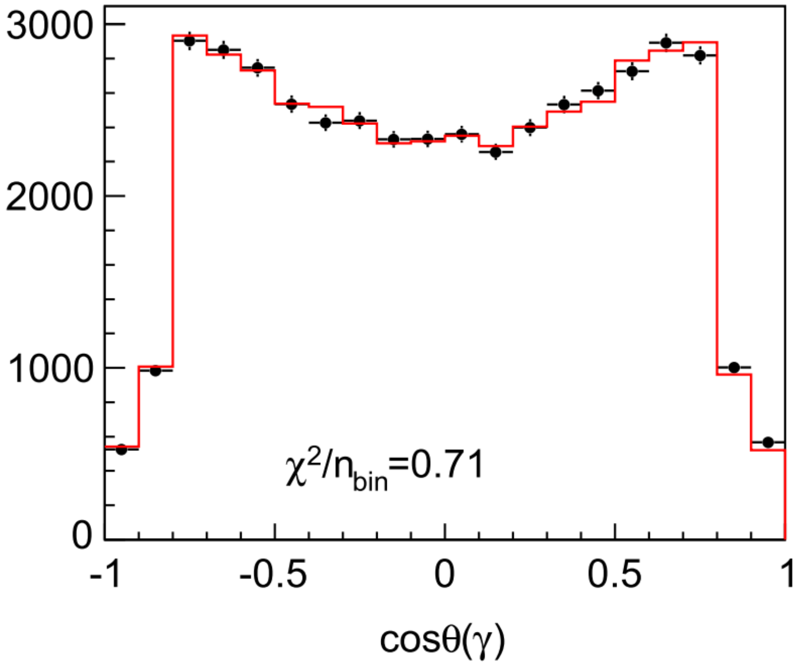
N. Berger, B. J. Liu, and J. K. Wang, J. Phys. Conf. Ser. 219, 042031(2010)

- A framework of PWA using GPU acceleration
- GPUPWA amplitude: the covariant tensor formalism in C++
- Combination of FDC-PWA
- Event-based ML fit to all observables
- Extract resonance properties with interference effects
- Well documented in manual and wiki page

Analyses with GPUPWA

- $J/\psi \to \gamma\phi\phi$ BESIII, PHYSICAL REVIEW D 93, 112011 (2016)





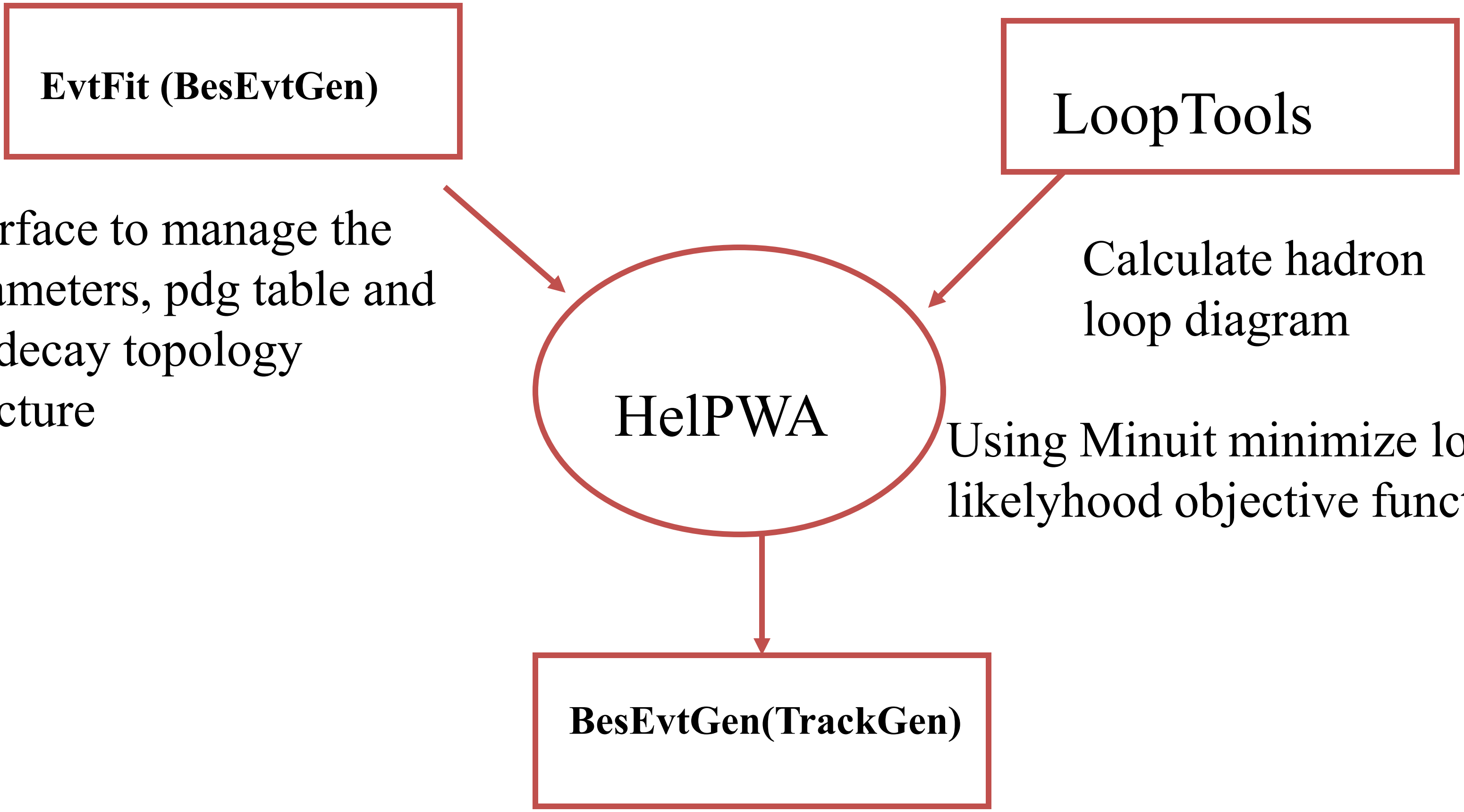
- 58,049 events are selected from 1.3 billion J/ψ decays
- ~10 times of PHSP events used to calculate the normalized factor
- Base line solution with 7 resonance

Helicity amplitude PWA (HelPAW)

- HelPWAAlg package implement the 3-, 4- and 5-body decays
- Amplitudes reconstructed automatically, user only need to provide decay chain and parameter list
- The signal yields and statistical error calculated automatically.
- MC events generated according to the amplitude model after the parameter obtained.
- The multi-thread technique used to accelerate the calculation.
- HelPWAAlg package dependent on the EvtFit (modified EvtGen) package and Looptool.
- Computer resources: 4 nodes, ~100 cores.

HelPWA package structure

The size of complied package is about 300 M



EvtFit (BesEvtGen)

Interface to manage the parameters, pdg table and the decay topology structure

LoopTools

Calculate hadron loop diagram

HelPWA

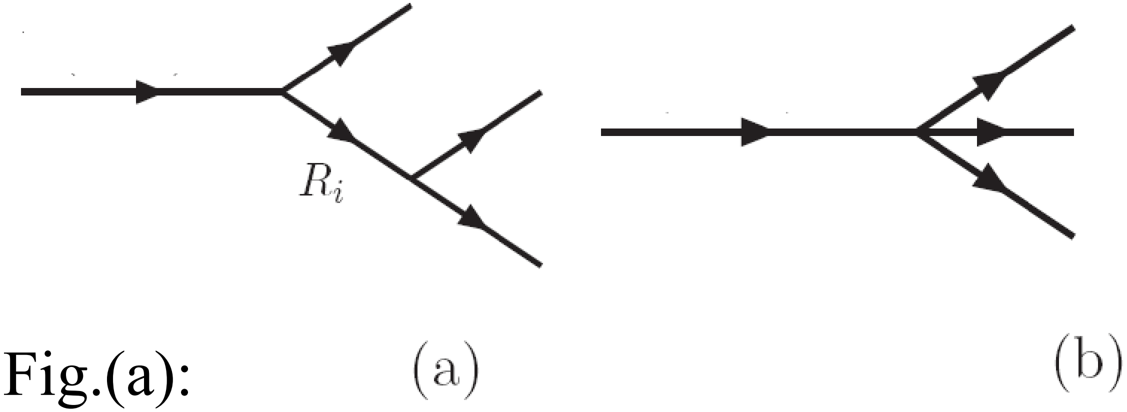
Using Minuit minimize log-likelyhood objective function

BesEvtGen(TrackGen)

Using the fitted amplitude to generate events and detector simulation

Amplitude of 3-body decays

- Topology of decay chain P(m)→$X_1(\lambda_1)X_2(\lambda_2)X_3(\lambda_3)$



(a) (b)

- Amplitudes for Fig.(a):

$$A_1(m) = \sum_{\lambda_R,\lambda'_1,\lambda'_2} F^J_{\lambda_R,\lambda_3} D^{J*}_{m,\lambda_R-\lambda_3}(\phi_0,\theta_0,0) BW(m_{12}) F^R_{\lambda'_1,\lambda'_2} D^{R*}_{\lambda_R,\lambda'_1-\lambda'_2}(\phi_1,\theta_1,0)$$

$$\boxed{\times d_{\lambda'_1,\lambda_1}({}^1\theta_1) d_{\lambda'_2,\lambda_2}({}^2\theta_1),}$$

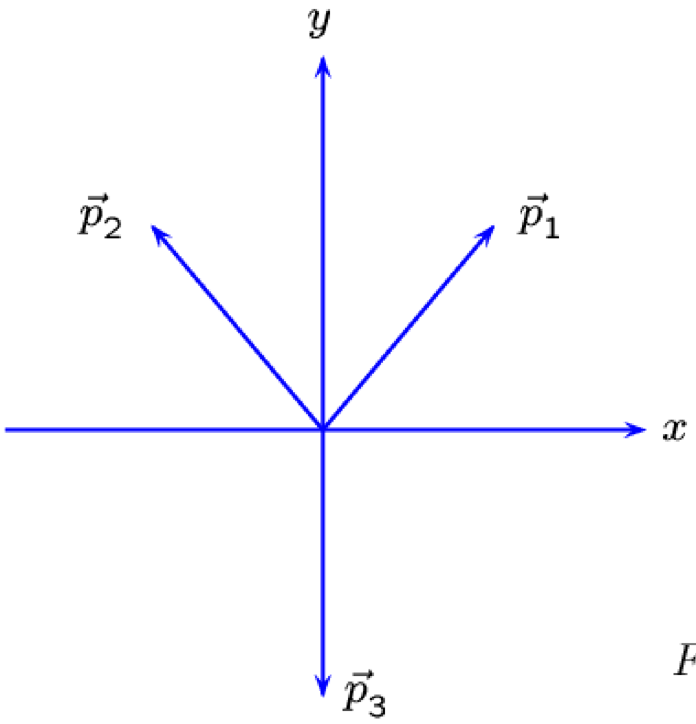
$$A_2(m) = \sum_{\lambda_R,\lambda'_1,\lambda'_3} F^J_{\lambda_R,\lambda_2} D^{J*}_{m,\lambda_R-\lambda_2}(\phi_2,\theta_2,0) BW(m_{13}) F^R_{\lambda'_1,\lambda'_3} D^{R*}_{\lambda_R,\lambda'_1-\lambda'_3}(\phi_3,\theta_3,0)$$

$$\boxed{\times d_{\lambda'_1,\lambda_1}({}^1\theta_3) d_{\lambda'_3,\lambda_3}({}^3\theta_3),}$$

$$A_3(m) = \sum_{\lambda_R,\lambda'_2,\lambda'_3} F^J_{\lambda_R,\lambda_1} D^{J*}_{m,\lambda_R-\lambda_1}(\phi_4,\theta_4,0) BW(m_{23}) F^R_{\lambda'_2,\lambda'_3} D^{R*}_{\lambda_R,\lambda'_2-\lambda'_3}(\phi_5,\theta_5,0)$$

$$\boxed{\times d_{\lambda'_2,\lambda_2}({}^2\theta_5) d_{\lambda'_3,\lambda_3}({}^3\theta_5),}$$

- Helicity for direct 3-body decays



$$A = \frac{N_J}{\sqrt{2\pi}} F_\mu^J(E_i\lambda_i) D_{M\mu}^{J*}(\alpha\beta\gamma)$$

Requirement of parity conservation

$$F_\mu^J(E_i\lambda_i) = \eta\eta_1\eta_2\eta_3(-)^{s_1+s_2+s_3+\mu} F_\mu^j(E_i\,-\lambda_i)$$

Requirement of identical particle symmetry

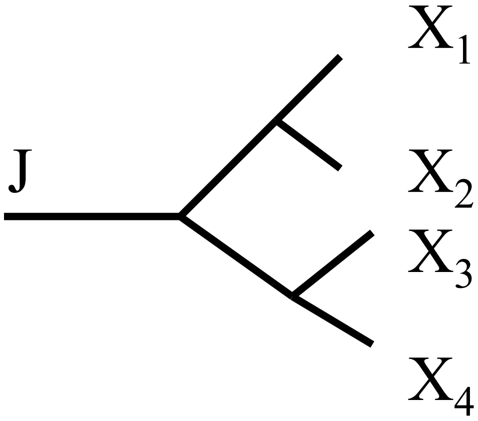
$$F_\mu^J(E_1\lambda_1, E_2\lambda_2, E_3\lambda_3) = \pm(-)^{J+\mu} F_{-\mu}^J(E_2\lambda_2, E_1\lambda_1, E_3\lambda_3)$$

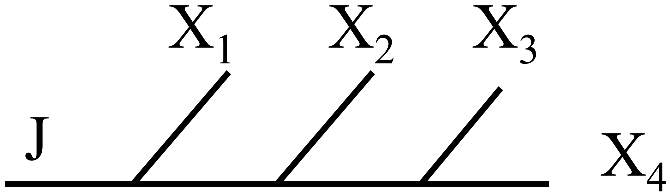
angles (α, β, γ) describe the orientation of the three-particle system

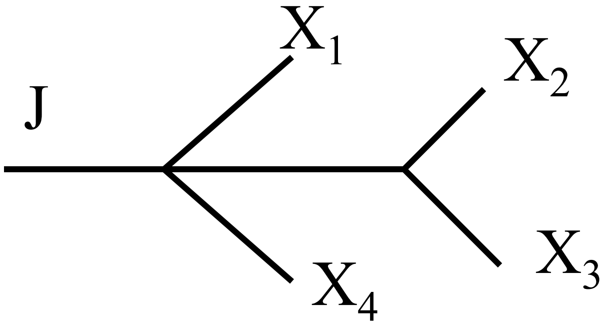
- Covariant tensor amplitudes can be added as required
  - Shifman formula
  - Tensor decays to 3-pseudoscalar

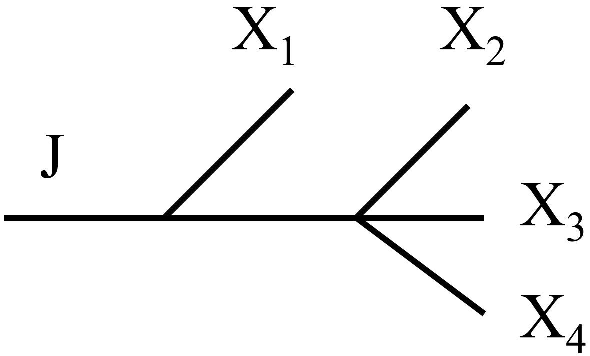
Topology for 4-body decays

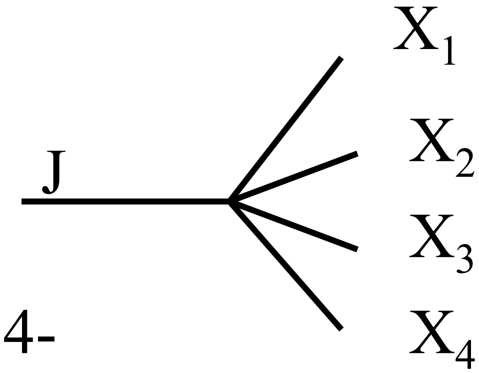
- Topology of decay chain





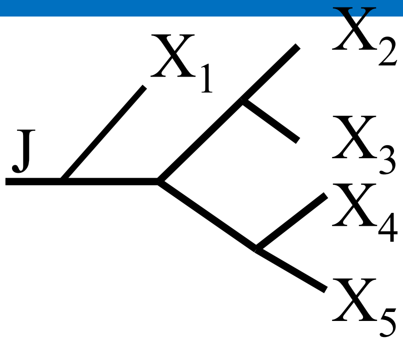


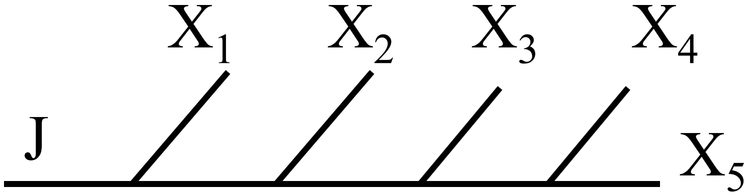


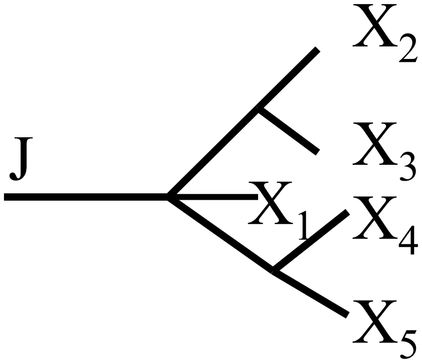


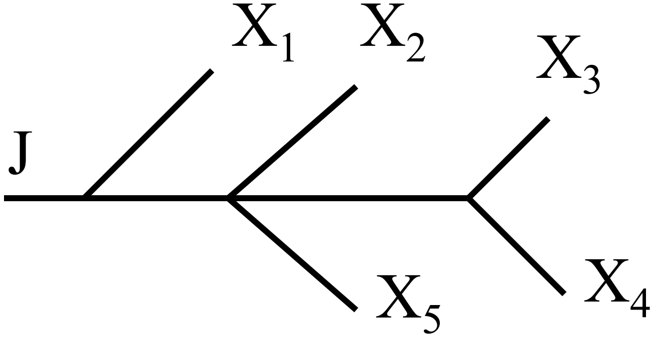
Specified amplitude for the direct 4-body decays need to be constructed.

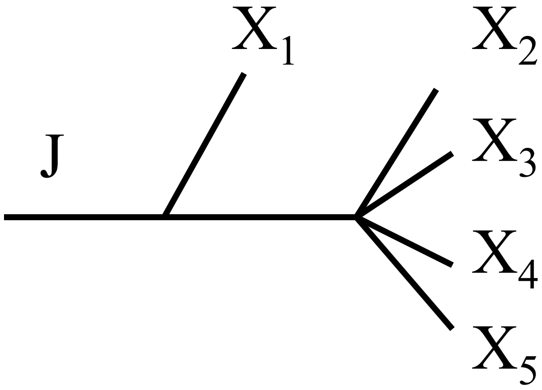
Topology for 5-body decays

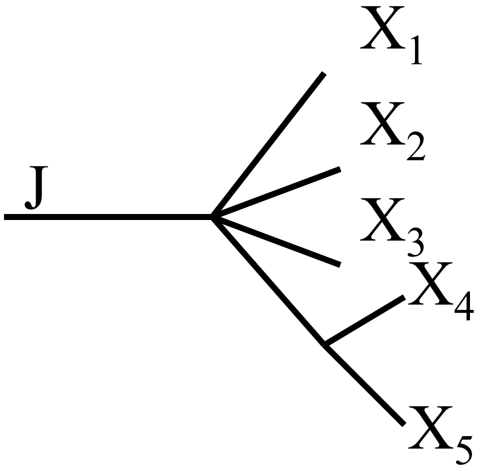


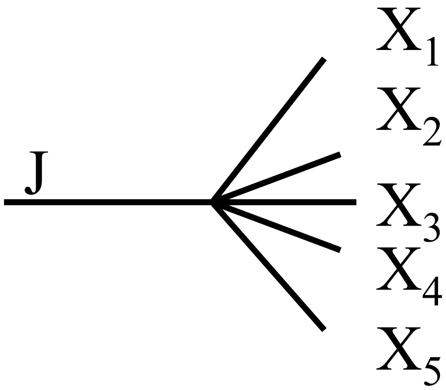








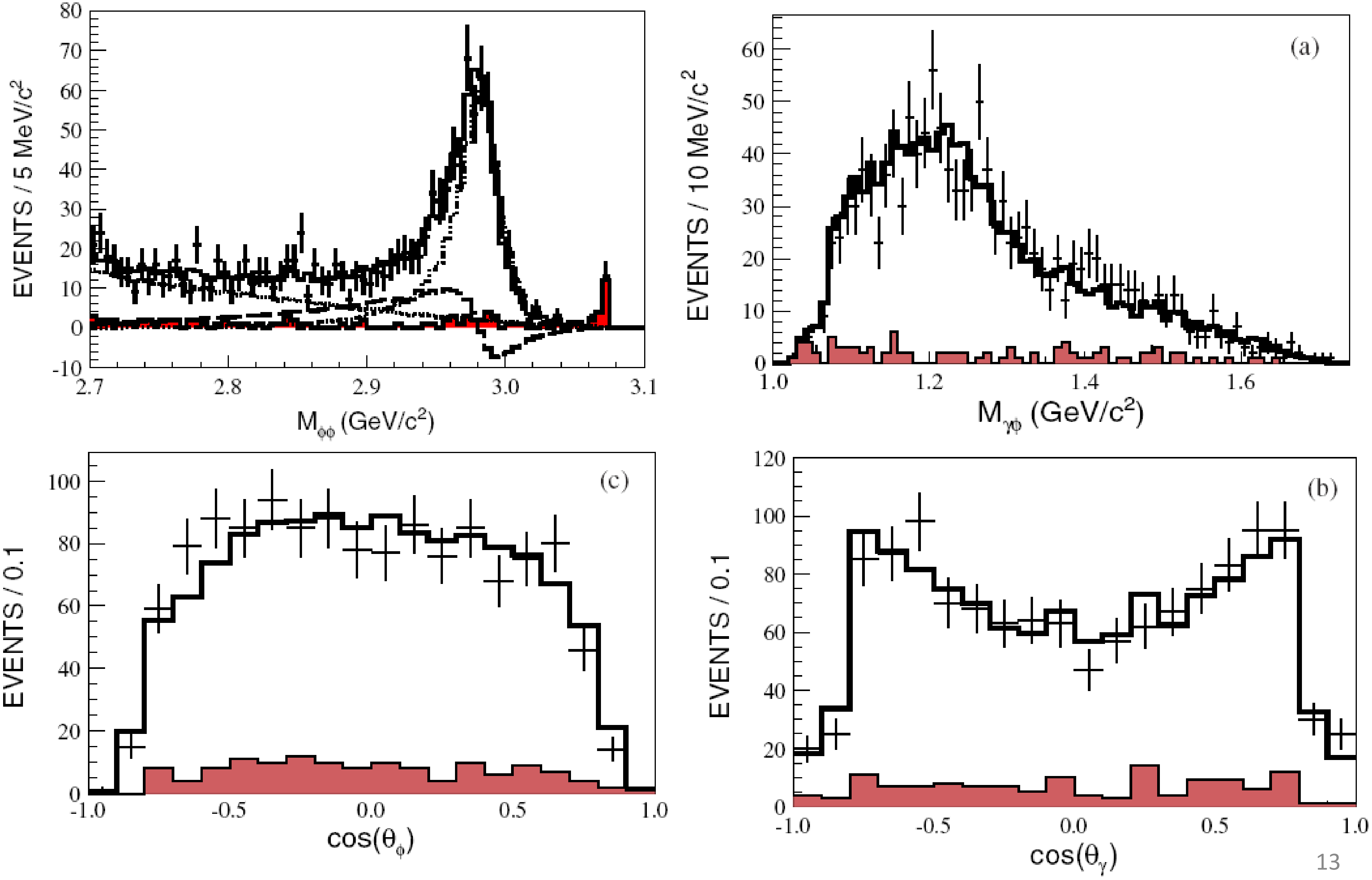




Specified amplitude for the direct 4-, 5-body decays need to be constructed.

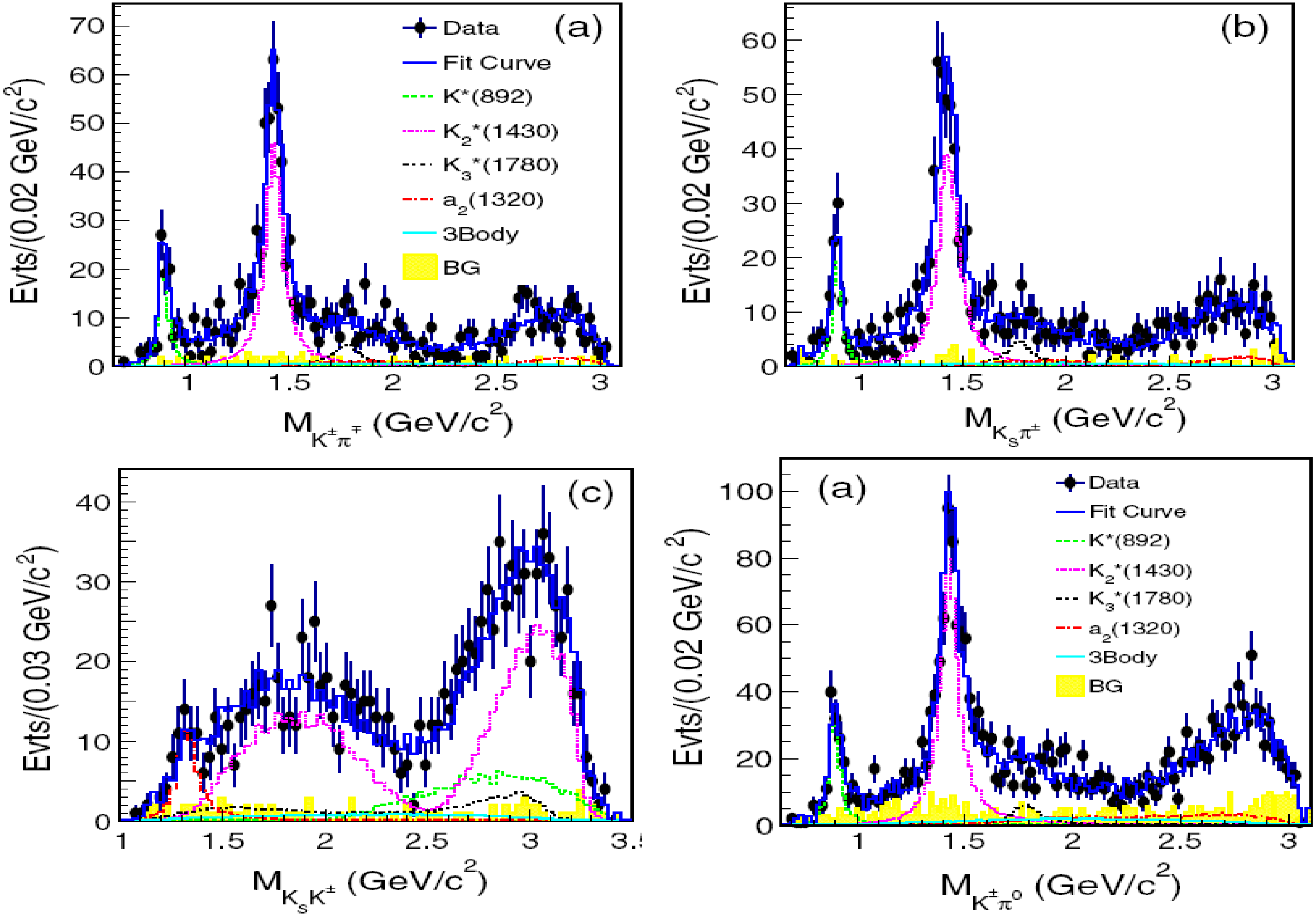
Analyses with HelPWA package

- $J/\psi \rightarrow \gamma\phi\phi$ (BESIII, Phys.Rev., D95, 092004 (2017))



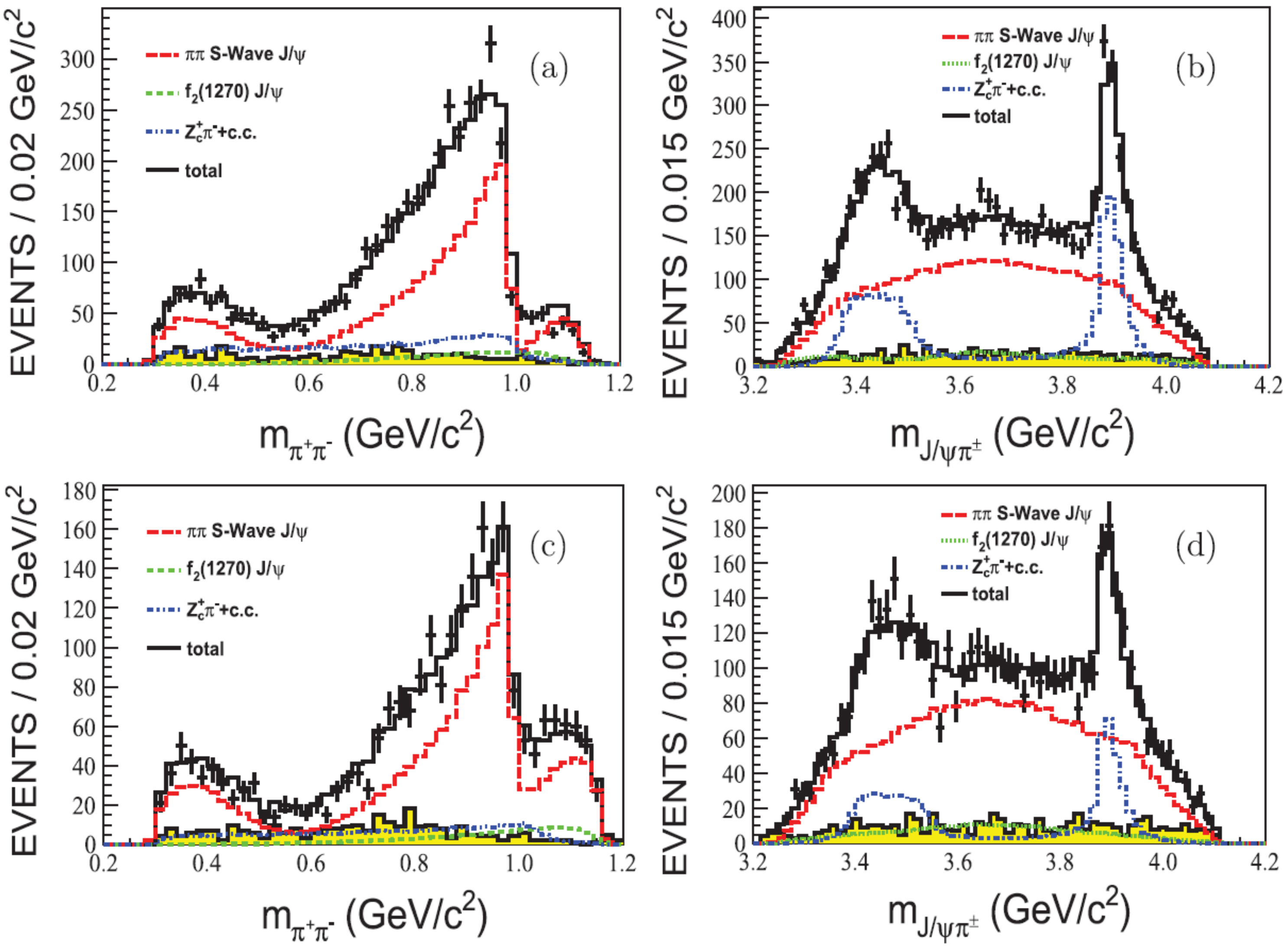
Analyses with HelPWA package

- $\chi_{c2} \rightarrow K_S K^+ \pi^- + c.c., K^+ K^- \pi^0$ +c.c.(BESIII, Phys.Rev., D 96, 111102(R) (2017))



Analyses with HelPWA package

- $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ (BESIII, Phys.Rev.Lett., 119, 119, 072001 (2017))



On Amplitude

- Unitary requirement

For scattering process, $i \to j, S_{ji} = \delta_{ji} - i(2\pi)^4 \delta^4 (P_j\text{-}P_i) T_{ji}$

Unitary equation: $S^{\dagger} S = 1 \quad \Rightarrow$ For L-wave T^L_{ji}, $i(T^L_{ji} - T^{L\dagger}_{ji}) = 2(T^L \rho T^{L\dagger})_{ji}$

- Isobar approximation in PWA amplitude

For example, one channel multi-resonance, $T_i^L = BW_i^L(s, M)$,

amplitude $A = \sum_{i,L} T_i^L$ not meets the unitary requirement

- Single BW meets the unitary condition

On amplitude

- But for multi-BWs in one channel, the unitary is violated at order of max(g_a,g_b)/(M_1^2-M_2^2). If unitary is preserved, it requires the amplitude is taken as

$$
\begin{aligned}
&\text{2 Res: } A(s) = BW_1(s) + BW_2(s) + 2iBW_1(s) * BW_2(s) \\
&\text{3 Res: } A(s) = BW_1(s) + BW_2(s) + BW_3(s) \\
&\qquad + 2i[BW_1(s) * BW_2(s) + BW_2(s) * BW_3(s) + BW_3(s) * BW_1(s)] \\
&\qquad - 4BW_1(s) * BW_2(s) * BW_3(s).
\end{aligned}
$$

Prog. Theo. Phys. 95, 745

- K-matrix guarantees the unitary for multi-resonance.

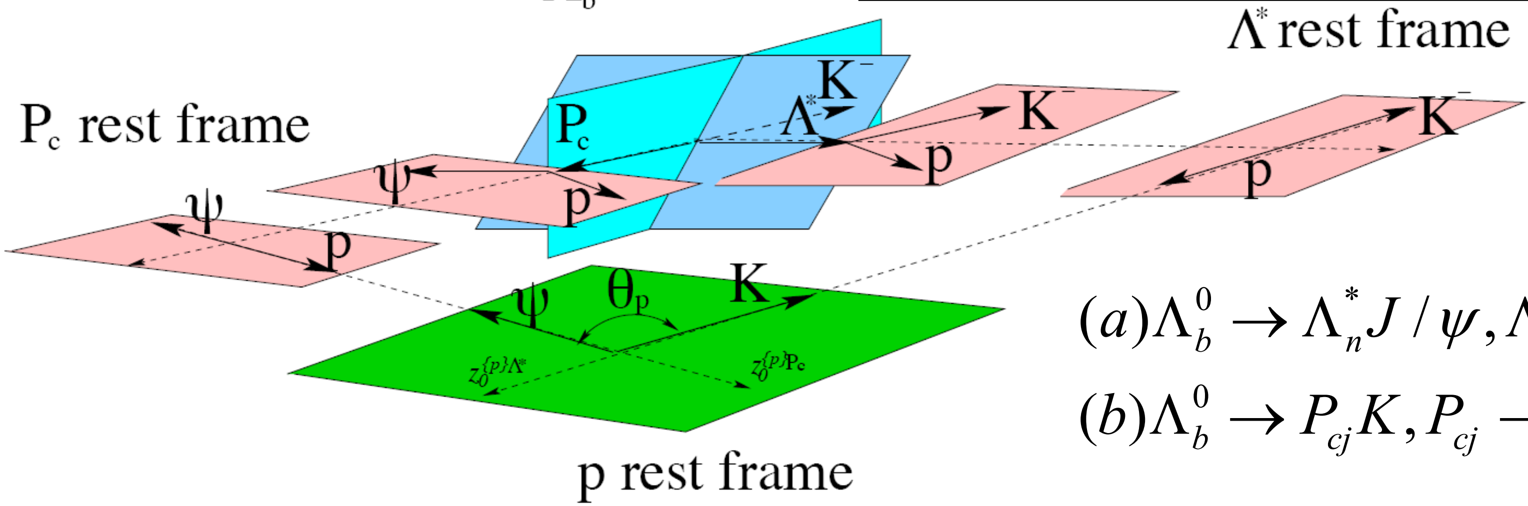
S.U.Chung, J. Brose, R. Hackmann, E. Klernpt'S,. Spanier, and C Strassbrger, Ann. Physik 4 (1995) 404-430

On amplitude

- To coherently sum of helicity amplitudes, introduce additional rotation to align the spin of final states in the same reference system

Belle, Phys. Rev. D 88, 074026 (2013). LHCb, Phys. Rev. Lett. 115, 072001 (2015).
BESIII, Phys.Rev.Lett., 119, 072001 (2017); H.Chen and R.G.Ping, PRD95, 076010 (2017)

$$\boxed{|\lambda_p\rangle = \sum_{\lambda_p^{P_c}} D^{J_p}_{\lambda_p^{P_c},\lambda_p}(0,\theta_p,0)^*|\lambda_p^{P_c}\rangle = \sum_{\lambda_p^{P_c}} d^{J_p}_{\lambda_p^{P_c},\lambda_p}(\theta_p)|\lambda_p^{P_c}\rangle.}$$



$(a)\Lambda_b^0 \to \Lambda_n^* J/\psi, \Lambda_n^* \to Kp$

$(b)\Lambda_b^0 \to P_{cj} K, P_{cj} \to J/\psi\, p$

Figure 9: Definition of the θ_p angle.

PWA techniques

- Likelihood function, Detection efficiency correction

$$P_i(\Omega_i) = \frac{I(\Omega_i)\varepsilon(\Omega_i)}{\int I(\Omega_i)\varepsilon(\Omega_i)d\Omega_i}, \quad \text{with } I(\Omega_i) = \frac{d\sigma}{d\Omega_i}$$

$$L = \prod_{i=1}^{n} P(\Omega_i) = \prod_{i=1}^{n} \frac{I(\Omega_i)\varepsilon(\Omega_i)}{\int I(\Omega_i)\varepsilon(\Omega_i)d\Omega_i}$$

Extended likelihood function

$$L = \frac{e^{-\mu}\mu^n}{n!} \prod_{i=1}^{n} \frac{I(\Omega_i)\varepsilon(\Omega_i)}{\int I(\Omega_i)\varepsilon(\Omega_i)d\Omega_i}$$

PWA techniques

- Minimization tools and Covariance matrix

  CERNLIB: MINUIT (fortran):

  provide MIGRAD, HESSE, and MINOS

  (see CERN Report 81-03, CERN, 1981)

  TMINUIT and TMINUIT2 (C++) in root package

  see http://www.cern.ch/minuit/doc/doc.html.

  FUMILI Minimization Package

- Signal yields

$$N_i = R_i * (N_{\rm obs} - N_{\rm bg}), \ \text{with} \ R_i = \frac{\sigma_i}{\sigma_{\rm tot}},$$

- Statistical error

$$\delta N_i^2 = \sum_{m=1}^{N_{\rm pars}} \sum_{n=1}^{N_{\rm pars}} \left(\frac{\partial N_i}{\partial X_m} \frac{\partial N_i}{\partial X_n} \right)_{\mathbf{X}=\mu} V_{mn}(\mathbf{X}),$$

V: the covariance matrix

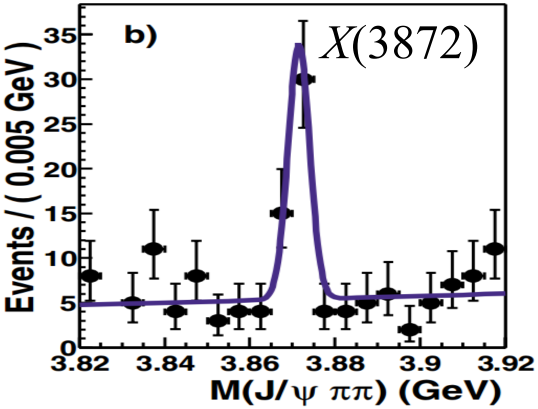
PWA techniques

- Mass resolution correction

➢ mass resolution distribution

$$R(\mathrm{m},\mathrm{mp}) = \mathrm{N}_1 * \frac{\sigma_1}{\pi}\frac{1}{(\mathrm{m}-\mathrm{mp})^2+\sigma_1^{\;2}} + N_2 * \frac{\sigma_2}{\pi}\frac{1}{(\mathrm{m}-\mathrm{mp})^2+\sigma_2^{\;2}}$$

$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$



Belle, Phys.Rev.Lett. 91 (2003) 26200

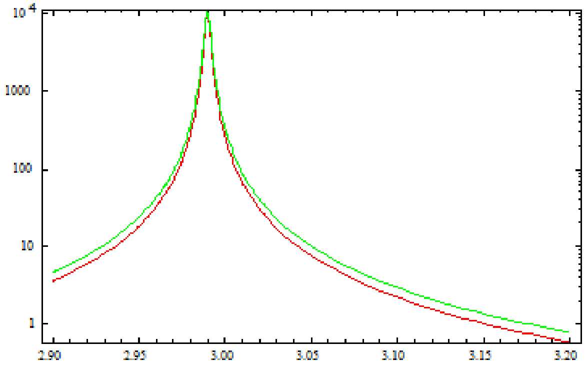
➢ Mass resolution calculation

$$|\,BW'(\mathrm{mp})\,|^2 = \int \left|\frac{1}{m^2 - M_0^{\;2} - I\Gamma M_0}\right|^2 R(m,mp)\,\mathrm{dm}$$

For example:

η_c with resolution (one BW) $\sigma_1 = 4\mathrm{MeV}$

Red: without resolution
Green: with resolution



PWA techniques

☐ MC check on the mass resolution

- MC: $e^+e^- \to \pi^+ Zc(3900)^-, \pi^+ Zc(4020)^- \to \pi^+\pi^- J/\psi$.
- Detector resolution is obtained from MC with zero width.
- MC pseudodata with the widths: $\Gamma_{Zc(3900)} = 46$ MeV, $\Gamma_{Zc(4020)} = 8$ MeV

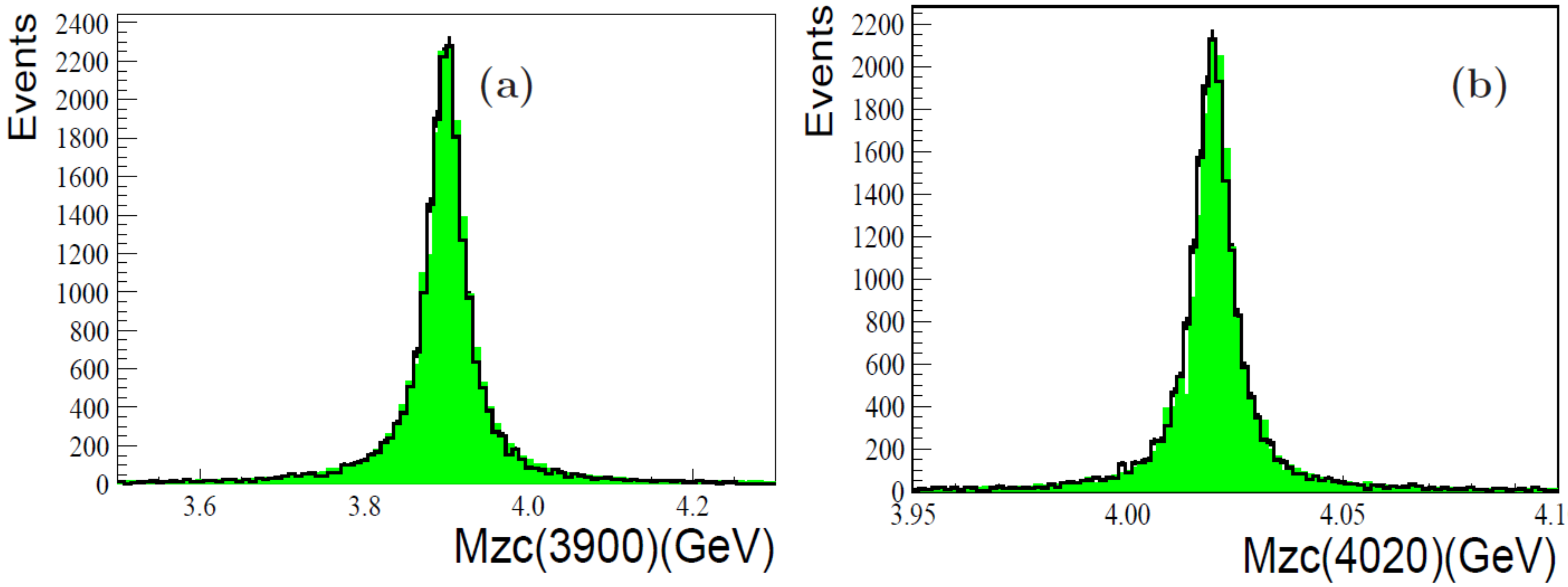


Fig. 10: Comparison of observed Z_c lineshape in the MC simulation (open histogram) and the distributions calculated with the Breit-Wigner function convoluted with the mass resolution functions (shaded histogram). (a): $Z_c(3900)$ lineshape, and (b): $Z_c(4020)$ lineshape.

Extract mass and width

- Width parametrization & Breit-Wigner

$$BW(s)=\frac{-g_a g_b}{s-M_{BW}^2+i\sqrt{s}\Gamma_{tot}(s)}$$

- Far away from threshold, $\Gamma_{tot}(s) \rightarrow \Gamma_{BW}$
- Narrow resonance, $\sqrt{s} \rightarrow M_{BW}$
- Near threshold, to take

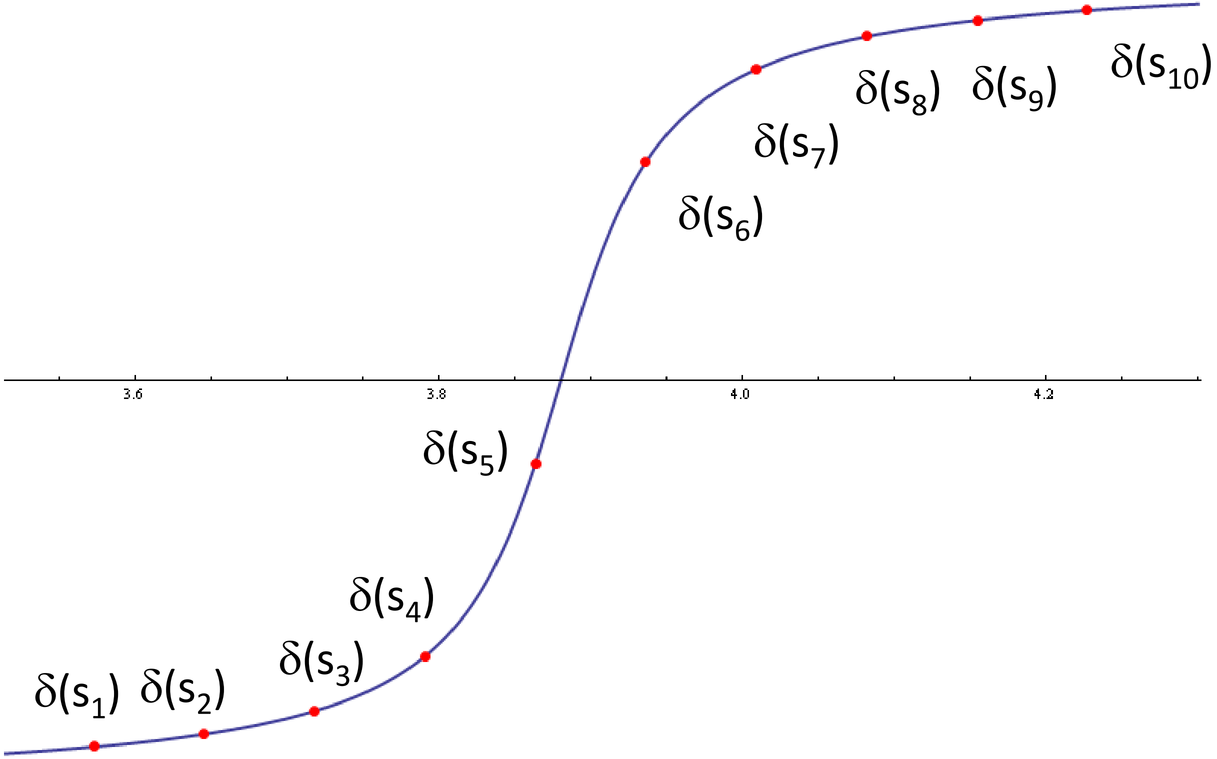
$$\Gamma(s)=\sum_c \Gamma_{R\to c}\left(\frac{q_c}{q_{R_c}}\right)^{2L+1}\left(\frac{F_{Lc}(q_c,q_0)}{F_{Lc}(q_{Rc},q_0)}\right)^2$$

- Flatte formula for $f_0(980)$

$$BW(s)=\frac{1}{s-m_0^2+i[g_1\rho_{\pi\pi}(s)+g_2\rho_{KK}(s)]}$$

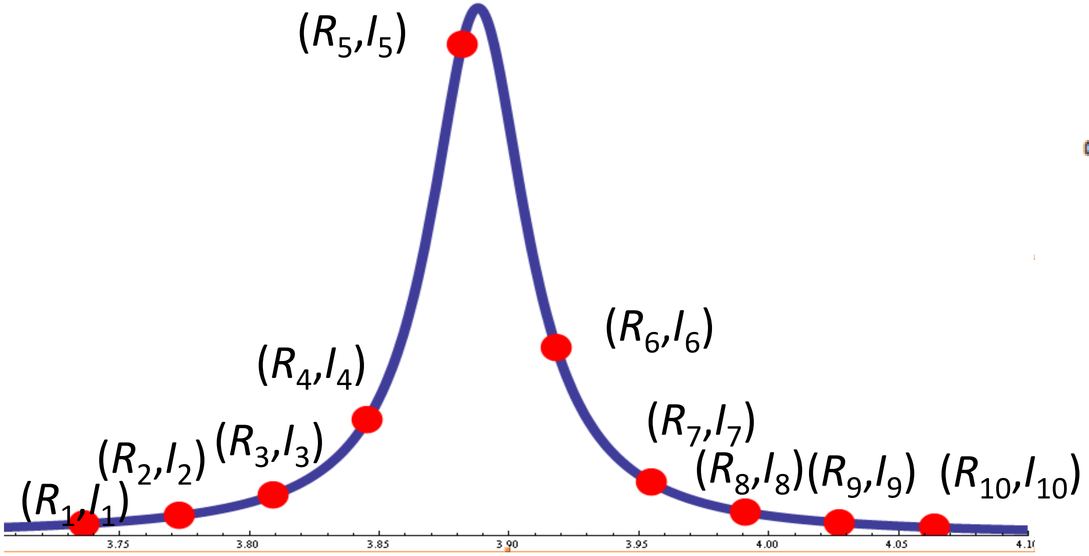
Phase shift $\delta(s)$ measurement

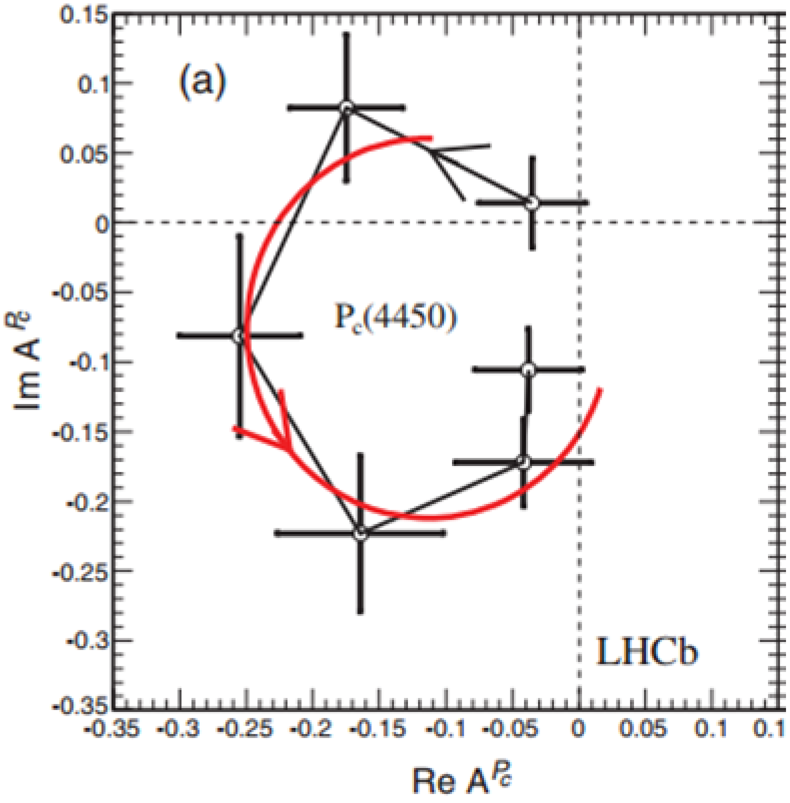
- Amplitude at s_i: $A(s_i)= \sin(\delta(s_i)) \exp(i\delta(s_i))$
- $\delta(s)$ available with interpolation among $\delta(s_i)$



Argand plot measurement

- set equal distance of amplitude point (R_i,I_i) in BW region
- $R(\sqrt{s})$=Interpolation(R_i), $I(\sqrt{s})$=Interpolation(I_i)
- Ampliude = $R(\sqrt{s})+i\, I_i(\sqrt{s})$





Spin and parity measurement

Null hypothesis H_0 : **data described with** $(\sigma_0, f_0(980), f_2(1270), f_0(1370), Zc(J^P))$

Alternative hypothesis H_1: data described with $(\sigma_0, f_0(980), f_2(1270), f_0(1370), Zc(1^+), \text{other } Zc(J^P))$

$$t \equiv -2\ln\lambda = 2[\ln L_{\max}(H_1) - \ln L_{\max}(H_0)],$$

See Ref.

Ilya Narsky, Nucl. Instr. Meth., A **450**, 444 (2000); Zhu Yong-Sheng, High Energy Physics and Nuclear Physics, **30**, 331 (2006).

$$p(t_{\rm obs}) = \int_{t_{\rm obs}}^{\infty} \chi^2(t;r)dt.$$

$$\int_{-S}^{S} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 - p(t_{\rm obs}) = \int_0^{t_{\rm obs}} \chi^2(t;r)dt.$$

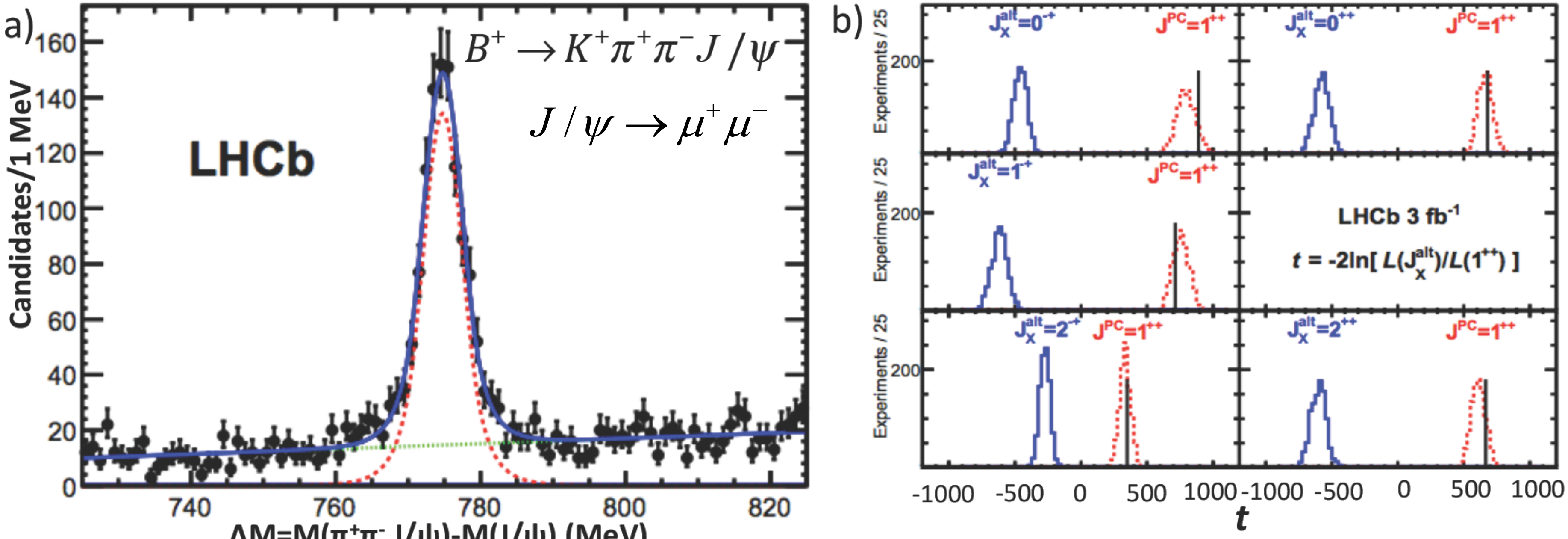
Significance to distinguish the quantum number 1^+ over other quantum numbers.

Hypothesis	$\Delta(-\ln L)$	$\Delta(ndf)$	significance
1^+ over 0^-	44.5	$4\times2+5$	7.3σ
1^+ over 1^-	107.0	$4\times2+5$	$>8.0\sigma$
1^+ over 2^-	51.8	$4\times2+5$	$>8.0\sigma$
1^+ over 2^+	193.5	$4\times2+5$	$>8.0\sigma$

Spin and parity measurement (Alternative)

- LHCb determined $J^{PC}=1^{++}$ (Toy MC)

LHCb, Phys. Rev. D92 (2015) 011102,
LHCb, Phys. Rev. Lett. 110 (2013) 222001



$$t = -2\ln(\mathcal{L}^{alt}/\mathcal{L}^{++})$$

- Data favors for 1^{++}
- $J^{PC} = 1^{++}$ assignment with significance $>16\sigma$

Coupled channel analysis

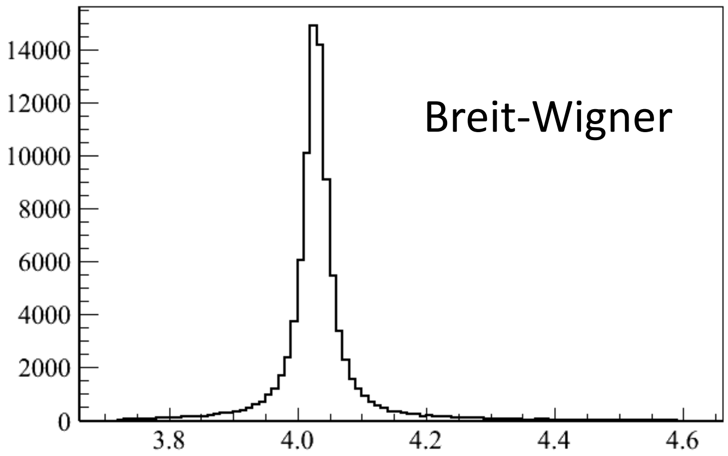
- ☐ One resonance, multi-channel decays

For example: Zc(4020)→ $D^*\underline{D}^*$, $\pi J/\psi$, πh_c, X

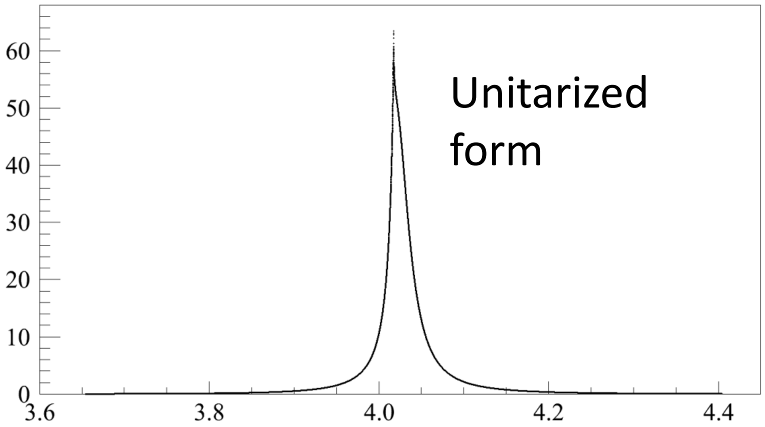
Line shape: Unitarized form

$$\Gamma_{\mathrm{mode}}(s) = \frac{1}{8\pi}\overline{\sum_{\lambda_i}} |\mu_{\mathrm{mode}}(\lambda_i)|^2 \frac{|P|}{s}$$

$$BW(s) = \frac{1}{s - M^2 + i\sqrt{s}\left[\Gamma_{D^*\bar{D}^*}(s) + \Gamma_{J/\psi\pi}(s) + \Gamma_{h_c\pi}(s) + \Gamma_X(s)\right]},$$



Breit-Wigner



Unitarized form

Summary

- tau – charm factory play important role in (exotic) hadron spectroscopy
- PWA as key tool to extract the hadron properties
- parallel computation techniques accelerate the calculation
- Develop physics model as input