Effects of triangle singularities in searching for P_c and P_s

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Mini-workshop on Baryon Spectroscopy at e^+e^- Colliders,
IHEP, Apr. 19–20, 2018

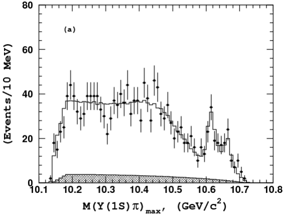
- Triangle singularities
- $P_c(4450)$ and P_s

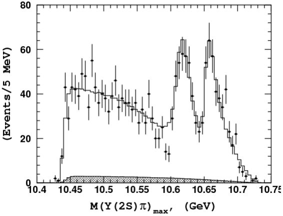
The search of resonances

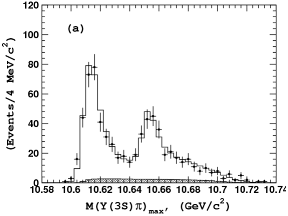
In practice, resonance hunting is normally the search of peaks.

Some famous peaks:

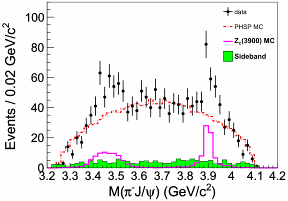
$Z_b(10610)$ and $Z_b(10650)$

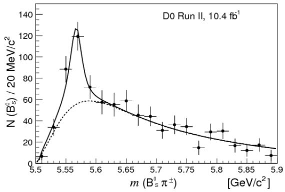


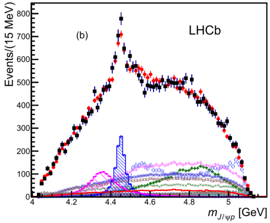




$Z_c(3900)$, $X(5568)$, $P_c(4380, 4450)$



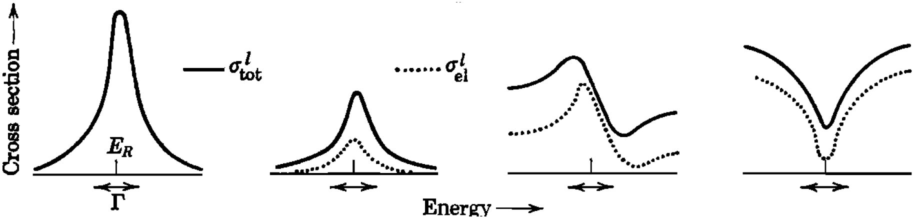




Resonances are not always peaks

Life is always harder than ideal

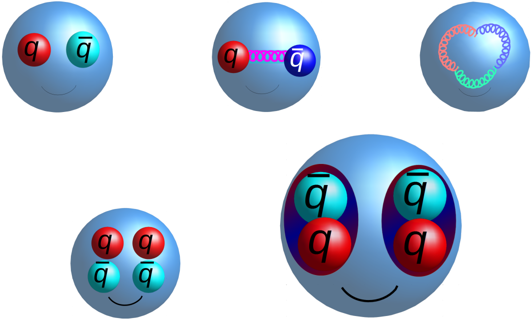
Resonances do not always appear as peaks:



J. R. Taylor, *Scattering Theory — The Quantum Theory on Nonrelativistic Collisions*

Peaks are not always resonances

- Dynamics ⇒ poles in the S-matrix (resonances): genuine physical states. The origins of the poles can be different:



- Kinematic effects ⇒ branching points of S-matrix
  - ☞ normal two-body threshold cusp
  - ☞ triangle singularity
  - ☞ ...

traps/tools in hadron spectroscopy

Peaks are not always resonances

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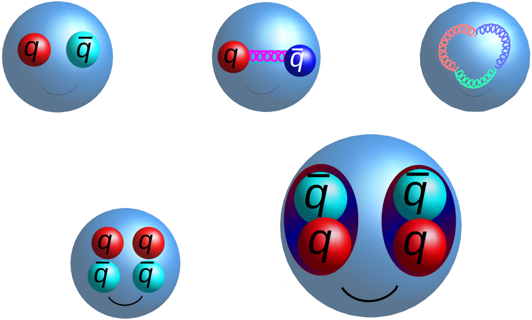

- Kinematic effects $\Rightarrow$ branching points of S-matrix
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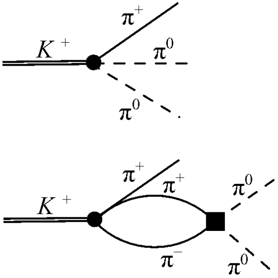
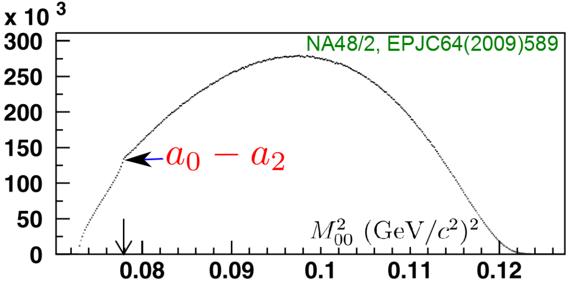
traps/tools in hadron spectroscopy

Threshold cusp

- There is always a cusp at an S-wave threshold
- Cusp effect as a useful tool for precise measurement:
  - ☞ example of the cusp in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$
  - ☞ strength of the cusp measures the interaction strength!

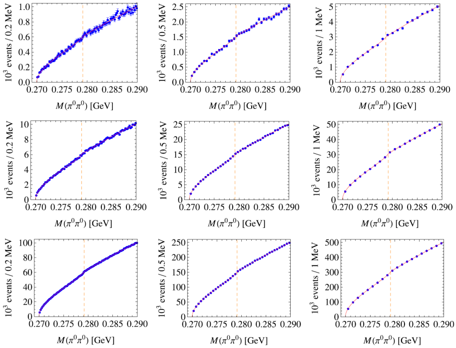
Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); . . .





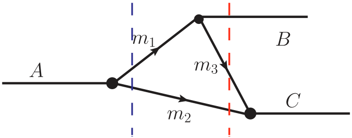
$\pi^+\pi^-$ cusp in $\Upsilon(3S) \to \Upsilon(1S)\pi^0\pi^0$

X.-H. Liu, FKG, E. Epelbaum, EPJC73(2013)2284



Bin width	Events	6×10^4	6×10^5	3×10^6	6×10^6
0.1 MeV	χ^2/dof	1.21	1.09	1.16	0.88
	$a_0 - a_2$	0.293 ± 0.036	0.260 ± 0.012	0.2717 ± 0.0048	0.2661 ± 0.0036
0.2 MeV	χ^2/dof	0.72	1.15	1.05	1.12
	$a_0 - a_2$	0.286 ± 0.035	0.251 ± 0.014	0.2722 ± 0.0048	0.2621 ± 0.0038
0.5 MeV	χ^2/dof	0.93	0.54	1.27	1.30
	$a_0 - a_2$	0.262 ± 0.026	0.256 ± 0.012	0.2659 ± 0.0051	0.2693 ± 0.0035
1 MeV	χ^2/dof	1.05	0.78	1.17	0.69
	$a_0 - a_2$	0.221 ± 0.054	0.291 ± 0.010	0.2658 ± 0.0054	0.2661 ± 0.0037
2 MeV	χ^2/dof	0.59	1.06	1.05	1.37
	$a_0 - a_2$	0.260 ± 0.040	0.262 ± 0.012	0.2592 ± 0.0055	0.2632 ± 0.0037

Triangle singularity



$$\frac{1}{2m_A}\sqrt{\lambda(m_A^2, m_1^2, m_2^2)} \equiv \boxed{p_{2,\text{left}} = p_{2,\text{right}}} \equiv \gamma\,(\beta\, E_2^* - p_2^*)$$

on-shell momentum of m_2 at the left and right cuts in the A rest frame

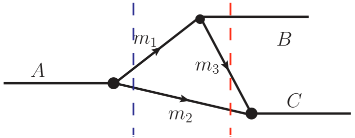
Bayar et al., PRD94(2016)074039

- $p_2 > 0$, $p_3 = \gamma\,(\beta\, E_3^* + p_2^*) > 0 \Rightarrow m_2$ and m_3 move in the same direction
- velocities in the A rest frame: $v_3 > \beta > v_2$

$$v_2 = \beta\,\frac{E_2^* - p_2^*/\beta}{E_2^* - \beta\, p_2^*} < \beta\,, \qquad v_3 = \beta\,\frac{E_3^* + p_2^*/\beta}{E_3^* + \beta\, p_2^*} > \beta$$

- Conditions (Coleman–Norton theorem): Coleman, Norton (1965); Bronzan (1964)
  - ☞ all three intermediate particles can go on shell simultaneously
  - ☞ $\vec{p}_2 \parallel \vec{p}_3$, particle-3 can catch up with particle-2 (as a classical process)
- needs very special kinematics $\Rightarrow$ process dependent! (contrary to pole position)

Triangle singularity



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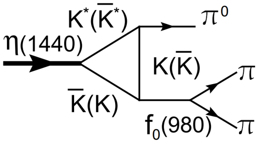
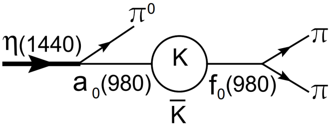
$\eta(1405/1475)$

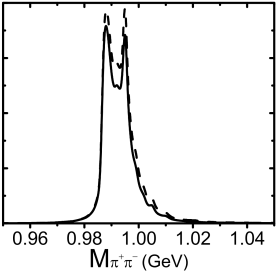
J.-J. Wu, X.-H. Liu, Q. Zhao and B.-S. Zou, PRL108(2012)081803;

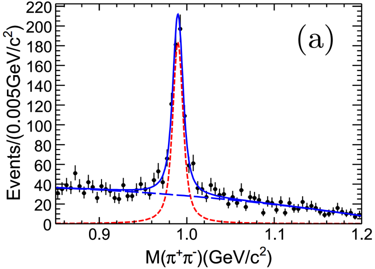
X.-G. Wu, J.-J. Wu, Q. Zhao and B.-S. Zou, PRD87(2013)014023(2013)





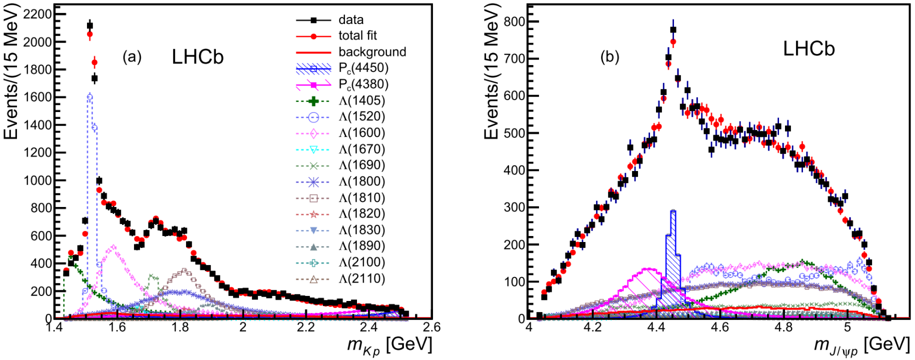
Unique consequence: huge isospin breaking, vary narrow $f_0(980)$ peak ~ 10 MeV





BESIII, PRL108(2012)182001

PRL115(2015)072001 [arXiv:1507.03414]



$$M_1 = (4380 \pm 8 \pm 29)\ \text{MeV}, \qquad \Gamma_1 = (205 \pm 18 \pm 86)\ \text{MeV},$$

$$M_2 = (4449.8 \pm 1.7 \pm 2.5)\ \text{MeV}, \qquad \Gamma_2 = (39 \pm 5 \pm 19)\ \text{MeV}.$$

LHCb's P_c (II)

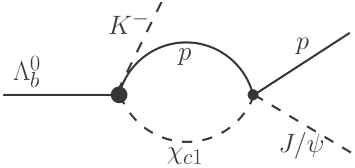
- Quantum numbers not fully determined, for ($P_c(4380), P_c(4450)$): $(3/2^-, 5/2^+)$, $(3/2^+, 5/2^-)$, $(5/2^+, 3/2^-)$, . . . (more see later slides)
- In $J/\psi\, p$ invariant mass distribution, with hidden charm
  $\Rightarrow$ pentaquarks if they are really hadron states
- Narrow pentaquark-like structures with hidden-charm had been predicted 5 years before (07.2010):
  Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV,
  J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, Phys. Rev. Lett. **105** (2010) 232001
- Pentaquark candidates! thus important to study in great details

Coincidence of $P_c(4450)$ with kinematic singularities

- Mass: $M_{P_c(4450)} = (4449.8 \pm 1.7 \pm 2.5)$ MeV
- Our trivial observation: $P_c(4450)$ coincides with the $\chi_{c1} p$ threshold:

$$M_{P_c(4450)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1)\ \text{MeV}$$

- Our non-trivial observation: there is a triangle singularity at the same time!
  Solving the equation $p_{2,\text{left}} = p_{2,\text{right}} \Rightarrow$
  to have a TS at $M_{J/\psi p} = M_{\chi_{c1}} + M_p$, we need $M_{\Lambda^*} \simeq 1.89$ GeV
  On shell $\Rightarrow \Lambda^*$ must be unstable, the TS is then a finite peak



More possible relevant TSs, see X.-H. Liu, Q. Wang, Q. Zhao, PLB757(2015)231

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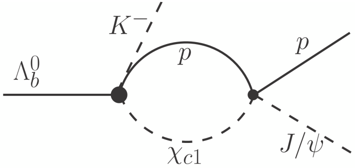
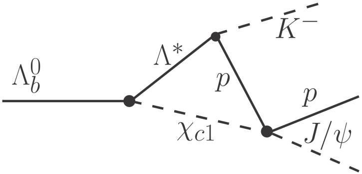
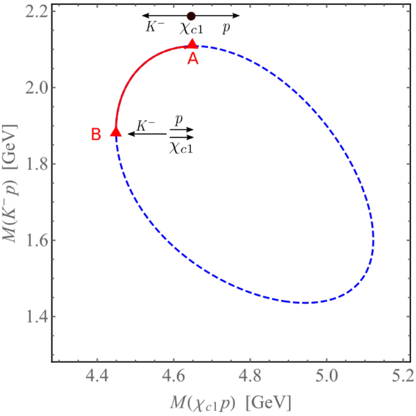


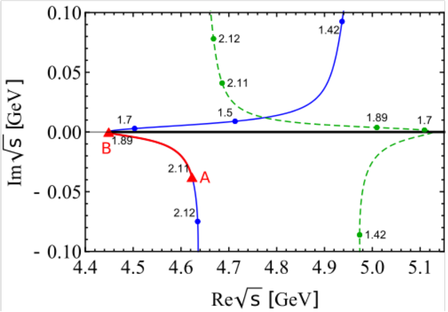


More possible relevant TSs, see X.-H. Liu, Q. Wang, Q. Zhao, PLB757(2015)231

Trajectories of triangle singularities in complex energy plane

Dalitz plot for $\Lambda_b \to \chi_{c1} p K^-$:





numbers: assumed masses for Λ^*

- blue: proton and χ_{c1} are parallel, in the 2nd Riemann sheet
- green: proton and χ_{c1} are anti-parallel

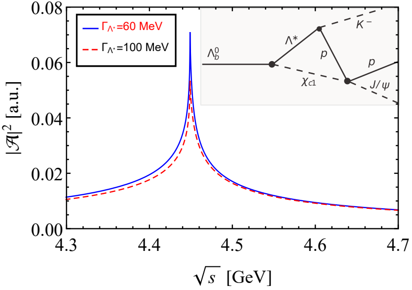
$M_{\Lambda_b} = 5.62$ GeV, $M_{\chi_{c1}} = 3.51$ GeV, $\sqrt{s} \equiv M(\chi_{c1} p)$

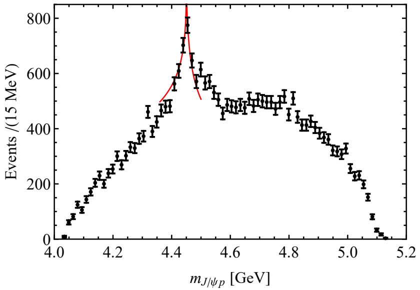
$$M_{K^-p,A} = M_{\Lambda_b} - M_{\chi_{c1}}, \qquad M_{K^-p,B} = \sqrt{\frac{M_{\Lambda_b}^2 M_p + M_K^2 M_{\chi_{c1}}}{M_{\chi_{c1}} + M_p} - M_{\chi_{c1}} M_p}$$

TS for $P_c(4450)$

FKG et al., PRD92(2015)071502(R); X.-H. Liu, Q. Wang, Q. Zhao, PLB757(2015)231

- When $M_{\Lambda^*} = 1.89$ GeV, TS is located exactly at the $\chi_{c1}p$ threshold, 4.449 GeV!
- Four-star baryon $\Lambda(1890)$: $J^P = 3/2^+$, $\Gamma:\ 60 - 200$ MeV
- triangle loop with S-wave $\chi_{c1}p$: $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$





- impossible to produce a narrow peak for $\chi_{c1}p$ in other partial waves

Bayar et al., PRD94(2016)074039; talk by M. Bayar [Monday]

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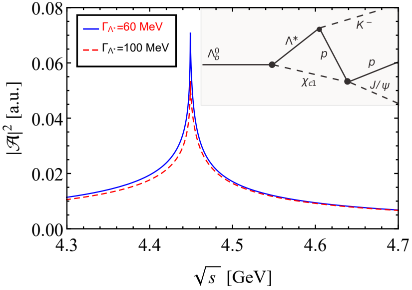

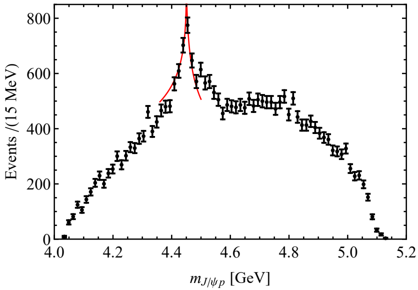

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TS for $P_c(4450)$: Comments

- Position of the TS completely fixed; shape also fixed
- but, strength of the TS is unknown
- operative in $J/\psi\pi$ quantum numbers $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$

		$P_c(4380)$		$P_c(4450)$	
$J^p(4380, 4450)$	$(\sqrt{\Delta(-2\ln\mathcal{L})})^2$	M_0	Γ_0	M_0	Γ_0
$(3/2-, 5/2^+)$ solution					
$3/2^-, 5/2^+$	$--$	4359	151	4450.1	49
Δ from $(3/2-, 5/2^+)$ solution					
$5/2^+, 3/2^-$	-3.6^2	10	-7	-1.6	-6
$5/2^-, 3/2^+$	-2.7^2	-4	-9	-3.6	-2
$3/2^-, 5/2^+$	–	–	–	–	–

from a reanalysis of the LHCb data using an extended Λ^* model

N. Jurik, CERN-THESIS-2016-086

- does not exclude the possibility of the existence of a pentaquark in addition

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$J^p(4380, 4450)$	$(\sqrt{\Delta(-2\ln\mathcal{L})})^2$	$P_c(4380)$ M_0	$P_c(4380)$ Γ_0	$P_c(4450)$ M_0	$P_c(4450)$ Γ_0
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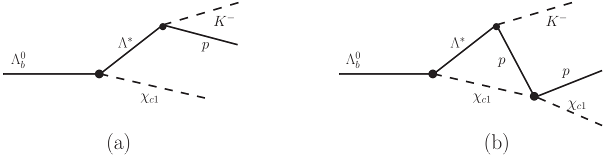
How to distinguish a TS from a genuine resonance?

- Schmid theorem: C. Schmid, Phys. Rev. 154 (1967) 1363

  see also, A. V. Anisovich, V. V. Anisovich, Phys. Lett. B 345 (1995) 321

  Triangle singularity cannot produce an additional peak in the invariant mass distribution of the elastic channel when neglecting inelasticity



  Nearby the effective singularity:

$$\mathcal{A}_{(a)+(b)}(s) \sim [1 + 2i\rho(s)T(s)]\,\mathcal{A}_{(a)}(s) = e^{2i\,\delta_{\chi_{c1}p}(s)}\mathcal{A}_{(a)}(s)$$

  here $\delta_{\chi_{c1}p}$ is the elastic $\chi_{c1}p$ scattering phase shift

- corrections from coupled channels A. Szczepaniak, PLB757(2016)61

How to distinguish a TS from a genuine resonance?

- determining quantum numbers unambiguously:
  TS as discussed here requires the $\chi_{c1}p$ in S-wave $\Rightarrow J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$
- processes (such as photoproduction) with a different kinematics
  Q. Wang, X.-H. Liu, Q. Zhao, PRD92(2015)034022;
  V. Kubarovsky, M. Voloshin, PRD92(2015)031502;
  M. Karliner, J. L. Rosner, PLB752(2015)329; ...
- measuring the process $\Lambda_b^0 \to \chi_{c1}\, p\, K^-$
  ☞ if a narrow near-threshold peak in $\chi_{c1}\, p \Rightarrow$ a real exotic resonance

  recently measured by LHCb in PRL119(2017)062001, no invariant mass distribution reported:

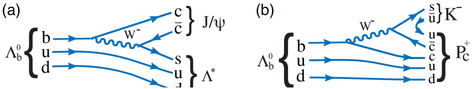
$$\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-) = \left(7.4 \pm 0.4 \pm 0.4 \pm 0.6^{+1.0}_{-0.7}\right) \times 10^{-5}$$

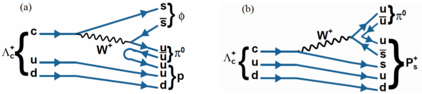
$$\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-) = \left(3.01 \pm 0.22^{+0.43}_{-0.27}\right) \times 10^{-4}$$

With LHC Run-1 data, statistics not enough N. Jurik, Mitsuyoshi Tanaka Dissertation Award Talk at the APS April Meeting 2018

P_s searching

- A ϕp bound state was predicted in several models with a mass ~ 2 GeV
  H. Gao, T.S.H. Lee, V. Marinov, PRC63(2001)022201;
  F. Huang, Z.-Y. Zhang, Y.-W. Yu, PRC73(2006)025207;
  H. Gao, H. Huang, T. Liu, J. Ping, F. Wang, Z. Zhao, PRC95(2017)055202
- Lattice evidence for strangenium-nucleon bound state at a large quark mass
  $m_{u,d,s}^{\text{Lat.}} = m_s^{\text{ph.}}$ $(M_\pi^{\text{Lat.}} \simeq 805$ MeV) S.R. Beane et al. [NPLQCD], PRD91(2015)114503
- Bump observed at $\sqrt{s} \sim 2$ GeV by LEPS and CLAS in $\gamma p \to \phi p$
  LEPS, PRL95(2005)182001; CLAS, PRC89(2014)055208, PRC90(2014)019901
- Suggestion to search for P_s in $\Lambda_c \to \pi^0 \phi p$ R. Lebed, PRD92(2015)114030





- No clear evidence was found in Belle searching Belle, PRD96(2017)051102(R)
- Difficult to search for P_s in $\Lambda_c \to \pi^0 \phi p$ J.-J. Xie, FKG, PLB774(2017)108

P_s searching

- A ϕp bound state was predicted in several models with a mass ~ 2 GeV

  H. Gao, T.S.H. Lee, V. Marinov, PRC63(2001)022201;

  F. Huang, Z.-Y. Zhang, Y.-W. Yu, PRC73(2006)025207;

  H. Gao, H. Huang, T. Liu, J. Ping, F. Wang, Z. Zhao, PRC95(2017)055202

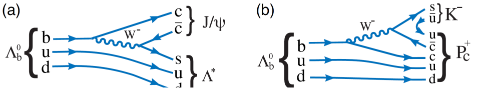
- Lattice evidence for strangenium-nucleon bound state at a large quark mass

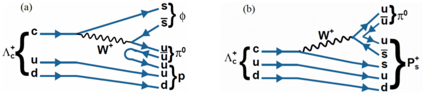
  $m_{u,d,s}^{\text{Lat.}} = m_s^{\text{ph.}}$ $(M_\pi^{\text{Lat.}} \simeq 805 \text{ MeV})$ S.R. Beane et al. [NPLQCD], PRD91(2015)114503

- Bump observed at $\sqrt{s} \sim 2$ GeV by LEPS and CLAS in $\gamma p \to \phi p$

  LEPS, PRL95(2005)182001; CLAS, PRC89(2014)055208, PRC90(2014)019901

- Suggestion to search for P_s in $\Lambda_c \to \pi^0 \phi p$ R. Lebed, PRD92(2015)114030





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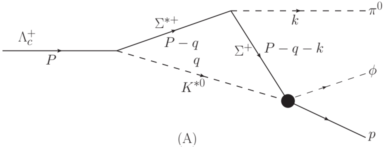
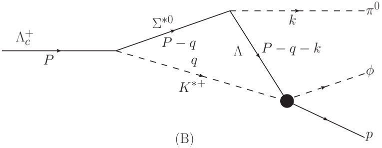
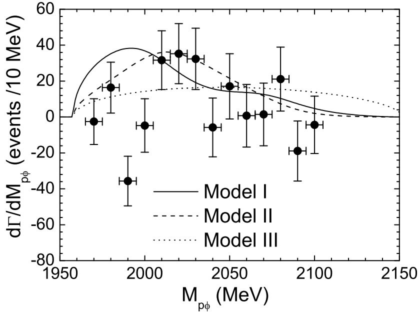
- Difficult to search for P_s in $\Lambda_c \to \pi^0 \phi p$ J.-J. Xie, FKG, PLB774(2017)108

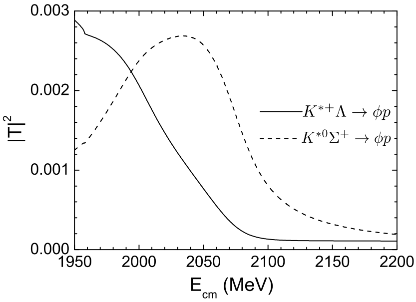
TS and P_s in $\Lambda_c \to p\phi\pi^0$

J.-J. Xie, FKG, PLB774(2017)108





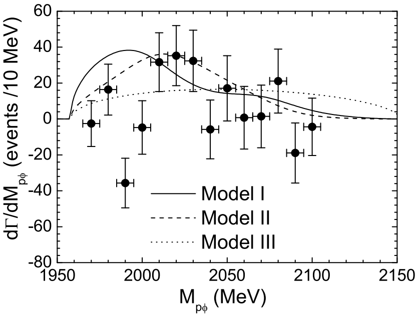




Model I: the BV interaction model (P_s generated) of A. Ramos, E. Oset, PLB727(2013)287;

Model II: no resonance, constant interaction; Model III: phase space

P_s (continued)



- TS produces a bump at around 2.02 GeV, width mainly from that of K^*
- P_s, if exists, could distort the line shape, but difficult to be distinguished from TS in this process
- A measurement of $\Lambda_c \to \Sigma^* K^*$ can help constrain the TS strength

To-do list

- To search for resonances in processes with different kinematics, and to measure the quantum numbers
- To estimate the strength of the TS contributions, a first try is being done in quark model by T. Burns and E. Swanson for the P_c talk by E. Swanson at the workshop “Bound states in strongly coupled systems”
- Analysis framework incorporating kinematic singularities

THANK YOU FOR YOUR ATTENTION!

Backup slides

Triangle singularity – literature

- Some recent works using triangle singularity to explain (part of) peak structures [$\eta(1405/1475)$, $a_1(1420)$, $\ldots$]:

J.-J. Wu, X.-H. Liu, Q. Zhao and B.-S. Zou, PRL108(2012)081803;

X.-G. Wu, J.-J. Wu, Q. Zhao and B.-S. Zou, PRD87(2013)014023(2013);

Q. Wang, C. Hanhart and Q. Zhao, PLB725(2013)106;

M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD91(2015)094015;

X.-H. Liu, M. Oka and Q. Zhao, PLB753(2016)297;

A. P. Szczepaniak, PLB747(2015)410; PLB757(2016)61;

F. Aceti, L.-R. Dai and E. Oset, PRD94(2016)096015;

A. E. Bondar and M. B. Voloshin, PRD93(2016)094008

V. R. Debastiani, F. Aceti, W.-H. Liang, E. Oset, PRD95(2017)034015

. . . . . .

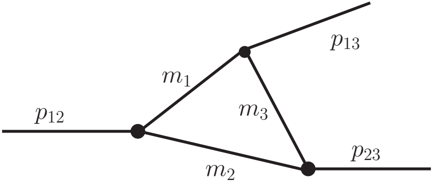
Recent reviews:

Q.Zhao, JPS Conf.Proc.13(2017)010008; FKG et al., arXiv:1705.00141

Recent lecture notes by one of the key players:

I. J. R. Aitchison, arXiv:1507.02697 [hep-ph], *Unitarity, Analyticity and Crossing Symmetry in Two- and Three-hadron Final State Interactions*

Landau equation for TS



- Triangle singularity: leading Landau singularity for a triangle diagram, anomalous threshold

  studied extensively in 1960s

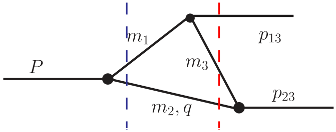
- Solutions of Landau equation: Landau (1959)

$$1 + 2\, y_{12}\, y_{23}\, y_{13} = y_{12}^2 + y_{23}^2 + y_{13}^2, \qquad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_{ij}^2}{2\, m_i\, m_j}$$

  quadratic equation of y_{ij}, always two solutions

- Do they affect the physical amplitude?

Some details



$$I \propto \int_0^\infty dq \; \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

$$f(q) = \int_{-1}^{1} dz \, \frac{1}{A(q,z)} \equiv \int_{-1}^{1} dz \, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

Singularities of the **integrand of I** in the rest frame of initial particle ($P^0 = M$):

- 1st cut: $M - \omega_1(l) - \omega_2(l) + i\,\epsilon = 0 \Rightarrow$

$$q_{\text{on}\pm} \equiv \pm \left(\frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)} + i\,\epsilon \right)$$

- 2nd cut: $A(q, \pm 1) = 0 \Rightarrow$ endpoint singularities of $f(q)$

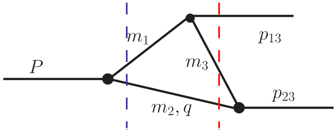
$$z = +1: \quad q_{a+} = \gamma\left(\beta\, E_2^* + p_2^*\right) + i\,\epsilon\,, \qquad q_{a-} = \gamma\left(\beta\, E_2^* - p_2^*\right) - i\,\epsilon\,,$$

$$z = -1: \quad q_{b+} = \gamma\left(-\beta\, E_2^* + p_2^*\right) + i\,\epsilon\,, \quad q_{b-} = -\gamma\left(\beta\, E_2^* + p_2^*\right) - i\,\epsilon$$

$$\beta = |\vec{p}_{23}|/E_{23}, \qquad \gamma = 1/\sqrt{1-\beta^2} = E_{23}/m_{23}$$

$E_2^*(p_2^*)$: energy (momentum) of particle-2 in the cmf of the (2,3) system

Some details



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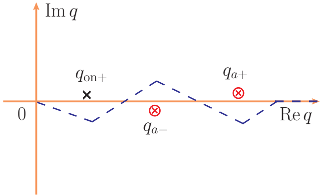
$E_2^*(p_2^*)$: energy (momentum) of particle-2 in the cmf of the (2,3) system

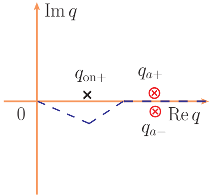
All singularities of the integrand of I:

$$q_{\rm on+}, \qquad q_{a+} = \gamma\,(\beta\,E_2^* + p_2^*) + i\,\epsilon, \qquad q_{a-} = \gamma\,(\beta\,E_2^* - p_2^*) - i\,\epsilon,$$

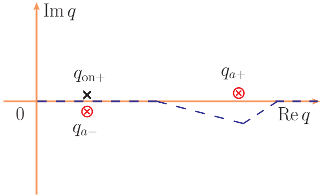
$$q_{\rm on-} < 0, \quad q_{b-} = -q_{a+} < 0 \text{ (for } \epsilon = 0), \qquad q_{b+} = -q_{a-},$$



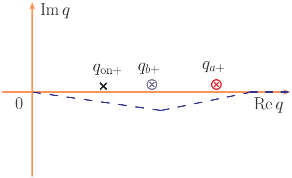
(a)



(b)



(c)

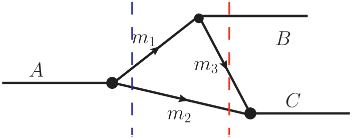


2-body threshold singularity at

$m_{23} = m_2 + m_3$

triangle singularity at

$$\boxed{q_{\rm on+} = q_{a-}}$$



$$\frac{1}{2m_A}\sqrt{\lambda(m_A^2, m_1^2, m_2^2)} \equiv \boxed{q_{\rm on+} = q_{a-}} \equiv \gamma\,(\beta\, E_2^* - p_2^*)$$

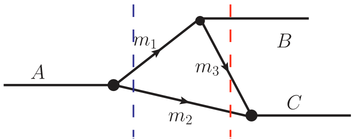
Bayar et al., PRD94(2016)074039

- Conditions (Coleman, Norton (1965); Bronzan (1964)):
  - ☞ all three intermediate particles can go on shell simultaneously
  - ☞ $\vec{p}_2 \parallel \vec{p}_3$, particle-3 can catch up with particle-2 (as a classical process)
- $p_3 = \gamma\,(\beta\, E_3^* + p_2^*) > 0 \Rightarrow$
  particles 2 and 3 move in the same direction in the rest frame of initial particle
- velocities in the rest frame of the initial particle: $v_3 > \beta > v_2$

$$v_2 = \beta\,\frac{E_2^* - p_2^*/\beta}{E_2^* - \beta\, p_2^*} < \beta\,, \qquad v_3 = \beta\,\frac{E_3^* + p_2^*/\beta}{E_3^* + \beta\, p_2^*} > \beta$$

particle 3 moves faster than particle 2 in the rest frame of initial particle

TS: kinematics



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Bayar et al., PRD94(2016)074039

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particle 3 moves faster than particle 2 in the rest frame of initial particle

TS: kinematics

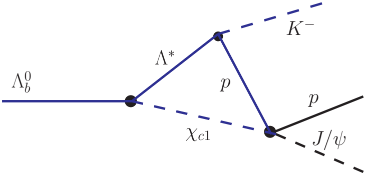
Dalitz plot for $\Lambda_b \to \chi_{c1}\Lambda^* \to \chi_{c1}p\bar{K}$:

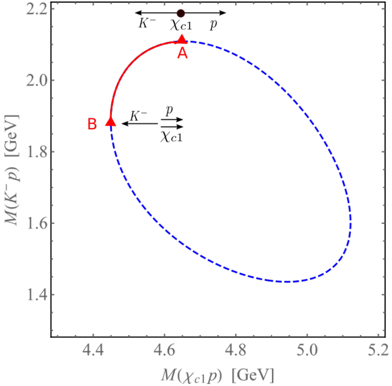
Starting from a large Λ^* mass, in Λ_b rest frame

- when $M_{\Lambda^*} > M_{\Lambda_b} - M_{\chi_c1}$, cannot go on-shell
- at point A, $M_{\Lambda^*} = M_{\Lambda_b} - M_{\chi_c1}$, χ_{c1} is at rest
- at point B, proton and χ_{c1} has the same velocity, $p\,\chi_{c1}$ threshold
- between A and B, $\vec{p}_p \parallel \vec{p}_{\chi_{c1}}$ and proton moves faster than χ_{c1}

$$M_{K^-p,A} = M_{\Lambda_b} - M_{\chi_{c1}},$$

$$M_{K^-p,B} = \sqrt{\frac{M_{\Lambda_b}^2 M_p + M_K^2 M_{\chi_{c1}}}{M_{\chi_{c1}} + M_p} - M_{\chi_{c1}} M_p}$$

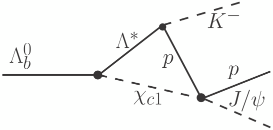


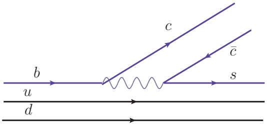


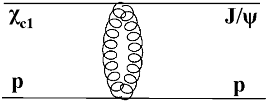
More comments

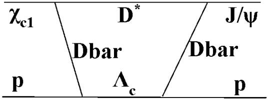
Strength of the triangle singularity is determined by

- couplings:
  - ☞ $\Lambda_b \to \Lambda^* \chi_{c1}$ is from $b \to c\bar{c}s$, not measured, but should have a sizeable branching fraction:
    Br$(B^+ \to J/\psi K^+) \simeq 1 \times 10^{-3}$,
    Br$(B^+ \to \chi_{c1} K^+) \simeq 0.5 \times 10^{-3}$
  - ☞ $\Lambda(1890) \to N\bar{K}$: largest branching fraction, Br $= 20 - 35\%$
  - ☞ $\chi_{c1} p \to J/\psi p$: unknown, OZI suppressed,
    $\mathcal{O}\left(1/N_c\right)$ [recall: OZI suppressed meson-meson scattering: $\mathcal{O}\left(1/N_c^2\right)$]









lattice QCD predicts possible $c\bar{c}$-nucleus bound states at $M_\pi = 805$ MeV

NPLQCD, PRD91(2015)114503