

Low momentum track reconstruction based on Hough transform for the BESIII drift chamber

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Abstract In order to overcome the difficulty brought by the curling charged tracks finding in the BESIII drift chamber, we introduce the Hough transform based tracking method. This method is used as the supplementary to find low transverse momentum tracks. Hough Transform is a mathematical method to transform hits in detector to parameter space which can find hits on track globally in the first step. This tracking algorithm is realized in C++ in BOSS (BESIII offline software system) and the performance has been checked by both Monte Carlo and data. The results show that this tracking method could enhance the reconstruction efficiency in the low transverse momentum region.

Keywords BESIII MDC · low transverse momentum · Hough transform · track reconstruction

1 Introduction

Detection of trajectories of charged particles and measurement of their momentum and vertexes from interactions are important tasks in particle and nuclear experiments. Usually these are realized by tracking detector (drift chamber [1], GEM [2] or other type) embedded in a magnetic field and by offline track reconstruction. The tracking qualities or performances such as detection efficiency, resolutions of momentum and

vertex rely not only on the design of detector including geometry and material but also on the reconstruction algorithms used.

The Beijing Spectrometer III(BESIII [3]) operated at the upgraded Beijing Electron Positron Collider (BEPCII [4]) is a high precision, general purpose detector designed for physics studies at the tau-charm energy region. The tracking system of the BESIII is a Multi-wire Drift Chamber(MDC) [5] within a 1 T solenoid field along z direction. The largest polar angle coverage is $|\cos\theta| \leq 0.93$. MDC is a small-cell drift chamber with 43 layers of sense wires. Three or four sense layers form a super-layer. The configuration of the axial super-layers(A) and stereo super-layers(U or V) from inner to outer is UVAAAUVUVAA. Axial wires are parallel to the z-axis while stereo wires with a small angle with z-axis can provide z position measurement. The distance between the trajectory and the sense wires called drift distance can be estimated with the drift time of ionization electrons from charged particle. We measure the curvature of trajectory through the signal hits and their drift distance. The particle's momentum and charge can be determined by the curvature of trajectory.

At BESIII, track reconstruction including two tasks: track finding and track fitting. Track finding is a pattern recognition problem and aims at dividing the measurements into subsets called track candidate. The goal of track fitting is to accurately estimate the state parameters of particles at the reference point which closest to the origin. Two track finding packages have been implemented for MDC: template pattern matching (PAT [6]) and track segment finding (TSF [7]). The PAT package is used to reconstruct tracks with transverse momentum $p_T > 250\text{MeV}/c$ and remained hits are used by the TSF algorithm to do track finding. Both of them search track segment from a super-layer and link

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the segments into tracks. The low transverse momentum tracks with $p_T < 120$ MeV/c can leave a curling trajectory in MDC. So a special tracking algorithm TCurlFinder [8] in TSF package is designed to reconstruct curling tracks. TCurlFinder form a seed track segment with the largest number of consecutive in one layer and merge hits on track segments in super-layer along seed tracks. The tracking efficiency of PAT, TSF and TCurlFinder algorithms (PATTSF) is over 95% for high transverse momentum tracks with $p_T \geq 120$ MeV/c, but for low transverse momentum tracks when $p_T < 120$ MeV/c it drops significantly, and the efficiency at $p_T=100$ MeV/c is about 80% [9] for barrel tracks with $|\cos\theta| < 0.8$.

All track finding algorithms worked for BESIII depend on the track segments which are found in local regions of super-layers and then link or merge the track segment along track road. That means hits are not treated globally or as a whole in track finding. The quality of track segment becomes worse if there are noises exist in a supers-layer and segment finding efficiency will reduced by the inefficiency of hits. For a low p_T particle ($p_T < 120$ MeV/c), if its longitudinal momentum p_z is small enough, the track becomes curling with multi-turn loops. So the hits from different turn may leave adjacent hits or overlapped hits in one super-layer. And due to larger energy loss for low p_T particles, the tracks are no longer perfect helix. As described above, the track finding is extremely difficult for low p_T particle. Figure 1 shows a real BESIII event, in which one of a curling track was not reconstructed.

Hough transform is a global track finding method which is widely used to detect lines and circles in HEP tracking [10] and computer vision. This method use all the available hits in detector at the finding stage. It map the hits in the detector space into track parameter space. Hits belong to a same track form a peak on parameter space, while the position of the peak indicate the track parameters. By finding local peaks in the parameter space, track and hits associated with them can be found more conveniently. Due to its merits such as noise-immunity and insensitive to the hits inefficiency, we develop a new tracking algorithm (HOUGH) based on Hough transform for low p_T tracks at BESIII. In this paper we represent the details of the HOUGH algorithm. In Section 2, the principle of Hough transform and its implementation in track finding are introduced. In Section 3.2, we compare the tracking performances between PATTSF algorithms and with the supplementary of HOUGH algorithm using the full Monte Carlo simulation and the experimental data.

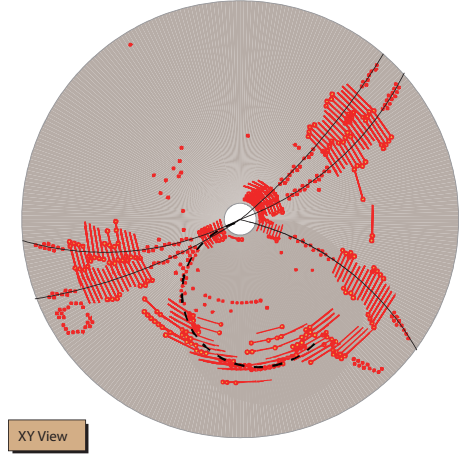


Fig. 1 a $Ds \rightarrow K^+ K^- \pi^+$ event with a lost curling track. The red points are hits from axial wires, the hits from straight line segments are projection of hits from stereo wires on $x-y$ plane, the black curves are the tracks reconstructed by current method, the dashed curve is a lost curling track.

2 Track finding based on Hough transform

The aim of HOUGH algorithm is to find curling tracks as supplementary of PATTSF packages. Figure 2 shows the flow chart of this algorithm. The HOUGH package does tracking with the MDC hits not used by previous algorithms and the ones on curling tracks. Considering the structure of the MDC detector, we perform tracking in each 2-Dimensional(2-D) stage and 3-Dimensional(3-D) stage. In 2-D stage, we perform a conformal mapping and Hough transform on axial hits to find the 2-D circle track, and in 3-D stage find stereo hits according to circle parameter. Global track fitting based on Least-Square method is done for 2-D and 3-D tracks respectively. Track candidates reconstructed by the PATTSF and the HOUGH algorithms are merged if necessary and then are further fitted precisely with the Runge Kutta and Kalman filter [11] methods. A Helix track can be described using five parameters, $\mathbf{a}=(d_\rho, \phi_0, \kappa, d_z, \tan\lambda)$ [6, 7], where d_ρ and d_z are the signed distance of the helix from the pivot to track in $x-y$ plane and in z direction; ϕ_0 is the azimuthal angle to specify the pivot with respect to the helix center; $\kappa=q/p_T$ where q is the charge of the track and p_T is the transverse momentum of the track; $\tan\lambda$ is the slope of the track where λ is the dip angle of track, Alternatively, $\theta = \pi/2 - \lambda$. In global track finding and fitting, the origin point is taken as the pivot.

2.1 Conformal mapping

The trajectory of a charged particle in a uniform magnetic field is a helix if its energy loss is negligible,

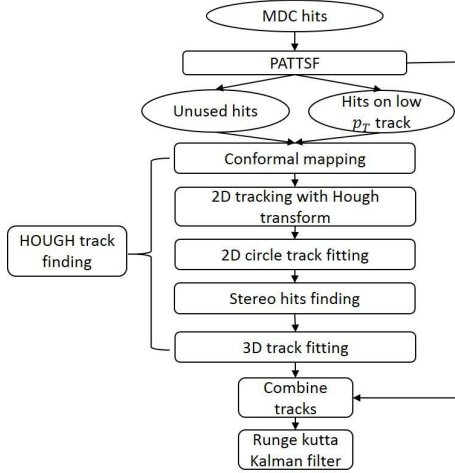


Fig. 2 the flowchart of HOUGH algorithm.

and the projection in the x - y plane perpendicular to the z direction is a circle. Suppose a particle comes out from the origin with a radius of R , a point (x, y) at the circle in detector space can be transformed into a point in conformal plane [12] (X, Y) by

$$X = \frac{2x}{x^2 + y^2}, Y = \frac{2y}{x^2 + y^2}, \quad (1)$$

After the mapping, a circle of track (the dashed curve in Figure 3) becomes a straight line (the dashed line in Figure 4) in the conformal space. In our detection, a signal hit is represented by a axial wire position (x', y') and a drift distance d to the track. The closest point between a track and a axial hit located on the circle, called drift circle, determined by the position of axial hit and it's drift distance. We can imagine a drift circle which is tangent to the circle of track as shown in Figure 3 (the circle Q). Since a hit circle is not originated from origin, after the conformal mapping, it is still a circle centered at point (X', Y') with radius r tangent to the line track as shown in Figure 4 (the circle Q'). Hits from outer layers deviate from the perfect curve or line due to the energy loss.

2.2 2-D track finding based on Hough transform

By conformal mapping, 2-D circle track finding on x - y detector plane becomes finding a straight line on the conformal plane. We use a Hough transform method for this straight line detecting. As we use the normal parameterization (ρ, α) to depict the straight line, the Hough transform can be described with the equation:

$$\rho = X \cos \alpha + Y \sin \alpha, \quad (2)$$

where (X, Y) is the coordinate of the points on the straight line, α is the polar angle of the normal vector

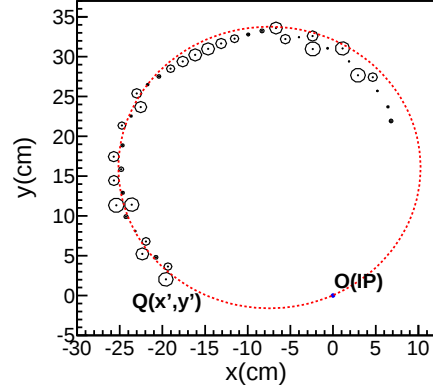


Fig. 3 a Monte Carlo π^- track of $p_T=60\text{MeV}/c$ on x - y detector plane with only axial hits, dashed curve is the track from Monte Carlo truth, the points are projection of axial hits on x - y plane, the circles are the drift circles of hits.

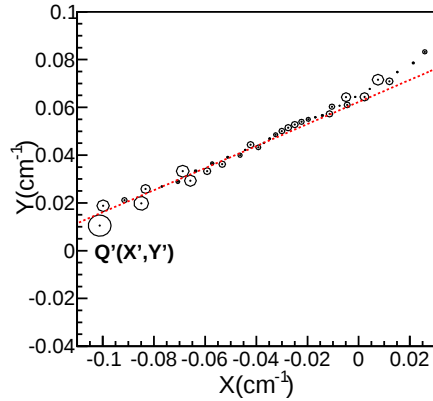


Fig. 4 the Monte Carlo π^- track of $p_T=60\text{MeV}/c$ on conformal plane, the dashed straight line is the track on conformal plane, the points are projection of axial hits after conformal mapping, the circles are the drift circles of hits after conformal mapping.

of the line, ρ is the line's sign distance from the origin. The equation indicate that one point on conformal plane transforms into a sinusoidal curve on the track parameter space (θ, ρ) , which means all the possible lines going through the point (X, Y) ; the collinear points transformed into a set of sinusoidal curves on parameter space, points which are collinear in conformal space all intersect at a common point in track parameter space and the coordinates of this parameter point characterizes the straight line connecting the points. Figure 5 and 6 show after Hough transform five collinear points transform into five sine curves that passing the same point.

Differing from the collinear points, the measurements of the MDC detector are hits and the drift circles and their drift circles are nearly tangent to a straight line

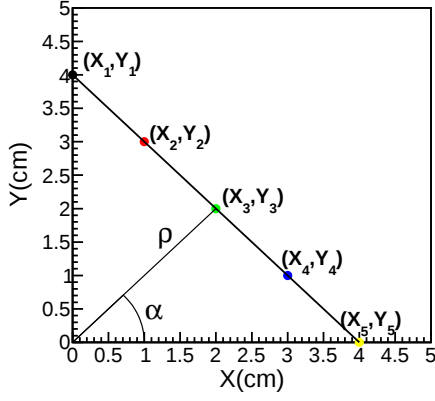


Fig. 5 a straight line on X-Y space, points with different color are lying on the line.

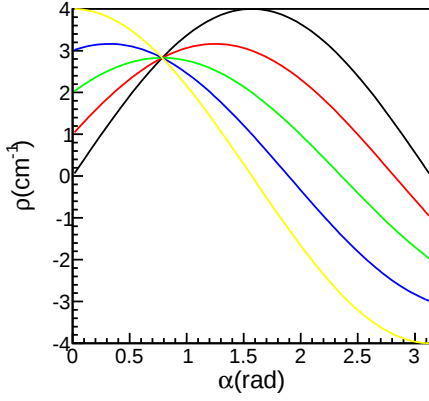


Fig. 6 representation of the five colinear points on line parameter space.

on conformal plane. We perform the a particular Hough transform [13] to take account in the measurement of drift distance. For each drift circle on conformal plane, the Hough transform becomes to transform all the straight lines that tangent to it into the parameter space. The point with the maximum contribution, in the track parameter space, represents the common tangent to the circles. The transformation can be described in the equations:

$$\begin{aligned} \rho &= X \cos \alpha + Y \sin \alpha + r, (\text{upper half circle}) \\ \rho &= X \cos \alpha + Y \sin \alpha - r, (\text{lower half circle}) \end{aligned} \quad (3)$$

where (X, Y) is the coordinate of axial hit on conformal plane, r is the radius of the drift circle after conformal mapping.

From the Eq.3, it can be seen that for one angle α , there are two straight lines tangent to the drift circle, and either of the line has a difference of one radius distance in the ρ direction from the straight line that passing the circle center with same angle α , as shown in

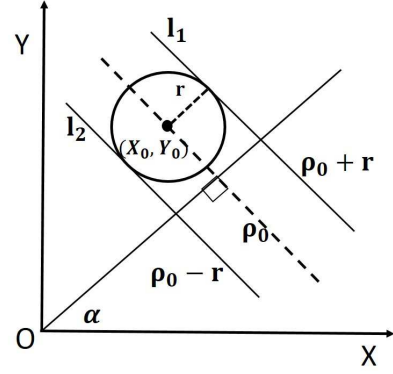


Fig. 7 a circle and its two example tangent lines on conformal plane, l_1, l_2 are two example tangent lines to in α direction, the dashed line is passing circle center (X_0, Y_0) .

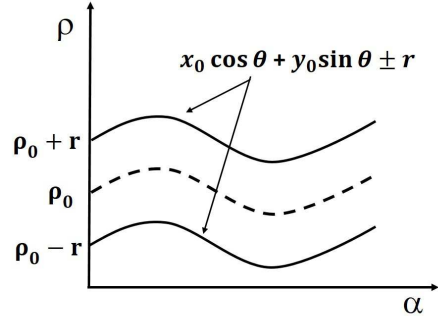


Fig. 8 representation of the circle in the track parameter space. The circle corresponds to two sinusoidal curves in the track parameter space. The dashed sinusoidal curves corresponds to the hit point on conformal plane.

Fig.7. Meanwhile, for one drift circle, after the Hough transform, it forms a pair of sinusoidal curves on the track parameter space, which means the parameter set of the tangent lines of all the angles α , and the distance between the two-pair curves are two times of the radius of the drift circle after conformal mapping, as shown in figure 8.

For the purpose of calculating the concentrated area on parameter space, the parameter space is divided into an array of discrete cells. We use 2-D histogram to describe the discrete cells. For each drift circle on conformal plane, we go through all bin along α axis, and fill the straight line's parameter in that direction into 2-D histogram by Eq.3. By this method, the bin that is passed by the sinusoidal curve of the correspond hit is incremented by one. When filling all the drift circles, there will be a peak on the 2-D histogram, the peak represents the straight line on the conformal plane, and the coordinate of the peak indicate the line's parameter.

The error of the peak position comes from the detector error of the hit and the constraint of the track passing origin, and has a close relationship with the

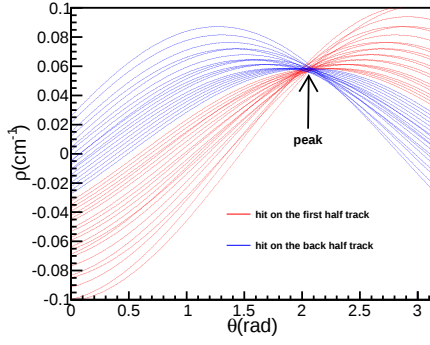


Fig. 9 representation of a curved track in parameter space, red color represent the first half hits, blue color represent the back half hits.

quantization of the 2-D histogram. When the width of the bin is too wide, the precision of the peak parameter is worse; when the width of the bin is too small, hits on the same track contribute to different bins and make it hard to find a peak. Monte Carlo study is performed to get a better resolution of the peak, the resolution $(\delta\alpha, \delta\rho)$ can be described as

$$\delta\alpha = \alpha_{peak} - \alpha_{track}, \delta\rho = \rho_{peak} - \rho_{track}, \quad (4)$$

where $(\alpha_{peak}, \rho_{peak})$ is the coordinate of the highest bin, and $(\alpha_{truth}, \rho_{truth})$ is calculated from Monte Carlo. The quantization of parameter space is varied to get a better resolution of the peak. We adopt the width of bin along α axis as $\pi/1000$ and the width of bin along ρ axis as $2\mu m^{-1}$. The other factor that influences the resolution of peak is the energy loss of the track. From Fig.2 we can see hits with longer flight length obviously deviate from the track, and we can also see this phenomenon on the conformal plane in Fig.3. It is shown in Fig.9 that the sign of sinusoidal curve's slope are opposite nearby the peak between the first half hits and the back half hits, so we use the sign of sinusoidal curves to separate the hits into two different 2-D histogram, then peak finding is implemented in two Hough map independently. These two tracks are compared after fitting and conserved with the one with better quality.

For finding the peak on parameter space, we performed a peak finding method on the 2-D histogram. First we loop the histogram to find the local maximum bin, of which height is higher or at least equal to the eight bins around it. When there appears two local maximum bins adjacent to each other, we suppose they are originate from the same track and just discard one of them, as shown in Figure 10. Then we set up a threshold, all local maximum bins under the threshold are discard. From the discussion above, the height of the bin indicates the number of sine curves passing it, that is the number of hits cross that bin. So this threshold

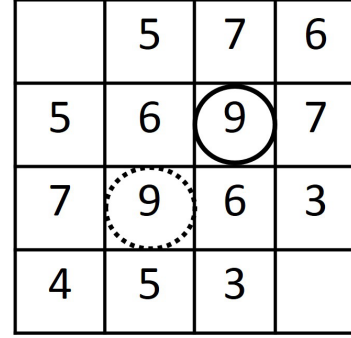


Fig. 10 local maximum found in histogram, the bin with dash circle is discard.

is related to the number of required hits of the track where appears in that bin. We set threshold according to the least number of hits of the track. The bins over threshold are considered candidate tracks, and the position of the candidate tracks can be calculated by:

$$r_c = \frac{1}{|\rho|}, x_c = \frac{1}{\rho} \cdot \cos \alpha, y_c = \frac{1}{\rho} \cdot \sin \alpha, \quad (5)$$

where (x_c, y_c) is the center of the circle track on detector plane, r_c is the track radius, (α, ρ) is the center position of the bin. In the next step we sort the candidate tracks by the height of its bin, and go through each candidate track with the order to collect axial hits. For each axial hit, we define *residual* as

$$residual = doca - drift, \quad (6)$$

where *dist* is the hit distance to the candidate track, and *drift* is the drift distance of the hit. The distribution of the residual in each layer can be fitted to a Gaussian function. In fact the σ in each layer are similar, so we use an average cut to collect axial hits which *residual* is inside 0.1cm. If two candidate tracks have more than 50% common hits, we consider they are similar tracks and discard the track with smaller height of bin. To accelerate peak finding algorithm, a simple iteration is applied. First we divide the parameter space into a histogram with larger bin width, and then divide the bin where may appears candidate peaks into more small bins, and do peak finding in this area.

In the last step of 2-D finder, all the candidate tracks are fitted by a global circle fitting method mentioned in Section 2.3 and give the helix track parameter of candidate track along with hits on the track.

2.3 3-D Track finding

The projection of helix into the s-z plane is straight line where s is the arc length between the point of closest approach on track in x-y plane, and z is z coordinate of hit. For each stereo wire, two hit positions in x-y plane along the track are calculated from the left and right ambiguity respectively and their arc length can be calculated by the circle track parameters [7]. Considering the energy loss of stereo layers in outer MDC for low p_T tracks, only stereo hits from inner eight layers of inner chamber are used to do the primary parameter fitting. We choose 3 hits from outer layers of inner chamber. With 2 ambiguity assumption for each hit, 8 line candidates are extracted as initial value and extrapolate to other layers. We select the closer one of the two ambiguity hits in the line fitting, meanwhile discards the hits with very big chi2. The best fitted line is chosen as initial value for 3-D helix.

2.4 track fitting

After 2-D and 3-D track finding, a least square method which is used by PAT [6] is performed to do the global 2-D circle and 3-D helix fitting without considering the material effect and non-uniformity of magnetic field. The corrections of these effects are taken care of by a subsequent software package after tracking algorithm, called Runge-Kutta as well as Kalman filter. Tracks that are finally used in physics analysis is fixed by these two packages.

2.5 track merging

Track merging is done for similar tracks from different turn of multi-turn track and from other tracking algorithm, we use the similar method with CurlFinder [8]. It is executed between track pairs and done iteratively until no similar track can be found. During merging the track close to the origin is kept.

3 Performance of tracking algorithm

The track finding based on Hough transform is implemented in the BESIII offline software system (BOSS [14]), in which the Monte Carlo simulation [15], data reconstruction and physics analysis can be performed. We compare the new tracking algorithm with the old PATTSF by MC and data.

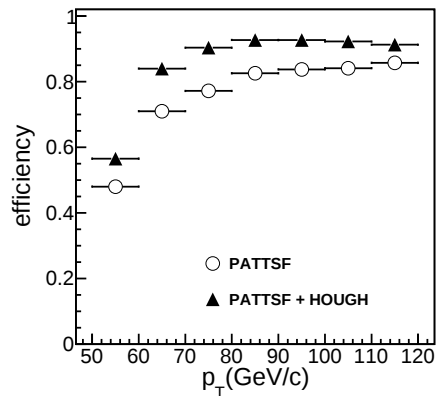


Fig. 11 The efficiency of the good track reconstruction for full simulated single π^- as a function of the transverse momentum p_T with and without the HOUGH algorithm.

3.1 Performance of MC event

The MC sample with negative π particle is generated with uniform distributions both at p_T [50-120 MeV/c] and $|\cos\theta| < 0.93$. The background is mixed with MC event to mimic a real environment during the generation. To avoid the decay of pion, the decay process is turned off. We reconstruct the MC sample using both PATTSF and with HOUGH package as the supplementary of PATTSF packages.

3.1.1 Tracking efficiency

The tracking efficiency is defined as:

$$\varepsilon = \frac{N_{rec}}{N_{mc}} \quad (7)$$

Where N_{rec} is the number events with one good track, and N_{mc} is the number of generated. A reconstructed track is defined as good if its charge is correct reconstructed and $|v_r| < 1cm, |v_z| < 10cm$, where v_r and v_z are vertex deviations from generation in x-y plane and z direction. Figure 11 and 12 show the efficiency as function of p_T and $\cos\theta$. The tracking efficiency relatively increases at an order of 10% at different $p_T < 120$ MeV/c, and it also increases at different angles especially at small angles where multi-turn tracks occur, indicating the Hough algorithm can reconstruct more curling tracks than PATTSF.

3.1.2 Momentum and Vertex resolutions

Figure 13, 14, 15 show the resolutions of p_T and vertex (v_r and v_z) at different p_T for good tracks reconstructed from a single Gaussian fit. The result of PATTSF and HOUGH combine tracking method show better momentum and vertex resolution.

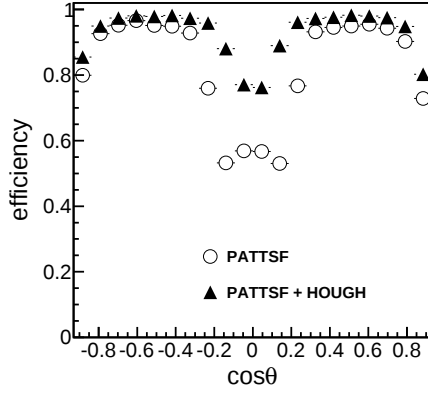


Fig. 12 The efficiency of the good track reconstruction for full simulated single π^- with p_T in region $[0.05, 0.12] \text{ GeV}/c$ as a function of the $\cos\theta$ with and without the HOUGH algorithm.

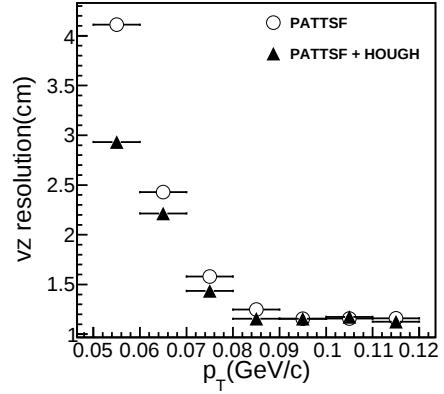


Fig. 15 The v_z resolution of the good track reconstruction for full simulated single π^- with p_T in region $[0.05, 0.12] \text{ GeV}/c$ with and without the HOUGH algorithm.

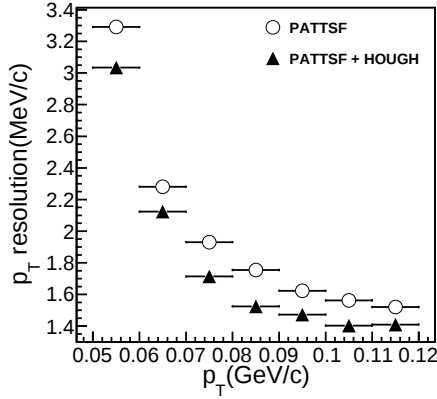


Fig. 13 The p_T resolution of the good track reconstruction for full simulated single π^- with p_T in region $[0.05, 0.12] \text{ GeV}/c$ with and without the HOUGH algorithm.

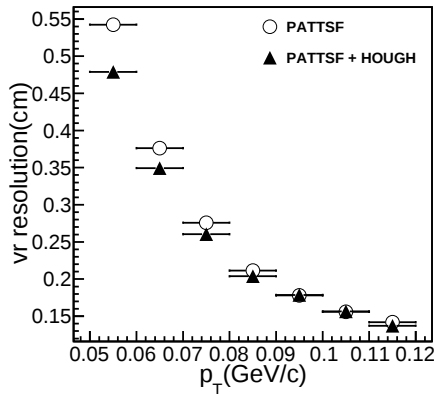


Fig. 14 The v_r resolution of the good track reconstruction for full simulated single π^- with p_T in region $[0.05, 0.12] \text{ GeV}/c$ with and without the HOUGH algorithm.

3.1.3 Fake track

The rate of fake track is defined as

$$\varepsilon_{fake} = \frac{N_{fake}}{N_{rec}}, \quad (8)$$

where N_{fake} is the number of events with more than one track reconstructed and N_{rec} is the number of event with at least one track reconstructed. The fake track rate is highly dependent on p_T , $\cos\theta$, track merging algorithm and even its definition. Here we list two cases, one case is we define event that reconstructed more than one track is a fake track event, the rate of fake track by HOUGH increases from 8.5% to 9.2%; and one case is we define event that reconstructed more than one good track is a fake track event, the rate increase from 0.3% to 1%.

3.1.4 Secondary particle tracking efficiency check with $K_s \rightarrow \pi^+\pi^-$

To study the tracking performance for decay channels involving secondary vertices, $K_s \rightarrow \pi^+\pi^-$ is used for analysis. Figure 20 shows the reconstruction efficiency of the $\pi^+\pi^-$ pair, versus the decay length of K_s generated from Monte Carlo. The tracking efficiency enhanced when decay length is less than 5cm, while when decay length is larger than 6cm, the tracking efficiency is slightly drop. This is because in the conformal mapping we constrain the track as originated from the origin, so in HOUGH algorithm with this constrain we will find fake tracks when a track long flight length and that will influence the reconstruction efficiency of K_s .

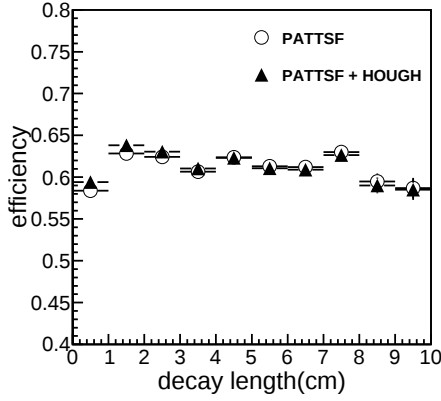


Fig. 16 efficiency of $\pi^+\pi^-$ reconstruction versus decay length of K_s , in the channel of $K_s \rightarrow \pi^+\pi^-$ with momentum of K_s in $[0.05, 0.5]\text{GeV}/c$.

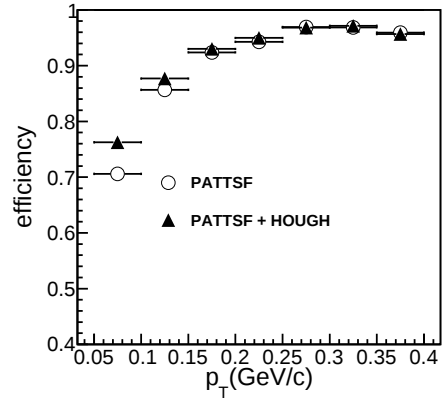


Fig. 17 Comparison of tracking efficiency vs p_T in $\psi \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+ l^-$ using real data.

3.2 Performance of physics events

3.2.1 $\psi \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+ l^-$

There are two soft charged pions ($p < 400\text{MeV}/c$) and two energetic leptons (electron or muon with $p > 1.4\text{GeV}/c$) in this decay channel. We select the event sample of $\psi \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+ l^-$ (e or μ) from experimental data taken at the peak of ψ' resonance. The numbers of such events with three tracks (π^- , l^+ , l^- , one π^+ missing), four tracks ($\pi^- \pi^+ l^+ l^-$) and five tracks ($\pi^- \pi^+ l^- l^+$), one extra charged track are denoted by n_3 , n_4 and n_5 , and the purity of the sample can be controlled at a level of 0.1%. We define the tracking efficiency of π^- as

$$\varepsilon = \frac{n_4}{n_3 + n_4 + n_5}, \quad (9)$$

we reconstruct the experimental raw data with PATTSF and HOUGH and use the same selection criteria to the samples of $\psi \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+ l^-$. The number of selected good events with the supplementary of HOUGH is 3% higher than that with PATTSF, and HOUGH costs more than 5% CPU time than PATTSF. The comparison of the tracking efficiencies as p_T is shown in figure 17. An average increase of 7% with HOUGH at low p_T can be observed.

We also generate MC events of $\psi \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+ l^-$, reconstruct them with PATTSF and HOUGH, and use the same criteria to select the samples. The tracking efficiencies from MC are shown in figure 18. The difference of tracking efficiencies of data and MC is the systematic uncertainty which is one of the important uncertainty sources on physics measurements. Figure 19 shows the differences between PATTSF and PATTSF with the supplementary of HOUGH. The HOUGH package.

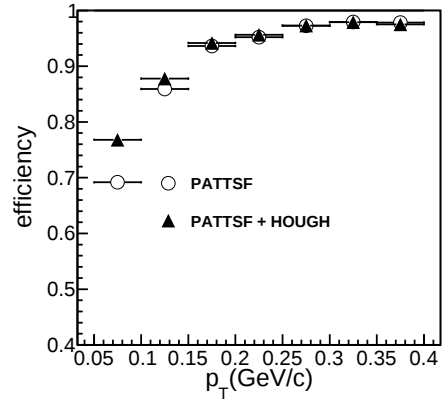


Fig. 18 Comparison of tracking efficiency vs p_T in $\psi \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+ l^-$ using Monte Carlo data.

tracking also improve the consistency between data and MC at low p_T .)

3.2.2 $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$

The tracking efficiency of the algorithm has also been tested with $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$ [9], the tracking efficiency ε_{trk} for a given particle is defined as

$$\varepsilon_{trk} = \frac{n}{N}, \quad (10)$$

where the denominator (N) is the number of signal events with all other particles in the final states required to be reconstructed except the one under study, no matter whether the studied particle is reconstructed or not; the numerator (n) is the number of events with all particles in the final states reconstructed, including the one under study. Figure 20 and 21 show the tracking efficiency from DATA and MC, we can see significant enhancement of the tracking efficiency with adding HOUGH package.

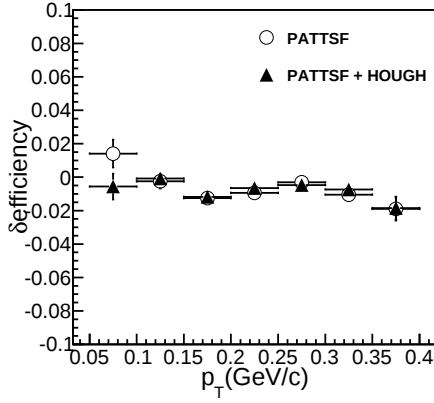


Fig. 19 the difference of tracking efficiency between data and Monte Carlo with only PATTSF and PATTSF+HOUGH.

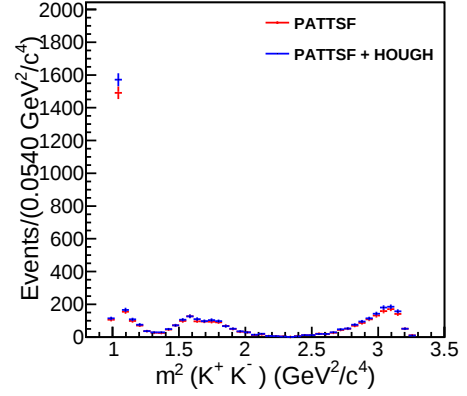


Fig. 22 $M(K^+ K^-)$ of all event

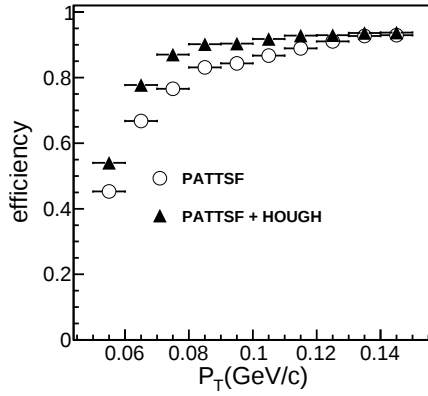


Fig. 20 Comparison of tracking efficiency vs p_T in $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$ using Monte Carlo data.

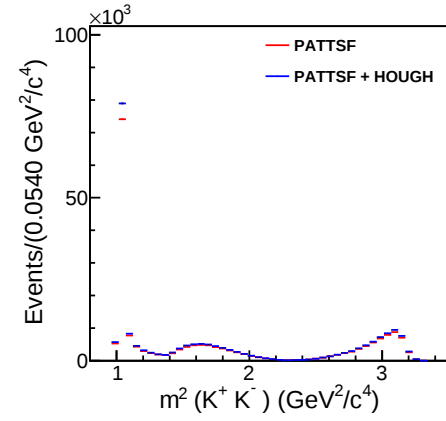


Fig. 23 $M(K^+ K^-)$ of different event

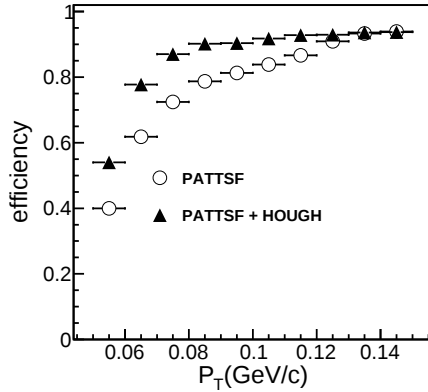


Fig. 21 Comparison of tracking efficiency vs p_T in $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$ using Monte Carlo data.

3.2.3 real data check in $D_s \rightarrow K^+ K^- \pi^+$

We demonstrate an analysis on the $D_s \rightarrow K^+ K^- \pi^+$, where the experimental raw data was taken at the $\sqrt{s} = 4.180 \text{ GeV}/c$. We require two D_s be reconstructed with one D_s decay to $K^+ K^- \pi^+$ and the other D_s decay to any of eight tag modes ($KK\pi, KsK, KsK^+\pi\pi, KK\pi - \pi_0, KsK^-\pi\pi, \pi\pi\pi, \eta\pi\pi_0, K\pi\pi$). The selected $D_s \rightarrow K^+ K^- \pi^+$ is comparing between two reconstruction algorithm. Figure 22 shows the mass $M(K^+ K^-)$ of all the selected events between PATTSF and PATTSF+HOUGH. We can see with HOUGH package, the number of signal event enhanced 6.5%. We also generate MC events with both $D_s \rightarrow KK\pi$, figure 23 shows the mass $M(K^+ K^-)$ of all the selected events between PATTSF and PATTSF with the supplementary of HOUGH. The the number of signal event enhanced 6.6%, and is similar with the result in real data.

4 Conclusion

In order to improve the tracking efficiency at low transverse momentum region. We develop a package based on Hough transform. From the performance validation of Monte Carlo and data, the tracking efficiency of low momentum region is improved, and signal of some physics channels is enhanced. The tracking method based on Hough transform has been accepted by BESIII collaboration. The HOUGH tracking package is released in the BOSS and is used for data reconstruction. For further study, we are trying to realize Hough transform in high transverse momentum tracks with CGEM-OUTER chamber detector and to improve the tracking efficiency of the secondary vertex events.

References

1. W. Farr, B. Granz, J. Heintze, R. Heuer, P. Lennert, T. Nozaki, H. Rieseberg, A. Wagner, Nuclear Instruments and Methods **156**(1), 283 (1978)
2. A. Balla, et al., Journal of Instrumentation **9**(01), C01014 (2014)
3. BESIII Collaboration, Nuclear Instruments and Methods in Physics Research Section A (2010)
4. BESIII Collaboration, Design Report of BEPCII (2003)
5. C. Chen, et al., IEEE Nuclear Science Symposium Conference Record (2007)
6. Zhang Yao, et al., HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS **31**(06), 570 (2007)
7. L. Qiu-Guang, Z. Yao, et al., Chinese Physics C **32**(7), 565 (2008)
8. J. Lu-Kui, Z. Yao, et al., Chinese Physics C **34**(12), 1866 (2010)
9. W.L. Yuan, et al., Chinese Physics C **40**(2) (2016)
10. C. Weiman. Method and means for recognizing patterns represented in logarithmic polar coordinates (1991). US Patent 5,063,604
11. W. Ji-Ke, et al., Chinese Physics C **33**(10) (2009)
12. E.W. Weisstein, From MathWorld—A Wolfram Web Resource.
13. T. Alexopoulos, M. Bachtis, E. Gazis, G. Tsipolitis, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **592**(3), 456 (2008)
14. Li Wei-Dong, Liu Huaimin, et al., Proceeding of CHEP06,Mumbai (2006)
15. D. ZY, C. GF, et al., HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS-CHINESE EDITION **30**(5), 371 (2006)