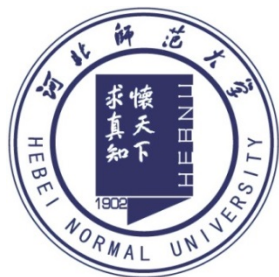


第十五届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会
丹东, 8.21-8.27.2018

Lattice Energy Levels
and Chiral Effective Field Theory



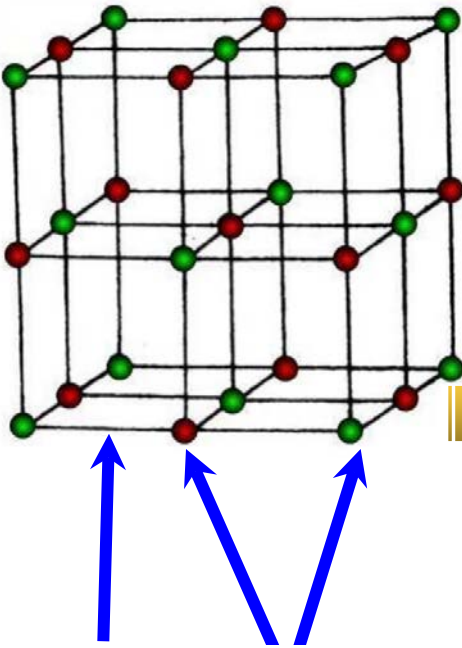
Zhi-Hui Guo (郭志辉)

Hebei Normal University (河北师范大学)

Outline:

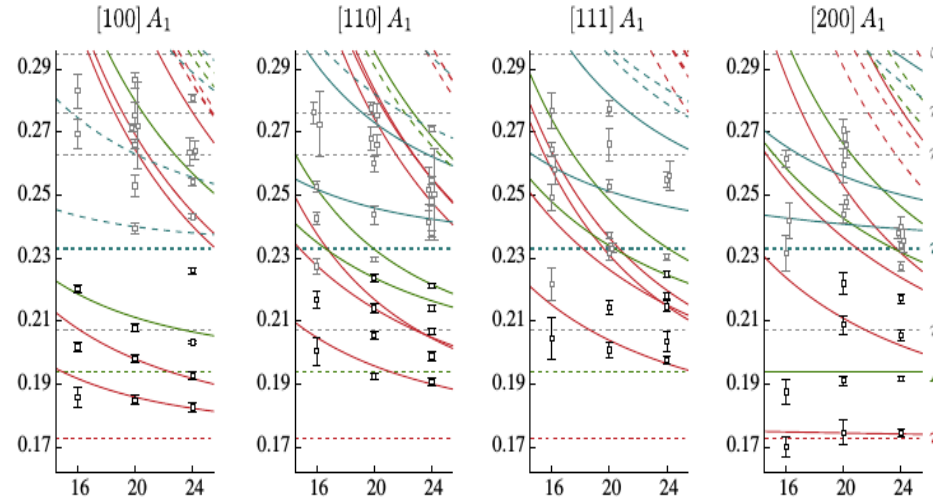
1. Introduction
2. ChPT amplitudes and Finite-volume effects
3. Results and Discussions
4. Summary

Introduction



Significant progress on meson-meson scattering in lattice simulation

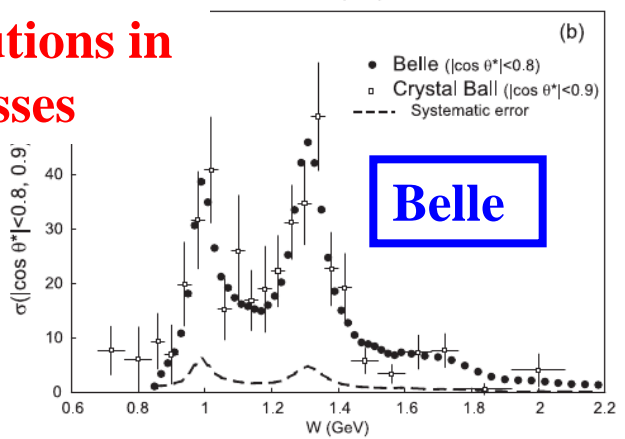
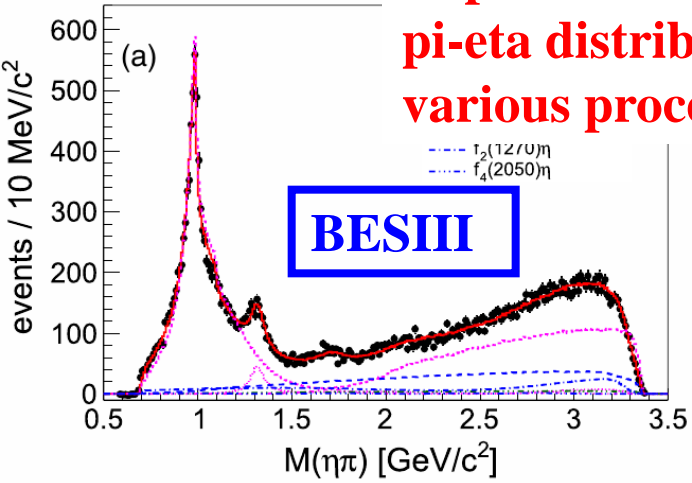
E.g. π - η , $K\bar{K}$, π - η' scattering
 [Dudek, Edwards, Wilson, PRD'16]



Gluon Quark

Data "measured" by lattice: finite-volume energy levels

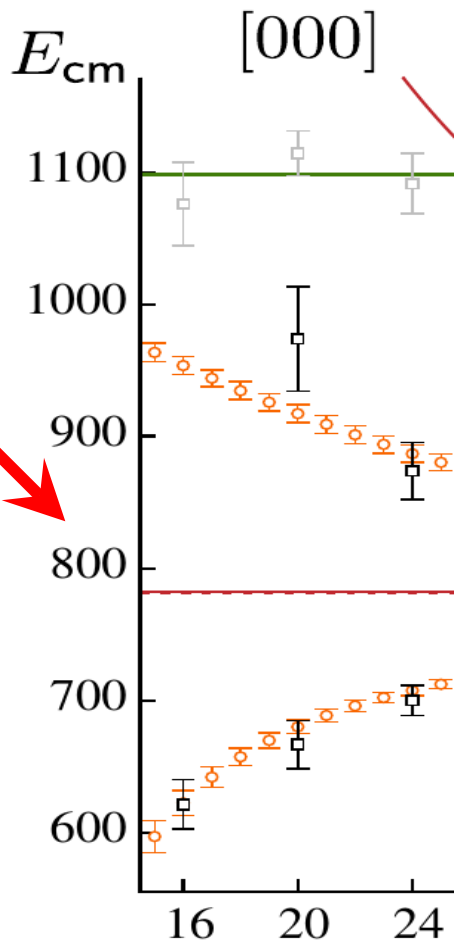
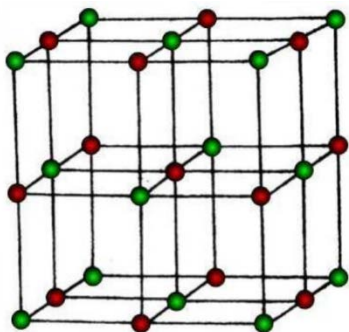
Exp measurement of π - η distributions in various processes



Lattice simulation data:

finite-volume spectra in meson-meson scattering

Eigenenergies
in finite box



An example:
Isoscalar S-wave pi-pi scattering

[Briceno, Dudek, Edwards, Wilson, PRL '17]

Length of
finite box

Luscher's Approach:

connect the discrete spectra in finite box to the scattering amplitudes in the infinite volume

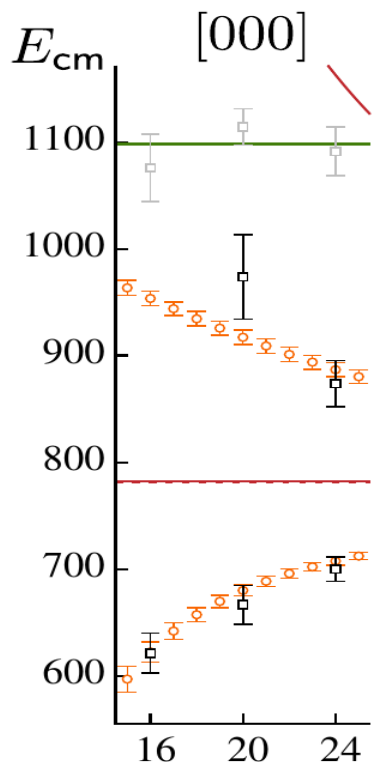
[Luscher, NPB '91]

Elastic scattering case:

$$\tan \phi(q) = - \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

Phase shifts

Luscher's Zeta function
(function of L , parameter free)



- For the elastic case, one has the one-to-one correspondance between the phase shifts and energy levels.
- The one-to-one correspondance will be lost in the inelastic case.

[He, Feng, Liu, JHEP'05] [Wilson, Briceno, Dudek, Edwards, Thomas, PRD '15]

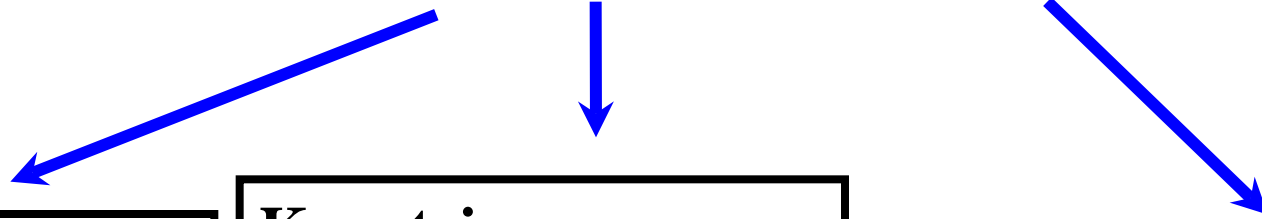
[Lang, Leskovec, Mohler, Prelovsek, Woloshyn, PRD'14] [Fu, PRD'12]

[Gockeler, Horsley, Lage, Meissner, Rakow, Rusetksy, Schierholz, Zanotti, PRD'12]

A widely used approach in the inelastic scattering case:

Luscher function + K matrix

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (\mathbf{1} + i\mathcal{M})] = 0$$



Kinematical factor

K matrix:
polynomial + pole terms

Luscher's finite-volume functions (complex objects)

- Free parameters in K matrix are determined by the finite-volume spectra. Then one can determine amplitudes in infinite volume.
- K matrix does not automatically respect the QCD symmetries, such as the chiral symmetry. It could be problematic for chiral extrapolation.

Our approach:

Step 1: Put chiral perturbation theory (ChPT) in finite volume.

Step 2: The free parameters in ChPT, which are independent of quark masses and volumes, are fitted to the finite-volume energy levels obtained at (un)physical quark masses.

Step 3: Perform the chiral extrapolation and give the predictions in infinite volume with physical quark masses, including phase shifts, inelasticities, resonance poles, etc.

I will mainly focus on the π - η , K - K bar and π - η' coupled-channel S-wave scattering and $a_0(980)$ in this talk.

Preliminary results for the D - π , D - η and D_s - K bar scattering will be also presented.

Status of lowest QCD scalars

$f_0(500)/\sigma$: precise $\pi\pi$ scattering data + dispersive technique + chiral EFT. Well determined pole positions !

[Xiao,Zheng, NPA01] [Caprini,Colangelo,Leutwyler,PRL06]
[Garcia-Martin, et al., PRD11]

$f_0(980)$: $\pi\pi$ scattering data + (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !

[Garcia-Martin, et al., PRD11] [Oller, Oset, Pelaez, PRD99]

$K^*_0(800)/\kappa$: πK scattering data+ (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !

[Zheng, Zhou, et al.,] [Descotes-Genon, Moussallam, EPJC06]
[Pelaez, Rodas, PRD16]

$a_0(980)$: Absence of the $\pi\eta$ scattering data.

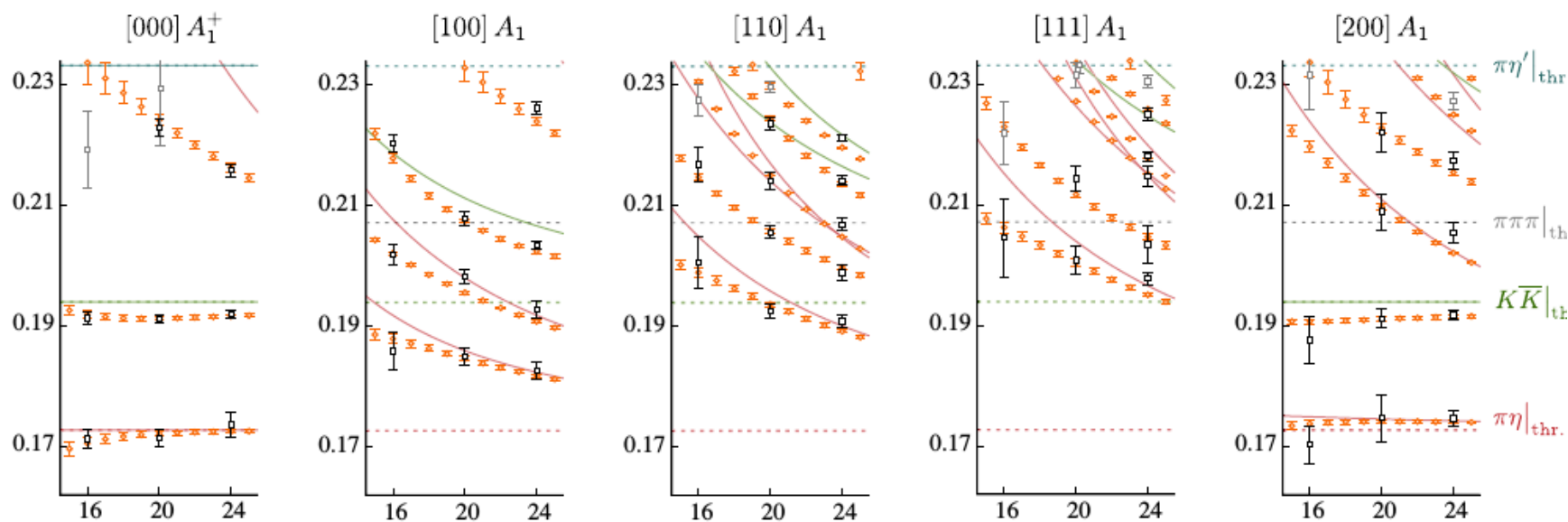
Still controversial: resonance pole or cusp effect ?

[Oller, Oset, PRD99] [Albaladejo, Moussallam, EPJC16] [Guo, Oller, PRD11]

Alternative way: Lattice QCD

The first lattice calculation for $\pi\eta$ scattering:

[Dudek, Edwards, Wilson, PRD16]



- Many finite-volume energy levels are obtained in CM & moving frames
- Lattice box is big enough ($m_\pi L > 3.8$)
- Only one large m_π is used in the simulation ($m_\pi = 391$ MeV)

Unitarized ChPT and its finite-volume effects

Three relevant coupled channels: $\pi\eta$, K - \bar{K} , $\pi\eta'$

In this case, it is essential to generalize from $SU(3)$ to $U(3)$ ChPT

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

Leading order:

$$\mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{3} M_0^2 \ln^2 \det u$$

Leads to a massive η_0

$$\begin{aligned} \eta_8 &= c_\theta \bar{\eta} + s_\theta \bar{\eta}' , \\ \eta_0 &= -s_\theta \bar{\eta} + c_\theta \bar{\eta}' , \end{aligned}$$

$$\sin \theta = - \left(\sqrt{1 + \frac{(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4})^2}{32\Delta^4}} \right)^{-1} \quad \Delta^2 = \bar{m}_K^2 - \bar{m}_\pi^2$$

Instead of using higher order local counterterms,
resonance saturations are assumed in our study.

$$\mathcal{L}_S = c_d \langle S_8 u_\mu u^\mu \rangle + c_m \langle S_8 \chi_+ \rangle + \tilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \tilde{c}_m S_1 \langle \chi_+ \rangle$$

$$\mathcal{L}_V = \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle,$$

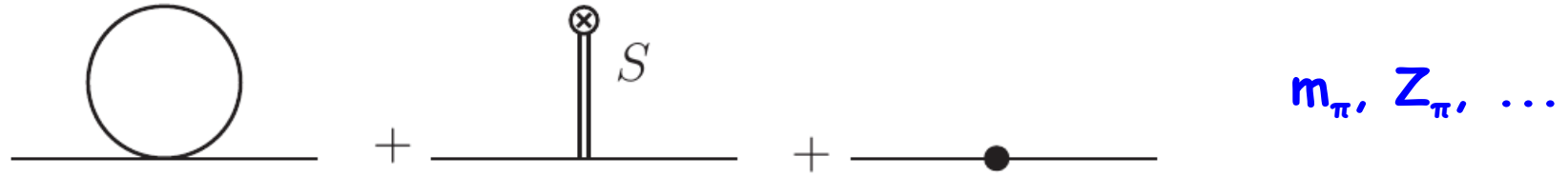
[Ecker, Gasser, Pich, de Rafael, NPB'89]

One important local U(3) operator is also considered:

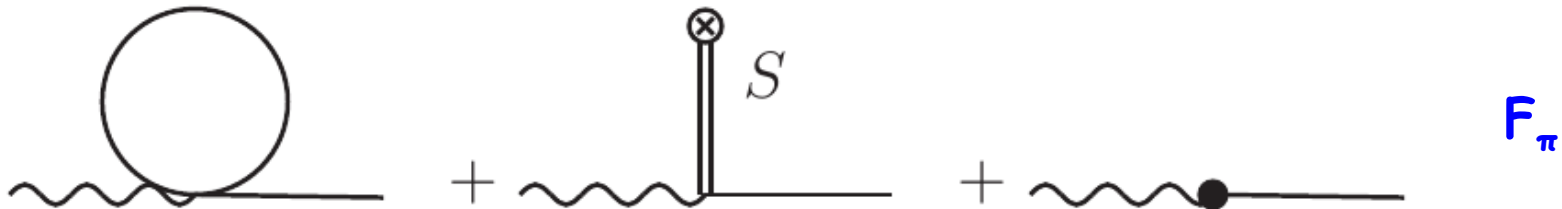
$$-\Lambda_2 \frac{F^2}{12} \langle U^+ \chi - \chi^+ U \rangle \ln \det u^2$$

Meson-meson scattering: $\pi\eta \rightarrow \pi\eta$, $\pi\eta \rightarrow \text{KKbar}$, $\pi\eta \rightarrow \pi\eta'$

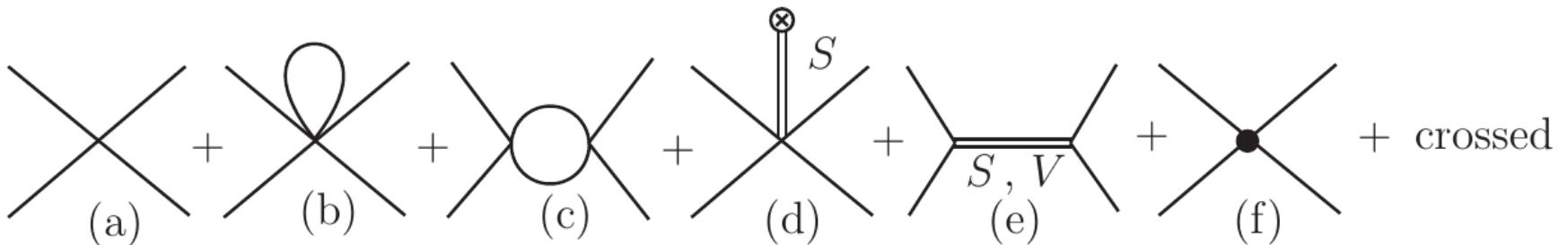
Self energy :



Goldstone decay constant :



Scattering amplitude :



Leading order amplitudes:

$$T_{J=0}^{I=1, \pi\eta \rightarrow \pi\eta}(s)^{(2)} = \frac{(c_\theta - \sqrt{2}s_\theta)^2 m_\pi^2}{3F_\pi^2},$$

$$T_{J=0}^{I=1, \pi\eta \rightarrow K\bar{K}}(s)^{(2)} = \frac{c_\theta(3m_\eta^2 + 8m_K^2 + m_\pi^2 - 9s) + 2\sqrt{2}s_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2},$$

$$T_{J=0}^{I=1, \pi\eta \rightarrow \pi\eta'}(s)^{(2)} = \frac{(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)m_\pi^2}{3F_\pi^2},$$

$$T_{J=0}^{I=1, K\bar{K} \rightarrow K\bar{K}}(s)^{(2)} = \frac{s}{4F_\pi^2},$$

$$T_{J=0}^{I=1, K\bar{K} \rightarrow \pi\eta'}(s)^{(2)} = \frac{s_\theta(3m_{\eta'}^2 + 8m_K^2 + m_\pi^2 - 9s) - 2\sqrt{2}c_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2},$$

$$T_{J=0}^{I=1, \pi\eta' \rightarrow \pi\eta'}(s)^{(2)} = \frac{(\sqrt{2}c_\theta + s_\theta)^2 m_\pi^2}{3F_\pi^2},$$

Unitarization: Algebraic approximation of N/D (a variant version of K-matrix) [Oller, Oset, PRD '99]

$$T_J(s) = \frac{N(s)}{1 + G(s) N(s)}$$

- **The s-channel unitarity is exact. The crossed-channel dynamics is included in a perturbative manner.**

- **Unitarity condition:** $\text{Im}G(s) = -\rho(s)$

$$G(s) = a^{SL}(s_0) - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds'$$

- **$N(s)$: given by the partial wave chiral amplitudes**

$$\mathcal{V}_{J, D_1\phi_1 \rightarrow D_2\phi_2}^{(S, I)}(s) = \frac{1}{2} \int_{-1}^{+1} d \cos \varphi P_J(\cos \varphi) V_{D_1\phi_1 \rightarrow D_2\phi_2}^{(S, I)}(s, t(s, \cos \varphi)).$$

Finite-volume effects

Two types of finite volume dependence of scattering amplitudes:

- Exponentially suppressed type $\propto \exp(-m_p L)$: *s, t, u* channels
- Power suppressed type $\propto 1/L^3$: **only s channel**

We ignore the exponentially suppressed terms, indicating that finite-volume effects only enter through s channel.

$$T_J(s) = \frac{N(s)}{1 + G(s) N(s)}$$

I.e. We only consider the finite-volume corrections for $\mathbf{G}(s)$.

$$G(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2 + i\epsilon)[(P - q)^2 - m_2^2 + i\epsilon]}, \quad s \equiv P^2$$

Sharp momentum cutoff to regularize $G(s)$

$$G(s)^{\text{cutoff}} = \int^{|\vec{q}| < q_{\text{max}}} \frac{d^3 \vec{q}}{(2\pi)^3} I(|\vec{q}|), \quad \begin{aligned} I(|\vec{q}|) &= \frac{w_1 + w_2}{2w_1 w_2 [E^2 - (w_1 + w_2)^2]}, \\ w_i &= \sqrt{|\vec{q}|^2 + m_i^2}, \quad s = E^2 \end{aligned}$$

$G(s)$ in a finite box of length L with periodic boundary condition

$$\tilde{G} = \frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}| < q_{\text{max}}} I(|\vec{q}|), \quad \vec{q} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

Finite-volume correction ΔG [Doring, Meissner, Oset, Rusetsky, EPJA11]

$$\begin{aligned} \Delta G &= \tilde{G} - G^{\text{cutoff}} \\ &= \left\{ \frac{1}{L^3} \sum_{\vec{q}}^{|\vec{q}| < q_{\text{max}}} - \int^{|\vec{q}| < q_{\text{max}}} \frac{d^3 \mathbf{q}}{(2\pi)^3} \right\} \frac{1}{2\omega_1(\mathbf{q})\omega_2(\mathbf{q})} \frac{\omega_1(\mathbf{q}) + \omega_2(\mathbf{q})}{E^2 - (\omega_1(\mathbf{q}) + \omega_2(\mathbf{q}))^2} \end{aligned}$$

Finite-volume effects in the moving frames

Lorentz invariance is lost in finite box. One needs to work out the explicit form of the loops when boosting from one frame to another.

transforming $\vec{q}_{i=1,2}$ to $\vec{q}_{i=1,2}^*$ \longrightarrow **CM quantities**

$$\vec{q}_i^* = \vec{q}_i + \left[\left(\frac{P^0}{E} - 1 \right) \frac{\vec{q}_i \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_i^0}{E} \right] \vec{P}$$

moving frame with total four-momentum $P^\mu = (P^0, \vec{P})$ $s = E^2 = (P^0)^2 - |\vec{P}|^2$

Impose on-shell condition

$$q_i^{*0} = \sqrt{|\vec{q}_i^*|^2 + m_i^2}$$

$$q_i^0 = \frac{q_i^{*0} E + \vec{q}_i \cdot \vec{P}}{P^0} \longrightarrow q_i^0 = \sqrt{|\vec{q}_i|^2 + m_i^2}$$

G function in the moving frame

$$\int_{|\vec{q}_1|^* < q_{\max}} \frac{d^3 \vec{q}_1^*}{(2\pi)^3} I(|\vec{q}_1^*|) \implies \tilde{G}^{\text{MV}} = \frac{E}{P^0 L^3} \sum_{\vec{q}_1}^{|\vec{q}_1^*| < q_{\max}} I(|\vec{q}_1^*(\vec{q}_1)|) \quad \begin{aligned} \vec{q}_1 &= \frac{2\pi}{L} \vec{n}, & \vec{n} &\in \mathbb{Z}^3, \\ \vec{P} &= \frac{2\pi}{L} \vec{N}, & \vec{N} &\in \mathbb{Z}^3 \end{aligned}$$

Finite-volume correction ΔG^{MV} :

$$\Delta G^{\text{MV}} = \tilde{G}^{\text{MV}} - G^{\text{cutoff}}$$

Mixing of different partial waves in finite volume

The mixing is absent in the infinite volume:
$$\int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta Y_{\ell m}(\theta, \phi) Y_{\ell' m'}^*(\theta, \phi) = \delta_{\ell\ell'} \delta_{mm'}$$

The mixing appears in finite-volume case, due to the absence of the general orthogonal conditions.

Finite-volume correction to G function:

$$\Delta G_{\ell m}^{\text{MV}} = \tilde{G}_{\ell m}^{\text{MV}} - G^{\text{cutoff}}$$

$$\tilde{G}_{\ell m}^{\text{MV}} = \sqrt{\frac{4\pi}{2\ell+1}} \frac{1}{L^3} \frac{E}{P^0} \sum_{\vec{n}}^{|\vec{q}^*| < q_{\text{max}}} \left(\frac{|\vec{q}^*|}{|\vec{q}^{\text{bn}*}|} \right)^\ell Y_{\ell m}(\hat{q}^*) I(|\vec{q}^*|)$$

Final expression for the G function:

$$\tilde{G}_{\ell m} = G^{\text{Infinite volume}} + \Delta G_{\ell m}^{\text{MV}}$$

To determine the energy levels in different frames with only S and P waves:

$$\mathbf{A}_1^+ (0,0,0): \det[I + N_0(s) \cdot \tilde{G}_{00}] = 0$$

$$\mathbf{T}_1^- (0,0,0): \det[I + N_1(s) \cdot (\tilde{G}_{00} + 2\tilde{G}_{20})] = 0$$

$$\mathbf{A}_1 (0,0,1): \det[I + N_{0,1} \cdot \mathcal{M}_{0,1}^{A_1}] = 0, \quad N_{0,1} = \begin{pmatrix} N_0 & 0 \\ 0 & N_1 \end{pmatrix}, \quad \mathcal{M}_{0,1}^{A_1} = \begin{pmatrix} \tilde{G}_{00} & i\sqrt{3}\tilde{G}_{10} \\ -i\sqrt{3}\tilde{G}_{10} & \tilde{G}_{00} + 2\tilde{G}_{20} \end{pmatrix}$$

.....

Results and Discussions

for π - η , $K\bar{K}$, π - η' scattering

Fits to lattice finite-volume energy levels

[Dudek, Edwards, Wilson, PRD16]

$$m_\pi = 391.3 \pm 0.7 \text{ MeV}, \quad m_K = 549.5 \pm 0.5 \text{ MeV}, \quad m_\eta = 587.2 \pm 1.1 \text{ MeV}, \quad m_{\eta'} = 929.8 \pm 5.7 \text{ MeV}$$

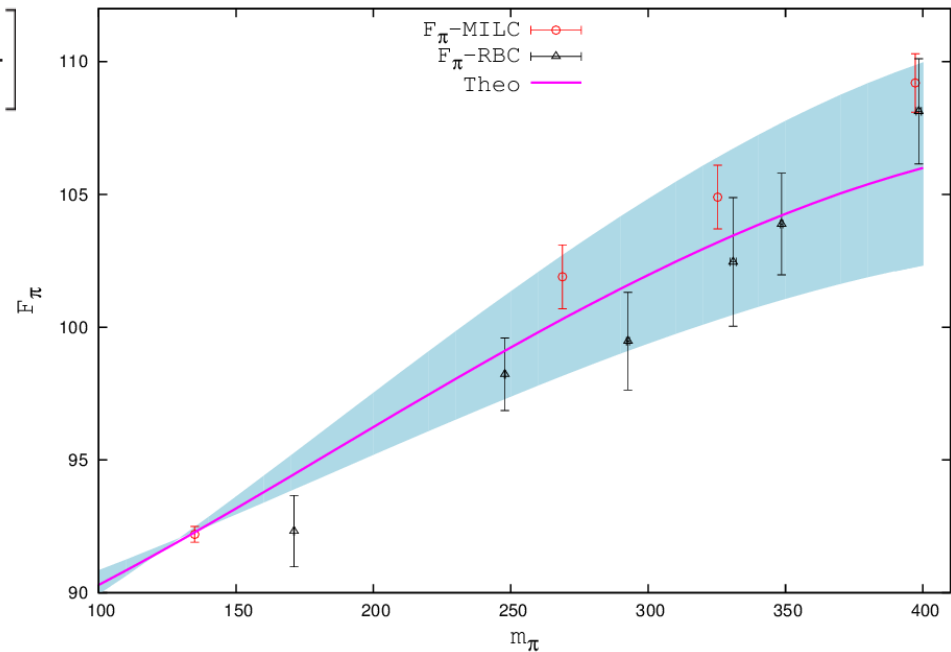
Our estimate of the leading order η - η' mixing angle at unphysical masses

$$\theta = (-10.0 \pm 0.1)^\circ \quad (\theta^{\text{phys}} = -16.2^\circ)$$

We also need to estimate F_π at the unphysical meson masses.

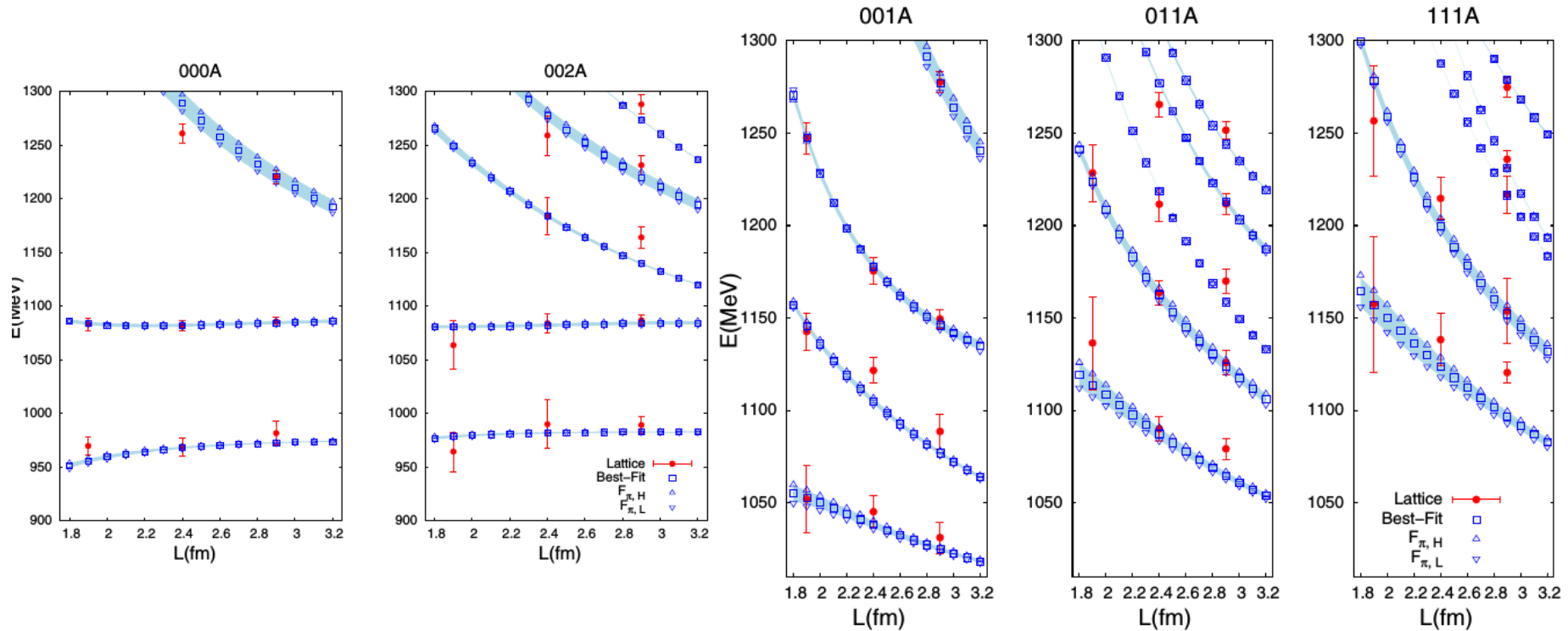
$$F_\pi = F \left\{ 1 - \frac{1}{16\pi^2 F^2} \left[m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} + \frac{m_K^2}{2} \ln \frac{m_K^2}{\mu^2} \right] + \left[\frac{4\tilde{c}_d \tilde{c}_m (m_\pi^2 + 2m_K^2)}{F^2 M_{S_1}^2} - \frac{8c_d c_m (m_K^2 - m_\pi^2)}{3F^2 M_{S_8}^2} \right] \right\}$$

[Guo, Oller, PRD'11]



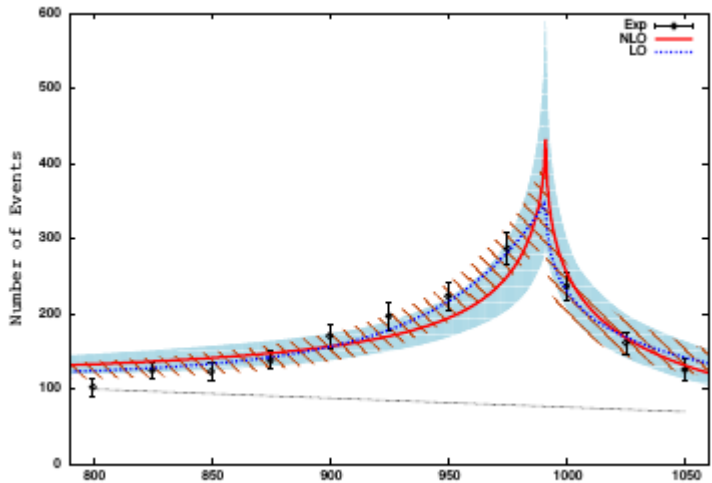
Leading order Fit (only LO amplitudes are included in the N(s) function.)

[ZHG, Liu, Meissner, Oller, Rusetsky, PRD'17]

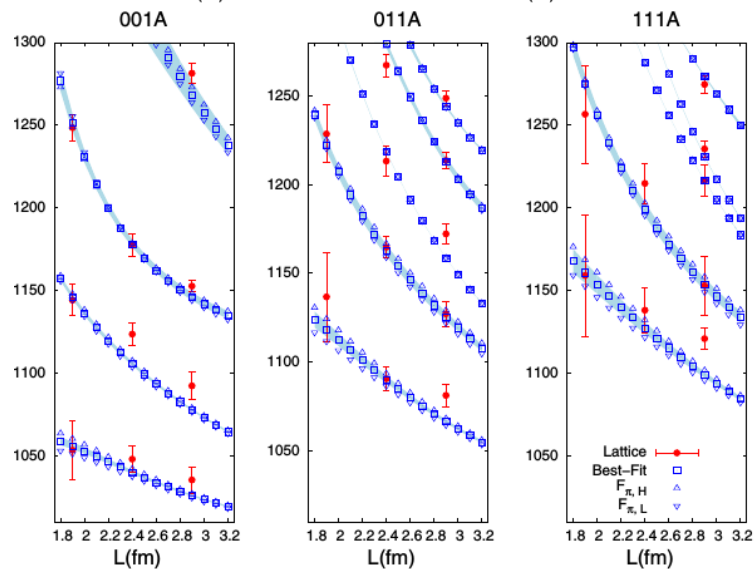
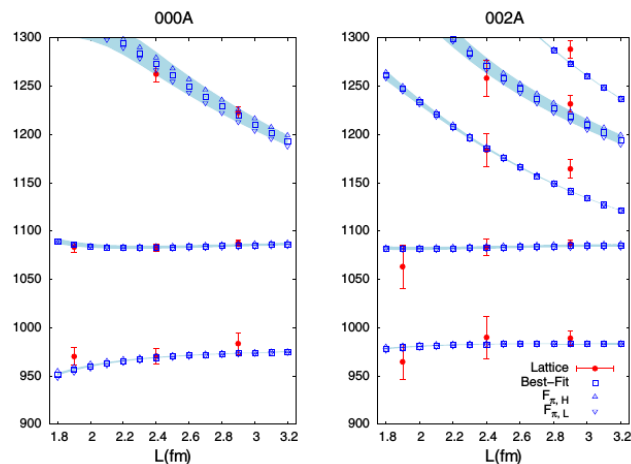
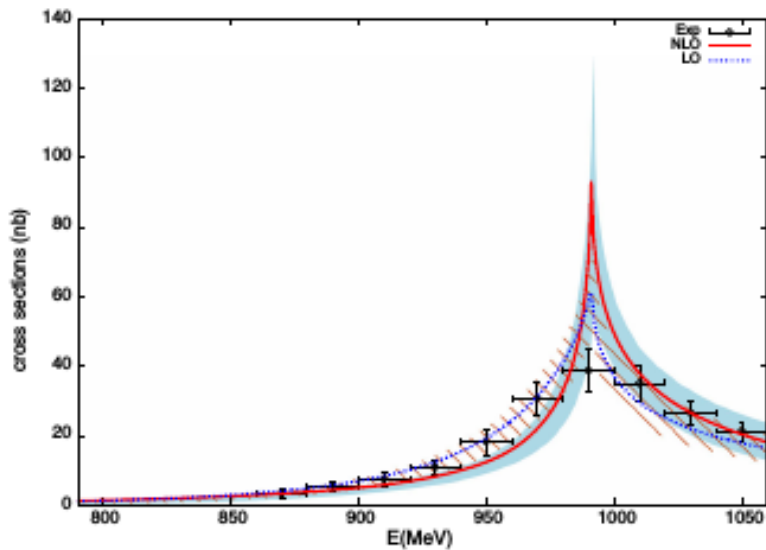


Remark: there is only one free parameter in the fits, i.e. the common subtraction constant !

$$\frac{dN_{\pi\eta}}{dE_{\pi\eta}} = q_{\pi\eta} |c_1 D^{-1}(s)_{\pi\eta \rightarrow \pi\eta} + c_2 D^{-1}(s)_{\pi\eta \rightarrow K\bar{K}}|^2$$



$$\sigma(s) = \frac{\alpha^2 q_{\pi\eta}}{2s^{3/2}} |c'_1 D^{-1}(s)_{\pi\eta \rightarrow \pi\eta} + c'_2 D^{-1}(s)_{\pi\eta \rightarrow K\bar{K}}|^2$$



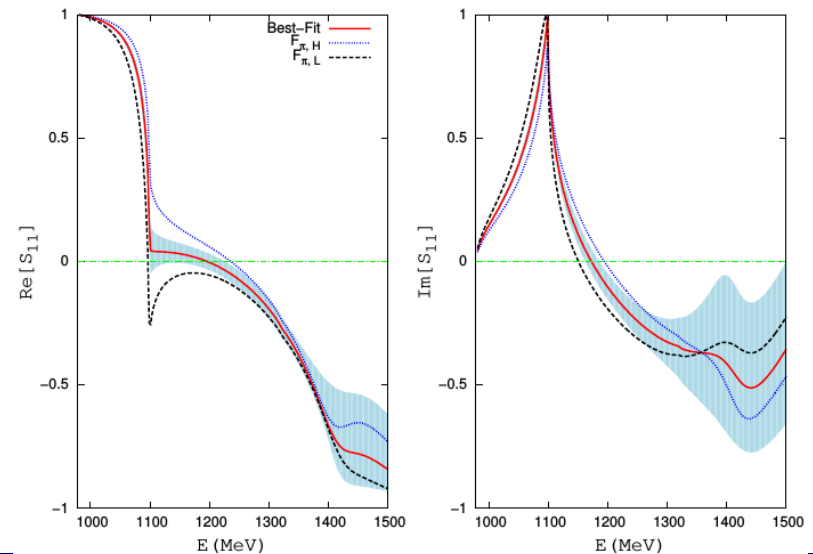
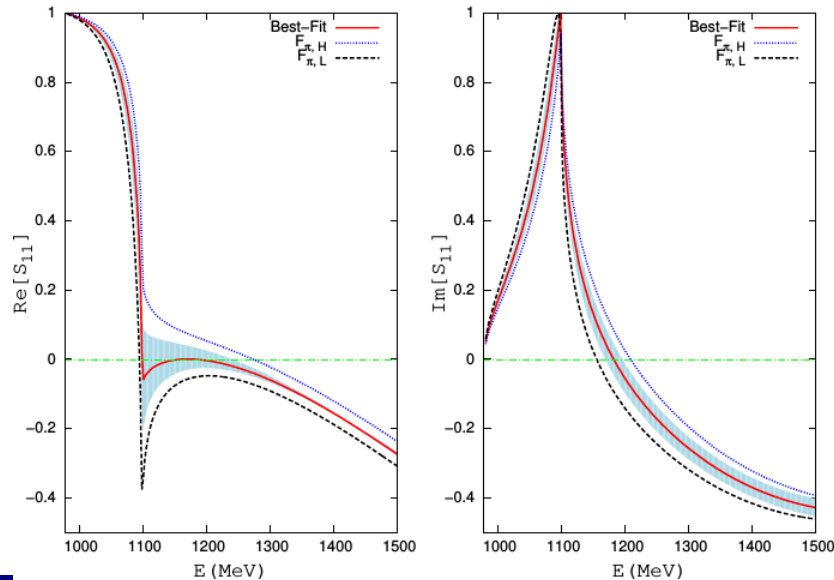
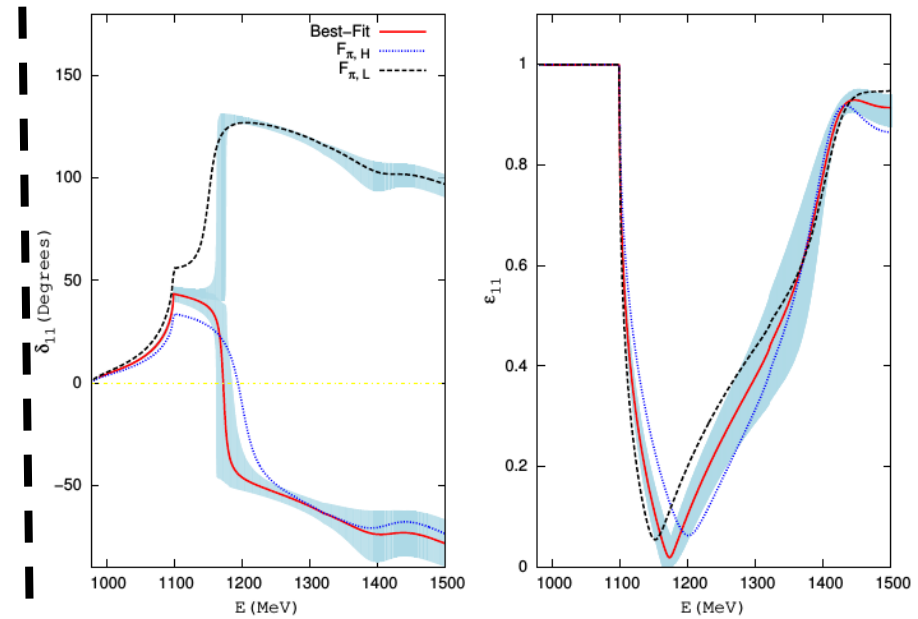
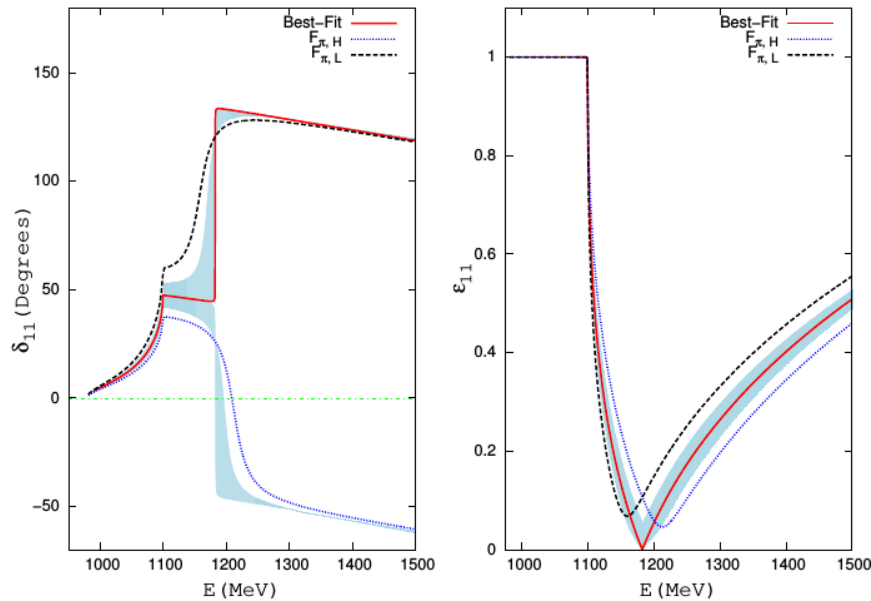
Next-to-Leading order Fit (Both loops and resonance exchanges are included in the N(s) function.) Similar fit quality from the two cases.

Phase shifts and inelasticities at unphysical meson masses

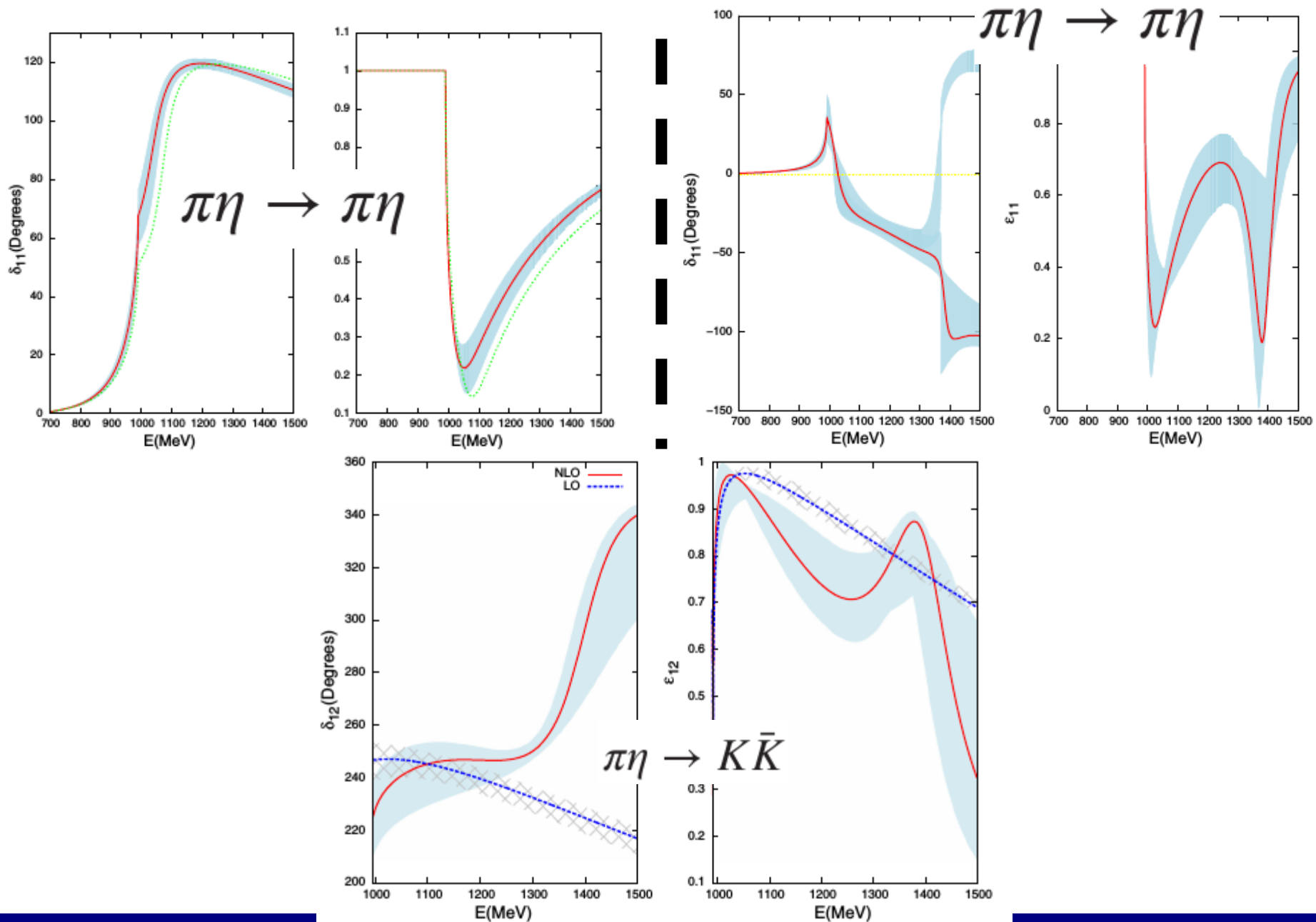
$$S = 1 + 2i\sqrt{\rho(s)} \cdot \mathcal{T}(s) \cdot \sqrt{\rho(s)}$$

$$S_{kk} = \varepsilon_{kk} e^{2i\delta_{kk}},$$

$$S_{kl} = i\varepsilon_{kl} e^{i\delta_{kl}}$$



Phase shifts and inelasticities at physical meson masses



Pole positions and residues at physical meson masses

Resonances are uniquely characterized by their poles and residues in the complex energy plane.

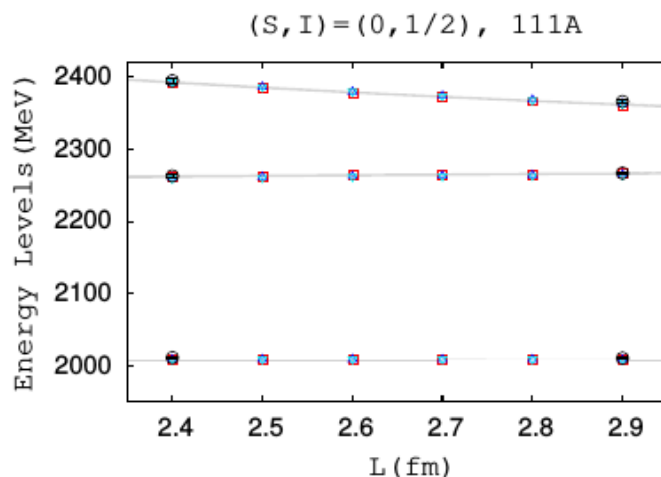
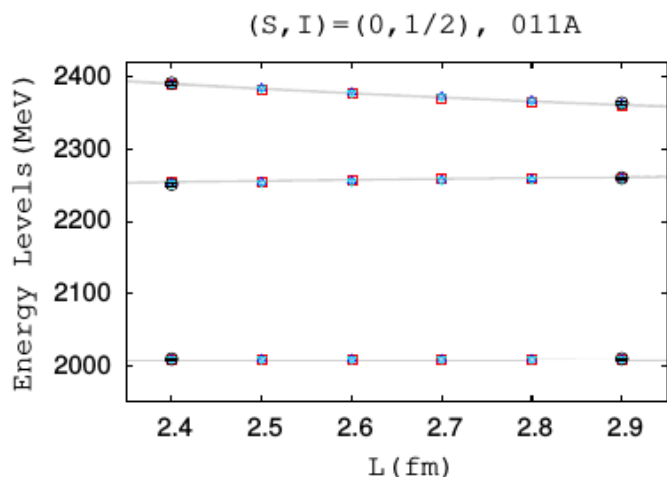
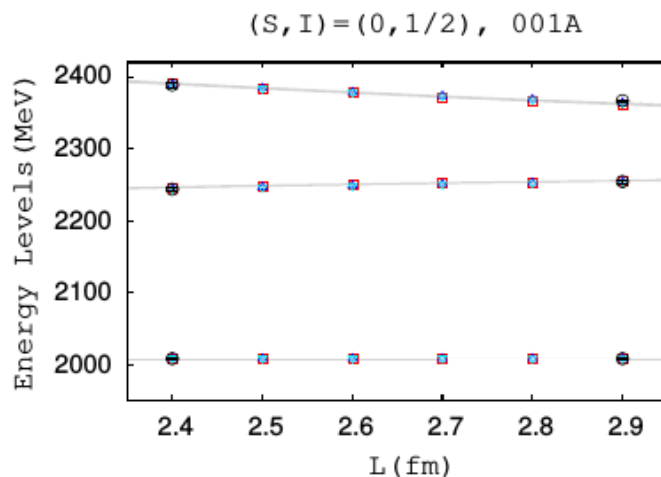
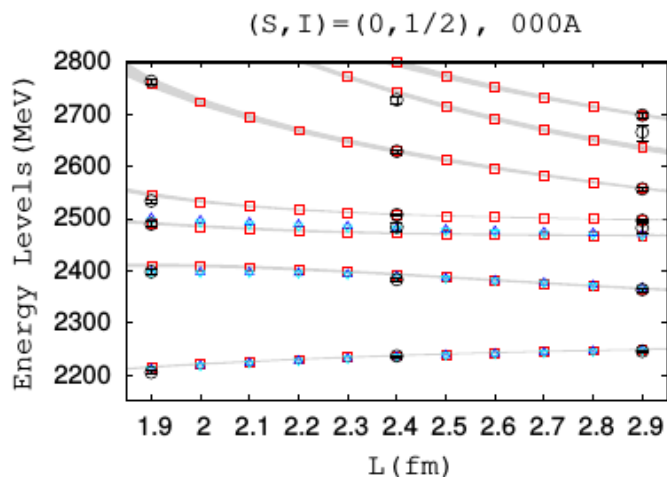
Resonance	RS	Mass (MeV)	Width/2 (MeV)	$ \text{Residue} _{\pi\eta}^{1/2}$ (GeV)	Ratios
LO					
$a_0(980)$	II	1037^{+17}_{-14}	44^{+6}_{-9}	$3.8^{+0.3}_{-0.2}$	$1.43^{+0.03}_{-0.03}$ ($K\bar{K}/\pi\eta$) $0.05^{+0.01}_{-0.01}$ ($\pi\eta'/\pi\eta$)
NLO					
$a_0(980)$	IV	1019^{+22}_{-8}	24^{+57}_{-17}	$2.8^{+1.4}_{-0.6}$	$1.8^{+0.1}_{-0.3}$ ($K\bar{K}/\pi\eta$) $0.01^{+0.06}_{-0.01}$ ($\pi\eta'/\pi\eta$)
$a_0(1450)$	V	1397^{+40}_{-27}	62^{+79}_{-8}	$1.7^{+0.3}_{-0.4}$	$1.4^{+2.4}_{-0.6}$ ($K\bar{K}/\pi\eta$) $0.9^{+0.8}_{-0.2}$ ($\pi\eta'/\pi\eta$)

[ZHG, Liu, Meissner, Oller, Rusetsky, PRD'17]

Preliminary results for the D-pi, D-eta, Ds-Kbar scattering

Reproduction of the finite-volume energy levels

[Moir, Peardon, Ryan, Thomas, Wilson, JHEP'16]



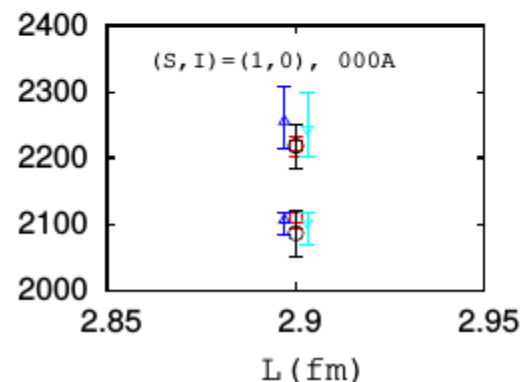
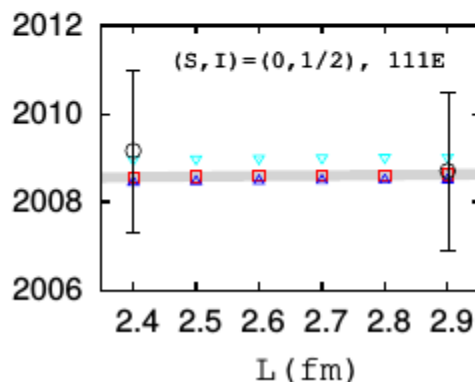
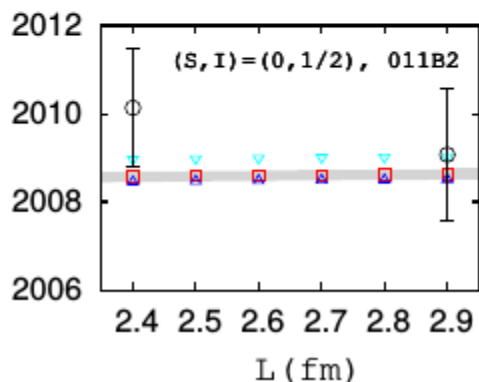
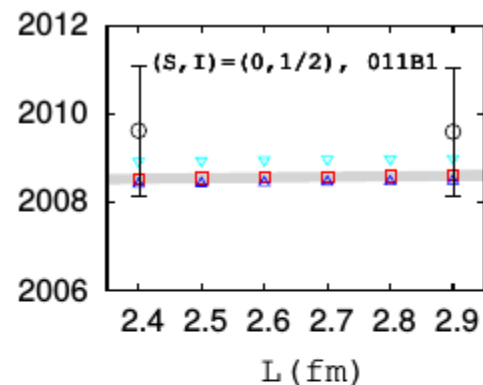
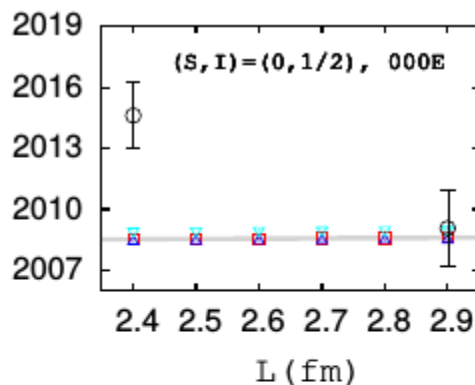
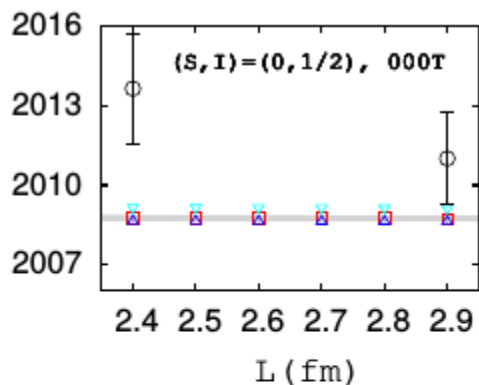
Preliminary

[ZHG, et al., in preparation]

Reproduction of the finite-volume energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]

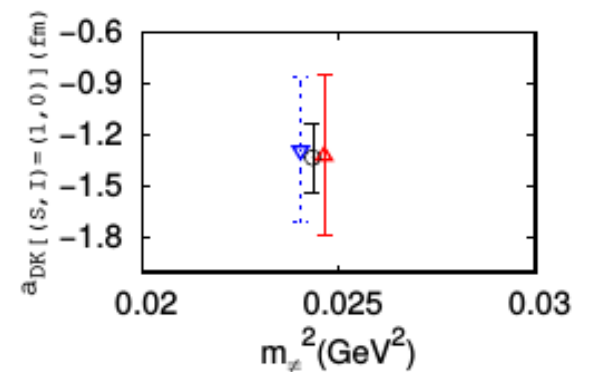
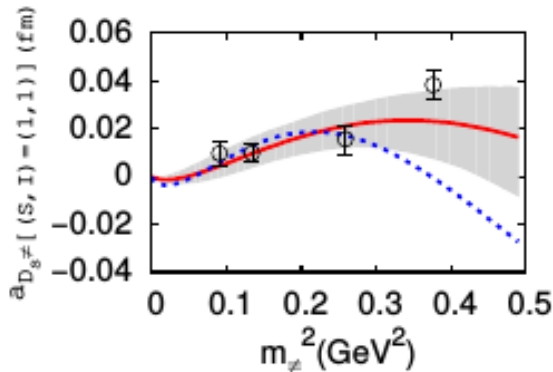
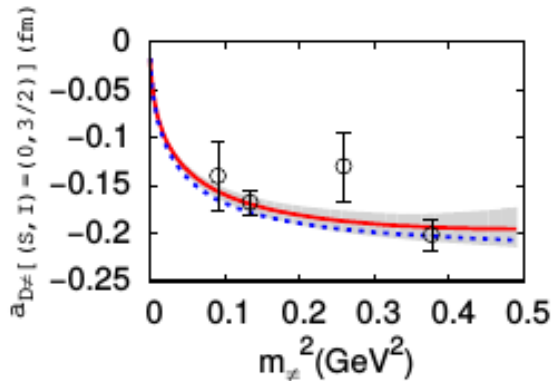
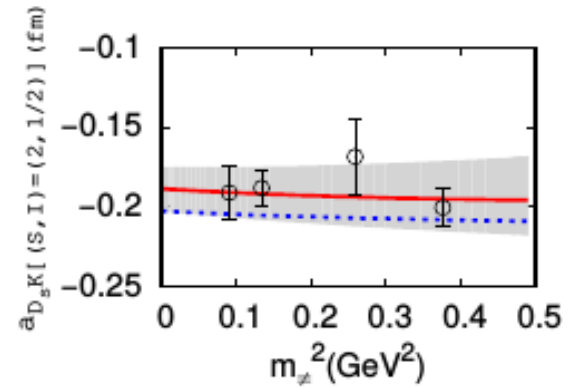
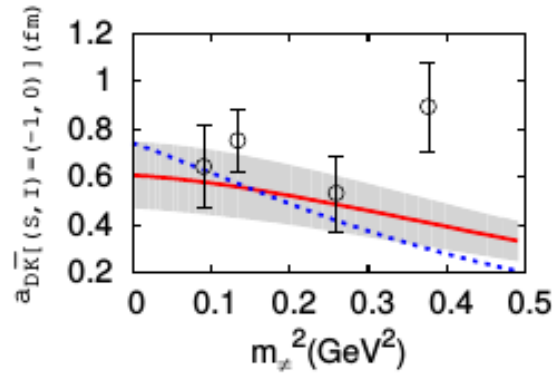
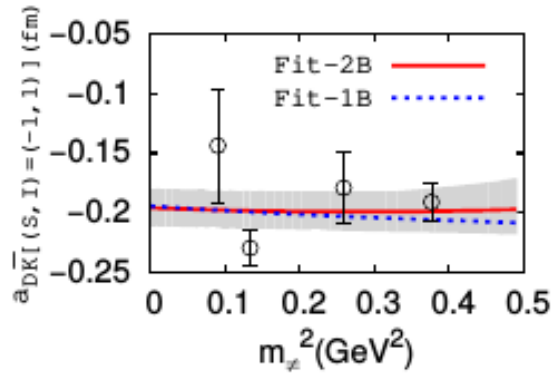


Preliminary

Reproduction of scattering lengths

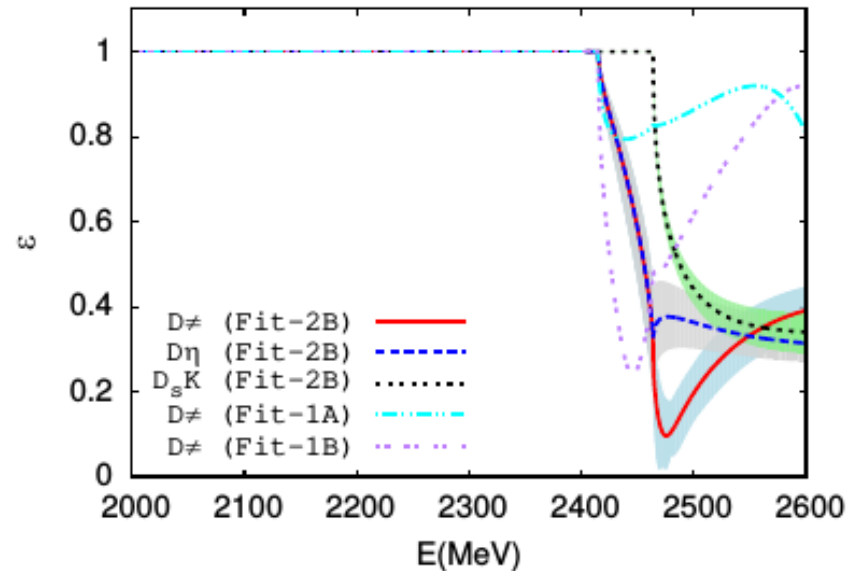
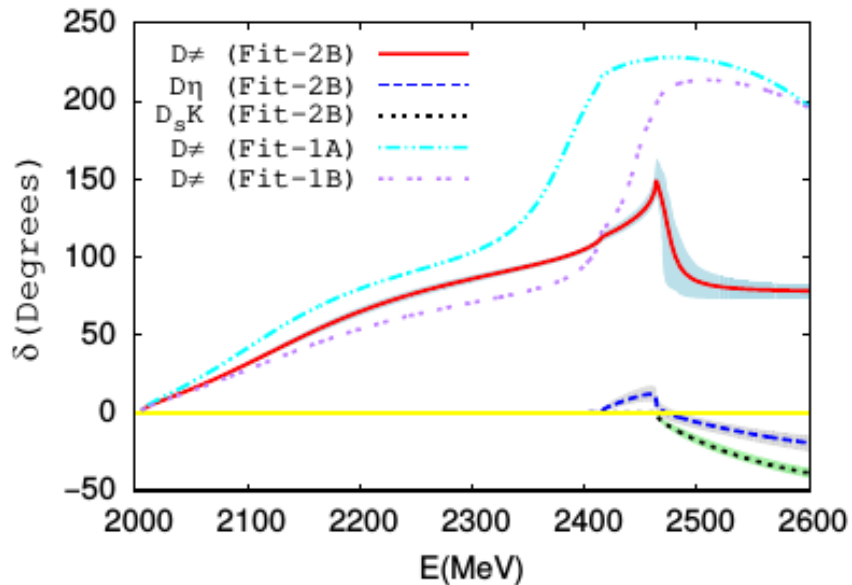
[L.Liu,Orginos,F.K.Guo,Meissner, PRD'13]

[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



Preliminary

Preliminary results of the D-pi phase shifts and inelasticities at physical meson masses



[ZHG, et al., in preparation]

Summary

- **The chiral approach illustrated in this talk provides an efficient way to study the finite-volume energy levels.**
- **It can build a bridge to connect the lattice eigenenergies in finite box obtained at unphysical masses with the physical observables, such as phase shifts, inelasticities, at physical meson masses.**
- **We have successfully applied this approach to the π - η , K - \bar{K} and π - η' coupled-channel scattering.**
- **Similar study in other systems is in progress.**

Thanks for your attention!