Chiral-scale perturbation theory and its applications in dense matter

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Aug. 21-27, 2018 Dandong，Liaoning
第十五届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会

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➢ Introduction

I. The scalar meson conundrum

Scalar meson with $m_\sigma \cong 600 MeV$,very important in nuclear physic.

- PDG 2016 $f_0(500)$:

m=400~550 MeV
Γ=400~700 MeV
But, a local bosonic field work;
e.g. Bonn boson exchange potential;
RMF

- QCD structure:

$\bar{q}q, (qq)(\bar{q}\bar{q}), G^2, mixing$?

In LσM, m≥1GeV, irrelevant for physics below $4\pi f_\pi$,
4^{th} component of chiral four-vector

Walecka model,chiral singlet

Chiral perturbation theory has been widely used in particle and nuclear physics.

$$SU(N_f)_L \times SU(N_f)_R$$

↓ Absence of parity partner

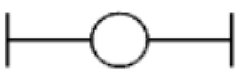
$$SU(N_f)_V + (N_f{}^2 - 1)\ \text{NGBs}$$
$$U(x) = e^{i\pi/f_\pi}$$

→ Chiral perturbation theory: power counting, derivative expansion — π,k...

Weinberg, 79; Gasser and Leutwyler, 84

↓

$$U(x) = \xi_L \quad h(x)\, h(x)^\dagger \quad \xi_R{}^\dagger$$



$K = 1: \quad \pi, \rho$

Son & Stephanov, 04

→ Effective theory of vector mesons, Hidden local symmetry — ρ,k*,ϕ...

Bando, *et al* 89; Harada & Yamawaki, 03

Chiral effective theory of baryon, extract the baryon mass in chiral limit, HBChPT， Chiral effective theory of heavy meson. — p,n...

How about the scalar meson,especially the lightest one $f_0(500)$?

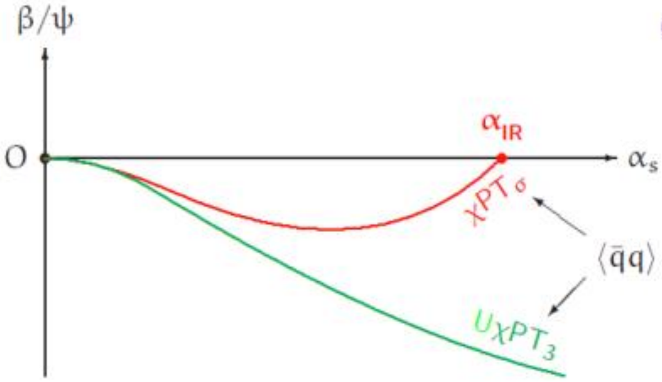
II. The lightest scalar meson as a NGB

$f_0(500)$ is a pNGB (noted $m_{f_0} \cong m_K$) arising from:

The SB of scale invariance + an explicit breaking of SI.

EB of SI: Departure of α_s from IRFP+ current quark mass.

Assumption: There is an IR fixed point in the running QCD coupling constant α_s .



$$\theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^2 + (1+\gamma_m) \sum_{q=u,d,s} m_q \bar{q} q$$

IR fixed point: $\beta(\alpha_{IR}) = 0$

$\chi PT_\sigma \Rightarrow$ expand in θ^μ_μ

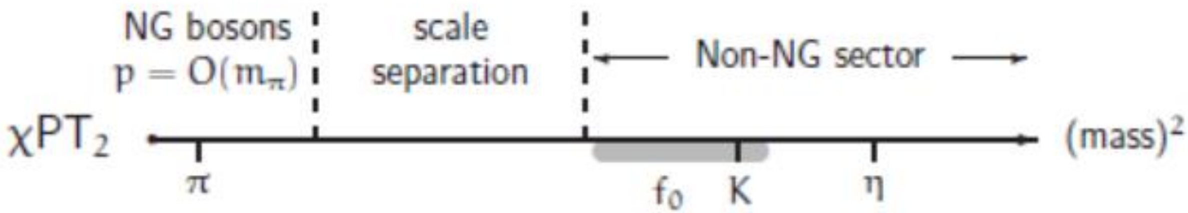
$\alpha_s \lesssim \alpha_{IR}$, $m_{u,d,s} \sim 0$

about scale-*dependent* $|vac\rangle$

$\Rightarrow$ NG bosons π, K, η, σ.

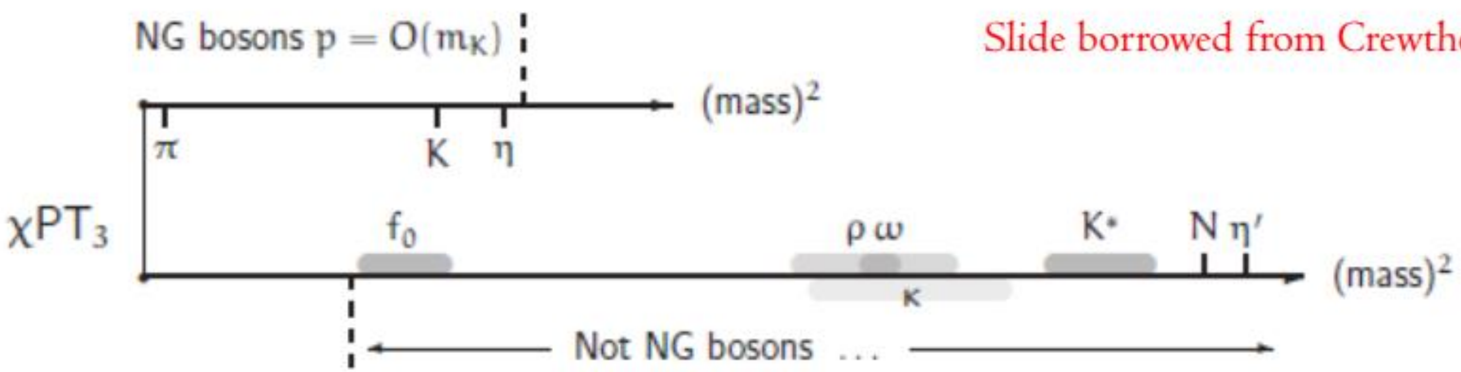
Crewther and Tunstall,
PRD91,034016

Different types of χPT

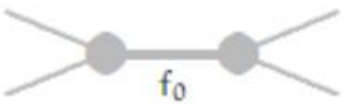


Standard χPT_2 OK: $m_s \neq 0$ stops θ^μ_μ from vanishing. Consistent with either χPT_3 or χPT_σ.

Slide borrowed from Crewther

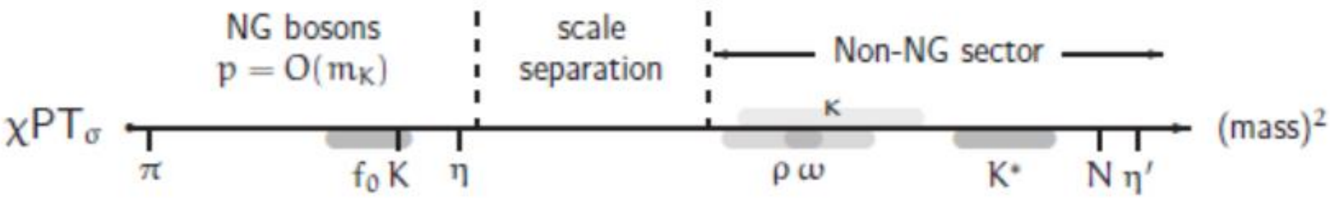


Assumes that α_{IR} does not exist. Only π, K, η are massless in the $SU(3)\times SU(3)$ limit. The small mass of the f_0 resonance is an unexplained accident $\Rightarrow$ lack of scale separation for $p = O(m_K)$ $\Rightarrow$ expansions in 0^{++} channels *diverge*.



$$\mathcal{A} = \left\{ \mathcal{A}_{LO} + \mathcal{A}_{NLO} + \cancel{\mathcal{A}_{NNLO}} + \cancel{\ldots} \right\}_{\chi PT_3} \quad \Rightarrow \quad U\chi PT_3$$

bad fit — large retrofit — ... abandon PT and unitarize



Use if α_{IR} exists. Promote f_0 to NG sector as dilaton σ: massless π, K, η and stable f_0/σ (decay phase space = 0) in the chiral-scale limit, so OK to use local field σ in $\mathcal{L}_{eff}$. Scale separation is restored. More ambitious than $U\chi PT_3$.

Let d denote the scaling dimension of operators used to construct an effective chiral-scale Lagrangian.

L_{inv}: Scale invariant term

L_{mass}: Simulate explicit breaking of chiral symmetry by the quark mass term

$$d_{\text{mass}} = 3 - \gamma_m(\alpha_{\text{IR}}), \qquad 1 \leq d_{\text{mass}} < 4$$

L_{anom}: Account for gluonic interactions responsible for the strong trace anomaly

$$d_{\text{anom}} = 4 + \beta'(\alpha_{\text{IR}}) > 4.$$

$$\mathcal{L}_{\chi\text{PT}_\sigma} = \mathcal{L}_{\text{inv}}^{\text{d=4}} + \mathcal{L}_{\text{anom}}^{\text{d>4}} + \mathcal{L}_{\text{mass}}^{\text{d<4}}.$$

➢ Chiral-scale perturbation theory

The procedure of the construction of χPT_{σ}:

- Since χPT_{σ}, the NGBs are π, K and σ, one first writes down all possible derivative terms acting on these particles and counting each derivative as chiral-scale order $O(p)$.
- In the same way as in the standard χPT, the current quark mass is counted as chiral-scale order $O(p^2)$.
- Moreover, since the theory is constructed for α_s below but near the IR fixed point, one should expand $\beta(\alpha_s)$ and the quark mass anomalous dimension $\gamma(\alpha_s)$ around the IR fixed point $\alpha_s(IR)$ and counting $\Delta\alpha_s = \alpha_s - \alpha_{IR}$ as chiral-scale order $O(p^2)$ since it is proportional to m_σ^2 .

◆ It's convenient to use conformal compensator χ

$$\chi_\mu \to \lambda^{-1}\chi_\mu ; \chi(x) \to \lambda\chi(\lambda^{-1}x); \chi(x) = f_\sigma e^{\sigma/f_\sigma};$$ chiral singlet

$$\sigma(x) \to \sigma(\lambda^{-1}x) + f_\sigma \ln\lambda$$ Nonlinear realization of scale symmetry

◆ Chiral field $U(x) = e^{i\pi/f_\pi}$

Chiral transformation: $U(x) \to g_L U(x) g_R^+$

Scale transformation: $U(x) \to U(\lambda^{-1}x)$

χPT_σ at leading order

Yan-Ling Li，Yong-Liang Ma, Mannque Rho.
Phys.Rev. D95 (2017) no.11, 114011

$$\mathcal{L}^{\rm LO}_{\chi{\rm PT}_\sigma} = \mathcal{L}^{d=4}_{\chi{\rm PT}_\sigma;{\rm inv}} + \mathcal{L}^{d>4}_{\chi{\rm PT}_\sigma;{\rm anom}} + \mathcal{L}^{d<4}_{\chi{\rm PT}_\sigma;{\rm mass}},$$

with

$$\mathcal{L}^{d=4}_{\rm inv,LO} = c_1 \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2}c_2 \partial_\mu\chi\partial^\mu\chi + c_3\left(\frac{\chi}{f_\sigma}\right)^4,$$

$$\mathcal{L}^{d>4}_{\rm anom,LO} = (1-c_1)\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{2+\beta'} {\rm Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2}(1-c_2)\left(\frac{\chi}{f_\sigma}\right)^{\beta'}\partial_\mu\chi\partial^\mu\chi + c_4\left(\frac{\chi}{f_\sigma}\right)^{4+\beta'},$$

$$\mathcal{L}^{d<4}_{\rm mass,LO} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m} {\rm Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right),$$

$$\begin{aligned} 4c_3 + (4+\beta')c_4 &= -(3-\gamma_m)\langle {\rm Tr}(MU^\dagger + UM^\dagger)\rangle_{\rm vac} \\ &= -(3-\gamma_m)F_\pi^2\left(m_K^2 + \frac{1}{2}m_\pi^2\right). \end{aligned}$$

c_3, c_4 are at $O(p^2)$

$\beta' \to 0$, turn off the explicit breaking of scale symmetry.

Both γ_m and β' are evaluated at α_{IR}

$$m_\sigma^2 F_\sigma^2 = F_\pi^2\left(m_K^2 + \frac{1}{2}m_\pi^2\right)(3-\gamma_m)(1+\gamma_m) - \beta'(4+\beta')c_4,$$

Analog to GMOR relation for pion

Consider chiral limit. Minima of the dilaton potential:

$$4c_3 + (4+\beta')c_4 = 0 \rightarrow c_3 = (4+\beta')\ c; c_4 = -4c$$

$$V(x) = -(4+\beta')\ c\left(\frac{\chi}{f_\sigma}\right)^4 + 4c\left(\frac{\chi}{f_\sigma}\right)^{4+\beta'}$$

$\beta' \neq 0,\quad$ NG mode

$\beta' = 0,\quad$ No SB

- ✓ The SB of scale symmetry and EB of scale symmetry are correlated and the SB is triggered by EB which agrees with that unlike chiral symmetry, SB of scale symmetry cannot take place in the absence of EB (Freund & Nambu,69).
- ✓ This implies that it is not possible to ``sit on'' the IR fixed point with both scale and chiral symmetries spontaneously broken. This is analogous to that one cannot ``sit'' on the VM fixed point in HLS theory (Harada & Yamawaki , 03).

- $\beta'(\alpha_{IR})$ is a small quantity,i.e., $\beta'(\alpha_{IR}) \ll 1$:

$$V(\chi) = \frac{m_\sigma^2 f_\sigma^2}{4}\left(\frac{\chi}{f_\sigma}\right)^4\left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right],$$

Coleman-Winberg type potential used in the literature,
Goldberger,et al,08

- Infinitesimal scale transformation:

$$\langle\theta_\mu^\mu\rangle = \langle\partial_\mu D^\mu\rangle = 4\beta' c = \frac{m_\sigma^2 f_\sigma^2}{4},$$

PCDC

A future application to nuclear physics: In a medium of baryonic matter, the vacuum is modified by density. Hence we expect that the decay constant of σ deviates from its vacuum value, i.e., $f_\sigma^* \neq f_\sigma = \langle\chi\rangle$. Since $f_\pi^* = f_\pi\langle\chi\rangle^*$, chiral symmetry breaking is locked to scale symmetry $f_\pi \approx f_\sigma$.

χPT_{σ} at next to leading order

The NLO Lagrangian = the higher chiral-scale order corrections due to the current quark mass and derivatives on the NGBs + the leading terms in the expansion of γ_m and β' in $\Delta\alpha_s$.

$$\chi^{\gamma_m(\alpha_{\rm IR})} \to \chi^{\gamma_m(\alpha_{\rm IR})}\left[1+\sum_{n=1}^{\infty} C_n\left(\Delta\alpha_s\Sigma\right)^n\right],$$

$$\Delta\alpha_s \sim O(p^2).$$

$$\mathrm{Tr}\left(\partial_\nu U \partial^\nu U^\dagger\right);\ \ \partial_\nu\chi\partial^\nu\chi;\ \ \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right).$$

The same applies to $\chi^{\beta'(\alpha_{IR})}$.

$$\mathcal{L}_{\chi\mathrm{PT}_\sigma}^{\mathrm{NLO}} = \mathcal{L}_{\chi\mathrm{PT}_\sigma}^{\mathrm{LO}\times\Delta\alpha_s} + \mathcal{L}_{\chi\mathrm{PT}_\sigma}^{O(p^4)}.$$

$$\begin{aligned}
\mathcal{L}_{\chi\mathrm{PT}_\sigma}^{\mathrm{LO}\times\Delta\alpha_s} = {} & \left[(1-c_1)\frac{f_\pi}{4}\left(\frac{\chi}{f_\sigma}\right)^2 \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2}(1-c_2)\partial_\mu\chi\partial^\mu\chi + c_4\chi^2\right]\left(\frac{\chi}{f_\sigma}\right)^{\beta'} \Sigma \\
& \times \left[D_1 \mathrm{Tr}\left(\partial_\nu U \partial^\nu U^\dagger\right) + D_2 \partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right) + D_3\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m} \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right] \\
& + \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m} \Sigma \\
& \times \left[D_4 \mathrm{Tr}\left(\partial_\nu U \partial^\nu U^\dagger\right) + D_5 \partial_\nu\left(\frac{\chi}{f_\sigma}\right)\partial^\nu\left(\frac{\chi}{f_\sigma}\right) + D_6\left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m} \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right].
\end{aligned}$$

- Does not transform homogeneously under scale transformation. This Lagrangian serves as renormalization counter terms in the loop expansion of χPT_σ .
- Vanishes when the explicit breaking of scale symmetry is absent.

$$\mathcal{L}^{O(p^4)}_{\chi\mathrm{PT}_\sigma} = \mathcal{L}^{O(p^4);d=4}_{\chi\mathrm{PT}_\sigma;\mathrm{inv}} + \mathcal{L}^{O(p^4);d>4}_{\chi\mathrm{PT}_\sigma;\mathrm{anom}} + \mathcal{L}^{O(p^4);d<4}_{\chi\mathrm{PT}_\sigma;\mathrm{mass}},$$

$$\begin{aligned}
\mathcal{L}^{O(p^4);d=4}_{\chi\mathrm{PT}_\sigma;\mathrm{inv}} = {} & L_1 \left[\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right]^2 + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right) \mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right) \\
& + J_1 \partial_\nu \left(\frac{\chi}{f_\sigma}\right) \partial^\nu \left(\frac{\chi}{f_\sigma}\right) \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + J_2 \partial_\mu \left(\frac{\chi}{f_\sigma}\right) \partial^\nu \left(\frac{\chi}{f_\sigma}\right) \mathrm{Tr}\left(\partial_\nu U \partial^\mu U^\dagger\right) \\
& + J_3 \partial_\mu \left(\frac{\chi}{f_\sigma}\right) \partial^\mu \left(\frac{\chi}{f_\sigma}\right) \partial_\nu \left(\frac{\chi}{f_\sigma}\right) \partial^\nu \left(\frac{\chi}{f_\sigma}\right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}^{O(p^4);d>4}_{\chi\mathrm{PT}_\sigma;\mathrm{anom}} = {} & \Bigg\{ (1-L_1) \left[\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right]^2 + (1-L_2) \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right) \mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) \\
& + (1-L_3) \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right) \\
& + (1-J_1) \partial_\nu \left(\frac{\chi}{f_\sigma}\right) \partial^\nu \left(\frac{\chi}{f_\sigma}\right) \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + (1-J_2) \partial_\mu \left(\frac{\chi}{f_\sigma}\right) \partial^\nu \left(\frac{\chi}{f_\sigma}\right) \mathrm{Tr}\left(\partial_\nu U \partial^\mu U^\dagger\right) \\
& + (1-J_3) \partial_\mu \left(\frac{\chi}{f_\sigma}\right) \partial^\mu \left(\frac{\chi}{f_\sigma}\right) \partial_\nu \left(\frac{\chi}{f_\sigma}\right) \partial^\nu \left(\frac{\chi}{f_\sigma}\right) \Bigg\} \left(\frac{\chi}{f_\sigma}\right)^{\beta'},
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}^{O(p^4);d<4}_{\chi\mathrm{PT}_\sigma;\mathrm{mass}} = {} & L_4 \left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right) + L_5 \left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m} \mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U \left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right] \\
& + L_6 \left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)} \left[\mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right]^2 + L_7 \left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)} \left[\mathrm{Tr}\left(\mathcal{M}^\dagger U - U^\dagger \mathcal{M}\right)\right]^2 \\
& + L_8 \left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)} \mathrm{Tr}\left(\mathcal{M}^\dagger U \mathcal{M}^\dagger U + U^\dagger \mathcal{M} U^\dagger \mathcal{M}\right) + H_2 \left(\frac{\chi}{f_\sigma}\right)^{2(3-\gamma_m)} \mathrm{Tr}\left(\mathcal{M}^\dagger \mathcal{M}\right) \\
& + J_4 \left(\frac{\chi}{f_\sigma}\right)^{1-\gamma_m} \partial_\mu \left(\frac{\chi}{f_\sigma}\right) \partial^\mu \left(\frac{\chi}{f_\sigma}\right) \mathrm{Tr}\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right).
\end{aligned}$$

Scale invariant hidden local symmetry HLS_σ

◆ Hidden Local Symmetry

M. Bando, T. Kugo, S. Uehara , K. Yamawaki and T. Yanagida. P.R.L 54,1215(1985)
M. Bando, T. Kugo and K. Yamawaki. Phys.Rept. 164 , 217 (1988)

$$[SU(N_f)_L \times SU(N_f)_R]_{global} \times [SU(N_f)_V]_{local} \rightarrow [SU(N_f)_V]_{global}$$

$$U = e^{2i\pi/F\pi} = \xi_L^{+} \; \xi_R$$

$$\xi_{L,R} = e^{i\sigma/F\sigma}\, e^{\pm i\pi/F\pi} \rightarrow h\, \xi_{L,R}\, g_{L,R}^{+}$$

h: $[SU(N_f)_V]_{local}$ $g_{L,R}^{+}$: $[SU(N_f)_{L,R}]_{global}$

◆ Particles

$V_\mu = V_\mu^{a} T_a$ ... HLS gauge boson

$\pi = \pi^a T_a$ ... NG boson of $[SU(N_f)_L \times SU(N_f)_R]_{global}$ symmetry breaking

$\sigma = \sigma^a T_a$ ... NG boson of $[SU(N_f)_V]_{local}$ symmetry breaking

- Maurer-Cartan 1-forms

$$\hat{\alpha}^{\mu}_{\perp,\parallel} = \left(D^{\mu}\xi_L \cdot \xi_L^{\dagger} \mp D^{\mu}\xi_R \cdot \xi_R^{\dagger} \right) / (2i)$$

$$D_{\mu}\xi_L = \partial_{\mu}\xi_L - iV_{\mu}\xi_L + i\xi_L \mathcal{L}_{\mu}$$

$$D_{\mu}\xi_R = \partial_{\mu}\xi_R - iV_{\mu}\xi_R + i\xi_R \mathcal{R}_{\mu}$$

V_{μ}: HLS gauge field

$\mathcal{L}_{\mu}, \mathcal{R}_{\mu}$: gauge fields of $SU(N_f)_{L,R}$

$$\hat{\alpha}^{\mu}_{\perp,\parallel} \to h\, \hat{\alpha}^{\mu}_{\perp,\parallel}\, h^{\dagger}$$

- Lagrangian

$$\mathcal{L}_{HLS} = F_{\pi}^2 \,\mathrm{tr}\,[\hat{\alpha}^{\mu}_{\perp}\hat{\alpha}_{\perp\mu}] + aF_{\pi}^2 \,\mathrm{tr}\left[\hat{\alpha}^{\mu}_{\parallel}\hat{\alpha}_{\parallel\mu}\right] - \frac{1}{2g^2}\mathrm{tr}\,[V_{\mu\nu}V^{\mu\nu}]$$

Where

F_{π} ... pion decay constant

g ... gauge coupling of the HLS $\quad m_{\rho}^2 = ag^2 (F_{\pi})^2$

$a = (F_{\sigma}/F_{\pi})^2$ ⟷ validity of the vector dominance

It's easy to incorporate the dilaton in HLS and obtain HLS_σ. As in the $\chi\mathrm{PT}_\sigma$ case, the leading-order HLS_σ Lagrangian consists of three components:

$$\mathcal{L}^{\rm LO}_{{\rm HLS}_\sigma} = \mathcal{L}^{d=4}_{{\rm HLS}_\sigma;{\rm inv}} + \mathcal{L}^{d>4}_{{\rm HLS}_\sigma;{\rm anom}} + \mathcal{L}^{d<4}_{{\rm HLS}_\sigma;{\rm mass}},$$

$$\begin{aligned}
\mathcal{L}^{d=4}_{{\rm HLS}_\sigma;{\rm inv}} &= f_\pi^2 c_1 \left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}[\hat{a}_{\perp\mu}\hat{a}_\perp^\mu] + a f_\pi^2 c_2 \left(\frac{\chi}{f_\sigma}\right)^2 {\rm Tr}[\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu] - \frac{1}{2g^2} c_3 {\rm Tr}[V_{\mu\nu}V^{\mu\nu}] \\
&\quad + \frac{1}{2} c_4 f_\sigma^2 \partial_\mu\chi\partial^\mu\chi + c_5 \left(\frac{\chi}{f_\sigma}\right)^4, \\
\mathcal{L}^{d>4}_{{\rm HLS}_\sigma;{\rm anom}} &= f_\pi^2(1-c_1)\left(\frac{\chi}{f_\sigma}\right)^{2+\beta'} {\rm Tr}[\hat{a}_{\perp\mu}\hat{a}_\perp^\mu] + (1-c_2) f_\sigma^2 \left(\frac{\chi}{f_\sigma}\right)^{2+\beta'} {\rm Tr}[\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu] \\
&\quad - \frac{1}{2g^2}(1-c_3)\left(\frac{\chi}{f_\sigma}\right)^{\beta'} {\rm Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2}(1-c_4) f_\sigma^2 \left(\frac{\chi}{f_\sigma}\right)^{\beta'} \partial_\mu\chi\partial^\mu\chi \\
&\quad + c_6 \left(\frac{\chi}{f_\sigma}\right)^{4+\beta'}, \\
\mathcal{L}^{d<4}_{{\rm HLS}_\sigma;{\rm mass}} &= \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\sigma}\right)^{3-\gamma_m} {\rm Tr}\left(\hat{\mathcal{M}} + \hat{\mathcal{M}}^\dagger\right).
\end{aligned}$$

NLO, operators account for $\Delta\alpha_s$:

$$\begin{aligned}
&{\rm Tr}(\hat{a}_{\perp\mu}\hat{a}_\perp^\mu); \quad {\rm Tr}(\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu); \quad \frac{1}{2g^2}{\rm Tr}(V_{\mu\nu}V^{\mu\nu}); \\
&\partial_\mu\chi\partial^\mu\chi; \quad {\rm Tr}\left(\hat{\mathcal{M}} + \hat{\mathcal{M}}^\dagger\right).
\end{aligned}$$

Scale invariant HLS with baryon octet: bsHLS$_\sigma$

$$B(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix},$$

$$B(x) \to h(x)B(x)h^\dagger(x), \qquad B(x) \to \lambda^{3/2}B(\lambda^{-1}x).$$

$$\mathcal{L}_{bs\mathrm{HLS}} = \mathcal{L}^{d=4}_{bs\mathrm{HLS;inv}} + \mathcal{L}^{d>4}_{bs\mathrm{HLS;anom}},$$

$$\begin{aligned} \mathcal{L}^{\mathrm{LO};d=4}_{bs\mathrm{HLS;inv}} = {} & c_1 \mathrm{Tr}\left(\bar{B} i\gamma_\mu D^\mu B\right) - \tilde{\mathring{m}}_B \frac{\chi}{f_\sigma} \mathrm{Tr}\left(\bar{B}B\right) - \tilde{g}_{A_1} \mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5 \left\{\hat{\alpha}^\mu_\perp, B\right\}\right) \\ & - \tilde{g}_{A_2} \mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5 \left[\hat{\alpha}^\mu_\perp, B\right]\right) + \tilde{g}_{V_1} \mathrm{Tr}\left(\bar{B}\gamma_\mu \left\{\hat{\alpha}^\mu_\parallel, B\right\}\right) \\ & + \tilde{g}_{V_2} \mathrm{Tr}\left(\bar{B}\gamma_\mu \left[\hat{\alpha}^\mu_\parallel, B\right]\right) \end{aligned}$$

$$\begin{aligned} \mathcal{L}^{\mathrm{LO};d>4}_{bs\mathrm{HLS;anom}} = {} & \Big[(1 - c_1)\mathrm{Tr}\left(\bar{B} i\gamma_\mu D^\mu B\right) - (\mathring{m}_B - \tilde{\mathring{m}}_B)\frac{\chi}{f_\sigma}\mathrm{Tr}\left(\bar{B}B\right) - (g_{A_1} \\ & - \tilde{g}_{A_1})\mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5 \left\{\hat{\alpha}^\mu_\perp, B\right\}\right) \\ & - (g_{A_2} - \tilde{g}_{A_2})\mathrm{Tr}\left(\bar{B}\gamma_\mu\gamma_5 \left[\hat{\alpha}^\mu_\perp, B\right]\right) + (g_{V_1} - \tilde{g}_{V_1})\mathrm{Tr}\left(\bar{B}\gamma_\mu \left\{\hat{\alpha}^\mu_\parallel, B\right\}\right) \\ & + (g_{V_2} - \tilde{g}_{V_2})\mathrm{Tr}\left(\bar{B}\gamma_\mu \left[\hat{\alpha}^\mu_\parallel, B\right]\right)\Big] \left(\frac{\chi}{f_\sigma}\right)^{\beta'}, \end{aligned}$$

Consider the terms contributing to the baryon mass in chiral limit:

$$\mathcal{L}_{bsHLS}^{\rm mass} = \left[\tilde{\mathring{m}}_B \frac{\chi}{f_\sigma} + (\mathring{m}_B - \tilde{\mathring{m}}_B)\frac{\chi}{f_\sigma}\left(\frac{\chi}{f_\sigma}\right)^{\beta'}\right] {\rm Tr}\left(\bar{B}B\right).$$

Meson fluctuation &small β' expansion

$$\mathcal{L}_{bsHLS}^{\rm mass} = \mathring{m}_B {\rm Tr}\left(\bar{B}B\right) + \mathring{m}_B \sum_{m=1}^{\infty}\frac{1}{m!}\Sigma^m {\rm Tr}\left(\bar{B}B\right) + (\mathring{m}_B - \tilde{\mathring{m}}_B)\sum_{n=1}^{\infty}\frac{1}{n!}\left(\beta'\Sigma\right)^n \times \left(1 + \sum_{m=1}^{\infty}\frac{1}{m!}\Sigma^m\right) {\rm Tr}\left(\bar{B}B\right).$$

Baryon mass in chiral limit

$\sigma^n BB$ coupling

E.g.,for n=1

$$g_{\sigma BB} = \mathring{m}_B \frac{1}{f_\sigma} + (\mathring{m}_B - \tilde{\mathring{m}}_B)\beta' \frac{1}{f_\sigma}$$

G-T type relation in the dilaton sector

- To finalize the heavy-baryon expansion, we should set up the chiral-scale counting of the interaction terms. Since the dilaton couples to baryons nonderivatively, one can't do the usual power counting as with the derivative in pion-nucleon coupling.
- In the absence of first-principle guidance, we establish the power counting using a numerical estimation.
- If we take the nucleon mass in the chiral limit $m_B \approx 900$ MeV, by taking $f_\pi \approx f_\sigma$, we obtain $g_{\sigma BB} \approx 10$ which is close to $g_{\pi BB} \approx 13$. This suggests that the other terms could be considered as of chiral-scale order $O(p)$. That is, in terms of the compensator χ

$$\mathring{m}_B\left(\frac{\chi}{f_\sigma}-1\right)+(\mathring{m}_B-\tilde{\mathring{m}}_B)\left[\left(\frac{\chi}{f_\sigma}\right)^{\beta'}-1\right]\frac{\chi}{f_\sigma}\sim O(p),$$

Then, the HBChPT including dilaton can be formulated in a straight forward way.

➢ The possible applications in dense matter

Yan-Ling Li, Yong-Liang Ma. arXiv:1710.02839

- Hadron interactions, medium modified dilepton decay widths......
- Extend the notion of BR scaling to a generalized framework, the information of the low energy constants.
- Arrive at the LOSS that can be confront with nature.

The external gauge fields $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ include W_μ,Z_μ and A_μ (photon) as

$$\begin{aligned} \mathcal{L}_\mu &= eQA_\mu + \frac{g_2}{\cos\theta_W}(T_z - \sin^2\theta_W)Z_\mu + \frac{g_2}{\sqrt{2}}\left(W_\mu^+ T_+ + W_\mu^- T_-\right), \\ \mathcal{R}_\mu &= eQA_\mu - \frac{g_2}{\cos\theta_W}\sin^2\theta_W Z_\mu. \end{aligned}$$

Expanding the 1-forms in term of π, one has

$$\begin{aligned} \widehat{\alpha}_{\perp\mu} &= \frac{1}{f_\pi}\partial_\mu\pi + \mathcal{A}_\mu - \frac{i}{f_\pi}\left[\mathcal{V}_\mu\,,\,\pi\right] - \frac{1}{6f_\pi^3}\Big[\left[\partial_\mu\pi\,,\,\pi\right]\,,\,\pi\Big] + \cdots\ , \\ \widehat{\alpha}_{\parallel\mu} &= -V_\mu + \mathcal{V}_\mu - \frac{i}{2f_\pi^2}\left[\partial_\mu\pi\,,\,\pi\right] - \frac{i}{f_\pi}\left[\mathcal{A}_\mu\,,\,\pi\right] + \cdots\ , \end{aligned}$$

Wave function renormalization of hadron fields in medium

Expectation value in medium is $\langle \chi \rangle^*$ .Then the expression of the kinetic term:

$$
\begin{aligned}
\mathcal{L}_{\rm kin} &= \left[h_1 + (1-h_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 {\rm Tr}\left(\partial_\mu \pi \partial^\mu \pi\right) \\
&+ \frac{1}{2}\left[h_4 + (1-h_4)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 \partial_\mu \sigma \partial^\mu \sigma \\
&- \frac{1}{2g^2}\left[h_3 + (1-h_3)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right] {\rm Tr}\left[V_{\mu\nu} V^{\mu\nu}\right]
\end{aligned}
$$

Then, the physical states $\tilde{\sigma}$ and $\tilde{\pi}$ can be defined through

$$
\tilde{\pi} = Z_{3\pi}\pi, \quad \tilde{\sigma} = Z_{3\sigma}\sigma, \quad \tilde{\rho}_\mu = Z_{3\rho}\rho_\mu,
$$

where the coefficients are

$$
\begin{aligned}
Z_{3\pi}^2 &= \left[h_1 + (1-h_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2, \\
Z_{3\sigma}^2 &= \left[h_4 + (1-h_4)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2, \\
Z_{3\rho}^2 &= \left[h_3 + (1-h_3)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right].
\end{aligned}
$$

meson decay constants：

$$f_\sigma^* = Z_{3\sigma} f_\sigma, \; f_\pi^* = Z_{3\pi} f_\pi$$

HLS gauge coupling constant：

$$g^* = Z_{3\rho}^{-1} g$$

the in-medium mass of mesons

$$\mathcal{L}^{\sigma}_{\rm mass} = -\frac{1}{2}\frac{1}{f_\sigma^2}\left[16c_3\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^4 + (4+\beta')^2 c_4\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\right]$$
$$= \frac{1}{2}\frac{4c\beta'(4+\beta')}{f_\sigma^2}\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\sigma^2.$$

redefinition →

$$\mathcal{L}^{\sigma}_{\rm mass} = \frac{1}{2}\frac{4c\beta'(4+\beta')}{f_\sigma^2}\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\frac{1}{Z_{3\sigma}^2}\tilde{\sigma}^2.$$

→

$$m_\sigma^{*2} = \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\frac{1}{Z_{3\sigma}^2}m_\sigma^2,$$

$$\mathcal{L}^{\pi,K}_{\rm mass} = -\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}\left(\frac{1}{2}m_\pi^2\pi^0\pi^0 + m_\pi^2\pi^+\pi^- + m_K^2K^+K^- + m_K^2\bar{K}^0K^0\right).$$

redefinition ↓

$$\mathcal{L}^{\pi,K}_{\rm mass} = -\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}\frac{1}{Z_{3\pi}^2}\left(\frac{1}{2}m_\pi^2\tilde{\pi}^0\tilde{\pi}^0 + m_\pi^2\tilde{\pi}^+\tilde{\pi}^- + m_K^2\tilde{K}^+\tilde{K}^- + m_K^2\tilde{\bar{K}}^0\tilde{K}^0\right)$$

↓

the physical pseudoscalar meson masses are

$$m_\pi^{*2} = \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}m_\pi^2\frac{1}{Z_{3\pi}^2},\quad m_K^{*2} = \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}m_K^2\frac{1}{Z_{3\pi}^2}.$$

$$\mathcal{L}^{V}_{\rm mass} = af_\pi^2\left[h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]{\rm Tr}[V_\mu V^\mu]$$

redefinition ↓

$$\mathcal{L}^{V}_{\rm mass} = m_\rho^2\left[h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{Z_{3\rho}^2}{\rm Tr}[\tilde{\rho}_\mu\tilde{\rho}^\mu].$$

↓

$$m_\rho^{*2} = m_\rho^2\left[h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{Z_{3\rho}^2}.$$

For the three-meson strong interactions ,we calculate
$\sigma-\pi-\pi$, $\sigma-\sigma-\sigma$, $\rho-\pi-\pi$, $\sigma-\rho-\rho$.

The relevant Lagrangians are expressed as

$$\begin{aligned}
\mathcal{L}_{\sigma\pi\pi} &= \left[2h_1\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (2+\beta')(1-h_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{f_\sigma}\sigma\mathrm{Tr}\left(\partial_\mu\pi\partial^\mu\pi\right) \\
&\quad - \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}\frac{(3-\gamma_m)}{f_\sigma}\sigma\left(\frac{1}{2}m_\pi^2\pi^0\pi^0 + m_\pi^2\pi^+\pi^- + m_K^2K^+K^- + m_K^2\bar{K}^0K^0\right) \\
\mathcal{L}_{\sigma\sigma\sigma} &= \left[h_4\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-h_4)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\left(1+\frac{1}{2}\beta'\right)\right]\frac{1}{f_\sigma}\sigma\partial_\mu\sigma\partial^\mu\sigma \\
&\quad + \frac{1}{6}\left[4c(4+\beta')\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\left[16-(4+\beta')^2\right]\right. \\
&\qquad \left. + \frac{f_\pi^2}{2}(3-\gamma_m)^3(2m_K^2+m_\pi^2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}\right]\left(\frac{\sigma}{f_\sigma}\right)^3, \\
\mathcal{L}_{\rho\pi\pi} &= iag\left[h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\mathrm{Tr}[\rho_\mu[\partial^\mu\pi,\pi]], \\
\mathcal{L}_{\sigma\rho\rho} &= ag^2f_\pi^2\left[2h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (2+\beta')(1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{f_\sigma}\sigma\mathrm{Tr}[\rho_\mu\rho^\mu] \\
&\quad - \frac{1}{2}\beta'(1-h_3)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\frac{1}{f_\sigma}\sigma\mathrm{Tr}\left[(\partial_\mu\rho_\nu-\partial_\nu\rho_\mu)(\partial^\mu\rho^\nu-\partial^\nu\rho^\mu)\right].
\end{aligned}$$

The relevant modification Lagrangians :

$$
\begin{aligned}
\mathcal{L}_{\sigma\pi\pi} &= \left[2h_1\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (2+\beta')(1-h_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{f_\sigma}\frac{1}{Z_{3\pi}^2 Z_{3\sigma}}\tilde{\sigma}\,\mathrm{Tr}\left(\partial_\mu\tilde{\pi}\partial^\mu\tilde{\pi}\right) \\
&\quad - \frac{(3-\gamma_m)}{f_\sigma}\frac{1}{Z_{3\sigma}}\tilde{\sigma}\left(\frac{1}{2}m_\pi^{*2}\tilde{\pi}^0\tilde{\pi}^0 + m_\pi^{*2}\tilde{\pi}^+\tilde{\pi}^- + m_K^{*2}\tilde{K}^+\tilde{K}^- + m_K^{*2}\tilde{\bar{K}}^0\tilde{K}^0\right) \\
\mathcal{L}_{\sigma\sigma\sigma} &= \left[c_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-c_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\left(1+\frac{1}{2}\beta'\right)\right]\frac{1}{f_\sigma}\frac{1}{Z_{3\sigma}^3}\tilde{\sigma}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} \\
&\quad + \frac{1}{6}\left[4c(4+\beta')\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\left[16-(4+\beta')^2\right]\right. \\
&\qquad \left. + \frac{f_\pi^2}{2}(3-\gamma_m)^3(2m_K^{*2}+m_\pi^{*2})Z_{3\pi}^2\right]\left(\frac{1}{f_\sigma}\right)^3\frac{1}{Z_{3\sigma}^3}\tilde{\sigma}^3, \\
\mathcal{L}_{\rho\pi\pi} &= iag^*\left[h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{Z_{3\pi}^2}\mathrm{Tr}[\tilde{\rho}_\mu\left[\partial^\mu\tilde{\pi},\tilde{\pi}\right]], \\
\mathcal{L}_{\sigma\rho\rho} &= ag^{*2}f_\pi^2\left[2h_2\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 + (2+\beta')(1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{2+\beta'}\right]\frac{1}{f_\sigma}\frac{1}{Z_{3\sigma}}\tilde{\sigma}\,\mathrm{Tr}[\tilde{\rho}_\mu\tilde{\rho}^\mu] \\
&\quad - \frac{1}{2}\beta'(1-h_3)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\frac{1}{f_\sigma}\frac{1}{Z_{3\rho}^2 Z_{3\sigma}}\tilde{\sigma}\,\mathrm{Tr}[(\partial_\mu\tilde{\rho}_\nu-\partial_\nu\tilde{\rho}_\mu)(\partial^\mu\tilde{\rho}^\nu-\partial^\nu\tilde{\rho}^\mu)].
\end{aligned}
$$

The EW term:

$$
\begin{aligned}
\mathcal{L}^{\rm EW}_{{\rm HLS}_\sigma} &= f_\pi^2 \left[h_1 + (1-h_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right] \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 \\
&\quad \times {\rm tr}\left[\left(\frac{1}{f_\pi}\partial_\mu \pi - ie\frac{1}{f_\pi}[A_\mu Q\,,\,\pi] + \frac{g_2}{2\sqrt{2}}\left(W_\mu^+ T_+ + W_\mu^- T_-\right) + \cdots \right)^2 \right] \\
&\quad + a f_\pi^2 \left[h_2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right] \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 {\rm tr}\left[\left(g\rho_\mu - eA_\mu Q + \frac{i}{2f_\pi^2}[\partial_\mu \pi\,,\,\pi] + \cdots \right)^2 \right] \\
&\quad + \cdots \\
&= \left[h_1 + (1-h_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right] \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 \left[\frac{g_2 f_\pi}{\sqrt{2}} {\rm tr}\left[\partial_\mu \pi \left(W_\mu^+ T_+ + W_\mu^- T_-\right)\right] \right] \\
&\quad - 2ie\left[1 - \frac{a}{2}\frac{1}{Z_{3\pi}^2} \left[h_2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right] \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 \right] {\rm tr}\left[\partial_\mu \tilde\pi\, [A_\mu Q\,,\,\tilde\pi]\right] \\
&\quad - 2ea \left[h_2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'} \right] \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 g f_\pi^2 A^\mu\, {\rm tr}\left[\rho_\mu Q\right] \\
&\quad + \cdots\,,
\end{aligned}
$$

Then, in terms of the normalized fields, the HLS_σ becomes:

$$
\begin{aligned}
\mathcal{L}_{{\rm HLS}_\sigma} &= \frac{g_2 f_\pi^*}{\sqrt{2}} {\rm tr}\left[\partial_\mu \tilde\pi \left(W_\mu^+ T_+ + W_\mu^- T_-\right)\right] - 2e\frac{m_\rho^{*2}}{g^*} A^\mu\, {\rm tr}\left[\tilde\rho_\mu Q\right] \\
&\quad - 2ie\left[1 - \frac{a}{2}\frac{Z_{3\rho}^2}{Z_{3\pi}^2}\frac{m_\rho^{*2}}{m_\rho^2} \right] {\rm tr}\left[\partial_\mu \tilde\pi\, [A_\mu Q\,,\,\tilde\pi]\right] \\
&\quad + \cdots\,.
\end{aligned}
$$

The in-medium mass of baryon:

$$\mathcal{L}_B^{\rm mass} = -\tilde{\mathring{m}}_B \frac{\langle\chi\rangle^*}{f_\sigma}{\rm Tr}\left(\bar{B}B\right) - \left(\mathring{m}_B - \tilde{\mathring{m}}_B\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{1+\beta'}{\rm Tr}\left(\bar{B}B\right)$$

$$= -\left[\tilde{\mathring{m}}_B \frac{\langle\chi\rangle^*}{f_\sigma} + \left(\mathring{m}_B - \tilde{\mathring{m}}_B\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{1+\beta'}\right]\frac{1}{Z_{3B}^2}{\rm Tr}\left(\bar{\tilde{B}}\tilde{B}\right),$$

the medium modified baryon mass as

$$m_B^* = \left[\tilde{\mathring{m}}_B \frac{\langle\chi\rangle^*}{f_\sigma} + \left(\mathring{m}_B - \tilde{\mathring{m}}_B\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{1+\beta'}\right]\frac{1}{Z_{3B}^2}.$$

The relevant Lagrangians including one meson are expressed as

$$\mathcal{L}_{\sigma\bar{B}B} = -\tilde{\mathring{m}}_B \frac{\langle\chi\rangle^*}{f_\sigma}\frac{1}{f_\sigma}\sigma{\rm Tr}\left(\bar{B}B\right) + (1-g_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\frac{\beta'}{f_\sigma}\sigma{\rm Tr}\left(\bar{B}i\gamma_\mu\partial^\mu B\right)$$

$$- \left(\mathring{m}_B - \tilde{\mathring{m}}_B\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{1+\beta'}\frac{1+\beta'}{f_\sigma}\sigma{\rm Tr}\left(\bar{B}B\right)$$

$$\mathcal{L}_{\pi\bar{B}B} = -\left[\tilde{g}_{A_1} + \left(g_{A_1} - \tilde{g}_{A_1}\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\frac{1}{f_\pi}{\rm Tr}\left(\bar{B}\gamma_\mu\gamma_5\left\{\partial^\mu\pi, B\right\}\right)$$

$$-\left[\tilde{g}_{A_2} + \left(g_{A_2} - \tilde{g}_{A_2}\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\frac{1}{f_\pi}{\rm Tr}\left(\bar{B}\gamma_\mu\gamma_5\left[\partial^\mu\pi, B\right]\right),$$

$$\mathcal{L}_{V\bar{B}B} = \left\{\left(g_1 - \tilde{g}_{V_2}\right) + \left[(1-g_1) - \left(g_{V_2} - \tilde{g}_{V_2}\right)\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right\}{\rm Tr}\left(\bar{B}\gamma_\mu\left[V^\mu, B\right]\right)$$

$$-\left\{\tilde{g}_{V_1} + \left(g_{V_1} - \tilde{g}_{V_1}\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right\}{\rm Tr}\left(\bar{B}\gamma_\mu\left\{V^\mu, B\right\}\right).$$

The modification relevant Lagrangians :

$$
\begin{aligned}
\mathcal{L}_{\sigma\bar{B}B} &= -\tilde{\mathring{m}}_B \frac{\langle\chi\rangle^*}{f_\sigma}\frac{1}{f_\sigma^*}\frac{1}{Z_{3B}^2}\tilde{\sigma}\,\mathrm{Tr}\left(\bar{\tilde{B}}\tilde{B}\right) + (1-g_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\frac{\beta'}{f_\sigma^*}\frac{1}{Z_{3B}^2}\tilde{\sigma}\,\mathrm{Tr}\left(\bar{\tilde{B}}i\gamma_\mu\partial^\mu\tilde{B}\right) \\
&\quad - \left(\mathring{m}_B - \tilde{\mathring{m}}_B\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{1+\beta'}\frac{1+\beta'}{f_\sigma^*}\frac{1}{Z_{3B}^2}\tilde{\sigma}\,\mathrm{Tr}\left(\bar{\tilde{B}}\tilde{B}\right) \\
\mathcal{L}_{\pi\bar{B}B} &= -\left[\tilde{g}_{A_1} + (g_{A_1} - \tilde{g}_{A_1})\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\frac{1}{f_\pi^*}\frac{1}{Z_{3B}^2}\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\gamma_5\left\{\partial^\mu\tilde{\pi},\tilde{B}\right\}\right) \\
&\quad - \left[\tilde{g}_{A_2} + (g_{A_2} - \tilde{g}_{A_2})\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\frac{1}{f_\pi^*}\frac{1}{Z_{3B}^2}\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\gamma_5\left[\partial^\mu\tilde{\pi},\tilde{B}\right]\right), \\
\mathcal{L}_{V\bar{B}B} &= \left\{(g_1 - \tilde{g}_{V_2}) + \left[(1-g_1) - (g_{V_2} - \tilde{g}_{V_2})\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right\}\frac{g^*}{Z_{3B}^2}\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\left[\tilde{\rho}^\mu,\tilde{B}\right]\right) \\
&\quad - \left\{\tilde{g}_{V_1} + (g_{V_1} - \tilde{g}_{V_1})\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right\}\frac{g^*}{Z_{3B}^2}\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\left\{\tilde{\rho}^\mu,\tilde{B}\right\}\right).
\end{aligned}
$$

The σ-B-B coupling canstant:

$$
g^*{}_{\sigma\bar{B}B} \approx \tilde{\mathring{m}}_B\frac{\langle\chi\rangle^*}{f_\sigma}\frac{1}{f_\sigma^*}\frac{1}{Z_{3B}^2} - (1-g_1)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\frac{\beta'}{f_\sigma^*}\frac{m_B^*}{Z_{3B}^2} + \left(\mathring{m}_B - \tilde{\mathring{m}}_B\right)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{1+\beta'}\frac{1+\beta'}{f_\sigma^*}\frac{1}{Z_{3B}^2}.
$$

Results：

From the above we can easily read the medium modified meson masses, the $\rho\pi\pi$ coupling constant $\tilde{g}_{\rho\pi\pi}$, the ρ–γ mixing strength $\tilde{g}_{\rho}$, the ϕKK coupling constant $\tilde{g}_{\phi KK}$, the ϕ–γ mixing strength $\tilde{g}_{\phi}$, the π–W mixing strength $\tilde{g}_{\pi}$ and the direct $\gamma\pi\pi$ coupling constant $\tilde{g}_{\gamma\pi\pi}$:

$$
\begin{aligned}
m_\pi^{*2} &= \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}\frac{1}{Z_{3\pi}^2}m_\pi^2\\
m_K^{*2} &= \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{3-\gamma_m}\frac{1}{Z_{3\pi}^2}m_K^2\\
m_\sigma^{*2} &= \left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{4+\beta'}\frac{1}{Z_{3\sigma}^2}m_\sigma^2\\
m_{\rho,\phi}^{*2} &= \left[h_2+(1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2\frac{1}{Z_{3\rho}^2}m_{\rho,\phi}^2\\
g_{\rho\pi\pi}^* &= \left[h_2+(1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2\frac{1}{Z_{3\rho}Z_{3\pi}^2}g_{\rho\pi\pi}\\
&= \frac{1}{2}ag^*\left[h_2+(1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2\frac{1}{Z_{3\pi}^2}\,,\\
g_\rho^* &= \left[h_2+(1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2\frac{g_\rho}{Z_{3\rho}}=\frac{m_\rho^{*2}}{g^*}\\
g_\pi^* &= \frac{g_2 f_\pi^*}{4}V_{\rm ud}\\
g_{\gamma\pi\pi}^* &= e\left[1-\frac{a}{2}\frac{Z_{3\rho}^2}{Z_{3\pi}^2}\frac{m_\rho^{*2}}{m_\rho^2}\right]\,.
\end{aligned}
$$

where:

$$
\begin{aligned}
m_\rho^2 &= ag^2 f_\pi^2\,,\\
g_{\rho\pi\pi} &= \frac{1}{2}ag\,,\\
g_\rho &= agf_\pi^2\,,\\
g_{\gamma\pi\pi} &= \left(1-\frac{a}{2}\right)e\,.
\end{aligned}
$$

the decay width:

$$
\begin{aligned}
\Gamma^*(\rho\to\pi\pi) &= \frac{|\vec{p}_\pi{}^*|^3}{6\pi m_\rho^{*2}}\left|g_{\rho\pi\pi}^*\right|^2\,,\quad |\vec{p}_\pi{}^*|=\sqrt{\frac{m_\rho^{*2}-4m_\pi^{*2}}{4}}\,,\\
\Gamma^*(\rho\to e^+e^-) &= \frac{4\pi\alpha^2}{3}\left|\frac{g_\rho^*}{m_\rho^{*2}}\right|^2\frac{m_\rho^{*2}+2m_e^2}{m_\rho^{*2}}\sqrt{m_\rho^{*2}-4m_e^2}\,,\\
\Gamma^*(\phi\to KK) &= \frac{\left|\vec{p}_\phi^*\right|^3}{6\pi m_\phi^{*2}}\left|g_{\phi KK}^*\right|^2\,,\quad |\vec{p}_\phi^*|=\sqrt{\frac{m_\phi^{*2}-4m_K^{*2}}{4}}\,,\\
\Gamma^*(\phi\to e^+e^-) &= \frac{4\pi\alpha^2}{3}\left|\frac{g_\phi^*}{m_\phi^{*2}}\right|^2\frac{m_\phi^{*2}+2m_e^2}{m_\phi^{*2}}\sqrt{m_\phi^{*2}-4m_e^2}\,,\\
\Gamma^*(\pi^-\to e^-\bar{\nu}_e) &= \frac{|g_\pi^*|^2}{12\pi}\left(\frac{m_\pi^{*2}-m_e^2}{m_\pi^{*2}-m_W^2}\right)^2\,.
\end{aligned}
$$

Implications of the low energy constants

The validity of the GMOR relation in medium, the quark condensate and pion decay constant are locked to each other.

$$\frac{m_\pi^{*2}}{m_\pi^2} = \frac{\Phi^{1-\gamma_m}}{h_1 + (1-h_1)\Phi^{\beta'}} = 1. \quad \xrightarrow{\gamma_m = 1} \quad h_1=1$$

If the in-medium ρ/ϕ mass is smaller than it in free space, then

F. Sakuma et al. (KEK-PS E325 Collaboration) Phys. Rev. Lett. 98, 152302 (2007)

A. Marin (Darmstadt, GSI) et al.(CERES Collaboration) PoS CPOD07 034 (2007)

$$\Phi = \frac{\langle\chi\rangle^*}{f_\sigma} \lesssim 1.$$

$$\frac{m_{\rho,\phi}^{*2}}{m_{\rho,\phi}^2} = \left[h_2 + (1-h_2)\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}\right]\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^2 \frac{1}{Z_{3\rho}^2} < 1$$

h_2, h_3=1

$$\frac{\Gamma^*(\phi \to e^+e^-)}{\Gamma(\phi \to e^+e^-)} = \left|\frac{g_\phi^*}{m_\phi^{*2}}\right|^2 \frac{m_\phi^*}{m_\phi}\left|\frac{g_\phi}{m_\phi^2}\right|^{-2} = \Phi$$

near chiral restoration, VM

$$\frac{a^*}{a} = \left[h_2 + (1-h_2)\Phi^{\beta'}\right] < 1,$$

h_1=1

then h_2<1, However, since our scaling relations as written are valid for the density $\leq n_{1/2} \approx 2n_0$, they cannot be relevant to the chiral restoration. Then, $a^*/a \simeq 1$ is reasonable and h_2=1.

For dilaton meson:

$$\frac{m_\sigma^{*2} f_\sigma^{*2}}{m_\sigma^2 f_\sigma^2} = \Phi^{4+\beta'}$$

h_4=1 →

$$\frac{m_\sigma^{*2}}{m_\sigma^2} = \Phi^{2+\beta'}, \quad \frac{f_\sigma^*}{f_\sigma} = \Phi.$$

GMOR relation $m_\sigma^2 f_\sigma^2 = 4c\beta'(4+\beta')$ still survives in medium, the parameter h_5 and/or h_6 are density dependent quantities.

$$h_5 = (4+\beta')c\left(\frac{\langle\chi\rangle^*}{f_\sigma}\right)^{\beta'}, \quad h_6 = -4c$$

$$\frac{c^*}{c} = \Phi^{4+\beta'}$$

→

$$\frac{h_5^*}{h_5} = \Phi^{4+2\beta'}, \quad \frac{h_6^*}{h_6} = \Phi^{4+\beta'}$$

And for baryons:

$$\frac{m_B^*}{m_B} = \frac{\left[\tilde{\mathring{m}}_B + (\mathring{m}_B - \tilde{\mathring{m}}_B)\Phi^{\beta'}\right]}{\mathring{m}_B\left[g_1 + (1-g_1)\Phi^{\beta'}\right]}\Phi.$$

$g_1 = 1$ and $\mathring{m}_B = \tilde{\mathring{m}}_B$ →

$$\frac{m_B^*}{m_B} = \Phi$$

Yan-Ling Li, Yong-Liang Ma, Mannque Rho. arXiv:1710.02840，arXiv:1804.00310

$$g_A^* = \frac{1}{Z_{3B}^2}\left[\tilde{g}_A + (g_A - \tilde{g}_A)\Phi^{\beta'}\right] \xrightarrow{g_1 = 1,\quad \tilde{g}_A = g_A}$$ g_A^* is a density invariant quantity

$$g^*{}_{\sigma\bar{B}B} \simeq m_B^* \frac{1}{f_\sigma^*} = \frac{m_B}{f_\sigma}$$

which is a scale independent quantity determined from the Goldberger-Trieman type relation in matter free space.

The results when we choose or fix the low energy constants:

1) For mesons:

$$\mathcal{L}_{\sigma\pi\pi} = 2\frac{1}{f_\sigma^*}\tilde{\sigma}\mathrm{Tr}\left(\partial_\mu\tilde{\Pi}\partial^\mu\tilde{\Pi}\right) - \frac{2}{f_\sigma^*}\tilde{\sigma}\mathrm{Tr}\left(\mathcal{M}^*\tilde{\Pi}^2\right)$$

$$\mathcal{L}_{\sigma\sigma\sigma} = \frac{1}{f_\sigma^*}\tilde{\sigma}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} + \frac{2}{3}\Big[c^*(4+\beta')\left[16-(4+\beta')^2\right] + f_\pi^2(2m_K^2+m_\pi^2)\Phi^2\Big]\frac{1}{f_\sigma^{*3}}\tilde{\sigma}^3,$$

$$\mathcal{L}_{\rho\pi\pi} = iag\mathrm{Tr}[\tilde{\rho}_\mu\left[\partial^\mu\tilde{\pi},\tilde{\pi}\right]],$$

$$\mathcal{L}_{\sigma\rho\rho} = 2ag^2 f_\pi^{*2}\frac{1}{f_\sigma^*}\tilde{\sigma}\mathrm{Tr}[\tilde{\rho}_\mu\tilde{\rho}^\mu].$$

$$\mathcal{L}_{\sigma\sigma\sigma} = \frac{1}{f_\sigma^*}\tilde{\sigma}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} + \frac{2}{3}\frac{c^*}{f_\sigma^{*3}}(4+\beta')\left[16-(4+\beta')^2\right]\tilde{\sigma}^3$$

$$g_{\rho\gamma}^* = g_{\rho\gamma}\Phi^2 = \frac{m_\rho^{*2}}{g^*},$$

$$g_{\gamma\pi\pi}^* = e\left(1-\frac{a}{2}\right).$$

2) For baryons:

$$\mathcal{L}_{\sigma\bar{B}B} = -\mathring{m}_B \frac{1}{f_\sigma}\tilde{\sigma}\,\mathrm{Tr}\left(\bar{\tilde{B}}\tilde{B}\right),$$

$$\mathcal{L}_{\pi\bar{B}B} = -\frac{1}{f_\pi^*} g_{A_1}\,\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\gamma_5\left\{\partial^\mu\tilde{\pi},\tilde{B}\right\}\right) - \frac{1}{f_\pi^*} g_{A_2}\,\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\gamma_5\left[\partial^\mu\tilde{\pi},\tilde{B}\right]\right),$$

$$\mathcal{L}_{V\bar{B}B} = \left(1-g_{V_2}\right) g\,\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\left[\tilde{\rho}^\mu,\tilde{B}\right]\right) - g_{V_1} g\,\mathrm{Tr}\left(\bar{\tilde{B}}\gamma_\mu\left\{\tilde{\rho}^\mu,\tilde{B}\right\}\right).$$

$$g^*_{\pi BB} = \Phi^{3-\gamma_m}\frac{1}{Z_{3B}^2 Z_{3\pi}}\frac{m_B^*}{f_\pi^*} g_A^* = \frac{m_B}{f_\pi} g_A,$$

In conclusion

$$\frac{m_\pi^*}{m_\pi} = \Phi^0, \quad \frac{m_\sigma^*}{m_\sigma} = \Phi^{\beta'/2+1}, \quad \frac{m_B^*}{m_B} = \frac{m_\rho^*}{m_\rho} = \frac{f_\pi^*}{f_\pi} = \Phi$$

$$\frac{h_5^*}{h_5} = \Phi^{4+2\beta'}, \quad \frac{h_6^*}{h_6} = \Phi^{4+\beta'},$$

$$\frac{g^*_{\sigma\pi\pi}}{g_{\sigma\pi\pi}} = \Phi^{-1}, \quad \frac{g^*_{\sigma\rho\rho}}{g_{\sigma\rho\rho}} = \Phi, \quad \frac{g^*_{\sigma\sigma\sigma}}{g_{\sigma\sigma\sigma}} = \Phi^{\beta'+1},$$

$$\frac{g^*_{\sigma BB}}{g_{\sigma BB}} = \frac{g^*_{\pi BB}}{g_{\pi BB}} = \frac{g^*_{\rho BB}}{g_{\rho BB}} = \frac{g^*_{\rho\pi\pi}}{g_{\rho\pi\pi}} = \Phi^0.$$

It is agreed with that given in PhysRevD.96.014031 , when we take β'<<1 upto the leading order expansion.

➢ Summary

- The chiral-scale effective theory discussed here can be used in the study of dense matter physics and going beyond the mean-field-based analysis.
- In the present construction, the explicit scale symmetry can be take into account by the derivation from the IRFP α_{IR} which give the Lagrangian used in above at the leading order of small β' so we believe the present Lagr. can yield a result closer to nature.
- Since the present chiral-scale effective theory is constructed with three flavors, it can provide a systematic way to study effects of strangeness in nuclear matter.
- the reduction to the LOSS from the general scale-chiral effective theory and the scaling behaviours obtained here is valid, probably only up to the density $\leq n_{1/2} \approx 2n_0$.

Thanks for your attention