QCD@LHC

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PLAN of the Lectures:

1. Spinor-helicity and soft/collinear factorization
2. NLO corrections to $e^+e^- \to q\bar{q}$ and $PP \to \gamma^* \to e^+e^-$
3. Large Logs and Resummation.

References:

1. hep-ph/9601359 ; 1101.2414
2. R. Field: Applications of Perturbative QCD

   Willenbrock: TASI Lecture 1989

1. Spinor-helicity and soft collinear fac.

massless fermion

$$\not{p}\, U(p) = 0 \qquad \not{p}\, V(p) = 0 \qquad , U, V = \begin{pmatrix} \times \\ \times \\ \times \\ \times \end{pmatrix}$$

Weyl basis :

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad , \quad \gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

$$\sigma^\mu = (1, \vec{\sigma}) \quad , \quad \bar{\sigma}^\mu = (1, -\vec{\sigma})$$

$$U_L = \frac{1}{2}(\mathbb{1} - \gamma^5) U = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix} U$$

$$U_R = \frac{1}{2}(\mathbb{1} + \gamma^5) U = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix} U$$

$$\therefore \; U(p) = \begin{pmatrix} u_L(p) \\ u_R(p) \end{pmatrix}$$

u_L, u_R : two component spinor

Dirac equation becomes

$$\begin{pmatrix} 0 & p\cdot\sigma \\ p\cdot\bar{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0 \Rightarrow \begin{cases} p\cdot\sigma\, u_R = 0 \\ p\cdot\bar{\sigma}\, u_L = 0 \end{cases}$$

exercise : check that $u_R(p) = i\sigma^2 u_L^*(p)$

Similarily, for antifermion field,

$$V(p) = \begin{pmatrix} v_L(p) \\ v_R(p) \end{pmatrix} \qquad \begin{array}{l} p\cdot\sigma\, v_R = 0 \\ p\cdot\bar{\sigma}\, v_L = 0 \end{array}$$

angle / square bracket notation :

$$u_R(p) = v_L(p) = |p\rangle \qquad u_L(p) = v_R(p) = |p]$$

$$\bar{u}_R(p) = \bar{v}_L(p) = [p| \qquad \bar{u}_L(p) = \bar{v}_R(p) = \langle p|$$

$$|p\rangle[p| = u_R\bar{u}_R = \frac{1}{2}(\mathbb{1}+\gamma^5)\sum_{pol} u\bar{u} = \frac{1}{2}(\mathbb{1}+\gamma^5)\not{p}$$

$$|p]\langle p| = u_L\bar{u}_L = \frac{1}{2}(\mathbb{1}-\gamma^5)\not{p}$$

Notation: For n particle amplitude with momenta $k_1, k_2, \cdots k_n$,

we will label the spinor by $|k_i\rangle \equiv |i\rangle$, $|k_j] \equiv |j]$

exercise : $(\langle ij\rangle)^* = [ji]$

Mandelstam variable s_{ij} can be represented by

$$\langle ij\rangle[ji] = \bar{u}_L(k_i)\,u_R(k_j)\,\bar{u}_R(k_j)\,u_L(k_i)$$

$$= \mathrm{Tr}\left[u_L(k_i)\bar{u}_L(k_i)\,u_R(k_j)\bar{u}_R(k_j)\right]$$

$$= \mathrm{Tr}\left[\frac{1}{2}(\mathbb{1}-\gamma^5)\not{k}_i\ \frac{1}{2}\cdot(1+\gamma^5)\not{k}_j\right]$$

$$= \frac{1}{2}\mathrm{Tr}\left[(\mathbb{1}-\gamma^5)\not{k}_i\not{k}_j\right]$$

$$= 2k_i\cdot k_j = s_{ij}$$

($\langle ij\rangle$ is "square root" of s_{ij} up to a phase!)

anti-symmetric :

$$\langle ij\rangle = \bar{u}_L(k_i)\,u_R(k_j)$$

$$= u_L^\dagger(k_i)\gamma^0 u_R(k_j) \qquad \text{4-component}$$

$$= u_L^\dagger(k_i)\,u_R(k_j) \qquad \text{2-component}$$

$$= u_L^\dagger(k_i)\, i\sigma^2 u_L^*(k_j)$$

$$= [u_L^*(k_i)]_\alpha\, (i\sigma^2)_{\alpha\beta}\, (u_L^*(k_j))_\beta$$

$$= -[u_L^*(k_i)]_\alpha\, (i\sigma^2)_{\beta\alpha}\, (u_L^*(k_j))_\beta$$

$$= -\langle j i \rangle$$

$$[ij] = -[ji], \qquad [ii] = \langle ii \rangle = 0$$

Charge conjugation:

$$[i|\gamma^\mu|j\rangle = \langle j|\gamma^\mu|i]$$

to prove ~~this~~, multiply ~~to~~ the left by an arbitrary null momentum k_m,

$$k_{m\mu}[i|\gamma^\mu|j\rangle = [i|\not{k}_m|j\rangle$$

$$= [im]\langle mj\rangle$$

$$= \langle jm\rangle [mi]$$

$$= k_{m\mu}\langle j|\gamma^\mu|i]$$

Fierz identity:

$$(\bar\sigma^\mu)_{ab}(\bar\sigma_\mu)_{cd} = 2(i\sigma^2)_{ac}(i\sigma^2)_{bd}$$

$$\langle i|\gamma^\mu|j]\langle k|\gamma_\mu|l] = \left[u_L^\dagger(k_i)\,\bar\sigma_\mu\, u_L(k_j)\right]\left[u_L^\dagger(k_k)\,\bar\sigma^\mu u_L(k_l)\right]$$

$$= 2\left[u_L^*(k_i)_a (i\sigma^2)_{ac}\, u_L^*(k_k)_c\right]$$

$$\times\left[u_L(k_j)_b\, (i\sigma^2)_{bd}\, u_L(k_l)_d\right]$$

$$= 2\langle ik\rangle [jl]$$

Schouten identify:

$$\langle ij\rangle\langle kl\rangle + \langle ik\rangle\langle lj\rangle + \langle il\rangle\langle jk\rangle = 0$$

$$[ij][kl] + [ik][lj] + [il][jk] = 0$$

Products of gamma matrices:

$$\langle i|\gamma^1\gamma^2\cdots\gamma^{2n+1}|j] = [j|\gamma^{2n+1}\cdots\gamma^2\gamma^1|i\rangle$$

$$[i|\gamma^1\gamma^2\cdots\gamma^{2n+1}|j\rangle = \langle j|\gamma^{2n+1}\cdots\gamma^2\gamma^1|i]$$

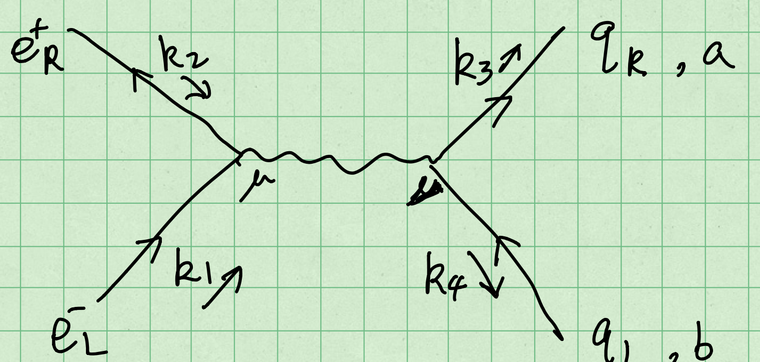
$$\langle i|\gamma^1\gamma^2\cdots\gamma^{2n}|j\rangle = -\langle j|\gamma^{2n}\cdots\gamma^2\gamma^1|i\rangle$$

$$[i|\gamma^1\gamma^2\cdots\gamma^{2n}|j] = -[j|\gamma^{2n}\cdots\gamma^2\gamma^1|i]$$

Use spinor-helicity to simplify amp calculation.

$e^+e^- \to \gamma^* \to q\bar{q}$ (the most famous Feynman Diagram)

Consider the helicity configuration: $e^+_R e^-_L \to q_R \bar{q}_L$ first



$$iM_4 = \bar{v}_R(k_2)\, ie\gamma_\mu u_L(k_1)\, \frac{-ig^{\mu\nu}}{(k_1+k_2)^2}\, \bar{u}_R(k_3)\, ie_q\delta_{ab} v_L(k_4)$$

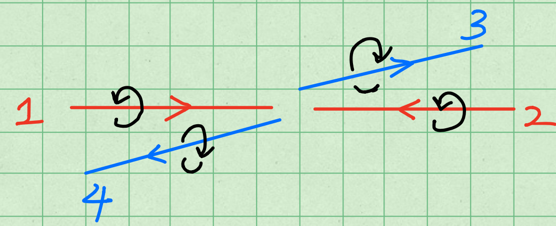
$= 2iee_q\,\delta_{ab}\,A_4$

$$A_4 = \frac{1}{2s_{12}} \langle 2|\gamma^\mu|1]\,[3|\gamma_\mu|4\rangle$$

$$\overset{\text{Fierz id.}}{=} \frac{1}{2s_{12}} 2\langle 24\rangle[13]$$

when $k_1 /\!/ k_3$, or $k_2 /\!/ k_4$

$A_4 = 0$: helicity suppression

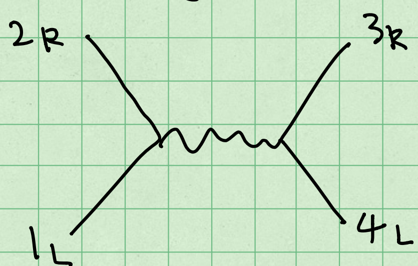


Exercise: show ~~that~~ A_4 can also be written as

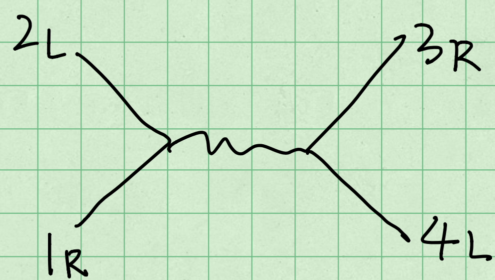
$$A_4 = \frac{\langle 24\rangle^2}{\langle 12\rangle\langle 34\rangle} \qquad \text{"holomorphic"}$$

$$= \frac{[13]^2}{[12][34]} \qquad \text{"anti-holomorphic"}$$

using charge conjugation + Parity can obtain all helicity combination.



$\xRightarrow[1\leftrightarrow 2]{C}$



$$A_4 = \frac{\langle 24\rangle^2}{\langle 12\rangle\langle 34\rangle} \qquad\qquad A_4 = \frac{-\langle 14\rangle^2}{\langle 12\rangle\langle 34\rangle}$$

$\Downarrow P \quad \langle\rangle \leftrightarrow [\,] \qquad\qquad \Downarrow P$

$\xRightarrow[1\leftrightarrow 2]{C}$

$$A_4 = \frac{[24]^2}{[12][34]} \qquad\qquad A_4 = \frac{-[14]^2}{[12][34]}$$

$$|A_4(1_L 2_R 3_R 4_L)|^2 = \frac{\langle 24\rangle^2 [42]^2}{\langle 12\rangle[21]\,\langle 34\rangle[43]}$$

$$= \frac{s_{24}^2}{s_{12}\, s_{34}} = \frac{s_{13}^2}{s_{12}^2}$$

$$|A_4(RLRL)|^2 = \frac{s_{14}^2}{s_{12}^2}$$

$$|A_4(RLLR)|^2 = \frac{s_{13}^2}{s_{12}^2}$$

$$|A_4(LRLR)|^2 = \frac{s_{14}^2}{s_{12}^2}$$

$$\therefore \sum_{\text{hel.}} |A_4|^2 = \frac{2}{s_{12}^2}\cdot\left(s_{14}^2 + s_{13}^2\right)$$

Spinor-helicity for massless vector (Chinese Magic)

final state spin-1 polarization vector

$$\varepsilon_R^{*\mu}(k) = \frac{1}{\sqrt{2}}\frac{\langle q|\gamma^\mu|k]}{\langle qk\rangle}, \quad \varepsilon_L^{*\mu}(k) = -\frac{1}{\sqrt{2}}\frac{[q|\gamma^\mu|k\rangle}{[qk]}$$

$k^2 = r^2 = 0, \quad k \neq r.$

transversality : $k_\mu \varepsilon_R^{*\mu}(k) = \frac{1}{\sqrt{2}}\frac{\langle q|\not{k}|k]}{\langle qk\rangle} = 0$

$$k_\mu \varepsilon_L^{*\mu}(k) = 0$$

bonus relation : $q_\mu \cdot \varepsilon_R^{*\mu}(k) = q_\mu \cdot \varepsilon_L^{*\mu}(k) = 0$

polarization sum:

$$\sum_{\text{pol.}} \varepsilon^{*\mu}\, \varepsilon^{\nu}(k) = \boxed{\frac{1}{\sqrt{2}} \cdot \frac{\langle q|\gamma^\mu|k]}{\langle qk\rangle}\, \frac{1}{\sqrt{2}}\, \frac{\langle k|\gamma^\nu|q]}{[kq]}}$$

$$+ \frac{1}{\sqrt{2}}\, \frac{[q|\gamma^\mu|k\rangle}{[qk]}\, \frac{1}{\sqrt{2}} \cdot \frac{[k|\gamma^\nu|q\rangle}{\langle kq\rangle}$$

first term $= \frac{1}{2} \cdot \frac{1}{2q\cdot k} \cdot \langle q|\gamma^\mu \not{k}\, \gamma^\nu\, |q]$

$$= \frac{1}{4q\cdot k} \cdot \langle q|\left(2k^\mu - \not{k}\,\gamma^\mu\right)\gamma^\nu|q]$$

Gordon identity:

$$\langle q|\gamma^\mu|q] = \mathrm{tr}[\, q]\langle q|\gamma^\mu\,]$$

$$= \mathrm{tr}\left[\tfrac{1}{2}(\mathbb{1} - \gamma^5)\, \not{q}\, \gamma^\mu\right]$$

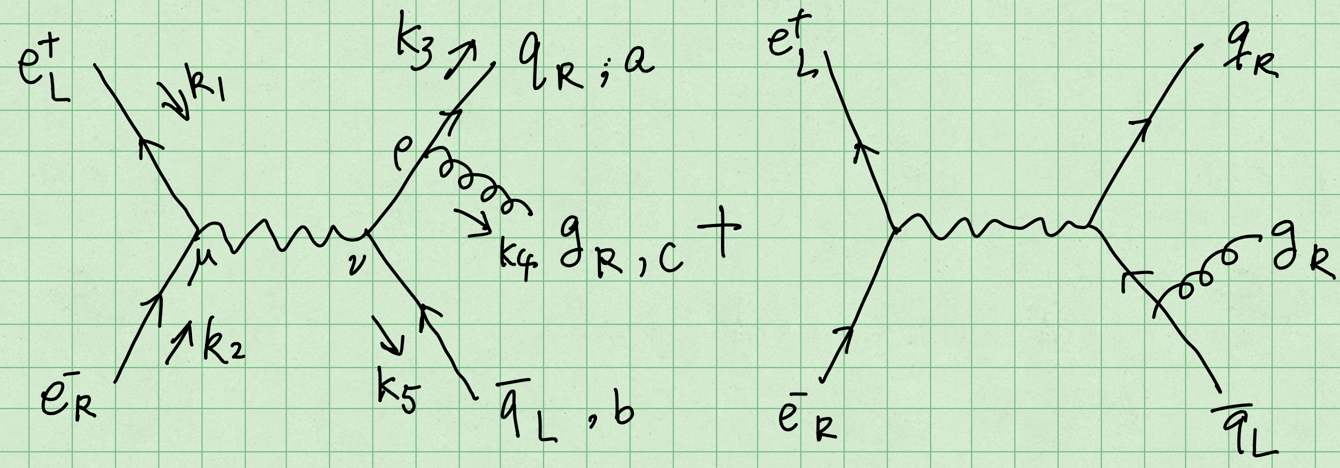
$$= 2q^\mu$$

first term $= \frac{k^\mu q^\nu}{k\cdot q} - \frac{\langle q|\not{k}\,\gamma^\mu\gamma^\nu|q]}{4q\cdot k}$

second term $= \frac{k^\nu q^\mu}{k\cdot q} - \frac{\langle q|\not{k}\,\gamma^\nu\gamma^\mu|q]}{4q\cdot k}$ exercise

first + second term

$$\sum_{\text{pol}} \varepsilon^{*\mu}\varepsilon^{\nu}(k) = \underbrace{-g^{\mu\nu} + \frac{k^\mu q^\nu + k^\nu q^\mu}{q\cdot k}}_{\text{light-cone gauge (physical)}}$$



$$iM_5 = \bar{V}_L(k_1)\, ie\gamma^\mu U_R(k_2)\, \frac{-ig_{\mu\nu}}{(k_1+k_2)^2}$$
$$\times \bar{U}_R(k_3)(-ig_s)\not{\varepsilon}^*_R(k_4)(t^c)_{ab}\frac{i}{(\not{k}_3+\not{k}_4)}\, ie_q\gamma^\nu V_L(k_5)$$
first diagram

$$+\bar{V}_L(k_1)\, ie\gamma^\mu U_R(k_2)\, \frac{-ig_{\mu\nu}}{(k_1+k_2)^2}$$
$$\times \bar{U}_R(k_3)\, ie_q\gamma^\nu\, \frac{-i}{(\not{k}_4+\not{k}_5)}\cdot(-ig_s)\, t^c_{ab}\cdot\not{\varepsilon}^*_R(k_4)V_L(k_5)$$
second diag.

let $iM_5 = 2iee_q g_s (t^c)_{ab} A_5$

$$A_5 = [1|\gamma^\mu|2\rangle\frac{1}{2s_{12}}\cdot[3|\not{\varepsilon}^*_R(k_4)\frac{(\not{k}_3+\not{k}_4)}{s_{34}}\gamma_\mu|5\rangle$$
$$-[1|\gamma^\mu|2\rangle\frac{1}{2s_{12}}[3|\gamma_\mu\frac{(\not{k}_4+\not{k}_5)}{s_{45}}\not{\varepsilon}^*_R(k_4)|5\rangle$$
$$= \frac{1}{s_{12}s_{34}}[1|(\not{k}_3+\not{k}_4)\not{\varepsilon}^*_R(k_4)|3]\langle 25\rangle$$
$$-\frac{1}{s_{12}s_{45}}[13]\langle 2|(\not{k}_4+\not{k}_5)\not{\varepsilon}^*_R(k_4)|5\rangle$$

recall that :

$$\varepsilon^\mu_R(k_4) = \frac{1}{\sqrt{2}}\frac{\langle q|\gamma^\mu|4]}{\langle q4\rangle} \qquad \varepsilon^\mu_L(k_4) = -\frac{1}{\sqrt{2}}\cdot\frac{[q|\gamma^\mu|4\rangle}{[q4]}$$

$$A_5 = \frac{1}{s_{12}s_{34}}\frac{1}{\sqrt{2}}\cdot[1|(\cancel{k}_3+\cancel{k}_4)\gamma^\mu|3]\langle 25\rangle\frac{\langle q|\gamma_\mu|4]}{\langle q4\rangle}$$

$$-\frac{1}{s_{12}s_{45}}\frac{1}{\sqrt{2}}\langle 2|(\cancel{k}_4+\cancel{k}_5)\gamma^\mu|5\rangle[13]\frac{\langle q|\gamma_\mu|4]}{\langle q4\rangle}$$

$$= \frac{2}{\sqrt{2}\,s_{12}s_{34}}\langle 25\rangle[1|(\cancel{k}_3+\cancel{k}_4)q\rangle\frac{[34]}{\langle q4\rangle}$$

$$-\frac{2}{\sqrt{2}\,s_{12}s_{45}}[13]\langle 2|(\cancel{k}_4+\cancel{k}_5)4]\frac{\langle 5q\rangle}{\langle q4\rangle}$$

let $\boxed{q=k_5}$, the second diagram can be dropped,

$$A_5\Big|_{q=k_5} = \frac{\sqrt{2}}{s_{12}s_{34}}\langle 25\rangle[1|(\cancel{k}_3+\cancel{k}_4)|5\rangle\frac{[34]}{\langle 54\rangle}$$

$$= \frac{\sqrt{2}}{s_{12}s_{34}}\langle 25\rangle[1|(\cancel{k}_1+\cancel{k}_2-\cancel{k}_5)|5\rangle\frac{[34]}{\langle 54\rangle}$$

$$= \frac{\sqrt{2}}{s_{12}s_{34}}\langle 25\rangle[12]\langle 25\rangle\cdot\frac{[34]}{\langle 54\rangle}$$

$$= \frac{\sqrt{2}\langle 25\rangle^2}{\langle 12\rangle\langle 34\rangle\langle 54\rangle} = -\frac{\sqrt{2}\langle 25\rangle^2}{\langle 12\rangle\langle 34\rangle\langle 45\rangle}$$

(课上漏了一个负号)

Similarly, we obtain for

$$A(1_R 2_L 3_R 4_L 5_L)\Big|_{q\to k_3} = \frac{\sqrt{2}}{s_{12}s_{45}}[13]\langle 2|(\cancel{k}_4+\cancel{k}_5)3]\frac{\langle 54\rangle}{[34]}$$

$$= \frac{\sqrt{2}}{s_{12}s_{45}}[13]\langle 21\rangle[13]\frac{\langle 54\rangle}{[34]}$$

$$= \frac{-\sqrt{2}[13]^2}{[12][34][45]}$$

exercise: use charge conjugation and parity to get the remainings.

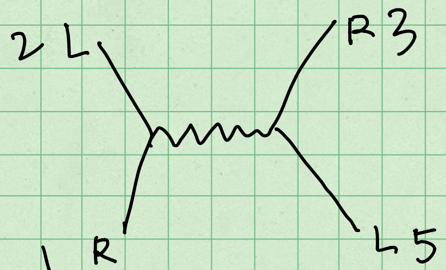
sum over spin and color, we obtain the square amplitude for $e^+e^- \to q(p_3)\,\bar{q}(p_5)\,g(p_4)$.

$$\sum |M|^2 = 16 e^2 e_q^2 g_s^2 \, tr[t^a t^a] \cdot \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12}\, s_{34}\, s_{45}}$$

$$= 48 e^2 e_q^2 g_s^2 C_F \cdot \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12}\, s_{34}\, s_{45}}$$

where $s_{ij} = 2k_i \cdot k_j$

Soft / collinear limit of amplitude.



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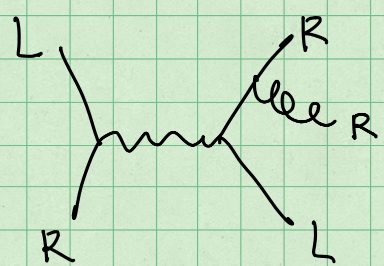

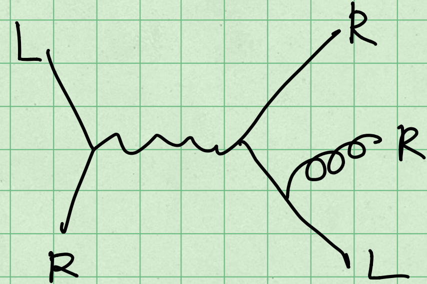

$$A_4 = \frac{\langle 25\rangle^2}{\langle 12\rangle\langle 35\rangle} \qquad A_5 = \frac{-\sqrt{2}\,\langle 25\rangle^2}{\langle 12\rangle\langle 34\rangle\langle 45\rangle}$$

soft gluon emission: $k_4^0 \to 0$

$$A_5 = \frac{-\sqrt{2}\,\langle 35\rangle}{\langle 34\rangle\langle 45\rangle} \cdot A_4$$

$$= \underbrace{S(RRL)}_{\text{eikonal amplitude.}} A_4$$

$$S(RLL) = \frac{-\sqrt{2}\,[35]}{[34][45]}$$