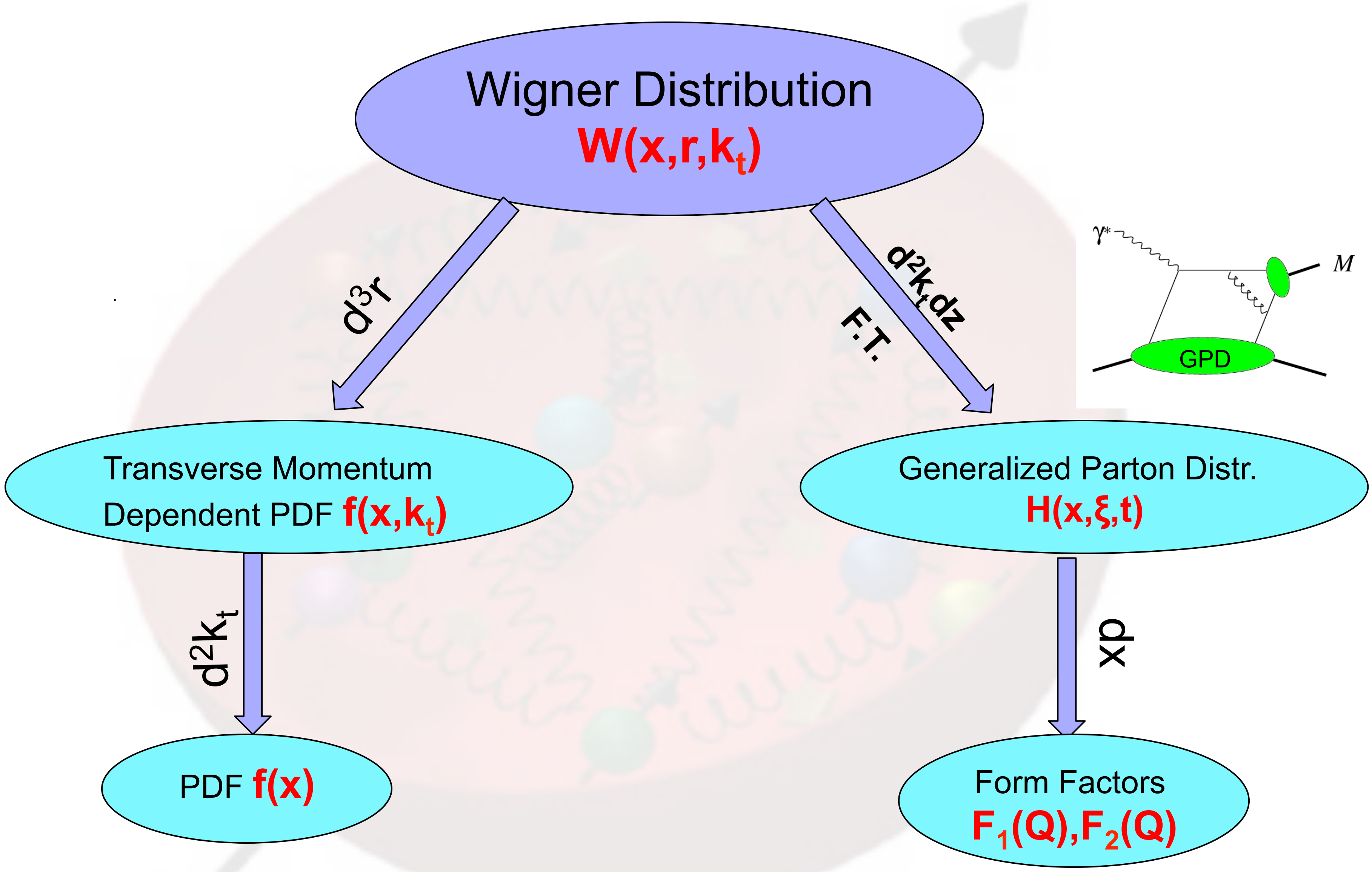
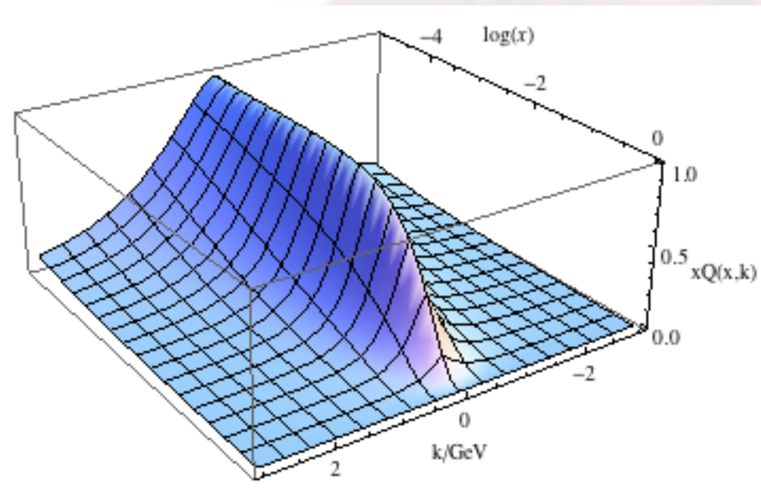
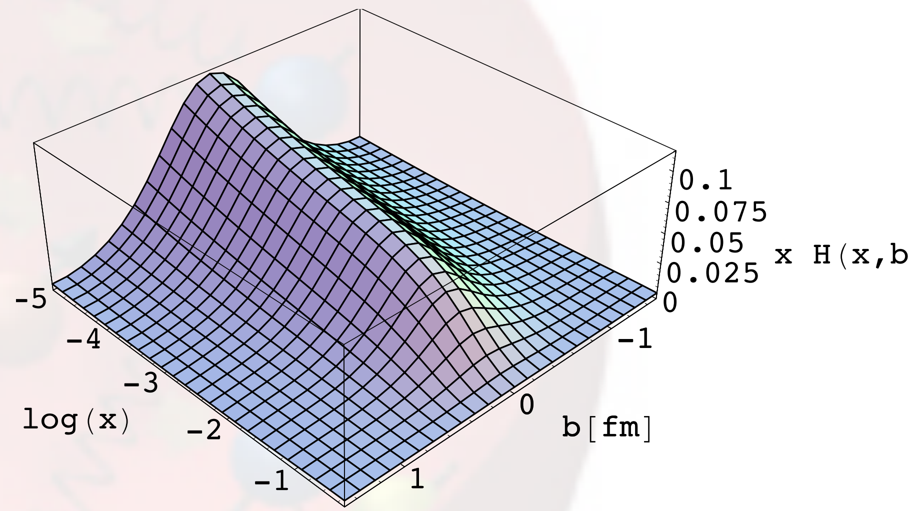


Wigner Distribution
W(x,r,k_t)
d^3r
d^2k_t dz
F.T.
γ*
M
GPD
Transverse Momentum Dependent PDF f(x,k_t)
Generalized Parton Distr. H(x,ξ,t)
d^2k_t
dx
PDF f(x)
Form Factors
F_1(Q),F_2(Q)

Transverse profile for the quark distribution: k_t vs b_t

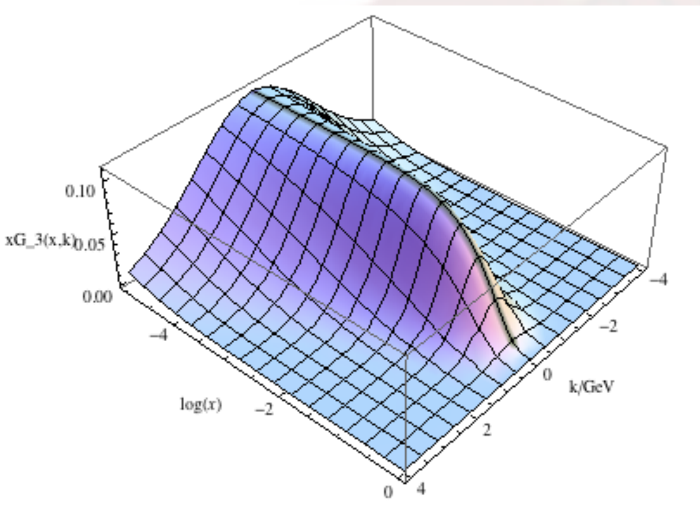


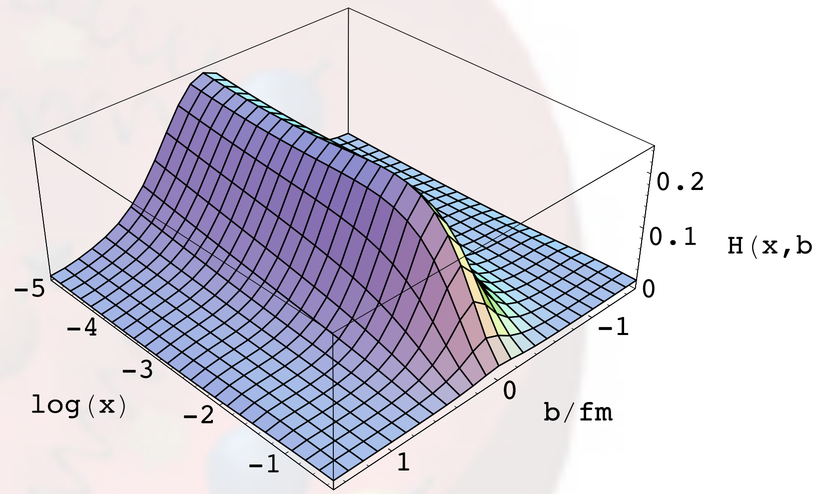


Quark distribution calculated from a saturation-inspired model
A.Mueller 99, McLerran-Venugopalan 99

GPD fit to the DVCS data from HERA, Kumerick-D.Mueller, 09,10

Gluon distribution

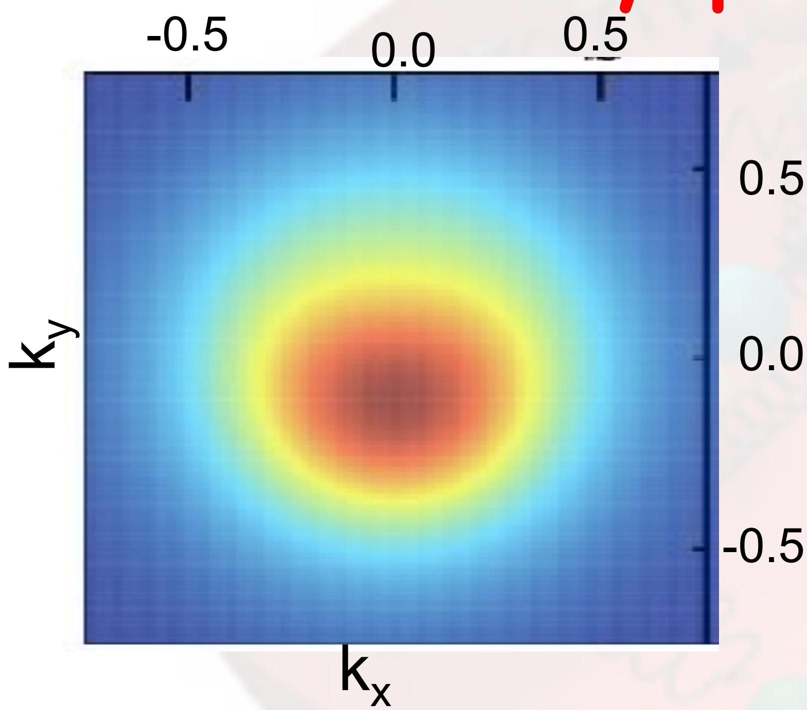


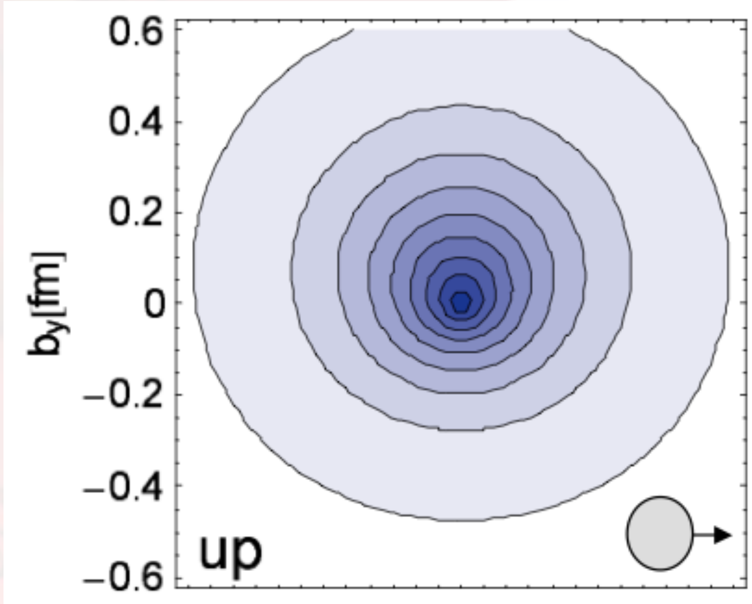


One of the TMD gluon distributions at small-x

GPD fit to the DVCS data from HERA, Kumerick-Mueller, 09,10

Deformation when nucleon is transversely polarized





Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 20009

Lattice Calculation of the IP density of Up quark, QCDSF/UKQCD Coll., 2006

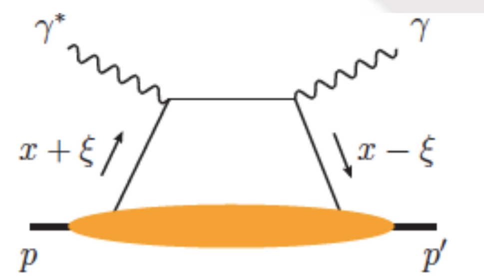
Generalized Parton Distributions

Mueller, et al. 1994; Ji, 1996, Radyushkin 1996

- Off-diagonal matrix elements of the quark operator (along light-cone)

$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{P} e^{-ig\int_{\lambda/2}^{-\lambda/2} d\alpha \; n\cdot A(\alpha n)} \psi_q\left(\frac{\lambda}{2}n\right) \right| P \right\rangle$$
$$= H_q(x,\xi,t)\; \frac{1}{2}\overline{U}(P')\not{n}U(P) + E_q(x,\xi,t)\; \frac{1}{2}\overline{U}(P')\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}U(P)\,.$$

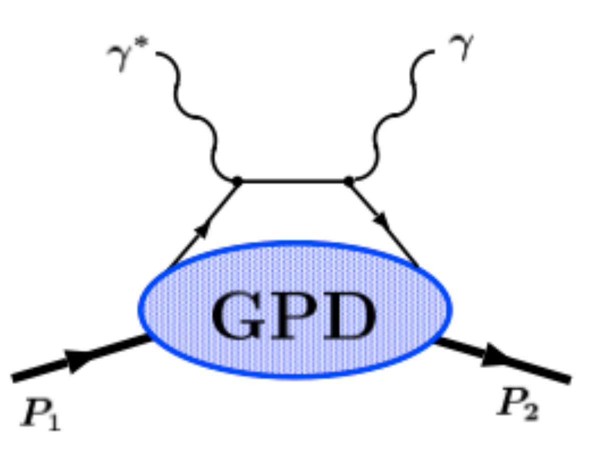
- It depends on quark momentum fraction x and skewness ξ, and nucleon momentum transfer t

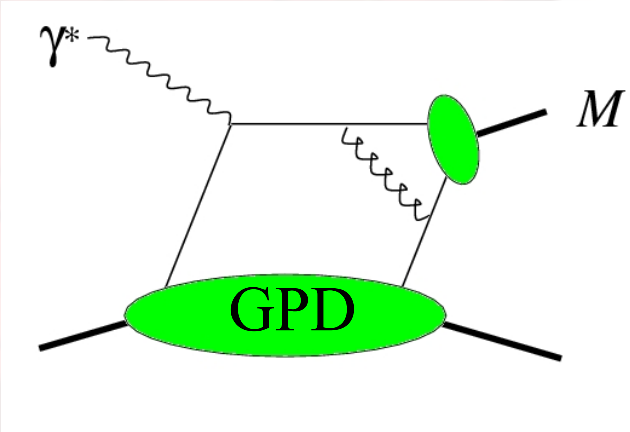


$$\xi = -n\cdot(P'-P)/2$$
$$t = \Delta^2 = (P-P')^2$$

Access the GPDs

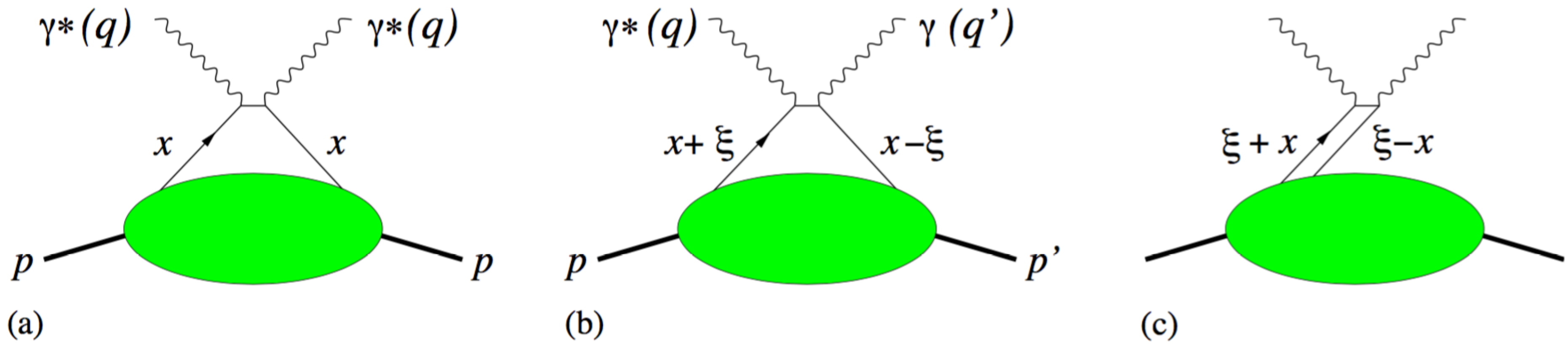
- Deeply virtual Compton Scattering (DVCS) and deeply virtual exclusive meson production (DVEM)

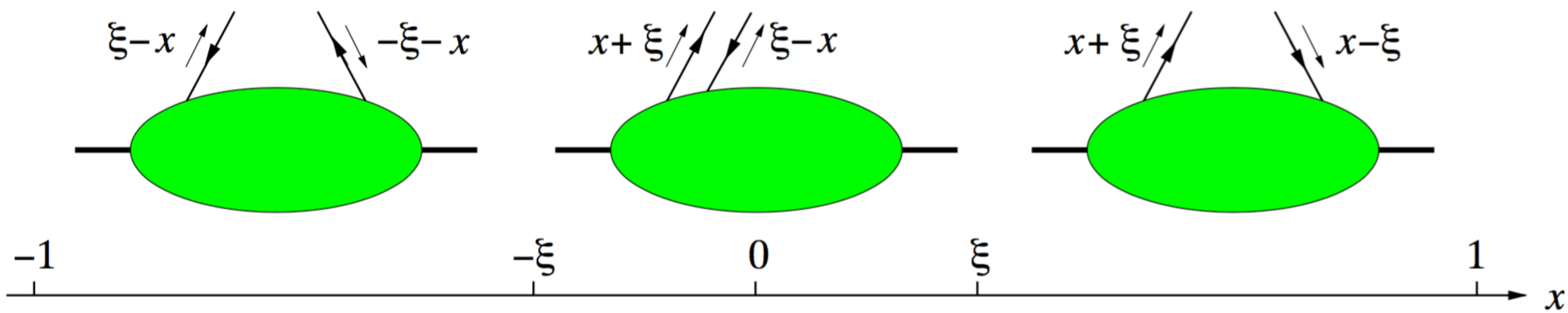




In the Bjorken limit: $Q^2 \gg (-t), \Lambda^2_{QCD}, M^2$

DVCS kinematics vs DIS





Anti-quark DGLAP

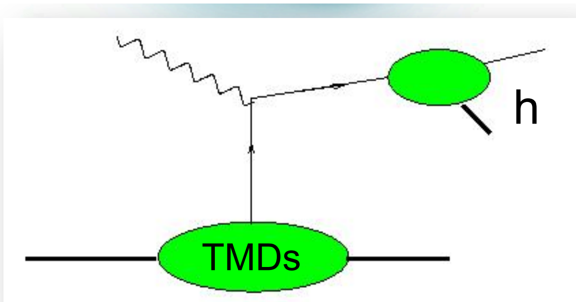
ERBL

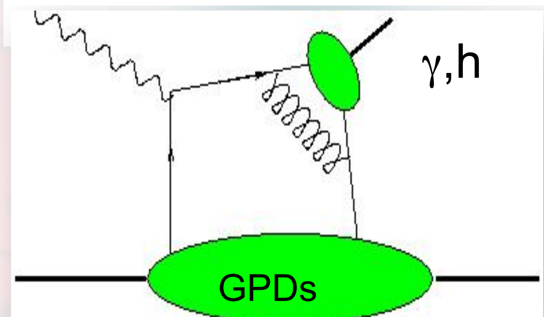
Quark DGLAP

Zoo of TMDs & GPDs

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$





- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of x_B, t, Q^2** (GPD) and **x_B, P_T, Q^2, z** (TMD)

Example: quark GPDs

- Unpolarized

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+ q(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

- Polarized

$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+\gamma_5 q(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2m}u(p)\right]$$

Forward limit

- Reduce to the normal PDFs

$$H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x) \qquad \text{for } x > 0$$

$$H^q(x,0,0) = -\bar{q}(-x), \quad \tilde{H}^q(x,0,0) = \Delta\bar{q}(-x) \quad \text{for } x < 0$$

Sum rules

- Integral over x lead to form factors

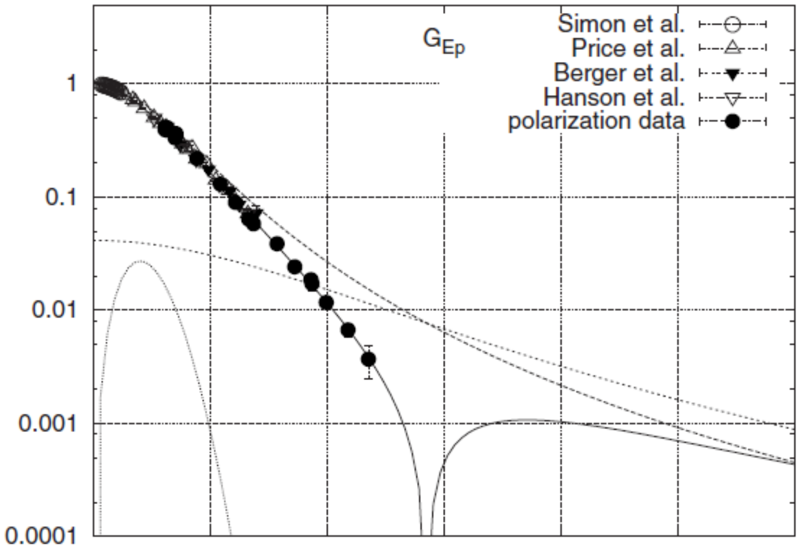
$$\int_{-1}^{1} \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t)$$

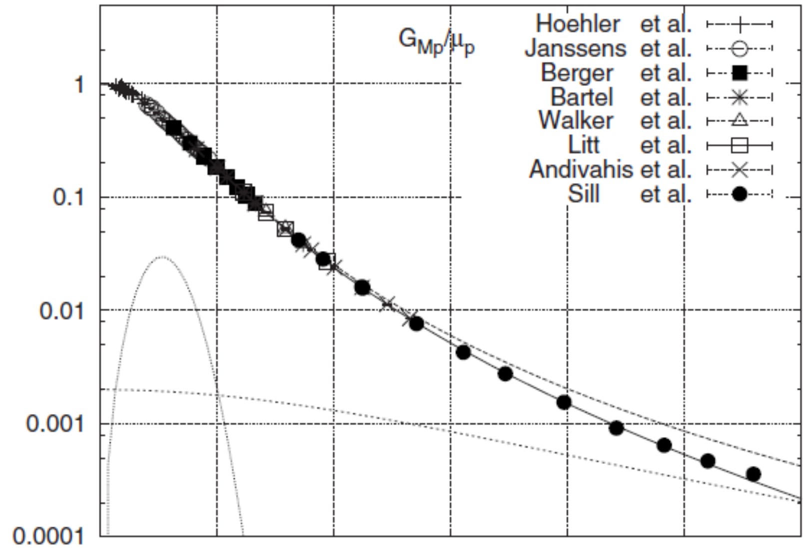
$$\langle p'|\bar{q}(0)\gamma^\mu q(0)|p\rangle = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{\mathrm{i}\sigma^{\mu\alpha}\Delta_\alpha}{2m}\right]u(p)$$

$$\int_{-1}^{1} \mathrm{d}x\, \tilde{H}^q(x,\xi,t) = g_A^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, \tilde{E}^q(x,\xi,t) = g_P^q(t)$$

$$\langle p'|\bar{q}(0)\gamma^\mu\gamma_5 q(0)|p\rangle = \bar{u}(p')\left[g_A^q(t)\gamma^\mu\gamma_5 + g_P^q(t)\,\frac{\gamma_5\Delta^\mu}{2m}\right]u(p)$$

Form factors have been used to constrain GPDs:





$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2),$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \qquad \tau = Q^2/4M^2$$

$$J^h_\mu = \bar{U}(P')\left[\gamma_\mu F_1(Q^2) + i(\sigma_{\mu\nu}q^\nu/2M)F_2(Q^2)\right]U(P)$$

Polynomiality:

- Moments (x) are polynomial in skewness

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x,\xi,t) = \sum_{\substack{i=0 \\ \text{even}}}^{n} (2\xi)^i A^q_{n+1,i}(t) + \mathrm{mod}(n,2)(2\xi)^{n+1} C^q_{n+1}(t)$$

$$\int_{-1}^{1} \mathrm{d}x\, x^n E^q(x,\xi,t) = \sum_{\substack{i=0 \\ \text{even}}}^{n} (2\xi)^i B^q_{n+1,i}(t) - \mathrm{mod}(n,2)(2\xi)^{n+1} C^q_{n+1}(t)$$

One particular example: Ji sum rule

$$\int (H + E)\, x\, dx = J_q = 1/2\, \Delta\Sigma + L_z$$

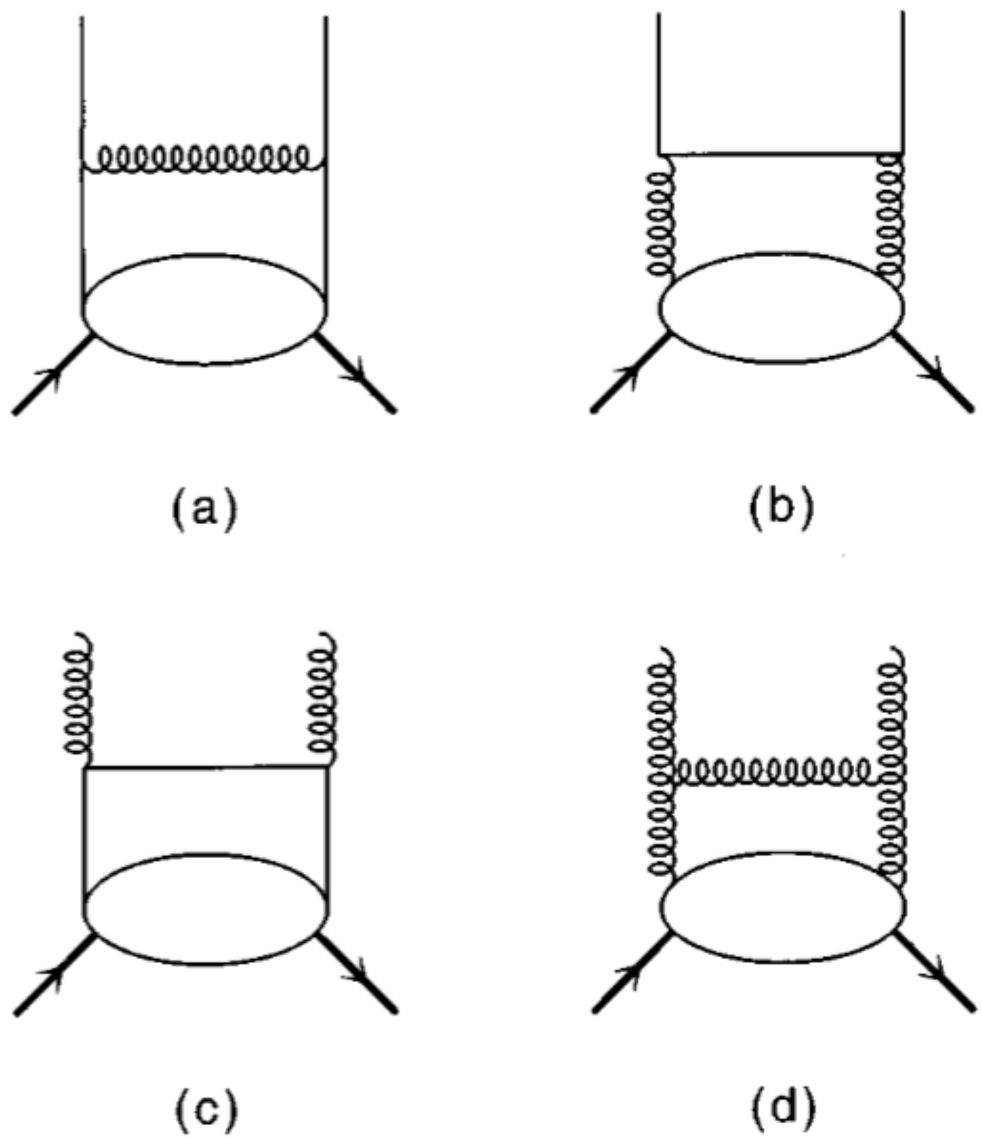
Ji,96

$$A_q(t) + B_q(t) = \int_{-1}^{1} \mathrm{d}x\, x [H_q(x,\xi,t) + E_q(x,\xi,t)]$$

- Define the gravitational form factors

$$\langle p' | T^{\mu\nu}_{q,g} | p \rangle = A_{q,g}(t) \bar{u} P^{(\mu} \gamma^{\nu)} u + B_{q,g}(t) \bar{u} \frac{P^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2m} u$$

Evolutions



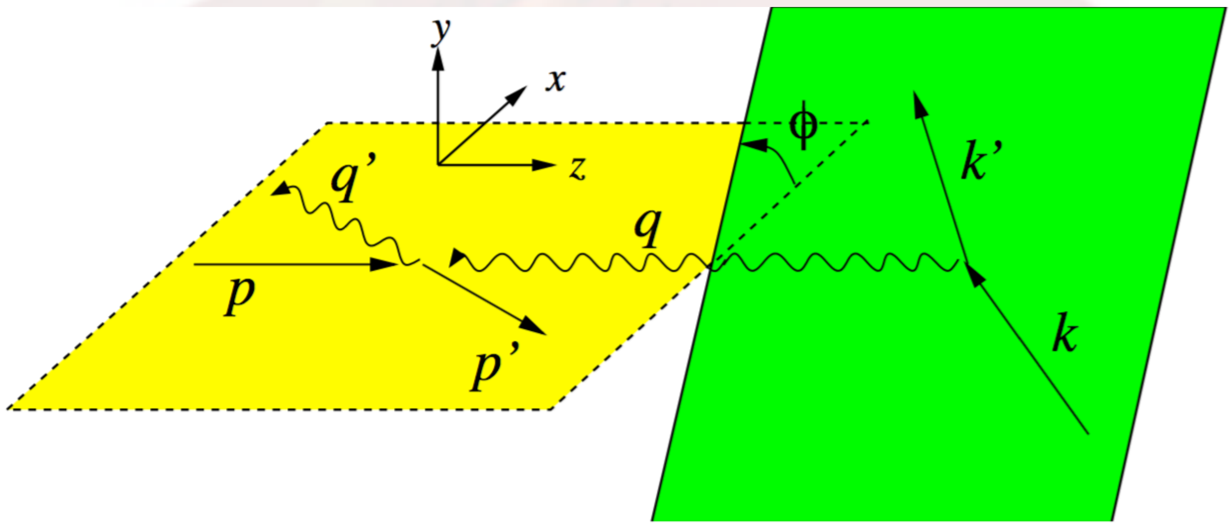
Example: non-singlet case

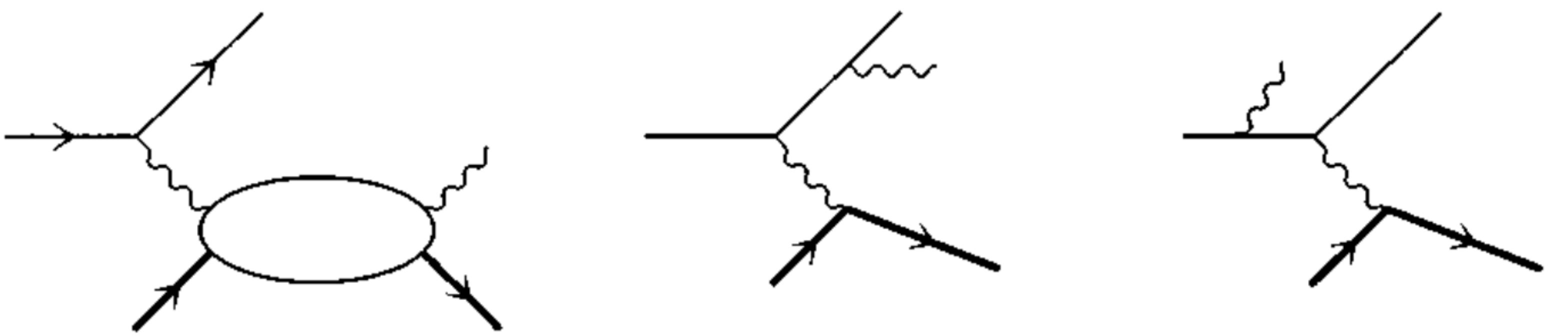
$$\frac{D_Q F_{NS}(x,\xi,Q^2)}{D\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y} P_{NS}\left(\frac{x}{y},\frac{\xi}{y},\frac{\epsilon}{y}\right) F_{NS}(y,\xi,Q^2)$$

$$P_{NS}(x,\xi,\epsilon) = C_F \frac{x^2+1-2\xi^2}{(1-x+\mathrm{i}\epsilon)(1-\xi^2)}$$

- Reduces to DGLAP evolution at $\xi=0$

Experiments: DVCS and BH





(a) (b) (c)

BH amplitude depends on form factors

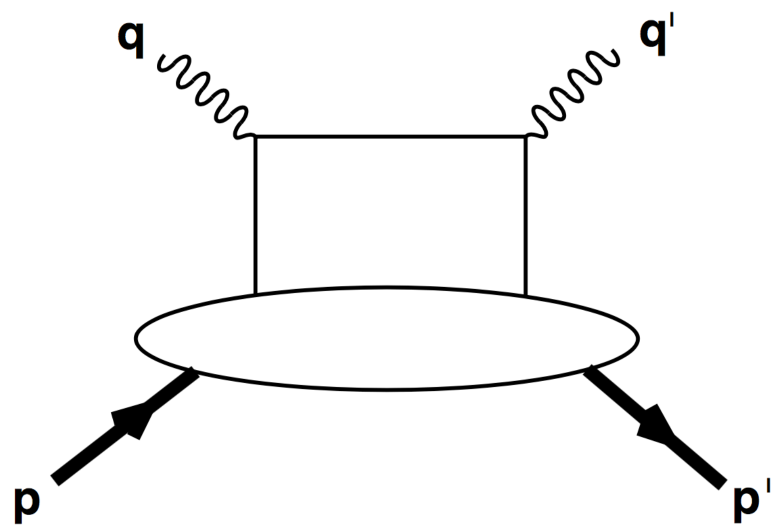
$$\mathcal{T}_2 = -e^3 \overline{u}(k') \left[\not{\epsilon}^* \frac{1}{\not{k} - \not{\Delta} - m_e + i\epsilon} \gamma^\mu \right.$$

$$\left. + \gamma^\mu \frac{1}{\not{k}' + \not{\Delta} - m_e + i\epsilon} \not{\epsilon}^* \right] u(k) \frac{1}{\Delta^2} \langle P' | J_\mu(0) | P \rangle$$

$$\langle P' | J_\mu(0) | P \rangle = \overline{U}(P') \left[\gamma_\mu F_1(\Delta^2) \right.$$

$$\left. + F_2(\Delta^2) \frac{i\sigma_{\mu\nu} \Delta \nu}{2M} \right] U(P)$$

Hand-back diagram for DVCS



+crossing diagram

$$T^{\mu\nu} = g_{\perp}^{\mu\nu} \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x - \xi + \mathrm{i}\epsilon} + \frac{1}{x + \xi - \mathrm{i}\epsilon} \right) \sum_q \mathrm{e}_q^2 F_q(x, \xi, t, Q^2)$$

- In the end, the differential cross section will depend on the BH, DVCS, and their interference

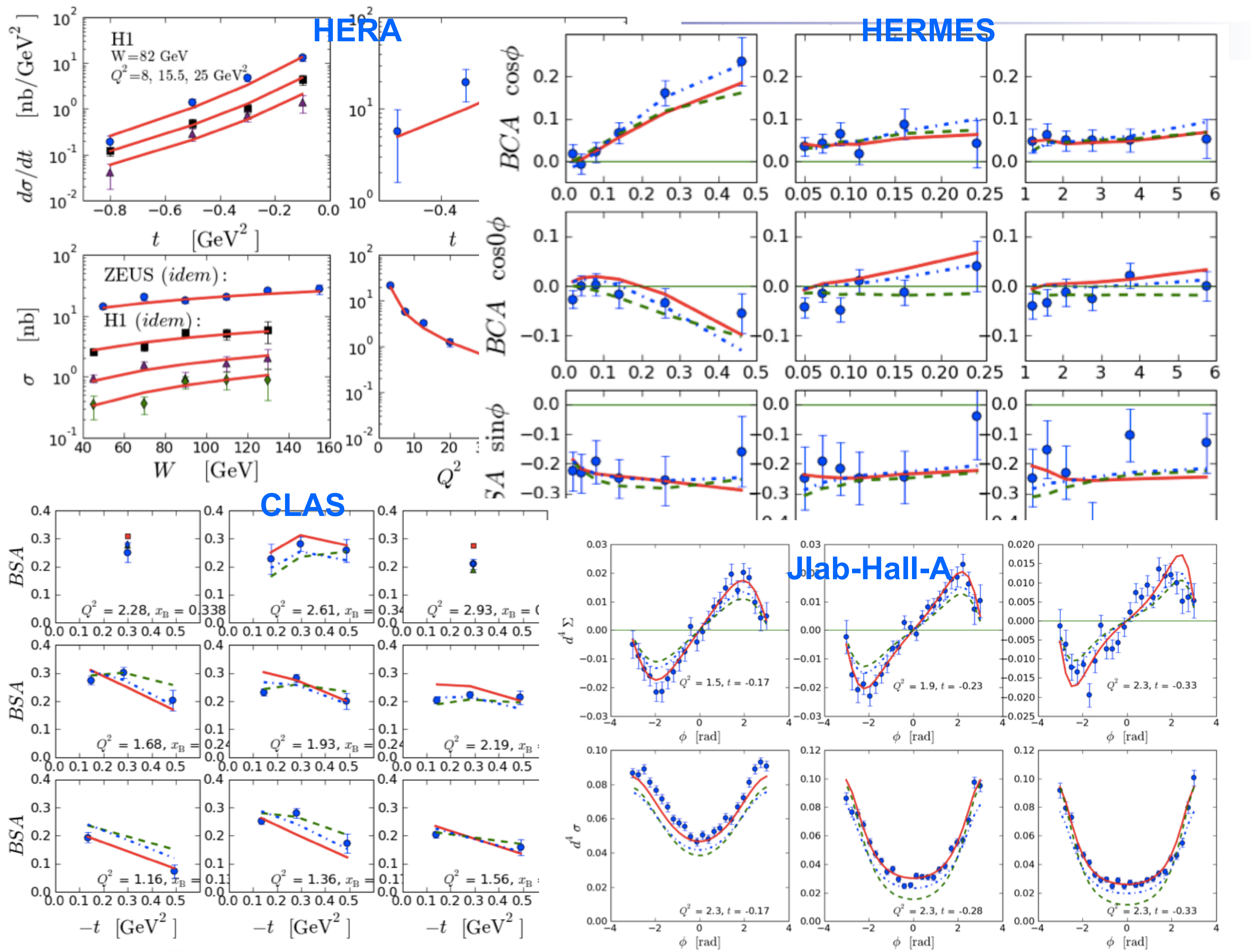
$$\mathcal{T}^2 = |\mathcal{T}_{\mathrm{BH}}|^2 + |\mathcal{T}_{\mathrm{DVCS}}|^2 + \mathcal{T}_{\mathrm{DVCS}}\mathcal{T}^*_{\mathrm{BH}} + \mathcal{T}^*_{\mathrm{DVCS}}\mathcal{T}_{\mathrm{BH}}$$

Azimuthal angular distribution

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{e^6}{x_{\mathrm{B}}^2 y^2 (1+\epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \times \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) + s_1^{\mathrm{BH}} \sin(\phi) \right\},$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 Q^2} \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} [c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi)] \right\}$$

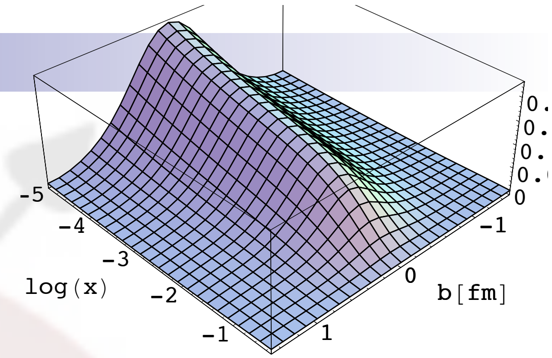
$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{B}} y^3 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\Delta^2} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} [c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi)] \right\},$$

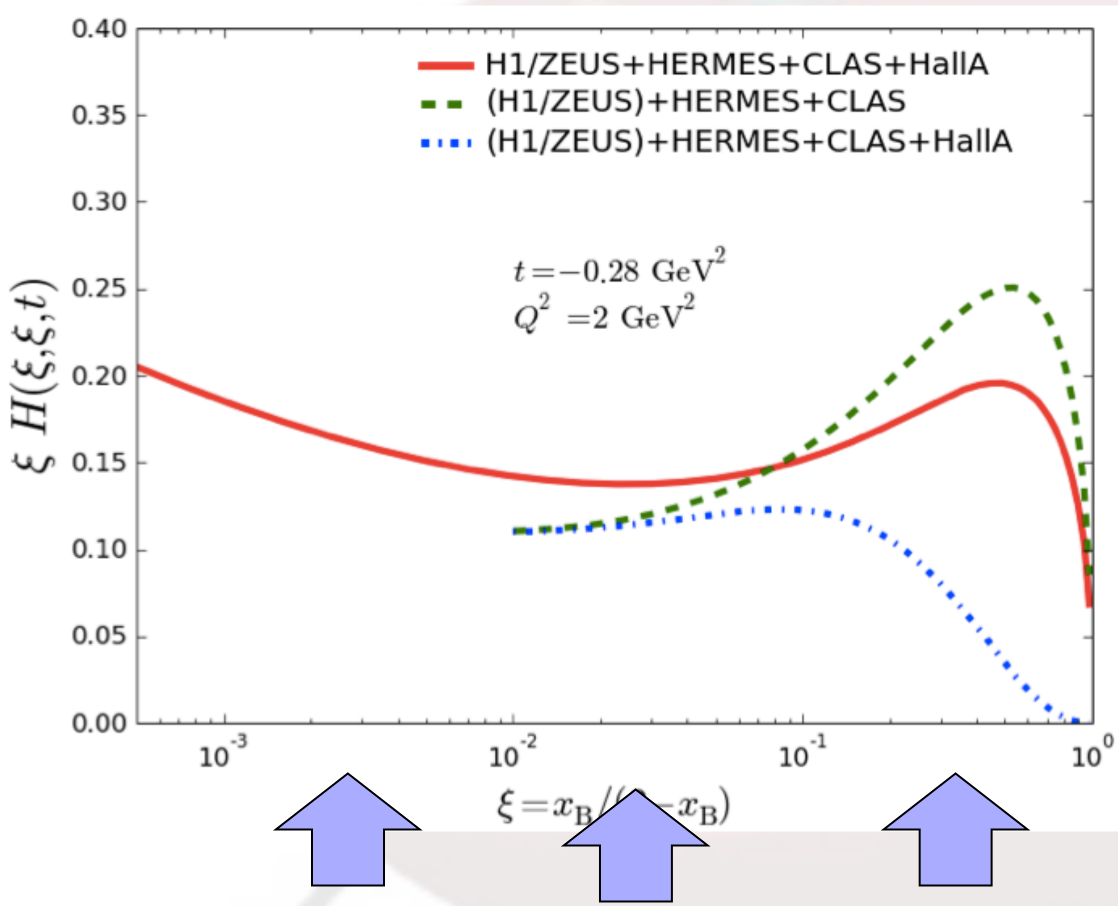

HERA
HERMES
CLAS
Jlab-Hall-A

Extract the GPDs

- The theoretical framework has been well established
  - Perturbative QCD corrections at NLO
  - However, GPDs depend on x, ξ, t, it is much more difficult than PDFs (only depends on x)
  - There will be model dependence at the beginning

One example: H(x,x,t)





HERA HERMES JLab

D. Mueller, et al, 09, 10

Small-x range constrained by HERA, uncertainties at large-x shall be very much reduced with Jlab 12 GeV COMPASS, and the planed EIC

Of course, there are also other GPDs, in particular, the GPD E

Power counting of Large x structure

- Drell-Yan-West (1970)

$$F_1(q^2) \xrightarrow[q^2 \to -\infty]{} (-1/q^2)^n \longleftrightarrow \nu W_2(x) \xrightarrow[x \to 1]{} (1-x)^{2n-1}$$

- Farrar-Jackson (1975)

$$\nu W_2^{\pi} \sim (1-x)^2 \quad \text{and} \quad \nu W_2^{p} \sim (1-x)^3$$

- Brodsky-Lepage (1979)

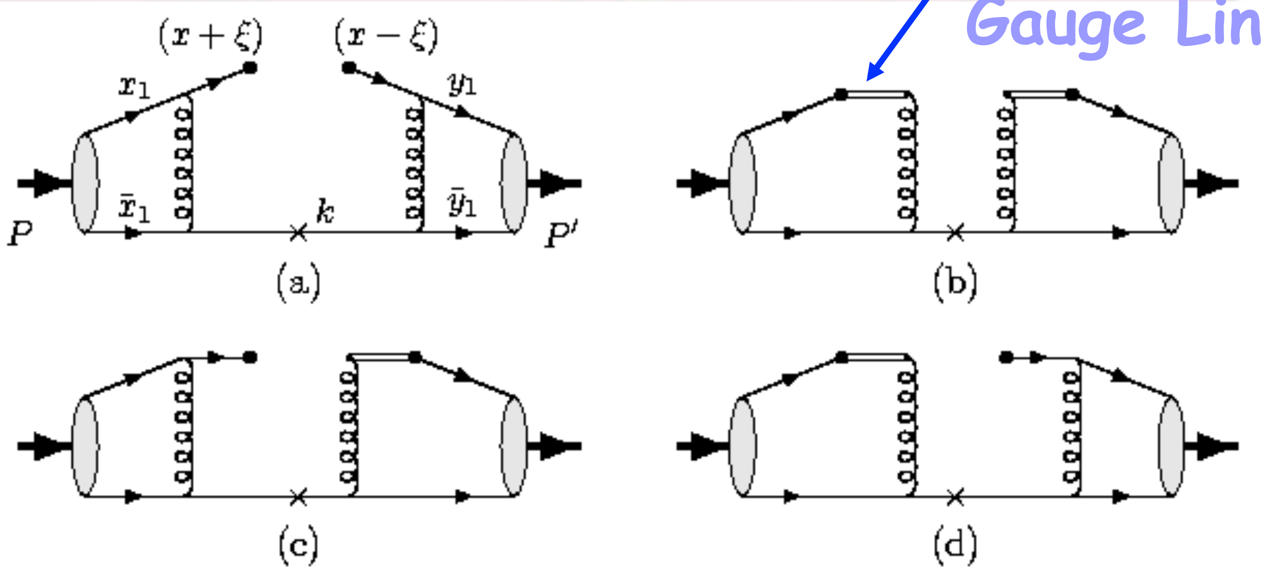
$$G_{q\uparrow/p\uparrow} \sim (1-x)^3 \quad ; \quad G_{q\downarrow/p\uparrow} \sim (1-x)^5$$

- Brodsky-Burkardt-Shmidt (1995) fit the polarized structure functions.

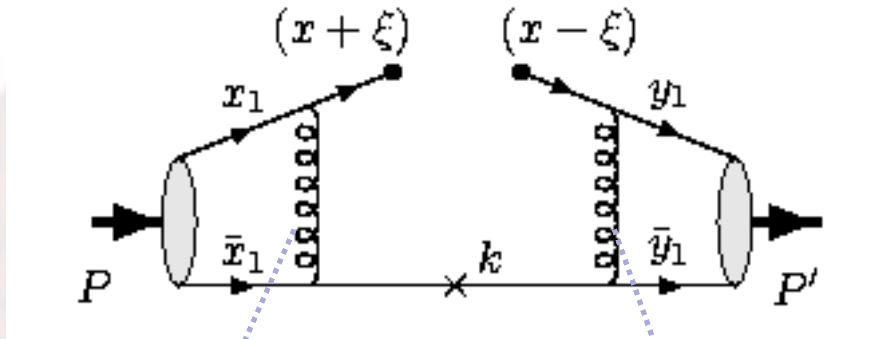
Large-x power counting for the GPDs

$$H_q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle \pi; P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{L} \psi_q\left(\frac{\lambda}{2}n\right) \right| \pi; P \right\rangle$$

Gauge Link



Where is the t-dependence



$$\frac{1}{2k\cdot P} = \frac{1-x}{\vec{k}_\perp^2(1+\xi)}\left[1+\frac{(1-x)^2(1-\xi^2)t}{4(1+\xi)^2\vec{k}_\perp^2}\right]$$

$$\frac{1}{2k\cdot P'} = \frac{1-x}{\vec{k}_\perp^2(1-\xi)}\left[1+\frac{(1-x)^2(1-\xi^2)t}{4(1-\xi)^2\vec{k}_\perp^2}\right]$$

- In the leading order, there is no t-dependence
- Any t-dependence is suppressed by a factor $(1-x)^2$

Power counting results for pion GPD

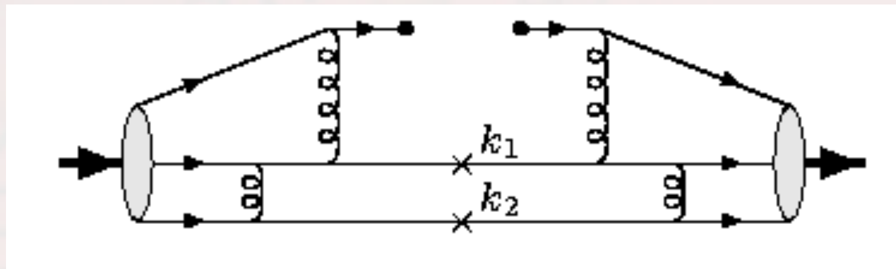
- in the limit of *x->1*,

$$H_q^{\pi}(x,\xi,t) \propto \frac{(1-x)^2}{1-\xi^2}$$

$$H_q^{\pi}(x,\xi,t) = \frac{1}{1-\xi^2} q^{\pi}(x)$$

- We can approximate the GPD with forward PDF at large *x*,

GPDs for nucleon



$$\mathcal{H}_{\lambda'\lambda} = \frac{1}{2\sqrt{1-\xi^2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P', \lambda' \left| \bar{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{L} \psi_q\left(\frac{\lambda}{2}n\right) \right| P, \lambda \right\rangle$$

Helicity nonflip

$$\mathcal{H}_{\uparrow\uparrow} = \mathcal{H}_{\downarrow\downarrow} = H_q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E_q(x,\xi,t) ,$$

Helicity flip

$$\mathcal{H}_{\downarrow\uparrow} = -\mathcal{H}^*_{\uparrow\downarrow} = \frac{\Delta^x + i\Delta^y}{2M_p(1-\xi^2)} E_q(x,\xi,t) .$$

Helicity non-flip amplitude

- The propagator

$$\frac{1}{2P\cdot(k_1+k_2)} = \frac{1-x}{\langle\vec{k}_\perp^2\rangle(1+\xi)}\left[1+\mathcal{O}((1-x)^2)\frac{t}{\langle\vec{k}_\perp^2\rangle}+\cdots\right]$$

- Power behavior

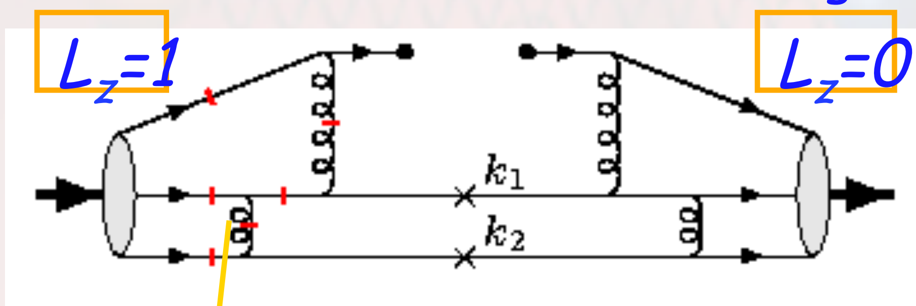
$$\mathcal{H}_{11} = \frac{1}{(1-\xi^2)^2}\underline{q(x)} \sim \frac{(1-x)^3}{(1-\xi^2)^2}$$

Forward PDF

BERKELEY LAB

Helicity flip amplitude

- Since hard scattering conserves quark helicity, to get the helicity flip amplitude, one needs to consider the hadron wave function with one-unit of orbital angular momentum



- In the expansion of the amplitude at small transverse momentum $l_{\perp}$, additional suppression of $(1-x)^2$ will arise

$$\frac{1}{(k_2 - x_3 P - l_\perp)^2} = \frac{1}{(k_2 - x_3 P)^2}\left[1 - \frac{\beta(1-x)^2 \vec{\Delta}_\perp \cdot \vec{l}_\perp}{(1+\xi)^2 \vec{k}_{2\perp}^2}\right]$$

- Two kinds of expansions

  Propagator: $(1-x)^5(1+\xi^2)/(1-\xi^2)^4$

  Wave function: $(1-x)^5/(1-\xi^2)^4$

- The power behavior for the helicity flip amplitude

$$\mathcal{H}_{\downarrow\uparrow} \sim (\Delta_\perp^x + i\Delta_\perp^y)\frac{(1-x)^5}{(1-\xi^2)^4}f(\xi)$$

- GPD *E*

$$E_q(x,\xi,t) \sim \frac{(1-x)^5}{(1-\xi^2)^3}f(\xi)$$

- GPD *H*

$$H_q(x,\xi,t) = \frac{1}{(1-\xi^2)^2}q(x)$$

Forward PDF

Summary for the GPDs' power prediction

- No t-dependence at leading order
- Power behavior at large x

$$H_q^\pi(x,\xi,t) = \frac{1}{1-\xi^2} q^\pi(x)$$ ~(1-x)^2

$$H_q(x,\xi,t) = \frac{1}{(1-\xi^2)^2} q(x)$$ ~ (1-x)^3

Forward PDF

$$E_q(x,\xi,t) = \frac{(1-x)^5}{(1-\xi^2)^3} f(\xi)$$

Log(1-x) should also show up

TMD Parton Distributions

- The definition contains explicitly the gauge links

Collins-Soper 1981,
Collins 2002,
Belitsky-Ji-Yuan 2002

$$f(x,k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp\cdot\vec{k}_\perp)} \times\langle PS|\overline{\psi}(\xi^-,\xi_\perp)L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

- The polarization and kt dependence provide rich structure in the quark and gluon distributions
  - Mulders-Tangerman 95, Boer-Mulders 98

Transverse-momentum-dependent (TMD) Parton distributions

- Generalize Feynman parton distribution $q(x)$ by including the transverse momentum.

  $$q(x,k_T)$$

- At small k_T, the transverse-momentum dependence is generated by soft non-perturbative physics.
- At large k_T, the k-dependence can be calculated in perturbative QCD and falls like powers of $1/k_T^2$

Transverse momentum dependent parton distribution

- Straightforward extension
  - Spin average, helicity, and transversity distributions
- Transverse momentum-spin correlations
  - Nontrivial distributions, $S_T X P_T$
  - In quark model, depends on S- and P-wave interference

Transverse momentum dependent parton distribution

Straightforward extension
- Spin average, helicity, and transversity distributions

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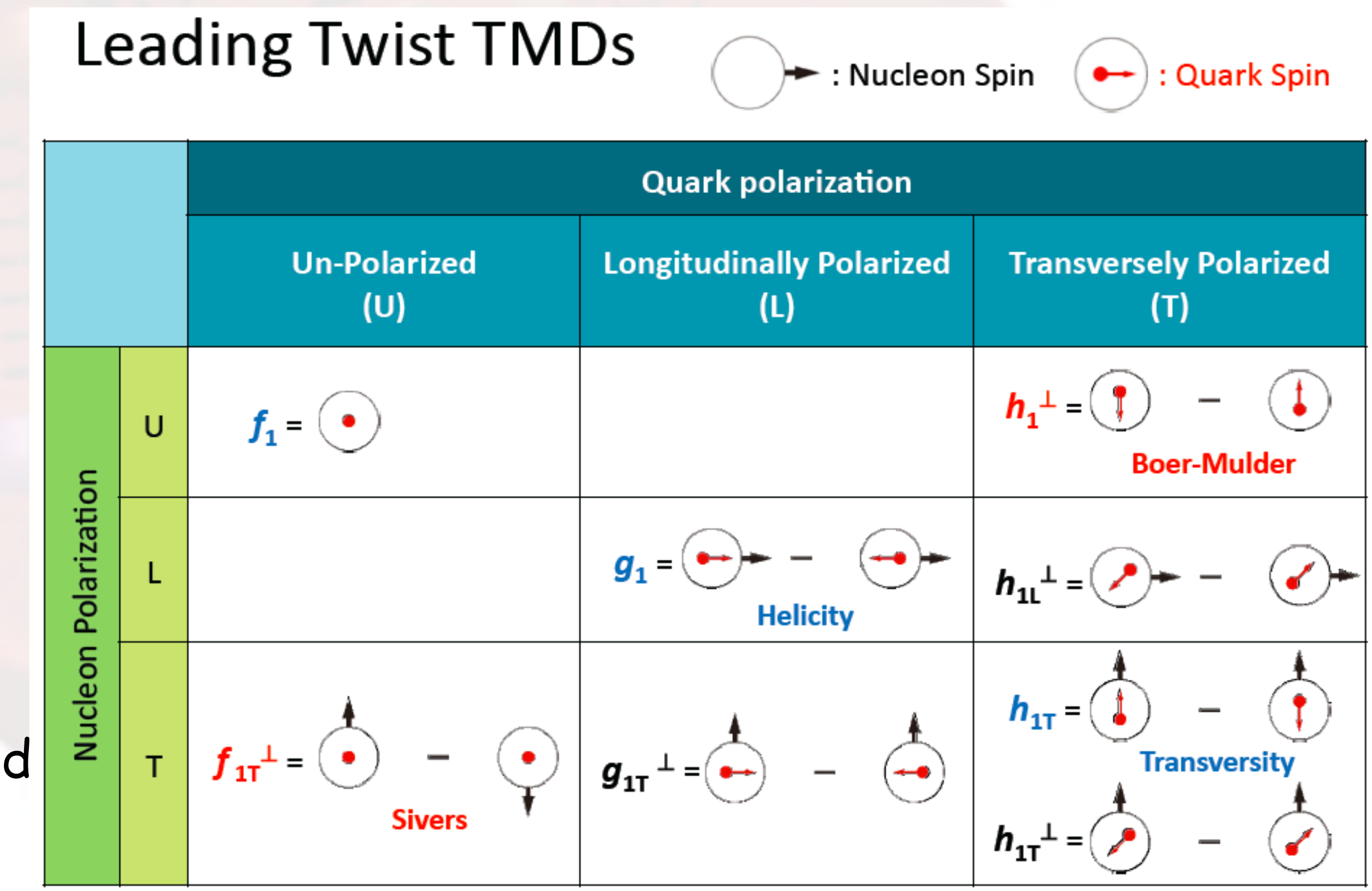
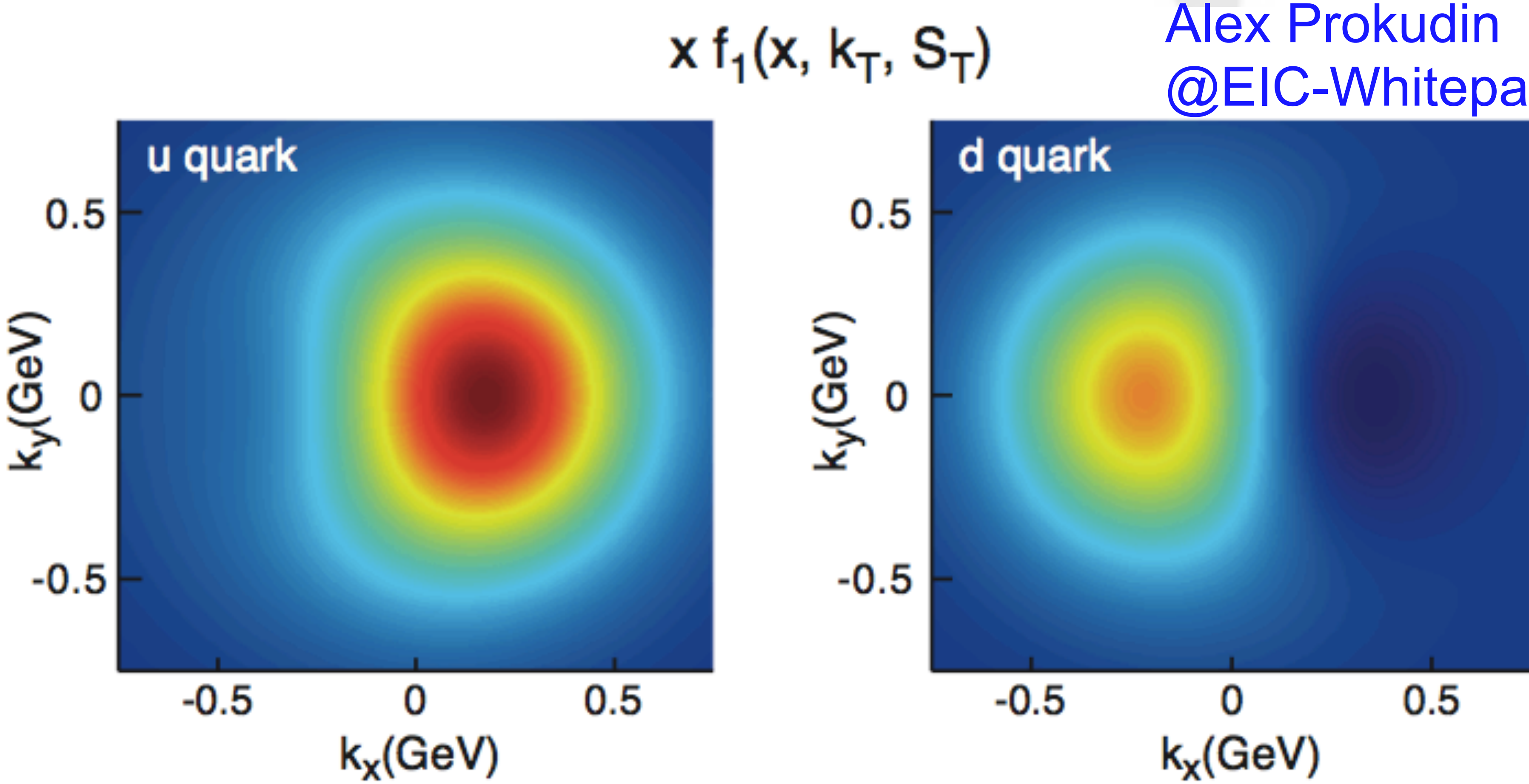
- Nontrivial distributions, $S_T X P_T$
- In quark model, depends on S- and P-wave interference



Alex Prokudin
@EIC-Whitepaper

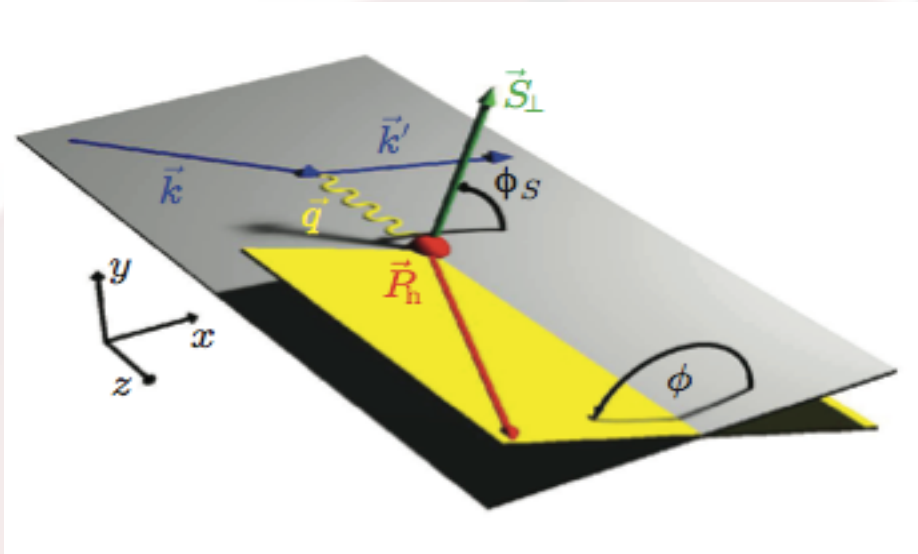


Quark Sivers function leads to an azimuthal asymmetric distribution of quark in the transverse plane

Where can we learn TMDs

- Semi-inclusive hadron production in deep inelastic scattering (SIDIS)
- Drell-Yan lepton pair, photon pair productions in pp scattering
- Dijet correlation in DIS
- Relevant e+e- annihilation processes
- …

Semi-inclusive DIS



- Novel Single Spin Asymmetries

Collins: $A_{UT}^{\sin(\phi+\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 \delta q(x)\, H_1^\perp(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$

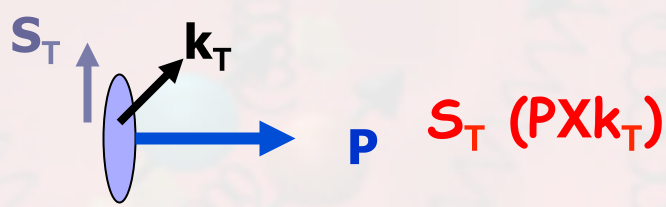
$z \stackrel{lab}{=} \frac{E_h}{\nu}$

Sivers: $A_{UT}^{\sin(\phi-\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 f_{1T}^{\perp,q}(x) \cdot D_1(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$

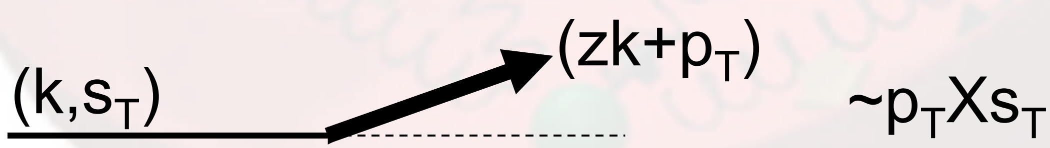
U: unpolarized beam
T: transversely polarized target

Two major contributions

- Sivers effect in the distribution

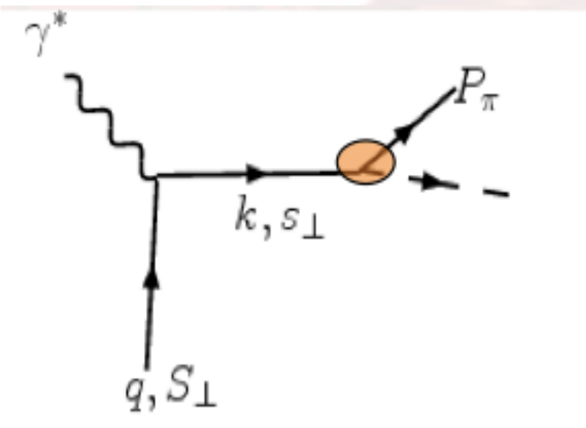


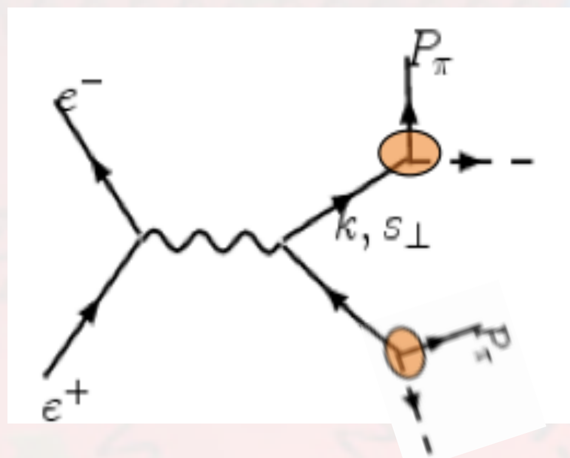
- Collins effect in the fragmentation

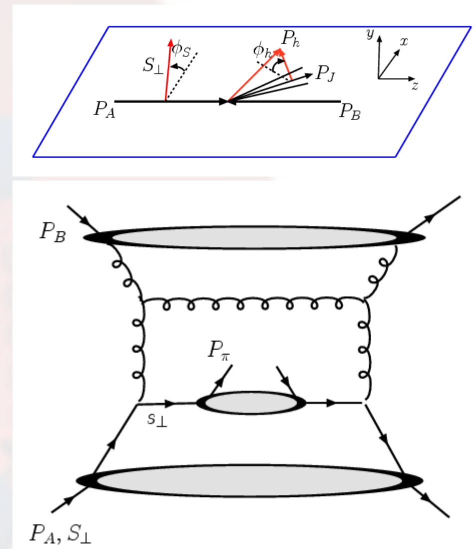


- Other contributions…

Universality of the Collins Fragmentation







ep--> e Pi X

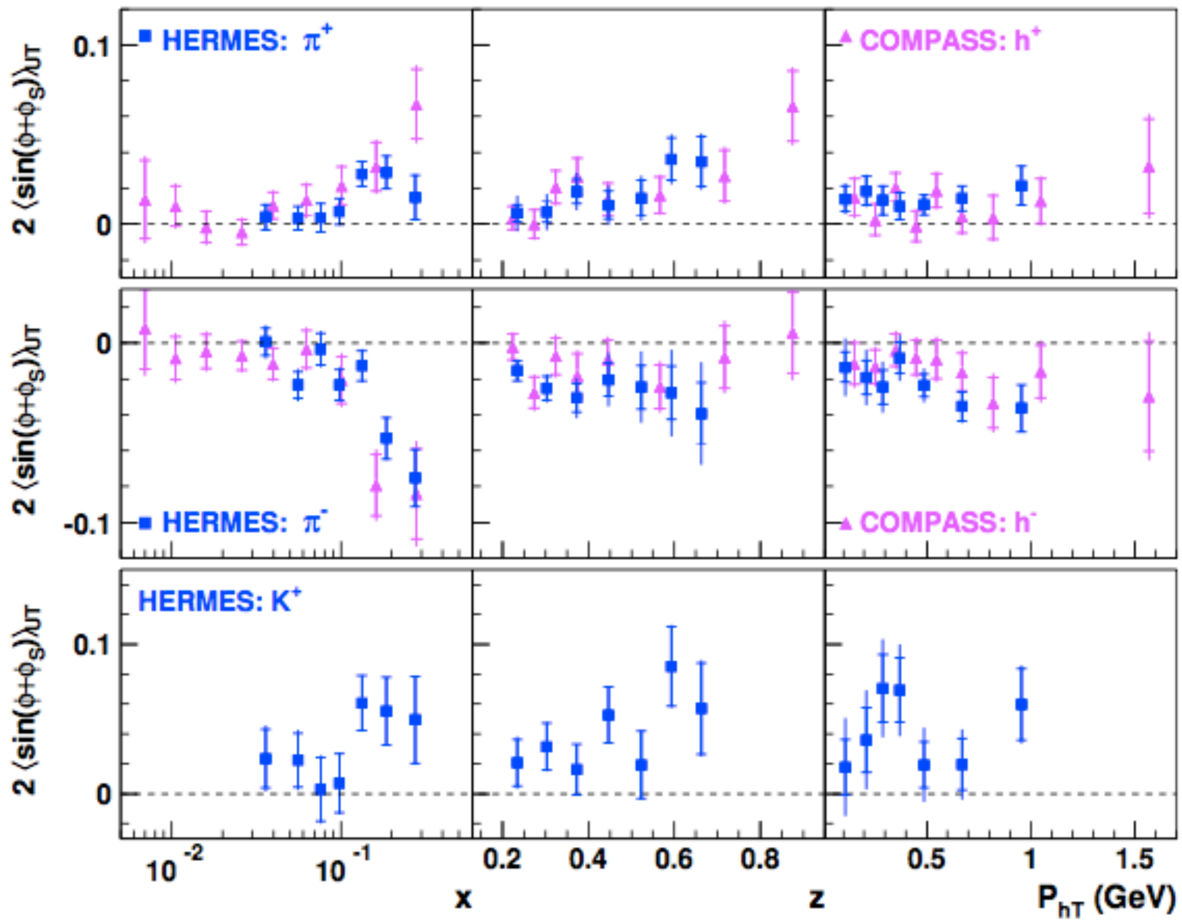
e^+e^---> Pi Pi X

pp--> jet(->Pi) X

- Metz 02, Collins-Metz 02, Yuan 07, Gamberg-Mukherjee-Mulders 08,10
- Meissner-Metz 0812.3783
- Yuan-Zhou, 0903.4680

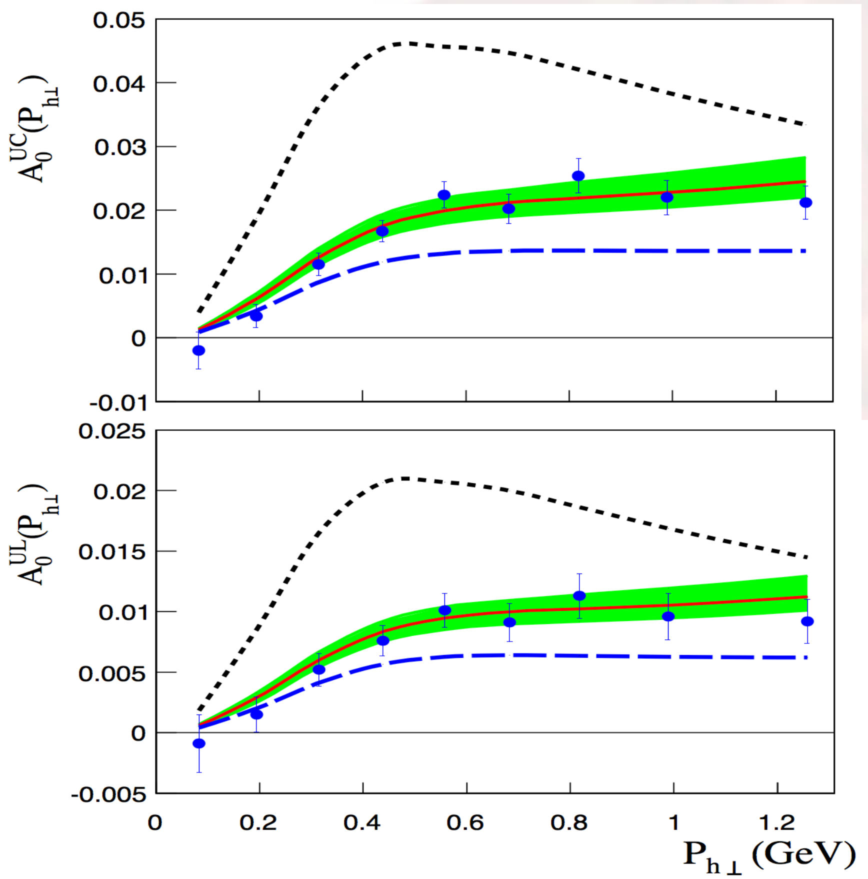
Exps: BELLE, BESIII, HERMES, JLab STAR at RHIC

Collins asymmetries in SIDIS

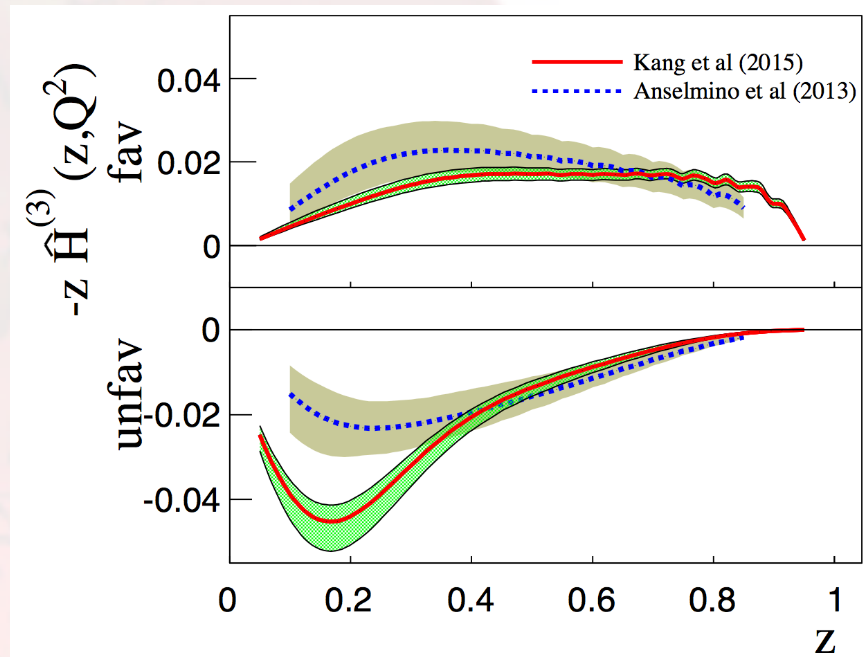


Summarized in the EIC Write-up

Collins effects in e^+e^-



BarBar Coll. 2014

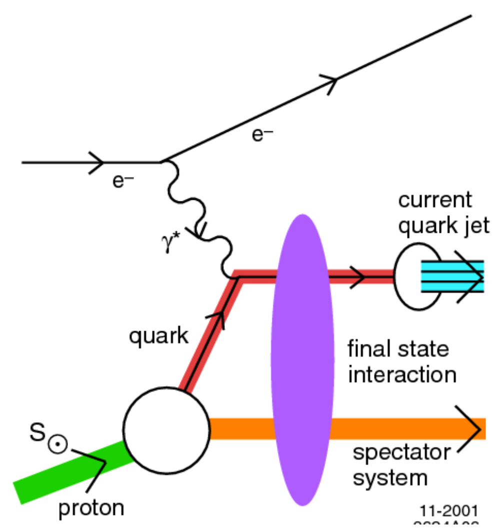


Collins functions extracted from the Data

Sivers effect is different

- It is the final state interaction providing the phase to a nonzero SSA
- Non-universality in general
- Only in special case, we have "Special Universality"

Brodsky,Hwang,Schmidt 02
Collins, 02;
Ji,Yuan,02;
Belitsky,Ji,Yuan,02

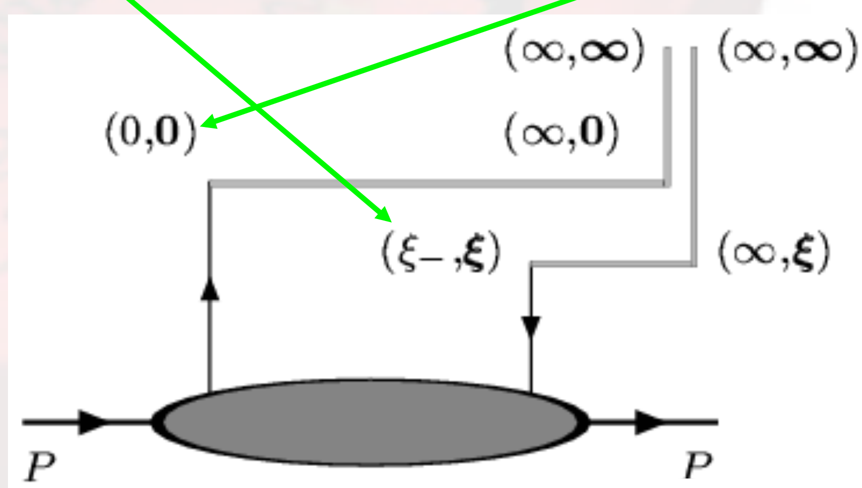


TMD Parton Distributions

- **The gauge invariant definition**

$$f(x,k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp\cdot\vec{k}_\perp)} \times \langle PS|\overline{\psi}(\xi^-,\xi_\perp)L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

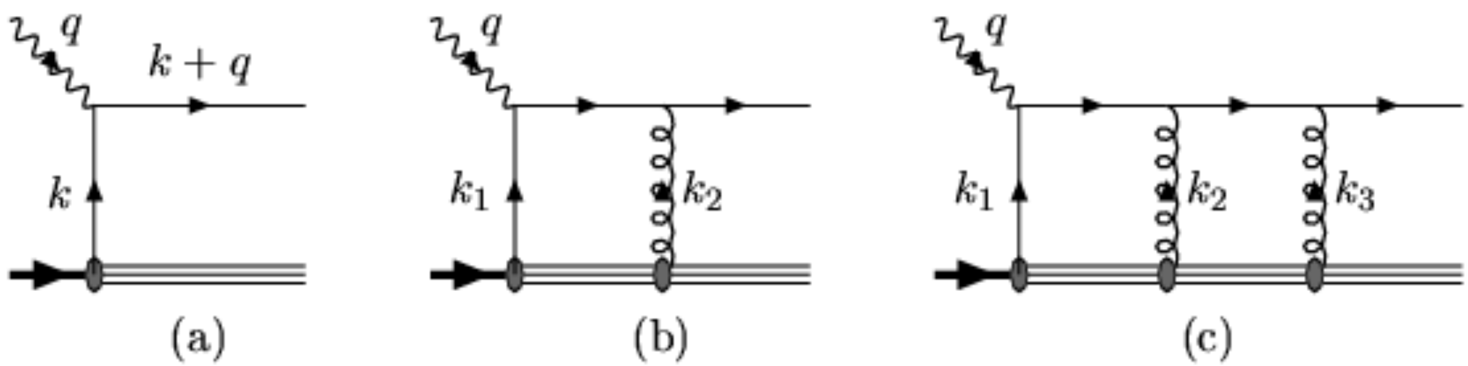
- **In Feynman gauge**



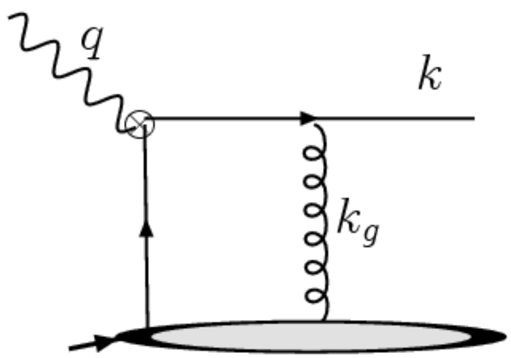
Belitsky, Ji, Yuan (03)

Where does the gauge link come from?

- Factorizable multiple gluon interactions



Example: FSI in DIS



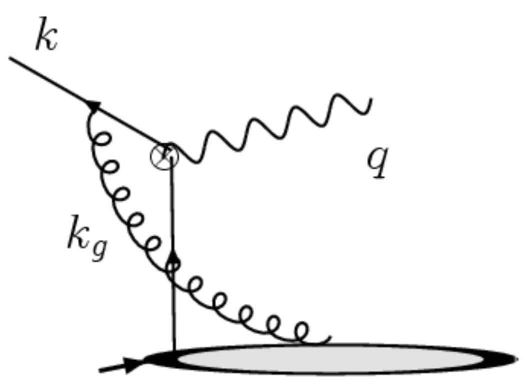
$$\int \frac{d^4k_g}{(2\pi)^4}\bar{u}(k)(-ig\gamma^\alpha T_a)\frac{i(\not k-\not k_g)}{(k-k_g)^2+i\epsilon}\Gamma \times\langle n|\psi(0)A_{a\alpha}(k_g)|P\rangle$$

The leading contribution comes from A^+, and taking the leading term with $k^- \to \infty$, we have

$$\int_{-\infty}^{+\infty} dk_g^+ \frac{i}{-k_g^+ + i\epsilon} A^+(k_g) = \int_0^\infty d\xi^- A^+(\xi^-)$$

- This is just the leading order expansion of the exponential gauge link
- Summing all final state gluon interactions will lead to the final gauge link in the parton distribution definition

Initial state interaction in Drell-Yan



$$\int \frac{d^4k_g}{(2\pi)^4}\bar{v}(k)(-ig\gamma^\alpha T_a)\frac{-i(\not{k}+\not{k}_g)}{(k+k_g)^2+i\epsilon}\Gamma$$
$$\times\langle n|\psi(0)A_{a\alpha}(k_g)|P\rangle$$

The leading contribution comes from A^+, and taking the leading term with $k^- \to \infty$, we have

$$\int_{-\infty}^{+\infty} dk_g^+ \frac{i}{-k_g^+ - i\epsilon} A^+(k_g) = \int_0^{-\infty} d\xi^- A^+(\xi^-)$$

- This leads to the gauge link in Drell-Yan process goes to -1, instead of +1 in DIS
- Consequence is the Sivers functions change sign for these two processes

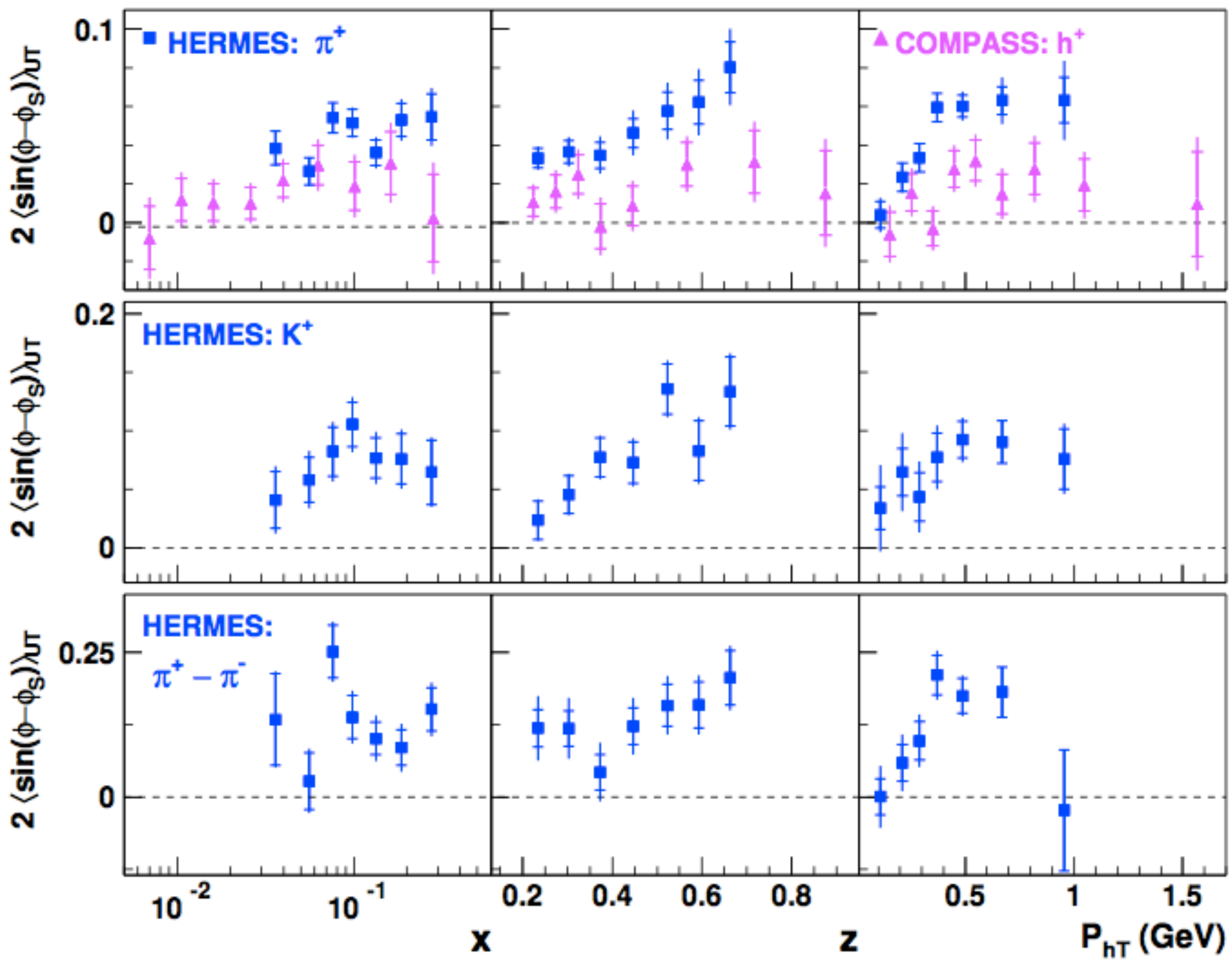
In light-cone gauge

- Additional gauge link is needed to ensure the gauge invariance of the definition

$$\Delta L = P \exp\left(-ig\int_0^\infty d\xi_\perp \cdot A_\perp(\xi^- = \infty, \xi_\perp)\right)$$

  - Which can also be derived from the previous diagrams

Sivers asymmetries in SIDIS

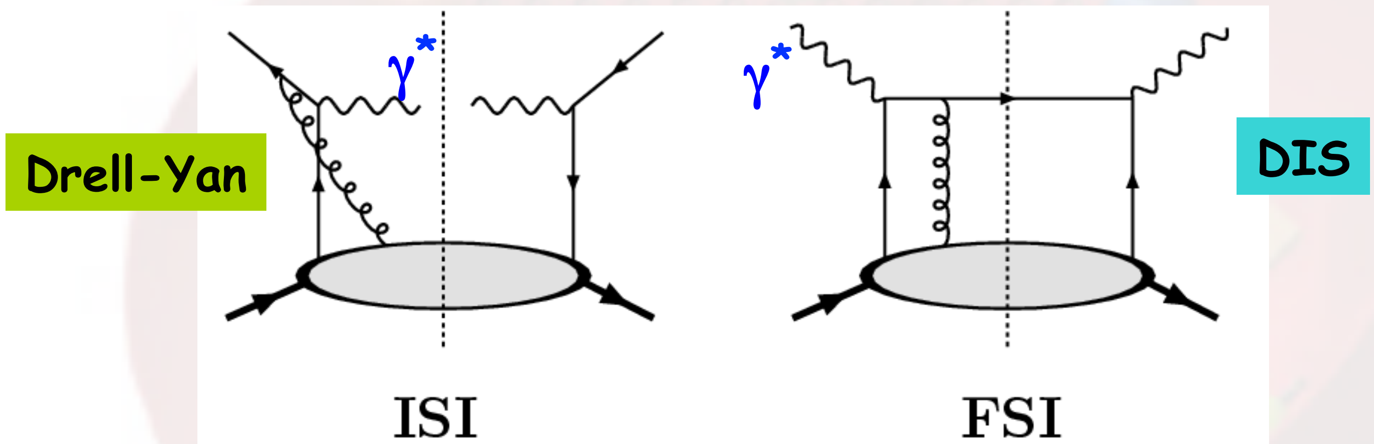


Jlab Hall A ^{3}He dat

Non-zero Sivers e
Observed in SIDI

DIS and Drell-Yan

- Initial state vs. final state interactions



$$\text{Sivers}|_{DY} = -\text{Sivers}|_{DIS}$$

HERMES

- "Universality": QCD prediction

TMD predictions rely on

- Non-perturbative TMDs constrained from experiments
- QCD evolutions, in particular, respect to the hard momentum scale Q
  - Strong theory/phenomenological efforts in the last few years
  - Need more exp. data/lattice calculations

Collins-Soper-Sterman Resummation

- Large Logs are resummed by solving the energy evolution equation of the TMDs

$$\frac{\partial}{\partial \ln Q} f(k_\perp, Q) = (K(q_\perp, \mu) + G(Q, \mu)) \otimes f(k_\perp, Q)$$

- K and G obey the renormalization group eq.

$$\frac{\partial}{\partial \ln \mu} K = -\gamma_K = \frac{\partial}{\partial \ln \mu} G$$

(Collins-Soper 81, Collins-Soper-Sterman 85)

Solving the evolution equations

$$\widetilde{f}_q^{(sub.)}(x,b,\zeta^2=\rho Q^2;\mu_F=Q) = e^{-S^q_{pert}(Q,b_*)-S^q_{NP}(Q,b)}\widetilde{\mathcal{F}}_q(\alpha_s(Q);\rho) \times \sum_i C_{q/i}(\mu_b/\mu)\otimes f_i(x,\mu) ,$$

Sudakov form factor (perturbative)

Non-perturbative input

- Universal C-function

$$C_{q/q'}(x)=\delta_{qq'}\left[\delta(1-x)+\frac{\alpha_s}{2\pi}C_F(1-x)\right]$$

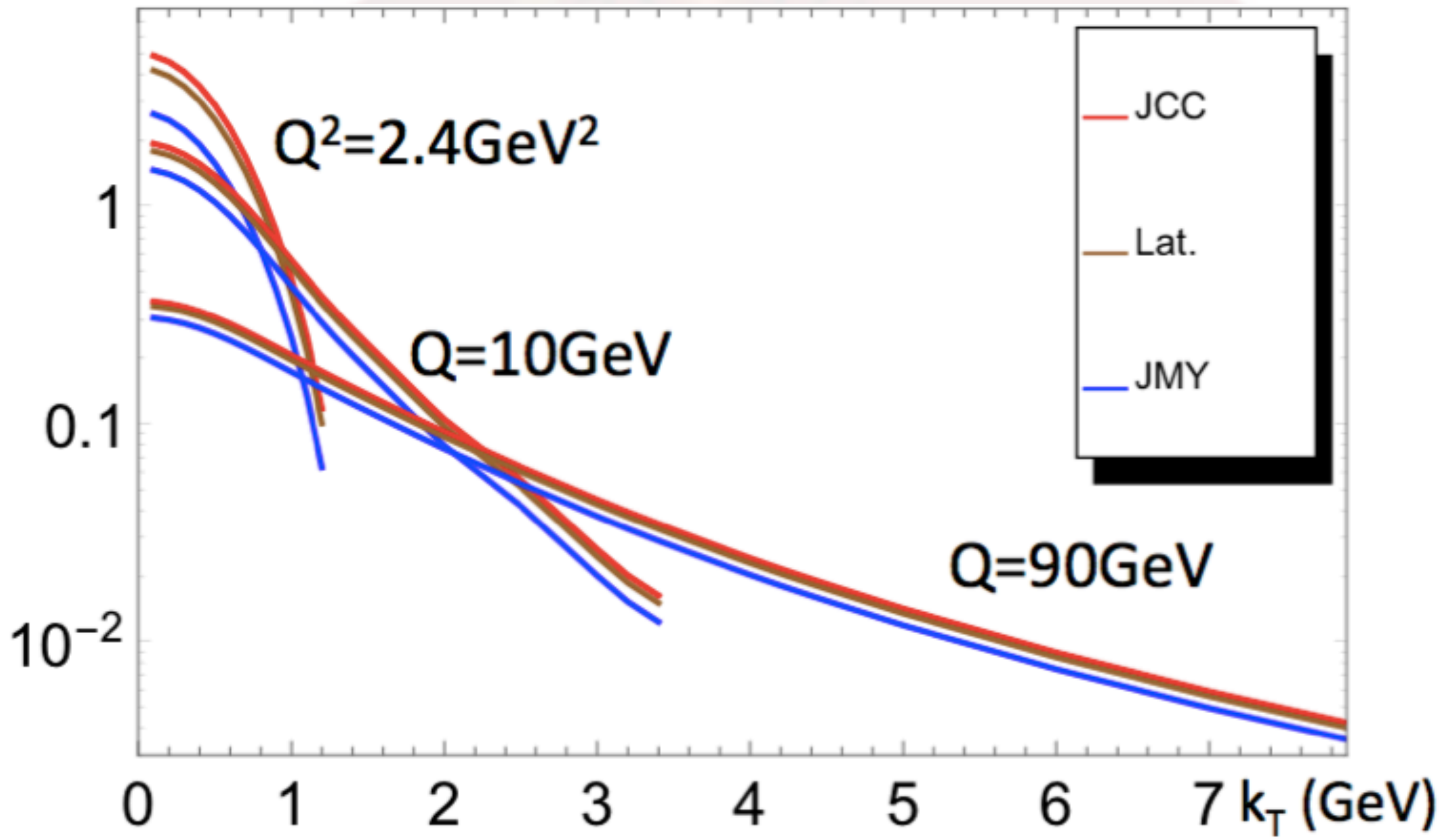
- Scheme-dept.

$$\widetilde{\mathcal{F}}_q^{\rm JCC}(\alpha_s(Q))=1+\mathcal{O}(\alpha_s^2)$$

$$\widetilde{\mathcal{F}}_q^{\rm JMY}(\alpha_s(Q);\rho)=1+\frac{\alpha_s}{2\pi}C_F\left(\ln\rho-\frac{\ln^2\rho}{2}-\frac{\pi^2}{2}-2\right)$$

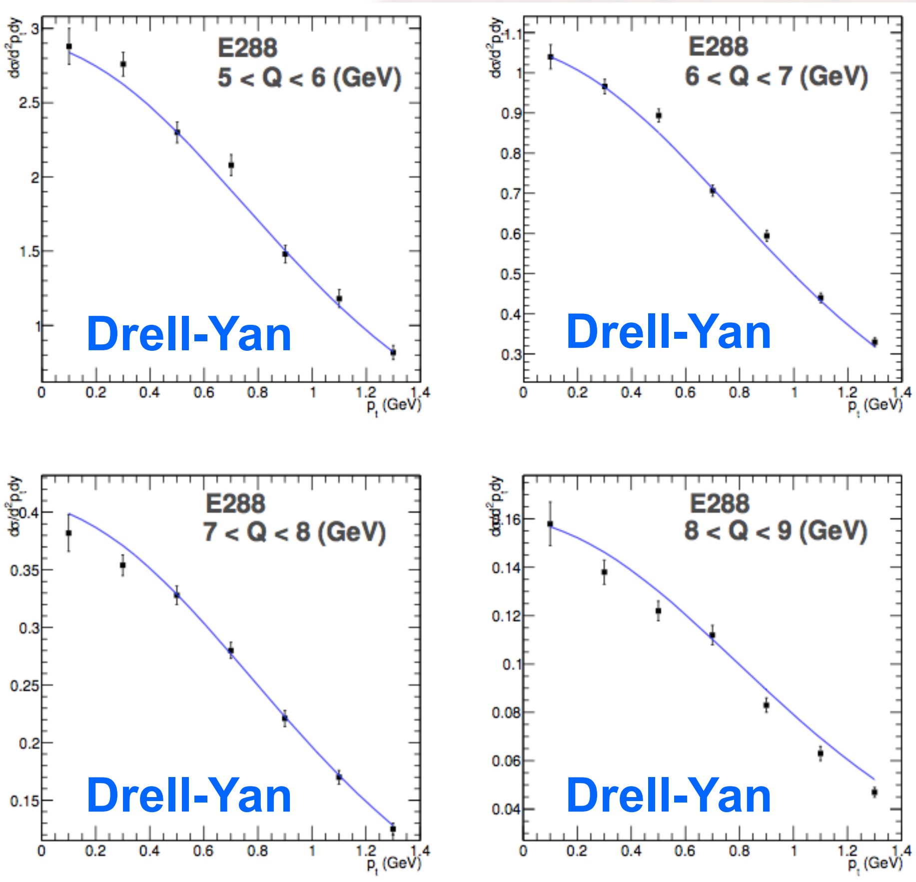
$$\widetilde{\mathcal{F}}_q^{\rm Lat.}(\alpha_s(Q))=1+\frac{\alpha_s}{2\pi}C_F(-2)$$

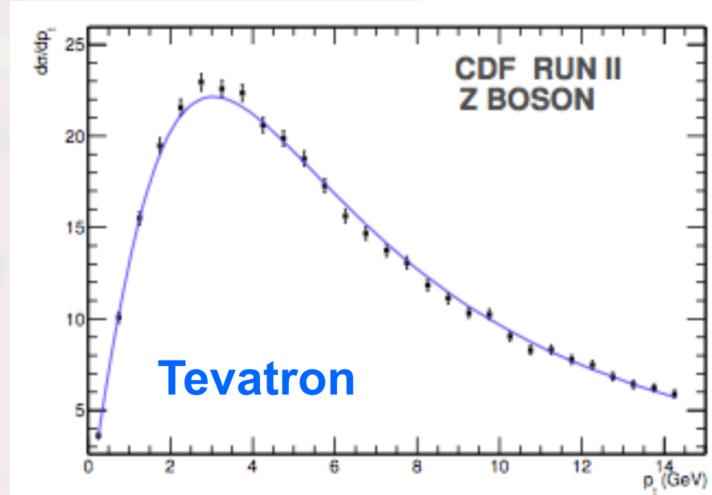
Unpolarized quark distribution

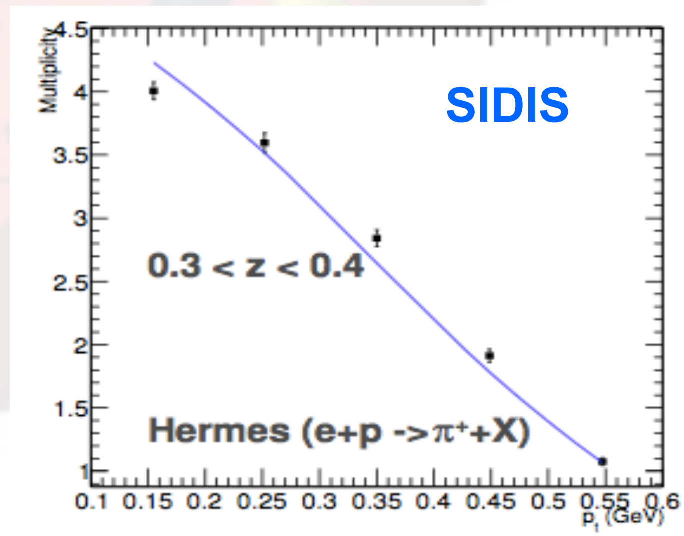


Describe well the exp. data

Sun-Issacson-Yuan-Yuan, 2014

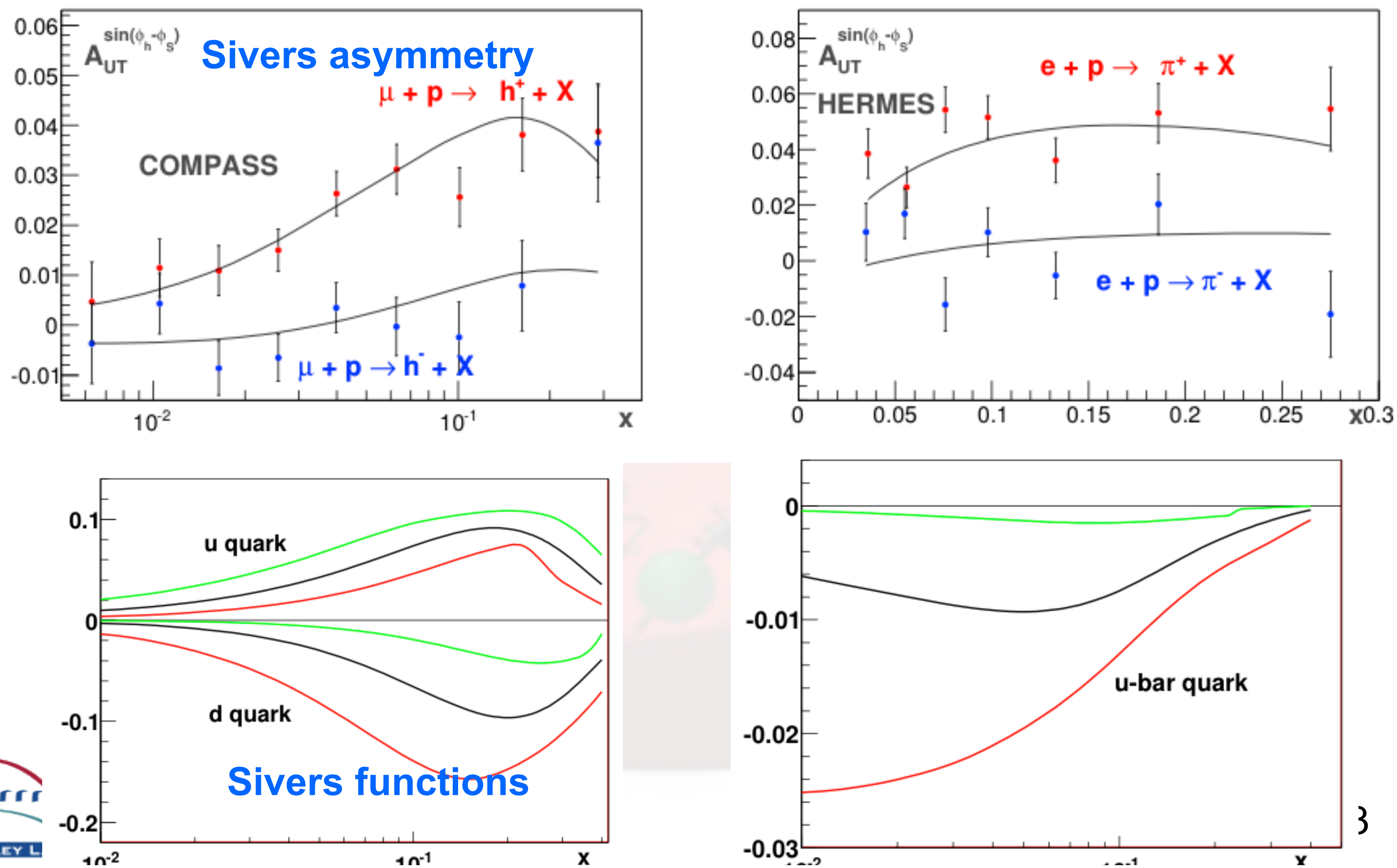






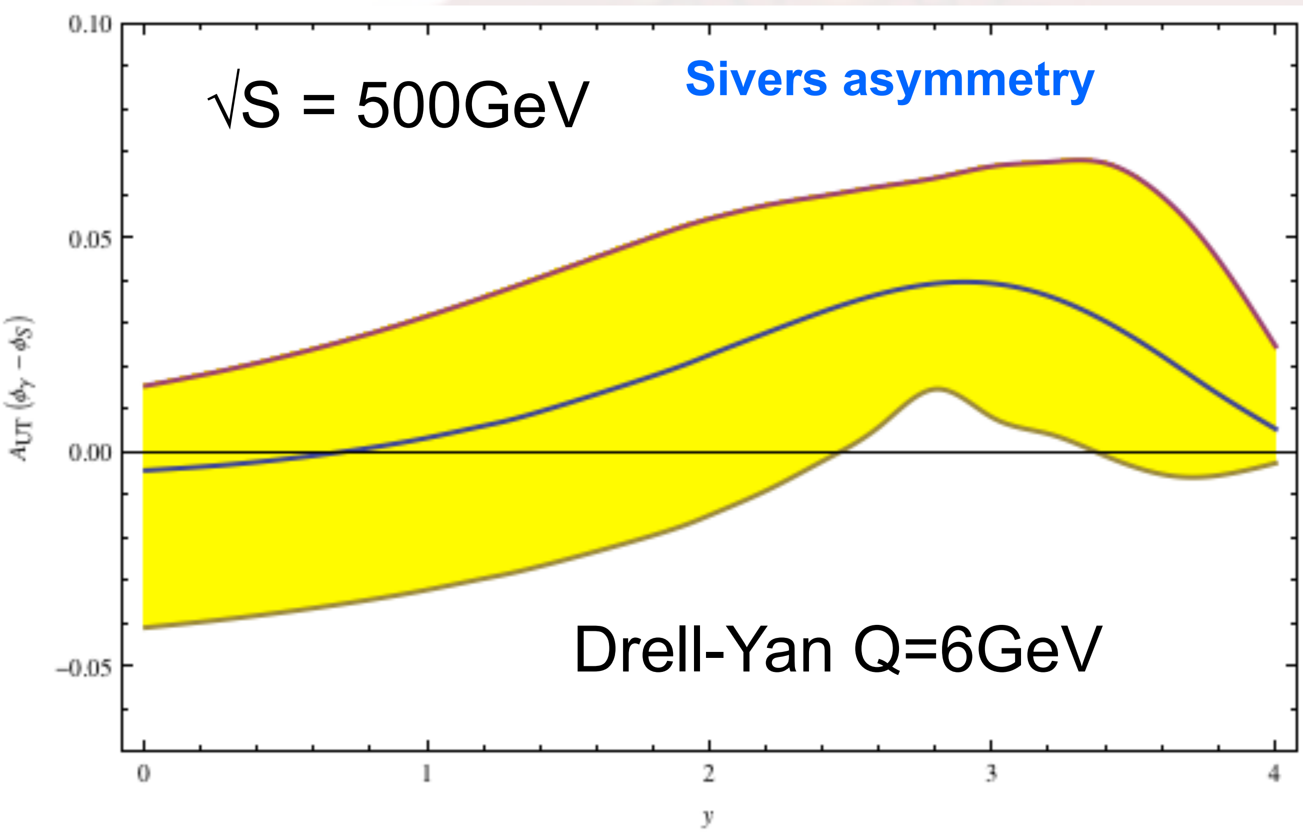
Sivers asymetries in SIDIS with Evolution

Sun, Yuan, PRD 2013
Prokudin-Sun-Yuan, in progress



Predictions at RHIC

Sun, Yuan, PRD 2013

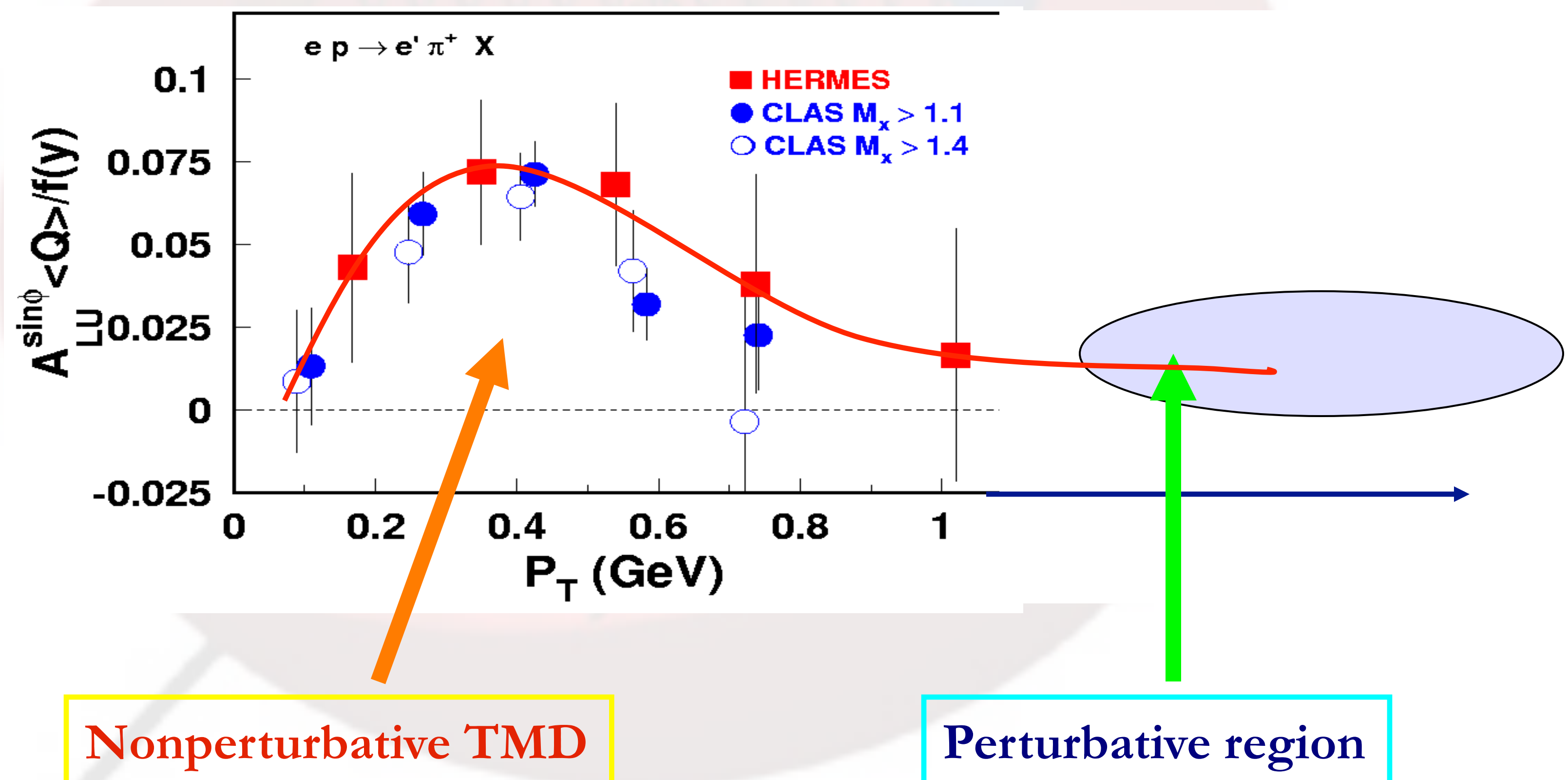


Additional theory uncertainties: x-dependence of the TMDs comes from a fit to fixed target drell-yan and w/z production at Tevatron

---Nadolsky et al.

Transition from Perturbative region to Nonperturbative region

- Compare different region of P_T



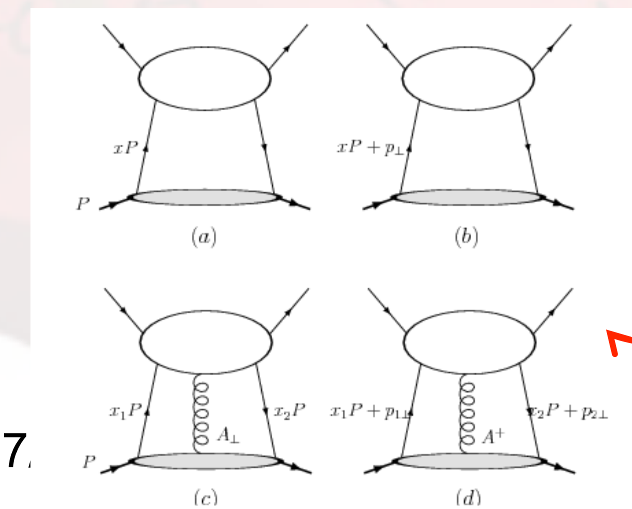
Perturbative tail is calculable

- Transverse momentum dependence

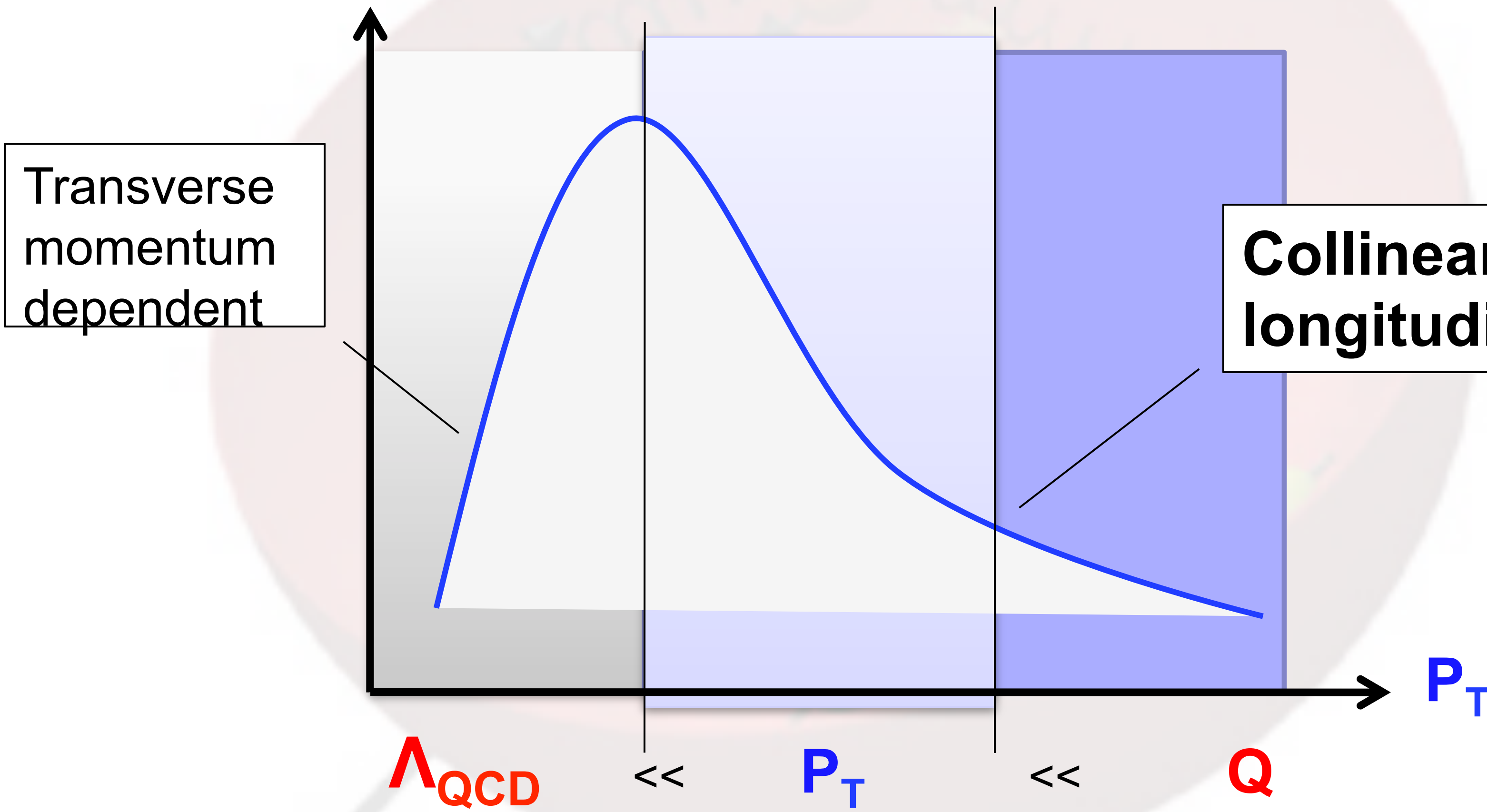
$$q(x,k_\perp)|_{k_\perp \gg \Lambda_{\rm QCD}} = \frac{1}{(k_\perp^2)^n}\int \frac{dx'}{x'} f_i(x') \times \mathcal{H}_{q/i}(x;x')$$

Power counting,
Brodsky-Farrar, 1973

Integrated Parton Distributions
Twist-three functions



A unified picture (leading pt/Q)



Ji-Qiu-Vogelsang-Yuan, 2006
Yuan-Zhou, 2009

Compared to the collinear factorization

- Simplification
  - Of the cross section in the region of pt<<Q, only keep the leading term
- Extension
  - To the small pt region, where the collinear factorization suffer large logarithms
  - Resummation can be done