Six lectures in the Standard Theory of Elementary Particle Physics

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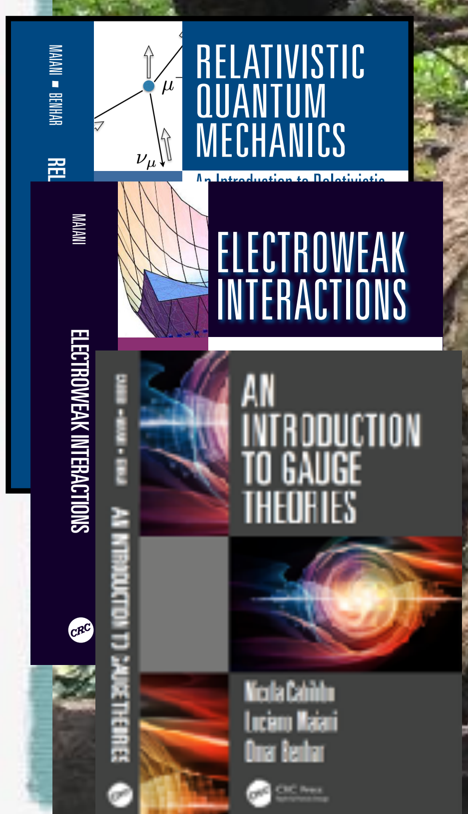
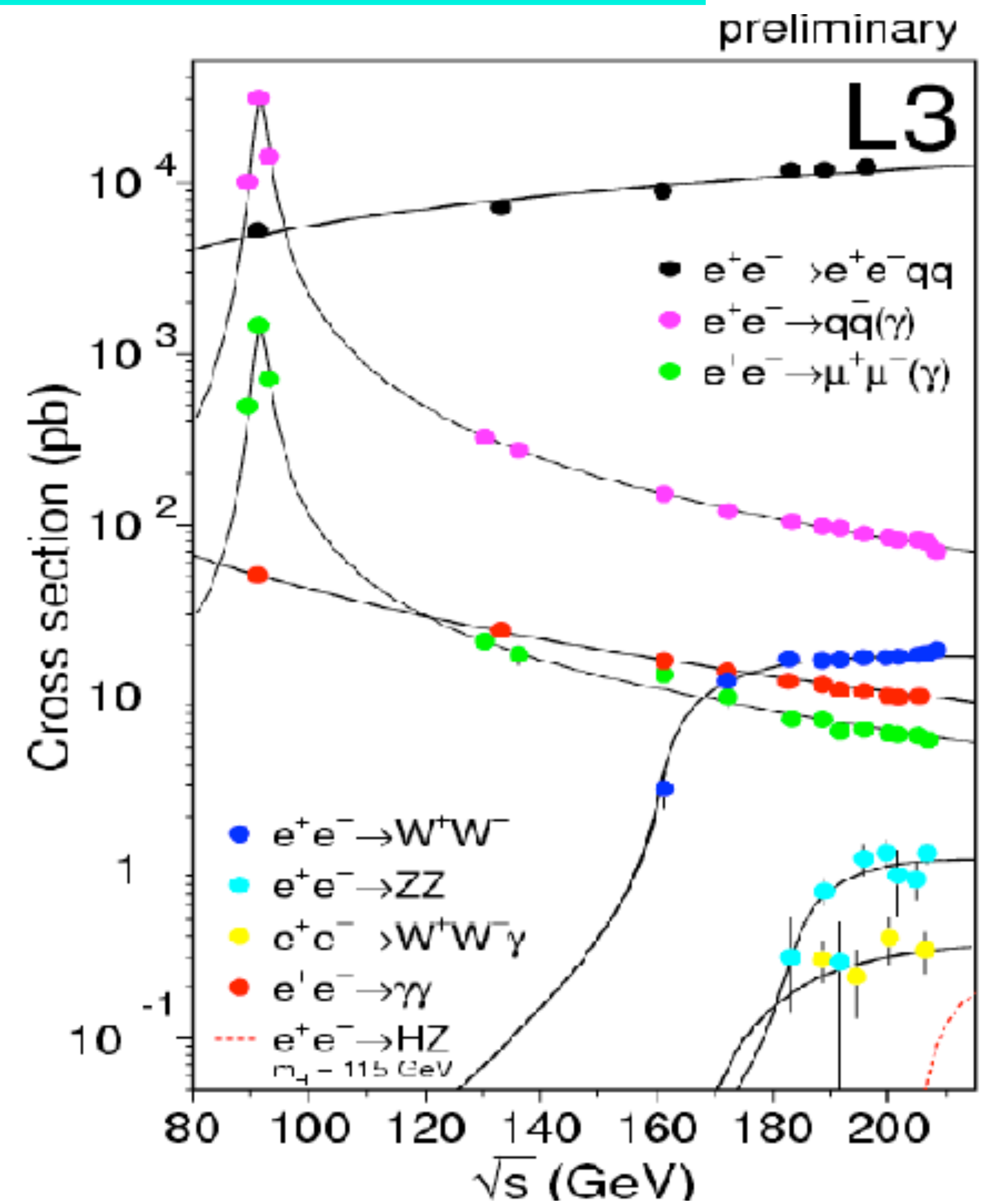


Lecture 1
Gauge Symmetry

1. Spinor Electro Dynamics

- Originally a theory of electrons and photons;
- μ and τ-particles behave the same;
- e, μ and τ numbers separately conserved;
- even in world made by electrons and photons only, $e^+ e^-$ annihilation gives access to muons, τ and to all other charged fermions.
- Spinor QED is determined by few general principles:
  - Lorentz invariance
  - Gauge invariance
  - matter particles are fermions with spin 1/2 (Dirac particles)
  - renormalizability
- The prototype of a fundamental field theory and an extraordinary success.



QED and local gauge invariance (abelian)

- Electric charge conservation follows from invariance of the lagrangian under ***global phase transformations*** $\psi(x) \to \psi'(x) = e^{i\phi}\psi(x)$
- We want to promote global to ***local phase (gauge) transformations***

$$\psi(x) \to \psi'(x) = e^{i\phi(x)}\psi(x); \quad A_\mu(x) \to A'_\mu(x) = A_\mu + \partial_\mu\phi(x)$$

- The minimal substitution produces a field which trasforms exactly like ψ:

$$\partial_\mu\psi \to D_\mu\psi = (\partial_\mu - iA_\mu)\psi$$

$$D'_\mu\psi'(x) = (\partial_\mu - iA'_\mu)\psi' = (\partial_\mu - iA_\mu - i\partial_\mu\phi)e^{i\phi}\psi = e^{i\phi}(\partial_\mu - iA_\mu)\psi = e^{i\phi}D_\mu\psi$$

- A_μ transforms in a complicated way, but the Maxwell's tensor is invariant:

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu; \quad F'_{\mu\nu} = F_{\mu\nu}$$

- Conclusion (W. Pauli): if $L_0(\psi,\partial_\mu\psi)$ is invariant under global phase transformations, the Lagrangian: $L_{QED} = L_0(\psi, D_\mu\psi) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}$
  - is invariant under local transformations;
  - rescaling A→ - eA, we see that "e" gives the strength of the interaction;
  - Symmetry determines the form of the photon-electron interaction (ex. g=2!!)
  - No photon mass is allowed!

QED histories

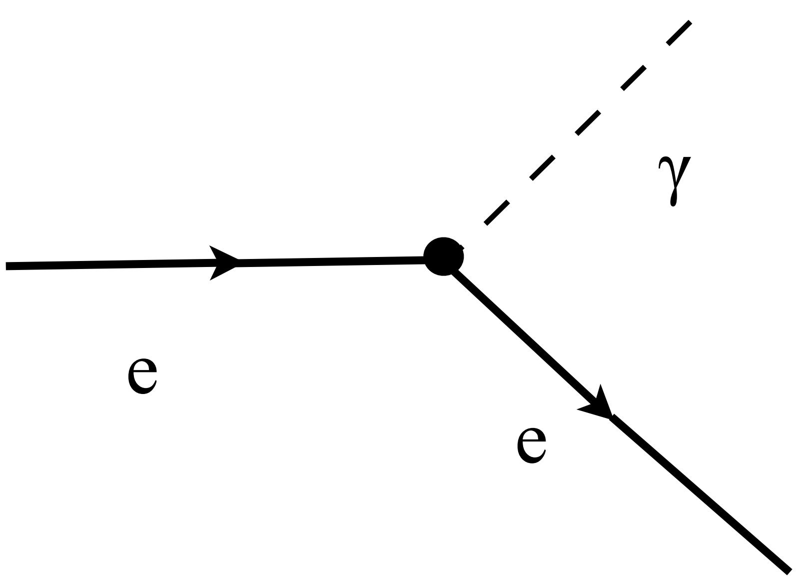
- Using the Dirac's Lagrangian, we get:

$$L_0(\psi, D_\mu\psi) = \bar{\psi}_e(i\partial_\mu\gamma^\mu - m)\psi_e \; - e\,\bar{\psi}_e A_\mu\gamma^\mu\psi_e +$$
$$+(\psi_e \to \psi_\mu) + (\psi_e \to \; \psi_\tau).$$

- that is:

$$L_I = -eA_\lambda[\bar{\psi}_e\gamma^\lambda\psi_e + \bar{\psi}_\mu\gamma^\lambda\psi_\mu + \bar{\psi}_\tau\gamma^\lambda\psi_\tau]$$

- QED hystories:
  - propagation of photons and electrons/mu/tau
  - vertices
  - lepton mass allowed by gauge invariance (because vector current!)



Gauge invariance and the photon propagator

- Gauge invariance means that not all components of A_μ are dynamical variables
- Thus we have to require a supplementary condition (gauge condition) to be obeyed by the field configurations over which we do the path integral.
- We can enforce the gauge condition by adding to the Lagrangian a Lagrange multiplier, to get:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2;\ F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu;$$

$$S = \int d^4x\, L = \int d^4x\, \frac{1}{2}\,(A^\mu K_{\mu\nu} A^\nu);$$

$$K_{\mu\nu} = g_{\mu\nu}(\partial)^2 - (1-\lambda)\partial_\mu\partial_\nu$$

- after Fourier transforming and taking the inverse, we easily find:

$$K^{-1}_{\mu\nu}(q) = \frac{1}{q^2 + i\varepsilon}\left(-g_{\mu\nu} + (1-\frac{1}{\lambda})\,\frac{q_\mu q_\nu}{q^2}\right)$$

Gauge invariance and the photon propagator

$$K^{-1}_{\mu\nu}(q) = \frac{1}{q^2 + i\varepsilon}(-g_{\mu\nu} + (1 - \frac{1}{\lambda})\,\frac{q_\mu q_\nu}{q^2})$$

- the λ dependence of the propagator disappears when computing physical amplitudes, on account of the fact that the photon propagator indices are always contracted with currents which are conserved: $q_\mu < J^\mu >= 0$
- the disappearence of the λ dependence happens only ***after we add up all Feynman diagrams related to each other by gauge invariance*** and it thus provides a powerful check of your calculation!
- useful gauges:
  - λ =1: Feynman gauge
  - λ =∞: Landau gauge (the propagator is explicitly transverse, $q^\mu K^{-1}_{\mu\nu}(q) = 0$

Non minimal couplings

- Using the Dirac's Lagrangian, we get:

$$L_0(\psi, D_\mu\psi) = \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi - \ eA_\mu\ \bar{\psi}\gamma^\mu\psi,\ (\psi = \psi_e,\ \psi_\mu,\ \psi_\tau)$$

- we could add non-minimal terms formed with $F_{\mu\nu}$ (Pauli): $\frac{\kappa}{4m}\ F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi$

- the magnetic moment would be anomalous $\mu = \frac{e}{2m}g\ S;$

$$g = 2(1+\kappa);\ S = spin$$

with respect to electrons and muons, which have g-2 very close to 0 (see later).

- The Pauli term makes the high-energy behaviour of scattering amplitudes more singular, so as to lead to a non-renormalizable theory;
- For particles which are not elementary, e.g proton and neutron, both criteria do not apply and non-minimal terms required by experiment.

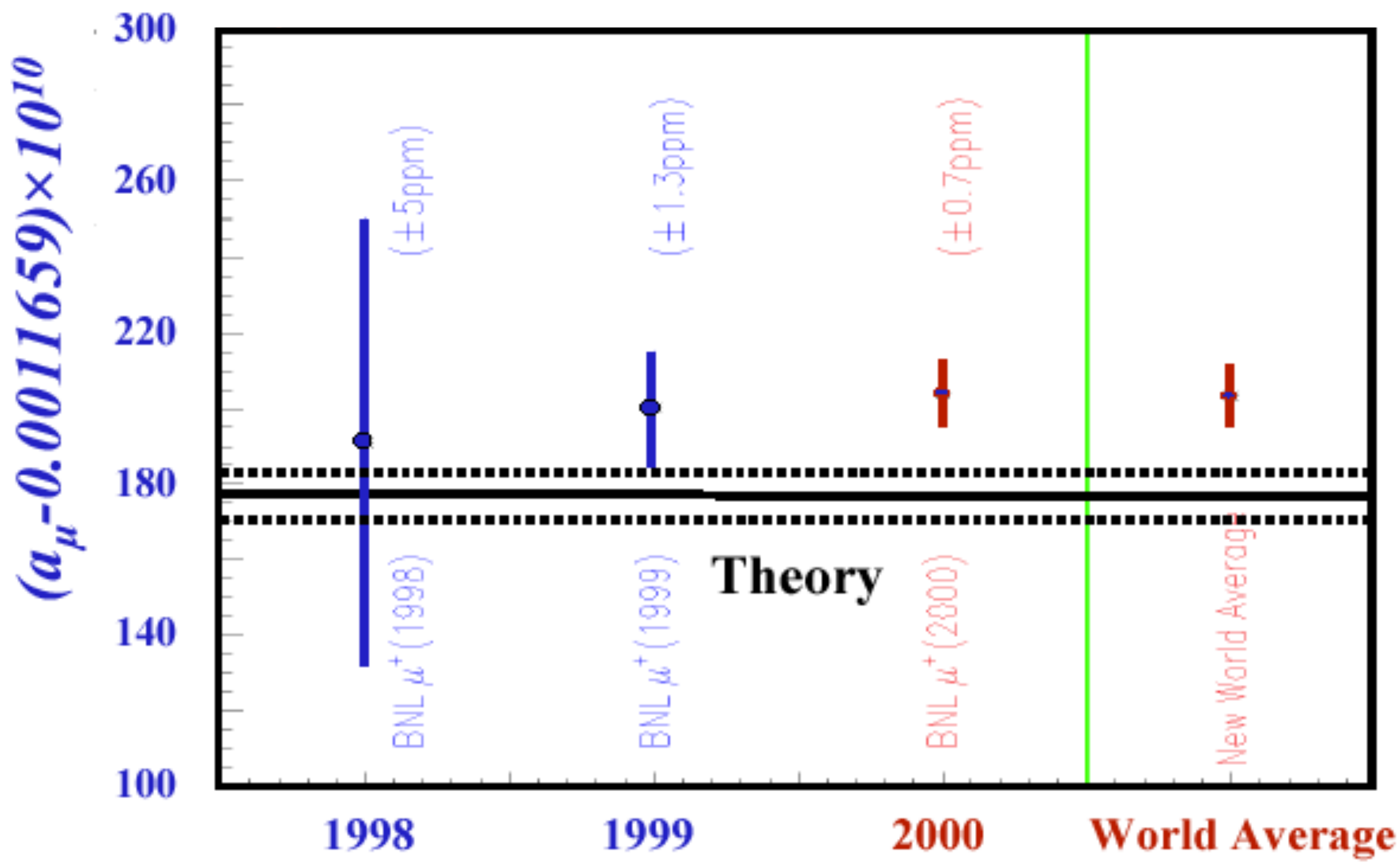
2. The muon g-2

Define: $a_\mu = g_\mu - 2$

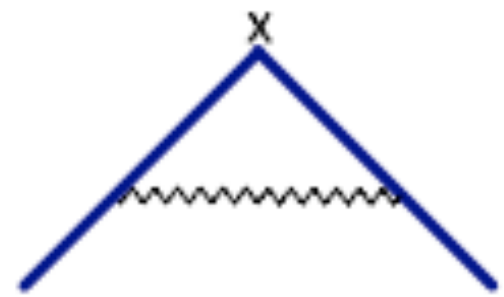
G. W. Bennett et al. [Muon G-2 Collab.], Phys. Rev. D 73 (2006) 072003

Brookhaven National Lab.

$$a_\mu^{\exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}$$



Pure QED



$$a_\mu^{(2)} = \frac{\alpha}{\pi} \quad \text{(J. Schwinger)}$$

Fig. 1 *Lowest Order QED Contribution*

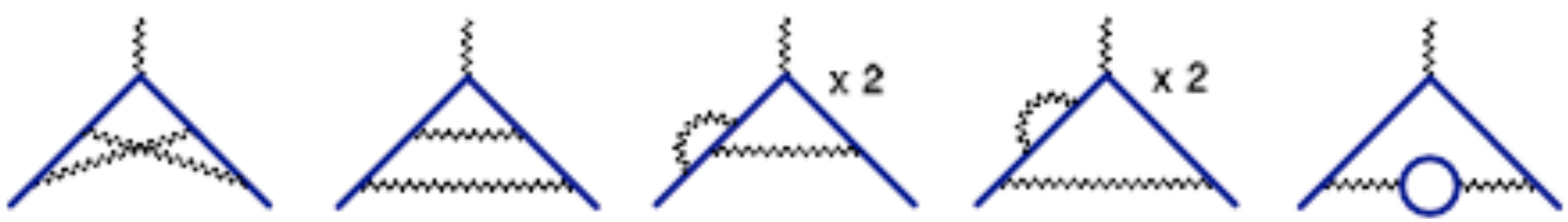
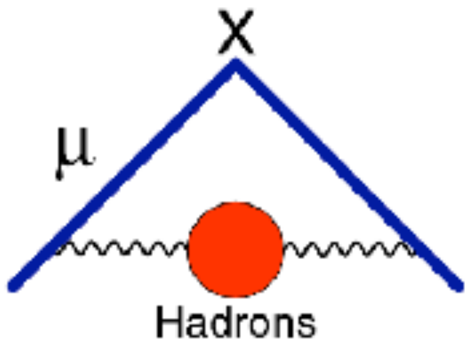


Fig. 2 *QED Contribution at the Two Loop Level*

$$a_l^{(4)} = \left\{ \frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4}\zeta(3) \right\} \left(\frac{\alpha}{\pi} \right)^2$$

Hadronic corrections



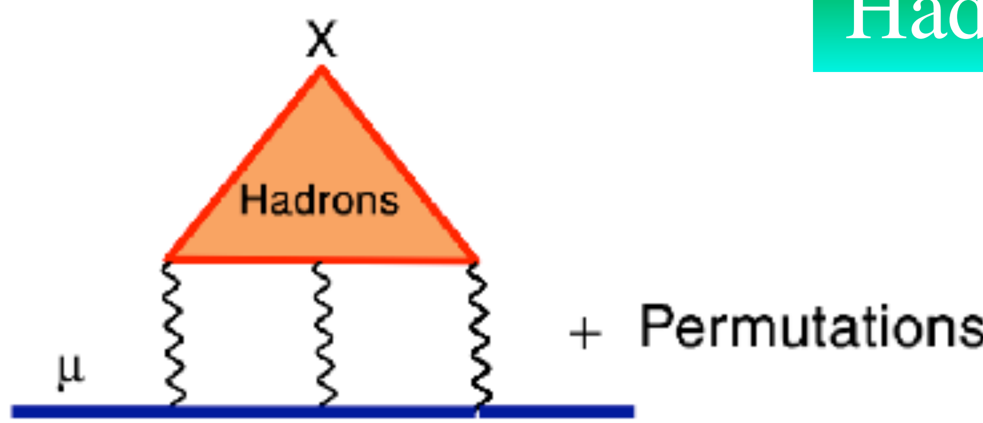
Hadronic Vacuum Polarization Contribution

Vacuum polarization

hadronic correction obtained from the experimental determination of:

$$\sigma(e^+e^- \to hadrons)$$

Hadronic light-by-light scattering



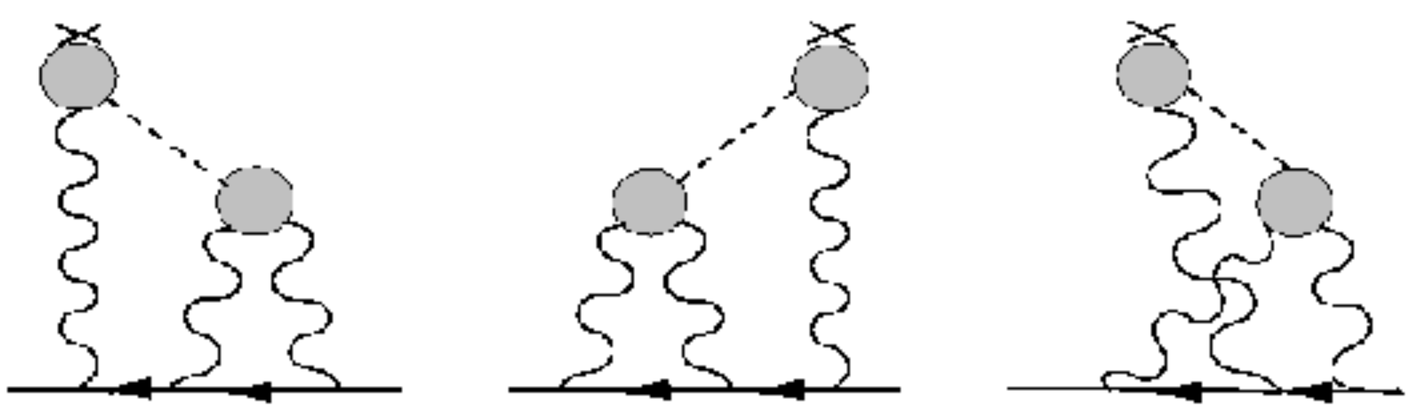


Figure 2: The pion-pole contributions to light-by-light scattering. The shaded blobs represent

Electroweak corrections

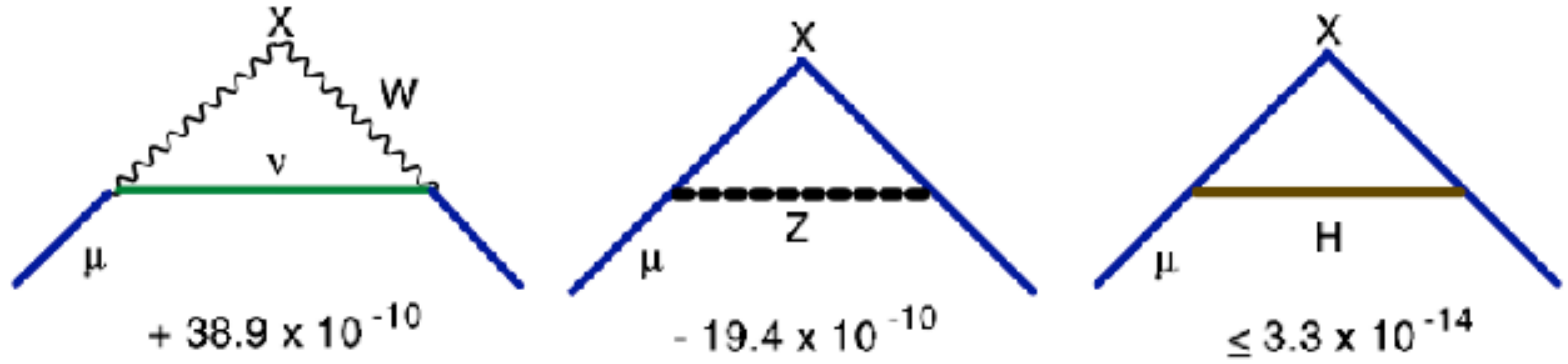


Fig. 12 *Weak Interactions at the one loop level*

1-loop calculations of the Electroweak effect have been done in the early '70s by several groups

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. **B46** (1972) 315; G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. **40B** (1972) 415; R. Jackiw and S. Weinberg, Phys. Rev. **D5** (1972) 2473; I. Bars and M. Yoshimura, Phys. Rev. **D6** (1972) 374; M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. **D6** (1972) 2923.

Result of 1+2 loops

$$a_\mu^{EW} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_W\right)^2 - \left(\frac{\alpha}{\pi}\right)(159 \pm 4)\right] = (15.2 \pm 0.1) \times 10^{-10}$$

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(63)(49) \times 10^{-11},$$

see: A. Hoecker, W.J. Marciano, PdG 2013
F. Jegerlehner, arXiv:1705.00263

3-4 σ discrepancy of experiment at BNL from Standard Theory prediction

The electron g-2

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based on the work carried out in collaboration with
M. Nio, T. Aoyama, M. Hayakawa, N. Watanabe, K. Asano.

presented at Nishina Hall, RIKEN
November 17, 2010

$a_e = a_e(\text{QED}) + a_e(\text{hadron}) + a_e(\text{electroweak})$, where

$$a_e(\text{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau)$$

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots,\ i = 1, 2, 3$$

- First four A_1 terms are known analytically or by numerical integration

$A_1^{(2)} = 0.5$ — 1 Feynman diagram (analytic)

$A_1^{(4)} = -0.328\,478\,965\ \ldots$ — 7 Feynman diagrams (analytic)

$A_1^{(6)} = 1.181\,241\,456\ \ldots$ — 72 Feynman diagrams (analytic,numerical)

Laporta, Remiddi, PLB 379, 283 (1996)
Kinoshita, PRL 75, 4728 (1995)

$A_1^{(8)} = -1.914\,4\ (35)$ — 891 Feynman diagrams (numerical)

Kinoshita,Nio, PRD 73, 013003 (2006)
Aoyama,Hayakawa,Kinoshita,Nio, PRD 77, 053012 (2008)

$A_1^{(10)} = 6.675\ (192)$ — *12,672 Feynman diagrams, 6354 dominant*

New! 10th order
T. Kinoshita et al.
arXiv:1712.06060v1

$$\begin{aligned} a_e(\text{Weak}) &= 0.030\,53\ (23) \times 10^{-12}, \\ a_e(\text{Hadron}) &= \{1.8490\ (108) - 0.2213\ (12) + 0.0280\ (2) + 0.037\ (5)\} \times 10^{-12} \\ &= 1.6927\ (120) \times 10^{-12}, \end{aligned}$$

$$a_e(\text{theory}) = 1\ 159\ 652\ 182.032\ (13)(12)(720) \times 10^{-12}$$

$$a_e(\text{HV08}) = 1\ 159\ 652\ 180.73\ (28) \times 10^{-12} \quad [0.24\text{ppb}]$$

Harvard group
D. Hanneke et al., PRL. **100**, 120801 (2008),
PR **A83**, 052122 (2011).

3. The Yang - Mills Theory

- Motivated by the global Isospin symmetry of the strong interactions:
- Can we have local (gauge) invariance under a non-abelian symmetry group?
- Consider local transformations of "matter fields" e.g. nucleons, under g∈G, compact, simple group. We take SU(2) (τ^A = Pauli matrices, A=1, 2, 3):

$$\psi(x) \to \psi'(x) = U(x)\psi(x);\ U(x) = e^{i\alpha^A(x)\cdot\tau^A/2}$$

- Introduce the gauge fields:

$$A_\mu(x) = \sum A^A_\mu(x)T^A;\ T^A = \frac{\tau^A}{2}$$

$$A'_\mu(x) = U(x)A_\mu(x)U(x)^\dagger + iU(x)\partial_\mu U(x)^\dagger$$

- Covariant derivative

$$D_\mu\psi(x) = (\partial_\mu - iA_\mu)\psi$$

- and….

$$D'_\mu\psi'(x) = \partial_\mu(U\psi) - i(UA_\mu U^\dagger + iU\partial_\mu U^\dagger)U\psi =$$
$$= (\partial_\mu U)\psi + U\partial_\mu\psi - iUA_\mu\psi + U(\partial_\mu U^\dagger)U\psi =$$
$$= U(\partial_\mu - iA_\mu)\psi = UD_\mu\psi$$

- if $L_0(\psi, \partial_\mu\psi)$ is invariant for global trasformations (isospin), $L_0(\psi, D_\mu\psi)$ is gauge invariant.
- We need the analog of the Mawxell term !!!????!!!!

Yang - Mills Lagrangian

- Define the Y-M tensor: $G_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu + i[A_\mu, A_\nu]$
- Prove that G transforms like the regular representation (a good exercise):

$$G'_{\mu\nu} = \partial_\nu A'_\mu - \partial_\mu A'_\nu + i[A'_\mu, A'_\nu] = gG_{\mu\nu}g^\dagger$$

- Yang-Mills lagrangian: invariant, quadratic in the derivatives of A:

$$L_{Y-M} = \frac{1}{8g^2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})$$

- Minimal substitution: given $L_0(\psi, \partial_\mu\psi)$ invariant under global transformations, the new Lagrangian:

$$L_{tot} = L_0(\psi, D_\mu\psi) - \frac{1}{8g^2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})$$

is invariant under local transformations

$$L_{tot} = L_0(\psi, D_\mu\psi) - \frac{1}{8g^2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})$$

- Rescaling A→gA, we see that "g" is the coupling constant (g=0, no interaction;
- There are as many g as there are simple components in G (es. $SU(2)_W \otimes U(1)_Y$ has 2 constants)
- ***No mass term for the gauge bosons is allowed.***
- ***The form of the interaction matter-gauge fields is fixed by the symmetry***
- Gauge fields are interacting, since they carry a non vanishing charge: unlike QED, pure Yang-Mills is non-trivial

Y-M is a toy model for gravity

- Spin 1/2 matter fields:

$$L_{int} = g(\bar{\psi}\gamma^\lambda T^A\psi)A^A_\lambda = gA^A_\lambda(J^{A\lambda})$$

J^A are the means by which matter interacts and the currents associated to the global symmetry (the CVC hypothesis was a long-neglected hint that the isospin currents had not to do with strong interactions!)

- Scalar matter fields: besides the current mediated interaction $(A^A)_\mu J^{A\mu}$ have a quadri-linear interaction (seagull): $L_{seagull} = \frac{g^2}{2}(\phi^\dagger g^{\mu\nu}\{T^A, T^B\}\phi)A^A_\mu A^B_\nu$
- Write L_0 for scalar fields

4 .Large q^2 behaviour in field theory: QED

- Asymptotic behaviour of deep inelastic processes, e.g.:

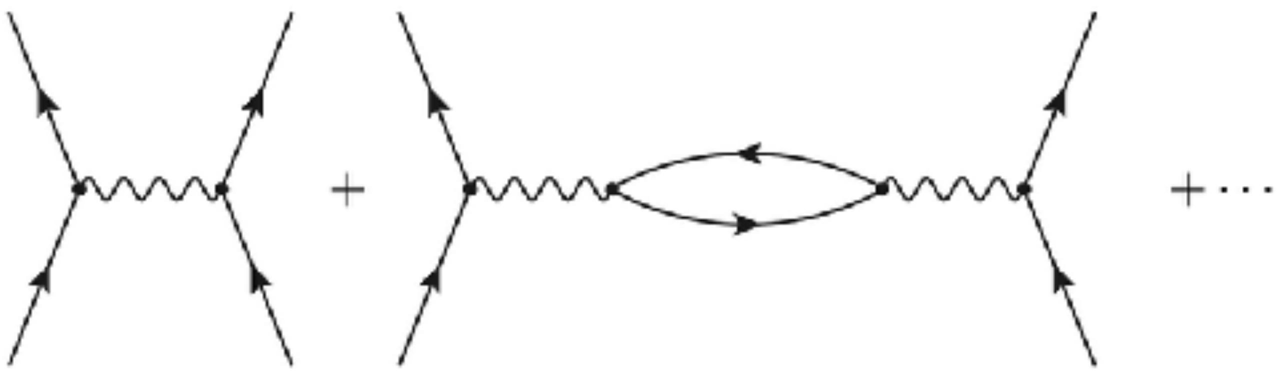
$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- The cross section for muon pairs scales with s like 1/s
- R = constant, above low-energy resonances, says that 1/s scaling is approximately true for hadron production.
- How could it be different? Naive scaling:
  - For large s we can set to 0 all particle masses
  - the dimension of the cross section is determined by the only dimensional variable at our disposal, s
  - $[s]=\text{lenght}^{-2}$, $[\sigma]=\text{lenght}^{2}$ σ =Const /s.

Failure of naive scaling

- The argument is wrong!
  - field theory needs to be computed with an UltraViolet cut off, Λ
  - renormalized physical quantities (e.g. charges) have to be defined at some value of q^2.
  - If we renormalize the physical quantities (e.g. charge) at $q^2=0$, we cannot send all masses to 0, because of mass singularities (i.e. $\log(q^2/m^2)$);
  - Alternatively, we can renormalize at a mass scale $q^2=-\mu^2 < 0$, and send masses to 0, to examine the limit $q^2 \to \infty$ or $s \to \infty$;
  - however, even in the massless theory, I can now have large logs of the type $[\log(-q^2/\mu^2)]^d$ or $[\log(s/\mu^2)]^d$ ***which spoil the naive scaling laws*** !
  - Scaling Laws in the asymptotic behaviour are non-trivial dynamical properties: why do we see approximate naive scaling to hold???

the β function of QED



$$\mathcal{A} = \frac{e_0^2}{q^2}\left(1 + e_0^2\Pi(q^2) + \cdots\right)$$

$$\Pi(q^2) - \Pi(\mu^2) = \text{finite} = \Pi_c(q^2, \mu^2) = \frac{1}{12\pi^2}\ln\frac{q^2}{\mu^2}; \quad (-q^2, -\mu^2 >> m^2)$$

- vertex and fermion propagator corrections carry a factor $Z_2/Z_1=1$ (Ward dentity). Define:

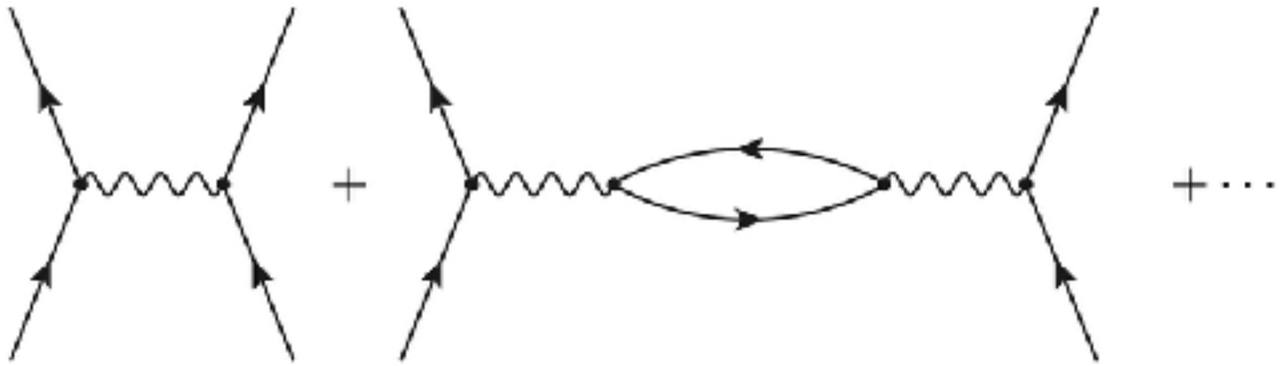
$$\mathbf{d} = q^2\mathcal{A} = e_0^2\left(1 + e_0^2\Pi(q^2) + \cdots\right) \sim e_0^2\left\{1 + e_0^2[\Pi(q^2) - \Pi(\mu^2)] + e_0^2\Pi(\mu^2)\right\} \sim$$
$$\sim e_0^2\left[1 + e_0^2\Pi(\mu^2)\right]\left[1 + e^2(\Pi_c(q^2,\mu^2)\right] = e^2(\mu)\left[1 + e^2(\mu)\Pi_c(q^2,\mu^2)\right]$$

- **d**(q^2, $e^2(\mu)$, μ^2) is a physical quantity, and it cannot depend from μ, which is arbitrary. A change of μ must be compensated by a change in e(μ):

$$0 = \mu\frac{d\mathbf{d}}{d\mu} = \mu\frac{\partial\mathbf{d}}{\partial e^2}\frac{\partial e^2}{\partial\mu} + \mu\frac{\partial\mathbf{d}}{\partial\mu}|_{q^2=0} \sim 2e\mu\frac{\partial e}{\partial\mu} - \frac{e^4}{6\pi^2}$$

$$\mu\frac{\partial e}{\partial\mu} = \beta(e) = +\frac{e^3}{12\pi^2}$$

QED, the Gell-Mann and Low equation for the running coupling



$$\mathcal{A} = \frac{\mathbf{d}(q^2, e(\mu^2), \mu^2)}{q^2}$$

- fix the subtractin point, μ
- **d** determines the strenght of the interaction at varying q^2, we call it the ***runnig coupling constant***
- denote it by $e^2(t)$, $t=\log(q^2/\mu^2)$ (q^2A can depend only on the ratio q^2/μ^2)

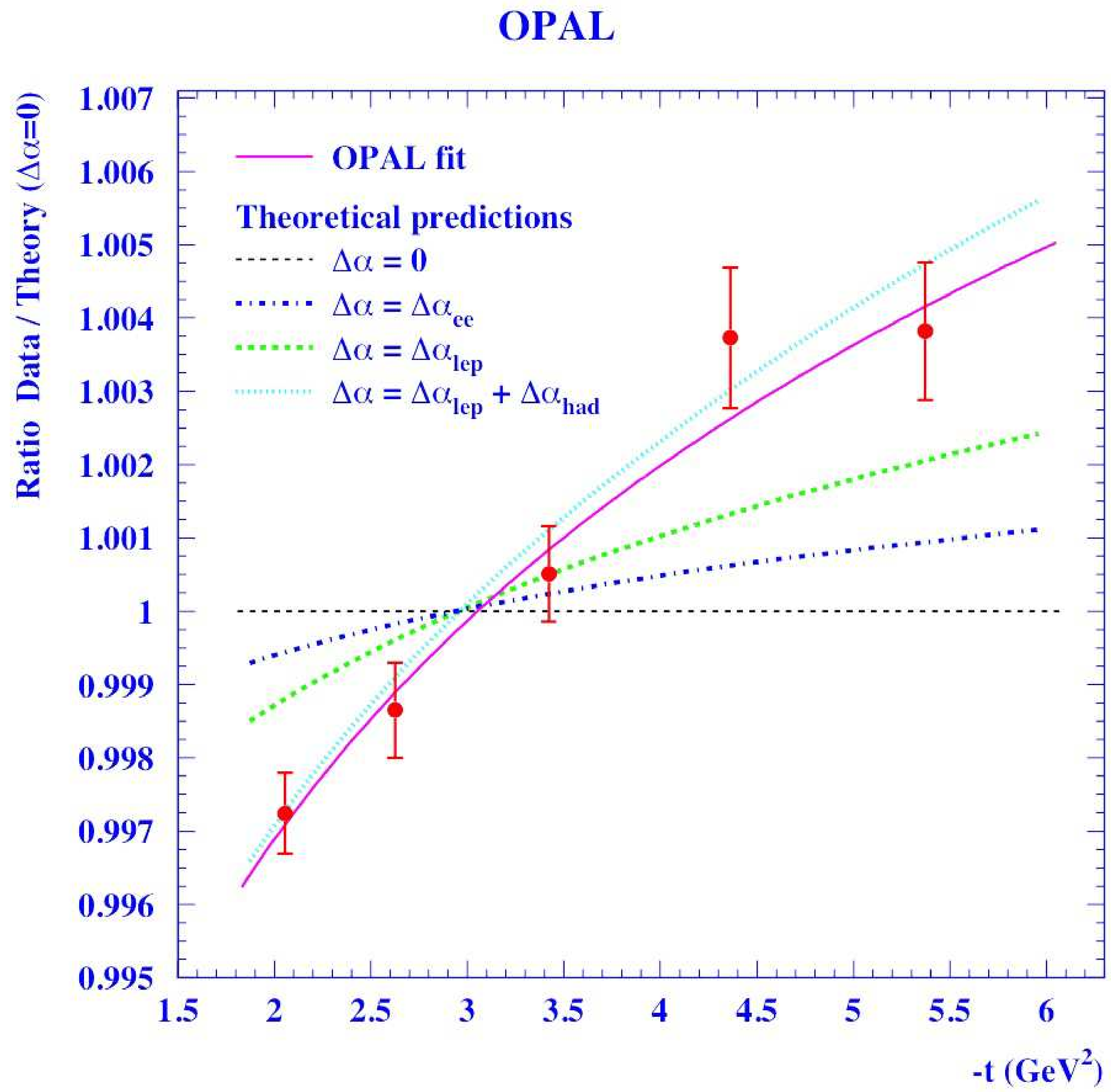
$$\frac{\partial \mathbf{d}}{\partial t} = q^2 \frac{\partial \mathbf{d}}{\partial q^2} = -\mu^2 \frac{\partial \mathbf{d}}{\partial \mu^2} = -\frac{1}{2}\mu \frac{\partial \mathbf{d}}{\partial \mu} = +e\beta(e) = \frac{e^4}{12\pi^2}$$

The Gell-Mann Low equation for the running fine structure constant $\alpha(t)=e^2(t)/4\pi$ is:

$$\frac{1}{4\pi}\frac{\partial \mathbf{d}}{\partial t} = \frac{d\alpha(t)}{dt} = \frac{\alpha^2}{3\pi} \quad \rightarrow \quad d(\frac{1}{\alpha}) = -\frac{dt}{3\pi}$$

$$\alpha(t) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi}\log(\frac{q^2}{\mu^2})}$$

increases with q^2



QED: the "running" fine structure constant from Bahba scattering. At the $-q^2 \sim (M_Z)^2$, $\alpha^{-1} \sim 128$

Dependence on $-t = Q^2$ of the ratio between the elastic scattering cross section for e^+–e^- (Bhabha scattering) measured at LEP and the results of theoretical calculations. The horizontal line corresponds to the case in which the value of α is kept constant as Q^2 varies

Bare vs. running coupling

$$\alpha(t) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi}\log(\frac{q^2}{\mu^2})}$$

If I put t=ln (Λ^2/μ^2) I get the coupling at the scale of the UV cutoff, i.e. the "bare coupling"

$$\alpha(\Lambda) = \frac{\alpha}{1 - \alpha\, b[log(\Lambda^2/\mu^2)]}$$

- BAD NEWS:
  - for fixed physical constant α, we cannot send $\Lambda \to \infty$: $\alpha(\Lambda)$ becomes negative above the Landau pole
  - the continuum theory is recovered only for $\alpha=0$
  - a similar phenomenon happens for the ϕ^4 theory, where we have much better control
- If b <0, $\alpha(\Lambda) \to 0$ ("asymptotic freedom")
- A reasonable conjecture: the limit $\Lambda \to \infty$ exists only for asymptotically free theories
- GOOD NEWS:
  - Yang-Mills theory with not too many fermions has b<0 and is asymptotically free. Asymptotic freedom provides the basis of scaling in deep inlastic scattering.