RG running of the SMEFT dim-7 operators and the relevant phenomenology

Xiaodong Ma
School of Physics, Nankai University, Tianjin
In collaboration with Prof. Yi Liao

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Overview

Convention

- (ν)SM field contents: $H, Q, L, u, d, e, B_\mu, W^I_\mu, G^A_\mu$ (+ sterile neutrino: N),
- Symmetry:

$$\text{Poincare} \otimes \text{Gauge} = T_{1,3} \ltimes SO^\uparrow_+(1,3) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- SM Lagrangian:

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger(D^\mu H) + \lambda v^2(H^\dagger H) - \lambda(H^\dagger H)^2 + \sum_{\Psi=Q,L,u,d,e} \bar{\Psi} i\not{D}\Psi - \left[\bar{Q}Y_u u\tilde{H} + \bar{Q}Y_d dH + \bar{L}Y_e eH + \text{h.c.}\right]$$

- νSM Lagrangian:

$$\mathcal{L}_{\nu\rm SM} = \mathcal{L}_{\rm SM} + \bar{N}\not{\partial}N - \left[\frac{1}{2}(NCM_N N) + \bar{L}Y_N N\tilde{H} + \text{h.c.}\right]$$

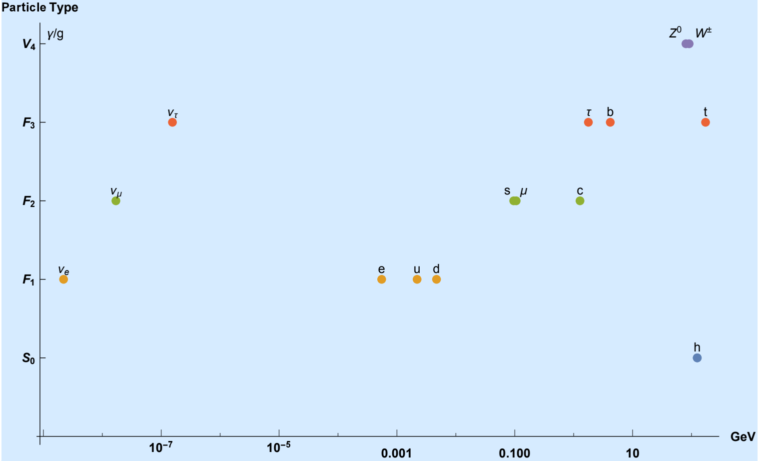
  - $D_\mu = \partial_\mu - ig_3 T^A G^A_\mu - ig_2 T^I W^I_\mu - ig_1 Y B_\mu, \quad \tilde{H}_i = \epsilon_{ij}H^*_j$
  - $Y_{u,d,e,N}$: the Yukawa couplings;
  - M_N: symmetric mass matrix for N.

SM fields and physical particles

SM	Before SSB			After SSB		
Spin	Fields	$SO^{\uparrow}_{+}(1,3)$	$U(1)_Y \times SU(2)_L \times SU(3)_C$	Physical Particles		
Scalar: 0	Higgs: H	(0,0)	$(\frac{1}{2}, 2, 1)$	h		
Spinor: $\frac{1}{2}$				1st Gen.	2d Gen.	3rd Gen.
	Quark: u	$(0, \frac{1}{2})$	$(\frac{2}{3}, 1, 3)$	u_r, u_b, u_g	c_r, c_b, c_g	t_r, t_b, t_g
	Quark: Q	$(\frac{1}{2}, 0)$	$(\frac{1}{6}, 2, 3)$			
	Quark: d	$(0, \frac{1}{2})$	$(-\frac{1}{3}, 1, 3)$	d_r, d_b, d_g	s_r, s_b, s_g	b_r, b_b, b_g
	-	-	-	ν_e	ν_μ	ν_τ
	Lepton: L	$(\frac{1}{2}, 0)$	$(-\frac{1}{2}, 2, 1)$			
	Lepton: e	$(0, \frac{1}{2})$	$(-1, 1, 1)$	e	μ	τ
Vector: 1	B_μ	$(\frac{1}{2}, \frac{1}{2})$	$(0, 0, 0)$	$\gamma, W^{\pm}, Z^0$		
	W^I_μ	$(\frac{1}{2}, \frac{1}{2})$	$(0, 3, 0)$			
	Gluon: G^A_μ	$(\frac{1}{2}, \frac{1}{2})$	$(0, 0, 8)$	$g^A, A = 1, \ldots, 8$		

$$SO^{\uparrow}_{+}(1,3) \sim SU(2)_l \times SU(2)_r$$

The spectra of SM particles



- A big gap between neutrinos and other particles;
- $M(F_1) < M(F_2) < M(F_3)$ when neutrino's mass ignored.

Problems unanswered by SM

- Experimental side:
  - Neutrino oscillation $\to$ nonvanishing neutrino mass;
  - The nature of dark matter;
  - Anomaly or bump from collider experiments: $g_\mu - 2$, $R_K(R_K^*)$, ...
  - ...
- Theoretical side:
  - Hierarchy problem: $\delta m_h^2 \propto \Lambda^2$;
  - Vacuum stability: $\lambda(\Lambda) \leq 0$;
  - Strong CP: $\theta_{\rm CP} < 10^{-10}$;
  - ...

Remark 1: Probably the anomalies are due to the uncertainties both from theory and experiment, and the naturalness problems from theory are people's biased taste of beauty.
Remark 2: Nevertheless, new physics(NP) must be involved from m_ν and DM: Top-down: Model building vs Bottom-up: EFT $\to$ SMEFT

From SM to SMEFT

- Assume NP beyond the SM exist, with $\Lambda_{\text{NP}} \gg v$;
- Ingredients: SM fields $+$ SM gauge symmetry;
- SMEFT$=$ all possible local, SM gauge invariant operators building from SM fields parameterized by the inverse power of Λ_{NP}, i.e.,

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda_{\text{NP}}^{D-4}} \sum_{D\geq 5} C_i^D \mathcal{O}_i^D,$$

  Wilson coefficients C_i^D encode the contribution from unknown NP.
- Merit: Model-independent;
- Connection with full theory: Integrating out the heavy d.o.f in full theory, then running and matching with SMEFT.

The state of the art for the basis of (ν)SMEFT

- SMEFT
  - Dim-5: $1(\not{L}\cap B)$ [Weinberg 79] $\to$ Dim-$D \in$ odd [Liao 10]

$$\mathcal{O}_{LH}^{D} = \left[(L^{\mathsf{T}}\epsilon H)C(L^{\mathsf{T}}\epsilon H)^{\mathsf{T}}\right](H^{\dagger}H)^{(D-5)/2}$$

  - Dim-6: $59(L\cap B) + 4(\not{L}\cap \not{B})$ [Buchmuller *et al* 86; Grzadkowski *et al* 10]
  - Dim-7: $12(\not{L}\cap B) + 6(\not{L}\cap \not{B})$ [Lehman 14; **1607.07309**]
  - Dim-8, 9, ... [Lehman *et al* 15; Henning *et al* 15, 17[1706.08520]]
- νSMEFT:
  - Dim-5: $2(\not{L}\cap B)$ [0904.3244]
  - Dim-6: $16(L\cap B) + 1(\not{L}\cap B) + 2(\not{L}\cap \not{B})$ [0806.0876; **1612.04527**]
  - Dim-7: $47(\not{L}\cap B) + 5(\not{L}\cap \not{B})$ [1505.05264; **1612.04527**]
- $D \in$ even: $|\not{B} + \not{L}| = 0$ vs $D \in$ odd: $|\not{B} - \not{L}| = 2$ for SMEFT;
- $D \in$ odd: L is violated;
- Note: The true counting of complete and independent operators needs the number of flavors, since many operators can be related with flavor symmetry.

The state of the art for the 1-loop RGE

- Dim-5: [Babu 93; Antusch *et al* 01; **1701.08019**]

$$16\pi^2\mu\frac{d}{d\mu}C_{LH}^{d\,pr}=\Big[(3d^2-18d+19)\lambda-\frac{3}{4}(d-5)g_1^2-\frac{3}{4}(3d-11)g_2^2+(d-3)W_H\Big]C_{LH}^{d\,pr}$$
$$-\frac{3}{2}\Big[(Y_eY_e^\dagger)_{vp}C_{LH}^{d\,vr}+(Y_eY_e^\dagger)_{vr}C_{LH}^{d\,pv}\Big],$$

$$W_H=\mathrm{Tr}[3(Y_u^\dagger Y_u)+3(Y_d^\dagger Y_d)+(Y_e^\dagger Y_e)]$$

- Dim-6: [J. Elias-Miro *et al* 13; Jenkins *et al* 13, 14]
- Dim-7: [**1607.07309** for 6($\not{L}\cap\not{B}$)+present work 12($\not{L}\cap B$)]
- Feature 1: holomorphic structure for the γ-matrix $\rightarrow$ many zeros in γ [1409.0868; Clifford Cheung and Chia-Hsien Shen: 1505.01844]
- Feature 2: Power counting for the γ-matrix [1309.0819;**1701.08019**].

Basis for dim-7 operators [Lehman 14; **1607.07309**]

	$\psi^2 H^4$ + h.c.		$\psi^2 H^3 D$ + h.c.
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m) H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\gamma_\mu e) H^j H^m i D^\mu H^n$
	$\psi^2 H^2 D^2$ + h.c.		$\psi^2 H^2 X$ + h.c.
$\mathcal{O}_{LHD1}$ $\mathcal{O}_{LHD2}$	$\epsilon_{ij}\epsilon_{mn}(L^i C D^\mu L^j) H^m (D_\mu H^n)$ $\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j) H^m (D_\mu H^n)$	$\mathcal{O}_{LHB}$ $\mathcal{O}_{LHW}$	$g_1 \epsilon_{ij}\epsilon_{mn}(L^i C\sigma_{\mu\nu} L^m) H^j H^n B^{\mu\nu}$ $g_2 \epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C\sigma_{\mu\nu} L^m) H^j H^n W^{I\mu\nu}$
	$\psi^4 D$ + h.c.		$\psi^4 H$ + h.c.
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C i D^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$ $\mathcal{O}_{\bar{d}LQLH1}$ $\mathcal{O}_{\bar{d}LQLH2}$ $\mathcal{O}_{\bar{d}LueH}$ $\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m) H^n$ $\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m) H^n$ $\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m) H^n$ $\epsilon_{ij}(\bar{d}L^i)(u C e) H^j$ $\epsilon_{ij}(\bar{Q}u)(L C L^i) H^j$
$\mathcal{O}_{\bar{L}QddD}$ $\mathcal{O}_{\bar{e}dddD}$	$(\bar{L}\gamma_\mu Q)(d C i D^\mu d)$ $(\bar{e}\gamma_\mu d)(d C i D^\mu d)$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$ $\mathcal{O}_{\bar{L}dddH}$ $\mathcal{O}_{\bar{e}Qdd\tilde{H}}$ $\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$(\bar{L}d)(u C d)\tilde{H}$ $(\bar{L}d)(d C d) H$ $\epsilon_{ij}(\bar{e}Q^i)(d C d)\tilde{H}^j$ $\epsilon_{ij}(\bar{L}d)(Q C Q^i)\tilde{H}^j$

- Neutrino mass: $\mathcal{O}_{LH}$;
- Magnetic moment: $\mathcal{O}_{LHB}$, $\mathcal{O}_{LHW}$;
- $0\nu\beta\beta$: $\mathcal{O}^5_{LH}$, $\mathcal{O}_{LH}$, $\mathcal{O}_{LeHD}$, $\mathcal{O}_{LHD1}$, $\mathcal{O}_{\bar{d}uLLD}$, $\mathcal{O}_{\bar{d}LQLH1}$, $\mathcal{O}_{\bar{d}LQLH2}$, $\mathcal{O}_{\bar{d}LueH}$, $\mathcal{O}_{\bar{Q}uLLH}$

Flavor structure of the basis

Class	Operator	flavor symmetries
$\psi^2 H^4$	$\mathcal{O}_{LH}$	$\mathcal{O}_{LH}^{pr} - p \leftrightarrow r = 0$
$\psi^2 H^3 D$	$\mathcal{O}_{LeHD}$	$\times$
$\psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}$	$(\mathcal{O}_{LDH1}^{pr} + \mathcal{K}^{pr}) - p \leftrightarrow r = 0$
	$\mathcal{O}_{LHD2}$	$\left[4\mathcal{O}_{LHD2}^{pr} + 2(Y_e)_{rv}\mathcal{O}_{LeHD}^{pv} - \mathcal{O}_{LHW}^{pr} + 2\mathcal{K}^{pr}\right] - p \leftrightarrow r = \mathcal{O}_{LHB}^{pr}$
$\psi^2 H^2 X$	$\mathcal{O}_{LHB}$	$\mathcal{O}_{LHB}^{pr} + p \leftrightarrow r = 0$
	$\mathcal{O}_{LHW}$	$\times$
$\psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}$	$(\mathcal{O}_{\bar{e}LLLH}^{prst} + r \leftrightarrow t) - r \leftrightarrow s = 0$
	$\mathcal{O}_{\bar{d}LQLH1}$	$\times$
	$\mathcal{O}_{\bar{d}LQLH2}$	$\times$
	$\mathcal{O}_{\bar{d}LueH}$	$\times$
	$\mathcal{O}_{\bar{Q}uLLH}$	$\times$
$\psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}$	$\left[\mathcal{O}_{\bar{d}uLLD}^{prst} + (Y_d)_{vp}\mathcal{O}_{\bar{Q}uLLH}^{vrst} - (Y_u^\dagger)_{rv}\mathcal{O}_{\bar{d}LQLH2}^{psvt}\right] - s \leftrightarrow t = 0$
$\not{B}:\ \psi^4 H$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$\times$
	$\mathcal{O}_{\bar{L}dddH}$	$\mathcal{O}_{\bar{L}dddH}^{prst} + s \leftrightarrow t = 0,\quad \mathcal{O}_{\bar{L}dddH}^{prst} + \mathcal{O}_{\bar{L}dddH}^{pstr} + \mathcal{O}_{\bar{L}dddH}^{ptrs} = 0$
	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst} + s \leftrightarrow t = 0$
	$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\times$
$\not{B}:\ \psi^4 D$	$\mathcal{O}_{\bar{L}QddD}$	$\left[\mathcal{O}_{\bar{L}QddD}^{prst} + (Y_u)_{rv}\mathcal{O}_{\bar{L}dud\tilde{H}}^{psvt}\right] - s \leftrightarrow t = -(Y_e^\dagger)_{vp}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{vrst} - (Y_d)_{rv}\mathcal{O}_{\bar{L}dddH}^{pvst}$
	$\mathcal{O}_{\bar{e}dddD}$	$\mathcal{O}_{\bar{e}dddD}^{prst} - r \leftrightarrow s = (Y_d^\dagger)_{tv}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{pvrs},\ (\mathcal{O}_{\bar{e}dddD}^{prst} + r \leftrightarrow t) - s \leftrightarrow t = (Y_e)_{vp}\mathcal{O}_{\bar{L}dddH}^{vrst}$

$$\mathcal{K}^{pr} \equiv (Y_u)_{vw}\mathcal{O}_{\bar{Q}uLLH}^{vwpr} - (Y_d^\dagger)_{vw}\mathcal{O}_{\bar{d}LQLH2}^{vpwr} - (Y_e^\dagger)_{vw}\mathcal{O}_{\bar{e}LLLH}^{vwpr}$$

Note: such relations make the RG running complicated, since we need to choose a basis with flavor specified to determine γ-matrix.

An example for determining the basis

- Determine the relation in class $\psi^4 D$:

$$
\begin{aligned}
&\mathcal{O}_{\bar{d}uLLD}^{prst} - s \leftrightarrow t \\
&= \epsilon_{ij} (\bar{d}_p \gamma_\mu u_r)(L_s^i C i D^\mu L_t^j) - s \leftrightarrow t \\
&\overset{\text{IBP}}{=\!=} -\epsilon_{ij} (\bar{d}_p i \overleftarrow{\not{D}} u_r)(L_s^i C L_t^j) - \epsilon_{ij} (\bar{d}_p i \not{D} u_r)(L_s^i C L_t^j) \\
&\overset{\text{EoMs}}{=\!=} (Y_d)_{vp} \left[\epsilon_{ij} \delta_{mn} (\bar{Q}_v^m u_r)(L_s^i C L_t^j) H^n \right] - (Y_u^\dagger)_{rv} \left[\epsilon_{ij} \epsilon_{mn} (\bar{d}_p Q_v^m)(L_s^i C L_t^j) H^n \right] \\
&\overset{\text{Fierz}}{=\!=} \left[-(Y_d)_{vp} \mathcal{O}_{\bar{Q}uLLH}^{vrst} + (Y_u^\dagger)_{rv} \mathcal{O}_{\bar{d}LQLH2}^{psvt} \right] - s \leftrightarrow t
\end{aligned}
$$

- IBP: Integration by part;
- EoM: Equation of motion $\sim$ field redefinition;
- Fierz: Fierz identity for fermion bilinears.
- Such relations indicate the true number of basis must include flavor.

Basics for RG evolution

Dimensional regularizaton($d = 4 - 2\epsilon$) + $\overline{\text{MS}}$ scheme + R_ξ-gauge

$$16\pi^2\beta_i = 16\pi^2\mu\frac{d}{d\mu}C_i = \sum_j \gamma_{ij}C_j$$

- Focus only on the correction from SM interactions: leading order in the expansion of Λ;
- Calculation of β-function or γ is boiling down to determine the counterterm:

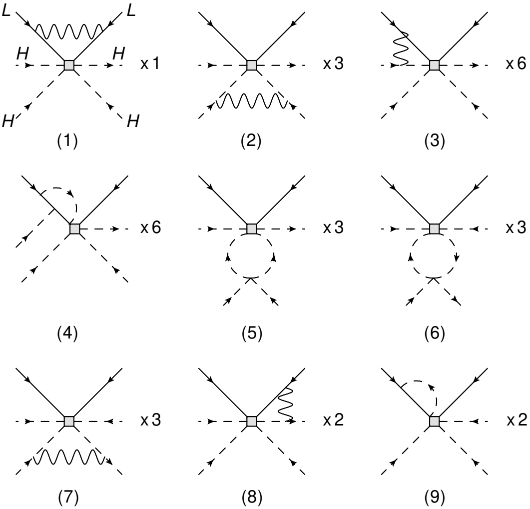
$$C^{\rm T}\mathcal{O}_b \xlongequal{\text{1-loop}} -\frac{1}{\epsilon}C^{\rm T}(P\mathcal{O}_b + R\mathcal{O}_r) \to -\frac{1}{\epsilon}C^{\rm T}(P + RM)\mathcal{O}_b \quad \text{with} \quad \mathcal{O}_r = M\mathcal{O}_b$$

$$\boxed{\gamma_{ij} = \sum_\alpha \rho_\alpha g_\alpha \left(\frac{\partial P_{ji}}{\partial g_\alpha} + \frac{\partial R_{jk}}{\partial g_\alpha}M_{ki} + R_{jk}\frac{\partial M_{ki}}{\partial g_\alpha}\right)}$$

$\rho_\alpha = 1$ for $g_\alpha \in \{g_{1,2,3}, Y_{e,d,u}\}$ and $\rho_\alpha = 2$ for $g_\alpha = \lambda$.

- ξ independent as a check for the calculation.

An example for 1-loop correction from SM interactions to $\mathcal{O}_{LH}$



The structure and power counting of γ-matrix

	$(\omega,\bar{\omega})$	$(5,3)$	$(5,3)$	$(5,3)$	$(5,5)$	$(5,5)$	$(5,5)$	$(7,3)$	$(7,3)$	$(7,3)$	$(7,3)$	$(7,3)$	$(7,5)$
$(\omega,\bar{\omega})$	γ_{ij}	$\mathcal{O}_{LHD1}$	$\mathcal{O}_{LHD2}$	$\mathcal{O}_{\bar{d}uLLD}$	$\mathcal{O}_{LeHD}$	$\mathcal{O}_{\bar{d}LueH}$	$\mathcal{O}_{\bar{Q}uLLH}$	$\mathcal{O}_{\bar{e}LLLH}$	$\mathcal{O}_{\bar{d}LQLH1}$	$\mathcal{O}_{\bar{d}LQLH2}$	$g_1\mathcal{O}_{LHB}$	$g_2\mathcal{O}_{LHW}$	$\mathcal{O}_{LH}$
$(5,3)$	$\mathcal{O}_{LHD1}$	g^2	g^2	g^2	0	0	0	0	0	0	0	0	0
$(5,3)$	$\mathcal{O}_{LHD2}$	g^2	g^2	0	0	0	0	0	0	0	0	0	0
$(5,3)$	$\mathcal{O}_{\bar{d}uLLD}$	g^2	g^2	g^2	0	0	0	0	0	0	0	0	0
$(5,5)$	$\mathcal{O}_{LeHD}$	g^3	g^3	0	g^2	g^2	0	0	0	0	$\sum\to 0$	$\sum\to 0$	0
$(5,5)$	$\mathcal{O}_{\bar{d}LueH}$	g^3	g^3	g^3	g^2	g^2	g^2	0	$Y_u Y_e$	$Y_u Y_e$	0	0	0
$(5,5)$	$\mathcal{O}_{\bar{Q}uLLH}$	g^3	g^3	g^3	0	g^2	g^2	$Y_u Y_e$	$Y_u Y_d$	$Y_u Y_d$	0	0	0
$(7,3)$	$\mathcal{O}_{\bar{e}LLLH}$	g^3	g^3	0	0	0	$Y_u^\dagger Y_e^\dagger$	g^2	g^2	g^2	g^3	g^3	0
$(7,3)$	$\mathcal{O}_{\bar{d}LQLH1}$	g^3	g^3	g^3	0	$Y_u^\dagger Y_e^\dagger$	$Y_u^\dagger Y_d^\dagger$	g^2	g^2	g^2	g^3	g^3	0
$(7,3)$	$\mathcal{O}_{\bar{d}LQLH2}$	g^3	g^3	g^3	0	$Y_u^\dagger Y_e^\dagger$	$Y_u^\dagger Y_d^\dagger$	g^2	g^2	g^2	g^3	g^3	0
$(7,3)$	$g_1\mathcal{O}_{LHB}$	g^2	g^2	0	$Y_e^\dagger$	0	0	g^1	g^1	0	g^2	g^2	0
$(7,3)$	$g_2\mathcal{O}_{LHW}$	g^2	g^2	0	$Y_e^\dagger$	0	0	g^1	g^1	g^1	g^2	g^2	0
$(7,5)$	$\mathcal{O}_{LH}$	g^4	g^4	0	g^3	0	g^3	g^3	g^3	0	0	g^4	g^2

- The zeros in the gray area can be explained using non-renormalization theorem in [1505.01844] A
- The power of $g \sim Y_{e,d,u} \sim g_{1,2,3} \sim \sqrt{\lambda}$ can be explained in [**1701.08019**], for n-loop the power is

$$\chi[\gamma_{ji}] = 2n + \chi[\mathcal{O}_i] - \chi[\mathcal{O}_j]$$

where $\chi[\Psi] = \frac{1}{2},\ \chi[X_{\mu\nu}] = 1,\ \chi[D_\mu] = 1,\ \chi[g_{1,2,3}] = \chi[Y] = 1,\ \chi[\lambda] = 2.$

Neutrino transition magnetic moment

After SSB,

$$\mathcal{L} \supset \frac{1}{2} v^2 e\Big(C_{LHB}^{pr} - C_{LHW}^{pr}\Big)(\nu_p C\sigma^{\mu\nu} P_L \nu_r) F_{\mu\nu} + \text{h.c.}$$

Define

$$\Lambda_{pr} = 2v^2 e\Big(C_{LHB}^{pr} - C_{LHW}^{pr}\Big)$$

Take the constraints from [1510.01684], we obtain

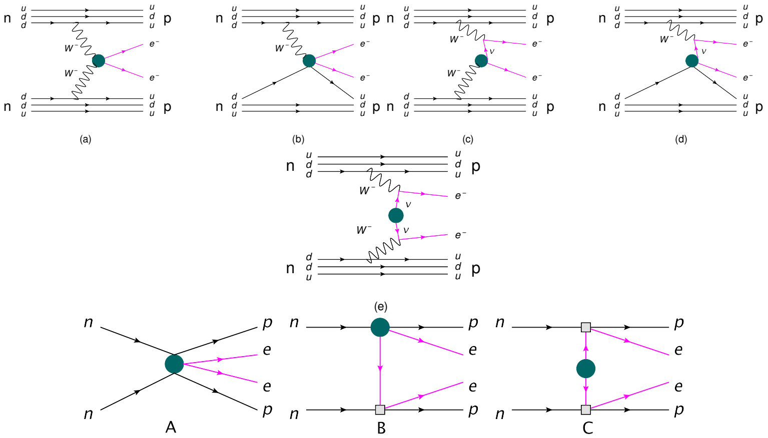
$$\left|\Big(U^{\mathrm{T}}(C_{LHB} - C_{LHW})U\Big)_{12}\right| \leq \frac{3.1\times 10^{-11}\mu_B}{ev^2} \sim 2\Big(\frac{1}{100\text{TeV}}\Big)^3,$$

$$\left|\Big(U^{\mathrm{T}}(C_{LHB} - C_{LHW})U\Big)_{13}\right| \leq \frac{4.0\times 10^{-11}\mu_B}{ev^2} \sim 2.6\Big(\frac{1}{100\text{TeV}}\Big)^3,$$

$$\left|\Big(U^{\mathrm{T}}(C_{LHB} - C_{LHW})U\Big)_{23}\right| \leq \frac{5.6\times 10^{-11}\mu_B}{ev^2} \sim 3.6\Big(\frac{1}{100\text{TeV}}\Big)^3.$$

where μ_B is the Bohr magneton.

The relevant Feynman diagrams for $0\nu\beta\beta$



- Short-range interactions: (a) and (b) $\rightarrow$ A;
- Long-range interactions: (c) and (d)$\rightarrow$ B;
- Mass mechanism: (e)$\rightarrow$ C;

Effective field theory description

After integrating out the W- gauge boson, the effective Lagrangian contributing to $0\nu\beta\beta$ is

- Short-range interaction: $\mathcal{L}_S = \sum_{n=1}^{2} C_{Sn}\mathcal{O}_{Sn}$ [Deppisch *et al* 1208.0727; Horoi *et al* 1706.05391]

$$\mathcal{O}_{S1} = \left[(\bar{u}\gamma^\mu P_L d)(\bar{u}\gamma_\mu P_L d)(\bar{e}P_R e^C)\right], \quad C_{S1} = -2\sqrt{2}G_F C_{LHD1}^{\dagger 11} = 4\frac{G_F^2}{m_p}\epsilon_3^{LLR},$$

$$\mathcal{O}_{S2} = \left[(\bar{u}\gamma^\mu P_L d)(\bar{u}\gamma_\mu P_R d)(\bar{e}P_R e^C)\right], \quad C_{S2} = -2\sqrt{2}G_F C_{\bar{d}uLLD}^{\dagger 1111} = 4\frac{G_F^2}{m_p}\epsilon_3^{LRR}.$$

- Long-range interaction: $\mathcal{L}_L = \sum_{n=0}^{5} C_{Ln}\mathcal{O}_n$

$$\mathcal{O}_0 = (\bar{u}\gamma^\mu P_L d)(\bar{e}\gamma_\mu P_L \nu), \quad C_{L0} = 2\sqrt{2}G_F,$$

$$\mathcal{O}_1 = (\bar{u}\gamma^\mu P_L d)(\bar{e}\gamma_\mu P_R \nu^C), \quad C_{L1} = -\frac{\sqrt{2}v}{2}C_{LeHD}^{11\dagger} = 2\sqrt{2}G_F\epsilon_{V-A}^{V+A},$$

$$\mathcal{O}_2 = (\bar{u}\gamma^\mu P_R d)(\bar{e}\gamma_\mu P_R \nu^C), \quad C_{L2} = \frac{\sqrt{2}v}{4}C_{\bar{d}LueH}^{1111\dagger} = 2\sqrt{2}G_F\epsilon_{V+A}^{V+A},$$

$$\mathcal{O}_3 = (\bar{u}P_L d)(\bar{e}P_R \nu^C), \quad C_{L3} = \frac{\sqrt{2}v}{2}C_{\bar{Q}uLLH}^{1111\dagger} = 2\sqrt{2}G_F\epsilon_{S-P}^{S+P},$$

$$\mathcal{O}_4 = (\bar{u}P_R d)(\bar{e}P_R \nu^C), \quad C_{L4} = \frac{\sqrt{2}v}{4}C_{\bar{d}LQLH1}^{1111\dagger} = 2\sqrt{2}G_F\epsilon_{S+P}^{S+P},$$

$$\mathcal{O}_5 = (\bar{u}\sigma^{\mu\nu} P_R d)(\bar{e}\sigma_{\mu\nu} P_R \nu^C), \quad C_{L5} = \frac{\sqrt{2}v}{16}\left[C_{\bar{d}LQLH1}^{1111\dagger} + 2C_{\bar{d}LQLH2}^{1111\dagger}\right] = 2\sqrt{2}G_F\epsilon_{TR}^{TR}.$$

The experimental constraints ▸ B

Assume only one operator dominates each time [Horoi *et al* 1706.05391]:

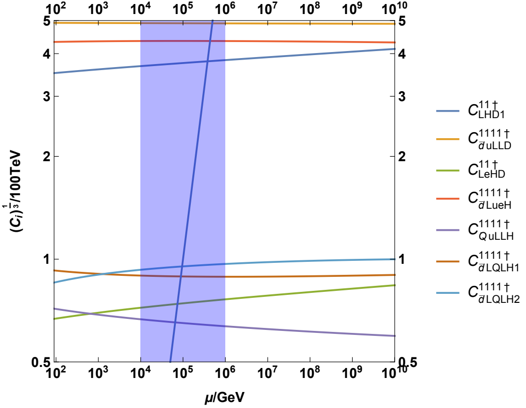
	^{48}Ca	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$\lvert\epsilon_3^{LLR}\rvert$	3.8×10^{-7}	8.9×10^{-9}	6.7×10^{-8}	2.0×10^{-8}	4.1×10^{-9}
$\lvert\epsilon_3^{LRR}\rvert$	6.3×10^{-7}	1.4×10^{-8}	1.1×10^{-7}	3.2×10^{-8}	6.7×10^{-9}
$\lvert\epsilon_{V-A}^{V+A}\rvert$	1.1×10^{-7}	2.2×10^{-9}	1.7×10^{-8}	5.1×10^{-9}	1.1×10^{-9}
$\lvert\epsilon_{V+A}^{V+A}\rvert$	1.3×10^{-5}	4.3×10^{-7}	2.2×10^{-6}	9.3×10^{-7}	2.0×10^{-7}
$\lvert\epsilon_{S\pm P}^{S+P}\rvert$	3.4×10^{-7}	7.9×10^{-9}	6.1×10^{-8}	1.4×10^{-8}	2.9×10^{-9}
$\lvert\epsilon_{TR}^{TR}\rvert$	1.8×10^{-8}	7.9×10^{-10}	5.9×10^{-9}	2.0×10^{-9}	4.2×10^{-10}

$\left(\frac{1}{100\text{TeV}}\right)^3$	^{48}Ca	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$\lvert C_{LHD1}^{11\dagger}\rvert$	6.68×10^3	0.156×10^3	1.178×10^3	0.352×10^3	0.072×10^3
$\lvert C_{\bar{d}uLLD}^{1111\dagger}\rvert$	11.076×10^3	0.246×10^3	1.934×10^3	0.563×10^3	0.118×10^3
$\lvert C_{LeHD}^{11\dagger}\rvert$	0.021×10^3	0.4	3.2	1.0	0.2
$\lvert C_{\bar{d}LueH}^{1111\dagger}\rvert$	4.927×10^3	0.163×10^3	0.834×10^3	0.352×10^3	0.076×10^3
$\lvert C_{\bar{Q}uLLH}^{1111\dagger}\rvert$	0.064×10^3	1.5	0.012×10^3	2.7	0.5
$\lvert C_{\bar{d}LQLH1}^{1111\dagger}\rvert$	0.129×10^3	3.0	0.023×10^3	5.3	1.1
$\lvert C_{\bar{d}LQLH2}^{1111\dagger}\rvert$	0.051×10^3	0.9	7.1	1.1	0.2

$\Rightarrow$ The bound for the NP scale is around $10-100$TeV if the $\mathcal{O}(1)$ coupling is assumed.

The evolution of the Wilson coefficient

In the RG equation we take all Yukawa couplings except top-Yukawa to zero, and neglect the threshold effect from EW scale to the hadron scale for simplicity:



- Point to the similar scale as the constraint from transition magnetic moment, i.e., around 100TeV;
- The running effect is small.

Other processes

- Nuclear muon-positron/anti-muon conversion: $\mu^- X \to e^+(\mu^+)X'$.
- Trimuon production from neutrino-muon collision: $\nu p \to \mu^-\mu^+\mu^+$.
- Production of Majorana neutrino from electron-proton collison: $e^+ p \to \overline{\nu} l_1^+ l_2^+ X$ from HERA.
- The tau lepton decaying into three muons: $\tau \to \mu^-\mu^-\mu^+$
- Kaon meson decaying into a pair of same charged leptons plus a pion: $K^\pm \to l^\pm l^\pm \pi^\mp$.
- Also for the lepton number violating decays from other mesons like D and B.

Summary

- Basis of Dim-7 operators in (ν)SMEFT determined up to Dim-7;
- 1-loop RGE of Dim-7 operators finished;
- New features of γ-matrix in SMEFT is analyzed;
- The rich phenomenologies explored, especially $0\nu\beta\beta$.

Thanks for your attention!

Non-renormalization theorem [1505.01844] ▸ A

Define the **holomorphic** and **anti-holomorphic** weight of an operator $\mathcal{O}$ as

$$\omega(\mathcal{O}) = n(\mathcal{O}) - H(\mathcal{O}), \bar{\omega}(\mathcal{O}) = n(\mathcal{O}) + H(\mathcal{O}).$$

$n(\mathcal{O})$: the minimal # of particles generated by the operator $\mathcal{O}$;
$h(\mathcal{O})$: the total helicity of the operator $\mathcal{O}$.
The weights of the SM fields:

SM fields	H	Ψ	$\bar{\Psi}$	D	X_-	X_+
weights $(\omega, \bar{\omega})$	$(1, 1)$	$(\frac{1}{2}, \frac{3}{2})$	$(\frac{3}{2}, \frac{1}{2})$	$(0, 0)$	$(0, 2)$	$(2, 0)$

Where Ψ is choosing as SM right handed fermion fields (Q^C, L^C, u, d, e) and $\bar{\Psi}$ as its left-handed counterpart, $X_{\pm}^{\mu\nu} = X^{\mu\nu} \mp i/2\epsilon^{\mu\nu\rho\sigma}X_{\rho\sigma}$.
Theorem: an operator $\mathcal{O}_i$ can only be renormalized by an operator $\mathcal{O}_j$ if $\omega_i \geq \omega_j \cup \bar{\omega}_i \geq \bar{\omega}_j$, i.e.,

$$\gamma_{ij} = 0 \quad \text{when} \quad \omega_i < \omega_j \quad \textit{or} \quad \bar{\omega}_i < \bar{\omega}_j,$$

and the anomalous case is from the non-holomorphic Yukawa coupling.