BPS Wilson Loops in 3d Super-Chern-Simons Theories

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Based on

- *Bin Chen, JW, 0809.2863*
- *Bin Chen, JW, Mengqi Zhu, 1410.2311*
- *Ouyang, JW, Jiaju Zhang, 1506.06192, 1510.05475, 1511.02967*
- *A. Mauri, Hao Ouyang, S. Penati, JW, Jiaju Zhang, 1808.01397*

Outline

- BPS Wilson loops in 4d $\mathcal{N}$=4 super Yang-Mills
- BPS Wilson loops in ABJM theory
- BPS Wilson loops in more general $\mathcal{N}$=2 Chern-Simons-matter theories

BPS WILSON LOOPS IN N=4 SUPER YANG-MILLS THEORIES

Wilson loops in gauge theories

- Wilson loops (WLs) are very important non-local observables in gauge theories.
- The vacuum expectation value(vev) of WL can be used as criterion for quark confinement.
- Loop equations encode the full dynamics.
- Null-gon WLs are dual to amplitudes in 4d $\mathcal{N}$=4 Suepr Yang-Mills.*[Alday, Maldacena, 07]*

[Drummond etal, 07][Brandhuber etal, 07]

Wilson-Maldacena Loops

- In 4d $\mathcal{N}$=4 Suepr Yang-Mills

$$W = \mathcal{P}\exp\left[-ig\int d\tau\,\left(A_\mu \dot{x}^\mu + \phi_I y^I |\dot{x}|\right)\right]$$

- In Eclidean space, let us consider straight line or circle. When y^I's are constants and $(dx/d\tau)^2=y^2$, this WL is half BPS. *[Maldacena 98][Rey, Yee 98]*

String theory dual

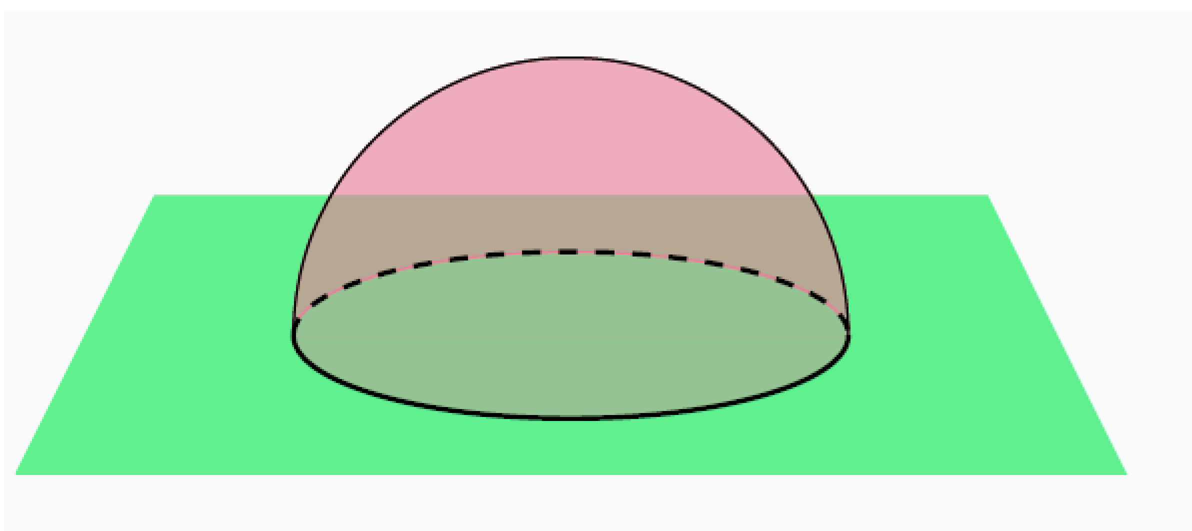
- 4d $\mathcal{N}$=4 Suepr Yang-Mills is dual to IIB string theory on $AdS_5 * S^5$.

- The gauge theory in the large N and large

 't Hooft coupling limit is dual to classical gravity/string theory.

Holographic Wlison loops

- When BPS WL loop is in fundamental representation, the gravity dual is a F-string in AdS_5 and the worldsheet has boundary along the circle. --> minimal surface



Circular WLs

- For half-BPS WLs **circular** WL, the computations of vev can be reduced to Gaussian matrix model.
- First, this was observed in perturbation theory.*[Erickson, Semenoff, Zarembo, 2000]*
- In **Landau** gauge, the lowest diagrams with vertices cancelled with each other. They conj. this to all loop.
- Contributions from ladder diagrams

→Gaussian matrix model

Nontrivial check of AdS/CFT

- The result from matrix model in the large N and large 't Hooft coupling limit

$$\langle W(C) \rangle_{\text{ladders}} \sim \frac{e^{\sqrt{g^2 N}}}{(\pi/2)^{1/2}(g^2 N)^{3/4}}.$$

- Consistent with holographic prediction

$$\langle W(C) \rangle_{\text{ladders}} \sim \frac{e^{\sqrt{g^2 N}}}{(\pi/2)^{1/2}(g^2 N)^{3/4}}.$$

Localization

- Supported by *Drukker and Gross (2000)* using conformal anomaly.

- This was proved by *Pestun (2007)* using supersymmetric localization.
- Generalization to less supersymmetric case:
- *Zarembo* loops *[2002]*
- DGRT loops *[Drukker, Giombi, Ricci, Trancanelli, 07]*

BPS WILSON LOOPS IN ABJM THEORY

WLs in CSM theories

- In 2007, *Gaiotto and Yin* constructed half (1/3)-BPS WLs in 3d $\mathcal{N}$=2 (3) Chern-Simons-matter(CSM) theories.

$$P\exp\left[\int d\tau\left(A_\mu\dot{x}^\mu+\sigma|\dot{\vec{x}}|\right)\right]$$

$$P\exp\left[\int d\tau\left(A_\mu\dot{x}^\mu+s_i\dot{y}^i\right)\right]$$

M2 branes

- During the 2nd string revolution, it was realized that five perturbative string theories are unified into M theory.
- There is the sixth corner: M-theory (in the narrow sense).
- Since 1995, CFTs from multi M2/M5 branes quite mysterious.
- N M2's-> $N^{3/2}$ d. o. f. (N M5's-> N^3).

M2 mini-revolution

- *Bagger-Lambert [06-07], Gustavsson[08],* 3d $\mathcal{N}$=8 theory based on 3-algebra.
- It is CSM theory. *[van Raamsdonk, 08]*
- *Aharony, Bergman, Jafferis, Maldacena*: 3d $\mathcal{N}$=6 CS-matter theories *(Jun. 6, 2008)*
- Low energy effective theory for M2's on C^4/Z_k.
- k=1, 2: non-perturbative supersymmetry enhancement to $\mathcal{N}$=8.
- Dual to M-theory $AdS_4{*}S^7/Z_k$, IIA on $AdS_4{*}CP^3$

ABJM theory

- Gauge group is U(N)*U(N) with Chern-Simons levels (k, -k).

- The matter fields are 4 scalars and 4 fermions in the bi-fundament representation.

WLs in ABJM theory

- Simple BPS Wilson loops

$$W = \mathcal{P} \exp\left(-i \int d\tau \mathcal{A}(\tau)\right),$$

$$\mathcal{A} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} M^I{}_J \phi_I \bar{\phi}^J |\dot{x}|.$$

- *[Drukker, Plefka, Young, 08][Chen, JW, 08]*
- *[Rey, Suyama, Yamaguchi 08]*
- Only 1/6 BPS! Enhancement of the theory from $\mathcal{N}$=3 to $\mathcal{N}$=6 do **not** carried by the WL!

Fermionic WLs

- But the dual string solution is half-BPS.
- The 1/2-BPS WL was constructed by *Drukker and Trancanelli in 2009*.

$$W = \mathcal{P}\exp\left(-i\int d\tau L(\tau)\right), \quad L = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix}.$$

$$\mathcal{A} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} M^I{}_J \phi_I \bar{\phi}^J |\dot{x}|, \quad \hat{\mathcal{A}} = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} N_I{}^J \bar{\phi}^I \phi_J |\dot{x}|,$$

$$\bar{f}_1 = \sqrt{\frac{2\pi}{k}} \bar{\zeta}_I \psi^I |\dot{x}|, \quad f_2 = \sqrt{\frac{2\pi}{k}} \bar{\psi}_I \eta^I |\dot{x}|.$$

Key point

- For WL to be BPS, it is enough to have Grassmann odd matrix

$$G = \begin{pmatrix} & \bar{g}_1 \\ g_2 & \end{pmatrix},$$

- such that

$$\delta L = \partial_\tau G + i[L, G].$$

Towards less supersymmetric theories

- Latter this is explained by Higgs mechanism *[K. Lee, S. Lee, 2010]*
- They also got 2/5 BPS fermionic WLs in $\mathcal{N}$=5 CSM theories.

- I feel that
- *1. Fermionic BPS WLs seems to have more supersymmetries than bosonic WLs.*
- *2. Theory with few supersymmetries seems not to admit Fermionic BPS WLs.*

- ***These speculations were disproved by my own work.***

BPS WILSON LOOPS IN GENERAL SUPER CHERN-SIMONS THEORIES

$\mathcal{N}$=3 case

- I began to think about the less amount of susy where the theory can admit fermionic BPS WLs.
- M-theory on AdS_4*N(1, 1)/Z_k is dual to $\mathcal{N}$=3 flavored ABJM theories. *[Hikka, Wei Li, Takayanagi, 09][Gaiotto, Jafferis, 09][Hohenegger, Kirsch, 09]*
- The M2 branes in AdS_4*N(1, 1)/Z_k dual to Wilson loops can be at most 1/3 BPS *[Chen, JW, Zhu, 14]*. The same as Gaiotto-Yin Loop. Seems to be bosonic.

$\mathcal{N}$=4 case

- We considered $\mathcal{N}$=4 orbifold ABJM theories the gravity dual is M-theory on $AdS_4*S^7/(Z_{nk}*Z_n)$. The M2 brane with worldvolume AdS_2*S^1 is half-BPS.
- The bosonic WLs are all 1/4 BPS.
- We searched for fermionic 1/2-BPS WLs and do find them *[Ouyang, JW, Zhang, 15]*
- *[Cooke, Drukker, Trancanelli, 15]* found more such WLs in general $\mathcal{N}$=4 CSM theories.

orbifold

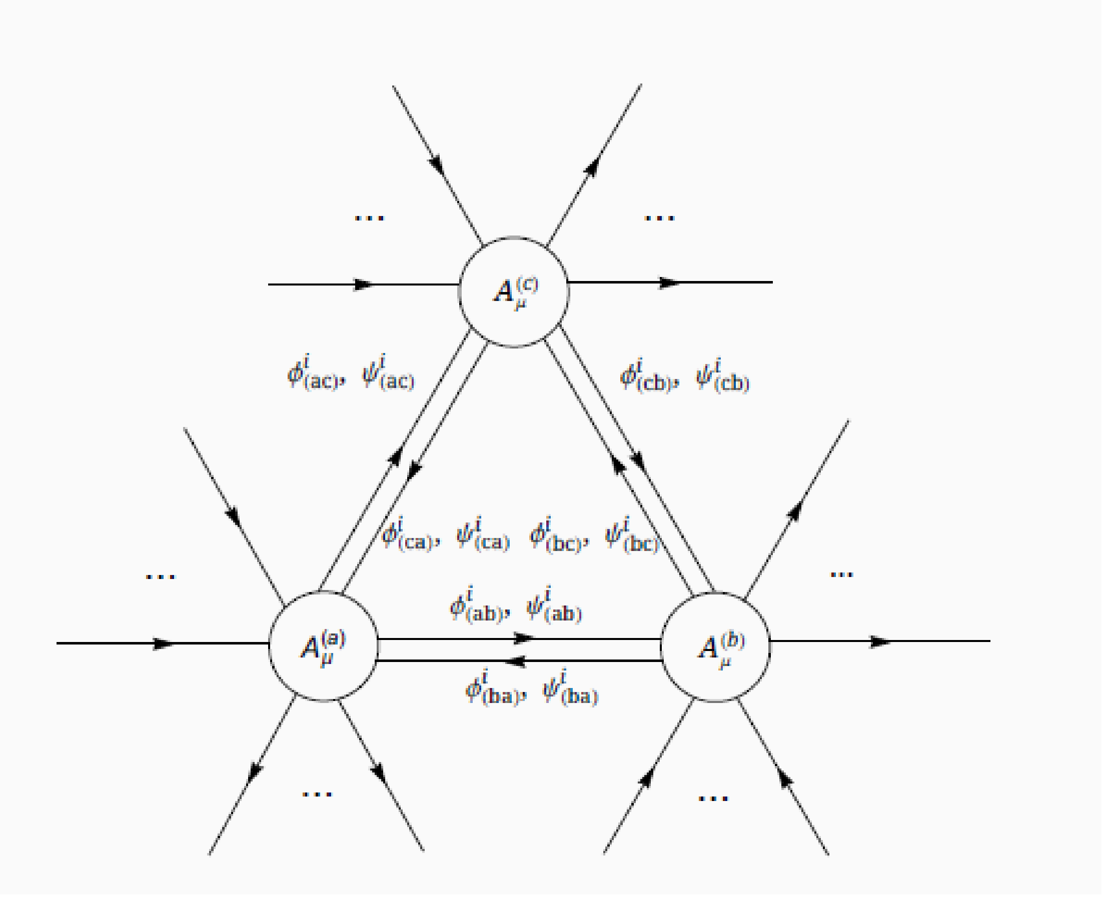
- This seems to the happy end of my own question. Theories with $\mathcal{N}\geq4$ have fermionic BPS WLs. But theories with $\mathcal{N}\leq3$ do not admit such WLs.
- **Wait…**
- There are also $\mathcal{N}=2$ orbifold ABJM theories. Why the construction for $\mathcal{N}=4$ orbifold theories cannot apply?

New results

- This leads to the construction of fermionic BPS WLs for generally $\mathcal{N}$=2 quiver CSM.

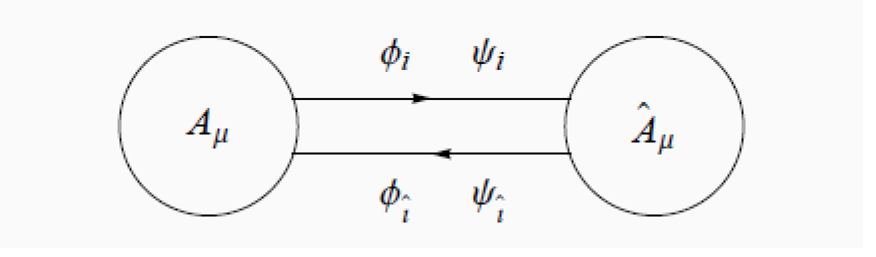
- Also less supersymmetric fermionic WL in ABJM theory with constant coupling to scalars.
- *[Ouyang, JW, Zhang, 15]*

General quiver theories



Two adjacent nodes

- Pick out two adjacent nodes in the quiver diagram.



Constructions

- For the line along *t* direction

$$W_{\mathrm{f}} = \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau\, L_{\mathrm{f}}(\tau)\right), \quad L_{\mathrm{f}} = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix},$$

$$\mathcal{A} = A_\mu \dot{x}^\mu + \sigma|\dot{x}| + \mathcal{B}|\dot{x}|, \quad \hat{\mathcal{A}} = \hat{A}_\mu \dot{x}^\mu + \hat{\sigma}|\dot{x}| + \hat{\mathcal{B}}|\dot{x}|,$$

$$\mathcal{B} = M^i{}_j \phi_i \bar{\phi}^j + M_{\hat{\imath}}{}^{\hat{\jmath}} \bar{\phi}^{\hat{\imath}} \phi_{\hat{\jmath}} + M^{i\hat{\imath}} \phi_i \phi_{\hat{\imath}} + M_{\hat{\imath} i} \bar{\phi}^{\hat{\imath}} \bar{\phi}^i,$$

$$\hat{\mathcal{B}} = N_i{}^j \bar{\phi}^i \phi_j + N^{\hat{\imath}}{}_{\hat{\jmath}} \phi_{\hat{\imath}} \bar{\phi}^{\hat{\jmath}} + N_{i\hat{\imath}} \bar{\phi}^i \bar{\phi}^{\hat{\imath}} + N^{\hat{\imath} i} \phi_{\hat{\imath}} \phi_i,$$

$$\bar{f}_1 = (\bar{\zeta}^i \psi_i + \bar{\psi}^{\hat{\imath}} \mu_{\hat{\imath}})|\dot{x}|, \quad f_2 = (\bar{\psi}^i \eta_i + \bar{\nu}^{\hat{\imath}} \psi_{\hat{\imath}})|\dot{x}|.$$

Constructions

- To preserve the following supersymmetry

$$\gamma_0\theta = \mathrm{i}\theta, \quad \bar{\theta}\gamma_0 = \mathrm{i}\bar{\theta},$$

- We need the existence of

$$G = \begin{pmatrix} & \bar{g}_1 \\ g_2 & \end{pmatrix},$$

such that

$$\delta L_\mathrm{f} = \partial_\tau G + \mathrm{i}[L_\mathrm{f}, G].$$

Constructions

- Use the parametrization

$$\bar{\zeta}^{i} = \bar{\alpha}^{i}\bar{\zeta}, \quad \mu_{\hat{\imath}} = \mu\gamma_{\hat{\imath}}, \quad \eta_{i} = \eta\beta_{i}, \quad \bar{\nu}^{\hat{\imath}} = \bar{\delta}^{\hat{\imath}}\bar{\nu},$$
$$\bar{\zeta}^{\alpha} = \bar{\nu}^{\alpha} = (1, \mathrm{i}), \quad \eta_{\alpha} = \mu_{\alpha} = (1, -\mathrm{i}),$$

- We get the four classes of solutions with
-

$$M^{i\hat{\imath}} = M_{\hat{\imath}i} = \bar{\alpha}^{i}\bar{\delta}^{\hat{\imath}} = \gamma_{\hat{\imath}}\beta_{i} = 0,$$
$$M^{i}{}_{j} = 2\mathrm{i}\bar{\alpha}^{i}\beta_{j}, \quad M_{\hat{\imath}}{}^{\hat{\jmath}} = 2\mathrm{i}\gamma_{\hat{\imath}}\bar{\delta}^{\hat{\jmath}}.$$

constructions

- Nonzero parameters in four classes

I	II	III	IV
$\bar{\alpha}^i, \beta_i$	$\bar{\delta}^{\hat{\imath}}, \gamma_{\hat{\imath}}$	$\bar{\alpha}^i, \gamma_{\hat{\imath}}$	$\bar{\delta}^{\hat{\imath}}, \beta_i$

- We also find fermionic 1/3BPS WLs in $\mathcal{N}$=3 theories.

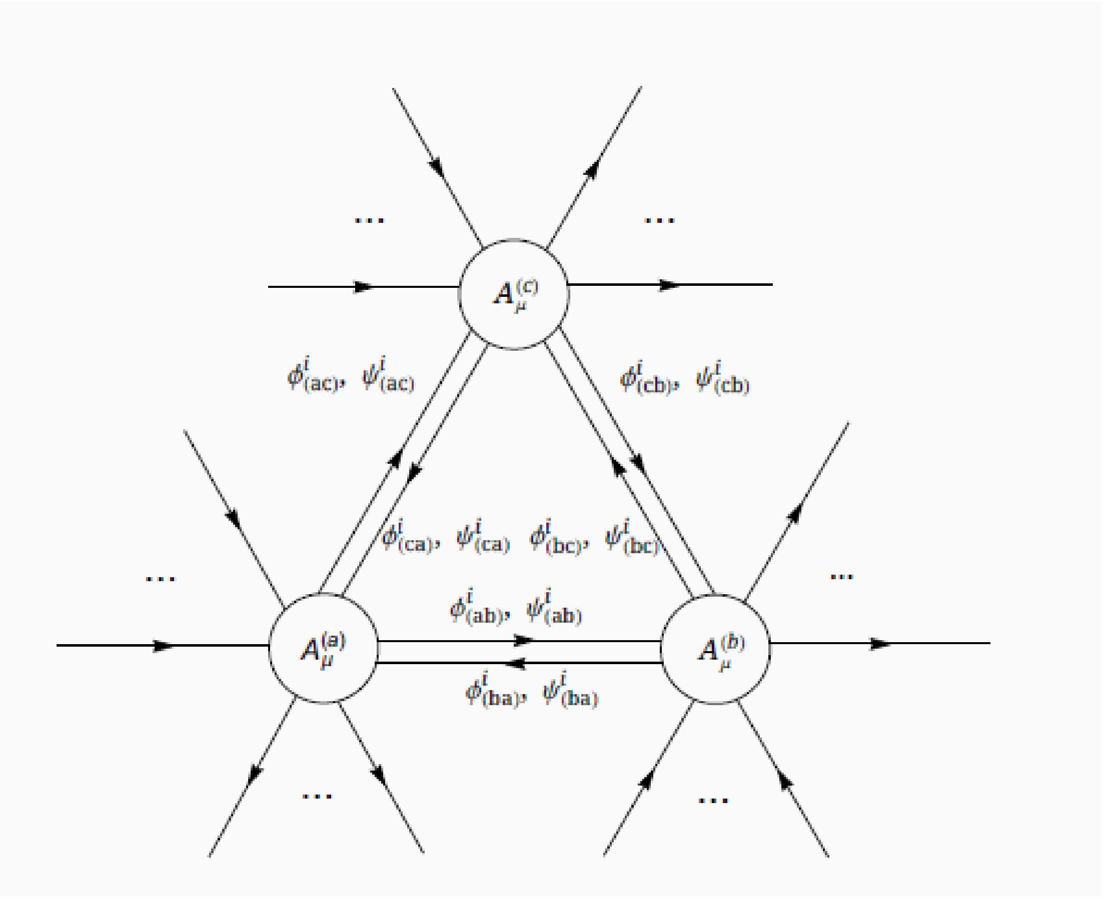
Back to ABJM

- In ABJM theory, we found similar four classes of fermionic 1/6-BPS WLs. Turning off coupling with fermions, this go back to the bosonic 1/6-BPS WLs.
- Special choice of parameters in classes I and II gives the fermionic 1/2-BPS WLs.
- Our fermionic 1/6-BPS Wls interpolates between bosonic 1/6-BPS WLs and fermionic 1/2-BPS WLs.

Newest results

- In above constructions, only two adjacent nodes in the quiver are used.
- Last Sep., the Milano group constructed WLs out of such class in the $\mathcal{N}$=4 orbifold ABJM theories. And some of these WLs have clear gravity dual. *[Mauri, Penati, Jiaju Zhang, 2017]*
- Again, this construction should be generalized to N=2 case. *[Mauri, Ouyang, Penati, JW, Jiaju Zhang, 2018]*

General quiver theories again



Bosonic WLs

- The gauge group is with factor $U(N_a)$.
- The bosonic BPS WLs along timelike line

(τ, 0, 0) is with the connection.

$$A = \mathrm{diag}(A_0^{(1)} + \sigma^{(1)}, A_0^{(2)} + \sigma^{(2)}, \cdots, A_0^{(n)} + \sigma^{(n)}).$$

- The preserved susy are

$$\gamma_0 \theta = \mathrm{i}\theta, \quad \bar{\theta}\gamma_0 = \mathrm{i}\bar{\theta}.$$

Fermionic WLs

- The ansatz for the connection in fermionic BPS WL is

$$L = A + B + F,$$

$$\begin{aligned} B_{(ab)} &= \sum_c (R^{(c)}_{(ab)}{}_i{}^j \phi^i_{(ac)} \bar{\phi}^{(cb)}_j + R^{(c)}_{(ab)}{}_{ij} \phi^i_{(ac)} \phi^j_{(cb)}, \\ &+ S^{(c)}_{(ab)}{}^i{}_j \bar{\phi}^{(ac)}_i \phi^j_{(cb)}) + S^{(c)}_{(ab)}{}^{ij} \bar{\phi}^{(ac)}_i \bar{\phi}^{(cb)}_j), \\ F_{(ab)} &= \bar{\xi}^{(ab)}_i \psi^i_{(ab)} + \bar{\psi}^{(ab)}_i \eta^i_{(ab)}. \end{aligned}$$

- The matrix is written in terms of n*n blocks.

constructions

- With the parameterization

$$\eta^i_{(ab)} = n^i_{(ab)}\eta, \quad \bar{\xi}^{(ab)}_i = \bar{m}^{(ab)}_i \bar{\xi},$$
$$\eta_\alpha = \frac{1}{\sqrt{2}}(1,-i), \quad \bar{\xi}^\alpha = \frac{1}{\sqrt{2}}(-i,1).$$

- To preserve the above supercharges, we need

$$R^{(c)}_{(ab)}{}_i{}^j = \bar{m}^{(ac)}_i n^j_{(cb)}, \quad S^{(c)}_{(ab)}{}^i{}_j = n^i_{(ac)} \bar{m}^{(cb)}_j,$$
$$R^{(c)}_{(ab)}{}_{ij} = S^{(c)}_{(ab)}{}^{ij} = \bar{m}^{(ac)}_i \bar{m}^{(cb)}_j = n^i_{(ac)} n^j_{(cb)} = 0.$$

- For all equations here, no summation over repeated node indices like c.

Feature of novel loops

- Diagonal blocks of the connections are always bosonic.
- For a, b not the same, $B_{(ab)}$, $F_{(ab)}$ can be non zero at the same time. So in general L is **not a superconnection of a supegroup**. This does not appear in ABJM or $\mathcal{N}$=4 orbifold ABJM cases. They appear only when there are triangles in the quivers.
- *Analog of higher dimensional representations of (super)group?*

Take home massages

- There are much more BPS Wilson loops in 3d super CSM theories than we thought before.
- Surprises always appear during research!
- Holography inspires many studies in field theory though these could be done without holography.
- =>String theory gives important hints to field theory. (KLT relation among amplitudes)

Further questions

- Are there Konish-like anomalies which make some of these WLs not truly BPS at the quantum level?
- To directly compare with prediction from supersymmetric localization, perturbation calculation at *framing one* at higher loop orders is needed. But this is quite complicated.

Further questions

- Precise map between WLs and probe membrane/string solutions.
- For $\mathcal{N}$=2, 3 theories, people (including us) thought bosonic BPS WL is dual to simple membrane/string solutions.
- Experiences from orbifold ABJM theories suggest the WL should be fermionic one.
- Has every truly BPS WL a simple gravity dual?

Thanks you very much!